

# 004 - Exploring Data - Part II

EPIB 607 - FALL 2020

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# Summarizing relationships between two variables

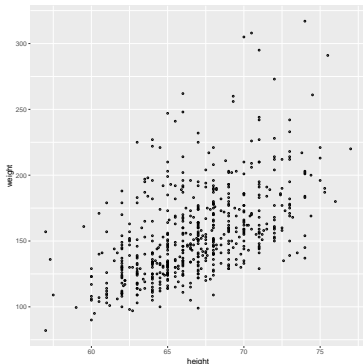
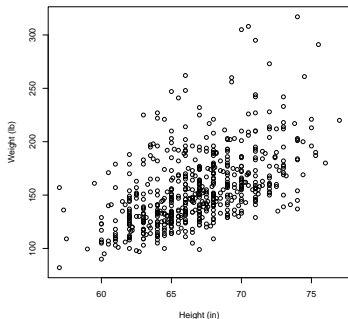
Approaches for summarizing relationships between two variables vary depending on variable types:

- Two numerical variables
- Two categorical variables
- One numerical variable and one categorical variable



# Scatterplots

```
library(ggplot2); library(oibioestat);  
data(famuss)  
  
plot(famuss$height, famuss$weight, xlab = "Height (in)", ylab = "Weight (lb)")  
  
ggplot(data = famuss, mapping = aes(x = height, y = weight)) +  
  geom_point(size = 0.8, pch = 21)
```



# Pearson's correlation coefficient

- The **sample** correlation ( $r$ ) between two variables  $X$  and  $Y$  is given by:

$$r = \frac{1}{n-1} \sum_{i=1}^n z_X \cdot z_Y \quad (1)$$

$$= \frac{1}{n-1} \sum_{i=1}^n \left( \frac{x_i - \bar{x}}{s_X} \right) \left( \frac{y_i - \bar{y}}{s_Y} \right) \quad (2)$$

- $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$  the  $n$  paired sample values of  $X$  and  $Y$
- $z_X$  and  $z_Y$  are the sample Z-scores of the  $X$  and  $Y$  variables, respectively
- $s_X$  and  $s_Y$  are the sample standard deviations of the  $X$  and  $Y$  variables, respectively
- $\bar{x}$  and  $\bar{y}$  are the sample means of the  $X$  and  $Y$  variables, respectively

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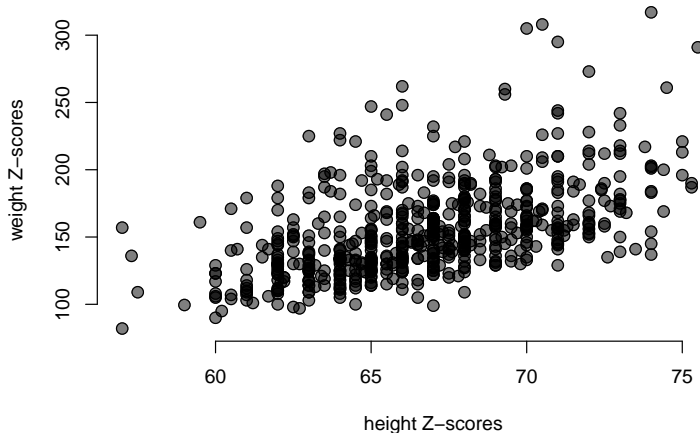
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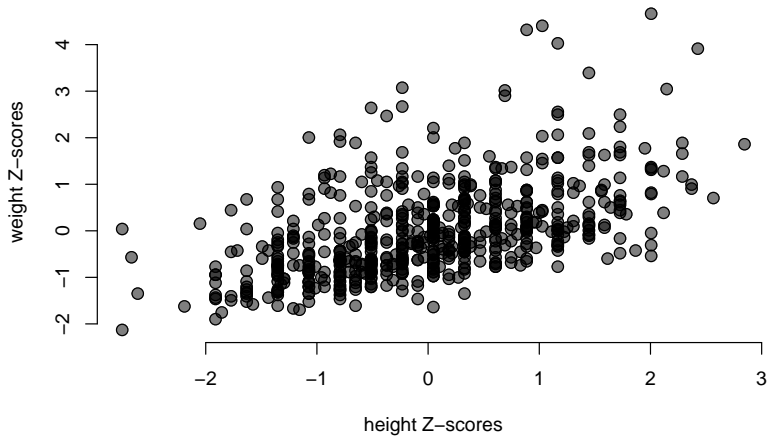
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- $\bar{x}$  and  $\bar{y}$  are the sample means of the  $X$  and  $Y$  variables, respectively
- The correlation coefficient quantifies the strength of a **linear** trend.

## Plot of weight vs. height in famuss dataset

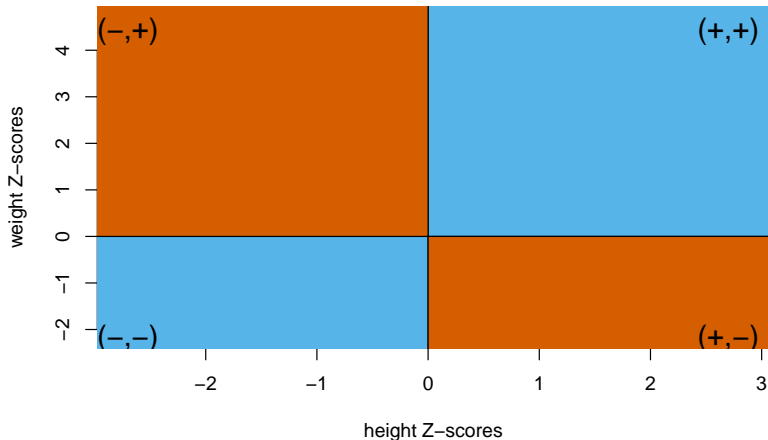


## Plot of Z-scores weight vs. Z-scores height in famuss dataset

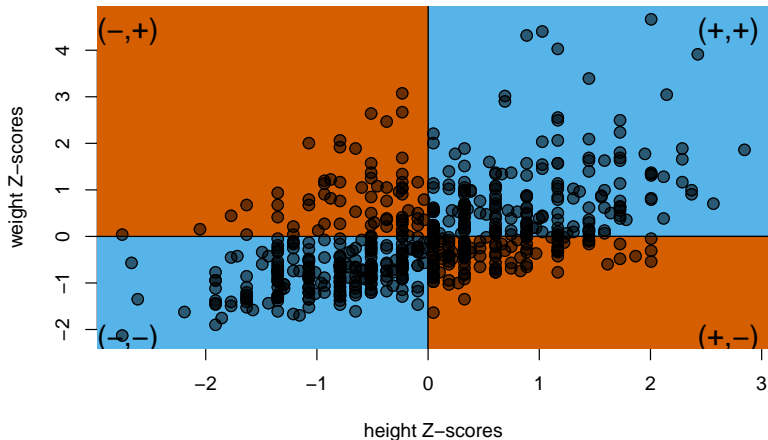




Partition the graph into four quadrants  $(x, y)$



# Correlation depends on which quadrants the points are on



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- Since the formula for calculating the correlation coefficient standardizes the variables, changes in scale or units of measurement will not affect its value

Exercise: Show mathematically that the correlation ( $r$ ) is bounded by -1 and 1

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Consider that we can't have higher correlation than when we compare a list to itself (perfect correlation).



# Correlation and Simple linear Regression

- If we are predicting a random variable  $Y$  knowing the value of another variable  $X = x$  using a regression line, then the formula for the regression can be given by:

$$\left( \frac{Y - \bar{y}}{s_Y} \right) = r \left( \frac{x - \bar{x}}{s_X} \right) \quad (3)$$

- This can be rewritten as:

$$Y = \bar{y} + r \left( \frac{x - \bar{x}}{s_X} \right) s_Y \quad (4)$$

# Correlation in R

- Correlation between weight and height in the famuss dataset:

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## [1] 0.53
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```
summary(lm(height ~ weight, data = famuss))
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  58.2952    0.5732   101.7   <2e-16 ***
## weight       0.0548    0.0036    15.2   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3 on 593 degrees of freedom
## Multiple R-squared:  0.282, Adjusted R-squared: 0.281
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- Exercise: calculate the correlation coefficient from the regression coefficient for weight.

# Let's remind ourselves about random variability

- In many cases, we do not observe data for the entire population of interest but rather for a random sample.
- As with the mean and standard deviation, the sample correlation is the most commonly used estimator of the population correlation.
- This implies that the correlation we compute and use as a summary is a random variable.

# Let's remind ourselves about random variability

Lets create a pseudo population from the 595 observations by sampling **with replacement**, and calculate the correlation. Lets repeat this process 1000 times:

```
B <- 1000; N <- 595
R <- replicate(B, {
  dplyr::sample_n(famuss, size = N, replace = TRUE) %>%
  dplyr::summarize(r = cor(height, weight)) %>%
  dplyr::pull(r)
})
```

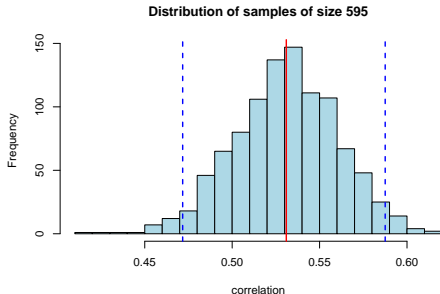
```
hist(R, breaks = 20, col = "lightblue", xlab = "correlation",
     main = "Distribution of samples of size 595")
abline(v = mean(R), col = "red", lwd = 2)
abline(v = quantile(R, probs = c(0.025, 0.975)), col = "blue",
      lty = 2, lwd = 2)
```

```
mean(R)

## [1] 0.53

quantile(R, probs = c(0.025, 0.975))

## 2.5% 98%
## 0.47 0.59
```



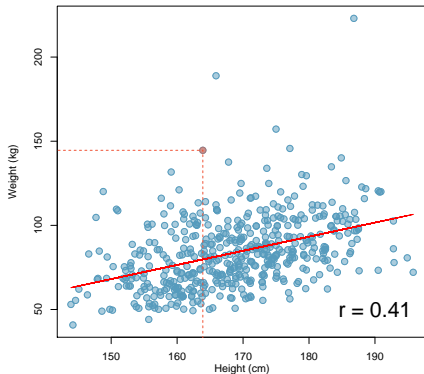
## Another example: NHANES<sup>2</sup>

- The National Health and Nutrition Examination Survey (NHANES) consists of a set of surveys and measurements conducted by the US CDC to assess the health and nutritional status of adults and children in the United States.
- The following example uses data from a sample of 500 adults (individuals ages 21 and older) from the NHANES dataset<sup>1</sup>.

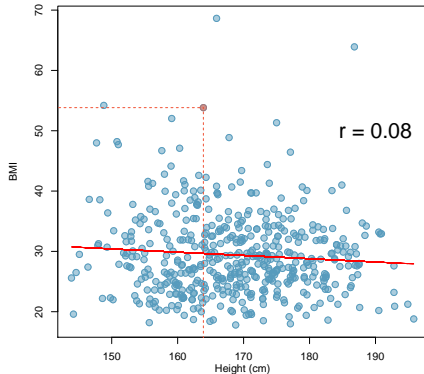
---

<sup>1</sup>The sample is available as `nhanes.samp.adult.500` in the R `oibiostat` package

<sup>2</sup><http://www.cdc.gov/nchs/nhanes.htm>



(a)



(b)

**Figure:** (a) A scatterplot showing height versus weight from the 500 individuals in the sample from NHANES. One participant 163.9 cm tall (about 5 ft, 4 in) and weighing 144.6 kg (about 319 lb) is highlighted. (b) A scatterplot showing height versus BMI from the 500 individuals in the sample from NHANES. The same individual highlighted in (a) is marked here, with BMI 53.83. Fitted regression lines are shown in red with correlation coefficient  $r$ .  $\text{BMI} = \text{weight}/\text{height}^2 \times 703$ .



## Cautionary notes

- The formulas above are for a particular sample, hence the lower case letters  $r, x, y$ . In statistical terms,  $r$  is the **estimator** for the population-level correlation  $\rho$  (the **estimand**) of the random variables  $X$  and  $Y$ . The actual value of the sample correlation is denoted by  $\hat{r}$  and is called the **estimate**

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- This implies that we are not 100% confident in our estimate and therefore should provide a confidence interval as well.
- A strong linear relationship is not necessarily a **causal** relationship, that is, just because  $r \approx 1$  (or  $r \approx -1$ ) does not mean that  $x$  **causes** changes in  $y$  (we may have a *spurious* correlation).

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- Just because  $r \approx 0$  does not mean that  $x$  and  $y$  are unrelated, merely that they are **uncorrelated**. That is, it is possible to construct examples where  $x$  and  $y$  have a strong functional relationship, but where  $r = 0$ .
- $X, Y$  independent  $\Rightarrow r_{XY} = 0$
- $r_{XY} = 0 \not\Rightarrow X, Y$  are independent

# Anscombe's quartet<sup>3</sup>

```
library(datasets); data("anscombe")
```

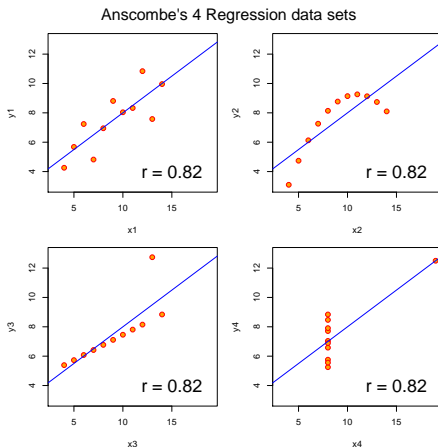
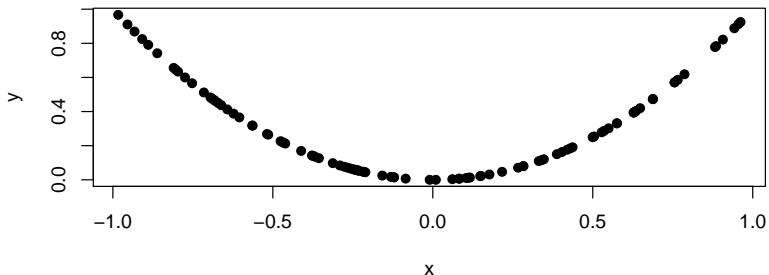


Figure: All four panels have the exact same linear correlation coefficient

<sup>3</sup> Anscombe, Francis J. (1973). Graphs in statistical analysis. The American Statistician, 27, 17–21. doi: 10.2307/2682899.

# Zero linear correlation does not imply independence

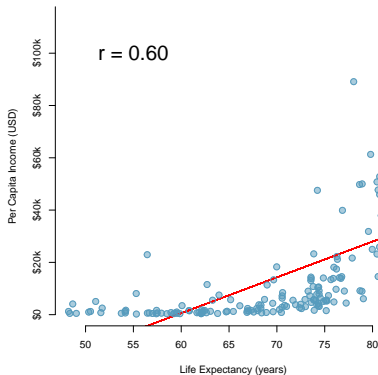
```
set.seed(12)
x <- runif(100,-1,1)
y <- x^2
plot(x,y, pch = 19)
```



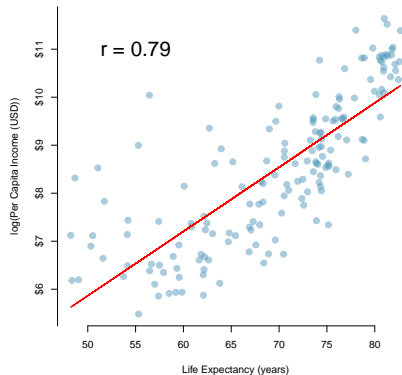
```
cor(x,y)
## [1] -0.023
```

Another example of same summary statistics but very different relationships

<https://www.autodeskresearch.com/publications/samestats>



(a)



(b)

**Figure:** (a) per capita income vs. life expectancy (b) log per capita income vs. life expectancy. Fitted regression line in red with correlation coefficient  $r$ .<sup>4</sup>

<sup>4</sup>The World Development Indicators (WDI) is a database of country-level variables (i.e., indicators) recording outcomes for a variety of topics, including economics, health, mortality, fertility, and education





# Two categorical variables

A contingency table summarizes data for two categorical variables:

```
tab1 <- table(famuss$race,  
              famuss$actn3.r577x)
```

```
tab1
```

```
##  
##           CC  CT  TT  
## African Am  16   6   5  
## Asian       21  18  16  
## Caucasian  125 216 126  
## Hispanic     4  10   9  
## Other        7  11   5
```

```
addmargins(tab1)
```

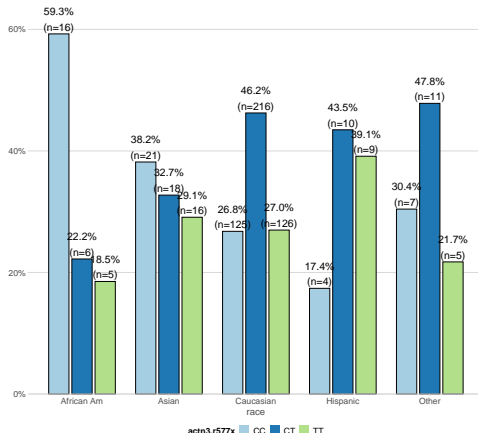
```
##  
##           CC  CT  TT Sum  
## African Am  16   6   5  27  
## Asian       21  18  16  55  
## Caucasian  125 216 126 467  
## Hispanic     4  10   9  23  
## Other        7  11   5  23  
## Sum        173 261 161 595
```

# Conditional distribution of genotype given race

The distributions we create this way are called **conditional distributions**, because they show the distribution of one variable for just those cases that satisfy a condition on another variable

```
addmargins(  
  prop.table(tab1, margin = 1)  
)  
  
##  
##          CC   CT   TT   Sum  
## African Am 0.59 0.22 0.19 1.00  
## Asian      0.38 0.33 0.29 1.00  
## Caucasian  0.27 0.46 0.27 1.00  
## Hispanic   0.17 0.43 0.39 1.00  
## Other      0.30 0.48 0.22 1.00  
## Sum        1.72 1.93 1.35 5.00
```

```
sjPlot::plot_xtab(famuss$race,  
  famuss$actn3.r577x,  
  margin = "row")
```

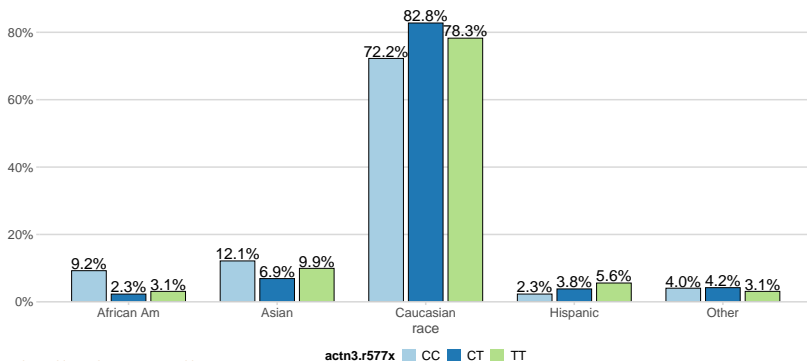


# Conditional distribution of race given genotype

```
addmargins(prop.table(tab1, margin = 2))
```

```
##  
##           CC      CT      TT      Sum  
## African Am 0.092 0.023 0.031 0.147  
## Asian      0.121 0.069 0.099 0.290  
## Caucasian  0.723 0.828 0.783 2.333  
## Hispanic   0.023 0.038 0.056 0.117  
## Other      0.040 0.042 0.031 0.114  
## Sum        1.000 1.000 1.000 3.000
```

```
sjPlot::plot_xtab(famuss$race, famuss$actn3.r577x, margin = "col", show.total = F, show.n = F)
```



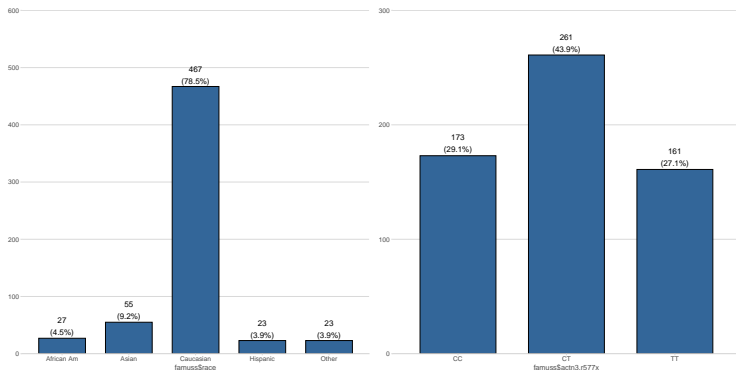
# Marginal distributions of race and genotype

Given a contingency table, the frequency distribution of one of the variables is called its **marginal distribution**.

```
table(famuss$race) / nrow(famuss)
```

```
##  
## African Am      Asian  Caucasian   Hispanic      Other  
##      0.045      0.092      0.785      0.039      0.039
```

```
sjPlot::plot_frq(famuss$race)  
sjPlot::plot_frq(famuss$actn3.r577x)
```

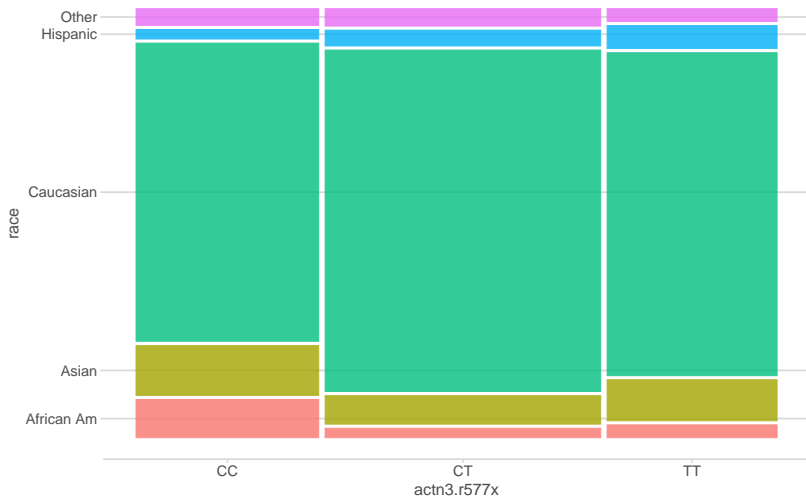


# Mosaic plots

- A mosaic plot is a graphical display that allows you to examine the relationship among two or more categorical variables.
- The mosaic plot starts as a square with length one. The square is divided first into horizontal bars whose widths are proportional to the probabilities associated with the first categorical variable.
- Then each bar is split vertically into bars that are proportional to the conditional probabilities of the second categorical variable. Additional splits can be made if wanted using a third, fourth variable, etc.

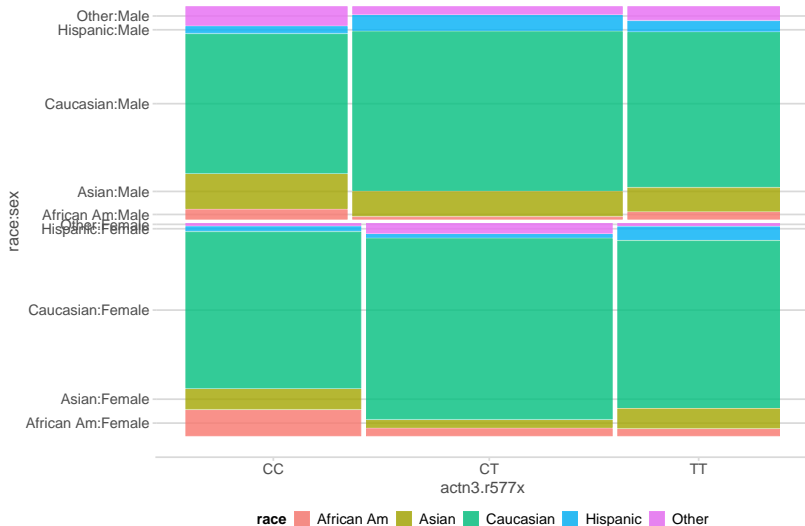
# Mosaic plots - race and genotype

```
# devtools::install_github("haleyjeppson/ggmosaic")
pacman::p_load(ggmosaic)
ggplot(data = famuss) +
  geom_mosaic(aes(x = product(race, actn3.r577x),
                    fill = race))
```



# Mosaic plots - race, genotype and sex

```
ggplot(data = famuss) +  
  geom_mosaic(aes(x = product(race, actn3.r577x),  
    fill = race, conds = product(sex)),  
    divider = mosaic("v"))
```





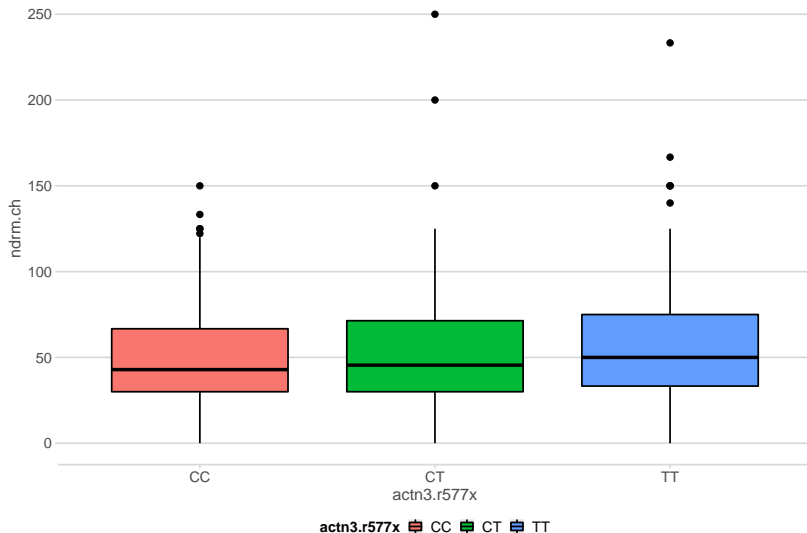


# A numerical variable and a categorical variable

- *FAMuSS* was designed to study the relationship between genotype at the location *r577x* in the gene *ACTN3* and muscle strength.
- Muscle strength was assessed by the percent change in non-dominant arm strength after resistance training (`ndrm.ch`).
- What visualization would be a good choice to make this comparison?

# A numerical variable and a categorical variable

```
ggplot(data = famuss, mapping = aes(x = actn3.r577x, y = ndrm.ch, fill = actn3.r577x)) +  
  geom_boxplot()
```





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- Measures of center include mean and median.
- Measures of spread include standard deviation, interquartile range
- Median and IQR are robust to outliers
- Histograms, boxplots, violin plots, and scatterplots are useful graphical summaries of numerical data, which can also be grouped by a categorical variable
- Bar plots, contingency tables, mosaic plots are useful summaries of categorical data



## Summary of exploring data slides *continued*

- Correlation coefficient ( $r$ ) quantifies the strength of a linear trend.
- The multiple R-squared in a simple linear regression output is equal to  $r^2$ .
- Transformation (e.g. log) can produce better linear associations for highly skewed data. But be careful about the interpretation!

## Summary of exploring data slides *continued*

- Correlation coefficient ( $r$ ) quantifies the strength of a linear trend.
- The multiple R-squared in a simple linear regression output is equal to  $r^2$ .
- Transformation (e.g. log) can produce better linear associations for highly skewed data. But be careful about the interpretation!
- Given a contingency table, the frequency distribution of one of the variables is called its marginal distribution
- Conditional distributions show the distribution of one variable for just those cases that satisfy a condition on another variable
- See <https://www.r-graph-gallery.com/> and <https://www.data-to-viz.com/> for a collection of graphical displays

# Session Info

```
R version 3.6.2 (2019-12-12)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Pop!_OS 19.10

Matrix products: default
BLAS:   /usr/lib/x86_64-linux-gnu/openblas/libblas.so.3
LAPACK: /usr/lib/x86_64-linux-gnu/libopenblas-r0.3.7.so

attached base packages:
[1] tools      stats      graphics  grDevices  utils      datasets  methods
[8] base

other attached packages:
[1] ggmosaic_0.3.0      cowplot_1.0.0      openintro_2.0.0
[4] usdata_0.1.0        cherryblossom_0.1.0 airports_0.1.0
[7] oibioestat_0.2.0    NCStats_0.4.7      FSA_0.8.30
[10] forcats_0.5.0       stringr_1.4.0      dplyr_1.0.2
[13] purrr_0.3.4         readr_1.3.1        tidyr_1.1.2
[16] tibble_3.0.3        ggplot2_3.3.2.9000 tidyverse_1.3.0
[19] knitr_1.29

loaded via a namespace (and not attached):
[1] nlme_3.1-143      fs_1.3.2          lubridate_1.7.4   RColorBrewer_1.1-2
[5] insight_0.8.1    httr_1.4.1        backports_1.1.9   R6_2.4.1
[9] sjlabelled_1.1.3 lazyeval_0.2.2    DBI_1.1.0         colorspace_1.4-1
[13] withr_2.2.0      tidyrselect_1.1.0 emmeans_1.4.5     compiler_3.6.2
[17] performance_0.4.4 cli_2.0.2         rvest_0.3.5       pacman_0.5.1
[21] xml2_1.3.0       plotly_4.9.2      sandwich_2.5-1    labeling_0.3
[25] bayestestR_0.5.2 scales_1.1.1      mvtnorm_1.0-12    digest_0.6.25
[29] minqa_1.2.4      htmltools_0.5.0   pkgconfig_2.0.3   lme4_1.1-21
[33] dbplyr_1.4.2     highr_0.8         htmlwidgets_1.5.1 rlang_0.4.7
[37] readxl_1.3.1     rstudioapi_0.11   farver_2.0.3      generics_0.0.2
[41] zoo_1.8-7        jsonlite_1.7.0    sjPlot_2.8.3      magrittr_1.5
[45] parameters_0.5.0 Matrix_1.2-18     Rcpp_1.0.4.6      munsell_0.5.0
[49] fansi_0.4.1      lifecycle_0.2.0   stringi_1.4.6     multcomp_1.4-12
[53] snakecase_0.11.0 MASS_7.3-51.5     plyr_1.8.6        grid_3.6.2
[57] sjmisc_2.8.3     crayon_1.3.4      lattice_0.20-38   ggeffects_0.14.1
[61] haven_2.3.1      splines_3.6.2     sjstats_0.17.9    hms_0.5.3
[65] pillar_1.4.6     boot_1.3-24       estimability_1.3   effectsize_0.2.0
[69] codetools_0.2-16 reprex_0.3.0      glue_1.4.2        evaluate_0.14
[73] data.table_1.12.8 modelr_0.1.5      vctrs_0.3.4       nlptr_1.2.2-1
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