Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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8.1 Stability

8.1. For a unity feedback system shown in Fig. 1

$$G(s) = \frac{K}{s(s+2)(s+4)(s+6)} \tag{8.1.1}$$

Design a lag lead compensator to yield a K_v = 2 and a phase margin of 30°. First we will design a lead compensator and for that whole system we will design a lag compensator which will finally be the lag lead compensator of the orignal transfer function.

Solution: For unity feedback we have Velocity error constant (K_v)

$$K_v = \lim_{s\to 0} sG(s) \tag{8.1.2}$$

$$\lim_{s\to 0}\left(\frac{K}{(2+s)(4+s)(6+s)}\right) = 2 \tag{8.1.3}$$

$$\implies K = 96 \tag{8.1.4}$$

Check the phase margin and gain crossover frequency by running the following code

```
codes/es17btech11019_1.py
```

- The Phase margin: 19.76°
- Gain Crossover Frequency:1.469 rad/sec

The plot of system is as shown,

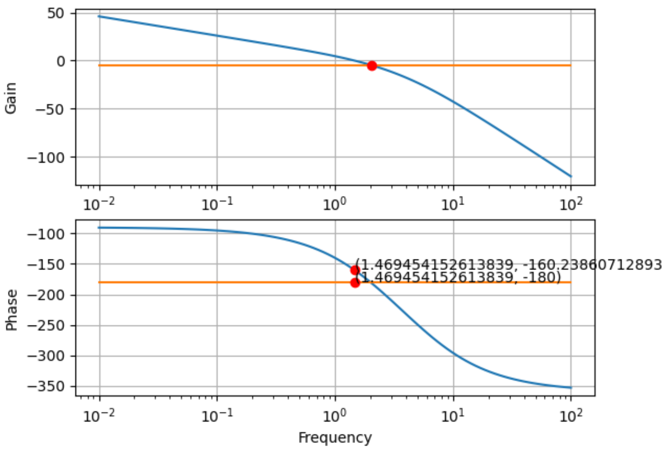

Fig. 8.1

Therefor amount of phase to be added: 30-19.76=10.24
Transfer function:

$$C(s) = \beta\left(\frac{1 + j\tau\omega}{1 + j\beta\tau\omega}\right) \tag{8.1.5}$$

Find the values of β and τ
Solution: The maximum phase lead compensated by a lead compensator is given by

$$\phi = \sin^{-1}\frac{1-\beta}{1+\beta} \tag{8.1.6}$$

at

$$\omega = \frac{1}{\sqrt{\beta}\tau} \tag{8.1.7}$$

Now we know that from Gain crossover frequency

$$\omega = 1.469 rad/sec \tag{8.1.8}$$

and the phase margin to be added:

$$\phi = 10.24^\circ \tag{8.1.9}$$

But to compensate for the added magnitude of lead compensator, a correction factor of 10° – 20° is added.Hence

$$\phi = 30.24^\circ \implies \beta = 0.33 \tag{8.1.10}$$

From the bode plot ω is chosen at which gain of original system is

$$-20\log\left(1/\sqrt{\beta}\right) = -4.81 \tag{8.1.11}$$

Find the plot using the following code

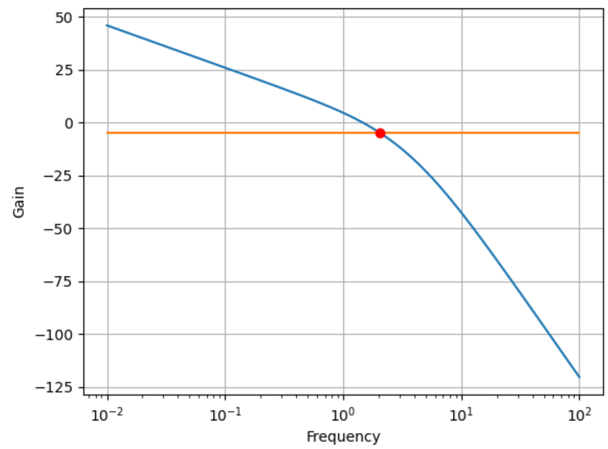

Fig. 8.1

```
codes/es17btech11019_2.py
```

From plot ω=2.009 rad/sec
Solving equations 8.1.6 and 8.1.7

$$\tau = 0.828 \tag{8.1.12}$$
$$\beta = 0.33 \tag{8.1.13}$$
$$\tag{8.1.14}$$

New Transfer Function:

$$G(s) = \frac{96\,(1 + 0.828s)}{(s)\,(2+s)\,(4+s)\,(6+s)\,(1+0.273s)} \tag{8.1.15}$$

8.2. Verify your results from the following code:

```
codes/es17btech11019_3.py
```

- The Phase margin: 29.269°
- The Gain Crossover Frequency: 2.02 rad/sec

The plot is as shown,

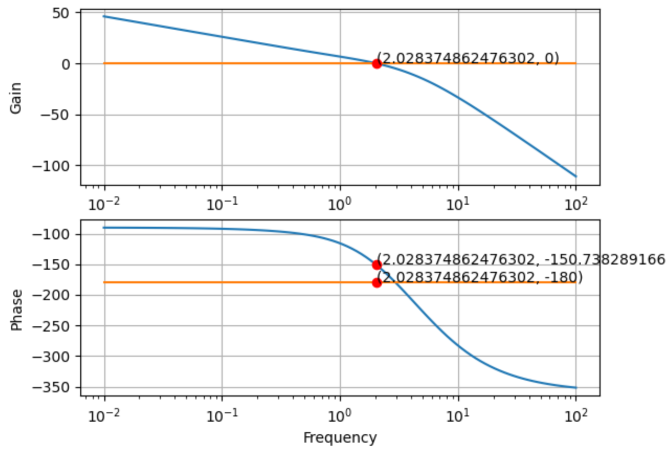

Fig. 8.2

Now for lag compensator of this whole lead compensated part Transfer function:

$$C'(s) = \left(\frac{1 + j\tau\omega}{1 + j\alpha\tau\omega}\right) \tag{8.2.1}$$

Find the values of α

Solution:

$$\alpha = \frac{1}{\beta} \tag{8.2.2}$$

Solving equations 8.2.2

$\alpha = 3.03$

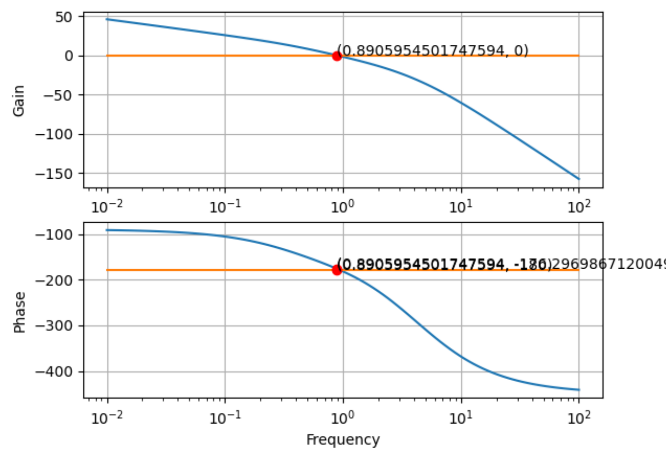

Fig. 8.2

New Transfer Function:

$$G(s) = \frac{96\,(1 + 0.828s)\,(1 + 0.828s)}{(s)\,(2 + s)\,(4 + s)\,(6 + s)\,(1 + 0.273s)\,(1 + 2.50884s)} \tag{8.2.3}$$

Final plot is,

Find the plot using the following code

codes/es17btech11019_4.py