1.) $\frac{15!}{(15-8)!}$ ← ways to choose

$\uparrow$ n-k

need to divide by total options: 15^8

$$\frac{\frac{15!}{7!}}{15^8} = \boxed{.101}$$

2.) options: 0-9

5 even numbers

5 odd numbers

$\underset{5\text{opt.}}{\underline{o}}\ \underset{5\text{opt.}}{\underline{o}}\ \underset{7\text{opt.}}{\underline{?}}\ \underset{6\text{opt.}}{\underline{?}}\ \underset{5\text{opt.}}{\underline{e}} = 4200$

total of numbers: 10^8

exactly 5: 10^5

$$\frac{4200}{10^5 \times 10^8} = \boxed{4.2 \times 10^{-10} \text{ chance}}$$

3.) $P(A) = \underline{\text{at least}}$ 2 dice 4 or higher $= P(2 \text{ dice} \geq 4) + P(3 \text{ dice} \geq 4)$

$P(B)$ = all three show same value

$$P(A) = \underbrace{\binom{3}{2}\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)}_{2 \text{ or more}} + \underbrace{\binom{3}{3}\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)}_{3} = \frac{1}{2}$$

$$P(B) = \underset{\substack{\uparrow \\ 1^{st} \text{ val can be} \\ \text{any } (\frac{6}{6})}}{1} \times \underbrace{\frac{1}{6} \times \frac{1}{6}}_{\text{next two same as first}} = \frac{1}{36}$$

$P(A) \cdot P(B)$ must $= P(A \cap B)$

$P(A \cap B) = \dfrac{3 \text{ ways to intersect}:\ 3}{6^3 \text{ ways to roll 3 dice}:\ 6^3} = \frac{1}{72}$ ✓

$\frac{1}{36} \cdot \frac{1}{2} = \frac{1}{72}$ ✓ $\boxed{\text{A and B are independent}}$

4.) total hands: $\binom{52}{5}$

ways to flush: 13 faces, pick 5 $\binom{13}{5}$

4 suits, pick 1 $\binom{4}{1}$

$$P(\text{flush}) = \frac{\binom{13}{5}\binom{4}{1}}{\binom{52}{5}}$$

expected number of flushes $= n \times p$

$\uparrow$ number of hands

$$1 = n \times \frac{\binom{13}{5}\binom{4}{1}}{\binom{52}{5}}$$

$$\boxed{n = 505 \text{ hands}}$$

5.) P(superstar played (A) | 4/5 games won (B))

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\text{not}A) \cdot P(A)} = \frac{(.36015)(.75)}{(.36015)(.75) + (.15625)(.25)} = .874$$

$P(A) = \frac{3}{4}$

$P(\text{not } A) = \frac{1}{4}$

$P(B|A) = \binom{n}{k} p^k (1-p)^{n-k} = \binom{5}{4}\left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right)^1 = .36015$

$P(B|\text{not}A) = \binom{5}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 = .15625$

$\boxed{87.4\% \text{ chance of star playing}}$