

EE2227: CONTROL SYSTEM

Presentation 1
GATE: 2017 ECE Q17

KOIDALA SURYA PRAKASH

EE18BTECH11026

. Consider the state space realization :

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 45 \end{bmatrix} u(t)$$

with the initial conditions

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

, where $u(t)$ denotes unit step function

The value of $\lim_{t \rightarrow \infty} |\sqrt{x_1^2(t) + x_2^2(t)}|$ is ?

State Space Representation :

A state-space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations.

For Linear systems :

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)$$

where $\dot{\mathbf{x}}(t) := \frac{d}{dt}\mathbf{x}(t)$

$$L\{\mathbf{x}(t)\} = \mathbf{X}(s)$$

$$L\{\dot{\mathbf{x}}(t)\} = s\mathbf{X}(s) - \mathbf{x}(0)$$

Solution

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 45 \end{bmatrix} u(t)$$

By applying Laplace transform on both sides, we get

$$sX_1(s) - x_1(0) = 0$$

$$X_1(s) = \frac{x_1(0)}{s} = 0 \quad \because x_1(0) = 0$$

$$\text{So, } x_1(t) = 0$$

$$\text{and } sX_2(s) - x_2(0) = -9X_2(s) + \frac{45}{s}$$

$$X_2(s) = \frac{45}{s(s+9)}$$

$$\text{Required value} = \lim_{t \rightarrow \infty} |\sqrt{x_1^2(t) + x_2^2(t)}| = |\lim_{t \rightarrow \infty} x_2(t)|$$

$$\lim_{t \rightarrow \infty} x_2(t) = \lim_{s \rightarrow 0} sX_2(s) = \frac{45}{9} = 5$$

$$\text{So Required value} = |5| = 5$$

Verification

