

# Control Systems

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## CONTENTS

<b>1</b>	<b>Signal Flow Graph</b>	<b>1</b>
<b>2</b>	<b>Gain of Feedback Circuits</b>	<b>1</b>
2.1	Voltage Amplifiers . . . . .	1
<b>3</b>	<b>Bode Plot</b>	<b>5</b>
<b>4</b>	<b>Second order System</b>	<b>5</b>
<b>5</b>	<b>Routh Hurwitz Criterion</b>	<b>5</b>
<b>6</b>	<b>State-Space Model</b>	<b>5</b>
<b>7</b>	<b>Nyquist Plot</b>	<b>5</b>
<b>8</b>	<b>Compensators</b>	<b>5</b>
<b>9</b>	<b>Gain Margin</b>	<b>5</b>
<b>10</b>	<b>Phase Margin</b>	<b>5</b>
<b>11</b>	<b>Oscillator</b>	<b>5</b>
<b>12</b>	<b>Root Locus</b>	<b>5</b>
<b>13</b>	<b>Polar Plot</b>	<b>5</b>
<b>14</b>	<b>PID Controller</b>	<b>5</b>

*Abstract*—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

svn co <https://github.com/gadepall/school/trunk/control/codes>

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## 1 SIGNAL FLOW GRAPH

### 2 GAIN OF FEEDBACK CIRCUITS

#### 2.1 Voltage Amplifiers

2.1.1. We are given with a feedback voltage amplifier shown in 2.1.1. We can neglect  $r_o$  and given with  $R_1 + R_2 \gg R_D$ .

(a) Find expressions for  $G$  and  $H$  hence amount of feedback.

(b) Noting that the feedback can be eliminated by removing  $R_1$  and  $R_2$  and connecting the gate of  $Q$  to a constant dc voltage (signal ground) give the input resistance  $R_i$  and output resistance  $R_o$  of the open-amplifier.

(c) Using standard circuit analysis (i.e. =, without invoking the feedback approach), find the input resistance  $R_{if}$  and the output resistance  $R_{of}$ . How does  $R_{if}$  relate to  $R_i$  and  $R_{of}$  relates to  $R_o$  ?

2.1.2. part(a) : We have to find the expressions for  $G$  (open loop gain) ,  $H$  (the feedback factor) and hence the amount of feedback.

**Solution:** For this , first we have to draw the Small-Signal Model for the above Circuit, we ground all constant voltage sources and open all constant current sources. All Small-Signal parameters are obtained from DC-Analysis of the circuit. In Small-Signal Analysis a N-MOSFET is modelled as a Current Source with value of current equal to  $g_m v_{gs}$  flowing from Drain to Source. Whereas a P-MOSFET is modelled as a Current Source with value of current equal to  $g_m v_{sg}$  flowing from Source to Drain.

2.1.3. For finding open loop gain ( $G$ ) and the feedback factor ( $H$ ).

**Solution:** For finding the open loop gain we have to remove  $R_2$  and  $R_1$  and the gate should be grounded.

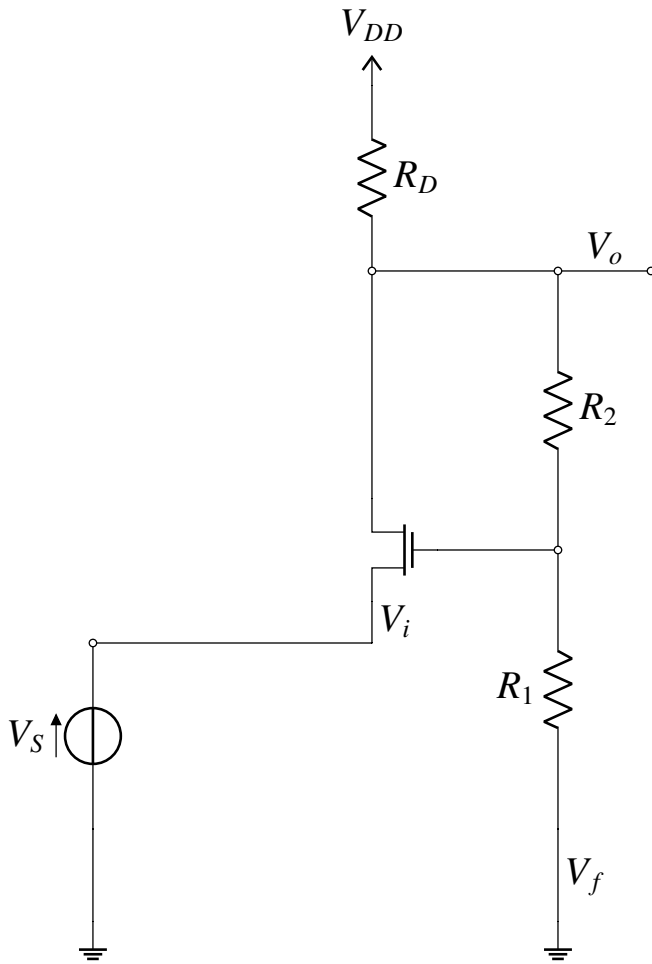


Fig. 2.1.1

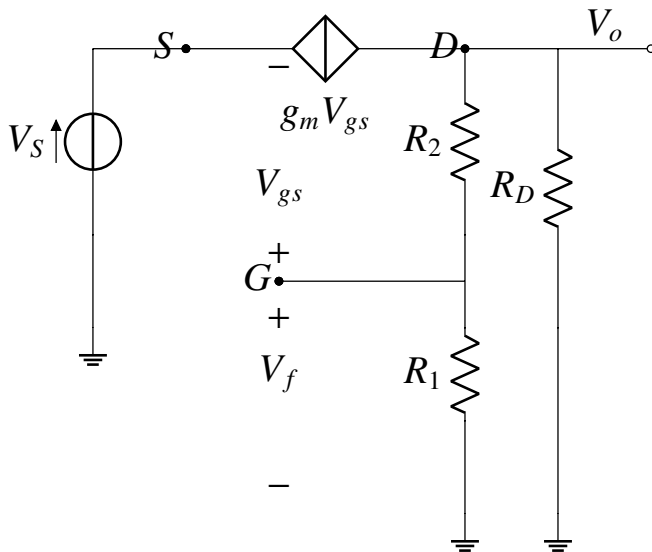


Fig. 2.1.2: Small Signal Model

**Solution:**

$$V_o = -g_m V_{gs} * R_D \quad (2.1.4.1)$$

$$V_{gs} = -V_S \quad (2.1.4.2)$$

$$G = \frac{V_o}{V_S} \quad (2.1.4.3)$$

$$G = g_m R_D \quad (2.1.4.4)$$

2.1.5. Finding the Expression for the feedback factor  $H$ .

**Solution:**

$$H = \frac{V_f}{V_o} \quad (2.1.5.1)$$

$$V_f = \frac{R_1}{R_1 + R_2} V_o \quad (2.1.5.2)$$

$$H = \frac{R_1}{R_1 + R_2} \quad (2.1.5.3)$$

Amount of feedback is defined as :  $1 + GH$

$$1 + GH = 1 + \frac{g_m R_D R_1}{R_1 + R_2} \quad (2.1.5.4)$$

The figure for part(b) :

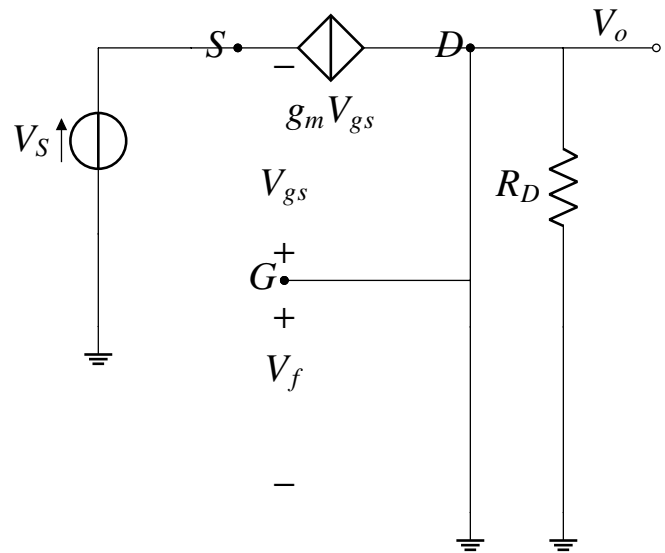


Fig. 2.1.5: CG amplifier

2.1.6. Part(b) : We have to eliminate the feedback by removing  $R_1$  and  $R_2$  and connecting the gate of Q to a constant DC voltage (signal ground). We have to find the expression of the input resistance  $R_i$  and the output resistance  $R_o$  of the open loop amplifier.

**Solution:**

2.1.4. Finding the open loop gain( $G$ )

When the  $R_1$  and  $R_2$  and gate of Q is connected to a constant DC voltage (signal ground) it becomes a CG(Common gate amplifier) without feedback. We can directly see from the 2.1.5 the expression of input resistance  $R_i$  and output resistance  $R_o$ .

For finding input resistance , output constant voltages are grounded and hence the only current flowing is  $g_m V_{gs}$ . Hence  $R_i$  is :

$$I_{in} = -g_m V_{gs} \quad (2.1.6.1)$$

$$V_{in} = V_S \quad (2.1.6.2)$$

$$V_S = -V_{gs} \quad (2.1.6.3)$$

$$R_i = \frac{V_{in}}{I_{in}} \quad (2.1.6.4)$$

$$R_i = \frac{1}{g_m} \quad (2.1.6.5)$$

Similarly , for finding output  $R_o$  ,  $V_{in}$  that is  $V_S$  will be zero and hence  $g_m V_{gs}$  will be zero. Hence only  $R_D$  will be left which is the output resistance.

$$R_o = R_D \quad (2.1.6.6)$$

2.1.7. Part(c) : Using standard circuit analysis that is without using feedback approach we have to find the input resistance  $R_{if}$  and output resistance  $R_{of}$  and how they relate to  $R_i$  and  $R_o$  , which we find earlier.

**Solution:**

We will find them one by one.

2.1.8. finding expression for  $R_{if}$

**Solution:**

To obtain  $R_{if}$  consider the figure 2.1.7 :

We gave test input voltage  $V_x$  and current  $I_x$  to find the input resistance from the input side to find  $R_i$ .

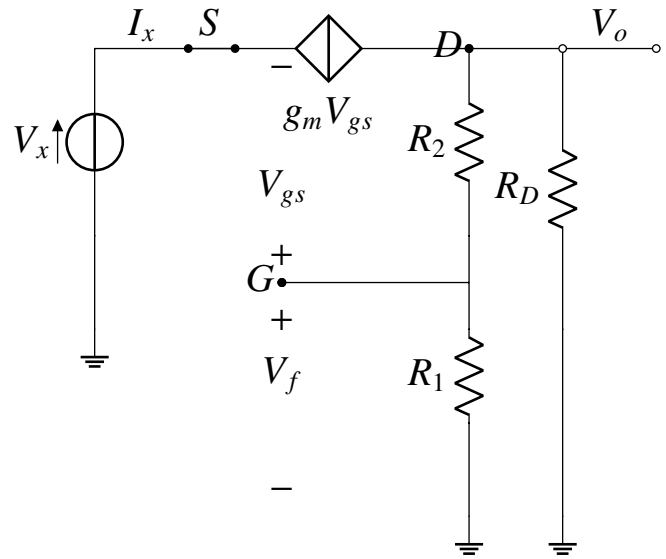


Fig. 2.1.7

$$R_{if} = \frac{V_x}{I_x} \quad (2.1.8.1)$$

$$I_x = -g_m V_{gs} \quad (2.1.8.2)$$

$$V_o = I_x R_D \quad (2.1.8.3)$$

$$V_f = \frac{V_o R_1}{R_1 + R_2} = \frac{I_x R_D R_1}{R_1 + R_2} \quad (2.1.8.4)$$

$$V_x = -V_{gs} + V_f \quad (2.1.8.5)$$

$$V_x = \frac{I_x}{g_m} + \frac{I_x R_D R_1}{R_1 + R_2} \quad (2.1.8.6)$$

$$\frac{V_x}{I_x} = \frac{1}{g_m} + \frac{R_D R_1}{R_1 + R_2} \quad (2.1.8.7)$$

$$\text{rearranging :} \quad (2.1.8.8)$$

$$R_{if} = \frac{1}{g_m} \left( 1 + \frac{g_m R_D R_1}{R_1 + R_2} \right) \quad (2.1.8.9)$$

$$R_{if} = R_i (1 + GH) \quad (2.1.8.10)$$

The input impedance is increased by a factor of  $(1 + GH)$ .  $R_{if}$  is related to  $R_i$  by :

$$R_{if} = R_i (1 + GH) \quad (2.1.8.11)$$

The figure for finding output resistance :

2.1.9. finding expression for  $R_{of}$

**Solution:**

To obtain  $R_{of}$  consider the figure 2.1.8 :

We gave test input voltage  $V_x$  and current

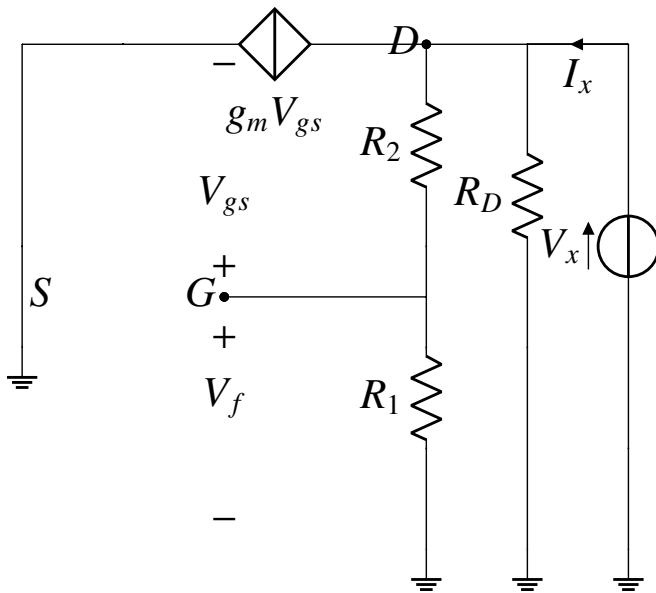


Fig. 2.1.8

$I_x$  from the output side to find the output resistance and made the input constant voltages as zero.

$$R_{of} = \frac{V_x}{I_x} \quad (2.1.9.1)$$

$$I_x = g_m V_{gs} \left( \frac{V_x}{R_1 + R_2} \right) + \left( \frac{V_x}{R_D} \right) \quad (2.1.9.2)$$

$$V_{gs} = \frac{R_1 V_x}{R_1 + R_2} \quad (2.1.9.3)$$

$$I_x = \frac{g_m R_1 V_x}{R_1 + R_2} + \frac{V_x}{R_1 + R_2} + \left( \frac{V_x}{R_D} \right) \quad (2.1.9.4)$$

$$I_x = V_x \left( \frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{R_D} \right) \quad (2.1.9.5)$$

$$R_{of} = \frac{V_x}{I_x} \quad (2.1.9.6)$$

$$R_{of} = \frac{1}{\frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{R_D}} \quad (2.1.9.7)$$

rearranging and multiply both the numerator and denominator by  $R_D$

$$R_{of} = \frac{R_D}{\frac{g_m R_1 R_D}{R_1 + R_2} + 1 + \frac{R_D}{R_1 + R_2}} \quad (2.1.9.8)$$

$$\text{since } R_1 + R_2 \gg R_D \Rightarrow \frac{R_D}{R_1 + R_2} = 0$$

$$R_{of} = \frac{R_D}{1 + \frac{g_m R_1 R_D}{R_1 + R_2}} \quad (2.1.9.9)$$

$$R_{of} = \frac{R_o}{1 + GH} \quad (2.1.9.10)$$

The output impedance is decreased by a factor of  $(1 + GH)$ .  $R_{of}$  is related to  $R_o$  by :

$$R_{of} = \frac{R_o}{1 + GH} \quad (2.1.9.11)$$

The table showing all the expressions we find out in this problem :

$G$	$\frac{g_m R_D}{R_1}$
$H$	$\frac{1}{R_1 + R_2}$
$R_i$	$\frac{1}{g_m}$
$R_o$	$R_D$
$R_{if}$	$\left( \frac{1}{g_m} \right) \left( 1 + \frac{g_m R_D R_1}{R_1 + R_2} \right)$
$R_{of}$	$\frac{R_D}{1 + \frac{g_m R_D R_1}{R_1 + R_2}}$

TABLE 2.1.9

2.1.10. Now for spice stimulation we will assume some values which are not given and see the spice simulation.

**Solution:**

For verifying  $G$  (The open loop Gain) we simulate the circuit and plot the ratio:  $\frac{V_o}{V_s}$

Finding  $g_m$  to be 0.09

$$G = g_m R_D \quad (2.1.10.1)$$

$$G = 0.93645 \quad (2.1.10.2)$$

For verifying  $H$  (The open loop Gain) we simulate the circuit and plot the ratio:  $\frac{V_f}{V_o}$

As in circuit we have taken values of  $R_1$  and  $R_2$  to be  $10k\Omega$  and  $10k\Omega$

$$H = \frac{R_1}{R_1 + R_2} \quad (2.1.10.3)$$

$$H = 0.5 \quad (2.1.10.4)$$

2.1.11. Simulate the circuit 2.1.1 using spice simulators and plot the generated output of the gains using python script

**Solution:** The plot for Open loop gain( $G$ ) is :

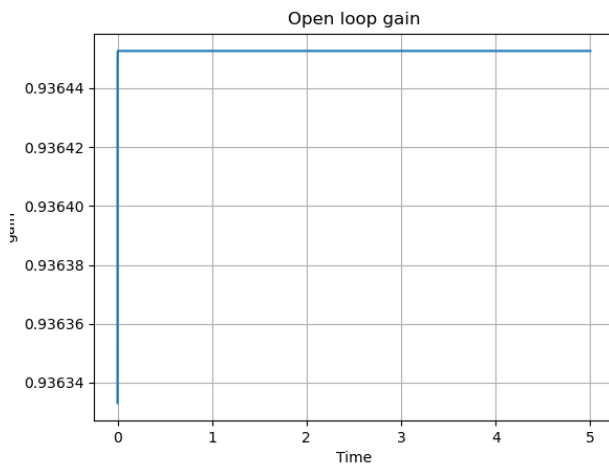


Fig. 2.1.11.6

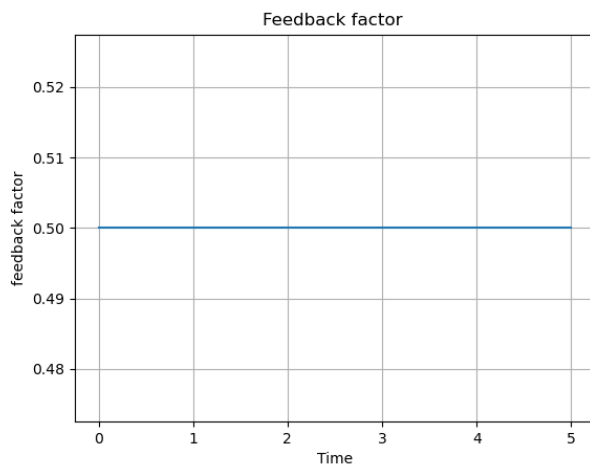


Fig. 2.1.11.7

Find the netlist of the simulated circuit here:

codes/es17btech11019/spice/  
es17btech11019.net

Python code used for generating the output:

codes/es17betch11019/spice/  
es17btech11019.py

### 3 BODE PLOT

### 4 SECOND ORDER SYSTEM

### 5 ROUTH HURWITZ CRITERION

### 6 STATE-SPACE MODEL

### 7 NYQUIST PLOT

### 8 COMPENSATORS

### 9 GAIN MARGIN

### 10 PHASE MARGIN

### 11 OSCILLATOR

### 12 ROOT LOCUS

### 13 POLAR PLOT

### 14 PID CONTROLLER