Control systems presentation-1

V.L.Narasimha Reddy - EE18BTECH11046

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Question

The root locus of the feedback control system having the characteristic equation $s^2 + 6Ks + 2s + 5 = 0$ where K>0, enters into the real axis at

(A) s = -1

(B) s = -$\sqrt{5}$

(C) s = -5

(D) s = $\sqrt{5}$

Solution

- Root Locus is a method of plotting the way the poles of transfer function moves by varying parameter K of the function from 0 to ∞, where K is the gain of system
- Let there be a Open-loop transfer function $KG(s)$ which can be rewritten as KN(s)/D(s), With unit gain negative feedback
- Then the closed-loop transfer function for that particular system is:

$$\frac{N'(s)}{D'(s)} = \frac{KG(s)}{1 + KG(s)H(s)}$$

- Then the characteristic equation which gives poles is: $1 + KG(s)H(s) = 0$
- The locus is symmetric about real axis

Solution(..continued)

- There are n branches of the locus, one for each Open loop transfer function pole
- A Root-Locus line starts at every pole
- We start from right hand side of graph and move towards left
- The locus starts (when $K = 0$) at poles of the open-loop transfer function, and ends (when $K = \infty$) at the zeros

Solution(..continued)

Given, the characteristic equation

$$s^2 + 6Ks + 2s + 5 = 0$$

Dividing with $s^2 + 2s + 5$ on both sides, we get

$$1 + \frac{6ks}{s^2 + 2s + 5} = 0$$

This is of form $1 + KG(s) = 0$ which is closed loop characteristic equation, so that along the root locus segments on the real axis $(s = \sigma)$

$$K = -\frac{1}{G(\sigma)} = -\frac{D(\sigma)}{N(\sigma)}$$

Solution(..continued)

- While varying K, the point where the Root Locus enters real axis is called a 'Breakaway Point'
- A breakaway point is the point on a real axis segment of the root locus between two real poles where the two real closed-loop poles meet and diverge to become complex conjugates
- Since a Breakaway point corresponds to the point where root locus meets real axis. As the root locus is symmetric about real axis there will be two roots at breakaway point.
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$$\frac{dK}{ds} = -\frac{d}{dK}\left(\frac{D(s)}{N(s)}\right) = 0$$

Solution(..continued)

- Once a pole breaks away from the real axis, they can either travel out towards infinity (to meet an implicit zero), or they can travel to meet an explicit zero, or they can re-join the real-axis to meet a zero that is located on the real-axis.

The breakaway points occur at

$$\frac{dK}{ds} = -\frac{d}{dK}\left(\frac{D(s)}{N(s)}\right) = 0$$

$$K = \frac{-\left(s^2 + 2s + 5\right)}{6s} = -\frac{1}{6}\left[s + 2 + \frac{5}{s}\right]$$

Final Result

And,

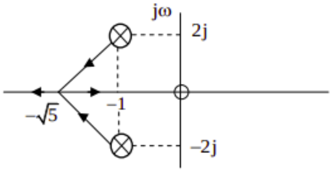
$$\frac{dk}{ds} = 0 \Rightarrow \left[1 - \frac{5}{s^2}\right] = 0$$

Solving for s, we get

$$s^2 - 5 = 0 \Rightarrow s = \pm\sqrt{5}$$

Since, 's' can only be in left half of plane

$$\boxed{s = -\sqrt{5}}$$



Root Locus Plot