Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

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1 Feedback Circuits

1.0.1. Complete the table entries given below :

Solution:

Transistor	$\mathbf{I}_E(mA)$	$\mathbf{r}_e(ohm)$	$\mathbf{g}_m(mA/V)$	$\mathbf{r}_\pi(K\Omega)$	β_o	$\mathbf{f}_T(MHz)$	$\mathbf{C}_\mu(pF)$	$\mathbf{C}_\pi(pF)$	$\mathbf{f}_\beta(MHz)$
(a)	2				100	500	2		
(b)		25					2	10.7	4
(c)				2.5	100	500		10.7	
(d)	10				100	500	2		
(e)	0.1				100	150	2		
(f)	1				10	500	2		
(g)						800	1	9	80

TABLE 1.0.1: Initial table

We have to fill this table from part a to part g. We can neglect r_π

1.0.2. We have to find the missing values of the table.

Solution: We will solve it part by part.

The Full Circuit referred to this problem is shown below which is a CC-CB amplifier :

Below is the circuit:

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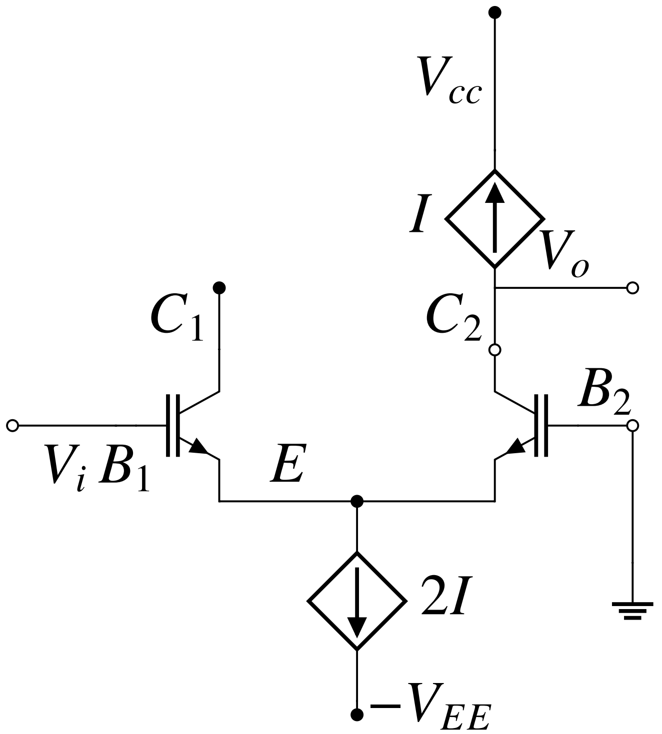

Fig. 1.0.2: Complete Circuit

1.0.3. First we will represent the given circuit using a Small Signal Equivalent Model.

Solution: The simplified small signal circuit for the above complete circuit is shown below :

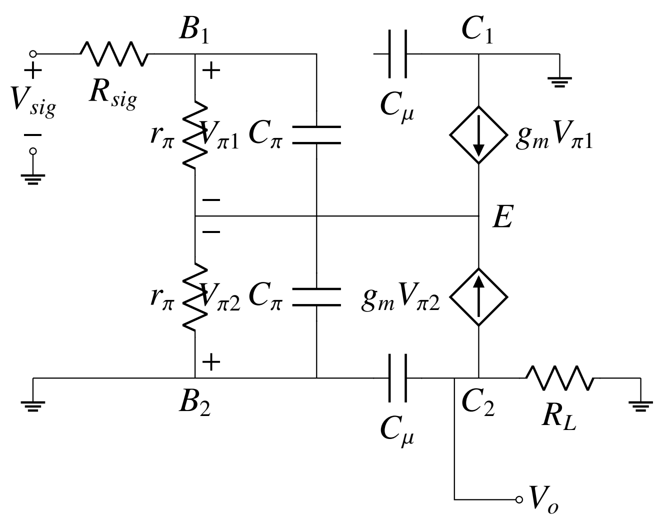

Fig. 1.0.3: Small signal Model

1.0.4. Part A : We have to find values of : r_e , g_m , r_π ,C_π , f_β
Solution:

$$r_e = \frac{V_T}{I_E}$$
$$= 25/2$$
$$= 12.5\Omega$$

$$r_e = 12.5\Omega$$

$$g_m = \frac{I_C}{V_T}$$
$$= (\frac{\beta}{\beta + 1})\frac{I_E}{V_T}$$
$$= (\frac{100}{100 + 1})\frac{2}{25}$$
$$g_m = 79.2\frac{mA}{V}$$

$$r_\pi = \frac{\beta}{g_m}$$
$$= \frac{100}{79.2 * 0.001}$$
$$= 1.26 \text{ k}\Omega$$

$$r_\pi = 1.26k\Omega$$

$$f_\beta = \frac{f_T}{\beta}$$
$$= \frac{500 * 10^6}{100}$$
$$= 5 \text{ Mhz}$$

$$f_\beta = 5Mhz$$

$C_\pi = \frac{g_m}{2_T} - C - \mu$

$= \frac{79.2 * 0.001}{2 * \pi * 10^8} - (2 * 10^-12)$

$= 23$ pF

$C_\pi = 23pF$

1.0.5. Part B : We have to find values of : I_e , g_m , r_π ,β_o , f_T
Solution:

$I_E = \frac{V_T}{r_e}$

$= \frac{25}{25}$

$= 1$ mA

$I_E = 1mA$

$g_m = \frac{I_E}{V_T}$

$= \frac{1}{25}$

$= 40$ mA/V

$g_m = 40\frac{mA}{V}$

$r_\pi = \frac{1}{2\pi(C_\pi + C_\mu)f_\beta}$

$= \frac{1}{2\pi(10.7 + 2) * 10^{-12} * 4 * 10^6}$

$= 3.13$ kΩ

$r_\pi = 3.13k\Omega$

$\beta_o = g_m r_\pi$

$= 3.13*10^3 * 40 * 10^{-3}$

$= 125$

$\beta_o = 125$

$f_T = \beta f_\beta$

$= 125 * 4 * 10^6$

$= 500$ MHz

$f_T = 500MHz$

1.0.6. Part C : We have to find values of : f_β , g_m , r_e ,I_E , C_μ

Solution:

$f_\beta = \frac{T}{\beta}$

$= \frac{500 * 10^6}{100}$

$= 5$ MHz

$f_\beta = 5MHz$

$$g_m = \frac{\beta}{r_\pi}$$
$$= \frac{100}{2500}$$
$$= 40 \text{ mA/V}$$
$$g_m = 40\frac{mA}{V}$$

$$I_E = g_m V_T$$
$$= 40 * 25$$
$$= 1 \text{ mA}$$
$$I_E = 1mA$$

$$r_e = \frac{V_T}{I_E}$$
$$= \frac{25}{1}$$
$$= 25\ \Omega$$
$$r_e = 25\Omega$$

$$C_\mu = \frac{g_m}{2_T} - C - \pi$$
$$= \frac{40 * 0.001}{2 * \pi * 500 * 10^8} - (10.7 * 10^{-12})$$
$$= 2.03 \text{ pF}$$
$$C_\mu = 2.03pF$$

1.0.7. Part D : We have to find values of : f_β , g_m , r_e ,r_π , C_π
Solution:

$$r_e = \frac{V_T}{I_E}$$
$$= \frac{25}{10}$$
$$= 2.5\ \Omega$$
$$r_e = 2.5\Omega$$

$$g_m = \frac{I_E}{V_T}$$
$$= \frac{10}{25}$$
$$= 0.4 \text{ A/V}$$
$$g_m = 0.4\frac{A}{V}$$

$$r_\pi = \frac{\beta}{g_m}$$
$$= \frac{100}{0.4}$$
$$= 250\Omega$$
$$r_\pi = 250\Omega$$

$$f_\beta = \frac{T}{\beta}$$

$$= \frac{500 * 10^6}{100}$$

$= 5$ MHz

$f_\beta = 5MHz$

$$C_\pi = \frac{g_m}{2\pi f_T} - C_\mu$$

$$= \frac{0.4}{2 * \pi * 500 * 10^6} - (2 * 10^{-12})$$

$= 125$ pF

$C_\pi = 125pF$

1.0.8. Part E : We have to find values of : f_β , g_m , r_e ,r_π , C_π

Solution:

$$r_e = \frac{V_T}{I_E}$$

$$= \frac{25}{0.1}$$

$= 250\ \Omega$

$r_e = 250\Omega$

$$g_m = \frac{I_E}{V_T}$$

$$= \frac{0.1}{25}$$

$= 4$ mA/V

$g_m = 4\frac{mA}{V}$

$$r_\pi = \frac{\beta}{g_m}$$

$$= \frac{100}{0.004}$$

$= 25\ K\Omega$

$r_\pi = 25K\Omega$

$$f_\beta = \frac{T}{\beta}$$

$$= \frac{150 * 10^6}{100}$$

$= 1.5$ MHz

$f_\beta = 1.5MHz$

$$C_\pi = \frac{g_m}{2\pi f_T} - C_\mu$$

$$= \frac{0.004}{2 * \pi * 150 * 10^6} - (2 * 10^{-12})$$

$= 2.24$ pF

$C_\pi = 2.24pF$

1.0.9. Part F : We have to find values of : f_β , g_m , r_e ,r_π , C_π

Solution:

$$r_e = \frac{V_T}{I_E}$$

$= \frac{25}{1}$

$= 25\ \Omega$

$r_e = 25\Omega$

$g_m = \frac{I_E}{V_T}$

$= \frac{1}{25}$

$= 40$ mA/V

$g_m = 40\frac{mA}{V}$

$r_\pi = \frac{\beta}{g_m}$

$= \frac{100}{0.004}$

$= 2500\ \Omega$

$r_\pi = 2500\Omega$

$f_\beta = \frac{T}{\beta}$

$= \frac{500 * 10^6}{10}$

$= 50$ MHz

$f_\beta = 50MHz$

$C_\pi = \frac{g_m}{2\pi f_T} - C_\mu$

$= \frac{0.04}{2 * \pi * 500 * 10^6} - (2 * 10^{-12})$

$= 10.7$ pF

$C_\pi = 10.7pF$

Part G : We have to find values of : β,f_β , g_m , r_e ,I_E

Solution:

$\beta = \frac{T}{f_\beta}$

$= \frac{800}{80}$

$= 10$

$\beta = 10$

$r_\pi = \frac{1}{2\pi(C_\pi + C_\mu)f_\beta}$

$= \frac{1}{2\pi(9 + 1) * 10^{-12} * 80 * 10^6}$

$= 199\ \Omega$

$r_\pi = 199\Omega$

$g_m = \frac{\beta}{r_\pi}$

$= 10_{\overline{199}}$
$= 50$ mA/V

$g_m = 50\frac{mA}{V}$

$I_E = g_m V_T$

$= 0.05 * 0.025$

$= 1.25$ mA

$I_E = 1.25 mA$

$r_e = \frac{V_T}{I_E}$

$= \frac{25}{1.25}$

$= 20\ \Omega$

$r_e = 20\Omega$

1.0.10. Given the following values the final table is :

Transistor	$\mathbf{I}_E(mA)$	$\mathbf{r}_e(ohm)$	$\mathbf{g}_m(mA/V)$	$\mathbf{r}_\pi(K)$	β_o	$\mathbf{f}_T(MHz)$	$\mathbf{C}_\mu(pF)$	$\mathbf{C}_\pi(pF)$	$\mathbf{f}_\beta(MHz)$
(a)	2	12.5	79.2	1.26	100	500	2	23	5
(b)	1	25	40	3.13	125	500	2	10.7	4
(c)	1	25	40	2.5	100	500	2.30	10.7	5
(d)	10	2.5	400	0.25	100	500	2	125	5
(e)	0.1	250	4	25	100	150	2	2.24	1.5
(f)	1	25	40	2.5	10	500	2	10.7	50
(g)	1.25	20	50	0.199	10	800	1	9	80

TABLE 1.0.10

1.0.11. Verify the above calculations using a Python code.

Solution:

```
codes/es17btech11019/es17btech11019_calc.py
```