RINGS

Dr. E.SURESH BABU

Assistant Professor

Computer Science and Engineering Department

National Institute of Technology, Warangal

Warangal

RINGS

Rings

- A **Ring** is typically denoted **{R,+,×}**
  - ✓ where **R denotes** the set of objects,
  - ✓ **'+' denotes** the operator with respect to R which is an **abelian group**
  - ✓ **'×' denotes** the additional operator needed for R to form a **ring.**

Rings

- If we can define **one more operation** on an **abelian group**, we have a **ring,** provided the **elements of the set** satisfy some **properties with respect to this new operation also.**

Properties of the Elements with Respect to the Ring Operator

- R must be **closed** with respect to the **additional operator ’×’**.
- R must exhibit **associativity** with respect to the **additional operator ‘×’**.
- The additional operator (that is, the “multiplication operator”) must **distribute over the group addition operator ’×’**. That is

$$\mathbf{a \times (b + c) = a \times b + a \times c}$$

$$\mathbf{(a + b) \times c = a \times c + b \times c}$$

Properties of the Elements with Respect to the Ring Operator

- The **"multiplication" operation** is frequently shown by just **concatenation** in such equations:

$$a(b + c) = ab + ac$$

$$(a + b)c = ac + bc$$

Commutative Rings

Commutative Rings

- A ring is commutative if the multiplication operation is commutative for **all elements in the ring**. That is, if all a and b in R satisfy the property

$$ab = ba$$

- if **multiplication operation is commutative,** it forms a **commutative ring**

Integral Domain

Integral Domain

- if multiplication operation has an **identity and no zero divisors**, it forms an integral domain
- An integral domain is a commutative ring with an identity (1) with no zero-divisors.

Integral Domain

- An integral domain {**R,+,×**} is a commutative ring that obeys the following two additional properties:

1. **Additional Property-1:** The **set R** must include **an identity element** for the multiplicative operation. **a1 = 1a = a**

2. **Additional Property-2:** Let **0 denote** the **identity element for the addition operation.** If a multiplication of any two elements **a and b of R** results in 0, that is if

   ab = 0 then **either a or b must be 0.**

Thank U