Gate Problem on Control Systems

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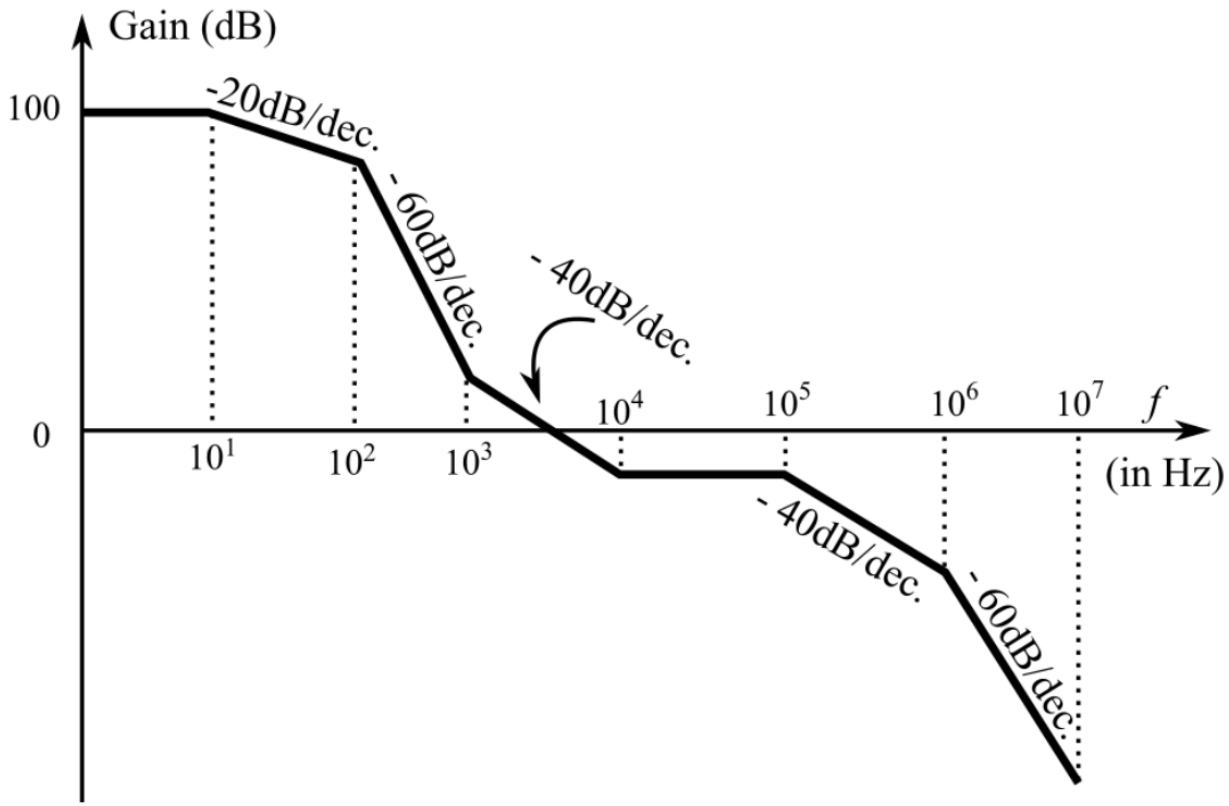
Gate 2019 EC Problem

Q.6 - For an LTI system, the Bode plot for its gain is as illustrated in the figure shown. The number of system poles N_p and number of system zeros N_z in the frequency range 1 Hz $\leq$ f $\leq 10^7$ Hz is



Solution

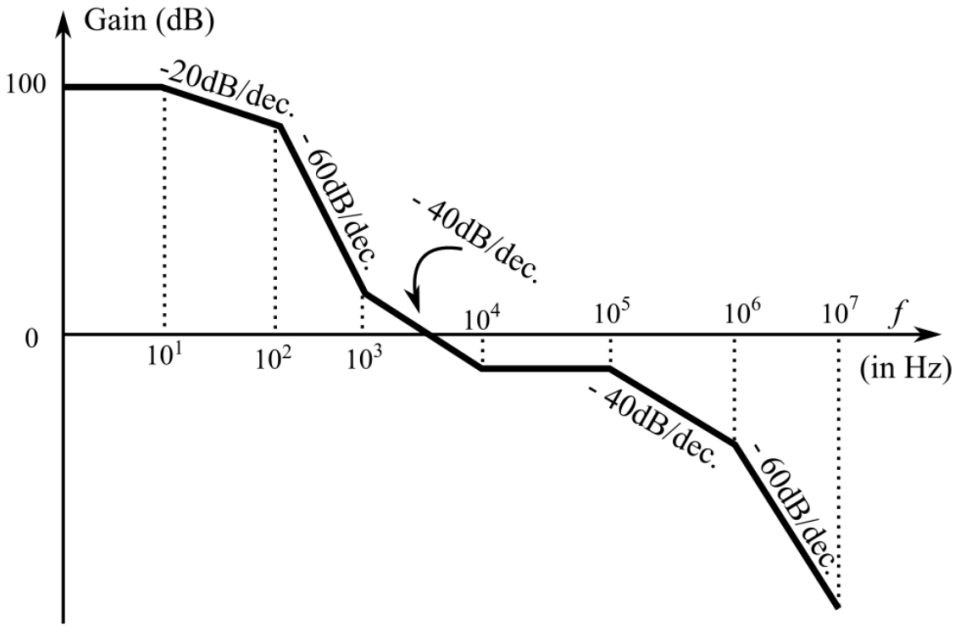
Let us consider a generalized transfer gain

$$H(s) = k\frac{(s-z_1)(s-z_2)...(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)....(s-p_{n-1})(s-p_n)}$$

$$Gain = 20log|H(s)| = 20log|k| + 20log|s-z_1| + 20log|s-z_2| + ... + 20log|s-z_m| - 20log|s-p_1| - 20log|s-p_2| - ..... - 20log|s-p_n|$$

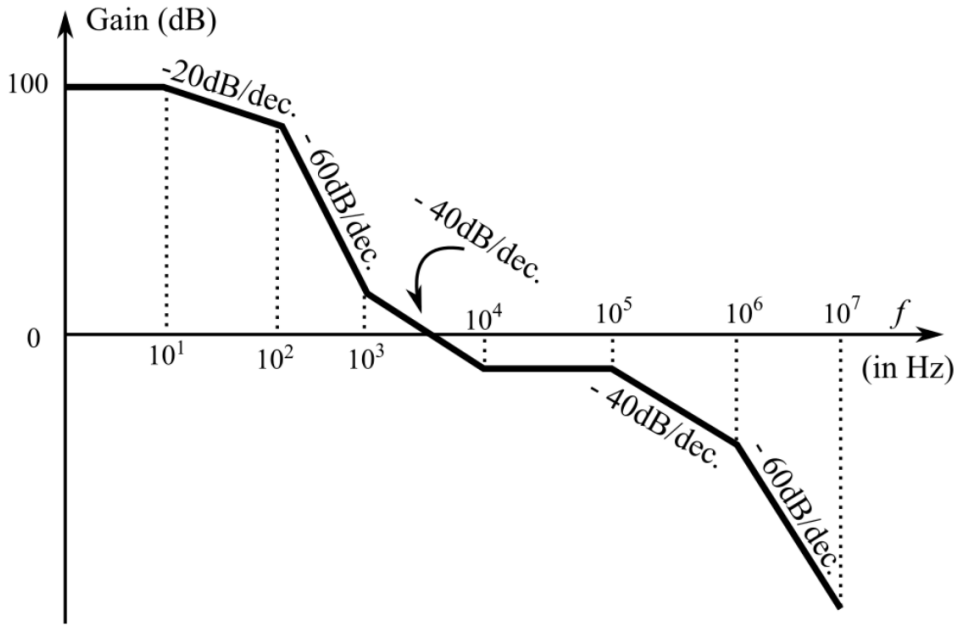
- When a pole is encountered the slope always decreases by -20 dB/decade
- When a zero is encountered the slope always increases by +20 dB/decade

Solution



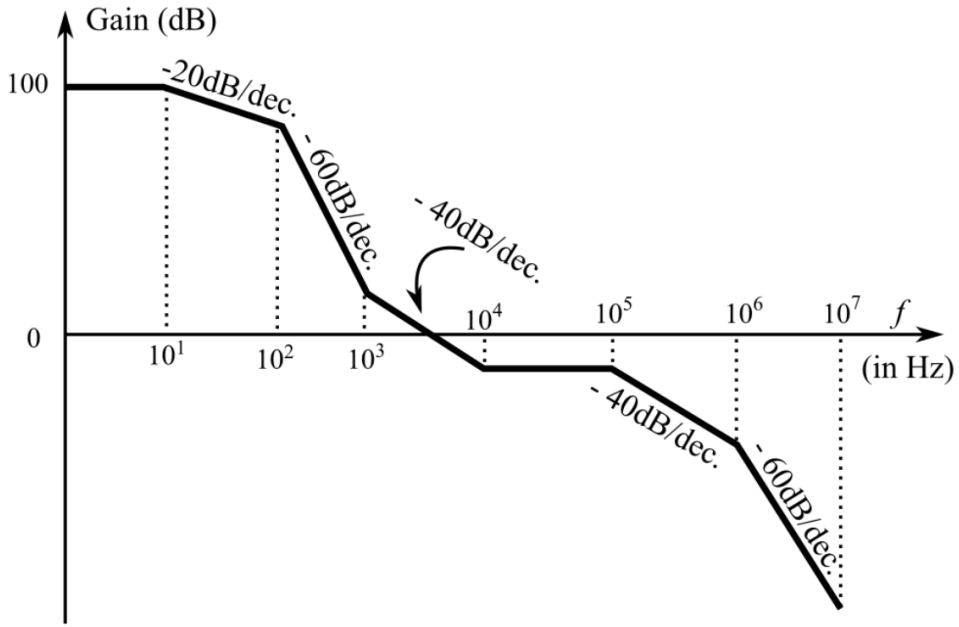
- At f = 10 Hz , change in slope = -20dB/sec, Hence we have 1 pole here
- At f = 10^2 Hz, Change in slope = -40dB/sec, Hence we have 2 poles here

Solution



- At f = 10^3 Hz, Change in slope = +20dB/sec, Hence we have 1 zero here
- At f = 10^4 Hz, Change in slope = +40dB/sec, Hence we have 2 zeros here

Solution



- At f = 10^5 Hz, Change in slope = -40dB/sec, Hence we have 2 poles here
- At f = 10^6 Hz, Change in slope = -20dB/sec, Hence we have 1 pole here

Answer

$$N_p = 6$$
$$N_z = 3$$