

# 023 - Poisson Regression

EPIB 607

Sahir Rai Bhatnagar

Department of Epidemiology, Biostatistics, and Occupational Health  
McGill University

`sahir.bhatnagar@mcgill.ca`

slides compiled on November 1, 2021





A total of 189,612 person-years of follow up were accumulated over the course of the study: 151,690 among infants who were being breastfed and 37,922 among infants not being breastfed. Over the course of follow up the investigators identified 514,230 incident cases of respiratory infection among breastfeeding infants and 140,312 among non-breastfeeding infants. Calculate the crude incidence rate difference and 95% CI comparing infants who were not breastfed with those who were.

```
fit <- glm(cases ~ -1 + PT + PT:not_breastfed, family = poisson(link = identity))
print(summary(fit), signif.stars = F)

##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## PT              3.390006   0.004727  717.10  <2e-16
## PT:not_breastfed 0.310010   0.010951   28.31  <2e-16
##
## (Dispersion parameter for poisson family taken to be 1)
##
##      Null deviance: Inf  on 2  degrees of freedom
## Residual deviance:  0  on 0  degrees of freedom
## AIC: 32.678
##
## Number of Fisher Scoring iterations: 2
```

---

<sup>1</sup> this page is intentionally left blank



Calculate the crude incidence rate ratio and 95% CI comparing infants who were not breastfed with those who were.

```
##
## Coefficients:
##           Estimate Std. Error z value Pr(>|z|)
## (Intercept)  1.220832   0.001395  875.46   <2e-16
## not_breastfed 0.087505   0.003012   29.05   <2e-16
##
## (Dispersion parameter for poisson family taken to be 1)
##
##      Null deviance: 8.3002e+02  on 1  degrees of freedom
## Residual deviance: 1.1533e-10  on 0  degrees of freedom
## AIC: 32.678
##
## Number of Fisher Scoring iterations: 2
```

---

<sup>1</sup> this page is intentionally left blank





See the 2018 Lancet article *Efficacy of Olyset Duo, a bednet containing pyriproxyfen and permethrin, versus a permethrin-only net against clinical malaria in an area with highly pyrethroid-resistant vectors in rural Burkina Faso: a cluster-randomised controlled trial* by Tiono et. al. Reproduce the Rate ratio (95% CI) in Table 2. Calculate the rate difference and 95% CI comparing PPF-treated to Standard long-lasting insecticidal nets. Check the goodness of fit.

```
## Call:
## glm(formula = cases ~ exposure + offset(log(years)), family = poisson(link = log),
##      data = df)
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.68314    0.02432  28.092 < 2e-16
## exposure    -0.26687    0.03286  -8.121 4.62e-16
##
## (Dispersion parameter for poisson family taken to be 1)
##
##      Null deviance: 1381.2  on 23  degrees of freedom
## Residual deviance: 1316.0  on 22  degrees of freedom
## AIC: 1476.7
##
## Number of Fisher Scoring iterations: 5
```

---

<sup>1</sup> this page is intentionally left blank



We can fit the following simple (multiplicative) rate ratio model to the patterns of mortality rates for 1980-1984 and 2000-2004. The reference cell is females 70-74, 1980-84.  $R$  = rate.  $M$  = multiplier.

Year	Age	Female (F)		Male (M)			
1980-1984	70-74	$R_F$		$R_F$	$\times M_M$		
	75-79	$R_F$	$\times M_{75}$	$R_F$	$\times M_{75}$	$\times M_M$	
	80-84	$R_F$	$\times M_{80}$	$R_F$	$\times M_{80}$	$\times M_M$	
	85-89	$R_F$	$\times M_{85}$	$R_F$	$\times M_{85}$	$\times M_M$	
2000-2004	70-74	$R_F$	$\times M_{20y}$	$R_F$	$\times M_M$	$\times M_{20y}$	
	75-79	$R_F$	$\times M_{75}$	$R_F$	$\times M_{75}$	$\times M_M$	$\times M_{20y}$
	80-84	$R_F$	$\times M_{80}$	$R_F$	$\times M_{80}$	$\times M_M$	$\times M_{20y}$
	85-89	$R_F$	$\times M_{85}$	$R_F$	$\times M_{85}$	$\times M_M$	$\times M_{20y}$

Year	Age	Female_deaths	Female_PT	Female_rate	Male_deaths	Male_PT	Male_rate
1980-1984	70-74	15989	586882.8	0.0272439	23810	456908.21	0.0521111
1980-1984	75-79	20838	454142.7	0.0458843	24707	300318.92	0.0822692
1980-1984	80-84	24073	297678.6	0.0808691	20319	167303.51	0.1214499
1980-1984	85-89	20216	147771.7	0.1368057	13524	74295.83	0.1820291
2000-2004	70-74	13912	521561.9	0.0266737	17360	436994.92	0.0397259
2000-2004	75-79	19731	471945.5	0.0418078	22477	341362.82	0.0658449
2000-2004	80-84	25541	369989.9	0.0690316	22992	217929.72	0.1055019
2000-2004	85-89	27135	226798.1	0.1196439	17444	104009.58	0.1677153
2005-2009	70-74	12179	540568.6	0.0225300	15782	472012.84	0.0334355
2005-2009	75-79	17273	444474.2	0.0388616	19547	344351.34	0.0567647
2005-2009	80-84	23513	363534.1	0.0646789	21781	230530.24	0.0944822
2005-2009	85-89	26842	237877.3	0.1128397	17811	114485.04	0.1555749

$$\begin{aligned}
 \text{Rate} &= \text{_____} \times \frac{\text{_____}}{\text{if } 75-79} \times \frac{\text{_____}}{\text{if } 80-84} \times \frac{\text{_____}}{\text{if } 85-89} \times \frac{\text{_____}}{\text{if male}} \times \frac{\text{_____}}{\text{if 2000-04}} \\
 \log[\text{Rate}] &= \text{_____} + \frac{\text{_____}}{\text{if } 75-79} + \frac{\text{_____}}{\text{if } 80-84} + \frac{\text{_____}}{\text{if } 85-89} + \frac{\text{_____}}{\text{if male}} + \frac{\text{_____}}{\text{if 2000-04}} \\
 \log[\text{Rate}] &= \text{_____} + \frac{\text{_____}}{\times I_{75-79}} + \frac{\text{_____}}{\times I_{80-84}} + \frac{\text{_____}}{\times I_{85-89}} + \frac{\text{_____}}{\times I_{\text{male}}} + \frac{\text{_____}}{\times I_{2000-04}}
 \end{aligned}$$

where each ' $I$ ' is a (0/1) indicator of the category in question. By using both the 0 and 1 values of each  $I$ , this 6-parameter equation produces a fitted value for each of the  $4 \times 2 \times 2 = 16$  cells.

---

<sup>1</sup> this page is intentionally left blank

# Session Info

```
R version 4.1.1 (2021-08-10)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Pop!_OS 21.04

Matrix products: default
BLAS:   /usr/lib/x86_64-linux-gnu/openblas-pthread/libblas.so.3
LAPACK: /usr/lib/x86_64-linux-gnu/openblas-pthread/libopenblaspr0.3.13.so

attached base packages:
[1] tools      stats      graphics  grDevices  utils      datasets  methods
[8] base

other attached packages:
[1] DT_0.16 mosaic_1.7.0 Matrix_1.3-2 mosaicData_0.20.1
[5] ggformula_0.9.4 ggstance_0.3.4 lattice_0.20-41 kableExtra_1.2.1
[9] socviz_1.2 gapminder_0.3.0 here_0.1 NCStats_0.4.7
[13] FSA_0.8.30 forcats_0.5.1 stringr_1.4.0 dplyr_1.0.7
[17] purrr_0.3.4 readr_1.4.0 tidyr_1.1.4 tibble_3.1.5
[21] ggplot2_3.3.5 tidyverse_1.3.0 knitr_1.36

loaded via a namespace (and not attached):
[1] fs_1.5.0 lubridate_1.7.9 webshot_0.5.2 httr_1.4.2
[5] rprojroot_2.0.2 backports_1.2.1 utf8_1.2.2 R6_2.5.1
[9] DBI_1.1.1 colorspace_2.0-2 withr_2.4.2 tidyselect_1.1.1
[13] gridExtra_2.3 leaflet_2.0.3 curl_4.3.2 compiler_4.1.1
[17] cli_3.0.1 rvest_1.0.0 pacman_0.5.1 xml2_1.3.2
[21] ggdendro_0.1.22 mosaicCore_0.8.0 scales_1.1.1 digest_0.6.28
[25] foreign_0.8-81 rmarkdown_2.11.3 rio_0.5.16 pkgconfig_2.0.3
[29] htmltools_0.5.2 highr_0.9 dbplyr_1.4.4 fastmap_1.1.0
[33] htmlwidgets_1.5.3 rlang_0.4.12 readxl_1.3.1 rstudioapi_0.13
[37] farver_2.1.0 generics_0.1.0 jsonlite_1.7.2 crosstalk_1.1.1
[41] zip_2.2.0 car_3.0-9 magrittr_2.0.1 Rcpp_1.0.7
[45] munsell_0.5.0 fansi_0.5.0 abind_1.4-5 lifecycle_1.0.1
[49] stringi_1.7.5 carData_3.0-4 MASS_7.3-53.1 plyr_1.8.6
[53] grid_4.1.1 blob_1.2.1 ggrepel_0.8.2 crayon_1.4.1
[57] cowplot_1.1.0 haven_2.3.1 splines_4.1.1 hms_1.1.1
[61] pillar_1.6.4 reprex_0.3.0 glue_1.4.2 evaluate_0.14
[65] data.table_1.14.2 modelr_0.1.8 vctrs_0.3.8 tweenr_1.0.1
[69] cellranger_1.1.0 gtable_0.3.0 polyclip_1.10-0 assertthat_0.2.1
[73] TeachingDemos_2.12 xfun_0.26 ggforce_0.3.2 openxlsx_4.1.5
[77] RColorBrewer_1.2-1 viridisLite_0.4.0 ellipsis_0.3.2
```