Control Systems

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Question

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A second-order LTI system is described by the following state equations:

- $\frac{\partial x_1(t)}{\partial t} - x_2(t) = 0$
- $\frac{\partial x_2(t)}{\partial t} + 2x_1(t) + 3x_2(t) = r(t)$

where $x_1(t)$ and $x_2(t)$ are the two state variables and $r(t)$ denotes the input. The output $c(t) = x_1(t)$. Identify the type of system.

Solution 1: State Space Analysis

- The corresponding state equations:
    1. $\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$
    2. $[c] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$
- The state space model of a LTI system is:
    1. State equation: $\dot{X} = AX + BU$
    2. Output equation: $Y = CX + DU$

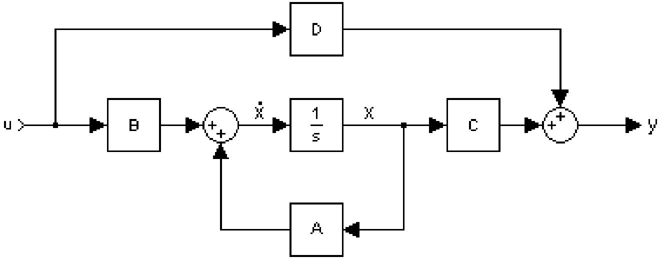

Figure 1: SSM Block Diagram

State Space Analysis

- Transfer Function from State Space model:
  $TF : H(s) = C[sI - A]^{-1}B + D = C\frac{Adj[sI-A]}{|sI-A|}B + D$
- $H(s) = \frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s(s+3)+2} = \frac{1}{s^2+3s+2}$
- Therefore the poles of the transfer function are: s = -1 and s = -2

Solution 2

- From first equation:

$$\frac{\partial x_1(t)}{\partial t} = x_2(t)$$

- Substitution in second equation results into the equation:
  $\frac{\partial^2 x_1}{\partial t^2} + 3\frac{\partial x_1(t)}{\partial t} + 2x_1(t) = r(t)$
- Taking Laplace transform on both sides:
  $s^2X_1(s) + 3sX_1(s) + 2X_1(s) - sx_1(0) - x_1'(0) - 3x_1(0) = R(s)$
- $H(s) = \frac{X_1(s)}{R(s)} = \frac{1}{s^2+3s+2}$
- Therefore the poles of the transfer function are: s = -1 and s = -2

Result and Conclusion

- Since the poles of the transfer function are real and distinct, the system is **OVERDAMPED**.
- Solution(Natural Response): $h(t) = L^{-1}(H(s)) = e^{-t} - e^{-2t}$