# Control System Problem

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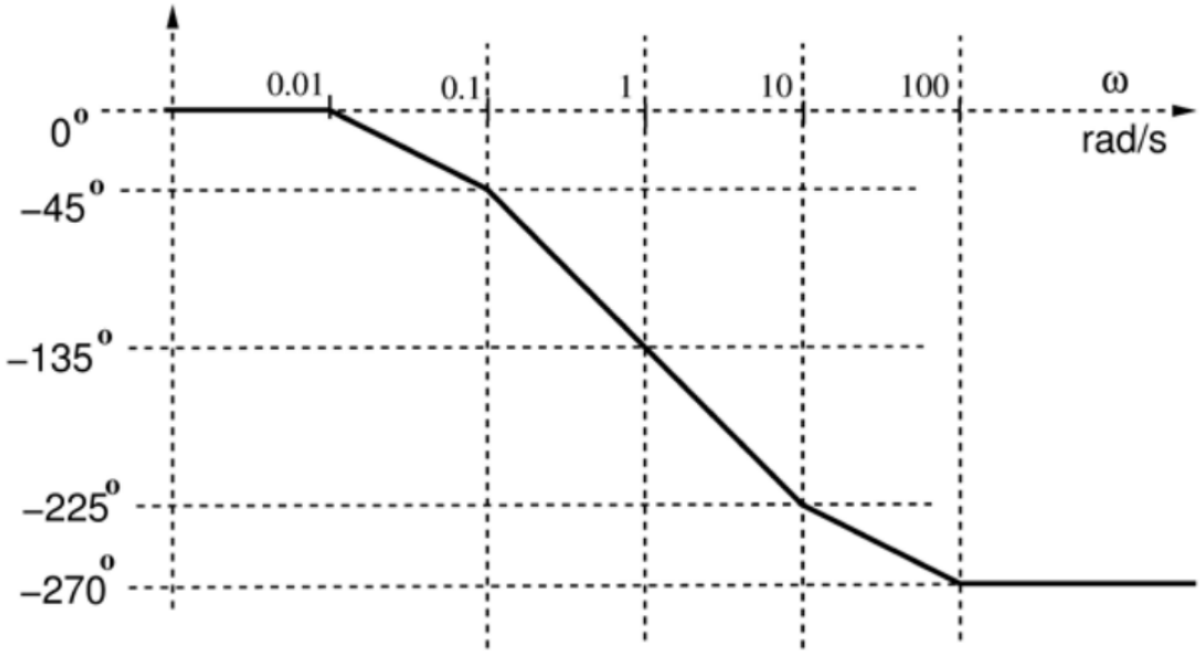
# Question

The asymptotic Bode phase plot of $\mathbf{G(s) = \frac{k}{(s+0.1)(s+10)(s+p1)}}$, with $k$ and $p_1$ both positive, is shown below.




Find the value of $p_1$.

# Theory Needed To Solve This Problem

Phase plot of a transfer function is calculated by substituting s with j$\omega$.
Since, the phase of a+ib is arctan($\frac{b}{a}$)
for a transfer function having $z_1, z_1$ has zeroes and $p_1, p_2, p_3$ has poles,

$$phase = \arctan(\frac{\omega}{z_1}) + \arctan(\frac{\omega}{z_2}) - \arctan(\frac{\omega}{p_1}) - \arctan(\frac{\omega}{p_2}) - \arctan(\frac{\omega}{p_3})$$

# Solution

Phase of this transfer function

$$\phi(\omega) = -\arctan(\frac{\omega}{0.1}) - \arctan(\frac{\omega}{10}) - \arctan(\frac{\omega}{p_1})$$

From the plot, at $\omega = 0.1$ $\phi$ *is* $-45^\circ$

$$-45^\circ = -\arctan(\frac{0.1}{0.1}) - \arctan(\frac{0.1}{10}) - \arctan(\frac{0.1}{p_1})$$

$$-45^\circ = -\arctan(1) - \arctan(\frac{1}{100}) - \arctan(\frac{1}{10p_1})$$

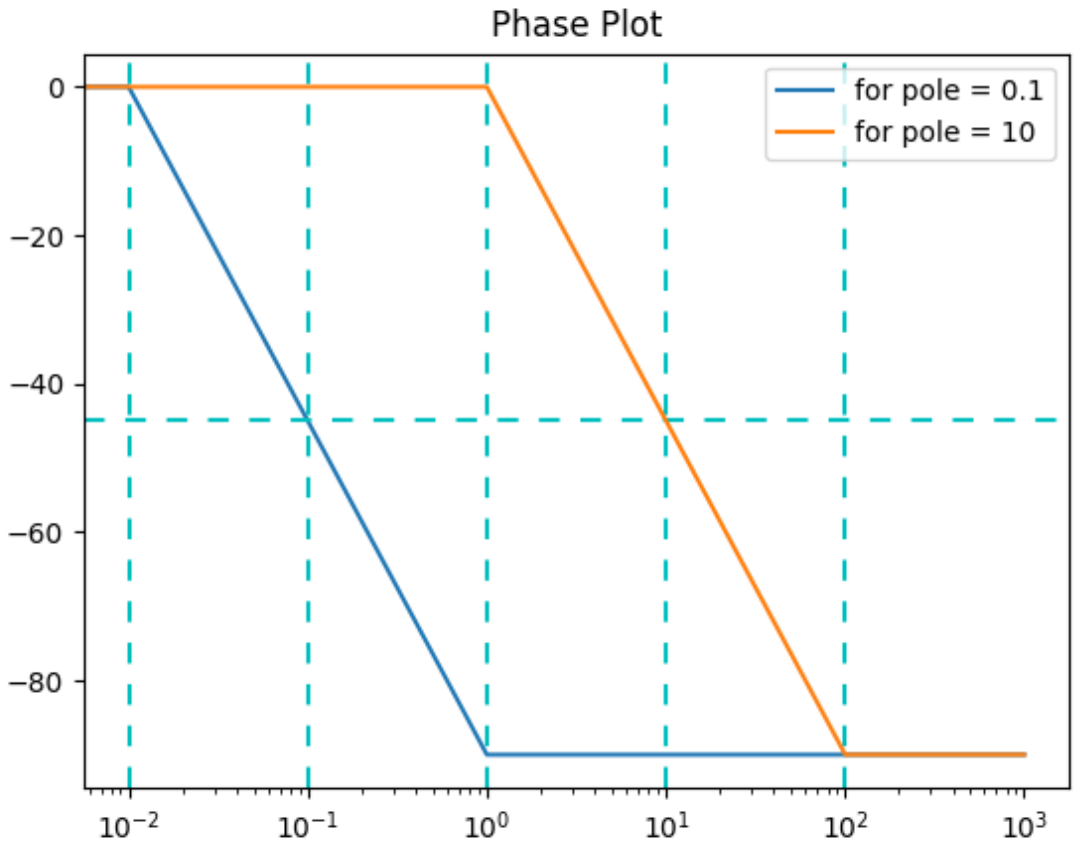
$$-45^\circ \approx -\arctan(\frac{0.1}{0.1}) - \arctan(\frac{0.1}{10}) - \arctan(\frac{0.1}{p_1})$$

# Solution

On Solving We get the $p_1$ is approximately 1, i.e, for $p_1$ in 0.95 to 1.05 the $\phi$ is approximately equals to $-45^\circ$.
Another way by intution, We know that in asymptotic Bode plot for a single pole has $-45^\circ$ at the pole and changes from 0 to -90 in 10 decades i.e, from $p/10$ to 10p

# Solution

So by adding the bodeplots corresponding to the 0.1,10 and $p_1$ poles we get the required bodeplot. By observing the bodeplots corresponding to 0.1 and 10,



# Solution

The values before the 0.1 does not change when compared to the given plot, so $p_1/10$ is greater than or equal to 0.1. In the plot obtained by adding these two plots the slope at 0.1 doesnt change, but in the given plot there is a change so $p/10 = 0.1 \implies p_1 = 1$

**THANK YOU!**