021 - Introduction to Regression

EPIB 607

Sahir Rai Bhatnagar
Department of Epidemiology, Biostatistics, and Occupational Health
McGill University

sahir.bhatnagar@mcgill.ca

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Parameter-contrasts

Fitting the regression equation with our sample data

Introduction to parameter-contrasts

- We started the course by talking about the case where there were no determinants, i.e., no subpopulations $\rightarrow$ there was one global parameter (μ, π, λ).

- Now we concern ourselves with determinants of the global parameter. For example:
  - μ_{north} vs. μ_{south}
  - π_{north} vs. π_{south}
  - λ_{north} vs. λ_{south}

- Today we introduce population parameter <u>contrasts</u> in a regression framework

Why regression for parameter-contrasts?

- Why do we start in a regression framework (as opposed to two-sample inference in DVB)?
- **Parameter contrasts are a special case of regression**

What is regression?

- How **parameters** relate to its determinants
- How to link the parameters between the different populations through generic equations, that looks like a regression equation.
- Then once you get data, you can actually fit or get your best estimates of those parameters

Linear regression: The Concept

- A regression model is said to be **linear** when it is of the form

$$\begin{aligned}\mu &= \mu_0 + \sum_{j=1}^{p} \beta_j X_j \\ &= \mu_0 + \beta_1 X_1 + \beta_1 X_1 + \cdots + \beta_p X_p\end{aligned}$$

- Which means that the value of the mean (μ) is viewed as a linear combination of the parameters $\mu_0, \beta_1, \beta_2, \ldots, \beta_p$, the coefficients of the linear combination being the realizations for the X's

Linear regression: Example

- Consider the depths of the ocean example
- Here, μ designates the true mean depth of the ocean
- For this parameter, one might consider the determinant
  - X which is an indicator variable defined by

$$X = \begin{cases} 1 & \text{if Southern hemisphere} \\ 0 & \text{if Northern hemisphere} \end{cases}$$

Linear regression: Example

- The model might be taken as

$$\mu_X = \mu_0 + \beta_1 \cdot X$$

  and provides the mean depth of the ocean given X

- The subscript X indicates that μ depends on the value of x

- The mean depth of the ocean μ_X is a linear combination of μ_0 and β_1

- If we had an infinite amount of data, the mean depth of the ocean would be determined by hemisphere:

$$\mu_X = \begin{cases} \mu_0 + \beta_1 & \text{if Southern hemisphere} \\ \mu_0 & \text{if Northern Hemisphere} \end{cases}$$

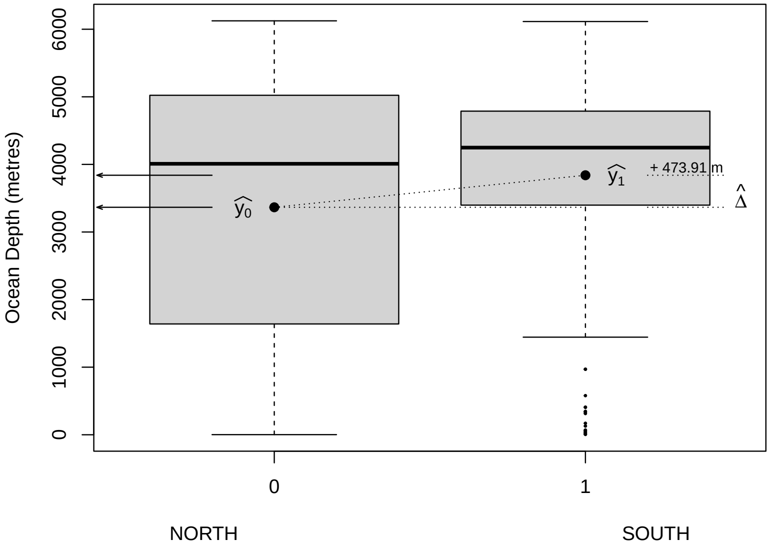
Parameter-contrasts

Fitting the regression equation with our sample data

Depths of the ocean: North vs. South Hemisphere

```r
# load function to get depths
source("https://raw.githubusercontent.com/sahirbhatnagar/EPIB607/master/inst/labs/
       003-ocean-depths/automate_water_task.R")

# get 1000 depths
set.seed(222333444)
depths <- automate_water_task(index = sample(1:50000, 1000),
student_id = 222333444, type = "depth")

# separate by north and south hemisphere
depths_north <- depths[which(depths$lat>0),]
depths_south <- depths[which(depths$lat<0),]

# restrict sample to 200 (at random)
depths_north <- depths_north[sample(1:nrow(depths_north), 200), ]
depths_south <- depths_south[sample(1:nrow(depths_south), 200), ]

# add indicator variable
depths_north$South <- 0
depths_south$South <- 1

# combine data
depths <- rbind(depths_north, depths_south)
head(depths)

# calculate mean and sd by hemisphere
    mean.sd <- depths %>% group_by(South) %>%
    summarise(means = mean(alt), sds = sd(alt))

    means <- mean.sd$means
    sds <- mean.sd$sds
```

Depths of the ocean: North vs. South Hemisphere



Standard error of the mean difference

To perform inference we first need to calculate the SE of the mean difference given by:

$$SE_{\bar{y}_1 - \bar{y}_0} = \sqrt{\frac{s_0^2}{n_0} + \frac{s_1^2}{n_1}} \tag{1}$$

```
n0 <- nrow(depths_north)
n1 <- nrow(depths_south)

mean0 <- mean(depths_north$alt)
mean1 <- mean(depths_south$alt)

var0 <- var(depths_north$alt)
var1 <- var(depths_south$alt)

(SEM <- sqrt(var0/n0 + var1/n1))

## [1] 171.4861
```

95% Confidence Interval for the Mean Difference

We can then calculate a 95% CI for the mean difference given by:

$$(\bar{y}_1 - \bar{y}_0) \pm t^{\star}_{(n_0+n_1-2)} \times SE_{\bar{y}_1 - \bar{y}_0} \tag{2}$$

```
# assuming equal variances
(mean1 - mean0) + qt(c(0.025, 0.975), df = n0 + n1 - 2) * SEM

## [1] 136.7782 811.0418

# similar to z interval
qnorm(c(0.025, 0.975), mean = mean1 - mean0, sd = SEM)

## [1] 137.8034 810.0166
```

Parameter contrasts with regression

Using the `lm` function in R:

```
# regression. lm assumes equal variances
fit <- lm(alt ~ South, data = depths)
summary(fit)

## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3365.6      121.3  27.755  < 2e-16 ***
## South          473.9      171.5   2.764  0.00598 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1715 on 398 degrees of freedom
## Multiple R-squared: 0.01883,^^IAdjusted R-squared: 0.01636
## F-statistic: 7.637 on 1 and 398 DF,  p-value: 0.005983
```

Confidence interval from regression fit

```
confint(fit)

##                 2.5 %    97.5 %
## (Intercept) 3127.2068 3603.9832
## South        136.7782  811.0418
```

Unequal variances using `stats::t.test`

`stats::t.test` assumes unequal variances by default:

```
stats::t.test(alt ~ South, data = depths, var.equal = FALSE)

## Welch Two Sample t-test with alt by South
## t = -2.7635, df = 356.262, p-value = 0.006015
## alternative hypothesis: true difference in means between group 0 and group 1 is not equa
## 95 percent confidence interval:
##  -811.1623 -136.6577
## sample estimates:
## mean in group 0 mean in group 1
##        3365.595        3839.505

(mean0 - mean1) + qt(c(0.025, 0.975), df = 349.61783) * SEM

## [1] -811.1841 -136.6359
```

Equal variances using stats::t.test

We can specify equal variance assumption in `stats::t.test`:

```
stats::t.test(alt ~ South, data = depths, var.equal = TRUE)

##  Two Sample t-test with alt by South
## t = -2.7635, df = 398, p-value = 0.005983
## alternative hypothesis: true difference in means between group 0 and group 1 is not equa
## 95 percent confidence interval:
##  -811.0418 -136.7782
## sample estimates:
## mean in group 0 mean in group 1
##        3365.595        3839.505

(mean0 - mean1) + qt(c(0.025, 0.975), df = n0 + n1 - 2) * SEM

## [1] -811.0418 -136.7782
```

Session Info

```
R version 4.1.1 (2021-08-10)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Pop!_OS 21.04

Matrix products: default
BLAS:   /usr/lib/x86_64-linux-gnu/openblas-pthread/libblas.so.3
LAPACK: /usr/lib/x86_64-linux-gnu/openblas-pthread/libopenblasp-r0.3.13.so

attached base packages:
[1] tools     stats     graphics  grDevices utils     datasets  methods
[8] base

other attached packages:
 [1] DT_0.16          mosaic_1.7.0      Matrix_1.3-2      mosaicData_0.20.1
 [5] ggformula_0.9.4  ggstance_0.3.4    lattice_0.20-41   kableExtra_1.2.1
 [9] socviz_1.2       gapminder_0.3.0   here_0.1          NCStats_0.4.7
[13] FSA_0.8.30       forcats_0.5.1     stringr_1.4.0     dplyr_1.0.7
[17] purrr_0.3.4      readr_1.4.0       tidyr_1.1.4       tibble_3.1.5
[21] ggplot2_3.3.5    tidyverse_1.3.0   knitr_1.36

loaded via a namespace (and not attached):
 [1] fs_1.5.0           lubridate_1.7.9    webshot_0.5.2      httr_1.4.2
 [5] rprojroot_2.0.2    backports_1.2.1    utf8_1.2.2         R6_2.5.1
 [9] DBI_1.1.1          colorspace_2.0-2   withr_2.4.2        tidyselect_1.1.1
[13] gridExtra_2.3      leaflet_2.0.3      curl_4.3.2         compiler_4.1.1
[17] cli_3.0.1          rvest_1.0.0        pacman_0.5.1       xml2_1.3.2
[21] ggdendro_0.1.22    mosaicCore_0.8.0   scales_1.1.1       digest_0.6.28
[25] foreign_0.8-81     rmarkdown_2.11.3   rio_0.5.16         pkgconfig_2.0.3
[29] htmltools_0.5.2    highr_0.9          dbplyr_1.4.4       fastmap_1.1.0
[33] htmlwidgets_1.5.3  rlang_0.4.12       readxl_1.3.1       rstudioapi_0.13
[37] farver_2.1.0       generics_0.1.0     jsonlite_1.7.2     crosstalk_1.1.1
[41] zip_2.2.0          car_3.0-9          magrittr_2.0.1     Rcpp_1.0.7
[45] munsell_0.5.0      fansi_0.5.0        abind_1.4-5        lifecycle_1.0.1
[49] stringi_1.7.5      carData_3.0-4      MASS_7.3-53.1      plyr_1.8.6
[53] grid_4.1.1         blob_1.2.1         ggrepel_0.8.2      crayon_1.4.1
[57] cowplot_1.1.0      haven_2.3.1        splines_4.1.1      hms_1.1.1
[61] pillar_1.6.4       reprex_0.3.0       glue_1.4.2         evaluate_0.14
[65] data.table_1.14.2  modelr_0.1.8       vctrs_0.3.8        tweenr_1.0.1
[69] cellranger_1.1.0   gtable_0.3.0       polyclip_1.10-0    assertthat_0.2.1
[73] TeachingDemos_2.12 xfun_0.26          ggforce_0.3.2      openxlsx_4.1.5
[77] broom_0.7.9        viridisLite_0.4.0  ellipsis_0.3.2
```