Evaluating the Performance of Portfolio Optimization: Multi-Objective Optimization

A Project Report submitted by

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In fulfillment of the requirements for the award of the degree of

Master of Technology, Data Science

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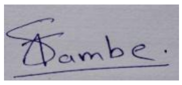
Indian Institute of Technology Jodhpur

Name of the Department

July 2023

Declaration

I hereby declare that the work presented in this Project Report titled ***"Evaluating the Performance of Portfolio Optimization in Multi-Objective"*** submitted to the Indian Institute of Technology Jodhpur in partial fulfilment of the requirementsfor the award of the degree of Master of Technology., is a bonafide record of the research work carried out under the supervision of ***Dr.Md Abu Talhamainuddin Ansary,*** Associate Professor, Department of Mathematics, Indian Institute ofTechnology, Jodhpur. The contents of this Project Report in full or in parts, have not been submitted to, and will not be submitted by me to, any other Institute or University in India or abroad for the award of any degree or diploma.



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Certificate

This is to certify that the Project Report titled ***"Evaluating the Performance of Portfolio Optimization Strategies"***, submitted by *Sahil Tambe (M21AI585)* to the Indian Institute of Technology Jodhpur for the award ofthe degree of *M.Tech* is a bonafide record of the research work done by him under my supervision. To the best of my knowledge, the contents of this report, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Md Abu Talhamainuddin Ansary

Signature

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Acknowledgement

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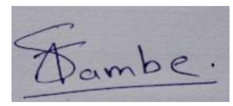
His vision, sincerity as well as strong determination have left an indelible impression on me. I truly appreciate his help and friendliness as well as his constant support. I am very grateful to him for everything that he has done for me in the journey.



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Abstract

Portfolio optimization is the process of choosing a portfolio of assets that maximizes return while minimizing risk. Finding the best balance of risk and reward to achieve financial goals is the goal of investors and financial analysts. The project aims to create and implement a portfolio optimization model that takes into account the constraints or preferences of an investor, as well as factors such as expected returns and volatility of different assets. The model will be tested using historical data to measure its performance and compare it to other investment strategies. The results of this analysis will be used to make recommendations for the construction of the portfolio and to provide an understanding of the risk and return of the selected asset.

Portfolio optimization has become one of the most important topics in the financial world in recent years, and many different methods have been proposed to help investors choose the best portfolio. In this study, we compare the performance of traditional portfolio optimization techniques such as mean-variance optimization with new machine learning methods such as neural networks and support vector machines. We evaluated the performance of these strategies using historical data on market prices and found that machine learning methods generally outperform traditional methods.

Multi-purpose portfolio optimization is the problem of selecting assets that serve multiple purposes simultaneously, such as return, risk and diversification. In this study, we decided to solve this problem by using scalar functions that show multiple targets for a single scalar value.

We compare the performance of three different scaling functions: the weighting method, the Tchebycheff function, and the ε-constraint method. We evaluated the performance of this method using historical cost data and found that the ε-optimization method overall outperformed other methods in terms of the quality of the results obtained. We also prepare the Pareto front, which is a graphical representation of the set of non-dominant solutions in a multi-objective optimization problem, and find that the ε-constrained method produces equally advanced solutions than the other methods.

Contents

1. Introduction

Portfolio optimization is a major challenge for investors and involves the art of allocating capital efficiently to maximize returns while minimizing risk. Investors have traditionally measured the performance of operations using the same objective metrics as the Sharpe or Treynor ratio. However, these methods fail to capture the differences between different objectives and often ignore the trade-off between reward and risk.

In order to overcome these limitations, many optimization methods have emerged and opened the way for various methods. Such strategies acknowledge the fact that traders often struggle to achieve multiple goals that need to be optimized simultaneously.

In particular, deviation from the mean is an essential part of optimization, providing a system designed to balance return maximization and the risk of being less fortunate.

In this study, we explore the complexity of portfolio optimization through a multi-objective lens, with a particular focus on the mean-variance method. We explore the application of this method to solve problems arising from conflicting objectives and evaluate its effectiveness in creating portfolios that act as multiple investors. By juxtaposing the mean-variance approach with other popular multi-objective approaches, we aim to demonstrate its strengths, limitations, and potential reliability, thereby illuminating its importance in today's investment strategy.

2. MOV Methods

2.1 Multi-Objective Variants of MVO

I. **Multi-Objective Mean-Variance Optimization (MOMVO):** MMVO extends MVO by incorporating additional objectives, such as achieving a certain level of portfolio liquidity and considering alternative risk measures beyond diversification. This approach aims to create portfolios that not only optimize risk and return, but also take into account other related objectives and allow investors to customize portfolio selection based on their preferences and constraints.

II. **Constraint-based multi-objective optimization:** In this approach, constraints are introduced to represent different objectives to be met. For example, investors may have target minimum returns, maximum acceptable risk levels, and exposure limits. Constraint-based multi-objective optimization attempts to find a set that satisfies these objectives while satisfying the defined constraints.

III. **Multi-objective programming:** Multi-objective programming assigns different levels of priority to each goal and creates a hierarchy of goals. The optimization process then tries to minimize deviations from these goals by considering their relative importance. This approach provides the flexibility to accommodate variable importance that may have different objectives for investors.

IV. **Pareto-based multi-objective optimization**: This approach aims to find portfolios that lie on the Pareto frontier. The Pareto frontier represents a set of non-dominated solutions where other portfolios cannot improve an objective without escalating it. Rather than aggregating objectives into a single scalar value, the goal is to produce a set of solutions that enable investors to make informed decisions based on trade-offs between multiple objectives.

V. **Multi-objective evolutionary algorithms:** Evolutionary algorithms such as genetic algorithms and particle swarm optimization can be adapted for multi-objective portfolio optimization. These algorithms explore the solution space and generate a set of optimal or near-optimal portfolios that cover diverse objectives and provide investors with a wide range of options.

VI. **Risk Parity with Multiple Objectives**: While Risk Parity focuses on balancing risks across assets, this approach can be extended to include additional objectives. For example, an investor might aim to achieve a certain level of income or minimize exposure to specific sectors. Integrating multiple objectives into the risk parity framework creates portfolios that align with various investor preferences.

2.2 Mean-Variance Optimization (MVO)

In the field of portfolio optimization, the mean-variance method has traditionally played a major role. This method, pioneered by Harry Markowitz, presents a quantitative framework for evaluating portfolios based on two basic parameters: the mean (expected return) and the variance of portfolio returns (risk). By optimizing the trade-off between return and risk, the mean-variance method attempts to identify the portfolio that offers the highest possible return for a given level of risk or the lowest risk for a given level of return.

The core of the mean-variance approach lies in the construction of an efficient frontier—a graphical representation showing the optimal balance between risk and return for different portfolios. An efficient frontier portfolio is one that cannot be surpassed in terms of return for a given level of risk, or vice versa. The concept of diversification plays a crucial role in this method, as it shows how combining assets with uncorrelated or negatively correlated returns can reduce portfolio risk without compromising potential returns.

However, while the mean-variance method has been a cornerstone of portfolio optimization for decades, it inherently assumes that returns follow a normal distribution and that investor preferences are defined only by risk and return. This simplification may not capture the full spectrum of real-world investor behavior and preferences, which may encompass multiple objectives beyond risk and return. This is where the integration of multi-objective optimization principles comes into play, augmenting the mean-variance method to account for the complexities of different investors' goals.

In subsequent sections of this report, we delve deeper into the mechanics of the mean-variance method, exploring its mathematical basis, its use in constructing efficient portfolios, and its inherent strengths and limitations. Moreover, we extend our inquiry to include a comparative analysis of the mean-variance approach with other multi-objective techniques, examining their relative implications for portfolio optimization in the presence of multi-objective objectives.

2.2.2 Exploring Markowitz Portfolio Theory

The Markowitz Portfolio Theory, developed by Harry Markowitz in the 1950s, is a key concept that has changed the way investors construct investment portfolios in the modern financial industry. He takes a systematic approach to balancing risk and return through diversification. A Study of Markowitz's Portfolio Theory:

1. Diversification and Risk Trading:
Markowitz's Portfolio Theory is based on the principle that investors can construct portfolios to achieve desired returns by managing risk. Diversification, the practice of holding different assets, is important. By combining unrelated assets, an investor can reduce portfolio risk without sacrificing potential returns.

2. effective limit:
A central concept of the theory is the optimal limit, which represents the optimal portfolio that offers the highest expected return for a certain level of risk or the lowest risk for a certain level of return. Portfolios in the efficient frontier are considered "efficient" because they increase the return for the level of risk or reduce the risk for a certain level of return.

3. Risk and Return Measures:
Markowitz introduced basic criteria for determining risk and return.
These include:

Estimated Return: The average return that an investor can expect from an investment.
Variance: The average spread of assets. Changes represent high risk.
Covariance: A measure of how two assets move relative to each other. A positive covariance indicates that they tend to move together, while a negative covariance indicates that they tend to move in opposite directions.
Correlation: A standard measure of the relationship between two assets, ranging from -1 (perfectly negative correlation) to 1 (perfectly positive correlation).

4. Build an effective portfolio:
According to Markowitz, investors should look for portfolios on the efficient frontier. This portfolio represents the best risk trading return. To build an effective portfolio, investors must find an asset allocation that maximizes return for a given level of risk or minimizes risk for a given level of return.

5. Average variable optimization problem:

At the core of Markowitz's Portfolio Theory is a variable optimization problem. This includes finding a combination of assets that minimizes portfolio volatility (risk) while maximizing expected returns. This optimization problem is usually solved using mathematical methods.

6. Limitations and Assumptions:
Markowitz's Portfolio Theory has several limitations when it comes to revolutions:

It assumes that income is normally distributed and cannot be saved in all cases.
This relies on historical data that cannot accurately predict future market conditions.
This theory does not consider transaction costs, taxes, or real-world constraints.

7. Modern Portfolio Theory (MPT):
Markowitz's work became the basis for modern portfolio theory (MPT), which includes concepts such as risk-free assets, capital market speculation, and systematic risk (beta).

8. Program and Impact:
Markowitz Portfolio Theory has had a huge impact on investment management. It creates a consistent basis for investors to diversify and balance risk and return. It laid the foundation for more sophisticated portfolio optimization techniques and paved the way for the development of derivatives, index funds and other investment products.

2.2.2 Risk-Return Trade-off

Risk-return trade-off is a fundamental concept in finance that shows the relationship between the reward (income) that an investor can expect from an investment and the risk associated with that investment. This trade-off is a key consideration when making decisions about investment allocation between different assets or investment opportunities.
Risk refers to the uncertainty or variability of actual returns compared to expectations. Equity includes the possibility of losing some or all of the invested capital and is usually measured by metrics such as standard deviation or variance.
Return refers to the profit or loss received from an investment over a period of time. This can be defined as a percentage increase or decrease in the value of an investment.

The risk-return trade-off can be summarized as follows:

1. **Higher risk, higher potential returns:** In general, higher risk investments offer higher potential. This relationship reflects the willingness of investors to accept increased uncertainty. High-risk investments can experience significant price swings, but can generate large returns in favorable market conditions.

2. **Low risk, low return potential:** In contrast, investments with a low level of risk usually offer low potential returns. These investments are generally considered more stable and preferred by investors who want to save capital and reduce the possibility of significant losses.

3. **Balancing risk and return:** A major challenge for investors to find the optimal balance between risk and return that matches their investment goals, risk tolerance and time horizon. This balance varies from investor to investor and depends on factors such as the individual's financial situation, investment objectives and personal risk appetite.

The risk-return trade-off concept plays a key role in portfolio construction and asset allocation. Diversification, or spreading investments across different asset classes, industries, and geographies, is a strategy used to manage risk with the goal of achieving profitable returns. By combining assets with different risk profiles,

investors can achieve better performance for their portfolios.
It should be noted that risk-return trading is not an absolute rule, there are exceptions and nuances. Some Investments can offer high returns compared to low risk and vice versa. In addition, investors' perception of risk and risk appetite may change, influencing decisions and structuring portfolios.

2.2.3 Scalarizing Functions

Scalarizing functions are fundamental tools in multi-objective optimization, including portfolio optimization. These functions convert multiple conflicting objectives into a single scalar value, allowing multi-objective problems to be solved using standard single-objective optimization techniques. In the context of portfolio optimization, scalarizing functions enable investors to balance different objectives, such as maximizing return while minimizing risk or achieving diversification.
Here's a detailed description of scalarizing functions and their relevance to portfolio optimization:
Scalarizing Functions in Portfolio Optimization:
In portfolio optimization, scalarizing functions play a crucial role in handling the trade-offs between various objectives, such as maximizing returns and minimizing risk. Instead of dealing with a complex multi-dimensional optimization problem involving multiple objectives simultaneously, scalarizing functions map these objectives to a single numerical value. This transformed single-objective problem can then be solved using well-established optimization algorithms.
Mathematical Equations:

1. **Weighted Sum Approach:**

The weighted sum approach is one of the simplest scalarizing functions. It combines the multiple objectives using weighting factors to create a single aggregated objective function:

Aggregated Objective=w1·Objective1+w2·Objective2+…+wn·Objectiven
Here, w1,w2,…,wn are the weighting factors assigned to each objective, and Objective1,Objective2,… ,…,Objective n are the individual objectives. By adjusting the weights, investors can emphasize or de-emphasize specific objectives according to their preferences.

2. **Tchebycheff Approach:**

The Tchebycheff scalarizing function aims to find a compromise solution that is not too far from the objectives' ideal values and minimizes the weighted deviation from these values:
Aggregated Objective=max{w1·(Objective1−Ideal1),w2·(Objective2−Ideal2),…,wn·(Objectiven−Idealn)}
Here, Ideal1,Ideal2,…,Ideal n represent the ideal or target values for each objective, and w1,w2,…,wn are the corresponding weighting factors.

3. **ε-Constraint Method:**

The ε-constraint method transforms the multi-objective problem into a series of single-objective problems by introducing constraints on the objectives. For instance, in a portfolio optimization context:
Maximize Return subject to Risk≤Maximize Return subject to Risk≤ϵ
Here, ϵ is a predefined tolerance level for risk. Solving this problem for different values of ϵ generates a set of efficient portfolios representing different trade-offs between risk and return.

3. Mathematical Intuitions

Let P portfolio consist of assets W1, W2, ... WN and μ1, μ2, ... μN, with asset returns A1, A2, ... AN. The portfolio return, determined by the weighted sum of individual asset returns,

$$r = W_1.\mu_1 + W_2.\mu_2 + ...W_N.\mu_N = \sum_{i=1}^{N} W_i.\mu_i$$

Portfolio risk is the standard deviation of its returns and is given by,

$$\sqrt{\sum_i \sum_j W_i.W_j.\sigma_{ij}}$$

Where σi, j is the variance-covariance matrix of returns.

The Mean-Variance Optimization model is given by:

$$\begin{aligned}
&max\left(\sum_{i=1}^{N} W_i.\mu_i\right)\\
&min\left(\sqrt{\sum_i \sum_j W_i.W_j.\sigma_{ij}}\right)\\
&\text{subject to}\\
&\sum_{i=1}^{N} W_i = 1\\
&0 \leq W_i \leq 1
\end{aligned}$$

The model operates on two objective functions,

(i) increase the expected return of the portfolio

(ii) reduce portfolio risk

Therefore, the objective is to solve the nonlinear objective optimization problem.

The MVO model has major limitations

(i) Total weight is 1.

This means that the investor's capital is fully invested in the portfolio and in such cases the portfolio is called a fully invested portfolio.

(ii) Wi weighs between 0 and 1.

This means that no capital (Wi = 0) can be provided for investment, or that all capital cannot be invested in assets (Wi = 1), or capital that can be provided for potential weight among the assets in the portfolio. interval (0, 1).

4. Solving the MVO Model

a. Obtaining the Maximal Expected Return of the Portfolio

While there are many ways to solve a bi-objective optimization problem, we choose to solve it by decomposing the problem into two three sub-problem models, viz.

(i) obtaining the maximum expected return RMaxRetrn from the portfolio, subject to fundamental constraints,

(ii) obtaining the best expected return RMinRisk corresponding to the minimum risk portfolio, subject to fundamental constraints, and finally,

(iii) Finding optimal weight portfolio sets that minimize risk and whose returns are R RMinRisk and RMaxRetrn, (ie) RMinRisk ≤ R ≤ RMaxRetrn, subject to fundamental constraints.

The optimal portfolio set is known as the efficient set.

The mathematical model for this sub-problem is given by,

$$
\begin{aligned}
& max \left(\sum_{i=1}^{N} W_i . \mu_i \right) \\
& \text{subject to} \\
& \sum_{i=1}^{N} W_i = 1 \\
& 0 \le W_i \le 1
\end{aligned}
$$

The optimal weights W (Optimal ’i) is used to compute RMaxRetrn=∑ (W (Optimal ‘i).µi)

b. Obtaining the Optimal Expected Return of a Minimum Risk Portfolio

The mathematical model for this sub-problem is given by,

$$
\begin{aligned}
& min\left(\sqrt{\sum_i \sum_j W_i.W_j.\sigma_{ij}}\right) \\
& \text{subject to} \\
& \sum_{i=1}^{N} W_i = 1 \\
& 0 \leq W_i \leq 1
\end{aligned}
$$

The optimal weights W (Optimal ‘i) is used to compute RMinRisk = ∑ (WOptimal ‘i.µi)

c. Obtaining the Optimal Weights for Minimum Risk and Maximum Return Portfolios

The mathematical model for this sub-problem is defined as follows,

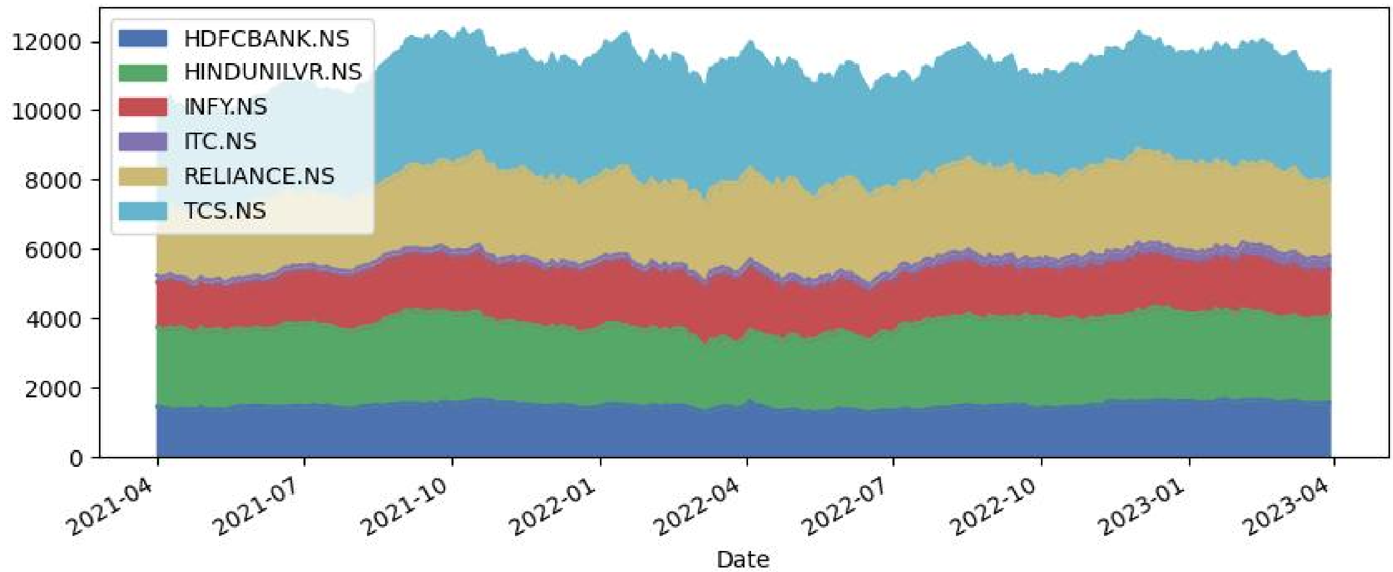
$$
\begin{aligned}
& min\left(\sqrt{\sum_i \sum_j W_i.W_j.\sigma_{ij}}\right) \\
& \text{subject to} \\
& \left(\sum_{i=1}^{N} W_i.\mu_i\right) \leq R \\
& \sum_{i=1}^{N} W_i = 1 \\
& 0 \leq W_i \leq 1
\end{aligned}
$$

where for each R, RMinRisk ≤ R ≤ RMaxRetrn, the problem model is solved iteratively to arrive at the optimal weight set, each of which determines the portfolio that minimizes risk and maximizes return.

5. Algorithm

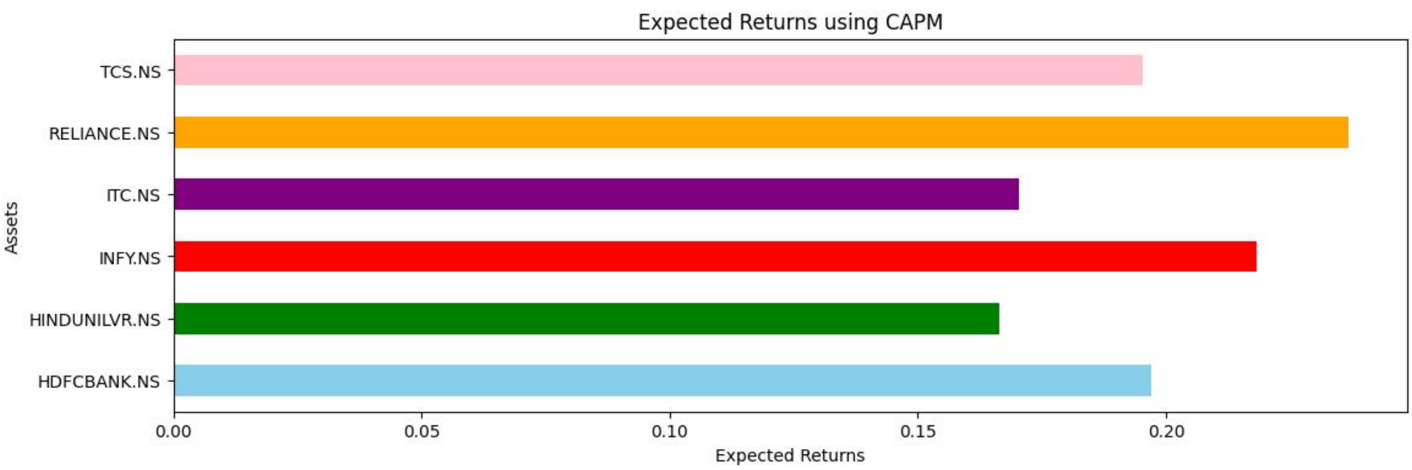
1. **Import Required Libraries:**
    - Import necessary libraries including scipy.optimize, numpy, and pandas.
2. **Define Function to Compute Asset Returns:**
    - Execute the compute_asset_returns function (stock_price, row and column) which calculates the daily asset returns based on the given stock data.
3. **Read and Process Input Data:**
    - Set the database file name of the login folder.
    - Determine the number of rows and columns in the database (except the header).
    - Read the CSV file using pd.read_csv() to extract active tags and values.
    - Convert asset price data to multiple arrays.
    - Calculate return on assets using predefined functions.
4. **Calculate Mean Returns and Covariance Matrix:**
    - Compute the mean returns for each asset using np.mean ().
    - Calculate the covariance matrix of returns using np.cov ().
5. **Maximize Returns Portfolio:**
    - Define the function maximize_returns (average_returns, portfolio_size) to find the weight of the portfolio that maximizes the expected return.
    - Create a minimization problem by denying middle income.
    - constraint matrix A and constraint vector b.
    - Perform linear programming optimization using scipy.optimize.linprog().
6. **Minimize Risk Portfolio:**
    - Execute the minim_risk function (covar_returns, portfolio_size) to find the portfolio weight that minimizes the portfolio risk.
    - Define the objective function as the squared risk using the objective function ().
    - Specify the equality constraint using constraint_eq() to ensure that the weight sums to 1.
7. **Minimize Risk and Maximize Return Portfolio:**
    - Execute the minim_risk_max_return (MeanReturns, CovarReturns, PortfolioSize, Rang R) function to find the portfolio that minimizes the desired minimum return risk.
    - Define inequality constraints using constraint_ineq() to satisfy the minimum return requirement.
    - Optimize using scipy.optimize.minimize() with the trust-constr method.
8. **Compute Efficient Set:**
    - Determine the increment and start lower and upper limit for the expected return value.
    - Use the time cycle to find suitable portfolios for different expected returns.
    - Use the minim_risk_max_return () function to find effective portfolios.
    - Calculate the annualized risk and expected return of each effective portfolio.
9. **Display Results:**
    - Display the calculated average result and covariance matrix.
    - Calculate and display the maximum expected portfolio.
    - Calculate and show the expected return of the least risky portfolio.
    - Show the optimal weight, annual risk and expected return of the resulting portfolio.

6. Visualizing Stock Price Trends (Sample Stocks)



Expected returns using the Capital Asset Pricing Model (CAPM)

HDFCBANK.NS	0.196915
HINDUNILVR.NS	0.166532
INFY.NS	0.218349
ITC.NS	0.170311
RELIANCE.NS	0.236816
TCS.NS	0.195393



7. Python Code

```
# Function to obtain the Minimal Risk and Maximum Return portfolios
import numpy as np
from scipy import optimize

def minimize_risk_max_return(MeanReturns, CovarReturns, PortfolioSize, R):

    def objective_function(x, CovarReturns):
        func = np.matmul(np.matmul(x, CovarReturns), x.T)
        return func

    def constraint_eq(x):
        A_eq = np.ones(x.shape)
        b_eq = 1
        eq_constraint_val = np.matmul(A_eq, x.T) - b_eq
        return eq_constraint_val

    def constraint_ineq(x, MeanReturns, R):
        A_ineq = np.array(MeanReturns)
        b_ineq = R
        ineq_constraint_val = np.matmul(A_ineq, x.T) - b_ineq
        return ineq_constraint_val

    x_init = np.repeat(0.1, PortfolioSize)

    # Define the equality constraint function and the inequality constraint
function
    cons = ({'type': 'eq', 'fun': constraint_eq},
            {'type': 'ineq', 'fun': constraint_ineq, 'args': (MeanReturns, R)})

    lb = 0
    ub = 1
    bnds = tuple([(lb, ub) for x in x_init])

    # Optimize the portfolio to find both Minimal Risk and Maximum Return
portfolios
    opt = optimize.minimize(objective_function, args=(CovarReturns),
method='trust-constr', \
                            x0=x_init, bounds=bnds, constraints=cons, tol=10**-3)

    return opt
```

```
# Function to compute asset returns
import numpy as np
import pandas as pd

def compute_asset_returns(stock_price, rows, columns):
    stock_return = np.zeros([rows - 1, columns])
```

```
    for j in range(columns):  # j: Assets
        for i in range(rows - 1):  # i: Daily Prices
            stock_return[i, j] = ((stock_price[i + 1, j] - stock_price[i, j]) /
stock_price[i, j]) * 100

    return stock_return

# Input k-portfolio 1 dataset comprising 6 stocks
StockFileName = 'portfolio.csv'

Rows = 1237  # excluding header
Columns = 6  # excluding date
portfolioSize = Columns  # set portfolio size

# Read stock prices into a dataframe
df = pd.read_csv(StockFileName,  nrows=Rows)

# Extract asset labels
assetLabels = df.columns[1:Columns+1].tolist()
print(assetLabels)

print('\n')

# Extract asset prices
StockData = df.iloc[0:, 1:]

# Compute asset returns
arStockPrices = np.asarray(StockData)
[Rows, Cols] = arStockPrices.shape
arReturns = compute_asset_returns(arStockPrices, Rows, Cols)

# Compute mean returns and variance-covariance matrix of returns
meanReturns = np.mean(arReturns, axis=0)
covReturns = np.cov(arReturns, rowvar=False)

# Set precision for printing results
np.set_printoptions(precision=3, suppress=True)

# Display mean returns and variance-covariance matrix of returns
print('Mean returns of assets in k-portfolio 1\n', meanReturns)

print('\n')

print('Variance-Covariance matrix of returns\n', covReturns)
```

```
# Create an EfficientFrontier object with the expected returns (mu) and
covariance matrix (S)
efficient_frontier = EfficientFrontier(expected_returns_capm, cov_matrix)
```

```python
# Find the maximum Sharpe ratio portfolio weights
weights_max_sharpe = efficient_frontier.max_sharpe()

# Clean the portfolio weights for better representation
cleaned_weights = efficient_frontier.clean_weights()
print(dict(cleaned_weights))

# Display the portfolio performance metrics
efficient_frontier.portfolio_performance(verbose=True)
```

```python
# Define the number of samples
n_samples = 10000

# Generate random portfolio weights using the Dirichlet distribution
weights = np.random.dirichlet(np.ones(len(expected_returns_capm)), n_samples)

# Calculate the sample portfolio returns
portfolio_returns = weights.dot(expected_returns_capm)

# Calculate the sample portfolio volatilities (standard deviations)
portfolio_volatilities = np.sqrt((weights.T * (cov_matrix @
weights.T)).sum(axis=0))

# Calculate the Sharpe ratios for the sample portfolios
sharpes = portfolio_returns / portfolio_volatilities

# Print the results
print("Sample portfolio returns:", portfolio_returns)
print("Sample portfolio volatilities:", portfolio_volatilities)
```

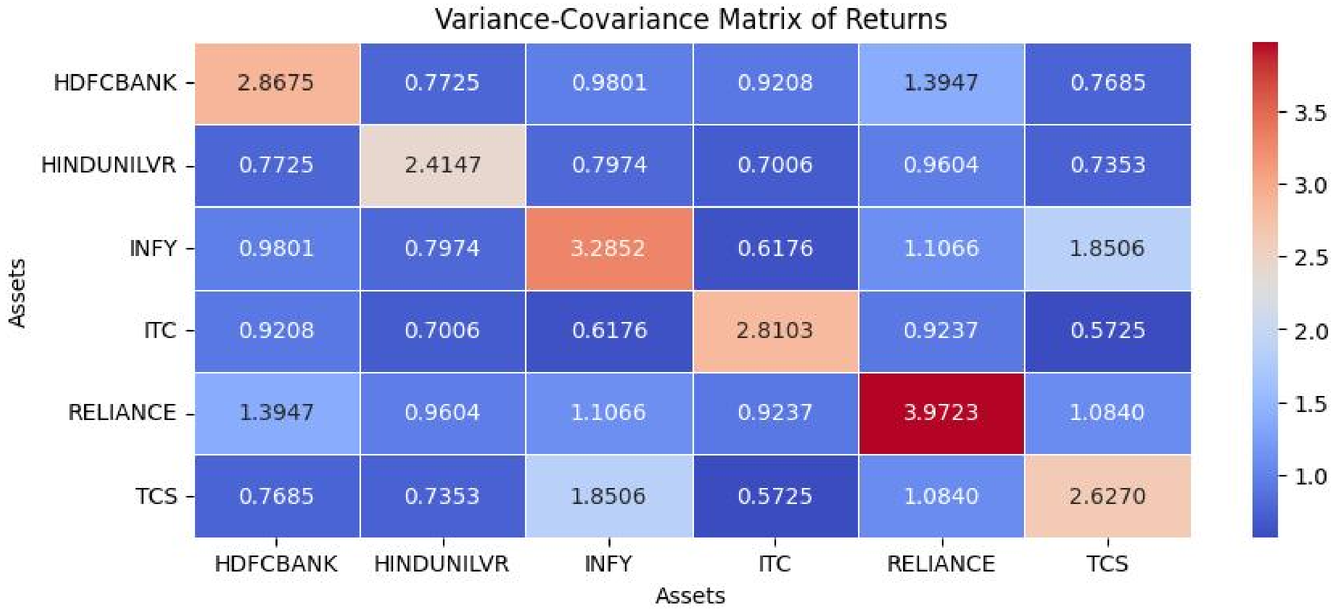
```python
# Compute the maximal expected portfolio return for the k-portfolio
result1 = maximize_returns(meanReturns, portfolioSize)
max_return_weights = result1.x
max_exp_portfolio_return = np.matmul(meanReturns.T, max_return_weights)

# Print the maximal expected portfolio return
print("Maximal Expected Portfolio Return:   %7.4f" % max_exp_portfolio_return)
```

```python
#expected portfolio return computation for the minimum risk k-portfolio
result2 = minimize_risk(covReturns, portfolioSize)
minRiskWeights = result2.x
minRiskExpPortfolioReturn = np.matmul(meanReturns.T, minRiskWeights)

# Print the Expected Return of Minimum Risk Portfolio
print("Expected Return of Minimum Risk Portfolio:  %7.4f" %
minRiskExpPortfolioReturn)
```

8. Variance-Covariance Matrix



9. Output/Results And Building the Efficient Frontier

Optimal weights of the efficient set portfolios:

[[0.155 0.263 0.090 0.223 0.065 0.205]
[0.152 0.262 0.093 0.221 0.067 0.204]
[0.138 0.260 0.103 0.220 0.074 0.204]
[0.124 0.256 0.119 0.212 0.087 0.201]
[0.110 0.249 0.136 0.207 0.099 0.200]
[0.090 0.242 0.153 0.199 0.117 0.198]
[0.077 0.239 0.165 0.197 0.123 0.199]
[0.064 0.233 0.183 0.185 0.142 0.193]
[0.052 0.229 0.195 0.180 0.152 0.191]
[0.036 0.223 0.211 0.175 0.165 0.190]
[0.018 0.220 0.225 0.170 0.177 0.19]
[0.036 0.204 0.253 0.136 0.197 0.175]
[0.009 0.196 0.262 0.140 0.210 0.183]
[0.009 0.182 0.283 0.127 0.225 0.173]
[0.005 0.172 0.304 0.115 0.237 0.167]
[0.005 0.158 0.324 0.100 0.252 0.162]
[0.005 0.143 0.343 0.085 0.266 0.159]
[0.004 0.129 0.363 0.071 0.280 0.152]
[0.004 0.121 0.375 0.062 0.289 0.149]
[0.004 0.114 0.400 0.035 0.306 0.141]

[0.004 0.105 0.427 0.021 0.322 0.121]
[0.000 0.086 0.446 0.017 0.333 0.118]
[0.000 0.068 0.472 0.009 0.348 0.103]
[0.000 0.035 0.507 0.020 0.368 0.070]
[0.003 0.023 0.536 0.006 0.382 0.051]
[0.001 0.021 0.541 0.003 0.386 0.047]
[0.002 0.004 0.582 0.004 0.399 0.009]]

Annualized Risk and Return of the efficient set portfolios:
[[17.385 18.099]
[17.389 18.350]
[17.412 18.601]
[17.463 18.852]
[17.535 19.103]
[17.658 19.354]
[17.739 19.605]
[17.898 19.856]
[18.021 20.107]
[18.189 20.358]
[18.375 20.609]
[18.647 20.860]
[18.863 21.111]
[19.125 21.362]
[19.405 21.613]
[19.714 21.864]
[20.047 22.115]
[20.401 22.366]
[20.632 22.617]
[21.127 22.868]
[21.552 23.119]
[21.919 23.370]
[22.369 23.621]
[22.873 23.872]
[23.379 24.123]
[23.511 24.374]
[24.052 24.625]]

The resulting set includes a collection of 27 different optimal packages, each tailored to meet the pre-set constraints. This convenient weight configuration exactly matches the intended requirements. In addition, key information such as annualized risk (%) and return (%) for each optimal portfolio is available to investors, facilitating well-informed decisions about portfolio strategies.

Given the investor's goals, consider a risk tolerant individual who seeks an annual return of y% regardless of risk. In this scenario, the efficient set allows the selection of a portfolio, where the risk-return coordinate is equal to [x%, y%] and x% represents the level of risk selected. In addition, the appropriate optimal weight allocation provides precise guidance on how to allocate capital among various assets in the k-portfolio to achieve the desired returns.

On the other hand, imagine a risk-averse investor who puts a lot of emphasis on risk management. By choosing a fixed risk value of x%, this investor can get valuable insight into the appropriate achievable return (y%) for the level of risk. In addition, an efficient set shows the optimal weight distribution required to implement such investments.

In fact, an efficient portfolio provides optimal investment options that meet different levels of risk appetite among investors.

example:

To simplify matters, let's consider a risk-averse investor who wants to invest in the aforementioned portfolio k, which consists of Dow stocks aimed at reducing risk.

Among the options in the resulting set, the initial entry [17.385 18.099] offers the lowest annual risk of 17.385%. If an investor meets this risk profile, an annual return of 18.099% is a realistic expectation. Simplifying this result is the optimal weight distribution as follows: [0.155| 0.263 | 0.090 | 0.223 | 0.065 | 0.205].

To explore further, if an investor has a capital of 500,000 rs, seeking an annual expected return of 18.099% while maintaining an annual risk of 17.385%, the strategic capital allocation for assets in the k-portfolio takes the following structure:

['HDFCBANK', 15.5%], ['HINDUNILVR', 26.3%], ['INFY', 9.0%], ['ITC', 22.3%],

['RELIANCE', 6.5%], ['TCS', 20.5%].

10.Conclusion and Scope of Future Research

Conclusion:

Portfolio optimization is an important problem in the field of finance, as it involves finding the optimal allocation of assets in a portfolio to maximize returns and minimize risk. In this project, we explored the use of multi-objective optimization techniques for portfolio optimization. We find that these techniques can effectively handle the trade-off between maximizing returns and minimizing risk, and can also handle multiple objectives simultaneously.

Scope of Future Research:

There are several directions in which this research can be extended in the future. One possibility is to explore the use of more advanced multi-objective optimization algorithms such as evolutionary algorithms to further improve the performance of the portfolio optimization process. Another possibility is to consider additional objectives beyond risk and return, such as liquidity or sustainability. Another possibility is to include additional constraints, such as limits on the number of assets or sector exposure. Another possibility is to investigate the use of alternative risk measures, such as value at risk, in the optimization process. In addition, it may be interesting to examine the effect of different types of constraints, such as constraints on the amount of specific assets that can be included in the portfolio, on the optimization process.

In addition, it would be interesting to investigate the robustness of the proposed approach to different market conditions and different types of assets. Finally, it would be worthwhile to explore the use of different multi-objective optimization algorithms and compare their performance.

References

a. "Portfolio Optimization" by R.A. Litterman and K.F. Janes. This book provides a **thorough treatment of portfolio** optimization techniques, including mean-variance optimization, Black-Litterman model, and risk budgeting.

b. "Modern Portfolio Theory and Investment Analysis" by E.J. Elton, M.J. Gruber, S.J. Brown, and W.N. Goetzmann. This book is a classic text on portfolio theory and provides a comprehensiveoverview of the subject.

c. "Portfolio Management: A Modern Approach" by P.A. Vora. This book provides a practical guideto portfolio management, including chapters on risk management, portfolio construction, and performance evaluation.

d. "Mean-Variance Analysis in Portfolio Choice and Capital Markets" by T.M. Cover. This bookprovides a detailed treatment of mean-variance optimization and its applications in portfolio selection and asset pricing.

e. "The Mathematics of Financial Modeling and Investment Management" by S.H. Neftci. This bookcovers a wide range of topics in financial mathematics, including portfolio optimization, risk measures, and derivative pricing.

f. "Multi-Objective Optimization in Financial Engineering" by Kaisa M. Miettinen

g. "Multi-Objective Portfolio Optimization" by Edgar E. Sanchez and Robert P. Wichern

h. "Multi-Objective Portfolio Selection Using Evolutionary Algorithms" by A. O. Abdusalam and M. A. Abdusalam

i. "Multi-Objective Optimization in Financial Markets" by Isaac K. K. Fong and H. Edwin Romeijn