1.) unusual: 7 letters, 5 unique letters

- unique subsets of 5 letters?

Order doesn't matter

uuusa = uuuas

one u: 4 letters remaining, pick 4, $\binom{4}{4} = 1$

two us: 4 letters remaining, pick 3, $\binom{4}{3} = 4$

three us: 4 letters remaining, pick 2 $\binom{4}{2} = 6$

$\binom{4}{4} + \binom{4}{3} + \binom{4}{2} =$ $\boxed{\text{11 unique substrings}}$

- how many different strings?

one u: one option (ulnsa), 5! different ways to arrange

two us: 4 options, 5! ways to arrange

- us are the same, so usual + usual is just one option
- divide by how many ways to swap the u's (2!)

three us: 6 options, 5! ways to arrange

- same as 2 u's, must divide by ways to swap us around (3!)

total strings: $5! + \frac{5!}{2!} \times 4 + \frac{5!}{3!} \times 6 =$ $\boxed{\text{480 unique strings}}$

2.) the pairs: 13 faces, picking 2
4 suits, picking 2 (for both) $\binom{13}{2}\binom{4}{2}\binom{4}{2}$

last card: 11 faces left (to be unique), picking 1 $\binom{11}{1}\binom{4}{1}$
4 suits, picking 1

$\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1} =$ $\boxed{\text{123, 552 ways}}$

3.) can play in any order, can play multiple songs for each, can get none

stars and bars

n = couples

k = songs

the fighting couple gets either 1 or no song

if fighting couple gets 1 song:

- violinist will have 15 songs left
- fighting already accounted for, so n=6

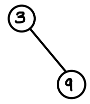
$\binom{6+15-1}{15} = \binom{20}{15}$

if fighting couple gets no song:

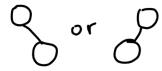
- violinist has 16 songs left

$\binom{6+16-1}{16} = \binom{21}{16}$

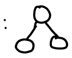
$\binom{20}{15} + \binom{21}{16} =$ $\boxed{\text{35, 853 ways}}$

4.) 

2 node tree: 2 ways



3 nodes: 5 ways

- each 2 node way has 2 different ways to add a third, plus the one overlapping: 

4 nodes: 10 + 4 = 14

each 3 node tree has 2 ways to add a 4th node so 5 x 2 = 10

plus the two ways stemming from the 2 node tree (that don't overlap): 2 x 2 = 4

5 nodes: 14 x 3 = 42

just a 4 node with three different starting options

so 14 x 3

12 x 2 = 24

4 x 3 x 2 = 24

14 x 5 x 2 = $\boxed{\text{420 ways}}$

5.) nurses are identical, order doesn't matter

4 nurses: get 1

6 left: $\binom{6}{4} = 15$

3 nurses: one nurse would get one extra (3 options)

15 + 3 = $\boxed{\text{18 ways}}$