Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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1 Signal Flow Graph

2 Gain of Feedback Circuits

2.1 Voltage Amplifiers

2.1.1. We are given with a feedback voltage amplifier shown in 2.1.1.We can neglect r_o and given with $R_1 + R_2 >> R_D$.

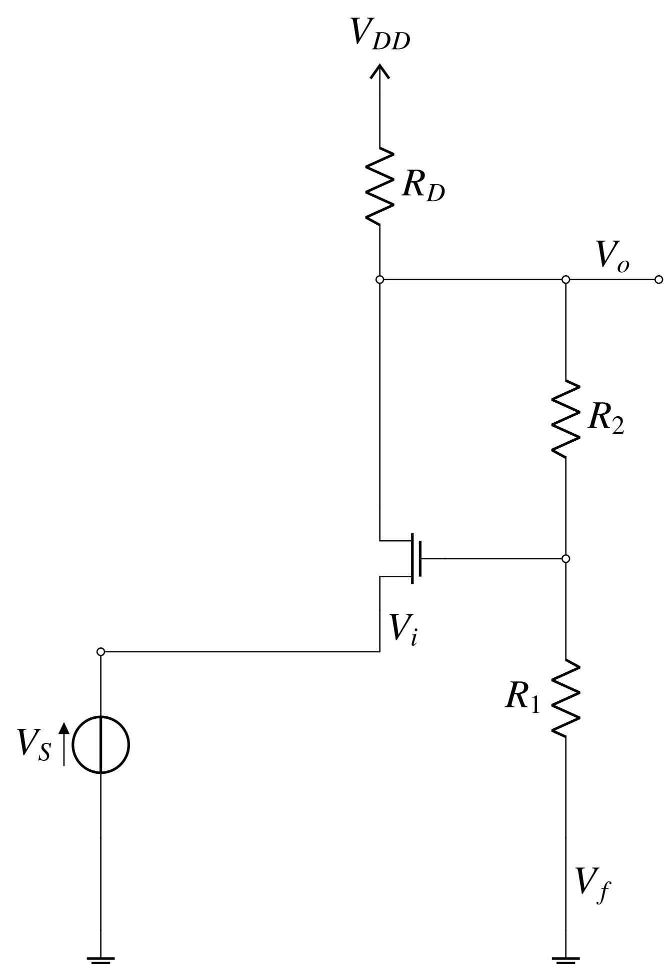

Fig. 2.1.1

2.1.2. part(a) : We have to find the expressions for G(open loop gain) , H(the feedback factor) and hence the amount of feedback.

Solution: For this , first we have to draw the Small-Signal Model for the above Circuit, we ground all constant voltage sources and open all constant current sources. All Small-Signal paramters are obtained from DC-Analysis of the circuit.In Small-Signal Analysis a N-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{gs}$ flowing from Drain to Source. Whereas a P-MOSFET is modelled as a Current Source with value of current equal to $g_m v_{sg}$ flowing from Source to Drain.

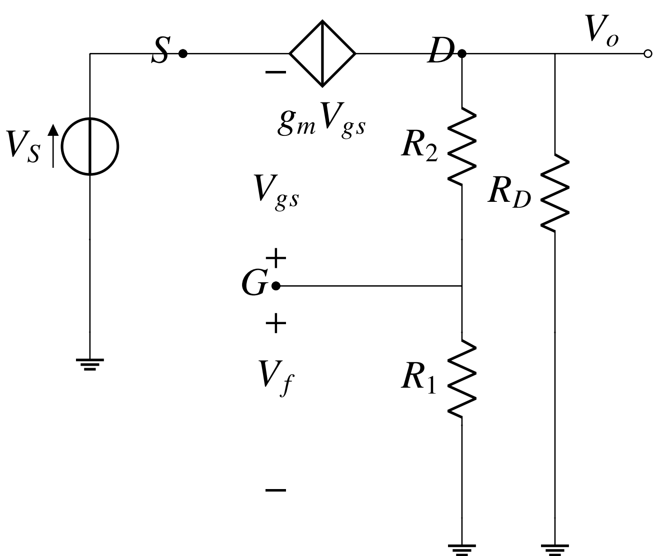

Fig. 2.1.2: Small Signal Model

2.1.3. For finding open loop gain (G) and the feedback factor (H).
Solution: For finding the open loop gain we have to remove R_2 and R_1 and the gate should be grounded.

2.1.4. Finding the open loop gain(G)
Solution:

$$V_o = -g_m V_{gs} * R_D \tag{2.1.4.1}$$

$$V_{gs} = -V_S \tag{2.1.4.2}$$

$$G = \frac{V_o}{V_s} \tag{2.1.4.3}$$

$$G = g_m R_D \tag{2.1.4.4}$$

2.1.5. Finding the Expression for the feedback factor H.
Solution:

$$H = \frac{V_f}{V_o} \tag{2.1.5.1}$$

$$V_f = \frac{R_1}{R_1 + R_2} V_o \tag{2.1.5.2}$$

$$H = \frac{R_1}{R_1 + R_2} \tag{2.1.5.3}$$

Amount of feedback is defined as : $1 + GH$

$$1 + GH = 1 + \frac{g_m R_D R_1}{R_1 + R2} \tag{2.1.5.4}$$

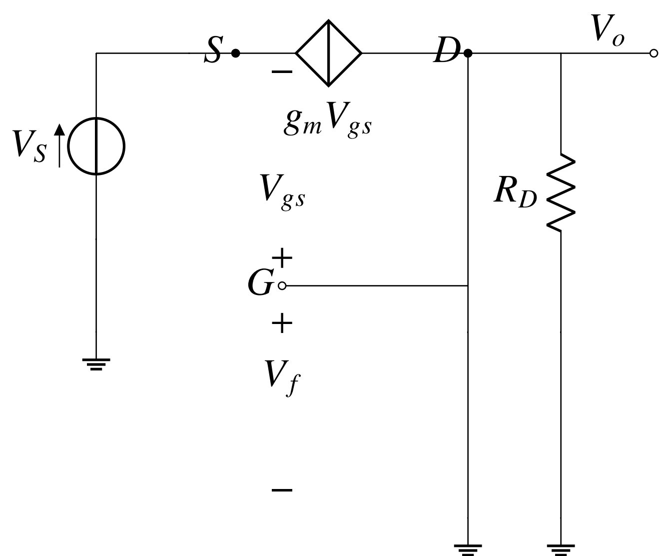

Fig. 2.1.5: CG amplifier

2.1.6. Part(b) : We have to eliminate the feedback by removing R_1 and R_2 and connecting the gate of Q to a constant DC voltage (signal ground).We have to find the expression of the input resistance R_i and the output resistance R_o of the open loop amplifier.
Solution:
When the R_1 and R_2 and gate of Q is connected to a constant DC voltage (signal ground) it becomes a CG(Common gate amplifier) without feedback. We can directly see from the 2.1.5 the expression of input resistance R_i and output resistance R_o.
For finding input resistance , output constant voltages are grounded and hence the only current flowing is $g_m V_{gs}$.Hence R_i is :

$$I_{in} = -g_m V_{gs} \tag{2.1.6.1}$$
$$V_{in} = V_S \tag{2.1.6.2}$$
$$V_S = -V_{gs} \tag{2.1.6.3}$$
$$R_i = \frac{V_{in}}{I_{in}} \tag{2.1.6.4}$$
$$R_i = \frac{1}{g_m} \tag{2.1.6.5}$$

Similarly , for finding output R_o , V_{in} that is V_S will be zero and hence $g_m V_{gs}$ will be zero. Hence only R_D will be left which is the output resistance.

$$R_o = R_D \tag{2.1.6.6}$$

2.1.7. Part(c) : Using standard circuit analysis that is without using feedback approach we have to find the input resistance R_{if} and output resistance R_{of} and how they relate to R_i and R_o , which we find earlier.
Solution:
We will find them one by one.

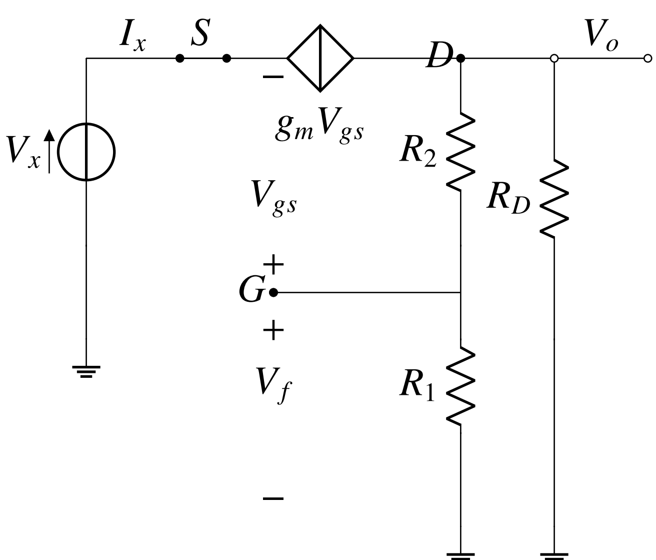

Fig. 2.1.7

2.1.8. finding expression for R_{if}
Solution:

To obtain R_{if} consider the figure 2.1.7 :
We gave test input voltage V_x and current I_x to find the input resistance from the input side to find R_i.

$$R_{if} = \frac{V_x}{I_x} \tag{2.1.8.1}$$
$$I_x = -g_m V_{gs} \tag{2.1.8.2}$$
$$V_o = I_x R_D \tag{2.1.8.3}$$
$$V_f = \frac{V_o R_1}{R_1 + R_2} = \frac{I_x R_D R_1}{R_1 + R_2} \tag{2.1.8.4}$$
$$V_x = -V_{gs} + V_f \tag{2.1.8.5}$$
$$V_x = \frac{I_x}{g_m} + \frac{I_x R_D R_1}{R_1 + R_2} \tag{2.1.8.6}$$
$$\frac{V_x}{I_x} = \frac{1}{g_m} + \frac{R_D R_1}{R_1 + R_2} \tag{2.1.8.7}$$
$$rearranging : \tag{2.1.8.8}$$
$$R_{if} = \frac{1}{g_m}\left(1 + \frac{g_m R_D R_1}{R_1 + R_2}\right) \tag{2.1.8.9}$$
$$R_{if} = R_i(1 + GH) \tag{2.1.8.10}$$

The input impedance is increased by a factor of $(1 + GH)$. R_{if} is related to R_i by :

$$R_{if} = R_i(1 + GH) \tag{2.1.8.11}$$

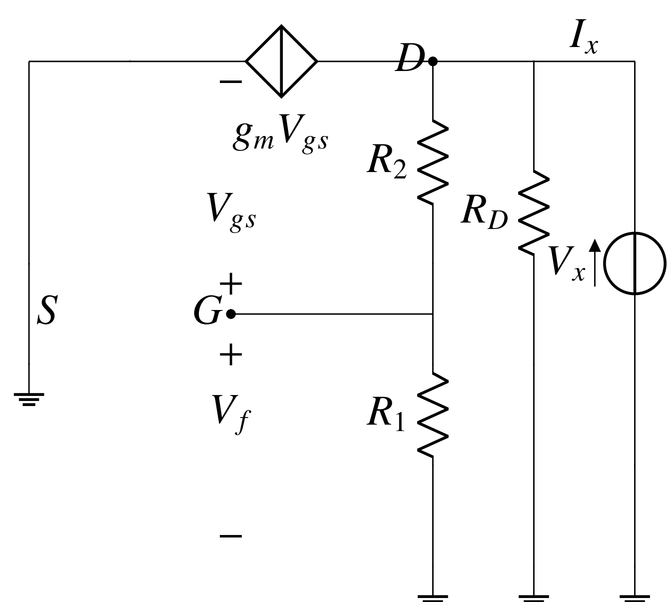

Fig. 2.1.8

2.1.9. finding expression for R_{of}
Solution:

To obtain R_{of} consider the figure 2.1.8 :
We gave test input voltage V_x and current

I_x from the output side to find the output resistance and made the input constant voltages as zero.

$$R_{of} = \frac{V_x}{I_x} \quad (2.1.9.1)$$

$$I_x = g_m V_{gs}(\frac{V_x}{R_1 + R_2}) + (\frac{V_x}{R_D}) \quad (2.1.9.2)$$

$$V_{gs} = \frac{R_1 V_x}{R_1 + R_2} \quad (2.1.9.3)$$

$$I_x = \frac{g_m R_1 V_x}{R_1 + R_2} + \frac{V_x}{R_1 + R_2} + (\frac{V_x}{R_D}) \quad (2.1.9.4)$$

$$I_x = V_x(\frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{R_D}) \quad (2.1.9.5)$$

$$R_{of} = \frac{V_x}{I_x} \quad (2.1.9.6)$$

$$R_{of} = \frac{1}{\frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{R_D}} \quad (2.1.9.7)$$

rearranging and multiply both the numerator and denominator by R_D

$$R_{of} = \frac{R_D}{\frac{g_m R_1 R_D}{R_1 + R_2} + 1 + \frac{R_D}{R_1 + R_2}} \quad (2.1.9.8)$$

since $R_1 + R_2 >> R_D \implies \frac{R_D}{R_1 + R_2} = 0$

$$R_{of} = \frac{R_D}{1 + \frac{g_m R_1 R_D}{R_1 + R_2}} \quad (2.1.9.9)$$

$$R_{of} = \frac{R_o}{1 + GH} \quad (2.1.9.10)$$

The output impedance is decreased by a factor of $(1 + GH)$. R_{of} is related to R_o by :

$$R_{of} = \frac{R_o}{1 + GH} \quad (2.1.9.11)$$

The table showing all the expressions we find out in this problem :

G	$g_m R_D$
H	$\frac{R_1}{R_1 + R_2}$
R_i	$\frac{1}{g_m}$
R_o	R_D
R_{if}	$(\frac{1}{g_m})(1 + \frac{g_m R_D R_1}{R_1 + R_2})$
R_{of}	$\frac{R_D}{1 + \frac{g_m R_D R_1}{R_1 + R_2}}$

TABLE 2.1.9

3 Bode Plot

4 Second order System

5 Routh Hurwitz Criterion

6 State-Space Model

7 Nyquist Plot

8 Compensators

9 Gain Margin

10 Phase Margin

11 Oscillator

12 Root Locus

13 Polar Plot

14 PID Controller