Gate 2018 Q.42 (EC)

Control Systems (EE2227)

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EE18BTECH11025

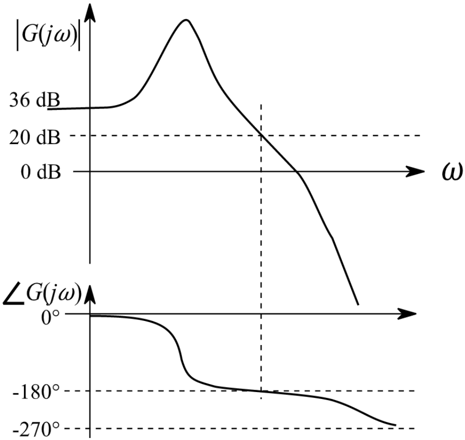
Question:

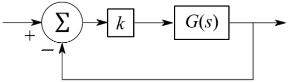
- The figure below shows the Bode magnitude and phase plots of a stable transfer function

$$G(s) = \frac{n_0}{s^3 + d_2 s^2 + d_1 s + d} \tag{1}$$

Consider the negative unity feedback configuration with gain k in the feedforward path. The closed loop is stable for $k < k_o$.
The maximum value of k_o *is*

Question:





Solution:

For a stable system, Gain margin at the phase cross-over frequency $>$ 0dB.

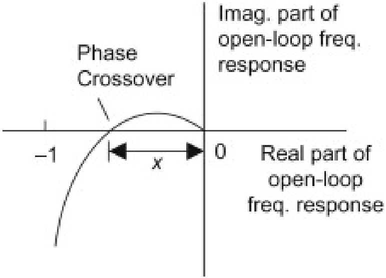
Phase crossover frequency (ω_{pc})

The phase crossover frequency is the frequency at which the phase angle first reaches -180 $^\circ$.

Gain Margin

- This is the factor by which the gain must be multiplied at the phase crossover to have the value 1.
- The gain margin refers to the amount of gain, which can be increased or decreased without making the system unstable.

Cross over frequency:



- The above shows nyquist plot of a stable transfer function.
- The phase crossover frequency is the frequency at which the phase angle first reaches -180°and thus is the point where the Nyquist plot crosses the real axis.

Gain Margin:

The gain margin is defined as

$$K_g = \frac{1}{|G(j\omega)|}$$

at the frequency at which the phase angle is -180°.
In terms of decibels:

$$K_g dB = -20 log(|G(j\omega)|) dB$$

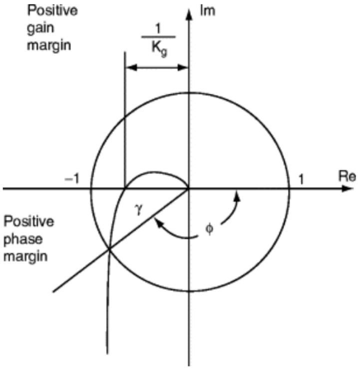


Fig: nyquist plot of stable transfer function

Gain Margin:

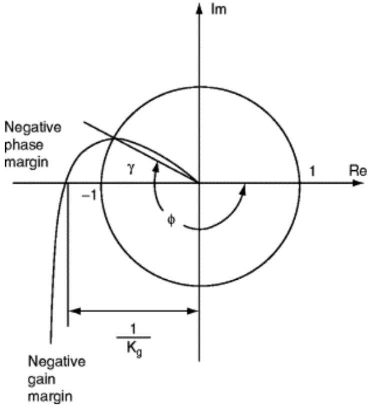


Fig: nyquist plot of unstable transfer function

Solution:

For a stable system, Gain margin at the phase cross-over frequency > 1. G(s) is cascaded with k, so,

$$G_1(s) = kG(s)$$

$$K_g = \frac{1}{|G_1(j\omega_{pc})|} > 1$$

$$\implies K_{g(dB)} = -20log(|G_1(j\omega_{pc})|) > 0dB$$

Solution:

$$\implies -20log(|G(j\omega_{pc})k|) > 0dB$$

$$\implies -20 - 20log(|k|) > 0dB$$

$$\implies 20log(k) < -20$$

$$\implies k < 10^{-1}$$

$$\implies k_{max} = 0.1$$

$\therefore k_o = 0.1$