8 – Puzzle using A* Algorithm

PROJECT DOCUMENTATION REPORT

PROGRAMMING PROJECT 1
ITCS 6150 - Intelligent Systems

DEPARTMENT OF COMPUTER SCIENCE

SUBMITTED TO
Dewan T. Ahmed, Ph.D.

SUBMITTED BY:

Urma Haldar — **801074045**

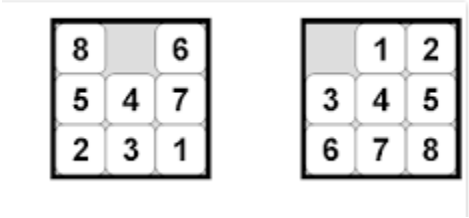
Sahil Sood — **801082267**

Table of Contents

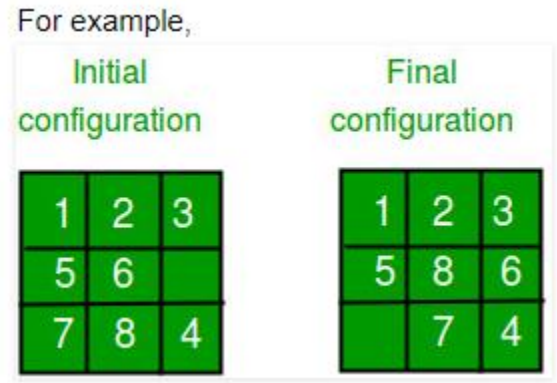
PROBLEM FORMULATION

1.1 INTRODUCTION

What is 8 – Puzzle Problem?
The 8-puzzle is a sliding puzzle that consists of a frame of numbered (1-8) square tiles in random order with one tile missing. The goal is usually to place those numbers in correct numerical order. *We can slide four adjacent (left, right, above and below) tiles into the empty space.* The more general n-puzzle is a classical problem which can be solved using graph search techniques. The problem of finding the optimal solution is NP-hard.



In our project we have user defined initial as well as output state.



A* Search technique:
The A* searching algorithm is a recursive algorithm that continuously calls itself until a winning state is found. It uses the sum of moves to current step and Manhattan priority function as cost function. A priority queue of search node containing number of moves, current board and previous search node is created. For each move, the search node with minimum cost is dequeued and neighboring nodes of this search node are then inserted into the priority queue. The sequence of moves using fewest number of moves to solve the puzzle is obtained when the goal board is ultimately dequeued (total number of moves is always at least its priority).

1.2 ALGORITHM PSEUDOCODE

```
boolean
Solution::search(PriorityQueue pq)
{ if pq.isEmpty() then
    return false
puz = pq.extract()
if puz.isGoal()
    return true
successors = puz.expand()
              // all possible successors to puz
for each suc in successors do
    pq.insert(suc)
if search(pq)
    return true
else
    return false
}
```

PROGRAM STRUCTURE

2.1 FUNCTIONS/PROCEDURES

Our code implements A* Search algorithm using Manhattan Distance Heuristic Calculation and also Misplaced Tiles Heuristic Calculation (separate codes) implementation using Python and priority queues.

Our codes handles just the heuristic calculation differently. The rest structure remains same. We use the following functions within the code:

h_misplaced_cost() – For calculating misplaced tiles heuristic (used in misplaced_tiles code)
mhd() – For calculation of Manhattan Distance (used in the one using Manhattan distance)
coor() – assign each digit of the state, the coordinate to calculate Manhattan distance
all() – to set goal elements
stepsoptimal() – to calculate best optimal steps to goal
solve() – to compare initial state with goal state and make moves towards the goal
main() – to read input/output states

2.2 GOLBAL/LOCAL VARIABLES

We use various global/local variables in our program:
The local variables within function are:
h_misplaced_cost – cost – Calculates misplaced heuristic
mhd – m – calculated manhattan distance
coor –

- c – for the array
- x, y – for the range

all -
stepsoptimal –

- optimal – storing optimal states
- last- length of state

solve –

- moves- up,down,right,left moves
- dtype- distance type
- dtstate -
- goalc – coordinate of goal state
- parent – parent state initialization
- gn - cost
- hn – heuristic calculation

- dtpriority – distance priotity
- priority – priority of digits
- pos - position
- fn - function
- loc - location
- succ – successor state
- q – priority queue

main-
The global variables are:

- goal - storing/passing goal state
- board –storing/passing initial state
- state – current state

2.3 LOGIC

We use priority queues as data structure. We use the indexes to calculate coordinates of each digit of the input and output and compare the difference and sort accordingly. We repeat this until our goal is reached. We calculate the heuristic based on these coordinates. It calculates the following information using both the heuristics:

- Goal achieved status
- Total generated nodes
- Total explored nodes
- Total optimized steps
- Time Taken

The code works perfectly for solvable states, but for unsolvable ones, it goes on in a loop.

CODE FOR EACH HEURISTIC

3.1 Manhattan Distance

```
import numpy as np
from copy import deepcopy
import time

# calculate Manhattan distance for each digit as per goal
def mhd(s, g):
    m = abs(s // 3 - g // 3) + abs(s % 3 - g % 3)
    return sum(m[1:])

# assign each digit the coordinate to calculate Manhattan
distance
def coor(s):
    c = np.array(range(9))
    for x, y in enumerate(s):
        c[y] = x
    return c

def all(s):
    #set = '012345678'
    set=string

    return 0 not in [c in s for c in set]

# generate board list as per optimized steps in sequence
def stepsoptimal(state):
    optimal = np.array([], int).reshape(-1, 9)
    last = len(state) - 1
    while last != -1:
```

```
        optimal = np.insert(optimal, 0, state[last]['board'], 0)
        last = int(state[last]['parent'])
    return optimal.reshape(-1, 3, 3)

# solve the board
def solve(board, goal):
    #
    moves = np.array(   [   ('u', [0, 1, 2], -3),
                            ('d', [6, 7, 8],  3),
                            ('l', [0, 3, 6], -1),
                            ('r', [2, 5, 8],  1)
                            ],
                dtype=  [   ('move',  str, 1),
                            ('pos',   list),
                            ('delta', int)
                            ]
                        )

    dtstate = [ ('board',  list),
                ('parent', int),
                ('gn',     int),
                ('hn',     int)
                ]

    goalc = coor(goal)
    # initial state values
    parent = -1 #initial parent state
    gn     = 0
    hn     = mhd(coor(board), goalc) #calculating manhattan
distance between initial and goal state
    state = np.array([(board, parent, gn, hn)], dtstate)
#initializing state
```

```
    #priority queue initialization
    dtpriority = [  ('pos', int),
                    ('fn', int)
                    ]

    priority = np.array( [(0, hn)], dtpriority)
    #
    while True:
        priority = np.sort(priority, kind='mergesort',
order=['fn', 'pos']) # sort priority queue
        pos, fn = priority[0]                    # pick out first
from sorted to explore
        priority = np.delete(priority, 0, 0)     # remove from
queue what we are exploring
        board, parent, gn, hn = state[pos]
        board = np.array(board)
        loc = int(np.where(board == 0)[0])       # locate '0'
(blank)
        gn = gn + 1                              # increase cost
g(n) by 1
        for m in moves:
            if loc not in m['pos']:
                succ = deepcopy(board)           # generate new
state as copy of current
                succ[loc], succ[loc + m['delta']] = succ[loc +
m['delta']], succ[loc]# do the move

                if ~(np.all(list(state['board']) == succ,
1)).any():#check if new (not repeat)
```

```
                    hn = mhd(coor(succ), goalc)
# calculate Manhattan distance
                    q = np.array(   [(succ, pos, gn, hn)],
dtstate)     # generate and add new state in the list
                    state = np.append(state, q, 0)
                    fn = gn + hn
# calculate f(n)
                    q = np.array([(len(state) - 1, fn)],
dtpriority)    # add to priority queue
                    priority = np.append(priority, q, 0)

                    if np.array_equal(succ, goal):
# is this goal state?
                        print('Goal achieved!')
                        return state, len(priority)



    return state, len(priority)

def main():
    print()
    print ("Using Manhattan Distance, solving the 8 puzzle:")
    print("Please enter the goal state: (Please enter a space
inbetween numbers)")
    goal = [int(x) for x in input().split()]# read goal state

    print('Enter initial board: (Please enter a space inbetween
numbers) ')# read initial state
    string = [int(x) for x in input().split()]# read goal state
```

```
    if len(string) != 9:
        print('incorrect input')
        return

    board = np.array(list(map(int, string)))
    print (board)



    t1=time.time()
    state, explored = solve(board, goal)
    t2=time.time()
    print()
    print('Total generated:', len(state))
    print('Total explored: ', len(state) - explored)
    print()
    # generate and show optimized steps
    optimal = stepsoptimal(state)
    print('Total optimized steps:', len(optimal) - 1)
    print()
    print(optimal)
    print()
    print ("The algorithm took " + str((t2-t1) * 1000)  + " ms
of time.")

# Main Program

if __name__ == '__main__':
    main()
```

3.1 Misplaced Tiles

```
import numpy as np
from copy import deepcopy
import time

def h_misplaced_cost(s, g): # calculating misplaced tiles
        cost = np.sum(s != g)-1
        #print (cost)# minus 1 to exclude the empty tile
        if cost > 0:
            return cost
        else:
            return 0

def all(s):
    #set = '012345678'
    set=string

    return 0 not in [c in s for c in set]

# generate board list as per optimized steps in sequence
def genoptimal(state):
    optimal = np.array([], int).reshape(-1, 9)
```

```
    last = len(state) - 1
    while last != -1:
        optimal = np.insert(optimal, 0, state[last]['board'], 0)
        last = int(state[last]['parent'])
    return optimal.reshape(-1, 3, 3)

# solve the board
def solve(board, goal):
    #
    moves = np.array(   [   ('u', [0, 1, 2], -3),
                            ('d', [6, 7, 8],  3),
                            ('l', [0, 3, 6], -1),
                            ('r', [2, 5, 8],  1)
                            ],
                dtype=  [   ('move',  str, 1),
                            ('pos',   list),
                            ('delta', int)
                            ]
                        )

    dtstate = [ ('board',  list),
                ('parent', int),
                ('gn',     int),
                ('hn',     int)
                ]

    # initial state values
    parent = -1 #initial parent state
    gn     = 0
    hn     = h_misplaced_cost(board, goal) #calculating
misplaced tiles between initial and goal state
```

```
    state = np.array([(board, parent, gn, hn)], dtstate)
#initializing state

    #priority queue initialization
    dtpriority = [  ('pos', int),
                    ('fn', int)
                    ]

    priority = np.array( [(0, hn)], dtpriority)
    #
    while True:
        priority = np.sort(priority, kind='mergesort',
order=['fn', 'pos']) # sort priority queue
        pos, fn = priority[0]                    # pick out first
from sorted to explore
        priority = np.delete(priority, 0, 0)    # remove from
queue what we are exploring
        board, parent, gn, hn = state[pos]
        board = np.array(board)
        loc = int(np.where(board == 0)[0])      # locate '0'
(blank)
        gn = gn + 1                              # increase cost
g(n) by 1
        for m in moves:
            if loc not in m['pos']:
                succ = deepcopy(board)           # generate new
state as copy of current
                succ[loc], succ[loc + m['delta']] = succ[loc +
m['delta']], succ[loc]# do the move

                if ~(np.all(list(state['board']) == succ,
1)).any():# check if new (not repeat)
```

```
                    hn = h_misplaced_cost(succ, goal)
# calculate Misplaced tiles
                    q = np.array(   [(succ, pos, gn, hn)],
dtstate)     # generate and add new state in the list
                    state = np.append(state, q, 0)
                    fn = gn + hn
# calculate f(n)
                    q = np.array([(len(state) - 1, fn)],
dtpriority)    # add to priority queue
                    priority = np.append(priority, q, 0)

                    if np.array_equal(succ, goal):
# is this goal state?
                        print('Goal achieved!')
                        return state, len(priority)


    return state, len(priority)

def main():
    print()
    alist = []
    print ("Using Misplaced Tiles, solving the 8 puzzle:")
    print("Please enter the goal state: (please enter a space
inbetween numbers)")
    alist = [int(x) for x in input().split()]# read goal state
    goal=alist
    print('Enter initial board (please enter a space inbetween
numbers): ')# read initial state
    string = [int(x) for x in input().split()]# read goal state

    if len(string) != 9:
```

```
        print('incorrect input')
        return

    board = np.array(list(map(int, string)))
    print (board)



    t1=time.time()
    state, explored = solve(board, goal)
    t2=time.time()
    print()
    print('Total generated:', len(state))
    print('Total explored: ', len(state) - explored)
    print()
    # generate and show optimized steps
    optimal = genoptimal(state)
    print('Total optimized steps:', len(optimal) - 1)
    print()
    print(optimal)
    print()
    print ("The algorithm took " + str((t2-t1) * 1000)  + " ms
of time.")

# Main Program

if __name__ == '__main__':
    main()
```

SAMPLE OUTPUT

4.1 Manhattan Distance

1. Please enter the goal state:
→ 3 2 1 8 0 4 7 5 6
Enter initial board:
→ 2 8 1 3 4 6 7 5 0
Goal achieved!
Total generated: 13
Total explored: 6
Total optimized steps: 6

[2 8 1] [3 4 6] → [7 5 0]	[2 8 1] [3 4 0] → [7 5 6]	[2 8 1] [3 0 4] → [7 5 6]	[2 0 1] [3 8 4] → [7 5 6]	[0 2 1] [3 8 4] → [7 5 6]	[3 2 1] [0 8 4] → [7 5 6]	[3 2 1] [8 0 4] [7 5 6]

The algorithm took 1.979827880859375 ms of time.

2. Please enter the goal state:
→ 1 2 3 8 6 4 7 5 0
Enter initial board:
→ 1 2 3 7 4 5 6 8 0
Goal achieved!
Total generated: 19
Total explored: 9
Total optimized steps: 8

[1 2 3] [7 4 5] → [6 8 0]	[1 2 3] [7 4 0] → [6 8 5]	[1 2 3] [7 0 4] → [6 8 5]	[1 2 3] [7 8 4] → [6 0 5]	[1 2 3] [7 8 4] → [0 6 5]	[1 2 3] [0 8 4] → [7 6 5]	[1 2 3] [8 0 4] → [7 6 5]
[1 2 3] [8 6 4] → [7 0 5]	[1 2 3] [8 6 4] [7 5 0]					

The algorithm took 3.980875015258789 ms of time.

4.2 Misplaced Tiles

1. Please enter the goal state:
→ 1 2 3 8 6 4 7 5 0
Enter initial board:
→ 1 2 3 7 4 5 6 8 0
Goal achieved!
Total generated: 44
Total explored: 23
Total optimized steps: 8

[1 2 3] [7 4 5] → [6 8 0]	[1 2 3] [7 4 0] → [6 8 5]	[1 2 3] [7 0 4] → [6 8 5]	[1 2 3] [7 8 4] → [6 0 5]	[1 2 3] [7 8 4] → [0 6 5]	[1 2 3] [0 8 4] → [7 6 5]	[1 2 3] [8 0 4] → [7 6 5]
[1 2 3] [8 6 4] → [7 0 5]	[1 2 3] [8 6 4] [7 5 0]					

The algorithm took 6.996631622314453 ms of time.

2. Please enter the goal state:
→ 3 2 1 8 0 4 7 5 6

Enter initial board:
→ 2 8 1 3 4 6 7 5 0

Goal achieved!
Total generated: 15
Total explored: 7
Total optimized steps: 6

[2 8 1] [3 4 6] → [7 5 0]	[2 8 1] [3 4 0] → [7 5 6]	[2 8 1] [3 0 4] → [7 5 6]	[2 0 1] [3 8 4] → [7 5 6]	[0 2 1] [3 8 4] → [7 5 6]	[3 2 1] [0 8 4] → [7 5 6]	[3 2 1] [8 0 4] [7 5 6]

The algorithm took 2.9981136322021484 ms of time.

CONCLUSION

The informed searches performs better as compared to uninformed search due to the information gained about the search in their heuristics. Both heuristics were admissible because they never overestimated the path costs (estimated and total). They did this by estimating the solution using a relaxed version of the puzzle. The algorithms were slightly more complex to implement, but reliably provided efficient and usable solutions for difficult puzzles.

Finally, from the sample outputs of both the informed techniques, we can infer which A* method is better for optimal use. Manhattan clearly generates less nodes to give output whereas misplaced tiles generates more number of nodes and also, it takes lesser time than misplaced tiles. Hence, Manhattan Distance is a better approach.

REFERENCES

[1] 8-Puzzle Problem, Heuristics, Wikipedia. Accessed: October 2018.

[2] 8 puzzle Problem using Branch And Bound, Geeks for Geeks Accessed: October 2018.

[3] hrishikeshparanjape/8-puzzle, Github. Accessed: October 2018.