

**EE2227: CONTROL SYSTEM**

**Presentation 1**  
**GATE: 2017 ECE Q17**

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## Question

. Consider the state space realization :

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 45 \end{bmatrix} u(t)$$

with the initial conditions

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

, where  $u(t)$  denotes unit step function

**The value of  $\lim_{t \rightarrow \infty} |\sqrt{x_1^2(t) + x_2^2(t)}|$  is ?**

### State Space Representation :

A state-space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations.

### For Linear systems :

$$\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t) + B(t)\mathbf{u}(t)$$

where  $\dot{\mathbf{x}}(t) := \frac{d}{dt}\mathbf{x}(t)$

$$L\{x(t)\} = X(s)$$

$$L\{\dot{x}(t)\} = sX(s) - x(0)$$

## Solution

$$\begin{bmatrix} \dot{x}_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 45 \end{bmatrix} u(t)$$

By applying Laplace transform on both sides, we get

$$sX_1(s) - x_1(0) = 0$$

$$X_1(s) = \frac{x_1(0)}{s} = 0 \quad \therefore x_1(0) = 0$$

$$\text{So, } x_1(t) = 0$$

$$\text{and } sX_2(s) - x_2(0) = -9X_2(s) + \frac{45}{s}$$

## Solution

$$X_2(s) = \frac{45}{s(s+9)}$$

$$\text{Required value} = \lim_{t \rightarrow \infty} |\sqrt{x_1^2(t) + x_2^2(t)}| = |\lim_{t \rightarrow \infty} x_2(t)|$$

$$\lim_{t \rightarrow \infty} x_2(t) = \lim_{s \rightarrow 0} s X_2(s) = \frac{45}{9} = 5$$

So Required value = |5| = 5

## Verification

