SET – 2

Series : BVM/1

कोड नं.
Code No. **65/1/2**

रोल नं.
Roll No. | | | | | | |

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **11** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **29** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **29** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे
Time allowed : 3 hours

अधिकतम अंक : 100
Maximum Marks : 100

सामान्य निर्देश :

(i) **सभी** प्रश्न अनिवार्य हैं ।

(ii) इस प्रश्न–पत्र में **29** प्रश्न हैं जो चार खण्डों में विभाजित हैं : अ, ब, स तथा द । खण्ड अ में **4** प्रश्न हैं जिनमें से प्रत्येक **एक अंक** का है । खण्ड ब में **8** प्रश्न हैं जिनमें से प्रत्येक **दो अंक** का है । खण्ड स में **11** प्रश्न हैं जिनमें से प्रत्येक **चार अंक** का है । खण्ड द में **6** प्रश्न हैं जिनमें से प्रत्येक **छः अंक** का है ।

(iii) खण्ड अ में सभी प्रश्नों के उत्तर एक शब्द, एक वाक्य अथवा प्रश्न की आवश्यकतानुसार दिए जा सकते हैं ।

(iv) पूर्ण प्रश्न–पत्र में विकल्प नहीं हैं । फिर भी खण्ड अ के **1** प्रश्न, खण्ड ब के **3** प्रश्नों में, खण्ड स के **3** प्रश्नों में तथा खण्ड द के **3** प्रश्नों में आंतरिक विकल्प हैं । ऐसे सभी प्रश्नों में से आपको **एक** ही विकल्प हल करना है ।

(v) कैलकुलेटर के प्रयोग की अनुमति **नहीं** है । यदि आवश्यक हो, तो आप लघुगणकीय सारणियाँ माँग सकते हैं ।

General Instructions :

(i) ***All*** *questions are compulsory.*

(ii) *This question paper contains* ***29*** *questions divided into four sections A, B, C and D. Section A comprises of 4 questions of* ***one mark*** *each, Section B comprises of 8 questions of* ***two marks*** *each, Section C comprises of 11 questions of* ***four marks*** *each and Section D comprises of 6 questions of* ***six marks*** *each.*

(iii) *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*

(iv) *There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only* ***one*** *of the alternatives in all such questions.*

(v) *Use of calculators is not permitted. You may ask logarithmic tables, if required.*

खण्ड – अ
SECTION – A

प्रश्न संख्या **1** से **4** तक के प्रत्येक प्रश्न **1** अंक का है ।

Question numbers **1** to **4** carry **1** mark each.

1. अवकल समीकरण $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$ की कोटि व घात ज्ञात कीजिए ।

Find the order and the degree of the differential equation $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$.

2. यदि $f(x) = x + 7$ और $g(x) = x - 7, x \in R$ हों, तो $\frac{d}{dx}(fog)(x)$ ज्ञात कीजिए।

If $f(x) = x + 7$ and $g(x) = x - 7, x \in R$, then find $\frac{d}{dx}(fog)(x)$.

3. यदि $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ है, तो $x - y$ का मान ज्ञात कीजिए।

Find the value of $x - y$, if

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}.$$

4. यदि एक रेखा x-अक्ष, y-अक्ष तथा z-अक्ष से क्रमश: 90°, 135°, 45° के कोण बनाती है। इस रेखा के दिक्-कोसाइन ज्ञात कीजिए।

अथवा

उस रेखा का सदिश समीकरण ज्ञात कीजिए जो बिन्दु (3, 4, 5) से गुजरती है तथा सदिश $2\hat{i} + 2\hat{j} - 3\hat{k}$ के समांतर है।

If a line makes angles 90°, 135°, 45° with the x, y and z axes respectively, find its direction cosines.

OR

Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.

खण्ड – ब

SECTION – B

प्रश्न संख्या **5** से **12** तक के प्रत्येक प्रश्न **2** अंक के हैं।

Question numbers **5** to **12** carry **2** marks each.

5. जाँच कीजिए कि क्या संक्रिया * जो R पर a * b = ab + 1 द्वारा परिभाषित है (i) द्वि–आधारी संक्रिया होगी या नहीं (ii) यदि यह द्वि–आधारी है, तो क्या यह साहचर्य होगी या नहीं ?

Examine whether the operation * defined on R by a * b = ab + 1 is (i) a binary or not. (ii) if a binary operation, is it associative or not ?

6. यदि $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ है, तो $(A^2 - 5A)$ ज्ञात कीजिए।

If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find $(A^2 - 5A)$.

7. ज्ञात कीजिए : $\int \sqrt{1 - \sin 2x}\, dx,\ \frac{\pi}{4} < x < \frac{\pi}{2}$

अथवा

ज्ञात कीजिए : $\int \sin^{-1}(2x)\, dx$.

Find : $\int \sqrt{1 - \sin 2x}\, dx,\ \frac{\pi}{4} < x < \frac{\pi}{2}$

OR

Find : $\int \sin^{-1}(2x)\, dx$.

8. वक्रों के कुल $y = e^{2x}(a + bx)$, जिसमें a, b स्वेच्छ अचर हैं, को निरूपित करने वाला अवकल समीकरण ज्ञात कीजिए।

Form the differential equation representing the family of curves $y = e^{2x}(a + bx)$, where 'a' and 'b' are arbitrary constants.

9. एक पासे को छः बार उछाला जाता है। यदि "पासे पर विषम संख्या प्राप्त होना" एक सफलता है, तो (i) 5 सफलताएँ (ii) अधिकतम 5 सफलताएँ, की प्रायिकताएँ क्या-क्या होंगी ?

अथवा

एक यादृच्छिक चर X का प्रायिकता बंटन P(X) निम्न प्रकार से है, जहाँ 'k' कोई संख्या है :

$$P(X = x) = \begin{cases} k, & \text{यदि } x = 0 \\ 2k, & \text{यदि } x = 1 \\ 3k, & \text{यदि } x = 2 \\ 0, & \text{अन्यथा} \end{cases}$$

'k' का मान ज्ञात कीजिए।

A die is thrown 6 times. If "getting an odd number" is a "success", what is the probability of (i) 5 successes ? (ii) atmost 5 successes ?

OR

The random variable X has a probability distribution P(X) of the following form, where 'k' is some number.

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Determine the value of 'k'.

10. एक पाँसा जिस पर 1, 2, 3 लाल रंग से तथा 4, 5, 6 हरे रंग से लिखा गया है, को उछाला जाता है । "संख्या सम होने" की घटना को A से व "संख्या लाल रंग में लिखी है" की घटना B से परिभाषित है । ज्ञात कीजिए कि क्या ये दो घटनाएँ A तथा B स्वतंत्र हैं या नहीं ।

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event "number is even" and B be the event "number is marked red". Find whether the events A and B are independent or not.

11. यदि दो मात्रक सदिशों का योग एक मात्रक सदिश हो, तो सिद्ध कीजिए कि उन दो सदिशों के अन्तर का परिमाण $\sqrt{3}$ होगा ।

अथवा

यदि $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ तथा $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$ है, तो $[\vec{a}\ \vec{b}\ \vec{c}]$ ज्ञात कीजिए ।

If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

OR

If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, find $[\vec{a}\ \vec{b}\ \vec{c}]$.

12. ज्ञात कीजिए : $\int \frac{\tan^2 x \sec^2 x}{1 - \tan^6 x}\, dx$.

Find : $\int \frac{\tan^2 x \sec^2 x}{1 - \tan^6 x}\, dx$.

खण्ड – स

SECTION – C

प्रश्न संख्या **13** से **23** तक प्रत्येक प्रश्न के **4** अंक हैं ।

Question numbers **13** to **23** carry **4** marks each.

13. x के लिए हल कीजिए : $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$.

Solve for x : $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$.

14. यदि $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$ हो, तो दर्शाइए कि $\frac{dy}{dx} = \frac{x + y}{x - y}$.

अथवा

यदि $x^y - y^x = a^b$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए ।

If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x + y}{x - y}$.

OR

If $x^y - y^x = a^b$, find $\frac{dy}{dx}$.

15. ज्ञात कीजिए : $\int \frac{3x + 5}{x^2 + 3x - 18}\, dx$.

Find : $\int \frac{3x + 5}{x^2 + 3x - 18}\, dx$.

16. सिद्ध कीजिए कि $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$, अतः $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x}\, dx$ का मान ज्ञात कीजिए ।

Prove that $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$, hence evaluate $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x}\, dx$.

17. यदि $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$ तथा $\hat{i} - 6\hat{j} - \hat{k}$ क्रमशः बिन्दु A, B, C और D के स्थिति सदिश हों तो सरल रेखाओं AB तथा CD के बीच का कोण ज्ञात कीजिए । ज्ञात कीजिए कि क्या सदिश $\overrightarrow{AB}$ तथा $\overrightarrow{CD}$ संरेख हैं या नहीं ।

If $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 5\hat{j}$, $3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not.

18. सारणिकों के गुणधर्मों के प्रयोग से, निम्न को सिद्ध कीजिए :

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a).$$

Using properties of determinants, prove the following :

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a).$$

19. यदि $x = \cos t + \log \tan\left(\frac{t}{2}\right)$, $y = \sin t$, तो $t = \frac{\pi}{4}$ के लिए $\frac{d^2y}{dt^2}$ तथा $\frac{d^2y}{dx^2}$ के मान ज्ञात कीजिए ।

If $x = \cos t + \log \tan\left(\frac{t}{2}\right)$, $y = \sin t$, then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

20. दिखाइए कि समुच्चय $\mathbb{R}$ में $R = \{(a, b) : a \le b\}$ द्वारा परिभाषित संबंध R स्वतुल्य व संक्रामक है, परन्तु सममित नहीं है ।

अथवा

सिद्ध कीजिए कि फलन $f : N \to N$, $f(x) = x^2 + x + 1$, द्वारा परिभाषित है, एक एकैकी फलन है किंतु आच्छादक नहीं ।
फलन $f : N \to S$, जहाँ S फलन f का परिसर है, का प्रतिलोम भी ज्ञात कीजिए ।

Show that the relation R on $\mathbb{R}$ defined as $R = \{(a, b) : a \le b\}$, is reflexive, and transitive but not symmetric.

OR

Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.
Find inverse of $f : N \to S$, where S is range of f.

21. वक्र $y = \sqrt{3x - 2}$ की उस स्पर्श-रेखा का समीकरण ज्ञात कीजिए जो रेखा $4x - 2y + 5 = 0$ के समान्तर है । स्पर्श बिन्दु से वक्र पर बने अभिलंब का समीकरण भी ज्ञात कीजिए ।

Find the equation of tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$. Also, write the equation of normal to the curve at the point of contact.

22. अवकल समीकरण : $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$ को हल कीजिए, दिया गया है $y = 0$ यदि $x = 1$.

अथवा

अवकल समीकरण : $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ को हल कीजिए, दिया गया है $y(0) = 0$.

Solve the differential equation : $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$, given that $y = 0$ when $x = 1$.

OR

Solve the differential equation : $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$, subject to the initial condition $y(0) = 0$.

23. λ का वह मान ज्ञात कीजिए जिसके लिए निम्न रेखाएँ लम्बवत हैं :
$\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ तथा $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$, यह भी ज्ञात कीजिए कि क्या ये रेखाएँ परस्पर प्रतिच्छेद करती हैं या नहीं ।

Find the value of λ, so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not.

खण्ड – द

SECTION – D

प्रश्न संख्या **24** से **29** तक प्रत्येक प्रश्न के **6** अंक हैं ।

Question numbers **24** to **29** carry **6** marks each.

24. सिद्ध कीजिए कि एक r त्रिज्या के गोले के अन्तर्गत अधिकतम आयतन के लंब वृत्तीय शंकु की ऊँचाई $\frac{4r}{3}$ है । शंकु का अधिकतम आयतन भी ज्ञात कीजिए ।

Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also find the maximum volume of cone.

25. यदि $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ हो, तो A^{-1} ज्ञात कीजिए । अत: निम्न समीकरण निकाय को हल कीजिए :

$2x - 3y + 5z = 11;\ 3x + 2y - 4z = -5;\ x + y - 2z = -3.$

अथवा

प्रारंभिक संक्रियाओं के प्रयोग द्वारा निम्नलिखित आव्यूह का व्युत्क्रम ज्ञात कीजिए :

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Hence solve the following system of equations :

$2x - 3y + 5z = 11,\ 3x + 2y - 4z = -5,\ x + y - 2z = -3.$

OR

Obtain the inverse of the following matrix using elementary operations :

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

26. एक निर्माता के पास तीन कारीगर A, B तथा C हैं । कारीगर A 1% त्रुटिपूर्ण इकाइयों का उत्पादन करता है, कारीगर B 5% तथा कारीगर C 7% त्रुटिपूर्ण इकाइयों का उत्पादन करते हैं । A सिर्फ 50% समय ही उत्पादन करता है, B सिर्फ 30% समय व C सिर्फ 20% समय ही उत्पादन करते हैं । यदि कुल उत्पादन का एक ढेर बना लिया जाता है और उस ढेर से यादृच्छया निकाली गई एक इकाई त्रुटिपूर्ण हो, तो इस इकाई के A द्वारा बनाई गई होने की प्रायिकता ज्ञात कीजिए ।

A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A ?

27. बिन्दुओं (2, 2, – 1), (3, 4, 2) तथा (7, 0, 6) से गुजरने वाले समतल के सदिश व कार्तीय समीकरण ज्ञात कीजिए । अत: उस समतल का समीकरण ज्ञात कीजिए जो बिन्दु (4, 3, 1) से गुजरता है और ऊपर प्राप्त समतल के समान्तर है ।

अथवा

उस समतल का सदिश समीकरण ज्ञात कीजिए जो रेखा $\vec{r} = (\hat{i} + \hat{j}) + \lambda (\hat{i} + 2\hat{j} - \hat{k})$ तथा बिंदु (–1, 3, – 4) को अंतर्विष्ट करता है । इस समतल पर बिन्दु (2, 1, 4) से डाले गए लंब की दूरी भी ज्ञात कीजिए ।

Find the vector and Cartesian equations of the plane passing through the points (2, 2 –1), (3, 4, 2) and (7, 0, 6). Also find the vector equation of a plane passing through (4, 3, 1) and parallel to the plane obtained above.

OR

Find the vector equation of the plane that contains the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda (\hat{i} + 2\hat{j} - \hat{k})$ and the point (–1, 3, – 4). Also, find the length of the perpendicular drawn from the point (2, 1, 4) to the plane thus obtained.

28. समाकलन विधि द्वारा, त्रिभुज ABC का क्षेत्रफल ज्ञात कीजिए, जहाँ A(2, 5), B (4, 7) तथा C(6, 2) त्रिभुज ABC के शीर्ष हैं ।

अथवा

समाकलन विधि से, x-अक्ष से ऊपर तथा वृत्त $x^2 + y^2 = 8x$ एवं परवलय $y^2 = 4x$ के अंतः भाग के मध्यवर्ती क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7) and C(6, 2).

OR

Find the area of the region lying about x-axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$.

29. एक निर्माता 5 कुशल व 10 अर्धकुशल कारीगरों को काम पर रखता है और एक उत्पाद के दो नमूने A और B बनाता है । नमूना A के प्रत्येक नग बनाने के लिए कुशल कारीगर को 2 घंटे व अर्धकुशल कारीगर को 2 घंटे काम करना पड़ता है । नमूना B के प्रत्येक नग बनाने के लिए कुशल कारीगर को 1 घंटा व अर्ध-कुशल कारीगर को 3 घंटे काम करना पड़ता है । दोनों ही कारीगरों में प्रत्येक को काम करने के लिए प्रतिदिन अधिकतम 8 घण्टे का समय उपलब्ध है । निर्माता को नमूने A के प्रत्येक नग पर ₹ 15 लाभ व नमूने B के प्रत्येक नग पर ₹ 10 का लाभ होता है । नमूना A और नमूना B के कितने नगों का अधिकतम लाभ कमाने के लिए, प्रतिदिन निर्माण करना चाहिए ? इस प्रश्न को रैखिक प्रोग्रामन समस्या के रूप में लिखिए और ग्राफ द्वारा हल कीजिए । अधिकतम लाभ भी ज्ञात कीजिए ।

A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is ₹ 15 and on an item of model B is ₹ 10. How many of items of each model should be made per day in order to maximize daily profit ? Formulate the above LPP and solve it graphically and find the maximum profit.

QUESTION PAPER CODE 65/1/2
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. order = 2, degree = 1 $\frac{1}{2}+\frac{1}{2}$

2. (fog) (x) = f(x – 7) = x $\frac{1}{2}$

$\Rightarrow \frac{d}{dx}[(fog)(x)] = 1$ $\frac{1}{2}$

3. $\begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$ $\frac{1}{2}$

$\Rightarrow x = 3, y = 3$ $\quad\quad \therefore x - y = 0$ $\frac{1}{2}$

4. d.c.'s = $<\cos 90°, \cos 135°, \cos 45°>$ $\frac{1}{2}$

$= <0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}>$ $\frac{1}{2}$

OR

$\vec{r} = (3\hat{i}+4\hat{j}+5\hat{k}) + \lambda(2\hat{i}+2\hat{j}-3\hat{k})$ 1

SECTION B

5. As a, b $\in$ R $\Rightarrow$ ab $\in$ R $\Rightarrow$ ab + 1 $\in$ R $\Rightarrow$ a*b $\in$ R $\Rightarrow$ * is binary. 1

For associative $(a*b)*c = (ab+1)*c = (ab+1)c+1 = abc+c+1$

also, $a*(b*c) = a*(bc+1) = a.(bc+1)+1 = abc+a+1$

In general $(a*b)*c \neq a*(b*c) \Rightarrow$ * is not associative. 1

6. $A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$ 1

$$A^2-5A=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}-\begin{bmatrix}10 & 0 & 5\\ 10 & 5 & 15\\ 5 & -5 & 0\end{bmatrix}=\begin{bmatrix}-5 & -1 & -3\\ -1 & -7 & -10\\ -5 & 4 & -2\end{bmatrix}$$ 1

7. Let $I=\int\sqrt{1-\sin 2x}\,dx$

$=\int(\sin x-\cos x)dx$ as $\sin x>\cos x$ when $x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ 1

$=-\cos x-\sin x+C$ 1

OR

$I=\int\sin^{-1}(2x).1\,dx$

$=x.\sin^{-1}(2x)-\int\frac{2x}{\sqrt{1-4x^2}}dx$ 1

$=x.\sin^{-1}(2x)+\frac{1}{4}\int\frac{-8x}{\sqrt{1-4x^2}}dx=x\sin^{-1}(2x)+\frac{1}{2}.\sqrt{1-4x^2}+C$ 1

8. $y'=be^{2x}+2y\Rightarrow b=\frac{y'-2y}{e^{2x}}$ $\frac{1}{2}$

differentiating again

$\frac{e^{2x}.(y''-2y')-(y'-2y).2x^{2x}}{(e^{2x})^2}=0$ 1

$\Rightarrow\ y''-4y'+4y=0$ or $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0$ $\frac{1}{2}$

9. Let X: getting an odd number

$p=\frac{1}{2},\ q=\frac{1}{2},\ n=6$ $\frac{1}{2}$

(i) $P(X=5)={}^6C_5\left(\frac{1}{2}\right)^6=\frac{3}{32}$ $\frac{1}{2}$

(ii) $P(X\le 5)=1-P(X=6)=1-\frac{1}{64}=\frac{63}{64}$ 1

OR

$k + 2k + 3k = 1$ 1

$\Rightarrow k = \frac{1}{6}$ 1

10. $A = \{2, 4, 6\}, \; B = \{1, 2, 3\}, \; A \cap B = \{2\}$

Now, $P(A) = \frac{1}{2}, \; P(B) = \frac{1}{2}, \; P(A \cap B) = \frac{1}{6}$ 1

as $P(A) \times P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \neq P(A \cap B)$ $\frac{1}{2}$

$\Rightarrow$ A and B are not independent. $\frac{1}{2}$

11. Given $|\hat{a} + \hat{b}| = 1$

As $|\hat{a} + \hat{b}|^2 + |\hat{a} - \hat{b}|^2 = 2(|\hat{a}|^2 + |\hat{b}|^2)$ 1

$\Rightarrow 1 + |\vec{a} - \vec{b}|^2 = 2(1 + 1)$

$\Rightarrow |\hat{a} - \hat{b}|^2 = 3 \Rightarrow |\hat{a} - \hat{b}| = \sqrt{3}$ 1

OR

$$[\vec{a} \;\; \vec{b} \;\; \vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$$ 1

$= -30$ 1

12. $I = \int \frac{\tan^2 x . \sec^2 x}{1 - (\tan^3 x)^2} dx$

Put $\tan^3 x = t \Rightarrow I = \frac{1}{3} \int \frac{dt}{1 - t^2}$ 1

$$= \frac{1}{6} \log \left| \frac{1+t}{1-t} \right| + C = \frac{1}{6} \log \left| \frac{1 + \tan^3 x}{1 - \tan^3 x} \right| + C$$ $\frac{1}{2} + \frac{1}{2}$

SECTION C

13. $\tan^{-1}\left(\frac{2x+3x}{1-(2x)(3x)}\right)=\frac{\pi}{4}$ 1

$\Rightarrow \frac{5x}{1-6x^2}=\tan\frac{\pi}{4}=1 \Rightarrow 6x^2+5x-1=0$ $1\frac{1}{2}$

$\Rightarrow x=-1 \text{ or } x=\frac{1}{6}$ 1

as $x = -1$ does not satisfy the given equation,

$\therefore x = \frac{1}{6}$ $\frac{1}{2}$

14. $\log(x^2+y^2)=2\tan^{-1}\left(\frac{y}{x}\right)$

differentiating both sides w.r.t. x,

$$\frac{1}{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right)=2\cdot\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\left(\frac{x\cdot\frac{dy}{dx}-y\cdot 1}{x^2}\right)$$ 2

$\Rightarrow \frac{2}{x^2+y^2}\left(x+y\frac{dy}{dx}\right)=\frac{2x^2}{x^2+y^2}\cdot\frac{1}{x^2}\cdot\left(x\frac{dy}{dx}-y\right)$ 1

$\Rightarrow (x+y)=(x-y)\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{x+y}{x-y}$ 1

OR

Let $u = x^y$, $v = y^x$. Then $u - v = a^b$

$\Rightarrow \frac{du}{dx}-\frac{dv}{dx} = 0$ ...(1) 1

Now, $\log u = y.\log x$

$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx}=y\cdot\frac{1}{x}+\log x\cdot\frac{dy}{dx}\Rightarrow\frac{du}{dx}=x^y\left(\frac{y}{x}+\log x\cdot\frac{dy}{dx}\right)$ ...(2) 1

Again, log v = x · log y

$$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = x\cdot\frac{1}{y}\cdot\frac{dy}{dx} + \log y\cdot 1 \Rightarrow \frac{dv}{dx} = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$$ ...(3) 1

From (1), (2) and (3)

$$x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) - y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$$ $\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{y^x\cdot\log y - x^{y-1}\cdot y}{x^y\cdot\log x - y^{x-1}\cdot x}$$ $\frac{1}{2}$

15. $$I = \int\frac{3x+5}{x^2+3x-18}\,dx = \frac{3}{2}\int\frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int\frac{1}{x^2+3x-18}dx$$ 1

$$= \frac{3}{2}\int\frac{2x+3}{x^2+3x-18}dx + \frac{1}{2}\int\frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}dx$$ 1

$$= \frac{3}{2}\log|x^2+3x-18| + \frac{1}{18}\log\left|\frac{x-3}{x+6}\right| + C$$ 1 + 1

16. Let $I = \int_0^a f(a-x)\,dx$

Put a – x = t ⇒ –dx = dt $\frac{1}{2}$

$$I = -\int_a^0 f(t)\,dt = \int_0^a f(t)\,dt = \int_0^a f(x)\,dx$$ $\frac{1}{2}$

II part.

$$I = \int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow \quad I = \int_0^\pi \frac{(\pi - x)\cdot\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi} \frac{\pi \cdot \sin x}{1+\cos^2 x}\, dx$$ $1\frac{1}{2}$

Put cos x = t ⇒ –sinx dx = dt

$$\Rightarrow I = -\frac{\pi}{2} \cdot \int_1^{-1} \frac{dt}{1+t^2} = \frac{\pi}{2} \times 2 \times \int_0^1 \frac{dt}{1+t^2}$$

$$= \pi[\tan^{-1} t]_0^1 = \frac{\pi^2}{4}$$ $1\frac{1}{2}$

17. $\overrightarrow{AB} = \hat{i} + 4\hat{j} - \hat{k}$ 1

$\overrightarrow{CD} = -2\hat{i} - 8\hat{j} + 2\hat{k}$ 1

Let required angle be θ.

Then $\cos\theta = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \dfrac{-2-32-2}{\sqrt{18}\sqrt{72}} = -1$ 1

⇒ θ = 180° or π $\frac{1}{2}$

Since θ = π so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. $\frac{1}{2}$

18. LHS = $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$

$C_2 \to C_2 + C_1, C_3 \to C_3 + C_1$

$= \begin{vmatrix} a+b+c & a+b & a+c \\ -c & a+b & -(a+c) \\ -b & -(a+b) & (a+c) \end{vmatrix}$ $1\frac{1}{2}$

$= (a+b)(a+c) \begin{vmatrix} a+b+c & 1 & 1 \\ -c & 1 & -1 \\ -b & -1 & 1 \end{vmatrix}$ $\frac{1}{2}$

$C_3 \to C_3 + C_2$

$$= (a+b)\,(a+c)\begin{vmatrix} a+b+c & 1 & 2 \\ -c & 1 & 0 \\ -b & -1 & 0 \end{vmatrix}$$ 1

$= 2(a+b)\,(b+c)\,(c+a) = \text{RHS}.$ 1

19. $\frac{dx}{dt} = -\sin t + \frac{1}{\tan\frac{t}{2}} \times \left(\sec^2\frac{t}{2} \times \frac{1}{2}\right) = \frac{\cos^2 t}{\sin t}$ 1

$\frac{dy}{dt} = \cos t$ $\frac{1}{2}$

$\frac{d^2y}{dt^2} = -\sin t \Rightarrow \left.\frac{d^2y}{dt^2}\right]_{t=\frac{\pi}{4}} = -\frac{1}{\sqrt{2}}$ 1

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t$ $\frac{1}{2}$

$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{dt}{dx} = \sec^4 t \cdot \sin t$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{4}} = 2\sqrt{2}$ 1

20. Clearly $a \le a \;\forall\, a \in \mathbb{R} \Rightarrow (a, a) \in R \Rightarrow R$ is reflexive. 1

For transitive:

Let $(a, b) \in R$ and $(b, c) \in R$, $a, b, c \in \mathbb{R}$

$\Rightarrow a \le b$ and $b \le c \Rightarrow a \le c \Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive. $1\frac{1}{2}$

For non-symmetric:

Let $a = 1$, $b = 2$. As $1 \le 2 \Rightarrow (1, 2) \in R$ but $2 \not\le 1 \Rightarrow (2, 1) \notin R$

$\Rightarrow R$ is non-symmetric. $1\frac{1}{2}$

OR

For one-one. Let $x_1, x_2 \in N$.

$f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 = x_2$ as $x_1 + x_2 + 1 \neq 0$ $\quad (\because x_1, x_2 \in N)$ $\quad 1\frac{1}{2}$

$\Rightarrow$ f is one-one.

For not onto.

for $y = 1 \in N$, there is no $x \in N$ for which $f(x) = 1$ $\quad 1\frac{1}{2}$

For f^{-1}: $y = f(x) \Rightarrow y = x^2 + x + 1 \Rightarrow y = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$

$$\Rightarrow x = \frac{\sqrt{4y-3}-1}{2}$$

$\therefore f^{-1}(y) = \dfrac{\sqrt{4y-3}-1}{2}$ or $f^{-1}(x) = \dfrac{\sqrt{4x-3}-1}{2}$ $\quad 1$

21. Let the point of contact be $P(x_1, y_1)$

$\dfrac{dy}{dx} = \dfrac{3}{2\sqrt{3x-2}}$ (slope of tangent)

$\Rightarrow m_1 = \left.\dfrac{dy}{dx}\right]_{(x_1, y_1)} = \dfrac{3}{2\sqrt{3x_1-2}}$ $\quad 1$

also, slope of given line $= 2 = m_2$

$m_1 = m_2 \Rightarrow x_1 = \dfrac{41}{48}$ $\quad 1$

when $x_1 = \dfrac{41}{48}$, $y_1 = \sqrt{\dfrac{41}{16}-2} = \dfrac{3}{4}$ $\quad \therefore P\left(\dfrac{41}{48}, \dfrac{3}{4}\right)$ $\quad \frac{1}{2}$

Equation of tangent is: $y - \dfrac{3}{4} = 2\left(x - \dfrac{41}{48}\right)$

$\Rightarrow 48x - 24y = 23$ 1

and, Equation of normal is: $y - \frac{3}{4} = \frac{-1}{2}\left(x - \frac{41}{48}\right)$

$\Rightarrow 48x + 96y = 113$ $\frac{1}{2}$

22. Writing $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} = \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2}$ $\frac{1}{2}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

Differential equation becomes $v + x\frac{dv}{dx} = v + \sqrt{1 + v^2}$

$\Rightarrow \int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$ 1

$\Rightarrow \log\left|v + \sqrt{1 + v^2}\right| = \log|x| + \log c$ 1

$\Rightarrow v + \sqrt{1 + v^2} = cx \Rightarrow y + \sqrt{x^2 + y^2} = cx^2$

when $x = 1, y = 0 \Rightarrow c = 1$ $\frac{1}{2}$

$\therefore y + \sqrt{x^2 + y^2} = x^2$ $\frac{1}{2}$

OR

Given equation is $\frac{dy}{dx} + \frac{2x}{1 + x^2} \cdot y = \frac{4x^2}{1 + x^2}$ $\frac{1}{2}$

I.F. $= e^{\int \frac{2x}{1+x^2} dx} = 1 + x^2$ 1

Solution is given by,

$y \cdot (1 + x^2) = \int \frac{4x^2}{1 + x^2} \cdot (1 + x^2) dx = \int 4x^2 \, dx$ 1

$$\Rightarrow\ y\cdot(1+x^2)=\frac{4x^3}{3}+c$$ $\frac{1}{2}$

when x = 0, y = 0 $\Rightarrow$ c = 0 $\frac{1}{2}$

$$y\cdot(1+x^2)=\frac{4x^3}{3}\ \text{ or }\ y=\frac{4x^3}{3(1+x^2)}$$ $\frac{1}{2}$

23. Given lines are: $\frac{x-1}{-3}=\frac{y-2}{\left(\frac{\lambda}{7}\right)}=\frac{z-3}{2}$ and $\frac{x-1}{\left(\frac{-3\lambda}{7}\right)}=\frac{y-5}{1}=\frac{z-6}{-5}$ 1

As lines are perpendicular,

$$(-3)\left(\frac{-3\lambda}{7}\right)+\left(\frac{\lambda}{7}\right)(1)+2(-5)=0\Rightarrow\lambda=7$$ 1

So, lines are

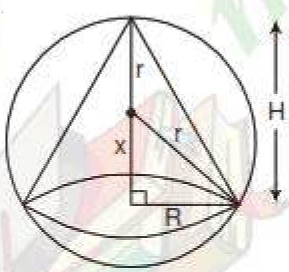
$$\frac{x-1}{-3}=\frac{y-2}{1}=\frac{x-3}{2}\ \text{ and }\ \frac{x-1}{-3}=\frac{y-5}{1}=\frac{z-6}{-5}$$ $\frac{1}{2}$

Consider $\Delta=\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=\begin{vmatrix}0 & 3 & 3\\ -3 & 1 & 2\\ -3 & 1 & -5\end{vmatrix}=-63$ 1

as $\Delta\neq 0\Rightarrow$ lines are not intersecting. $\frac{1}{2}$

SECTION D

24.



Correct Figure 1

$r^2 = x^2 + R^2$

Now, $V=\frac{1}{3}\pi R^2 H$

$$=\frac{1}{3}\pi(r^2-x^2)(r+x)$$

$$=\frac{1}{3}\pi(r+x)^2\,(r-x)$$ 1

$$\frac{dV}{dx} = \frac{1}{3}\pi\left[(r+x)^2(-1) + (r-x).2(r+x)\right]$$

$$= \frac{1}{3}\pi(r+x)(r-3x)$$ 1

$$\frac{dV}{dx} = 0 \Rightarrow x = -r \text{ or } x = \frac{r}{3}$$ $\frac{1}{2}$

(Rejected)

$$\frac{d^2V}{dx^2} = \frac{1}{3}\pi\left[(r+x)(-3) + (r-3x)\right] = -\pi H < 0$$ 1

$\Rightarrow$ V is maximum when $x = \frac{r}{3}$.

$$H = r + x = r + \frac{r}{3} = \frac{4r}{3}$$ $\frac{1}{2}$

Maximum volume $V = \frac{1}{3}\pi\left(r+\frac{r}{3}\right)^2\left(r-\frac{r}{3}\right) = \frac{32}{81}\pi r^3$ 1

25. $|A| = -1 \neq 0 \Rightarrow A^{-1}$ exists. 1

$$\text{adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$ 2

$$A^{-1} = \frac{1}{|A|}\cdot \text{adj } A = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$ $\frac{1}{2}$

Given system of equations can be written as AX = B where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Now, $X = A^{-1}B$ 1

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ 1

$\Rightarrow x = 1,\ y = 2,\ z = 3$ $\frac{1}{2}$

OR

A = I.A

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$ 1

$R_1 \leftrightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$R_2 \to R_2 + R_1,\ R_3 \to R_3 - 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} \cdot A$$

$R_2 \to \frac{R_2}{3}$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5/3 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1/3 & 1/3 & 0 \\ 0 & -3 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 - 2R_2,\ R_3 \to R_3 + 5R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1/3 \end{bmatrix} = \begin{bmatrix} -2/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 \\ 5/3 & -4/3 & 1 \end{bmatrix} \cdot A$$ 4

$R_3 \to 3R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2/3 & 1/3 & 0 \\ 1/3 & 1/3 & 0 \\ 5 & -4 & 3 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + \frac{1}{3}R_3,\ R_2 \to R_2 - \frac{5}{3}R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \cdot A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$$ 1

26. $\left.\begin{array}{l} \text{Let } E_1 : \text{item is produced by A} \\ E_2 : \text{item is produced by B} \\ E_3 : \text{item is produced by C} \\ A : \text{defective item is found.} \end{array}\right\}$ 1

$P(E_1) = \dfrac{50}{100}, P(E_2) = \dfrac{30}{100}, P(E_3) = \dfrac{20}{100}$ 1

$P(A/E_1) = \dfrac{1}{100}, P(A/E_2) = \dfrac{5}{100}, P(A|E_3) = \dfrac{7}{100}$ 1

$$P(E_1|A) = \frac{\frac{50}{100} \times \frac{1}{100}}{\frac{50}{100} \times \frac{1}{100} + \frac{30}{100} \times \frac{5}{100} + \frac{20}{100} \times \frac{7}{100}}$$ 2

$$= \frac{5}{34}$$ 1

27. Equation of plane is $\begin{vmatrix} x-2 & y-2 & z+1 \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 0$ 2

$\Rightarrow 5x + 2y - 3z = 17$ (Cartesian equation) 1

Vector equation is $\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17$ 1

Equation of required parallel plane is

$\vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} + 3\hat{j} + \hat{k}) \cdot (5\hat{i} + 2\hat{j} - 3\hat{k})$ 1

$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} - 3\hat{k}) = 23$ 1

OR

Let required plane be $a(x + 1) + b(y - 3) + c(z + 4) = 0$ ...(1) 1

Plane contains the given line, so it will also contain the point (1, 1, 0).

So, $2a - 2b + 4c = 0$ or $a - b + 2c = 0$ ...(2) 1

Also, $a + 2b - c = 0$ ...(3) 1

From (2) and (3),

$$\frac{a}{-3} = \frac{b}{3} = \frac{c}{3}$$ 1

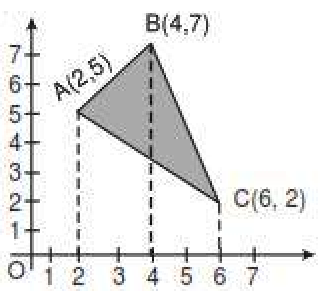
$\therefore$ Required plane is $-3(x + 1) + 3(y - 3) + 3(z + 4) = 0$

$\therefore$ $-x + y + z = 0$

Also vector equation is: $\vec{r}.(-\hat{i} + \hat{j} + \hat{k}) = 0$ 1

Length of perpendicular from (2, 1, 4) $= \dfrac{|-2+1+4|}{\sqrt{(-1)^2 + 1^2 + 1^2}} = \sqrt{3}$ 1

28.



Correct Figure 1

Equation of AB : $y = x + 3$

Equation of BC : $y = \dfrac{-5x}{2} + 17$

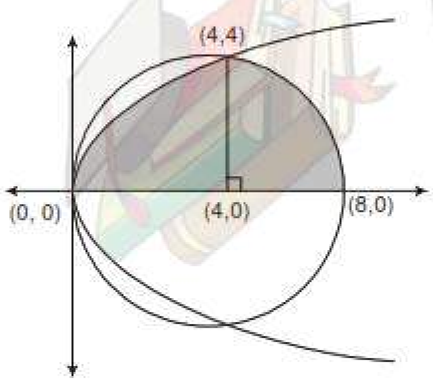
Equation of AC : $y = \dfrac{-3x}{4} + \dfrac{13}{2}$ $1\frac{1}{2}$

Required Area $= \displaystyle\int_2^4 (x+3)\,dx + \int_4^6 \left(\frac{-5x}{2} + 17\right) dx - \int_2^6 \left(\frac{-3x}{4} + \frac{13}{2}\right) dx$ $1\frac{1}{2}$

$$= \left[\frac{(x+3)^2}{2}\right]_2^4 + \left[\frac{-5x^2}{4} + 17x\right]_4^6 - \left[\frac{-3x^2}{8} + \frac{13x}{2}\right]_2^6$$ $1\frac{1}{2}$

$= 7$ $\frac{1}{2}$

OR



Correct Figure 1

Given circle $x^2 - 8x + y^2 = 0$

or $(x-4)^2 + y^2 = 4^2$

Point of intersection (0, 0) and (4, 4) 1

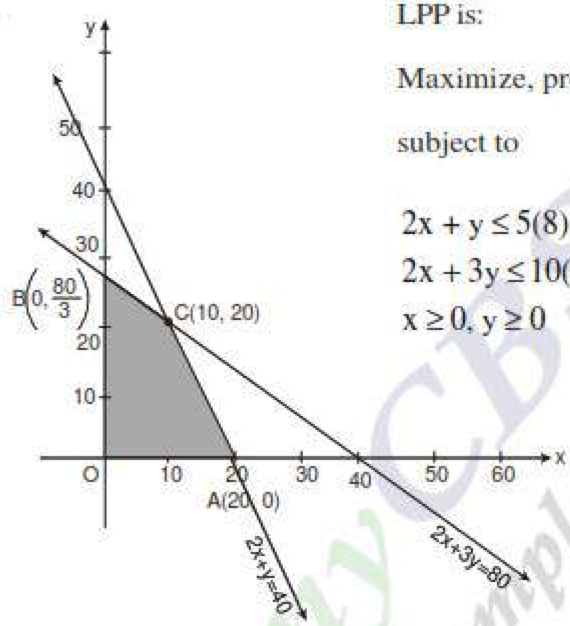
$$\text{Required Area} = \int_0^4 2\sqrt{x}\,dx + \int_4^8 \sqrt{4^2-(x-4)^2}\,dx \qquad 1\tfrac{1}{2}$$

$$= \left[\frac{4}{3}x^{3/2}\right]_0^4 + \left[\frac{x-4}{2}\sqrt{16-(x-4)^2} + \frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8 \qquad 1\tfrac{1}{2}$$

$$= \left(4\pi + \frac{32}{3}\right) \qquad 1$$

Note: A student may also arrive at the answer $\left(8\pi+\frac{64}{3}\right)$ which is double $\left(4\pi+\frac{32}{3}\right)$ because of 'about x-axis'. He/she may be given full marks.

29. Let number of items produced of model A be x and that of model B be y.



LPP is:

Maximize, profit $z = 15x + 10y$ 1

subject to

$$\left.\begin{array}{ll} 2x + y \le 5(8) & \text{i.e., } 2x + y \le 40 \\ 2x + 3y \le 10(8) & \text{i.e., } 2x + 3y \le 80 \\ x \ge 0,\ y \ge 0 & \end{array}\right\} \qquad 2$$

Correct Figure 2

Corner point	$z = 15x + 10y$	
A(20, 0)	300	
B$\left(0, \frac{80}{3}\right)$	$\frac{800}{3} \approx 266.6$	$\frac{1}{2}$
C(10, 20)	350 ← maximum	

Maximum profit = ₹ 350

when x = 10, y = 20. $\frac{1}{2}$

If a student has interpreted the language of the question in a different way, then the LPP will be of the type:

Maximise profit $z = 15x + 10y$

Subject to $2x + y \le 8$

$2x + 3y \le 8$

$x \ge 0,\ y \ge 0$

This is be accepted and marks may be given accordingly.

SET-1

Series BVM/2

कोड नं.
Code No. **65/2/1**

रोल नं.
Roll No.

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **11** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **29** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **29** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 100*

Time allowed : 3 hours | *Maximum Marks : 100*

सामान्य निर्देश :

(i) **सभी** प्रश्न अनिवार्य हैं ।

(ii) इस प्रश्न-पत्र में **29** प्रश्न हैं जो चार खण्डों में विभाजित हैं : अ, ब, स तथा द । खण्ड अ में **4** प्रश्न हैं जिनमें से प्रत्येक **एक अंक** का है । खण्ड ब में **8** प्रश्न हैं जिनमें से प्रत्येक **दो अंक** का है । खण्ड स में **11** प्रश्न हैं जिनमें से प्रत्येक **चार अंक** का है । खण्ड द में **6** प्रश्न हैं जिनमें से प्रत्येक **छः अंक** का है ।

(iii) खण्ड अ में सभी प्रश्नों के उत्तर एक शब्द, एक वाक्य अथवा प्रश्न की आवश्यकतानुसार दिए जा सकते हैं ।

(iv) पूर्ण प्रश्न-पत्र में विकल्प नहीं हैं । फिर भी खण्ड अ के 1 प्रश्न में, खण्ड ब के 3 प्रश्नों में, खण्ड स के 3 प्रश्नों में तथा खण्ड द के 3 प्रश्नों में आन्तरिक विकल्प है । ऐसे सभी प्रश्नों में से आपको एक ही विकल्प हल करना है ।

(v) कैलकुलेटर के प्रयोग की अनुमति **नहीं** है । यदि आवश्यक हो, तो आप लघुगणकीय सारणियाँ माँग सकते हैं ।

General Instructions :

*(i) **All** questions are compulsory.*

*(ii) The question paper consists of **29** questions divided into four sections A, B, C and D. Section A comprises of **4** questions of **one mark** each, Section B comprises of **8** questions of **two marks** each, Section C comprises of **11** questions of **four marks** each and Section D comprises of **6** questions of **six marks** each.*

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only one of the alternatives in all such questions.

*(v) Use of calculators is **not** permitted. You may ask for logarithmic tables, if required.*

खण्ड अ

SECTION A

प्रश्न संख्या 1 से 4 तक प्रत्येक प्रश्न 1 अंक का है ।

Question numbers 1 to 4 carry 1 mark each.

1. यदि A एक वर्ग आव्यूह है जिसमें $A'A = I$ है, तो $|A|$ का मान लिखिए ।

If A is a square matrix satisfying $A'A = I$, write the value of $|A|$.

2. यदि $y = x|x|$ है, तो $x < 0$ के लिए, $\frac{dy}{dx}$ ज्ञात कीजिए ।

If $y = x|x|$, find $\frac{dy}{dx}$ for $x < 0$.

3. निम्न अवकल समीकरण की कोटि व घात (यदि परिभाषित है) ज्ञात कीजिए :

$$\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = 2x^2 \log\left(\frac{d^2y}{dx^2}\right)$$

Find the order and degree (if defined) of the differential equation

$$\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = 2x^2 \log\left(\frac{d^2y}{dx^2}\right)$$

4. उस रेखा के दिक्-कोसाइन ज्ञात कीजिए जो निर्देशांक अक्षों से समान कोण बनाती है ।

अथवा

एक रेखा किसी एक बिन्दु, जिसका स्थिति सदिश $2\hat{i} - \hat{j} + 4\hat{k}$ है, से गुज़रती है और सदिश $\hat{i} + \hat{j} - 2\hat{k}$ की दिशा में है । इस रेखा का कार्तीय समीकरण ज्ञात कीजिए ।

Find the direction cosines of a line which makes equal angles with the coordinate axes.

OR

A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form.

खण्ड ब

SECTION B

प्रश्न संख्या 5 से 12 तक प्रत्येक प्रश्न के 2 अंक हैं ।

Question numbers 5 to 12 carry 2 marks each.

5. सभी वास्तविक संख्याओं के समुच्चय $\mathbb{R}$ पर परिभाषित संक्रिया $*$: $a * b = \sqrt{a^2 + b^2}$ क्या द्विआधारी है, इसकी जाँच कीजिए । यदि यह द्विआधारी है, तो ज्ञात कीजिए कि क्या यह साहचर्य है या नहीं ।

Examine whether the operation $*$ defined on $\mathbb{R}$, the set of all real numbers, by $a * b = \sqrt{a^2 + b^2}$ is a binary operation or not, and if it is a binary operation, find whether it is associative or not.

6. यदि $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ है, तो दर्शाइए कि $(A - 2I)(A - 3I) = 0$.

If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, show that $(A - 2I)(A - 3I) = 0$.

7. ज्ञात कीजिए :

$$\int \sqrt{3 - 2x - x^2}\, dx$$

Find :

$$\int \sqrt{3 - 2x - x^2}\, dx$$

8. ज्ञात कीजिए :

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} \, dx$$

अथवा

ज्ञात कीजिए :

$$\int \frac{x-3}{(x-1)^3} e^x \, dx$$

Find :

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} \, dx$$

OR

Find :

$$\int \frac{x-3}{(x-1)^3} e^x \, dx$$

9. वक्रों के कुल $y = Ae^{2x} + Be^{-2x}$, जहाँ A, B स्वेच्छ अचर हैं, को निरूपित करने वाला अवकल समीकरण ज्ञात कीजिए।

Find the differential equation of the family of curves $y = Ae^{2x} + Be^{-2x}$, where A and B are arbitrary constants.

10. यदि $|\vec{a}| = 2$, $|\vec{b}| = 7$ तथा $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ है, तो $\vec{a}$ और $\vec{b}$ के बीच का कोण ज्ञात कीजिए।

अथवा

उस घनाभ का आयतन ज्ञात कीजिए जिसके किनारे $-3\hat{i} + 7\hat{j} + 5\hat{k}$, $-5\hat{i} + 7\hat{j} - 3\hat{k}$ तथा $7\hat{i} - 5\hat{j} - 3\hat{k}$ द्वारा दिए गए हैं।

If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

OR

Find the volume of a cuboid whose edges are given by $-3\hat{i} + 7\hat{j} + 5\hat{k}$, $-5\hat{i} + 7\hat{j} - 3\hat{k}$ and $7\hat{i} - 5\hat{j} - 3\hat{k}$.

11. यदि P(A नहीं) = 0·7, P(B) = 0·7 तथा P(B/A) = 0·5 है, तो P(A/B) ज्ञात कीजिए।

If P(not A) = 0·7, P(B) = 0·7 and P(B/A) = 0·5, then find P(A/B).

12. एक सिक्का 5 बार उछाला गया । (i) 3 बार चित आने की प्रायिकता क्या है ? (ii) अधिकतम 3 बार चित आने की प्रायिकता क्या है ?

अथवा

दो सिक्कों को एक बार एक साथ उछालने पर चितों की संख्या, X का प्रायिकता बंटन ज्ञात कीजिए ।

A coin is tossed 5 times. What is the probability of getting (i) 3 heads, (ii) at most 3 heads ?

OR

Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.

खण्ड स

SECTION C

प्रश्न संख्या 13 से 23 तक प्रत्येक प्रश्न के 4 अंक हैं ।

Question numbers 13 to 23 carry 4 marks each.

13. जाँच कीजिए कि क्या समुच्चय $A = \{1, 2, 3, 4, 5, 6\}$ पर परिभाषित संबंध $R = \{(a, b) : b = a + 1\}$ स्वतुल्य, सममित या संक्रामक है ।

अथवा

मान लीजिए कि $f : N \to Y$, $f(x) = 4x + 3$, द्वारा परिभाषित एक फलन है, जहाँ $Y = \{y \in N : y = 4x + 3$, किसी $x \in N$ के लिए$\}$ है । सिद्ध कीजिए कि f व्युत्क्रमणीय है । इसका प्रतिलोम फलन भी ज्ञात कीजिए ।

Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

OR

Let $f : N \to Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N : y = 4x + 3, \text{ for some } x \in N\}$. Show that f is invertible. Find its inverse.

14. $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$ का मान ज्ञात कीजिए ।

Find the value of $\sin\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right)$.

15. सारणिकों के गुणधर्मों का प्रयोग करके, दर्शाइए कि

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

Using properties of determinants, show that

$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

16. यदि $x\sqrt{1+y} + y\sqrt{1+x} = 0$ और $x \neq y$ है, तो सिद्ध कीजिए कि $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

अथवा

यदि $(\cos x)^y = (\sin y)^x$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए।

If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$.

OR

If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

17. यदि, किसी $c > 0$ के लिए, $(x-a)^2 + (y-b)^2 = c^2$ है, तो सिद्ध कीजिए कि

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$, a और b से स्वतंत्र एक स्थिर राशि है।

If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$ is a constant independent of a and b.

18. वक्र $x^2 = 4y$ पर उस अभिलंब का समीकरण ज्ञात कीजिए, जो बिन्दु $(-1, 4)$ से गुज़रता है।

Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point $(-1, 4)$.

19. ज्ञात कीजिए :

$$\int \frac{x^2 + x + 1}{(x + 2)(x^2 + 1)} \, dx$$

Find :

$$\int \frac{x^2 + x + 1}{(x + 2)(x^2 + 1)} \, dx$$

20. सिद्ध कीजिए कि

$$\int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx$$

अत:

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\,dx$$

का मूल्यांकन कीजिए ।

Prove that

$$\int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx$$

and hence evaluate

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\,dx$$

21. अवकल समीकरण को हल कीजिए :

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

अथवा

अवकल समीकरण को हल कीजिए :

$$\frac{dy}{dx} = -\left[\frac{x + y\cos x}{1 + \sin x}\right]$$

Solve the differential equation :

$$x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right)$$

OR

Solve the differential equation :

$$\frac{dy}{dx} = -\left[\frac{x + y \cos x}{1 + \sin x}\right]$$

22. सदिशों $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ और $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ के लिए, सदिश $\vec{b} + \vec{c}$ के अनुदिश मात्रक सदिश व सदिश $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ का अदिश गुणनफल 1 है । λ का मान ज्ञात कीजिए और अत: $\vec{b} + \vec{c}$ के अनुदिश मात्रक सदिश भी ज्ञात कीजिए ।

The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

23. यदि रेखाएँ $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ और $\frac{x-1}{3\lambda} = \frac{y-1}{2} = \frac{z-6}{-5}$ परस्पर लम्बवत् हों, तो λ का मान ज्ञात कीजिए । अत: ज्ञात कीजिए कि क्या ये रेखाएँ एक-दूसरे को काटती हैं या नहीं ।

If the lines $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{2} = \frac{z-6}{-5}$ are perpendicular, find the value of λ. Hence find whether the lines are intersecting or not.

खण्ड द

SECTION D

प्रश्न संख्या 24 से 29 तक प्रत्येक प्रश्न के 6 अंक हैं ।

Question numbers 24 to 29 carry 6 marks each.

24. यदि $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए ।

अत: निम्न समीकरण निकाय का हल ज्ञात कीजिए :

$$x + 3y + 4z = 8$$
$$2x + y + 2z = 5$$
और $5x + y + z = 7$

अथवा

प्रारंभिक रूपांतरणों द्वारा, निम्न आव्यूह का व्युत्क्रम ज्ञात कीजिए :

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}$, find A^{-1}.

Hence solve the system of equations

$$x + 3y + 4z = 8$$
$$2x + y + 2z = 5$$
and $5x + y + z = 7$

OR

Find the inverse of the following matrix, using elementary transformations :

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

25. सिद्ध कीजिए कि एक R त्रिज्या के गोले के अंतर्गत अधिकतम आयतन के बेलन की ऊँचाई $\frac{2R}{\sqrt{3}}$ है । अधिकतम आयतन भी ज्ञात कीजिए ।

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

26. समाकलन विधि से उस त्रिभुज का क्षेत्रफल ज्ञात कीजिए जिसके शीर्ष (1, 0), (2, 2) और (3, 1) हैं ।

अथवा

समाकलन विधि से, दो वृत्तों $x^2 + y^2 = 4$ तथा $(x - 2)^2 + y^2 = 4$ के बीच घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

Using method of integration, find the area of the triangle whose vertices are (1, 0), (2, 2) and (3, 1).

OR

Using method of integration, find the area of the region enclosed between two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

27. बिन्दुओं, जिनके स्थिति सदिश $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ तथा $\hat{i} + 2\hat{j} + \hat{k}$ हैं, से गुज़रने वाले समतल का सदिश व कार्तीय समीकरण ज्ञात कीजिए । उपर्युक्त समतल के समांतर समतल, जो बिन्दु (2, 3, 7) से गुज़रता है, का समीकरण भी लिखिए । अत:, दोनों समांतर समतलों के बीच की दूरी ज्ञात कीजिए ।

अथवा

बिन्दुओं (2, – 1, 2) तथा (5, 3, 4) से गुज़रने वाली रेखा का समीकरण ज्ञात कीजिए तथा बिन्दुओं (2, 0, 3), (1, 1, 5) तथा (3, 2, 4) से गुज़रने वाले समतल का समीकरण भी ज्ञात कीजिए । रेखा व समतल का प्रतिच्छेदन बिन्दु भी ज्ञात कीजिए ।

Find the vector and cartesian equations of the plane passing through the points having position vectors $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$. Write the equation of a plane passing through a point (2, 3, 7) and parallel to the plane obtained above. Hence, find the distance between the two parallel planes.

OR

Find the equation of the line passing through (2, – 1, 2) and (5, 3, 4) and of the plane passing through (2, 0, 3), (1, 1, 5) and (3, 2, 4). Also, find their point of intersection.

28. तीन सिक्के दिए गए हैं । एक सिक्के के दोनों ओर चित ही है । दूसरा सिक्का अभिनत है जिसमें चित 75% बार प्रकट होता है और तीसरा अनभिनत सिक्का है । तीनों में से एक सिक्का यादृच्छया चुना गया और उसे उछाला गया है । यदि सिक्के पर चित प्रकट हुआ हो, तो क्या प्रायिकता है कि वह दोनों तरफ चित वाला सिक्का है ?

There are three coins. One is a two-headed coin, another is a biased coin that comes up heads 75% of the time and the third is an unbiased coin. One of the three coins is chosen at random and tossed. If it shows heads, what is the probability that it is the two-headed coin ?

29. एक कंपनी दो प्रकार का सामान, A और B बनाती है, जिसमें सोने व चाँदी का उपयोग होता है । प्रकार A की प्रत्येक इकाई में 3 g चाँदी व 1 g सोना, तथा प्रकार B की प्रत्येक इकाई में 1 g चाँदी व 2 g सोना प्रयोग में आता है । कंपनी ज्यादा-से-ज्यादा 9 g चाँदी व 8 g सोने का ही प्रयोग कर सकती है । यदि प्रकार A की एक इकाई से ₹ 40 का लाभ व प्रकार B की एक इकाई से ₹ 50 का लाभ कमाया जाता है, तो अधिकतम लाभ अर्जित करने हेतु कंपनी को दोनों प्रकारों की कितनी-कितनी इकाइयाँ बनानी चाहिएँ ? उपर्युक्त समस्या को रैखिक प्रोग्रामन समस्या में परिवर्तित करके आलेख विधि से हल कीजिए तथा अधिकतम लाभ भी ज्ञात कीजिए ।

A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of ₹ 40 and that of type B ₹ 50, find the number of units of each type that the company should produce to maximize profit. Formulate the above LPP and solve it graphically and also find the maximum profit.

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Senior School Certificate Examination

March 2019

Marking Scheme — Mathematics (041) 65/2/1, 65/2/2, 65/2/3

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
5. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
6. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
7. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
8. A full scale of marks 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
9. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 25 answer books per day.
10. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
11. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.
12. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
13. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
14. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
15. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/2/1
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|A'||A| = |I| \Rightarrow |A|^2 = 1$ $\frac{1}{2}$

$\therefore$ $|A| = 1$ or $|A| = -1$ $\frac{1}{2}$

2. For $x < 0$, $y = x\,|x| = -x^2$ $\frac{1}{2}$

$\therefore$ $\frac{dy}{dx} = -2x$ $\frac{1}{2}$

3. Order = 2, Degree not defined $\frac{1}{2}+\frac{1}{2}$

4. D. Rs are 1, 1, 1
$\therefore$ Direction cosines of the line are:

$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ 1

OR

Equation of the line is:

$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$ 1

SECTION B

5. $\left.\begin{array}{l}\forall\, a, b \in \mathbb{R}, \sqrt{a^2+b^2} \in \mathbb{R} \\ \therefore * \text{ is a binary operation on } \mathbb{R}\end{array}\right\}$ 1

Also,

$$\left.\begin{array}{l} a*(b*c) = a*\sqrt{b^2+c^2} = \sqrt{a^2+b^2+c^2} \\ (a*b)*c = \sqrt{a^2+b^2}*c = \sqrt{a^2+b^2+c^2} \end{array}\right\} \left.\begin{array}{l} \Rightarrow a*(b*c) = (a*b)*c \\ \therefore \ * \text{ is Associative} \end{array}\right\}$$ 1

6. $(A-2I)(A-3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ 1

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$ 1

7. $\int\sqrt{3-2x-x^2}\,dx=\int\sqrt{2^2-(x+1)^2}\,dx$ 1

$$=\frac{x+1}{2}\sqrt{3-2x-x^2}+2\sin^{-1}\left(\frac{x+1}{2}\right)+c$$ 1

8. $\int\frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}dx=\int(\sec x\cdot\tan x+\operatorname{cosec} x\cdot\cot x)\,dx$ 1

$$=\sec x-\operatorname{cosec} x+c$$ 1

OR

$\int\frac{x-3}{(x-1)^3}e^x dx=\int e^x\{(x-1)^{-2}-2(x-1)^{-3}\}dx$ 1

$$\left.\begin{array}{c} =e^x(x-1)^{-2}+c \\ \text{or} \\ \dfrac{e^x}{(x-1)^2}+c \end{array}\right\}$$ 1

9. Differentiating $y=Ae^{2x}+Be^{-2x}$, we get

$\frac{dy}{dx}=2Ae^{2x}-2Be^{-2x}$, differentiate again to get, 1

$\frac{d^2y}{dx^2}=4Ae^{2x}+4Be^{-2x}=4y$ or $\frac{d^2y}{dx^2}-4y=0$ 1

10. Let θ be the angle between $\vec{a}$ & $\vec{b}$, then

$\sin\theta=\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}=\frac{7}{2.7}=\frac{1}{2}$ $1\frac{1}{2}$

$\Rightarrow\ \theta=\frac{\pi}{6}$ or $\frac{5\pi}{6}$ $\frac{1}{2}$

OR

$(-3\hat{i}+7\hat{j}+5\hat{k})\cdot\{(-5\hat{i}+7\hat{j}-3\hat{k})\times(7\hat{i}-5\hat{j}-3\hat{k})\}=\begin{vmatrix}-3 & 7 & 5\\ -5 & 7 & -3\\ 7 & -5 & -3\end{vmatrix}$ 1

$$=-264$$ $\frac{1}{2}$

$\therefore$ Volume of cuboid = 264 cubic units $\frac{1}{2}$

11. $P(\bar{A}) = 0.7 \Rightarrow 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3$ $\frac{1}{2}$

$P(A \cap B) = P(A) \cdot P(B|A) = 0.3 \times 0.5 = 0.15$ $\frac{1}{2}$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.15}{0.7} = \dfrac{15}{70}$ or $\dfrac{3}{14}$ 1

12. (i) $P(3 \text{ heads}) = {}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \frac{5}{16}$ 1

(ii) $P(\text{At most 3 heads}) = P\ (r \le 3)$

$$\left.\begin{aligned} &= 1 - P(4 \text{ heads or } 5 \text{ heads}) \\ &= 1 - \left\{ {}^5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) + {}^5C_5\left(\frac{1}{2}\right)^5 \right\} \\ &= \frac{26}{32} \text{ or } \frac{13}{16} \end{aligned}\right\}$$ 1

OR

X = No. of heads in simultaneous toss of two coins.

X:	0	1	2	$\frac{1}{2}$
P(x):	1/4	1/2	1/4	$1\frac{1}{2}$

SECTION C

13. $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

For $1 \in A$, $(1, 1) \notin R \Rightarrow R$ is not reflexive 1

For $1, 2 \in A$, $(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow R$ is not symmetric $1\frac{1}{2}$

For $1, 2, 3 \in A$, $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R \Rightarrow R$ is not transitive $1\frac{1}{2}$

OR

One-One: Let for $x_1, x_2 \in N$, $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$ 2

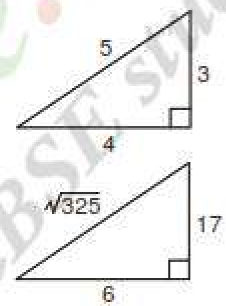
$\therefore$ 'f ' is one-one

Onto: co-domain of f = Range of f = Y 1

$\therefore$ 'f ' is onto

$\therefore$ f is invertible with, $f^{-1}: Y \to N$ and $f^{-1}(y) = \dfrac{y-3}{4}$ or $f^{-1}(x) = \dfrac{x-3}{4}, x \in Y$ 1

14. $\sin\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$



$= \sin\left[\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ 1

$= \sin\left[\tan^{-1}\left(\frac{3/4 + 2/3}{1 - 3/4 \cdot 2/3}\right)\right] = \sin\left[\tan^{-1}\left(\frac{17}{6}\right)\right]$ $1\frac{1}{2}$

$= \sin\left[\sin^{-1}\left(\frac{17}{\sqrt{325}}\right)\right] = \frac{17}{\sqrt{325}}$ $1\frac{1}{2}$

15. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$

$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} \begin{pmatrix} \text{By applying,} \\ C_1 \to C_1 + C_2 + C_3 \end{pmatrix}$ 1

$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix} \begin{cases} \text{By applying,} \\ R_2 \to R_2 - R_1; \\ R_3 \to R_3 - R_1 \end{cases}$ 2

$= (a + b + c) \{4bc + 2ab + 2ac + a^2 - (a^2 - ac - ba + bc)\}$

$= 3(a + b + c)(ab + bc + ca)$ 1

16. $x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$ $\frac{1}{2}$

Squaring to get: $x^2(1+y) = y^2(1+x)$ $\frac{1}{2}$

Simplifying to get: $(x-y)(x+y+xy) = 0$ 1

As, $x \neq y \quad \therefore \quad y = -\dfrac{x}{1+x}$ 1

Differentiating w.r.t. 'x', we get:

$$\frac{dy}{dx} = \frac{-1(1+x)-(-x)\cdot 1}{(1+x)^2} = \frac{-1}{(1+x)^2}$$ 1

OR

$(\cos x)^y = (\sin y)^x \Rightarrow y \cdot \log(\cos x) = x \cdot \log(\sin y)$ 1

Differentiating w.r.t 'x'

$$\Rightarrow \frac{dy}{dx} \cdot \log(\cos x) + y(-\tan x) = \log(\sin y) + x \cdot \cot y \cdot \frac{dy}{dx}$$ 2

$$\Rightarrow \frac{dy}{dx} = \frac{y \cdot \tan x + \log(\sin y)}{\log(\cos x) - x\cot y}$$ 1

17. $(x-a)^2 + (y-b)^2 = c^2,\ c > 0$

Differentiating both sides with respect to 'x', we get

$$2(x-a) + 2(y-b) \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x-a}{y-b}$$ $1\frac{1}{2}$

Differentiating again with respect to 'x', we get;

$$\frac{d^2y}{dx^2} = -\frac{(y-b)-(x-a)\cdot\frac{dy}{dx}}{(y-b)^2} = \frac{-c^2}{(y-b)^3} \quad \left(\text{By substituting } `\frac{dy}{dx}\text{'}\right)$$ $1+\frac{1}{2}$

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+\frac{(x-a)^2}{(y-b)^2}\right]^{3/2}}{-\frac{c^2}{(y-b)^3}} = \frac{\frac{c^3}{(y-b)^3}}{-\frac{c^2}{(y-b)^3}} = -c$$ 1

Which is a constant independent of 'a' & 'b'.

18. Let (α, β) be the point on the curve where normal

passes through $(-1, 4)$ $\therefore$ $\alpha^2 = 4\beta$, also $\frac{dy}{dx} = \frac{x}{2}$

Slope of normal at $(\alpha, \beta) = \dfrac{-1}{\left.\frac{dy}{dx}\right]_{(\alpha,\beta)}} = \dfrac{-1}{\frac{\alpha}{2}} = \dfrac{-2}{\alpha}$ 1

Equation of normal: $y - 4 = \frac{-2}{\alpha}(x+1)$ $\frac{1}{2}$

(α, β) lies on the normal $\Rightarrow \beta - 4 = \frac{-2}{\alpha}(\alpha + 1)$

Putting $\beta = \frac{\alpha^2}{4}$, we get; $\alpha^3 - 8\alpha + 8 = 0 \Rightarrow (\alpha - 2)(\alpha^2 + 2\alpha - 4) = 0$ 1

For $\alpha = 2$ Equation of normal is: $x + y - 3 = 0$ 1

For $\alpha = \pm\sqrt{5} - 1$; Equation of normal is: $y - 4 = \frac{-2}{\pm\sqrt{5}-1}(x+1)$ $\frac{1}{2}$

19. $\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx = \frac{3}{5}\int\frac{1}{x+2}dx + \frac{1}{5}\int\frac{2x}{x^2+1}dx + \frac{1}{5}\int\frac{1}{x^2+1}dx$ 2

$= \frac{3}{5}\log|x+2| + \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1}x + c$ 2

20. $\int_0^a f(x)\,dx = -\int_a^0 f(a-t)dx$ Put $x = a - t, dx = -dt$

Upper limit $= t = a - x = a - a = 0$

Lower limit $= t = a - x = a - 0 = a$ 1

$= \int_0^a f(a-t)\,dt = \int_0^a f(a-x)\,dx$

Let $I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x}dx$ ...(i)

$\Rightarrow I = \int_0^{\pi/2} \frac{\pi/2 - x}{\sin\left(\frac{\pi}{2}-x\right) + \cos\left(\frac{\pi}{2}-x\right)}dx$ $\therefore$ $I = \int_0^{\pi/2}\frac{\pi/2 - x}{\cos x + \sin x}dx$ ...(ii) 1

Adding, (i) and (ii) we get

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\cos x + \sin x}dx = \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)}dx$$

$$= \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right)dx$$ $\frac{1}{2}$

$$\Rightarrow 2I = \frac{\pi}{2\sqrt{2}}\left\{\log\left|\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right|\right\}_0^{\frac{\pi}{2}}$$ $\frac{1}{2}$

$$= \frac{\pi}{2\sqrt{2}}\{\log(\sqrt{2}+1) - \log(\sqrt{2}-1)\}$$ $\frac{1}{2}$

$$\Rightarrow I = \frac{\pi}{4\sqrt{2}}\{\log(\sqrt{2}+1) - \log(\sqrt{2}-1)\} \text{ or } \frac{\pi}{4\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$ $\frac{1}{2}$

21. The given differential equation can be written as:

$$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$$ $\frac{1}{2}$

Put $\frac{y}{x} = v$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, to get 1

$$v + x\frac{dv}{dx} = v - \tan v \Rightarrow x\frac{dv}{dx} = -\tan v$$

$$\Rightarrow \cot v\ dv = -\frac{1}{x}dx,$$ 1

Integrating both sides we get,

$$\left.\begin{aligned} &\log|\sin v| = -\log|x| + \log c \\ \Rightarrow\ &\log|\sin v| = \log\left|\frac{c}{x}\right| \end{aligned}\right\}$$ 1

$\therefore$ Solution of differential equation is

$$\sin\left(\frac{y}{x}\right) = \frac{c}{x} \text{ or } x\cdot\sin\left(\frac{y}{x}\right) = c$$ $\frac{1}{2}$

OR

The given differential equation can be written as:

$$\frac{dy}{dx}+\frac{\cos x}{1+\sin x}\cdot y=\frac{-x}{1+\sin x};$$ $1\frac{1}{2}$

$$\text{I.F.} = e^{\int\frac{\cos x}{1+\sin x}dx}=e^{\log(1+\sin x)}=1+\sin x$$ 1

$\therefore$ Solution of the given differential equation is:

$$y(1+\sin x)=\int\frac{-x}{1+\sin x}\times(1+\sin x)\,dx+c$$ 1

$$\Rightarrow y(1+\sin x)=\frac{-x^2}{2}+c \text{ or } y=\frac{-x^2}{2(1+\sin x)}+\frac{c}{1+\sin x}$$ $\frac{1}{2}$

22. $\vec{a}\cdot\frac{(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|}=1$ 1

$$\Rightarrow (\hat{i}+\hat{j}+\hat{k})\cdot\{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}\}=\sqrt{(2+\lambda)^2+36+4}$$ $1\frac{1}{2}$

$$\Rightarrow \lambda+6=\sqrt{(2+\lambda)^2+40}$$

Squaring to get

$$\lambda^2+12\lambda+36=\lambda^2+4\lambda+44 \Rightarrow \lambda=1$$ $\frac{1}{2}$

$\therefore$ Unit vector along $(\vec{b}+\vec{c})$ is $\frac{3}{7}\hat{i}+\frac{6}{7}\hat{j}-\frac{2}{7}\hat{k}$ 1

23. Lines are perpendicular

$\therefore$ $-3(3\lambda)+2\lambda(2)+2(-5)=0 \Rightarrow \lambda=-2$ 2

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_3 \end{vmatrix}=\begin{vmatrix} 1-1 & 1-2 & 6-3 \\ -3 & 2(-2) & 2 \\ 3(-2) & 2 & -5 \end{vmatrix}=-63\neq 0$$ $1\frac{1}{2}$

$\therefore$ Lines are not intersecting $\frac{1}{2}$

SECTION D

24. $|A| = 11$; $\text{Adj (A)} = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ 1+2

$\therefore \quad A^{-1} = \frac{1}{|A|} \cdot \text{Adj } A = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ $\frac{1}{2}$

Taking: $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$

The system of equations in matrix form is

$A \cdot X = B \qquad \therefore X = A^{-1} \cdot B$ 1

$\therefore$ Solution is:

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ 1

$\therefore \quad x = 1, y = 1, z = 1$ $\frac{1}{2}$

OR

We know: $A = IA$

$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1

$R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + 2R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} A$$ 4

$R_2 \leftrightarrow R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_2 \to R_2 - 3R_3$

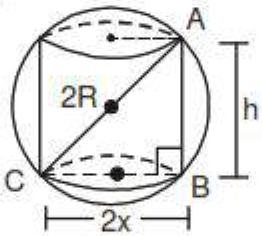
$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_1 \to R_1 - R_2 - 2R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore \; A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$ 1

25. Correct Figure 1



In rt. ΔABC; $4x^2 + h^2 = 4R^2$, $x^2 = \dfrac{4R^2 - h^2}{4}$ 1

V(Volume of cylinder) $= \pi x^2 h = \dfrac{\pi}{4}(4R^2 h - h^3)$ 1

$V'(h) = \dfrac{\pi}{4}(4R^2 - 3h^2)$; $V''(h) = \dfrac{\pi}{4}(-6h)$ $\frac{1}{2}+\frac{1}{2}$

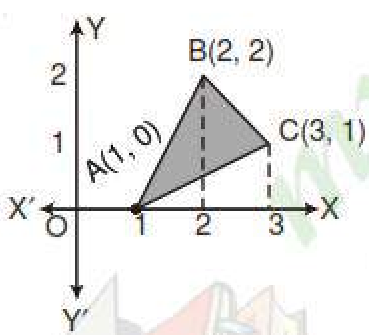
$V'(h) = 0 \Rightarrow h = \dfrac{2R}{\sqrt{3}}$ 1

$V''\left(\dfrac{2R}{\sqrt{3}}\right) = \dfrac{-6\pi}{4}\left(\dfrac{2R}{\sqrt{3}}\right) < 0 \Rightarrow$ Volume 'V' is max. $\frac{1}{2}$

for $h = \dfrac{2R}{\sqrt{3}}$

Max. Volume: $V = \dfrac{4}{3\sqrt{3}}\pi R^3$ $\frac{1}{2}$

26. Correct Figure 1



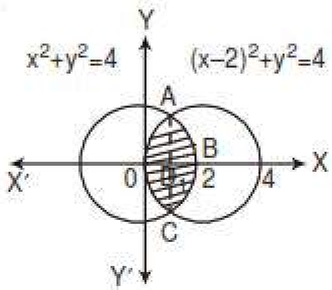
$$\left.\begin{aligned}&\text{Equation of line AB}: y = 2(x-1)\\&\text{Equation of line BC}: y = 4 - x\\&\text{Equation of line AC}: y = \frac{1}{2}(x-1)\end{aligned}\right\}$$ $1\frac{1}{2}$

$\text{ar}(\Delta ABC) = 2\int_1^2 (x-1)\,dx + \int_2^3 (4-x)\,dx - \dfrac{1}{2}\int_1^3 (x-1)\,dx$ $1\frac{1}{2}$

$= (x-1)^2\Big]_1^2 - \dfrac{1}{2}(4-x)^2\Big]_2^3 - \dfrac{1}{4}(x-1)^2\Big]_1^3$ $1\frac{1}{2}$

$= 1 + \dfrac{3}{2} - 1 = \dfrac{3}{2}$ $\frac{1}{2}$

OR



Correct Figure 1

Getting the point of intersection as x = 1 1

Area (OABCO) = 4 × ar(ABD)

$= 4\int_1^2 \sqrt{2^2 - x^2}\, dx$ 2

$= 4\left\{\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right\}_1^2$ 1

$= \left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ 1

27. Let $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

$(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 0 & 1 & 3 \end{vmatrix} = -9\hat{i} - 3\hat{j} + \hat{k}$ 2

Vector equation of plane is:

$\{\vec{r} - (\hat{i} + \hat{j} - 2\hat{k})\} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) = 0$

$\Rightarrow \vec{r} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) + 14 = 0$ 1

Cartesian Equation of plane is: $-9x - 3y + z + 14 = 0$ 1

Equation of plane through (2, 3, 7) and parallel to above plane is:

$-9(x - 2) - 3(y - 3) + (z - 7) = 0$

$\Rightarrow -9x - 3y + z + 20 = 0$ 1

Distance between parallel planes $= \left|\frac{-14 + 20}{\sqrt{91}}\right| = \frac{6}{\sqrt{91}}$ 1

OR

Equation of line: $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$ 1

Equation of plane: $\begin{vmatrix} x-2 & y & z-3 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}=0$ 1

$\Rightarrow \; -3(x-2) + 3y - 3(z-3) = 0$

$\Rightarrow \; x - y + z - 5 = 0$ 1

General point on line: $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}$

is: P(3k + 2, 4k − 1, 2k + 2); Putting in the equation of plane 1

we get, $3k + 2 - 4k + 1 + 2k + 2 = 5 \Rightarrow k = 0$ 1

$\therefore$ Point of intersection is: (2, −1, 2) 1

28. Let E_1 = Event that two-headed coin is chosen
E_2 = Event that biased coin is chosen
E_3 = Event that unbiased coin is chosen
A = Event that coin tossed shows head 1

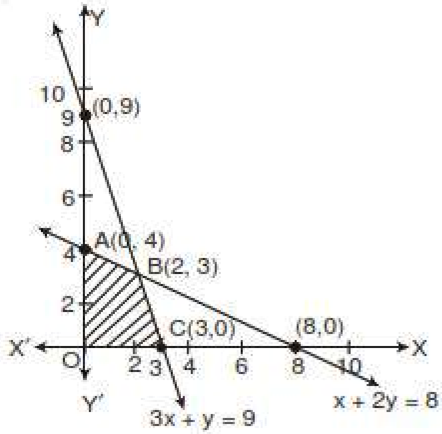
Then, $P(E_1) = P(E_2) = P(E_3) = 1/3$ 1

$P(A|E_1) = 1,\; P(A|E_2)=\frac{75}{100}=\frac{3}{4},\; P(A|E_3)=\frac{1}{2}$ $1\frac{1}{2}$

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1)+P(E_2)\cdot P(A|E_2)+P(E_3)\cdot P(A|E_3)}$$

$$=\frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1+\frac{1}{3}\cdot\frac{3}{4}+\frac{1}{3}\cdot\frac{1}{2}}=\frac{4}{9}$$ $2+\frac{1}{2}$

29.



let the company produce: Goods A = x units

Goods B = y units

then, the linear programming problem is:

Maximize profit: z = 40x + 50y (In ₹) $\frac{1}{2}$

Subject to constraints:

$$\left.\begin{array}{l} 3x + y \le 9 \\ x + 2y \le 8 \\ x, y \ge 0 \end{array}\right\} \quad 2\frac{1}{2}$$

Correct graph: 2

Corner point	Value of z (₹)	
A(0, 4)	200	
B(2, 3)	230 (Max)	$\frac{1}{2}$
C(3, 0)	120	

∴ Maximum profit = ₹ 230 at:

Goods A produced = 2 units, Goods B produced = 3 units $\frac{1}{2}$

QUESTION PAPER CODE 65/2/2
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|A| = |B| = 0$ $\frac{1}{2}$

$\Rightarrow |AB| = 0$ $\frac{1}{2}$

2. $\frac{d}{dx}(e^{\sqrt{3x}}) = \frac{\sqrt{3}}{2\sqrt{x}} e^{\sqrt{3x}}$ 1

3. Order = 2, Degree not defined $\frac{1}{2}+\frac{1}{2}$

4. D. Rs are 1, 1, 1
$\therefore$ Direction cosines of the line are:

$$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$ 1

OR

Equation of the line is:

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$$ 1

SECTION B

5. $\int \sqrt{3-2x-x^2}\,dx = \int \sqrt{2^2-(x+1)^2}\,dx$ 1

$$= \frac{x+1}{2}\sqrt{3-2x-x^2} + 2\sin^{-1}\left(\frac{x+1}{2}\right) + c$$ 1

6. $|A| = p^2 - 4$ $\frac{1}{2}$

$|A^3| = 125 \Rightarrow |A|^3 = 125 \Rightarrow |A| = 5$ 1

$\therefore\ p^2 - 4 = 5 \Rightarrow p = \pm 3$ $\frac{1}{2}$

7. $\left.\begin{array}{l}\forall\, a, b \in \mathbb{R}, \sqrt{a^2+b^2} \in \mathbb{R} \\ \therefore * \text{ is a binary operation on } \mathbb{R}\end{array}\right\}$ 1

Also,

$$\left.\begin{array}{l} a*(b*c)=a*\sqrt{b^2+c^2}=\sqrt{a^2+b^2+c^2} \\ (a*b)*c=\sqrt{a^2+b^2}*c=\sqrt{a^2+b^2+c^2} \end{array}\right\} \left.\begin{array}{l} \Rightarrow\ a*(b*c)=(a*b)*c \\ \therefore\ \ *\text{ is Associative} \end{array}\right\}$$ 1

8. Let θ be the angle between $\vec{a}$ & $\vec{b}$, then

$$\sin\theta=\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}=\frac{7}{2.7}=\frac{1}{2}$$ $1\frac{1}{2}$

$$\Rightarrow\ \theta=\frac{\pi}{6}\text{ or }\frac{5\pi}{6}$$ $\frac{1}{2}$

OR

$$(-3\hat{i}+7\hat{j}+5\hat{k})\cdot\{(-5\hat{i}+7\hat{j}-3\hat{k})\times(7\hat{i}-5\hat{j}-3\hat{k})\}=\begin{vmatrix}-3 & 7 & 5\\ -5 & 7 & -3\\ 7 & -5 & -3\end{vmatrix}$$ 1

$$=-264$$ $\frac{1}{2}$

$\therefore$ Volume of cuboid = 264 cubic units $\frac{1}{2}$

9. (i) P(3 heads) = ${}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2=\frac{5}{16}$ 1

(ii) P(At most 3 heads) = P $(r \le 3)$

$$\left.\begin{array}{l} =1-P(4\text{ heads or }5\text{ heads}) \\ =1-\left\{{}^5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)+{}^5C_5\left(\frac{1}{2}\right)^5\right\} \\ =\frac{26}{32}\text{ or }\frac{13}{16} \end{array}\right\}$$ 1

OR

X = No. of heads in simultaneous toss of two coins.

X:	0	1	2	$\frac{1}{2}$
P(x):	1/4	1/2	1/4	$1\frac{1}{2}$

10. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\sec x \cdot \tan x + \text{cosec } x \cdot \cot x) dx$ 1

$= \sec x - \text{cosec } x + c$ 1

OR

$\int \frac{x-3}{(x-1)^3} e^x dx = \int e^x \{(x-1)^{-2} - 2(x-1)^{-3}\} dx$ 1

$= e^x (x-1)^{-2} + c$
or
$\frac{e^x}{(x-1)^2} + c$ 1

11. $P(\bar{A}) = 0.7 \Rightarrow 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3$ $\frac{1}{2}$

$P(A \cap B) = P(A) \cdot P(B/A) = 0.3 \times 0.5 = 0.15$ $\frac{1}{2}$

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.7} = \frac{15}{70}$ or $\frac{3}{14}$ 1

12. Given differential equation can be written as:

$\frac{dy}{dx} = e^x \cdot e^y \Rightarrow e^{-y} dy = e^x dx$ 1

Integrating both sides, we get

$-e^{-y} = e^x + c$ 1

SECTION C

13. $\sin\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$

$= \sin\left[\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ 1

$= \sin\left[\tan^{-1}\left(\frac{3/4 + 2/3}{1 - 3/4 \cdot 2/3}\right)\right] = \sin\left[\tan^{-1}\left(\frac{17}{6}\right)\right]$ $1\frac{1}{2}$

$= \sin\left[\sin^{-1}\left(\frac{17}{\sqrt{325}}\right)\right] = \frac{17}{\sqrt{325}}$ $1\frac{1}{2}$

14. $$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} \begin{pmatrix} \text{By applying,} \\ C_1 \to C_1 + C_2 + C_3 \end{pmatrix}$$ 1

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix} \left\{ \begin{array}{l} \text{By applying,} \\ R_2 \to R_2 - R_1; \\ R_3 \to R_3 - R_1 \end{array} \right.$$ 2

$= (a + b + c) \{4bc + 2ab + 2ac + a^2 - (a^2 - ac - ba + bc)\}$

$= 3(a + b + c) (ab + bc + ca)$ 1

15. $\bar{R} = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

For $1 \in A$, $(1, 1) \notin R \Rightarrow R$ is not reflexive 1

For $1, 2 \in A$, $(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow R$ is not symmetric $1\frac{1}{2}$

For $1, 2, 3 \in A$, $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R \Rightarrow R$ is not transitive $1\frac{1}{2}$

OR

One-One: Let for $x_1, x_2 \in N$, $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$ 2

$\therefore$ 'f' is one-one

Onto: co-domain of f = Range of f = Y 1

$\therefore$ 'f' is onto

$\therefore$ f is invertible with, $f^{-1}: Y \to N$ and $f^{-1}(y) = \dfrac{y-3}{4}$ or $f^{-1}(x) = \dfrac{x-3}{4}, x \in Y$ 1

16. $x\sqrt{1+y} + y\sqrt{1+x} = 0 \Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$ $\frac{1}{2}$

Squaring to get: $x^2 (1 + y) = y^2(1 + x)$ $\frac{1}{2}$

Simplifying to get: $(x - y) (x + y + xy) = 0$ 1

As, $x \neq y \quad \therefore \; y = -\dfrac{x}{1+x}$ 1

Differentiating w.r.t. 'x', we get:

$$\frac{dy}{dx} = \frac{-1(1+x)-(-x)\cdot 1}{(1+x)^2} = \frac{-1}{(1+x)^2} \qquad 1$$

OR

$$(\cos x)^y = (\sin y)^x \Rightarrow y \cdot \log(\cos x) = x \cdot \log(\sin y) \qquad 1$$

Differentiating w.r.t 'x'

$$\Rightarrow \frac{dy}{dx} \cdot \log(\cos x) + y(-\tan x) = \log(\sin y) + x \cdot \cot y \cdot \frac{dy}{dx} \qquad 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{y \cdot \tan x + \log(\sin y)}{\log(\cos x) - x \cot y} \qquad 1$$

17. $\int_0^a f(x)\,dx = -\int_a^0 f(a-t)\,dt$ Put $x = a - t$, $dx = -dt$

Upper limit $= t = a - x = a - a = 0$

Lower limit $= t = a - x = a - 0 = a$

$$= \int_0^a f(a-t)\,dt = \int_0^a f(a-x)\,dx \qquad 1$$

$$\text{Let } I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x}\,dx \qquad \text{...(i)}$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\pi/2 - x}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}\,dx \qquad \therefore\ I = \int_0^{\pi/2} \frac{\pi/2 - x}{\cos x + \sin x}\,dx \qquad \text{...(ii)} \qquad 1$$

Adding (i) and (ii) we get

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\cos x + \sin x}\,dx = \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)}\,dx$$

$$= \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right)dx \qquad \frac{1}{2}$$

$$\Rightarrow 2I = \frac{\pi}{2\sqrt{2}}\left\{\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|\right\}_0^{\pi/2}$$ $\frac{1}{2}$

$$= \frac{\pi}{2\sqrt{2}}\{\log|\sqrt{2}+1|-\log|\sqrt{2}-1|\}$$ $\frac{1}{2}$

$$\Rightarrow I = \frac{\pi}{4\sqrt{2}}\{\log(\sqrt{2}+1)-\log(\sqrt{2}-1)\} \text{ or } \frac{\pi}{4\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$$ $\frac{1}{2}$

18. $\int\frac{x^2+x+1}{(x+2)(x^2+1)}dx = \frac{3}{5}\int\frac{1}{x+2}dx+\frac{1}{5}\int\frac{2x}{x^2+1}dx+\frac{1}{5}\int\frac{1}{x^2+1}dx$ 2

$$= \frac{3}{5}\log|x+2|+\frac{1}{5}\log|x^2+1|+\frac{1}{5}\tan^{-1}x+c$$ 2

19. The given differential equation can be written as:

$$\frac{dy}{dx}=\frac{y}{x}-\tan\left(\frac{y}{x}\right)$$ $\frac{1}{2}$

Put $\frac{y}{x}=v$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$, to get 1

$$v+x\frac{dv}{dx}=v-\tan v \Rightarrow x\frac{dv}{dx}=-\tan v$$

$$\Rightarrow \cot v\, dv = -\frac{1}{x}dx,$$ 1

Integrating both sides we get,

$$\left.\begin{array}{l}\log|\sin v|=-\log|x|+\log c\\ \Rightarrow \log|\sin v|=\log\left|\frac{c}{x}\right|\end{array}\right\}$$ 1

$\therefore$ Solution of differential equation is

$$\sin\left(\frac{y}{x}\right)=\frac{c}{x} \text{ or } x\cdot\sin\left(\frac{y}{x}\right)=c$$ $\frac{1}{2}$

OR

The given differential equation can be written as:

$$\frac{dy}{dx}+\frac{\cos x}{1+\sin x}\cdot y=\frac{-x}{1+\sin x};$$ $1\frac{1}{2}$

$$\text{I.F.} = e^{\int \frac{\cos x}{1+\sin x}dx} = e^{\log(1+\sin x)} = 1 + \sin x$$ 1

$\therefore$ Solution of the given differential equation is:

$$y(1+\sin x) = \int \frac{-x}{1+\sin x} \times (1+\sin x)\,dx + c$$ 1

$$\Rightarrow y(1+\sin x) = \frac{-x^2}{2} + c \text{ or } y = \frac{-x^2}{2(1+\sin x)} + \frac{c}{(1+\sin x)}$$ $\frac{1}{2}$

20. $\vec{a} \cdot \frac{(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|} = 1$ 1

$$\Rightarrow (\hat{i}+\hat{j}+\hat{k}) \cdot \{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}\} = \sqrt{(2+\lambda)^2 + 36 + 4}$$ $1\frac{1}{2}$

$$\Rightarrow \lambda + 6 = \sqrt{(2+\lambda)^2 + 40}$$

Squaring to get

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \Rightarrow \lambda = 1$$ $\frac{1}{2}$

$\therefore$ Unit vector along $(\vec{b}+\vec{c})$ is $\frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$ 1

21. $e^{y/x} = \frac{x}{a+bx}$, taking log, on both sides, we get $\frac{y}{x} = \log x - \log(a+bx)$ 1

Differentiating with respect to 'x'

$$\left.\begin{aligned} &\frac{x \cdot y' - y}{x^2} = \frac{1}{x} - \frac{b}{a+bx} = \frac{a}{(a+bx)x} \\ &\Rightarrow x \cdot y' - y = \frac{ax}{a+bx} \quad \text{...(i)} \end{aligned}\right\}$$ $1\frac{1}{2}$

Differentiating with respect to 'x'

$$\Rightarrow x \cdot y'' + y' - y' = \frac{(a+bx) \cdot a - ax \cdot b}{(a+bx)^2} = \left(\frac{a}{a+bx}\right)^2$$ 1

$$\left.\begin{aligned} &\Rightarrow x \cdot y'' = \left(\frac{a}{a+bx}\right)^2 \Rightarrow x^3 \cdot y'' = \left(\frac{ax}{a+bx}\right)^2 \\ &\Rightarrow x^3 \frac{d^2y}{dx^2} = \left\{x \cdot \frac{dy}{dx} - y\right\}^2 \quad \text{(Using (i))} \end{aligned}\right\}$$ $\frac{1}{2}$

22. Let Edge = x cm, then

V(Volume of cube) = x^3, S(Surface area) = $6x^2$ 1

$$\frac{dv}{dt} = 8\,\text{cm}^3/\text{s} \Rightarrow 3x^2\frac{dx}{dt} = 8 \Rightarrow \frac{dx}{dt} = \frac{8}{3x^2}$$ 1

$$\frac{ds}{dt} = 12x\frac{dx}{dt} = 12x \cdot \frac{8}{3x^2} = \frac{32}{x}$$ 1

$$\therefore \left.\frac{ds}{dt}\right]_{x=12} = \frac{32}{12} = \frac{8}{3}\,\text{cm}^2/\text{s}$$ 1

23. Equation of plane:

$$\left.\begin{aligned} &\begin{vmatrix} x-2 & y-5 & z+3 \\ -2-2 & -3-5 & 5+3 \\ 5-2 & 3-5 & -3+3 \end{vmatrix} = 0 \\ \Rightarrow &\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0 \end{aligned}\right\}$$ 2

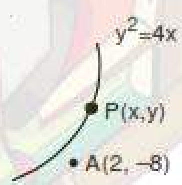
$\Rightarrow$ (x − 2)(16) − (y − 5)(−24) + (z + 3)(32) = 0

$\Rightarrow$ 2x + 3y + 4z − 7 = 0 1

Vector form: $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) - 7 = 0$ 1

SECTION D

24.



Let P(x, y) be any point on the curve $y^2 = 4x$

$$z = AP = \sqrt{(x-2)^2 + (y+8)^2}$$ 1

$$\text{let } s = z^2 = \left(\frac{y^2}{4} - 2\right)^2 + (y+8)^2$$ 1

$$\frac{ds}{dy} = 2\left(\frac{y^2}{4} - 2\right)\left(\frac{y}{2}\right) + 2(y+8) = \frac{y^3}{4} + 16$$ 1

$$\frac{d^2s}{dy^2} = \frac{3y^2}{4}$$ $\frac{1}{2}$

Let $\frac{ds}{dy} = 0 \Rightarrow y^3 = -64 \Rightarrow y = -4$ 1

$$\left.\frac{d^2s}{dy^2}\right]_{y=-4} = \frac{3(16)}{4} > 0$$ $\frac{1}{2}$

$\therefore$ s or z is minimum at y = –4; $x = \frac{y^2}{4} = 4$

$\therefore$ The nearest point is P(4, –4) 1

25. Let $f(x) = x^2 + 2 + e^{2x}$, a = 1, b = 3, nh = 2 1

then,

$$f(1) + f(1+h) + f(1+2h) + ... + f(1+\overline{n-1}h)$$

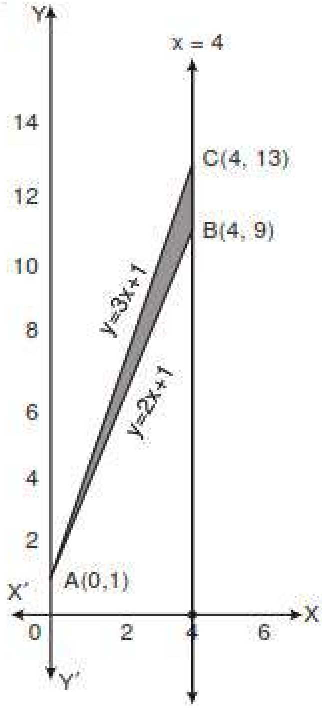
$$= 3n + h^2[1^2 + 2^2 + ... + (n-1)^2] + 2h[1 + 2 + ... + (n-1)] + e^2[1 + e^{2h} + e^{4h} + ... + e^{2(n-1)h}]$$ 1

$$= 3n + h^2\frac{(n-1)(n)(2n-1)}{6} + \frac{2h(n-1)(n)}{2} + \frac{e^2(e^{2nh}-1)}{e^{2h}-1}$$ 2

$$\int_1^3 (x^2 + 2 + e^{2x})dx = \lim_{h\to 0}\left[3(nh) + \frac{(nh-h)(nh)(2nh-h)}{6} + (nh)(nh-h) + \frac{e^2}{2}\cdot\frac{2h}{e^{2h}-1}(e^{2nh}-1)\right]$$ 1

$$\left.\begin{aligned} &= 6 + \frac{2\times 2\times 4}{6} + 2\times 2 + \frac{e^2}{2}\times 1\times(e^4-1) \\ &= \frac{38}{3} + \frac{e^2(e^4-1)}{2} \end{aligned}\right\}$$ 1

OR



Correct figure 1

Points of intersection of given lines

are A(0, 1), B(4, 9), C(4, 13) $1\frac{1}{2}$

$\therefore$ Req. Area $= \int_0^4 (3x+1)\,dx - \int_0^4 (2x+1)\,dx$ 2

$= \int_0^4 x\,dx = \frac{1}{2}x^2\Big]_0^4 = 8$ $1\frac{1}{2}$

26. $|A| = 11$; Adj (A) $= \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ 1+2

$\therefore \quad A^{-1} = \frac{1}{|A|} \cdot \text{Adj}\, A = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}$ $\frac{1}{2}$

Taking: $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$

The system of equations in matrix form is

$A \cdot X = B \qquad \therefore X = A^{-1} \cdot B$ 1

$\therefore$ Solution is:

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ 1

$\therefore \quad x = 1, y = 1, z = 1$ $\frac{1}{2}$

OR

We know: A = IA

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1

$R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + 2R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix} A$$ 4

$R_2 \leftrightarrow R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_2 \to R_2 - 3R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$R_1 \to R_1 - R_2 - 2R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$ 1

27. let the company produce: Goods A = x units

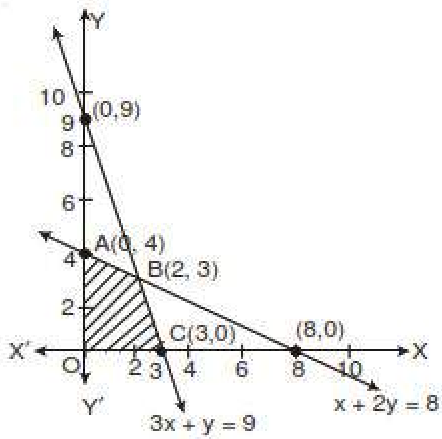
Goods B = y units

then, the linear programming problem is:

Maximize profit: z = 40x + 50y (In ₹) $\frac{1}{2}$

Subject to constraints:

$$\left.\begin{array}{l} 3x + y \le 9 \\ x + 2y \le 8 \\ x, y \ge 0 \end{array}\right\}$$ $2\frac{1}{2}$



Correct graph. 2

Corner point	Value of z (₹)	
A(0, 4)	200	
B(2, 3)	230 (Max)	$\frac{1}{2}$
C(3, 0)	120	

$\therefore$ Maximum profit = ₹ 230 at:

Goods A produced = 2 units, Goods B produced = 3 units $\frac{1}{2}$

28. Let E_1 = Event that two-headed coin is chosen

E_2 = Event that biased coin is chosen

E_3 = Event that unbiased coin is chosen

A = Event that coin tossed shows head 1

Then, $P(E_1) = P(E_2) = P(E_3) = 1/3$ 1

$P(A|E_1) = 1,\ P(A|E_2) = \frac{75}{100} = \frac{3}{4},\ P(A|E_3) = \frac{1}{2}$ $1\frac{1}{2}$

$$P(E_1|A) = \frac{P(E_1)\cdot P(A|E_1)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\cdot P(A|E_3)}$$

$$= \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{1}{2}} = \frac{4}{9}$$ $2+\frac{1}{2}$

29. Let $\vec{a}=\hat{i}+\hat{j}-2\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{c}=\hat{i}+2\hat{j}+\hat{k}$

$$(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 3\\ 0 & 1 & 3\end{vmatrix}=-9\hat{i}-3\hat{j}+\hat{k}$$ 2

Vector equation of plane is:

$$\{\vec{r}-(\hat{i}+\hat{j}-2\hat{k})\}\cdot(-9\hat{i}-3\hat{j}+\hat{k})=0$$

$\Rightarrow\ \vec{r}\cdot(-9\hat{i}-3\hat{j}+\hat{k})+14=0$ 1

Cartesian Equation of plane is: $-9x-3y+z+14=0$ 1

Equation of plane through (2, 3, 7) and parallel to above plane is:

$-9(x-2)-3(y-3)+(z-7)=0$

$\Rightarrow\ -9x-3y+z+20=0$ 1

Distance between parallel planes $=\left|\dfrac{-14+20}{\sqrt{91}}\right|=\dfrac{6}{\sqrt{91}}$ 1

OR

Equation of line: $\dfrac{x-2}{3}=\dfrac{y+1}{4}=\dfrac{z-2}{2}$ 1

Equation of plane: $\begin{vmatrix}x-2 & y & z-3\\ -1 & 1 & 2\\ 1 & 2 & 1\end{vmatrix}=0$ 1

$\Rightarrow\ -3(x-2)+3y-3(z-3)=0$

$\Rightarrow\ x-y+z-5=0$ 1

General point on line: $\dfrac{x-2}{3}=\dfrac{y+1}{4}=\dfrac{z-2}{2}$

is: $P(3k+2,\ 4k-1,\ 2k+2)$; Putting in the equation of plane 1

we get, $3k+2-4k+1+2k+2=5\Rightarrow k=0$ 1

$\therefore$ Point of intersection is: $(2,-1,2)$ 1

QUESTION PAPER CODE 65/2/3
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\frac{dy}{dx} = 2ae^{2x} \Rightarrow \frac{dy}{dx} = 2(y-5)$ $\frac{1}{2}+\frac{1}{2}$

2. $\frac{dy}{dx} = -\frac{\sqrt{3}\sin(\sqrt{3x})}{2\sqrt{x}}$ 1

3. $|A'||A| = |I| \Rightarrow |A|^2 = 1$ $\frac{1}{2}$

$\Rightarrow |A| = 1$ or $|A| = -1$ $\frac{1}{2}$

4. D. Rs are 1, 1, 1
$\therefore$ Direction cosines of the line are:

$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ 1

OR

Equation of the line is:

$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$ 1

SECTION B

5. $\overrightarrow{AB} = 3\hat{i} - \hat{j} - 2\hat{k}$; $\overrightarrow{AC} = 9\hat{i} - 3\hat{j} - 6\hat{k}$ 1

Clearly, $\overrightarrow{AC} = 3 \cdot \overrightarrow{AB} \Rightarrow \overrightarrow{AC} \parallel \overrightarrow{AB}$, $\therefore$ A, B & C are Collinear 1

OR

$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix} = -17\hat{i} + 13\hat{j} + 7\hat{k}$ $1\frac{1}{2}$

$\therefore$ $|\vec{a}\times\vec{b}| = \sqrt{289+169+49} = \sqrt{507}$ $\frac{1}{2}$

6. $\int \frac{x-5}{(x-3)^3} \cdot e^x \, dx = \int e^x \left[\frac{(x-3)-2}{(x-3)^3} \right] dx$ $\frac{1}{2}$

$= \int e^x [(x-3)^{-2} - 2(x-3)^{-3}] dx$ $\frac{1}{2}$

$= e^x (x-3)^{-2} + c$ or $\frac{e^x}{(x-3)^2} + c$ 1

7. $\left.\begin{array}{l} \forall\, a, b \in \mathbb{R}, \sqrt{a^2+b^2} \in \mathbb{R} \\ \therefore * \text{ is a binary operation on } \mathbb{R} \end{array}\right\}$ 1

Also,

$$\left.\begin{array}{l} a*(b*c) = a*\sqrt{b^2+c^2} = \sqrt{a^2+b^2+c^2} \\ (a*b)*c = \sqrt{a^2+b^2}*c = \sqrt{a^2+b^2+c^2} \end{array}\right\} \left.\begin{array}{l} \Rightarrow\ a*(b*c) = (a*b)*c \\ \therefore\ \ * \text{ is Associative} \end{array}\right\}$$ 1

8. $(A-2I)(A-3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ 1

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$ 1

9. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\sec x \cdot \tan x + \operatorname{cosec} x \cdot \cot x) dx$ 1

$= \sec x - \operatorname{cosec} x + c$ 1

OR

$\int \frac{x-3}{(x-1)^3} e^x dx = \int e^x \{(x-1)^{-2} - 2(x-1)^{-3}\} dx$ 1

$$\left.\begin{array}{c} = e^x (x-1)^{-2} + c \\ \text{or} \\ \frac{e^x}{(x-1)^2} + c \end{array}\right\}$$ 1

10. (i) $P(3 \text{ heads}) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = \frac{5}{16}$ 1

(ii) P(At most 3 heads) = P (r ≤ 3)

$$= 1 - P(4 \text{ heads or } 5 \text{ heads})$$
$$= 1 - \left\{ {}^5C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right) + {}^5C_5 \left(\frac{1}{2}\right)^5 \right\}$$
$$= \frac{26}{32} \text{ or } \frac{13}{16}$$ 1

OR

X = No. of heads in simultaneous toss of two coins.

X:	0	1	2	$\frac{1}{2}$
P(x):	1/4	1/2	1/4	$1\frac{1}{2}$

11. $P(\bar{A}) = 0.7 \Rightarrow 1 - P(A) = 0.7 \Rightarrow P(A) = 0.3$ $\frac{1}{2}$

$P(A \cap B) = P(A) \cdot P(B/A) = 0.5 \times 0.3 = 0.15$ $\frac{1}{2}$

$P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.15}{0.7} = \dfrac{15}{70}$ or $\dfrac{3}{14}$ 1

12. Differenatiae $y = Ae^{2x} + Be^{-2x}$, we get

$\dfrac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}$, differenatiate again to get, 1

$\dfrac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x} = 4y$ or $\dfrac{d^2y}{dx^2} - 4y = 0$ 1

SECTION C

13. $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\dfrac{8}{31}$

$\Rightarrow \tan^{-1}\left\{\dfrac{2x}{1-(x^2-1)}\right\} = \tan^{-1}\dfrac{8}{31}$ 2

$\Rightarrow 4x^2 + 31x - 8 = 0 \Rightarrow x = \dfrac{1}{4}$ or $x = -8$ 1

$x = -8$ does not satisty the given equation so $x = \dfrac{1}{4}$ 1

14. $x = ae^t (\sin t + \cos t);\ y = ae^t (\sin t - \cos t)$

then

$$\left.\begin{aligned}\frac{dy}{dt} &= ae^t(\sin t - \cos t) + ae^t(\cos t + \sin t)\\ &= y + x\end{aligned}\right\}$$ $1\frac{1}{2}$

$$\left.\begin{aligned}\frac{dx}{dt} &= ae^t(\sin t + \cos t) + ae^t(\cos t - \sin t)\\ &= x - y\end{aligned}\right\}$$ $1\frac{1}{2}$

$\therefore\ \frac{dy}{dx} = \frac{y+x}{x-y}$ or $\frac{x+y}{x-y}$ 1

OR

Let, $y = x^{\sin x} + (\sin x)^{\cos x} = u + v;\ \therefore\ \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ 1

$u = x^{\sin x} \Rightarrow \log u = \sin x \cdot \log x \Rightarrow \frac{du}{dx} = x^{\sin x}\left\{\cos x \cdot \log x + \frac{\sin x}{x}\right\}$ $1\frac{1}{2}$

$v = (\sin x)^{\cos x} \Rightarrow \log v = \cos x \cdot \log(\sin x) \Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}\{\cos x \cdot \cot x - \sin x \cdot \log \sin x\}$ 1

$\therefore\ \frac{dy}{dx} = x^{\sin x}\left\{\cos x \cdot \log x + \frac{\sin x}{x}\right\} + (\sin x)^{\cos x}\{\cos x \cdot \cot x - \sin x . \log \sin x\}$ $\frac{1}{2}$

15. $\int \frac{2\cos x}{(1-\sin x)(2-\cos^2 x)}dx = \int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}dx = \int \frac{2}{(1-t)(1+t^2)}dt$, where $\sin x = t$, $\cos x\, dx = dt$

1

$\int \frac{2}{(1-t)(1+t^2)}dt = \int \frac{1}{1-t}dt + \int \frac{t+1}{t^2+1}dt$ $1\frac{1}{2}$

$= -\log(1-t) + \frac{1}{2}\log(t^2+1) + \tan^{-1}(t) + c$ 1

$= -\log|1-\sin x| + \frac{1}{2}\log(\sin^2 x + 1) + \tan^{-1}(\sin x) + c$ $\frac{1}{2}$

16. $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

For $1 \in A$, $(1, 1) \notin R \Rightarrow R$ is not reflexive 1

For $1, 2 \in A$, $(1, 2) \in R$ but $(2, 1) \notin R \Rightarrow R$ is not symmetric $1\frac{1}{2}$

For $1, 2, 3 \in A$, $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R \Rightarrow R$ is not transitive $1\frac{1}{2}$

OR

One-One: Let for $x_1, x_2 \in N$, $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$ 2

$\therefore$ 'f' is one-one

Onto: co-domain of f = Range of f = Y 1

$\therefore$ 'f' is onto

$\therefore$ f is invertible with, $f^{-1}: Y \to N$ and $f^{-1}(y) = \frac{y-3}{4}$ or $f^{-1}(x) = \frac{x-3}{4}, x \in Y$ 1

17. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix} \begin{pmatrix} \text{By applying,} \\ C_1 \to C_1 + C_2 + C_3 \end{pmatrix}$$ 1

$$= \begin{vmatrix} a+b+c & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix} \left\{ \begin{array}{l} \text{By applying,} \\ R_2 \to R_2 - R_1; \\ R_3 \to R_3 - R_1 \end{array} \right.$$ 2

$= (a + b + c) \{4bc + 2ab + 2ac + a^2 - (a^2 - ac - ba + bc)\}$

$= 3(a + b + c)(ab + bc + ca)$ 1

18. $(x - a)^2 + (y - b)^2 = c^2, c > 0$

Differentiating both sides with respect to 'x', we get

$$2(x - a) + 2(y - b) \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x-a}{y-b}$$ $1\frac{1}{2}$

Differentiating again with respect to 'x', we get;

$$\frac{d^2y}{dx^2} = -\frac{(y-b)-(x-a)\cdot\frac{dy}{dx}}{(y-b)^2} = \frac{-c^2}{(y-b)^3} \text{ (By substituting '}\frac{dy}{dx}\text{')}$$ $1+\frac{1}{2}$

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+\frac{(x-a)^2}{(y-b)^2}\right]^{3/2}}{-\frac{c^2}{(y-b)^3}} = \frac{\frac{c^3}{(y-b)^3}}{-\frac{c^2}{(y-b)^3}} = -c$$ 1

Which is a constant independent of 'a' & 'b'.

19. Let (α, β) be the point on the curve where normal

passes through $(-1, 4)$ $\therefore$ $\alpha^2 = 4\beta$, also $\frac{dy}{dx} = \frac{x}{2}$

Slope of normal at $(\alpha, \beta) = \frac{-1}{\left.\frac{dy}{dx}\right]_{(\alpha,\beta)}} = \frac{-1}{\frac{\alpha}{2}} = \frac{-2}{\alpha}$ 1

Equation of normal: $y - 4 = \frac{-2}{\alpha}(x+1)$ $\frac{1}{2}$

(α, β) lies on normal $\Rightarrow \beta - 4 = \frac{-2}{\alpha}(\alpha+1)$

Putting $\beta = \frac{\alpha^2}{4}$, we get; $\alpha^3 - 8\alpha + 8 = 0 \Rightarrow (\alpha - 2)(\alpha^2 + 2\alpha - 4) = 0$ 1

For $\alpha = 2$ Equation of normal is: $x + y - 3 = 0$ 1

For $\alpha = \pm\sqrt{5} - 1$; Equation of normal is: $y - 4 = \frac{-2}{\pm\sqrt{5}-1}(x+1)$ $\frac{1}{2}$

20. $\int_0^a f(x)\,dx = -\int_a^0 f(a-t)\,dx$ Put $x = a - t$, $dx = -dt$

Upper limit $= t = a - x = a - a = 0$

Lower limit $= t = a - x = a - 0 = a$

$$= \int_0^a f(a-t)\,dt = \int_0^a f(a-x)\,dx$$ 1

Let $I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$ ...(i)

$$\Rightarrow I = \int_0^{\pi/2} \frac{\pi/2 - x}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$\therefore\ I = \int_0^{\pi/2} \frac{\pi/2 - x}{\cos x + \sin x} dx$ ...(ii) 1

Adding (i) and (ii) we get

$$2I = \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\cos x + \sin x} dx = \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)} dx$$

$= \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx$ $\frac{1}{2}$

$\Rightarrow\ 2I = \frac{\pi}{2\sqrt{2}}\left\{\log\left|\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right|\right\}_0^{\pi/2}$ $\frac{1}{2}$

$= \frac{\pi}{2\sqrt{2}}\{\log|\sqrt{2}+1| - \log|\sqrt{2}-1|\}$ $\frac{1}{2}$

$\Rightarrow\ I = \frac{\pi}{4\sqrt{2}}\{\log(\sqrt{2}+1) - \log(\sqrt{2}-1)$ or $\frac{\pi}{4\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$ $\frac{1}{2}$

21. The given differential equation can be written as:

$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$ $\frac{1}{2}$

Put $\frac{y}{x} = v$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, to get 1

$$v + x\frac{dv}{dx} = v - \tan v \Rightarrow x\frac{dv}{dx} = -\tan v$$

$\Rightarrow\ \cot v\ dv = -\frac{1}{x}dx,$ 1

Integrating both sides we get,

$$\left.\begin{aligned} &\log|\sin v| = -\log|x| + \log c \\ \Rightarrow\ &\log|\sin v| = \log\left|\frac{c}{x}\right| \end{aligned}\right\}$$ 1

$\therefore$ Solution of differential equation is

$$\sin\left(\frac{y}{x}\right) = \frac{c}{x} \text{ or } x \cdot \sin\left(\frac{y}{x}\right) = c$$ $\frac{1}{2}$

OR

The given differential equation can be written as:

$$\frac{dy}{dx} + \frac{\cos x}{1+\sin x} \cdot y = \frac{-x}{1+\sin x};$$ $1\frac{1}{2}$

$$\text{I.F.} = e^{\int \frac{\cos x}{1+\sin x} dx} = e^{\log(1+\sin x)} = 1 + \sin x$$ 1

$\therefore$ Solution of the given differential equation is:

$$y(1 + \sin x) = \int \frac{-x}{1+\sin x} \times (1+\sin x)\,dx + c$$ 1

$$\Rightarrow y(1 + \sin x) = \frac{-x^2}{2} + c \text{ or } y = \frac{-x^2}{2(1+\sin x)} + \frac{c}{1+\sin x}$$ $\frac{1}{2}$

22. Lines are perpendicular

$\therefore$ $-3(3\lambda) + 2\lambda(2) + 2(-5) = 0 \Rightarrow \lambda = -2$ 2

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_3 \end{vmatrix} = \begin{vmatrix} 1-1 & 1-2 & 6-3 \\ -3 & 2(-2) & 2 \\ 3(-2) & 2 & -5 \end{vmatrix} = -63 \neq 0$$ $1\frac{1}{2}$

$\therefore$ Lines are not intersecting $\frac{1}{2}$

23. $\vec{a} \cdot \frac{(\vec{b}+\vec{c})}{|\vec{b}+\vec{c}|} = 1$ 1

$$\Rightarrow (\hat{i}+\hat{j}+\hat{k}) \cdot \{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}\} = \sqrt{(2+\lambda)^2 + 36 + 4}$$ $1\frac{1}{2}$

$$\Rightarrow \lambda + 6 = \sqrt{(2+\lambda)^2 + 40}$$

Squaring to get

$$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \Rightarrow \lambda = 1$$ $\frac{1}{2}$

$\therefore$ Unit vector along $(\vec{b} + \vec{c})$ is $\frac{3}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{2}{7}\hat{k}$ 1

SECTION D

24. $A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$ $1\frac{1}{2}$

$A^3 = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$ $1\frac{1}{2}$

LHS = $A^3 - 6A^2 + 5A + 11I$

$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ 1

$= \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} = O = \text{R.H.S.}$

$A^3 - 6A^2 + 5A + 11I = O$. Pre-multiplying by A^{-1}

$\Rightarrow A^2 - 6A + 5I + 11A^{-1} = O \Rightarrow A^{-1} = -\frac{1}{11}(A^2 - 6A + 5I)$ 1

$\therefore A^{-1} = \begin{bmatrix} -3/11 & 4/11 & 5/11 \\ 9/11 & -1/11 & -4/11 \\ 5/11 & -3/11 & -1/11 \end{bmatrix}$ 1

OR

Let,

$A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$

System of equation in Matrix form: A · X = B

$|A| = 3(2-3) + 2(4+4) + 3(-6-4) = -17 \neq 0$ 1

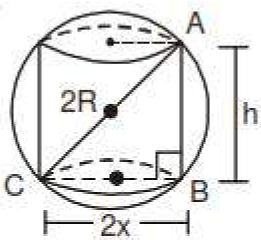
Solution matrix, $X = A^{-1} \cdot B$. 1

$$(\text{adj } A) = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$ 2

$$\therefore \quad A^{-1} = \frac{-1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$ $\frac{1}{2}$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow x = 1, y = 2, x = 3$$ $1\frac{1}{2}$

25. Correct Figure 1



In rt. ΔABC; $4x^2 + h^2 = 4R^2$, $x^2 = \dfrac{4R^2 - h^2}{4}$ 1

V(Volume of cylinder) $= \pi x^2 h = \dfrac{\pi}{4}(4R^2 h - h^3)$ 1

$V'(h) = \dfrac{\pi}{4}(4R^2 - 3h^2)$; $V''(h) = \dfrac{\pi}{4}(-6h)$ $\frac{1}{2}+\frac{1}{2}$

$V'(h) = 0 \Rightarrow h = \dfrac{2R}{\sqrt{3}}$ 1

$V''\left(\dfrac{2R}{\sqrt{3}}\right) = \dfrac{-6\pi}{4}\left(\dfrac{2R}{\sqrt{3}}\right) < 0 \Rightarrow$ Volume 'V' is max. $\frac{1}{2}$

for $h = \dfrac{2R}{\sqrt{3}}$

Max. Volume: $V = \dfrac{4}{3\sqrt{3}}\pi R^3$ $\frac{1}{2}$

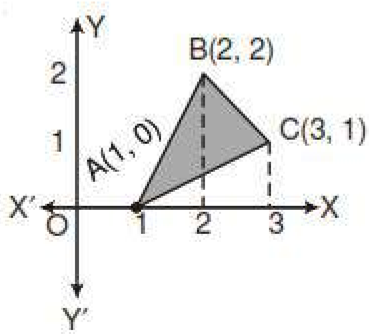
26. $\left.\begin{array}{l} E_1 = \text{Event of selecting first bag} \\ E_2 = \text{Event of selecting second bag} \\ A = \text{Event both balls drawn are red.} \end{array}\right\}$ 1

$$P(E_1) = P(E_2) = \frac{1}{2}; P(A|E_1) = \frac{{}^5C_2}{{}^9C_2} = \frac{20}{72}; P(A|E_2) = \frac{{}^3C_2}{{}^9C_2} = \frac{6}{72}$$ 1+2

$$P(E_2|A) = \frac{P(E_2)\cdot P(A|E_2)}{P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{6}{72}}{\frac{1}{2}\cdot\frac{20}{72}+\frac{1}{2}\cdot\frac{6}{72}} = \frac{6}{26} = \frac{3}{13}$$ $1\frac{1}{2}+\frac{1}{2}$

27. Correct Figure 1



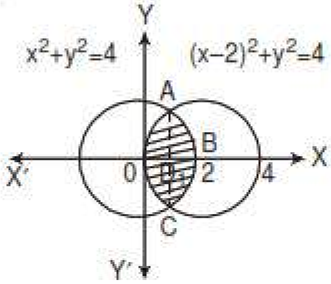
$\left.\begin{array}{l} \text{Equation of line AB}: y = 2(x-1) \\ \text{Equation of line BC}: y = 4 - x \\ \text{Equation of line AC}: y = \frac{1}{2}(x-1) \end{array}\right\}$ $1\frac{1}{2}$

$$\text{ar}(\Delta ABC) = 2\int_1^2 (x-1)\,dx + \int_2^3 (4-x)\,dx - \frac{1}{2}\int_1^3 (x-1)\,dx$$ $1\frac{1}{2}$

$$= (x-1)^2\Big]_1^2 - \frac{1}{2}(4-x)^2\Big]_2^3 - \frac{1}{4}(x-1)^2\Big]_1^3$$ $1\frac{1}{2}$

$$= 1 + \frac{3}{2} - 1 = \frac{3}{2} \text{ sq. units}$$ $\frac{1}{2}$

OR



Correct Figure 1

Getting the point of intersection as x = 1 1

Area (OABCO) = 4 × ar(ABD)

$$= 4\int_1^2 \sqrt{2^2 - x^2}\, dx \qquad 2$$

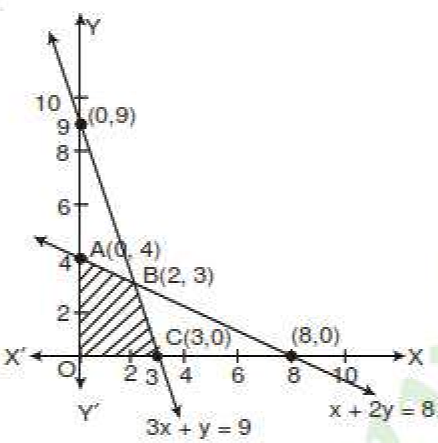
$$= 4\left\{\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right\}_1^2 \qquad 1$$

$$= \left(\frac{8\pi}{3} - 2\sqrt{3}\right) \qquad 1$$

28. let the company produce: Goods A = x units

Goods B = y units

then, the linear programming problem is:



Maximize profit: z = 40x + 50y (In ₹) $\frac{1}{2}$

Subject to constraints:

$$\left.\begin{array}{l} 3x + y \le 9 \\ x + 2y \le 8 \\ x, y \ge 0 \end{array}\right\} \qquad 2\frac{1}{2}$$

Correct graph. 2

Corner point	Value of z (₹)	
A(0, 4)	200	
B(2, 3)	230 (Max)	$\frac{1}{2}$
C(3, 0)	120	

∴ Maximum profit = ₹ 230 at:

Goods A produced = 2 units, Goods B produced = 3 units $\frac{1}{2}$

29. Let $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

$$(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 0 & 1 & 3 \end{vmatrix} = -9\hat{i} - 3\hat{j} + \hat{k}$$ 2

Vector equation of plane is:

$$\{\vec{r} - (\hat{i} + \hat{j} - 2\hat{k})\} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) = 0$$

$\Rightarrow \vec{r} \cdot (-9\hat{i} - 3\hat{j} + \hat{k}) + 14 = 0$ 1

Cartesian Equation of plane is: $-9x - 3y + z + 14 = 0$ 1

Equation of plane through (2, 3, 7) and parallel to above plane is:

$-9(x - 2) - 3(y - 3) + (z - 7) = 0$

$\Rightarrow -9x - 3y + z + 20 = 0$ 1

Distance between parallel planes $= \left|\dfrac{-14 + 20}{\sqrt{91}}\right| = \dfrac{6}{\sqrt{91}}$ 1

OR

Equation of line: $\dfrac{x-2}{3} = \dfrac{y+1}{4} = \dfrac{z-2}{2}$ 1

Equation of plane: $\begin{vmatrix} x-2 & y & z-3 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0$ 1

$\Rightarrow -3(x - 2) + 3y - 3(z - 3) = 0$

$\Rightarrow x - y + z - 5 = 0$ 1

General point on line: $\dfrac{x-2}{3} = \dfrac{y+1}{4} = \dfrac{z-2}{2} = k$ (say)

is: $P(3k + 2, 4k - 1, 2k + 2)$; Putting in the equation of plane 1

we get, $3k + 2 - 4k + 1 + 2k + 2 = 5 \Rightarrow k = 0$ 1

$\therefore$ Point of intersection is: $(2, -1, 2)$ 1

SET-1

Series BVM/3

कोड नं.
Code No. **65/3/1**

रोल नं.
Roll No.

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **11** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **29** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **29** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 100*

Time allowed : 3 hours | *Maximum Marks : 100*

सामान्य निर्देश :

(i) **सभी** प्रश्न अनिवार्य हैं ।

(ii) इस प्रश्न-पत्र में **29** प्रश्न हैं जो चार खण्डों में विभाजित हैं : अ, ब, स तथा द । खण्ड अ में **4** प्रश्न हैं जिनमें से प्रत्येक **एक अंक** का है । खण्ड ब में **8** प्रश्न हैं जिनमें से प्रत्येक **दो अंक** का है । खण्ड स में **11** प्रश्न हैं जिनमें से प्रत्येक **चार अंक** का है । खण्ड द में **6** प्रश्न हैं जिनमें से प्रत्येक **छः अंक** का है ।

(iii) खण्ड अ में सभी प्रश्नों के उत्तर एक शब्द, एक वाक्य अथवा प्रश्न की आवश्यकतानुसार दिए जा सकते हैं ।

(iv) पूर्ण प्रश्न-पत्र में विकल्प नहीं हैं । फिर भी खण्ड अ के 1 प्रश्न में, खण्ड ब के 3 प्रश्नों में, खण्ड स के 3 प्रश्नों में तथा खण्ड द के 3 प्रश्नों में आन्तरिक विकल्प है । ऐसे सभी प्रश्नों में से आपको एक ही विकल्प हल करना है ।

(v) कैल्कुलेटर के प्रयोग की अनुमति **नहीं** है । यदि आवश्यक हो, तो आप लघुगणकीय सारणियाँ माँग सकते हैं ।

General Instructions :

(i) **All** questions are compulsory.

(ii) The question paper consists of **29** questions divided into four sections : A, B, C and D. Section A comprises of **4** questions of **one mark** each, Section B comprises of **8** questions of **two marks** each, Section C comprises of **11** questions of **four marks** each and Section D comprises of **6** questions of **six marks** each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is **not** permitted. You may ask for logarithmic tables, if required.

खण्ड अ

SECTION A

प्रश्न संख्या 1 से 4 तक प्रत्येक प्रश्न 1 अंक का है ।

Questions number 1 to 4 carry 1 mark each.

1. यदि वर्ग आव्यूह A की कोटि 3 और $|A| = 4$ है, तो $|-2A|$ का मान लिखिए ।

If A is a square matrix of order 3 with $|A| = 4$, then write the value of $|-2A|$.

2. यदि $y = \sin^{-1} x + \cos^{-1} x$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए ।

If $y = \sin^{-1} x + \cos^{-1} x$, find $\frac{dy}{dx}$.

3. अवकल समीकरण

$$\left(\frac{d^4y}{dx^4}\right)^2 = \left[x + \left(\frac{dy}{dx}\right)^2\right]^3$$

की कोटि व घात लिखिए ।

Write the order and the degree of the differential equation

$$\left(\frac{d^4y}{dx^4}\right)^2 = \left[x + \left(\frac{dy}{dx}\right)^2\right]^3.$$

4. यदि एक रेखा के दिक्-अनुपात – 18, 12, – 4 हैं, तो इसके दिक्-कोसाइन क्या हैं ?

अथवा

बिन्दु (– 2, 4, – 5) से गुज़रने वाली उस रेखा का कार्तीय समीकरण ज्ञात कीजिए जो रेखा $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$ के समांतर है ।

If a line has the direction ratios – 18, 12, – 4, then what are its direction cosines ?

OR

Find the cartesian equation of the line which passes through the point (– 2, 4, – 5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

खण्ड ब

SECTION B

प्रश्न संख्या 5 से 12 तक प्रत्येक प्रश्न के 2 अंक हैं ।
Questions number 5 to 12 carry 2 marks each.

5. सभी वास्तविक संख्याओं के समुच्चय $\mathbb{R}$ पर परिभाषित संक्रिया $* : a * b = \sqrt{a^2 + b^2}$ है । $\mathbb{R}$ में $*$ के सापेक्ष तत्समक अवयव, यदि इसका अस्तित्व है, ज्ञात कीजिए ।

If $*$ is defined on the set $\mathbb{R}$ of all real numbers by $* : a * b = \sqrt{a^2 + b^2}$, find the identity element, if it exists in $\mathbb{R}$ with respect to $*$.

6. यदि $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ तथा $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$ है, तो k, a और b के मान ज्ञात कीजिए ।

If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the values of k, a and b.

7. ज्ञात कीजिए :

$$\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}}\, dx, \; 0 < x < \pi/2$$

Find :

$$\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}}\, dx,\ \ 0 < x < \pi/2$$

8. ज्ञात कीजिए :

$$\int \frac{\sin (x - a)}{\sin (x + a)}\, dx$$

अथवा

ज्ञात कीजिए :

$$\int (\log x)^2\, dx$$

Find :

$$\int \frac{\sin (x - a)}{\sin (x + a)}\, dx$$

OR

Find :

$$\int (\log x)^2\, dx$$

9. स्वेच्छ अचरों 'm' तथा 'a' को विलुप्त करते हुए वक्रों के कुल $y^2 = m\,(a^2 - x^2)$ को निरूपित करने वाला अवकल समीकरण बनाइए ।

Form the differential equation representing the family of curves $y^2 = m\,(a^2 - x^2)$ by eliminating the arbitrary constants 'm' and 'a'.

10. सदिशों $\vec{a}$ तथा $\vec{b}$, जहाँ $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ तथा $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$, दोनों के लम्बवत् एक मात्रक सदिश ज्ञात कीजिए ।

अथवा

दिखाइए कि सदिश $\hat{i} - 2\hat{j} + 3\hat{k}, -2\hat{i} + 3\hat{j} - 4\hat{k}$ तथा $\hat{i} - 3\hat{j} + 5\hat{k}$ समतलीय हैं ।

Find a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

OR

Show that the vectors $\hat{i} - 2\hat{j} + 3\hat{k}, -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar.

11. एक परिवार की फोटो हेतु माँ, पिता व बेटे को एक लाइन में यादृच्छया खड़ा किया जाता है । यदि दो घटनाएँ A और B निम्न रूप में परिभाषित हों, तो P(B/A) ज्ञात कीजिए :

घटना A : बेटा एक किनारे पर, घटना B : पिता बीच में

Mother, father and son line up at random for a family photo. If A and B are two events given by A = Son on one end, B = Father in the middle, find P(B/A).

12. मान लीजिए X एक यादृच्छिक चर है जिसके संभावित मूल्य x_1, x_2, x_3, x_4 इस प्रकार हैं :

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4).$$

X का प्रायिकता बंटन ज्ञात कीजिए ।

अथवा

एक सिक्का 5 बार उछाला जाता है । (i) कम-से-कम 4 चित, और (ii) अधिक-से-अधिक 4 चित प्राप्त करने की प्रायिकता ज्ञात कीजिए ।

Let X be a random variable which assumes values x_1, x_2, x_3, x_4 such that

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4).$$

Find the probability distribution of X.

OR

A coin is tossed 5 times. Find the probability of getting (i) at least 4 heads, and (ii) at most 4 heads.

खण्ड स
SECTION C

प्रश्न संख्या 13 से 23 तक प्रत्येक प्रश्न के 4 अंक हैं ।
Questions number 13 to 23 carry 4 marks each.

13. दिखाइए कि पूर्णांक समुच्चय Z पर परिभाषित संबंध R = {(a, b) : (a – b), 2 से विभाजित है} एक तुल्यता संबंध है ।

अथवा

यदि $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$ है, तो दिखाइए कि सभी $x \neq \frac{2}{3}$ के लिए, $fof(x) = x$ है ।

f का प्रतिलोम भी ज्ञात कीजिए ।

Show that the relation R on the set Z of all integers, given by R = {(a, b) : 2 divides (a – b)} is an equivalence relation.

OR

If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $fof(x) = x$ for all $x \neq \frac{2}{3}$. Also, find the inverse of f.

14. यदि $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, $x > 0$ है, तो x का मान ज्ञात कीजिए और अत: $\sec^{-1}\left(\frac{2}{x}\right)$ का मान ज्ञात कीजिए ।

If $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, $x > 0$, find the value of x and hence find the value of $\sec^{-1}\left(\frac{2}{x}\right)$.

15. सारणिकों के गुणधर्मों का प्रयोग करके, सिद्ध कीजिए कि

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

Using properties of determinants, prove that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

16. यदि $\sin y = x \sin (a + y)$ है, तो सिद्ध कीजिए कि

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

अथवा

यदि $(\sin x)^y = x + y$ है, तो $\frac{dy}{dx}$ ज्ञात कीजिए ।

If $\sin y = x \sin (a + y)$, prove that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

OR

If $(\sin x)^y = x + y$, find $\frac{dy}{dx}$.

17. यदि $y = (\sec^{-1} x)^2,\ x > 0$ हो, तो दिखाइए कि

$$x^2 (x^2 - 1) \frac{d^2y}{dx^2} + (2x^3 - x) \frac{dy}{dx} - 2 = 0$$

If $y = (\sec^{-1} x)^2,\ x > 0$, show that

$$x^2 (x^2 - 1) \frac{d^2y}{dx^2} + (2x^3 - x) \frac{dy}{dx} - 2 = 0$$

18. वक्र $y = \frac{x - 7}{(x - 2)(x - 3)}$ जहाँ x-अक्ष को काटता है, उस बिन्दु से वक्र पर स्पर्श-रेखा व अभिलंब के समीकरण ज्ञात कीजिए ।

Find the equations of the tangent and the normal to the curve $y = \frac{x - 7}{(x - 2)(x - 3)}$ at the point where it cuts the x-axis.

19. ज्ञात कीजिए :

$$\int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$$

Find :

$$\int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$$

20. सिद्ध कीजिए कि

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$

अत:

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$$ का मूल्यांकन कीजिए ।

Prove that

$$\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$$ and hence evaluate

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}.$$

21. अवकल समीकरण $\frac{dy}{dx} = \frac{x+y}{x-y}$ को हल कीजिए ।

अथवा

अवकल समीकरण हल कीजिए :

$$(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx$$

Solve the differential equation :

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

OR

Solve the differential equation :

$$(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx$$

22. मान लीजिए $\vec{a}$, $\vec{b}$ और $\vec{c}$ ऐसे तीन सदिश हैं जिनके लिए $|\vec{a}| = 1$, $|\vec{b}| = 2$ तथा $|\vec{c}| = 3$ हैं । यदि सदिश $\vec{b}$ का सदिश $\vec{a}$ पर प्रक्षेप और सदिश $\vec{c}$ का सदिश $\vec{a}$ पर प्रक्षेप एक-दूसरे के बराबर हैं तथा सदिश $\vec{b}$ और $\vec{c}$ लम्बवत् हों, तो $|3\vec{a} - 2\vec{b} + 2\vec{c}|$ का मान ज्ञात कीजिए ।

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $|\vec{c}| = 3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$; and $\vec{b}$, $\vec{c}$ are perpendicular to each other, then find $|3\vec{a} - 2\vec{b} + 2\vec{c}|$.

23. λ का मान ज्ञात कीजिए जिसके लिए निम्नलिखित रेखाएँ परस्पर लम्बवत् हैं :

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}; \quad \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$

अत: ज्ञात कीजिए कि क्या ये रेखाएँ एक-दूसरे को काटती हैं या नहीं ।

Find the value of λ for which the following lines are perpendicular to each other :

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}; \quad \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$

Hence, find whether the lines intersect or not.

खण्ड द

SECTION D

प्रश्न संख्या 24 से 29 तक प्रत्येक प्रश्न के 6 अंक हैं ।

Questions number 24 to 29 carry 6 marks each.

24. यदि $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$ है, तो A^{-1} ज्ञात कीजिए ।

अत: निम्न समीकरण निकाय को हल कीजिए :

$x + y + z = 6,$

$y + 3z = 11$

तथा $x - 2y + z = 0$

अथवा

प्रारंभिक रूपांतरणों द्वारा, निम्नलिखित आव्यूह का व्युत्क्रम ज्ञात कीजिए :

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, find A^{-1}.

Hence, solve the following system of equations :

$x + y + z = 6,$

$y + 3z = 11$

and $x - 2y + z = 0$

OR

Find the inverse of the following matrix, using elementary transformations :

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

25. दिखाइए कि अधिकतम आयतन के और दिए गए पृष्ठीय क्षेत्रफल के बेलन (जिसका ऊपरी भाग खुला हो) की ऊँचाई, बेलन के आधार की त्रिज्या के बराबर होगी ।

Show that the height of a cylinder, which is open at the top, having a given surface area and greatest volume, is equal to the radius of its base.

26. समाकलन के प्रयोग से, उस त्रिभुज का क्षेत्रफल ज्ञात कीजिए जिसके शीर्ष (–1, 1), (0, 5) तथा (3, 2) हैं ।

अथवा

समाकलन के प्रयोग से वक्रों $(x - 1)^2 + y^2 = 1$ तथा $x^2 + y^2 = 1$ से परिबद्ध क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

Find the area of the triangle whose vertices are (–1, 1), (0, 5) and (3, 2), using integration.

OR

Find the area of the region bounded by the curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$, using integration.

27. बिन्दुओं (2, 5, – 3), (– 2, – 3, 5) और (5, 3, – 3) से गुज़रने वाले समतल के सदिश व कार्तीय समीकरण ज्ञात कीजिए । यह समतल, एक रेखा, जो बिन्दुओं (3, 1, 5) तथा (– 1, – 3, – 1) से गुज़रती है, को जिस बिन्दु पर काटता है उसे भी ज्ञात कीजिए ।

अथवा

समतलों $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ तथा $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ के प्रतिच्छेदन से होकर जाने वाले उस समतल का समीकरण ज्ञात कीजिए, जो x-अक्ष के समांतर हो । अत: इस समतल की x-अक्ष से दूरी ज्ञात कीजिए ।

Find the vector and cartesian equations of the plane passing through the points (2, 5, – 3), (– 2, – 3, 5) and (5, 3, – 3). Also, find the point of intersection of this plane with the line passing through points (3, 1, 5) and (– 1, – 3, – 1).

OR

Find the equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis. Hence, find the distance of the plane from x-axis.

28. दो डिब्बे I और II दिए गए हैं । डिब्बे I में 3 लाल व 6 काली गेंदें हैं । डिब्बे II में 5 लाल व 'n' काली गेंदें हैं । दोनों डिब्बों I और II में से एक डिब्बे को यादृच्छया चुना जाता है और उसमें से यादृच्छया एक गेंद निकाली जाती है । यदि निकाली गई गेंद लाल है और उसके डिब्बे II से आने की प्रायिकता $\frac{3}{5}$ हो, तो 'n' का मान ज्ञात कीजिए ।

There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and 'n' black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$, find the value of 'n'.

29. एक कंपनी प्लाइवुड के दो प्रकार के अनूठे स्मृति-चिह्न का निर्माण करती है । A प्रकार के प्रति स्मृति-चिह्न के निर्माण में 5 मिनट काटने और 10 मिनट जोड़ने में लगते हैं । B प्रकार के प्रति स्मृति-चिह्न के लिए 8 मिनट काटने और 8 मिनट जोड़ने में लगते हैं । दिया गया है कि काटने के लिए कुल समय 3 घंटे 20 मिनट तथा जोड़ने के लिए 4 घंटे उपलब्ध हैं । प्रत्येक A प्रकार के स्मृति-चिह्न पर ₹ 50 और प्रत्येक B प्रकार के स्मृति-चिह्न पर ₹ 60 का लाभ होना है । ज्ञात कीजिए कि लाभ के अधिकतमीकरण के लिए प्रत्येक प्रकार के कितने-कितने स्मृति-चिह्नों का कंपनी द्वारा निर्माण होना चाहिए । इस उपर्युक्त समस्या को रैखिक प्रोग्रामन समस्या में परिवर्तित करके आलेख विधि से हल कीजिए तथा अधिकतम लाभ भी ज्ञात कीजिए ।

A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. The profit is ₹ 50 each for type A and ₹ 60 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize profit ? Formulate the above LPP and solve it graphically and also find the maximum profit.

Strictly Confidential — (For Internal and Restricted Use Only)

Senior School Certificate Examination

March 2019

Marking Scheme — Mathematics (041) 65/3/1, 65/3/2, 65/3/3

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. **Evaluation is a 10-12 days mission for all of us. Hence, it is necessary that you put in your best efforts in this process.**
2. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.**
3. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
4. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled.
5. If a question does not have any parts, marks must be awarded in the left hand margin and encircled.
6. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
7. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
8. A full scale of marks 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.
9. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 25 answer books per day.
10. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
11. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as (X) and awarded zero (0) Marks.
12. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
13. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
14. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
15. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/3/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|-2A| = (-2)^3 \cdot |A|$ $\frac{1}{2}$

$= -8 \cdot 4 = -32$ $\frac{1}{2}$

2. $y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$ $\frac{1}{2}+\frac{1}{2}$

3. order 4, degree 2 $\frac{1}{2}+\frac{1}{2}$

4. $\sqrt{(-18)^2+(12)^2+(-4)^2} = 22$ $\frac{1}{2}$

$\therefore$ DC's are $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$ or $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$ $\frac{1}{2}$

OR

D·R's of required line are 3, –5, 6 $\frac{1}{2}$

Equation of line is $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$ $\frac{1}{2}$

SECTION B

5. Let $e \in \mathbb{R}$ be the identity element.

then $a*e = e*a = a$ 1

$\Rightarrow a^2 + e^2 = e^2 + a^2 = a^2 \Rightarrow e^2 = 0 \Rightarrow e = 0.$ 1

$\therefore$ Identity element is $0 \in \mathbb{R}$

6. $kA = k\begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix}$

$\therefore \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix} \Rightarrow 2k = 3a, 3k = 2b \text{ and } -4k = 24$ $\frac{1}{2}$

$\Rightarrow k = -6,\ a = \frac{-12}{3} = -4,\ b = \frac{-18}{2} = -9$ $1\frac{1}{2}$

7. $\int \frac{\sin x - \cos x}{\sqrt{1+\sin 2x}} dx = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ 1

$= -\log|\sin x + \cos x| + c$ 1

8. $\int \frac{\sin(x-a)}{\sin(x+a)}dx = \int \frac{\sin(x+a-2a)}{\sin(x+a)}dx$ $\frac{1}{2}$

$= \int\left[\frac{\sin(x+a)\cos 2a}{\sin(x+a)} - \frac{\cos(x+a)\sin 2a}{\sin(x+a)}\right]dx$ $\frac{1}{2}$

$= x\cdot\cos 2a - \sin 2a\cdot\log|\sin(x+a)| + c$ $\frac{1}{2}+\frac{1}{2}$

OR

$\int(\log x)^2\cdot 1dx = (\log x)^2\cdot x - \int 2\cdot\log x\cdot\frac{1}{x}\cdot x\,dx$ 1

$= x\cdot(\log x)^2 - \left\{\log x\cdot 2x - \int\frac{1}{x}\cdot 2xdx\right\}$ $\frac{1}{2}$

$= x(\log x)^2 - 2x\log x + 2x + c$ $\frac{1}{2}$

9. $y^2 = m(a^2 - x^2) \Rightarrow 2y\frac{dy}{dx} = -2mx$ $\frac{1}{2}$

or $y\frac{dy}{dx} = -mx$ ...(i)

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -m$ ...(ii) $\frac{1}{2}$

form (i) and (ii) we get $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$ 1

or $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

10. A vector perpendicular to both $\vec{a}$ and $\vec{b} = \vec{a}\times\vec{b} = 19\hat{j}+19\hat{k}$ or $\hat{j}+\hat{k}$ 1

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b} = \frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$ 1

OR

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a},\vec{b},\vec{c}$ are coplanar if $\vec{a}\cdot\vec{b}\times\vec{c} = 0$ $\frac{1}{2}$

$$\vec{a}\cdot\vec{b}\times\vec{c}=\begin{vmatrix}1 & -2 & 3\\ -2 & 3 & -4\\ 1 & -3 & 5\end{vmatrix}=1(3)+2(-6)+3(3)$$
$$=3-12+9=0$$ $1+\frac{1}{2}$

Hence $\vec{a},\vec{b},\vec{c}$ are coplanar

11. $A=\{(S, F, M), (S, M, F), (M, F, S), (F, M, S)\}$
$B=\{(S, F, M), (M, F, S)\}$ 1

Total number of possible arrangements = 6

$$P(B|A)=\frac{P(B\cap A)}{P(A)}$$

$$=\frac{2/6}{4/6}=\frac{1}{2}$$ 1

12. Given $2\,P(X=x_1)=3P(X=x_2)=P(X=x_3)=5P(X=x_4)$

Let $P(X=x_3)=k$, then $P(X=x_1)=\frac{k}{2}, P(X=x_2)=\frac{k}{3}$ and $P(X=x_4)=\frac{k}{5}$ $\frac{1}{2}$

$\therefore\ k+\frac{k}{2}+\frac{k}{3}+\frac{k}{5}=1\Rightarrow k=\frac{30}{61}$ 1

$\therefore$ Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

$\frac{1}{2}$

OR

(i) P(at least 4 heads) = $P(r\ge 4)=P(4)+P(5)$

$$={}^5C_4\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4+{}^5C_5\left(\frac{1}{2}\right)^5=6\left(\frac{1}{2}\right)^5=\frac{6}{32}\text{ or }\frac{3}{16}$$ 1

(ii) P(at most 4 heads) = $P(r\le 4)=1-P(5)$

$$=1-\left(\frac{1}{2}\right)^5=\frac{31}{32}$$ 1

SECTION C

13. (i) For $a \in Z, (a, a) \in R \because a - a = 0$ is divisible by 2

$\therefore$ R is reflexive ...(i) 1

Let $(a, b) \in R$ for $a, b \in Z$, then $a - b$ is divisible by 2

$\Rightarrow (b - a)$ is also divisible by 2

$\therefore (b, a) \in R \Rightarrow$ R is symmetric ...(ii) 1

For $a, b, c, \in Z$, Let $(a, b) \in R$ and $(b, c) \in R$

$\therefore a - b = 2p, p \in Z$, and $b - c = 2q, q \in Z$,

adding, $a - c = 2(p + q) \Rightarrow (a - c)$ is divisible by 2

$\Rightarrow (a, c) \in R$, so R is transitive ...(iii) $1\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow$ R is an equivalence relation. $\frac{1}{2}$

OR

$\text{fof}(x) = f\left(\frac{4x+3}{6x-4}\right)$ 1

$$= \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$ 1

$\Rightarrow \text{fof}(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{34x}{34} = x$ 1

Since $\text{fof}(x) = x \Rightarrow \text{fof} = I \Rightarrow f^{-1} = f$ 1

14. Given $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right), x > 0$

$\Rightarrow \tan^{-1}x - \left(\frac{\pi}{2} - \tan^{-1}x\right) = \frac{\pi}{6}$ 1

$\Rightarrow 2\tan^{-1}x = \frac{2\pi}{3} \Rightarrow \tan^{-1}x = \frac{\pi}{3}$ 1

$\Rightarrow x = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ $\therefore \sec^{-1}\frac{2}{x} = \sec^{-1}\frac{2}{\sqrt{3}} = \frac{\pi}{6}$ 1+1

15. Let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$R_1 \to R_1 - (R_2 + R_3) \Rightarrow \Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ 1

$\therefore \ \Delta = -2\begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = -\frac{2}{b}\begin{vmatrix} 0 & bc & b \\ b & bc+ab & b \\ c & bc & a+b \end{vmatrix}$ $1+\frac{1}{2}$

$C_2 \to C_2 - cC_3$

$\Rightarrow \Delta = -\frac{2}{b}\begin{vmatrix} 0 & 0 & b \\ b & ab & b \\ c & -ac & a+b \end{vmatrix}$ 1

$= -\frac{2}{b} \cdot b \cdot (-abc - abc) = 4abc.$ $\frac{1}{2}$

16. $\sin y = x \cdot \sin(a+y) \Rightarrow x = \frac{\sin y}{\sin(a+y)}$ $\frac{1}{2}$

differentiating w.r.t. y, we get

$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$ $1\frac{1}{2}$

$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$ $1\frac{1}{2}$

$\therefore \ \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ $\frac{1}{2}$

OR

$(\sin x)^y = (x+y) \Rightarrow y \cdot \log \sin x = \log(x+y)$ 1

differentiating w.r.t. x, we get

$y \cdot \cot x + \log \sin x \cdot \frac{dy}{dx} = \frac{1}{x+y}\left(1 + \frac{dy}{dx}\right)$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x+y} - y\cot x}{\log \sin x - \frac{1}{x+y}}$$ 1

$$= \frac{1 - y(x+y)\cot x}{(x+y)\log \sin x - 1}$$ $\frac{1}{2}$

17. $y = (\sec^{-1} x)^2,\ x > 0$

$$\frac{dy}{dx} = 2\sec^{-1} x \cdot \frac{1}{x\sqrt{x^2-1}}$$ 1

$$\Rightarrow x\sqrt{x^2-1}\frac{dy}{dx} = 2\sqrt{y}$$ $\frac{1}{2}$

squaring both sides, we get

$$x^2(x^2-1)\left(\frac{dy}{dx}\right)^2 = 4y \quad \text{or} \quad (x^4 - x^2)\left(\frac{dy}{dx}\right)^2 = 4y$$ $\frac{1}{2}$

differentiating w.r.t. x.

$$(x^4 - x^2)2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + (4x^3 - 2x)\left(\frac{dy}{dx}\right)^2 = 4 \cdot \frac{dy}{dx}$$ $1\frac{1}{2}$

$$\Rightarrow x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} - 2 = 0$$ $\frac{1}{2}$

18. Curve $y = \frac{x-7}{(x-2)(x-3)}$ cuts at x – axis at the point x = 7, y = 0 i.e. (7, 0) $\frac{1}{2}$

$$\frac{dy}{dx} = \frac{(x^2 - 5x + 6)\cdot 1 - (x-7)(2x-5)}{(x^2-5x+6)^2}$$ $\frac{1}{2}$

at (7, 0) $\frac{dy}{dx} = \frac{20}{(20)^2} = \frac{1}{20}$ $\frac{1}{2}$

$\therefore$ Slope of tangent at (7, 0) is $\frac{1}{20}$ $\frac{1}{2}$

and slope of Normal at (7, 0) is –20 $\frac{1}{2}$

Equation of tangent at (7, 0) is $y - 0 = \frac{1}{20}(x - 7)$

or $x - 20y - 7 = 0$ 1

Equation of Normal at (7, 0) is $y - 0 = -20(x - 7)$

or $20x + y = 140$. $\frac{1}{2}$

19. $I = \int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$

Put $\sin^2 x = t \Rightarrow \sin 2x\, dx = dt$ $\frac{1}{2}$

$\therefore\ I = \int \frac{dt}{(t+1)(t+3)} = \int \left(\frac{1/2}{t+1} + \frac{-1/2}{t+3} \right) dt$ $1\frac{1}{2}$

$= \frac{1}{2} \log|t+1| - \frac{1}{2} \log|t+3| + c$ $1\frac{1}{2}$

$= \frac{1}{2} \log(\sin^2 x + 1) - \frac{1}{2} \log(\sin^2 + 3) + c.$ $\frac{1}{2}$

20. RHS $= \int_a^b f(a+b-x)\, dx = -\int_b^a f(t)\, dt$, where $a + b - x = t$, $dx = -dt$ $\frac{1}{2}$

$= \int_a^b f(t)\, dt = \int_a^b f(x)\, dx =$ LHS $\frac{1}{2}$

Let $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ ...(i) $\frac{1}{2}$

$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/2 - x)}}{\sqrt{\cos(\pi/2 - x)} + \sqrt{\sin(\pi/2 - x)}} dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ ...(ii) $1\frac{1}{2}$

adding (i) and (ii) to get $2I = \int_{\pi/6}^{\pi/3} 1 \cdot dx = x]_{\pi/6}^{\pi/3} = \pi/6.$ $\frac{1}{2}$

$\Rightarrow I = \frac{\pi}{12}$ $\frac{1}{2}$

21. $\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$

Put y/x = v so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

$\therefore \; v + x\frac{dv}{dx} = \frac{1+v}{1-v} \Rightarrow \; x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v}$

$\Rightarrow \int \frac{1-v}{1+v^2} dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2} dv - \frac{1}{2}\int \frac{2v}{1+v^2} dv = \int \frac{dx}{x}$ $1+\frac{1}{2}$

$\Rightarrow \; \tan^{-1} v = \frac{1}{2}\log\left|1+v^2\right| + \log|x| + c$ 1

$\Rightarrow \; \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x| + c$ 1

or $\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log|x^2+y^2| + c$

OR

$(1+x^2)dy + 2xy\,dx = \cot x.dx.$

$\Rightarrow \; \frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{\cot x}{1+x^2}$ 1

I.F. = $e^{\int\frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$ 1

$\therefore$ Solution is, $y\cdot(1+x^2) = \int \cot x\,dx = \log|\sin x| + c$ 1+1

or $y = \frac{1}{1+x^2}\cdot\log|\sin x| + \frac{c}{1+x^2}$

22. Given $\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|} \;\therefore\; \vec{b}\cdot\vec{a} = \vec{c}\cdot\vec{a}$ ...(i) $\frac{1}{2}$

$\vec{b} \perp \vec{c} \Rightarrow \vec{b}\cdot\vec{c} = 0$ ...(ii) $\frac{1}{2}$

$(|3\vec{a}-2\vec{b}+2\vec{c}|)^2 = 9|\vec{a}|^2+4|\vec{b}|^2+4|\vec{c}|^2-12\,\vec{a}\cdot\vec{b}-8\,\vec{b}\cdot\vec{c}+12\,\vec{a}\cdot\vec{c}$ 1

$= 9(1)^2+4(2)^2+4(3)^2$ [using (i) and (ii)] 1

$= 9 + 16 + 36 = 61$

$\Rightarrow |3\vec{a}-2\vec{b}+2\vec{c}| = \sqrt{61}$ 1

23. Writing the equations of given lines in standard form, as

$\dfrac{x-5}{5\lambda+2}=\dfrac{y-2}{-5}=\dfrac{z-1}{1};\ \dfrac{x}{1}=\dfrac{y+\frac{1}{2}}{2\lambda}=\dfrac{z-1}{3}$ $\frac{1}{2}$

lines are perpendicular to each other,

$\Rightarrow (5\lambda+2)\cdot 1+(-5)(2\lambda)+1(3)=0$ $\frac{1}{2}$

$-5\lambda+5=0 \Rightarrow \lambda=1$ $\frac{1}{2}$

$\therefore$ lines are $\dfrac{x-5}{7}=\dfrac{y-2}{-5}-\dfrac{z-1}{1};\ \dfrac{x}{1}=\dfrac{y+\frac{1}{2}}{2}=\dfrac{z-1}{3}$ $\frac{1}{2}$

Shortest distance between these lines $= \dfrac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|} = \dfrac{\left|\left(5\hat{i}+\frac{5}{2}\hat{j}\right)\cdot(-17\hat{i}-20\hat{j}+19\hat{k})\right|}{|\vec{b}_1\times\vec{b}_2|}$ 1

$= \dfrac{135}{|\vec{b}_1\times\vec{b}_2|} \neq 0$ $\frac{1}{2}$

$\therefore$ lines are not intersecting. $\frac{1}{2}$

SECTION D

24. $|A| = 1(7)-1(-3)+1(-1)=9$ 1

$(\text{adj } A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$ 2

$\Rightarrow A^{-1} = \dfrac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$ $\frac{1}{2}$

Given equations can be written as $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

or $AX = B \Rightarrow X = A^{-1}B$ $\frac{1}{2}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ $1\frac{1}{2}$

$\therefore$ x = 1, y = 2, z = 3 $\frac{1}{2}$

OR

Let: $\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1

$$R_1 \leftrightarrow R_3 \begin{bmatrix} 3 & 7 & 2 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$$

$$\begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{matrix} \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$$

$$R_3 \to R_3 - 2R_1 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & -5 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & 0 & -2 \end{bmatrix} A$$

$$R_3 \to R_3 + 5R_2 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ -2 & 5 & -2 \end{bmatrix} A$$

$$\begin{matrix} R_1 \to R_1 - 4R_2 \\ R_3 \to -R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

$$R_1 \to R_1 - R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

4

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$$ 1

25. Let Given surface area of open cylinder be S.

Then $S = 2\pi rh + \pi r^2$

$\Rightarrow h = \dfrac{S - \pi r^2}{2\pi r}$ 1

Volume $V = \pi r^2 h$ $\frac{1}{2}$

$V = \pi r^2\left[\dfrac{S-\pi r^2}{2\pi r}\right] = \dfrac{1}{2}[Sr - \pi r^3]$ 1

$\dfrac{dV}{dr} = \dfrac{1}{2}[S - 3\pi r^2]$ $\frac{1}{2}$

$\dfrac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2$ or $2\pi rh + \pi r^2 = 3\pi r^2$ 1

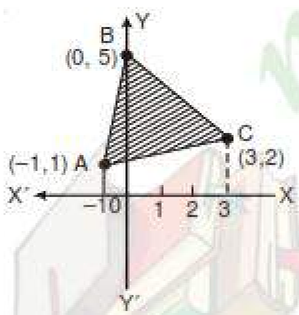
$\Rightarrow 2\pi rh = 2\pi r^2$ $\qquad \Rightarrow h = r$ 1

$\dfrac{d^2V}{dr^2} = -6\pi r < 0$ 1

$\therefore$ For volume to be maximum, height = radius

26. Let the points be A (–1, 1), B (0, 5) and C (3, 2)

Correct Figure 1



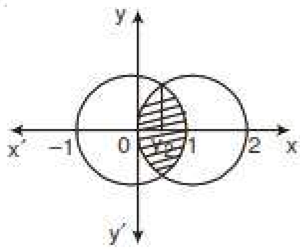
$$\left.\begin{aligned} \text{Equation of } AB &: y = 4x + 5 \\ BC &: y = 5 - x \\ AC &: y = \frac{1}{4}(x+5) \end{aligned}\right\}$$ $1\frac{1}{2}$

Req. Area $= \displaystyle\int_{-1}^{0}(4x+5)\,dx + \int_{0}^{3}(5-x)\,dx - \int_{-1}^{3}\frac{1}{4}(x+5)\,dx$ $1\frac{1}{2}$

$\therefore\ A = \left[\dfrac{(4x+5)^2}{8}\right]_{-1}^{0} + \left[\dfrac{(5-x)^2}{-2}\right]_{0}^{3} - \dfrac{1}{4}\left[\dfrac{(x+5)^2}{2}\right]_{-1}^{3}$ 1

$$= 3 + \frac{21}{2} - 6 = \frac{15}{2}$$ 1

OR



Correct Figure 1

$$(x-1)^2 + y^2 = 1$$

and $x^2 + y^2 = 1 \Rightarrow (x-1)^2 = x^2$

$$\Rightarrow x = \frac{1}{2}$$ 1

$$\therefore \text{ Required area } = 2\left[\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2}\,dx + \int_{\frac{1}{2}}^{1} \sqrt{1-x^2}\,dx\right]$$ 2

$$= 2\left[\frac{x-1}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}} + 2\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$ 1

$$= 2\left[\frac{-\sqrt{3}}{8} + \frac{\pi}{6}\right] + 2\left[\frac{-\sqrt{3}}{8} + \frac{\pi}{6}\right] = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$ 1

27. Equation of plane passing through (2, 5, –3), (–2, –3, 5) and (5, 3, –3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$ 1

$$\Rightarrow 16(x-2) + 24(y-5) + 32(z+3) = 0$$

i.e. $2x + 3y + 4z - 7 = 0$ ...(i) 1

which in vector form can be written as $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = 7$ 1

Equation of line passing through (3, 1, 5) and (–1, –3, –1) is

$$\frac{x-3}{4} = \frac{y-1}{4} = \frac{z-5}{6} \text{ or } \frac{x-3}{2} = \frac{y-1}{2} = \frac{z-5}{3}$$ ...(ii) 1

Any point on (ii) is $(2\lambda + 3, 2\lambda + 1, 3\lambda + 5)$ $\frac{1}{2}$

If this is point of intersection with plane (i), then

$2(2\lambda+3)+3(2\lambda+1)+4(3\lambda+5)-7=0$ $\frac{1}{2}$

$22\lambda+22=0 \Rightarrow \lambda=-1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (1, –1, 2) $\frac{1}{2}$

OR

Equation of plane through the intersection of planes

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$, is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4]=0$ 1

$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]-1+4\lambda=0$ ...(i) 1

Plane (i) is ∥ to x–axis $\Rightarrow 1+2\lambda=0 \Rightarrow \lambda=\frac{-1}{2}$ $1\frac{1}{2}$

$\therefore$ Equation of plane is $\vec{r}\cdot\left(-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-3=0$ $1\frac{1}{2}$

or $\vec{r}\cdot(-\hat{j}+3\hat{k})-6=0$

Distance of this plane from x–axis

$=\frac{|-6|}{\sqrt{(-1)^2+(3)^2}}=\frac{6}{\sqrt{10}}$ units 1

28. Let the events be

$\left.\begin{array}{l} E_1 : \text{bag I is selected} \\ E_2 : \text{bag II is selected} \\ A : \text{getting a red ball} \end{array}\right\}$ 1

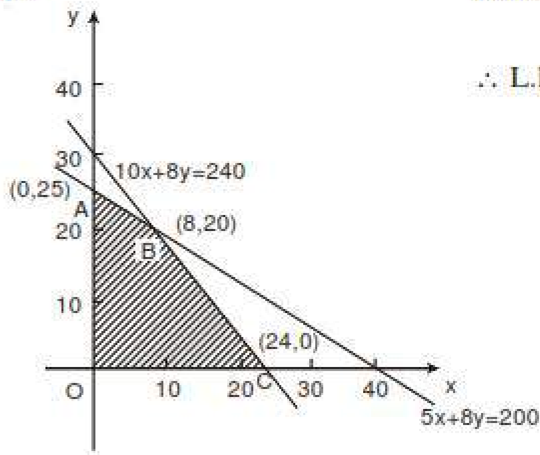
$P(E_1)=P(E_2)=\frac{1}{2}$ $\frac{1}{2}$

$P(A/E_1)=\frac{3}{9}=\frac{1}{3};\ \ P(A/E_2)=\frac{5}{5+n}$ $\frac{1}{2}+1$

$$P(E_2/A) = \frac{3}{5} = \frac{\frac{1}{2}\cdot\frac{5}{5+n}}{\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{5}{5+n}}$$ 2

$$\Rightarrow \frac{3}{5} = \frac{15}{5+n+15} \Rightarrow n = 5.$$ 1

29. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise $P = 50x + 60y$ $\frac{1}{2}$

such that $\left.\begin{array}{r} 5x+8y \le 200 \\ 10x+8y \le 240 \\ x, y \ge 0 \end{array}\right\}$ $2\frac{1}{2}$

Correct Graph 2

P(at A) = ₹1500

P(at B) = ₹(400 + 1200) = ₹1600

P(at C) = ₹(1200)

$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20. 1

QUESTION PAPER CODE 65/3/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\frac{dy}{dx} - \frac{2}{x} \cdot y = 2x \Rightarrow \text{I.F.} = e^{-2\log x} = \frac{1}{x^2}$ $\frac{1}{2}+\frac{1}{2}$

2. $y^2 + 2xy\frac{dy}{dx} - 2x = 0 \Rightarrow \frac{dy}{dx} = \frac{2x - y^2}{2xy}$ $\frac{1}{2}+\frac{1}{2}$

3. $|-2A| = (-2)^3 \cdot |A|$ $\frac{1}{2}$

$= -8 \times 4 = -32$ $\frac{1}{2}$

4. $\sqrt{(-18)^2 + (12)^2 + (-4)^2} = 22$ $\frac{1}{2}$

$\therefore$ DC's are $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$ or $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$ $\frac{1}{2}$

OR

D·R's of required line are 3, –5, 6 $\frac{1}{2}$

Equation of line is $\frac{x+2}{3} = \frac{y-4}{-5} = \frac{z+5}{6}$ $\frac{1}{2}$

SECTION B

5. Let a = 2, b = 3 $\Rightarrow 2*3 = \frac{2}{4} = \frac{1}{2}$, $3*2 = \frac{3}{3} = 1 \Rightarrow 2*3 \neq 3*2$. 1

$(2*3)*4 = \frac{1}{2}*4 = \frac{\frac{1}{2}}{4+1} = \frac{1}{10}, 2*(3*4) = 2*\frac{3}{5} = \frac{2}{8/5} = \frac{5}{4}$

$\Rightarrow (2*3)*4 \neq 2*(3*4)$ 1

6. $A^2 = \begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix} = A$ 1

$\Rightarrow A^3 = A^2 \cdot A = A \cdot A = A^2 = A$ 1

7. $y^2 = m(a^2 - x^2) \Rightarrow 2y\frac{dy}{dx} = -2mx$ $\frac{1}{2}$

or $\quad y\dfrac{dy}{dx} = -mx \qquad ...(i)$

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = -m \qquad ...(ii)$$ $\frac{1}{2}$

form (i) and (ii) we get $y\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2 = \dfrac{y}{x}\dfrac{dy}{dx}$ 1

or $\quad xy\dfrac{d^2y}{dx^2}+x\left(\dfrac{dy}{dx}\right)^2-y\dfrac{dy}{dx} = 0$

8. $\displaystyle\int\frac{\sin x-\cos x}{\sqrt{1+\sin 2x}}dx = \int\frac{\sin x-\cos x}{\sin x+\cos x}dx$ 1

$= -\log|\sin x+\cos x|+c$ 1

9. $\displaystyle\int\frac{\sin(x-a)}{\sin(x+a)}dx = \int\frac{\sin(x+a-2a)}{\sin(x+a)}dx$ $\frac{1}{2}$

$= \displaystyle\int\left[\frac{\sin(x+a)\cdot\cos 2a}{\sin(x+a)}-\frac{\cos(x+a)\sin 2a}{\sin(x+a)}\right]dx$ $\frac{1}{2}$

$= x\cdot\cos 2a-\sin 2a\cdot\log|\sin(x+a)|+c$ $\frac{1}{2}+\frac{1}{2}$

OR

$\displaystyle\int(\log x)^2\cdot 1dx = (\log x)^2\cdot x-\int 2\cdot\log x\cdot\frac{1}{x}\cdot x\,dx$ 1

$= x\cdot(\log x)^2-\left\{\log x\cdot 2x-\displaystyle\int\frac{1}{x}\cdot 2xdx\right\}$ $\frac{1}{2}$

$= x(\log x)^2-2x\log x+2x+c$ $\frac{1}{2}$

10. $\left.\begin{array}{l}A = \{(S, F, M), (S, M, F), (M, F, S), (F, M, S)\}\\ B = \{(S, F, M), (M, F, S)\}\end{array}\right\}$ 1

Total number of possible arrangements = 6

$P(B|A) = \dfrac{P(B\cap A)}{P(A)}$

$$= \frac{2/6}{4/6} = \frac{1}{2}$$ 1

11. Given $2\,P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$

Let $P(X = x_3) = k$, then $P(X = x_1) = \frac{k}{2}, P(X = x_2) = \frac{k}{3}$ and $P(X = x_4) = \frac{k}{5}$ $\frac{1}{2}$

$\therefore\ k + \frac{k}{2} + \frac{k}{3} + \frac{k}{5} = 1 \Rightarrow k = \frac{30}{61}$ 1

$\therefore$ Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

$\frac{1}{2}$

OR

(i) P(at least 4 heads) = $P(r \geq 4) = P(4) + P(5)$

$$= {}^5C_4\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 + {}^5C_5\left(\frac{1}{2}\right)^5 = 6\left(\frac{1}{2}\right)^5 = \frac{6}{32} \text{ or } \frac{3}{16}$$ 1

(ii) P(at most 4 heads) = $P(r \leq 4) = 1 - P(5)$

$$= 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32}$$ 1

12. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\vec{a} \times \vec{b} = 19\hat{j} + 19\hat{k}$ or $\hat{j} + \hat{k}$ 1

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$ 1

OR

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\vec{a} \cdot \vec{b} \times \vec{c} = 0$ $\frac{1}{2}$

$$\left.\begin{aligned} \vec{a} \cdot \vec{b} \times \vec{c} &= \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 1(3) + 2(-6) + 3(3) \\ &= 3 - 12 + 9 = 0 \end{aligned}\right\}$$ $1 + \frac{1}{2}$

Hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar

SECTION C

13. (i) For $a \in Z, (a, a) \in R \because a - a = 0$ is divisible by 2

$\therefore$ R is reflexive ...(i) 1

Let (a, b) $\in$ R for a, b $\in$ Z, then a – b is divisible by 2

$\Rightarrow$ (b – a) is also divisible by 2

$\therefore$ (b, a) $\in$ R $\Rightarrow$ R is symmetric ...(ii) 1

For a, b, c, $\in$ Z, Let (a, b) $\in$ R and (b, c) $\in$ R

$\therefore$ a – b = 2p, p $\in$ Z, and b – c = 2q, q $\in$ Z,

adding, a – c = 2 (p + q) $\Rightarrow$ (a – c) is divisible by 2

$\Rightarrow$ (a, c) $\in$ R, so R is transitive ...(iii) $1\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow$ R is an equivalence relation. $\frac{1}{2}$

OR

$$fof(x) = f\left(\frac{4x+3}{6x-4}\right)$$ 1

$$= \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$ 1

$$\Rightarrow fof(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{34x}{34} = x$$ 1

Since fof(x) = x $\Rightarrow$ fof = I $\Rightarrow f^{-1} = f$ 1

14. $\sin y = x \cdot \sin(a+y) \Rightarrow x = \frac{\sin y}{\sin(a+y)}$ $\frac{1}{2}$

differentiating w.r.t. y, we get

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(a+y)}{\sin^2(a+y)}$$ $1\frac{1}{2}$

$$\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$$ $1\frac{1}{2}$

$$\therefore\ \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$ $\frac{1}{2}$

OR

$(\sin x)^y = (x+y) \Rightarrow y\cdot\log\sin x = \log(x+y)$ 1

differentiating w.r.t. x, we get

$$y\cdot\cot x + \log\sin x\cdot\frac{dy}{dx} = \frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{x+y} - y\cot x}{\log\sin x - \frac{1}{x+y}}$$ 1

$$= \frac{1-y(x+y)\cot x}{(x+y)\log\sin x - 1}$$ $\frac{1}{2}$

15. $\sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2} \Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \frac{\pi}{2} - \sin^{-1}\frac{4}{x} = \cos^{-1}\frac{4}{x}$ 1

$$\Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \sin^{-1}\left(\sqrt{1-\frac{16}{x^2}}\right) \Rightarrow \left(\frac{3}{x}\right)^2 = \frac{x^2-16}{x^2}$$ $1\frac{1}{2}$

$\Rightarrow x^2 = 25 \Rightarrow x = \pm 5,\ x = -5$ (rejected) $\therefore\ x = 5$ $\frac{1}{2}+1$

16. $$\text{LHS} = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a(a^2+1) & a^2b & a^2c \\ ab^2 & b(b^2+1) & b^2c \\ ac^2 & bc^2 & c(c^2+1) \end{vmatrix} \left\{\begin{array}{l}\text{Applying} \\ R_1 \to aR_1 \\ R_2 \to bR_2 \\ R_3 \to cR_3\end{array}\right.$$ 1

$$= \begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix} = (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix} \{R_1 \to R_1+R_2+R_3$$ $\frac{1}{2}+1$

$$= (1+a^2+b^2+c^2)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix} = 1+a^2+b^2+c^2. \begin{cases} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{cases}$$ $1\frac{1}{2}$

$= \text{RHS}$

17. $y = (\cot^{-1} x)^2 \Rightarrow \dfrac{dy}{dx} = 2\cot^{-1} x \cdot \left(\dfrac{-1}{1+x^2}\right)$ 1

$\Rightarrow (1+x^2)\dfrac{dy}{dx} = -2\cot^{-1} x = -2\sqrt{y}$ $\frac{1}{2}$

squaring both sides, we get

$(1+x^2)^2 \cdot \left(\dfrac{dy}{dx}\right)^2 = 4y$ $\frac{1}{2}$

differentiating, w.r.t. x,

$2(1+x^2)2x \cdot \left(\dfrac{dy}{dx}\right)^2 + 2(1+x^2)^2 \cdot \dfrac{dy}{dx} \cdot \dfrac{d^2y}{dx^2} = 4 \cdot \dfrac{dy}{dx}$ $1\frac{1}{2}$

$\Rightarrow 2x(1+x^2)\dfrac{dy}{dx} + (1+x^2)^2 \dfrac{d^2y}{dx^2} = 2.$ $\frac{1}{2}$

18. $I = \displaystyle\int \frac{\sin 2x}{(\sin^2 x+1)(\sin^2 x+3)}\,dx$

Put $\sin^2 x = t \Rightarrow \sin 2x\,dx = dt$ $\frac{1}{2}$

$\therefore\ I = \displaystyle\int \frac{dt}{(t+1)(t+3)} = \int\left(\frac{1/2}{t+1} + \frac{-1/2}{t+3}\right)dt$ $1\frac{1}{2}$

$= \dfrac{1}{2}\log|t+1| - \dfrac{1}{2}\log|t+3| + c$ $1\frac{1}{2}$

$= \dfrac{1}{2}\log(\sin^2 x+1) - \dfrac{1}{2}\log(\sin^2 + 3) + c.$ $\frac{1}{2}$

19. RHS $= \displaystyle\int_a^b f(a+b-x)\,dx = -\int_b^a f(t)\,dt$, where $a + b - x = t$, $dx = -dt$ $\frac{1}{2}$

$= \displaystyle\int_a^b f(t)\,dt = \int_a^b f(x)\,dx = \text{LHS}$ $\frac{1}{2}$

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$ ...(i) $\frac{1}{2}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ ...(ii) $1\frac{1}{2}$

adding (i) and (ii) to get $2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1\cdot dx = x]_{\pi/6}^{\pi/3} = \pi/6.$ $\frac{1}{2}$

$\Rightarrow I = \frac{\pi}{12}$ $\frac{1}{2}$

20. $\frac{dy}{dx} = \frac{x+y}{x-y} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$

Put y/x = v so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2}$

$\therefore\ v + x\frac{dv}{dx} = \frac{1+v}{1-v} \Rightarrow x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v}$

$\Rightarrow \int \frac{1-v}{1+v^2}dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$ $1+\frac{1}{2}$

$\Rightarrow \tan^{-1} v = \frac{1}{2}\log\left|1+v^2\right| + \log|x| + c$ 1

$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x| + c$ 1

or $\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log|x^2+y^2| + c$

OR

$(1+x^2)dy + 2xy\,dx = \cot x.dx.$

$$\Rightarrow \frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{\cot x}{1+x^2}$$ 1

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$$ 1

$\therefore$ Solution is, $y\cdot(1+x^2)=\int \cot x\, dx = \log|\sin x|+c$ 1+1

or $y = \frac{1}{1+x^2}\cdot \log|\sin x|+\frac{c}{1+x^2}$

21. Writing the equations of given lines in standard form, as

$$\frac{x-5}{5\lambda+2}=\frac{y-2}{-5}=\frac{z-1}{1};\frac{x}{1}=\frac{y+\frac{1}{2}}{2\lambda}=\frac{z-1}{3}$$ $\frac{1}{2}$

lines are perpendicular to each other,

$$\Rightarrow (5\lambda+2)\cdot 1+(-5)(2\lambda)+1(3)=0$$ $\frac{1}{2}$

$$-5\lambda+5=0 \Rightarrow \lambda=1$$ $\frac{1}{2}$

$\therefore$ lines are $\frac{x-5}{7}=\frac{y-2}{-5}=\frac{z-1}{1};\frac{x}{1}=\frac{y+\frac{1}{2}}{2}=\frac{z-1}{3}$ $\frac{1}{2}$

Shortest distance between these lines $=\frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}=\frac{\left|\left(5\hat{i}+\frac{5}{2}\hat{j}\right)\cdot(-17\hat{i}-20\hat{j}+19\hat{k})\right|}{|\vec{b}_1\times\vec{b}_2|}$ 1

$$=\frac{135}{|\vec{b}_1\times\vec{b}_2|}\neq 0$$ $\frac{1}{2}$

$\therefore$ lines are not intersecting. $\frac{1}{2}$

22. Given $\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}=\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}$ $\therefore$ $\vec{b}\cdot\vec{a}=\vec{c}\cdot\vec{a}$ ...(i) $\frac{1}{2}$

$\vec{b}\perp\vec{c} \Rightarrow \vec{b}\cdot\vec{c}=0$ ...(ii) $\frac{1}{2}$

$(|3\vec{a}-2\vec{b}+2\vec{c}|)^2=9|\vec{a}|^2+4|\vec{b}|^2+4|\vec{c}|^2-12\,\vec{a}\cdot\vec{b}-8\,\vec{b}\cdot\vec{c}+12\,\vec{a}\cdot\vec{c}$ 1

$= 9(1)^2 + 4(2)^2 + 4(3)^2$ [using (i) and (ii)] 1

$= 9 + 16 + 36 = 61$

$\Rightarrow |3\vec{a} - 2\vec{b} + 2\vec{c}| = \sqrt{61}$ 1

23. Curve $y = \dfrac{x-7}{(x-2)(x-3)}$ cuts at x – axis at the point x = 7, y = 0 i.e. (7, 0) $\frac{1}{2}$

$\dfrac{dy}{dx} = \dfrac{(x^2-5x+6)\cdot 1-(x-7)(2x-5)}{(x^2-5x+6)^2}$ $\frac{1}{2}$

at (7, 0) $\dfrac{dy}{dx} = \dfrac{20}{(20)^2} = \dfrac{1}{20}$ $\frac{1}{2}$

$\therefore$ Slope of tangent at (7, 0) is $\dfrac{1}{20}$ $\frac{1}{2}$

and slope of Normal at (7, 0) is –20 $\frac{1}{2}$

Equation of tangent at (7, 0) is $y - 0 = \dfrac{1}{20}(x - 7)$

or $x - 20y - 7 = 0$ 1

Equation of Normal at (7, 0) is $y - 0 = -20(x - 7)$

or $20x + y = 140$. $\frac{1}{2}$

SECTION D

24. $f(x) = \sin x + \frac{1}{2}\cos 2x \Rightarrow f'(x) = \cos x - \sin 2x$ 1

$f'(x) = 0 \Rightarrow \cos x - 2\sin x \cos x = 0$

$\Rightarrow \cos x(1 - 2\sin x) = 0$

$\Rightarrow x = \dfrac{\pi}{2}$ or $x = \dfrac{\pi}{6}$ 1

$x = \dfrac{\pi}{6} \in \left(0, \dfrac{\pi}{2}\right)$ 1

$f''(x) = -\sin x - 2\cos 2x$ 1

$f''(\pi/6) < 0 \Rightarrow \ x = \dfrac{\pi}{6}$ is a local maxima. 1

Local Max. Value = $f(\pi/6) = \dfrac{3}{4}$ 1

Local extreme values do exist at end points $x = 0$, $x = \dfrac{\pi}{2}$ but no marks are alotted here for that

25. $A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ 2

$A^2 \cdot A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ 3

or $A \cdot A^2 = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\Rightarrow A^2 = A^{-1}$ 1

OR

Given System of equation can be written as

$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ -2 \\ -2 \end{bmatrix}$ or $AX = B$ 1

$|A| = 2(0) + 3(-2) + 5(1) = -1 \neq 0$ 1

$\therefore \ X = A^{-1} \cdot B$

$(\text{adj. } A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ 2

$A^{-1} = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ $\frac{1}{2}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 13 \\ -2 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 2,\ y = 2,\ z = 3.$ $1\frac{1}{2}$

26. Let the events be

$$\left.\begin{array}{l} E_1 : \text{bag I is selected} \\ E_2 : \text{bag II is selected} \\ A : \text{getting a red ball} \end{array}\right\}$$ 1

$P(E_1) = P(E_2) = \frac{1}{2}$ $\frac{1}{2}$

$P(A/E_1) = \frac{3}{9} = \frac{1}{3};\ \ P(A/E_2) = \frac{5}{5+n}$ $\frac{1}{2}+1$

$$P(E_2/A) = \frac{3}{5} = \frac{\frac{1}{2}\cdot\frac{5}{5+n}}{\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{5}{5+n}}$$ 2

$\Rightarrow \frac{3}{5} = \frac{15}{5+n+15} \Rightarrow n = 5.$ 1

27. Equation of plane passing through (2, 5, –3), (–2, –3, 5) and (5, 3, –3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$ 1

$\Rightarrow 16(x-2)+24(y-5)+32(z+3)=0$

i.e. $2x + 3y + 4z - 7 = 0$ ...(i) 1

which in vector form can be written as $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k}) = 7$ 1

Equation of line passing through (3, 1, 5) and (–1, –3, –1) is

$\frac{x-3}{4} = \frac{y-1}{4} = \frac{z-5}{6}$ or $\frac{x-3}{2} = \frac{y-1}{2} = \frac{z-5}{3}$ ...(ii) 1

Any point on (ii) is $(2\lambda+3,\ 2\lambda+1,\ 3\lambda+5)$ $\frac{1}{2}$

If this is point of intersection with plane (i), then

$2(2\lambda+3)+3(2\lambda+1)+4(3\lambda+5)-7=0$ $\frac{1}{2}$

$22\lambda+22=0 \Rightarrow \lambda=-1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (1, –1, 2) $\frac{1}{2}$

OR

Equation of plane through the intersection of planes

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0,$ is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4]=0$ 1

$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]-1+4\lambda=0$ ...(i) 1

Plane (i) is ∥ to x–axis $\Rightarrow 1+2\lambda=0 \Rightarrow \lambda=\frac{-1}{2}$ $1\frac{1}{2}$

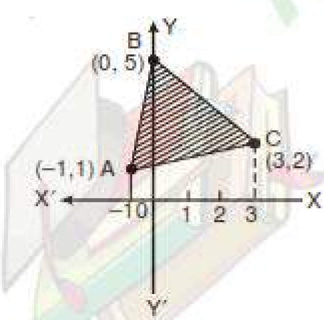
$\therefore$ Equation of plane is $\vec{r}\cdot\left(-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-3=0$ $1\frac{1}{2}$

or $\vec{r}\cdot(-\hat{j}+3\hat{k})-6=0$

Distance of this plane from x–axis

$=\frac{|-6|}{\sqrt{(-1)^2+(3)^2}}=\frac{6}{\sqrt{10}}$ units 1

28. Let the points be A (–1, 1), B (0, 5) and C (3, 2)



Correct Figure 1

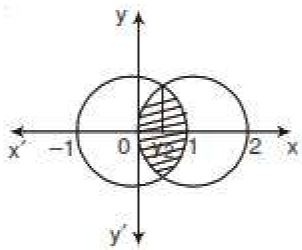
$$\left.\begin{aligned}\text{Equation of AB}&: y=4x+5\\ \text{BC}&: y=5-x\\ \text{AC}&: y=\frac{1}{4}(x+5)\end{aligned}\right\}$$ $1\frac{1}{2}$

Req. Area $=\int_{-1}^{0}(4x+5)dx+\int_{0}^{3}(5-x)dx-\int_{-1}^{3}\frac{1}{4}(x+5)dx$ $1\frac{1}{2}$

$$\therefore\ A=\left[\frac{(4x+5)^2}{8}\right]_{-1}^{0}+\left[\frac{(5-x)^2}{-2}\right]_{0}^{3}-\frac{1}{4}\left[\frac{(x+5)^2}{2}\right]_{-1}^{3}$$ 1

$$=3+\frac{21}{2}-6=\frac{15}{2}$$ 1

OR



Correct Figure 1

$$(x-1)^2+y^2=1$$

and $x^2+y^2=1 \Rightarrow (x-1)^2=x^2$

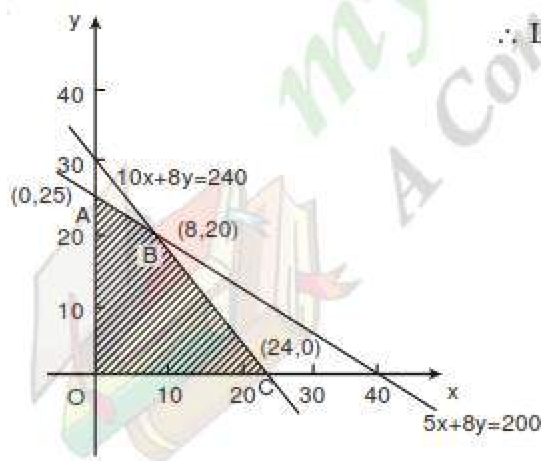
$$\Rightarrow x=\frac{1}{2}$$ 1

$$\therefore \text{ Required area } = 2\left[\int_0^{\frac{1}{2}}\sqrt{1-(x-1)^2}\,dx+\int_{\frac{1}{2}}^{1}\sqrt{1-x^2}\,dx\right]$$ 2

$$=2\left[\frac{x-1}{2}\sqrt{1-(x-1)^2}+\frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}}+2\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$ 1

$$=2\left[\frac{-\sqrt{3}}{8}+\frac{\pi}{6}\right]+2\left[\frac{-\sqrt{3}}{8}+\frac{\pi}{6}\right]=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$$ 1

29. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise $P = 50x + 60y$ $\frac{1}{2}$

$$\left.\begin{aligned}\text{such that } 5x+8y &\le 200\\ 10x+8y &\le 240\\ x, y &\ge 0\end{aligned}\right\}$$ $2\frac{1}{2}$

Correct Graph 2

P(at A) = ₹1500

P(at B) = ₹(400 + 1200) = ₹1600

P(at C) = ₹(1200)

$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20.

QUESTION PAPER CODE 65/3/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. DRs are (6, 2, 3) $\therefore$ DC's are $\left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$ $\frac{1}{2}+\frac{1}{2}$

OR

$\frac{x-1}{-3} = \frac{y-7}{p} = \frac{z-3}{2}; \frac{x-1}{-3p} = \frac{y-5}{1} = \frac{z-6}{-5}$ $\frac{1}{2}$

$\Rightarrow 9p + p - 10 = 0 \Rightarrow p = 1$ $\frac{1}{2}$

2. $\frac{dy}{dx} - \frac{2}{x} \cdot y = 2x$ $\frac{1}{2}$

$\Rightarrow$ I.F. $= e^{-2\log x} = x^{-2} = \frac{1}{x^2}$ $\frac{1}{2}$

3. $|-2A| = (-2)^3 \cdot |A|$ $\frac{1}{2}$

$= -8 \times 4 = -32$ $\frac{1}{2}$

4. $y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$ $\frac{1}{2}+\frac{1}{2}$

SECTION B

5. $B'A' = \begin{bmatrix} 4 & 7 & 2 \\ 0 & 1 & 2 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 3 & 1 & 7 \\ 9 & 8 & 5 \\ 0 & -2 & 4 \end{bmatrix}$ 1

$= \begin{bmatrix} 75 & 56 & 71 \\ 9 & 4 & 13 \\ 42 & 22 & 58 \end{bmatrix}$ 1

6. $\int_a^b \frac{\log x}{x} dx = \left[\frac{1}{2}(\log x)^2\right]_a^b$ 1

$= \frac{1}{2}[(\log b)^2 - (\log a)^2]$ 1

7. $y^2 = m(a^2 - x^2) \Rightarrow 2y\frac{dy}{dx} = -2mx$ $\frac{1}{2}$

or $y\frac{dy}{dx} = -mx$ ...(i)

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -m$ ...(ii) $\frac{1}{2}$

form (i) and (ii) we get $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$ 1

or $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

8. $\int\frac{\sin(x-a)}{\sin(x+a)}dx = \int\frac{\sin(x+a-2a)}{\sin(x+a)}dx$ $\frac{1}{2}$

$= \int\left[\frac{\sin(x+a)\cdot\cos 2a}{\sin(x+a)} - \frac{\cos(x+a)\sin 2a}{\sin(x+a)}\right]dx$ $\frac{1}{2}$

$= x\cdot\cos 2a - \sin 2a\cdot\log|\sin(x+a)| + c$ $\frac{1}{2}+\frac{1}{2}$

OR

$\int(\log x)^2\cdot 1dx = (\log x)^2\cdot x - \int 2\cdot\log x\cdot\frac{1}{x}\cdot x\,dx$ 1

$= x\cdot(\log x)^2 - \left\{\log x\cdot 2x - \int\frac{1}{x}\cdot 2xdx\right\}$ $\frac{1}{2}$

$= x(\log x)^2 - 2x\log x + 2x + c$ $\frac{1}{2}$

9. A vector perpendicular to both $\vec{a}$ and $\vec{b} = \vec{a}\times\vec{b} = 19\hat{j} + 19\hat{k}$ or $\hat{j}+\hat{k}$ 1

$\therefore$ Unit vector perpendicular to both $\vec{a}$ and $\vec{b} = \frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$ 1

OR

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\vec{a}\cdot\vec{b}\times\vec{c} = 0$ $\frac{1}{2}$

$$\vec{a}\cdot\vec{b}\times\vec{c} = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 1(3)+2(-6)+3(3)$$
$$= 3-12+9 = 0$$ $1+\frac{1}{2}$

Hence $\vec{a},\vec{b},\vec{c}$ are coplanar

10. $A = \{(S, F, M), (S, M, F), (M, F, S), (F, M, S)\}$
$B = \{(S, F, M), (M, F, S)\}$ 1

Total number of possible arrangements = 6

$$P(B|A) = \frac{P(B\cap A)}{P(A)}$$

$$= \frac{2/6}{4/6} = \frac{1}{2}$$ 1

11. Given $2\,P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$

Let $P(X = x_3) = k$, then $P(X = x_1) = \frac{k}{2}, P(X = x_2) = \frac{k}{3}$ and $P(X = x_4) = \frac{k}{5}$ $\frac{1}{2}$

$\therefore\ k + \frac{k}{2} + \frac{k}{3} + \frac{k}{5} = 1 \Rightarrow k = \frac{30}{61}$ 1

$\therefore$ Probability distribution is

X	x_1	x_2	x_3	x_4
P(X)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

$\frac{1}{2}$

OR

(i) P(at least 4 heads) = $P(r \geq 4) = P(4) + P(5)$

$$= {}^5C_4\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 + {}^5C_5\left(\frac{1}{2}\right)^5 = 6\left(\frac{1}{2}\right)^5 = \frac{6}{32} \text{ or } \frac{3}{16}$$ 1

(ii) P(at most 4 heads) = $P(r \leq 4) = 1 - P(5)$

$$= 1 - \left(\frac{1}{2}\right)^5 = \frac{31}{32}$$ 1

12. Let $e \in \mathbb{R}$ be the identity element.

then $a*e = e*a = a$ 1

$\Rightarrow a^2 + e^2 = e^2 + a^2 = a^2 \Rightarrow e^2 = 0 \Rightarrow e = 0.$ 1

$\therefore$ Identity element is $0 \in \mathbb{R}$

SECTION C

13. $\tan(\sec^{-1}\frac{1}{x}) = \sin(\tan^{-1}2) \Rightarrow \tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$ 2

$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}} \Rightarrow \frac{1-x^2}{x^2} = \frac{4}{5}$ 1

$\Rightarrow 9x^2 = 5 \Rightarrow x^2 = \frac{5}{9} \Rightarrow x = \frac{\sqrt{5}}{3}, \{x > 0\}$ 1

14. $e^y \cdot (x+1) = 1 \Rightarrow e^y \cdot 1 + (x+1)e^y \cdot \frac{dy}{dx} = 0$ 1

$\Rightarrow \frac{dy}{dx} = -\frac{1}{x+1}$ 1

$\Rightarrow \frac{d^2y}{dx^2} = +\frac{1}{(x+1)^2}$ 1

$\therefore \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ 1

OR

$y = \sin^{-1}\left(\frac{2 \cdot 2^x}{1+(2^x)^2}\right) = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$, where $t = 2^x$ 1

$\Rightarrow y = 2\tan^{-1}t$ 1

$\frac{dy}{dt} = \frac{2}{1+t^2}$ and $\frac{dt}{dx} = 2^x \cdot \log 2.$ 1

$\Rightarrow \frac{dy}{dx} = \frac{2}{1+t^2} \cdot 2^x \cdot \log 2 = \frac{2^{x+1} \cdot \log 2}{1+4^x}$ 1

15. $f(x) = 4x^3 - 6x^2 - 72x + 30$

$\Rightarrow f'(x) = 12x^2 - 12x - 72 = 12(x-3)(x+2)$ 1

$f'(x) = 0 \Rightarrow x = -2, x = 3$ 1

$\therefore$ possible intervals are $(-\infty, -2), (-2, 3), (3, \infty)$ $\frac{1}{2}$

$f'(x) < 0$ in $(-2, 3)$

and $f'(x) > 0$ in $(-\infty, -2)$ and $(3, \infty)$

$\Rightarrow$ f(x) is strictly increasing in $(-\infty, -2), (3, \infty)$ or $(-\infty, 2], [3, \infty)$ 1

and strictly decreasing in $(-2, 3)$ or $[-2, 3]$ $\frac{1}{2}$

16. (i) For $a \in Z, (a, a) \in R$ $\because$ $a - a = 0$ is divisible by 2

$\therefore$ R is reflexive ...(i) 1

Let $(a, b) \in R$ for $a, b \in Z$, then $a - b$ is divisible by 2

$\Rightarrow (b - a)$ is also divisible by 2

$\therefore (b, a) \in R \Rightarrow R$ is symmetric ...(ii) 1

For $a, b, c, \in Z$, Let $(a, b) \in R$ and $(b, c) \in R$

$\therefore a - b = 2p, p \in Z$, and $b - c = 2q, q \in Z$,

adding, $a - c = 2(p + q) \Rightarrow (a - c)$ is divisible by 2

$\Rightarrow (a, c) \in R$, so R is transitive ...(iii) $1\frac{1}{2}$

(i), (ii), and (iii) $\Rightarrow$ R is an equivalence relation. $\frac{1}{2}$

OR

$fof(x) = f\left(\frac{4x+3}{6x-4}\right)$ 1

$$= \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$ 1

$$\Rightarrow fof(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{34x}{34} = x$$ 1

Since $fof(x) = x \Rightarrow fof = I \Rightarrow f^{-1} = f$ 1

17. Let $\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

$R_1 \to R_1 - (R_2 + R_3) \Rightarrow \Delta = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ 1

$\therefore \Delta = -2\begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = -\frac{2}{b}\begin{vmatrix} 0 & bc & b \\ b & bc+ab & b \\ c & bc & a+b \end{vmatrix}$ $1+\frac{1}{2}$

$C_2 \to C_2 - cC_3$

$\Rightarrow \Delta = -\frac{2}{b}\begin{vmatrix} 0 & 0 & b \\ b & ab & b \\ c & -ac & a+b \end{vmatrix}$ 1

$= -\frac{2}{b} \cdot b \cdot (-abc - abc) = 4abc.$ $\frac{1}{2}$

18. $y = (\sec^{-1} x)^2, x > 0$

$\frac{dy}{dx} = 2\sec^{-1} x \cdot \frac{1}{x\sqrt{x^2-1}}$ 1

$\Rightarrow x\sqrt{x^2-1}\frac{dy}{dx} = 2\sqrt{y}$ $\frac{1}{2}$

squaring both sides, we get

$x^2(x^2-1)\left(\frac{dy}{dx}\right)^2 = 4y$ or $(x^4-x^2)\left(\frac{dy}{dx}\right)^2 = 4y$ $\frac{1}{2}$

differentiating w.r.t. x.

$(x^4-x^2)2\cdot\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + (4x^3-2x)\left(\frac{dy}{dx}\right)^2 = 4\cdot\frac{dy}{dx}$ 1

$$\Rightarrow x^2(x^2-1)\frac{d^2y}{dx^2}+(2x^3-x)\frac{dy}{dx}-2=0$$ 1

19. RHS $=\int_a^b f(a+b-x)\,dx=-\int_b^a f(t)\,dt$, where $a+b-x=t$, $dx=-dt$ $\frac{1}{2}$

$$=\int_a^b f(t)\,dt=\int_a^b f(x)\,dx=\text{LHS}$$ $\frac{1}{2}$

Let $I=\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ...(i) $\frac{1}{2}$

$$=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}}dx=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$ ...(ii) $1\frac{1}{2}$

adding (i) and (ii) to get $2I=\int_{\pi/6}^{\pi/3}1\cdot dx=x]_{\pi/6}^{\pi/3}=\pi/6.$ $\frac{1}{2}$

$\Rightarrow I=\frac{\pi}{12}$ $\frac{1}{2}$

20. $I=\int\frac{\sin 2x}{(\sin^2 x+1)(\sin^2 x+3)}dx$

Put $\sin^2 x=t \Rightarrow \sin 2x\,dx=dt$ $\frac{1}{2}$

$\therefore I=\int\frac{dt}{(t+1)(t+3)}=\int\left(\frac{1/2}{t+1}+\frac{-1/2}{t+3}\right)dt$ $1\frac{1}{2}$

$$=\frac{1}{2}\log|t+1|-\frac{1}{2}\log|t+3|+c$$ $1\frac{1}{2}$

$$=\frac{1}{2}\log(\sin^2 x+1)-\frac{1}{2}\log(\sin^2+3)+c.$$ $\frac{1}{2}$

21. Writing the equations of given lines in standard form, as

$$\frac{x-5}{5\lambda+2}=\frac{y-2}{-5}=\frac{z-1}{1};\ \frac{x}{1}=\frac{y+\frac{1}{2}}{2\lambda}=\frac{z-1}{3}$$ $\frac{1}{2}$

lines are perpendicular to each other,

$\Rightarrow (5\lambda+2)\cdot 1+(-5)(2\lambda)+1(3)=0$ $\frac{1}{2}$

$-5\lambda+5=0 \Rightarrow \lambda=1$ $\frac{1}{2}$

$\therefore$ lines are $\frac{x-5}{7}=\frac{y-2}{-5}-\frac{z-1}{1}; \frac{x}{1}=\frac{y+\frac{1}{2}}{2}=\frac{z-1}{3}$ $\frac{1}{2}$

Shortest distance between these lines $=\frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}=\frac{\left|\left(5\hat{i}+\frac{5}{2}\hat{j}\right)\cdot(-17\hat{i}-20\hat{j}+19\hat{k})\right|}{|\vec{b}_1\times\vec{b}_2|}$ 1

$=\frac{135}{|\vec{b}_1\times\vec{b}_2|}\neq 0$ $\frac{1}{2}$

$\therefore$ lines are not intersecting. $\frac{1}{2}$

22. Given $\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}=\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}$ $\therefore$ $\vec{b}\cdot\vec{a}=\vec{c}\cdot\vec{a}$ ...(i) $\frac{1}{2}$

$\vec{b}\perp\vec{c} \Rightarrow \vec{b}\cdot\vec{c}=0$ ...(ii) $\frac{1}{2}$

$(|3\vec{a}-2\vec{b}+2\vec{c}|)^2=9|\vec{a}|^2+4|\vec{b}|^2+4|\vec{c}|^2-12\,\vec{a}\cdot\vec{b}-8\,\vec{b}\cdot\vec{c}+12\,\vec{a}\cdot\vec{c}$ 1

$=9(1)^2+4(2)^2+4(3)^2$ [using (i) and (ii)] 1

$= 9 + 16 + 36 = 61$

$\Rightarrow |3\vec{a}-2\vec{b}+2\vec{c}|=\sqrt{61}$ 1

23. $\frac{dy}{dx}=\frac{x+y}{x-y}=\frac{1+\frac{y}{x}}{1-\frac{y}{x}}$

Put y/x = v so that $\frac{dy}{dx}=v+x\frac{dv}{dx}$ $\frac{1}{2}$

$\therefore\ v+x\frac{dv}{dx}=\frac{1+v}{1-v} \Rightarrow x\frac{dv}{dx}=\frac{1+v}{1-v}-v=\frac{1+v-v+v^2}{1-v}$

$$\Rightarrow \int \frac{1-v}{1+v^2}dv = \int \frac{dx}{x} \Rightarrow \int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$$ $1+\frac{1}{2}$

$$\Rightarrow \tan^{-1} v = \frac{1}{2}\log\left|1+v^2\right| + \log|x| + c$$ 1

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log\left|\frac{x^2+y^2}{x^2}\right| + \log|x| + c$$ 1

$$\text{or } \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log|x^2+y^2| + c$$

OR

$$(1+x^2)dy + 2xy\,dx = \cot x.dx.$$

$$\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{\cot x}{1+x^2}$$ 1

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$$ 1

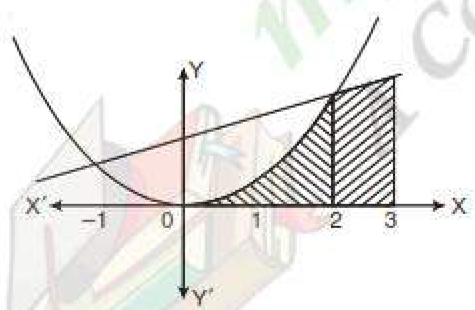
$$\therefore \text{ Solution is, } y\cdot(1+x^2) = \int \cot x\,dx = \log|\sin x| + c$$ 1+1

$$\text{or } y = \frac{1}{1+x^2}\cdot\log|\sin x| + \frac{c}{1+x^2}$$

SECTION D

24. $\{(x,y): 0 \le y \le x^2, 0 \le y \le x+2, -1 \le x \le 3\}$

Correct Figure 1



$$\text{Area} = \int_{-1}^{2} x^2 dx + \int_{2}^{3}(x+2)dx$$ 2

$$= \left.\frac{x^3}{3}\right]_{-1}^{2} + \left.\frac{(x+2)^2}{2}\right]_{2}^{3}$$ 2

$$= 3 + \frac{9}{2} = \frac{15}{2}$$ 1

OR

$$I = \lim_{h\to 0} h[f(1) + f(1+h) + f(1+2h) + \cdots + f(1+\overline{n-1})n)]$$ 1

where $h = \dfrac{3}{n}$ or $nh = 3$ $\frac{1}{2}$

$$= \lim_{h\to 0} h[(2+e^2) + (2+h+e^{2+2h}) + (2+2h+e^{2+4h}) + \cdots + (2+(n-1)h) + e^{2+2(n-1)h}]$$ 1

$$= \lim_{h\to 0} h\left[2n + h\cdot\frac{n(n-1)}{2}\right] + \lim_{h\to 0} h\cdot e^2[1+e^{2h}+e^{4h}+\cdots e^{2(n-1)h}]$$ 1

$$= \lim_{h\to 0}\left[2nh + \frac{nh(nh-h)}{2}\right] + \lim_{h\to 0} \cancel{h}\cdot\frac{e^2}{2\cancel{h}}\,\frac{e^{2nh}-1}{\frac{e^{2h}-1}{2h}}$$ $1\frac{1}{2}$

$$= 6 + \frac{9}{2} + \frac{e^2(e^6-1)}{2} = \frac{21}{2} + \frac{e^2(e^6-1)}{2}$$ 1

25. p = (prob. of doublet) = 1/6 $\quad\therefore q = 5/6$ 1

X	0	1	2	3	4
P(X)	$\left(\frac{5}{6}\right)^4 = \frac{625}{1296}$	$4\cdot\left(\frac{5}{6}\right)^3\frac{1}{6} = \frac{500}{1296}$	$6\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2$ $\frac{150}{1296}$	$4\frac{5}{6}\left(\frac{1}{6}\right)^3$ $\frac{20}{1296}$	$\left(\frac{1}{6}\right)^4$ $\frac{1}{1296}$

1

XP(X)	0	$\frac{500}{1296}$	$\frac{300}{1296}$	$\frac{60}{1296}$	$\frac{4}{1296}$
X^2P(X)	0	$\frac{500}{1296}$	$\frac{600}{1296}$	$\frac{180}{1296}$	$\frac{16}{1296}$

1

1

Mean $= \Sigma XP(X) = \dfrac{864}{1296} = \dfrac{2}{3}$ 1

Variance $= \Sigma X^2\cdot P(X) - [\Sigma X\cdot P(X)]^2 = 1 - \dfrac{4}{9} = \dfrac{5}{9}$ 1

26. Equation of plane passing through (2, 5, –3), (–2, –3, 5) and (5, 3, –3) is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$ 1

$\Rightarrow\ 16(x-2)+24(y-5)+32(z+3)=0$

i.e. $2x + 3y + 4z - 7 = 0$ ...(i) 1

which in vector form can be written as $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=7$ 1

Equation of line passing through (3, 1, 5) and (–1, –3, –1) is

$\frac{x-3}{4}=\frac{y-1}{4}=\frac{z-5}{6}$ or $\frac{x-3}{2}=\frac{y-1}{2}=\frac{z-5}{3}$ ...(ii) 1

Any point on (ii) is $(2\lambda+3, 2\lambda+1, 3\lambda+5)$ $\frac{1}{2}$

If this is point of intersection with plane (i), then

$2(2\lambda+3)+3(2\lambda+1)+4(3\lambda+4)-7=0$ $\frac{1}{2}$

$22\lambda+22=0 \Rightarrow \lambda=-1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (1, –1, 2) $\frac{1}{2}$

OR

Equation of plane through the intersection of planes

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$, is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4]=0$ 1

$\Rightarrow\ \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]-1+4\lambda=0$ ...(i) 1

Plane (i) is ∥ to x–axis $\Rightarrow 1+2\lambda=0 \ \Rightarrow\ \lambda=\frac{-1}{2}$ $1\frac{1}{2}$

$\therefore$ Equation of plane is $\vec{r}\cdot\left(-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right)-3=0$ $1\frac{1}{2}$

or $\vec{r}\cdot(-\hat{j}+3\hat{k})-6=0$

Distance of this plane from x–axis

$=\frac{|-6|}{\sqrt{(-1)^2+(3)^2}}=\frac{6}{\sqrt{10}}$ units 1

27. Let Given surface area of open cylinder be S.

Then S = $2\pi rh + \pi r^2$

$$\Rightarrow h = \frac{S - \pi r^2}{2\pi r}$$ 1

Volume V = $\pi r^2 h$ $\frac{1}{2}$

$$V = \pi r^2 \left[\frac{S - \pi r^2}{2\pi r}\right] = \frac{1}{2}[Sr - \pi r^3]$$ 1

$$\frac{dV}{dr} = \frac{1}{2}[S - 3\pi r^2]$$ $\frac{1}{2}$

$$\frac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2 \text{ or } 2\pi rh + \pi r^2 = 3\pi r^2$$ 1

$$\Rightarrow 2\pi rh = 2\pi r^2 \qquad \Rightarrow h = r$$ 1

$$\frac{d^2V}{dr^2} = -6\pi r < 0$$ 1

$\therefore$ For volume to be maximum, height = radius

28. $|A| = 1(7) - 1(-3) + 1(-1) = 9$ 1

$$(\text{adj } A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$ 2

$$\Rightarrow A^{-1} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$ $\frac{1}{2}$

Given equations can be written as $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

or $AX = B \Rightarrow X = A^{-1}B$ $\frac{1}{2}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ $1\frac{1}{2}$

$\therefore$ x = 1, y = 2, z = 3 $\frac{1}{2}$

OR

$$\text{Let: } \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1

$$R_1 \leftrightarrow R_3 \begin{bmatrix} 3 & 7 & 2 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$$

$$\begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{matrix} \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$$

$$R_3 \to R_3 - 2R_1 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & -5 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & 0 & -2 \end{bmatrix} A$$

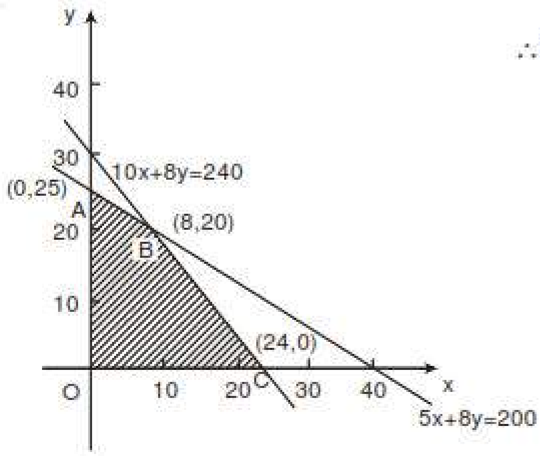
$$R_3 \to R_3 + 5R_2 \begin{bmatrix} 1 & 4 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ -2 & 5 & -2 \end{bmatrix} A$$ 4

$$\begin{matrix} R_1 \to R_1 - 4R_2 \\ R_3 \to -R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

$$R_1 \to R_1 - R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$$ 1

29. Let number of Souvenirs of type A be x, and that of type B be y.



$\therefore$ L.P.P is maximise $P = 50x + 60y$ $\frac{1}{2}$

such that $5x + 8y \le 200$
$10x + 8y \le 240$
$x, y \ge 0$ $2\frac{1}{2}$

Correct Graph 2

P(at A) = ₹1500

P(at B) = ₹(400 + 1200) = ₹1600

P(at C) = ₹(1200)

$\therefore$ Max Profit = ₹ 1600, when number of Souvenirs of type A = 8 and number of Souvenirs of type B = 20. 1