Gate 2015 ECE

Q. 48

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Question

A plant transfer function is given as G(s) $= (K_P + \frac{K_I}{s})(\frac{1}{s(s+2)})$. When the plant operates in a unity feedback configuration, the condition for the stability of the closed loop system is

1. $K_P > \frac{K_I}{2} > 0$
2. $2K_I > K_P > 0$
3. $2K_I < K_P$
4. $2K_I > K_P$

Solution

The closed loop transfer function for unity feedback is

$$\frac{G(s)}{1+G(s)} = \frac{(K_p s+K_i)}{s^2(s+2)+(K_s s+K_i)}$$

$$= \frac{(K_p s+K_i)}{s^3+2s^2+K_p s+K_i}$$

Using Routh's tabular form:

$$
\begin{array}{c|cc}
S^3 & 1 & K_P \\
S^2 & 2 & K_I \\
S^1 & \frac{2K_p - K_i}{2} & 0 \\
S^0 & K_p &
\end{array}
$$

contd...

Sufficient condition for Routh-Hurwitz Stability Criterion:The sufficient condition is that all the elements of the first column of the Routh array should have the same sign.
For system to be Stable: $\quad \mathrm{K}_P > 0$

and $\frac{2K_P - K_I}{2} > 0$

or

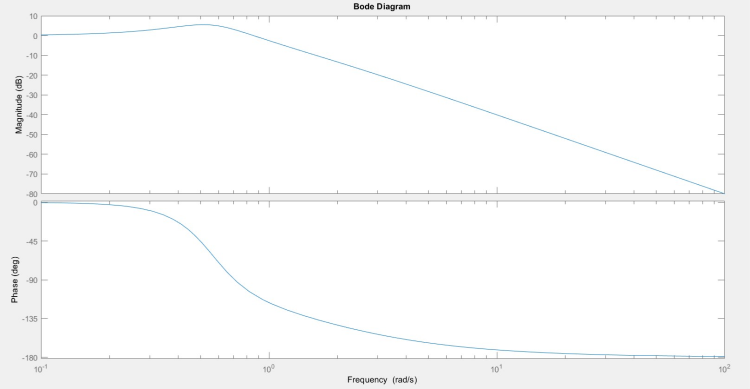
$2\mathrm{K}_P - K_I > 0$

or

$\mathrm{K}_P > \frac{K_I}{2} > 0$

So, 1st option is correct.

For a system to be stable Gain Margin should be greater than 1 and Phase Margin Should be Positive.
for $K_P = 1$ and $K_I = 0.5$
Gain Margin = infinity
Phase Margin = 73.07



for $K_P = -1$ and $K_I = -0.5$
Gain Margin = infinity
Phase Margin = -180
PM is negative indicates that system is unstable.