# Logic Synthesis & Verification, Fall 2024

**National Taiwan University**

## Problem Set 1

Due on 2024/10/6 23:59 on NTU Cool.

## 1 [Characteristic Function]

(10%) Consider the state transition graph in Figure 1, where the label "$i/j$" on an edge indicates the transition condition under input $i$ and output $j$. Give a six-variable (including two current-state variables, one input variable, two next-state variables, and one output variable) Boolean function that characterizes the state transition relation in the graph. Let states $S_0, S_1, S_2, S_3$ be encoded as 00, 01, 10, 11, respectively. Show the Boolean function with a truth table.

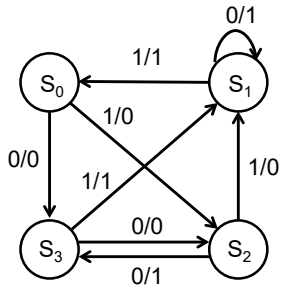


**Fig. 1.** A state transition graph.

## 2 [Boolean Algebra Definition]

(10%) Does the tuple $(\{0,1\}, \oplus, \cdot, 0, 1)$, where $\oplus$ and $\cdot$ stand for Boolean XOR and AND operations, respectively, form a Boolean algebra? Check the five postulates and determine their satisfaction/nonsatisfaction.

## 3 [Uniqueness of Complement]

(10%) Given any Boolean algebra $(\mathbb{B}, \cdot, +, \underline{0}, \underline{1})$, show that for any $a \in \mathbb{B}$, there is a unique $a' \in \mathbb{B}$ such that both equalities $a + a' = \underline{1}$ and $a \cdot a' = \underline{0}$ hold.

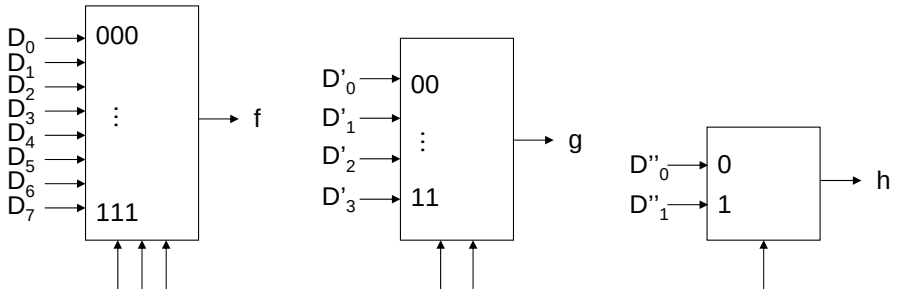


**Fig. 2.** Mux implementation of Boolean functions

## 4 [Properties of Boolean Algebra]

(24%) Prove the following equalities using ONLY the five postulates of Boolean algebra (or other properties that you have proven using the postulates). Please specify clearly which postulate is applied in each step of your derivation.

(a) $a + a' \cdot b = a + b$
(b) $(a + b)' = (a' \cdot b')$
(c) $(a + b) \cdot (a' + c) \cdot (b + c) = (a + b) \cdot (a' + c)$

## 5 [Boole's Expansion Theorem]

(16%) Use Boole's expansion theorem to prove the following equalities.

(a) $uf(u, v, w) = uf(1, v, w)$
(b) $u' + f(u, v, w) = u' + f(1, v, w)$

## 6 [Boolean Functions]

(12%) Let $g$ and $h$ be single-variable Boolean functions. For each of the following cases, express $f(0)$ and $f(1)$ as simplified formulas involving $g(0), g(1), h(0)$, and $h(1)$.

(a) $f(x) = g(h(x))$
(b) $f(x) = g(g'(x))$

## 7 [Boolean Algebra Application]

(18%) Let Boolean function $f(x, y, z) = xy + x'y' + y'z'$ for $\mathbb{B} = \{0, 1\}$.

(a) Consider the multiplexor implementation of $f$ in Figure 2 (a). What are the values of $D_i$?

(b) Consider implementing $f(x, y, z)$ by another Boolean function $g(y, z)$ using the multiplexor of Figure 2 (b).
- What is the new Boolean algebra? Please define the five-tuple $(\mathbb{B}, +, \cdot, \underline{0}, \underline{1})$.
- What are the possible values of variables $y$ and $z$? Why the multiplexor assumes $y$ and $z$ have only values $\{0, 1\}$?
- Please explain in what sense $f(x, y, z)$ and $g(y, z)$ can be equivalent. What should the values $D_i'$ be?

(c) Consider implementing $f(x, y, z)$ by yet another Boolean function $h(z)$ using the multiplexor of Figure 2 (c). What is the new Boolean algebra? Please define the the five-tuple $(\mathbb{B}, +, \cdot, \underline{0}, \underline{1})$. What are the values $D_i''$?