EE2227 Control Systems

EE18BTECH11050

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Question 20/GATE EC-2015

Question

A unity negative feedback system has the open loop transfer function

$$G(s) = \frac{K}{s(s+1)(s+3)}$$

The value of the gain K (>0) at which the root locus crosses the imaginary axis is ?

Root Locus

- The Root locus is the locus of the roots of the characteristic equation, which are the poles of closed loop transfer function, by varying system gain K from 0 to ∞.
- The characteristic equation of the closed loop control system is:
$$1 + G(s)H(s) = 0$$
- The points on the root locus branches must satisfy the **angle condition.**
- We can find the value of K for the points on the root locus branches by using **magnitude condition.**

Conditions for Root Locus

- Angle Condition: Given the Characteristic equation:

$$1 + G(s)H(s) = 0$$

$\implies$

$$G(s)H(s) = -1 + j0$$

The phase angle of G(s)H(s) is: $\angle$G(s)H(s) =

$$\arctan\left(\frac{0}{-1}\right) = (2n+1)\pi$$

- The angle condition is the point at which the angle of the transfer function is an odd multiple of 180.

Conditions for Root Locus

- Magnitude Condition: Magnitude of G(s)H(s) is:

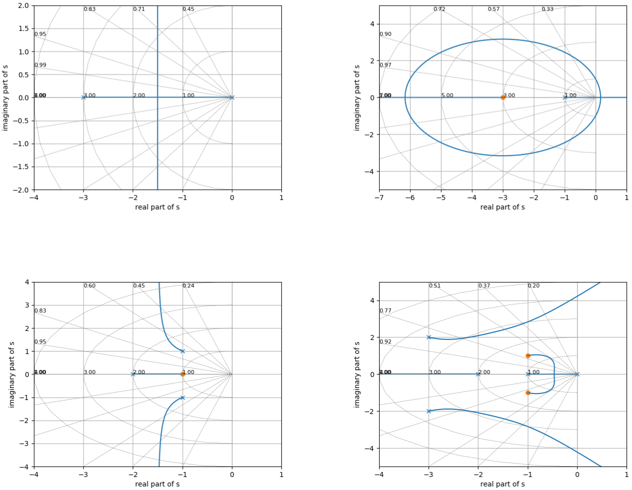
$$|G(s)H(s)| = \sqrt{(-1)^2 + 0^2}$$

$\Longrightarrow$

$$|G(s)H(s)| = 1$$

- The magnitude condition is that the point (which satisfied the angle condition) at which the magnitude of the transfer function is one.

Root Locus Examples



Solution

Given open loop transfer function

$$G(s) = \frac{K}{s(s+1)(s+3)}$$

we have P = 3 poles, at s = 0,-1,-3 and Z = 0 zeroes.

For unity negative feedback, closed loop transfer function is:

$$T(s) = \frac{G(s)H(s)}{1 + G(s)H(s)}$$

Here H(s) = 1

$\Longrightarrow$

$$T(s) = \frac{K}{s(s+1)(s+3) + K}$$

Solution

Poles of closed loop transfer function are the roots of the Characteristic Equation.
Given characteristic Equation is:

$$1 + G(s)H(s) = 0$$

$\Longrightarrow$

$$s^3 + 4s^2 + 3s + K = 0$$

If all elements of any row of the Routh array table are zero, then the root locus branch intersects the imaginary axis

Solution

Routh Array Table:

Order	Coefficients	
s^3	1	3
s^2	4	K
s^1	(12-K)/4	0
s^0	K	

For poles to be on imaginary axis, row s^1 should be zero.
So,

$$\frac{12 - K}{4} = 0$$

Hence,

$$\mathbf{K = 12}$$

Verification

Auxilliary equation:

$$4s^2 + K = 0$$

$\implies$

$$4s^2 + 12 = 0$$

$\implies$

$$s = -j\sqrt{3}, +j\sqrt{3}$$

Thus a pair of poles lie on imaginary axis for K = 12.

Plot

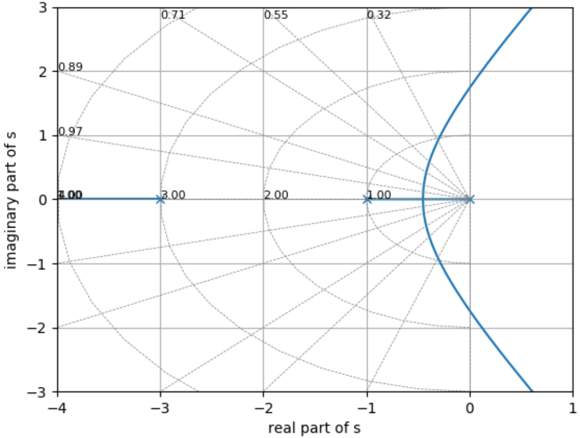


Figure 1: Root Locus Plot