CONTROL SYSTEMS

HOMEWORK-1

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Question 33

Let the state-space representation of an LTI system be.

$\dot{x}(t) = Ax(t) + Bu(t)$

y(t)=Cx(t)+Du(t)

A,B,C are matrices, D is scalar, u(t) is input to the system and y(t) is output to the system. let

$$b1 = \begin{vmatrix} 0 & 0 & 1 \end{vmatrix}$$

$b1^T = B$

and D=0. Find A and C.

$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1}$

Solution

STATE MODEL

Let U1(t) and U2(t) are the inputs of the MIMO system and y1(t),y2(t) are the output of the system and x1(t) and x2(t) are the state variables.

so output equation is,

$$y1(t) = C_{11} \times x1(t) + C_{12} \times x2(t) + d_{11} \times U1(t) + d_{12} \times U2(t) \quad (1)$$

$$y2(t) = C_{21} \times x1(t) + C_{22} \times x2(t) + d_{21} \times U1(t) + d_{22} \times U2(t) \quad (2)$$

$$\begin{bmatrix} y1(t) \\ y2(t) \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{11} & C_{12} \end{bmatrix} \times \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} \\ d_{11} & d_{12} \end{bmatrix} \times \begin{bmatrix} U1(t) \\ U2(t) \end{bmatrix}$$

therefore Y(t)=C.X(t)+D.U(t)

$$x\dot{1}(t) = a_{11} \times x1(t) + a_{12} \times x2(t) + b_{11} \times U1(t) + b_{12} \times U2(tx \quad (3)$$

$$x\dot{2}(t) = a_{21} \times x1(t) + a_{22} \times x2(t) + b_{21} \times U1(t) + b_{22} \times U2(t) \quad (4)$$

$$\begin{bmatrix} \dot{x1}(t) \\ \dot{x2}(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{11} & a_{12} \end{bmatrix} \times \begin{bmatrix} x1(t) \\ x2(t) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{11} & b_{12} \end{bmatrix} \times \begin{bmatrix} U1(t) \\ U2(t) \end{bmatrix}$$

therefore $\dot{X}(t)$=A.X(t)+B.U(t)

FINDING TRANSFER FUNCTION

So, $\dot{X}(t)$=A.X(t)+B.U(t) *be equation* 1
and Y(t)=C.X(t)+D.U(t) *be equation* 2
by applying laplace transforms on both sides of equation 1
we get
S.X(S)-X(0)=A.X(S)+B.U(S)
S.X(S)-A.X(S)=B.U(S)+X(0)
(SI-A)X(S)=X(0)+B.U(S)
X(S)=X(0)([SI-A])$^{-}1 + B.([SI - A])^{-}1.U(S)$
Laplace transform of equation 2 *and sub* $X(s)$

Y(S)=C.X(S)+D.U(S)
Y(S)=C.[X(0)([SI-A])$^{-}1 + B.([SI - A])^{-}1.U(S)] + D.U(S)$
If X(0)=0
then Y(S)=C.[B.([SI-A])$^{-}1.U(S)] + D.U(S)$
$\frac{Y(S)}{U(S)} = C.[B.([SI - A])^{-}1] + D = H(S)$

As we know that

$$H(s) = \frac{Y(S)}{U(s)} = \frac{1}{s^3 + 3s^2 + 2s + 1}$$

let X(S)= $\frac{U(S)}{denominator}$

Y(S)=X(S)$\times numerotor$

s$^3X(s) + 3s^2X(s) + 2sX(s) + X(s) = U(S)$
$\dddot{X}(t)$ $+3\ddot{X}(t) + 2\dot{X}(t) + X(t) = U(t)$

X(t)=x_1 and $\dot{x_n} = x_{n+1}$
X=x_1
$\dot{X} = \dot{x_1} = x_2$
$\ddot{X} = \ddot{x_1} = \dot{x_2} = x_3$
$\dddot{X} = \dddot{x_1} = \ddot{x_2} = \dot{x_3}$

$$\dot{x_3} = -x_1 - 2x_2 - 3x_3 + U$$

$\dot{x_1} = x_2$, $\dot{x_2} = x_3$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times U$$

therfore A= $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$

Since Y(S)=X(S)$\times$*numerator*
therefore Y(S)=X(S); Y(t)=X(t)

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$