### 001 - Introduction to Inferential Statistics

### EPIB 607 - FALL 2020

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slides compiled on September 2, 2020



### Objectives

Visualize/Analyze/Interpret data using statistical methods with a computer

Gather data into analysis ready format

Learn regression

Understand the statistical results in a scientific pape

Learn the tools for creating reproducible analyses

Where does this course fit in my life?

## Objectives for this course

- Visualize/Analyze/Interpret data using statistical methods with a computer
- 2. Gather data into analysis ready format
- 3. Learn regression
- 4. Understand the statistical results in a scientific paper
- 5. Learn the tools for creating reproducible analyses

Objectives 3/39.

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Where does this course fit in my life?

## Data is the new oil<sup>1</sup>

### Fuel of the future

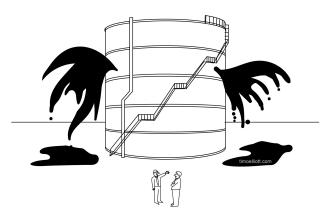
## Data is giving rise to a new economy

How is it shaping up?



 $<sup>1\\ \</sup>texttt{https://www.economist.com/briefing/2017/05/06/data-is-giving-rise-to-a-new-economy}$ 

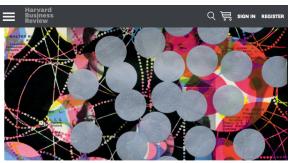
## Danger<sup>2</sup>



"Data is the new oil? Absolutely—toxic if mishandled!..."

 $<sup>2</sup>_{\tt https://timoelliott.com/blog/2018/03/data-is-the-new-oil-yes-toxic-if-mish and led.html}$ 

## Data science<sup>3</sup>



ARTWORK: TAMAR COHEN, ANDREW J BUBOLTZ, 2011, SILK SCREEN ON A PAGE FROM A HIGH SCHOOL YEARBOOK, 8.5" X 12"

DATA

# Data Scientist: The Sexiest Job of the 21st Century

by Thomas H. Davenport and D.J. Patil

FROM THE OCTOBER 2012 ISSUE

 $<sup>3</sup>_{\rm https://hbr.org/2012/10/data-scientist-the-sexiest-job-of-the-21st-century}$ 

## Why R?

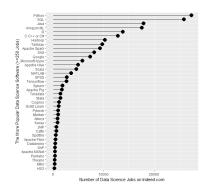


Figure: Data as of May 2019

http://r4stats.com/articles/popularity/

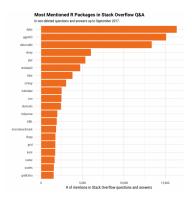


Figure: Popular R packages

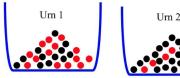
https://stackoverflow.blog/2017/10/10/impressive-growth-r/

## First day in a statistics course

### **Example:**

We have two urns. Urn 1 contains 14 red balls and 12 black balls. Urn 2 contains 6 red balls and 20 black balls.

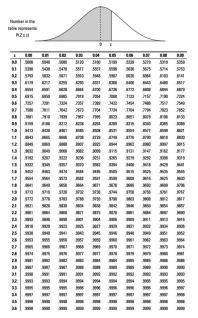
An Urn is selected at random and a ball is selected from that urn.



If the ball turns out to be red what is the probability that it came from the first urn?

	Sepal.Length <sup>‡</sup>	Sepal.Width <sup>‡</sup>	Petal.Length <sup>‡</sup>	Petal.Width <sup>‡</sup>	Species <sup>‡</sup>
1	5.1	3.5	1.4	0.2	setosa
2	4.9	3.0	1.4	0.2	setosa
3	4.7	3.2	1.3	0.2	setosa
4	4.6	3.1	1.5	0.2	setosa
5	5.0	3.6	1.4	0.2	setosa
6	5.4	3.9	1.7	0.4	setosa
7	4.6	3.4	1.4	0.3	setosa
8	5.0	3.4	1.5	0.2	setosa
9	4.4	2.9	1.4	0.2	setosa
10	4.9	3.1	1.5	0.1	setosa
- 11	5.4	3.7	1.5	0.2	setosa
12	4.8	3.4	1.6	0.2	setosa
13	4.8	3.0	1.4	0.1	setosa
14	4.3	3.0	1.1	0.1	setosa
15	5.8	4.0	1.2	0.2	setosa

## Second day in a statistics course



### Objectives

Visualize/Analyze/Interpret data using statistical methods with a computer

### Gather data into analysis ready forma

Learn regression

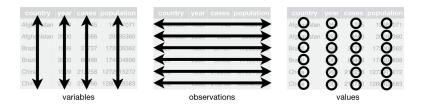
Understand the statistical results in a scientific paper

Learn the tools for creating reproducible analyses

Where does this course fit in my life?

## Tidy data

- Each variable forms a column.
- Each observation forms a row.
- Each type of observational units forms a table
- Tidy data is ready for regression routines and plotting



## Example: Does a full moon affect behaviour?

- Many people believe that the moon influences the actions of some individuals.
- A study of dementia patients in nursing homes recorded various types of disruptive behaviors every day for 12 weeks.
- Days were classified as moon days if they were in a 3-day period centered at the day of the full moon.
- For each patient, the average number of disruptive behaviors was computed for moon days and for all otherdays.

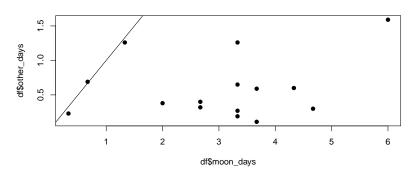
patient	moon_days	other_days
1	3.33	0.27
2	3.67	0.59
3	2.67	0.32
4	3.33	0.19
5	3.33	1.26
6	3.67	0.11
7	4.67	0.30

Is it tidy?

patient	moon_days	other_days
1	3.33	0.27
2	3.67	0.59
3	2.67	0.32

## Question: Can I plot the data?

plot(df\$moon\_days, df\$other\_days, pch = 19)
abline(a=0,b=1)



## Is it tidy?

patient	moon_days	other_days
1	3.33	0.27
2	3.67	0.59
3	2.67	0.32
4	3.33	0.19
5	3.33	1.26

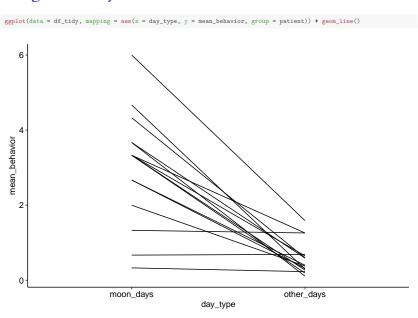
# Question: Can I fit a meaningful regression model directly to the variables in the data?

```
## Call: lm(formula = moon days ~ other days, data = df)
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                 2.56 0.66
                                     3.9 0.002
## other days
                 0.79
                           0.91
                                    0.9 0.402
##
## Residual standard error: 1.5 on 13 degrees of freedom
## Multiple R-squared: 0.055, ^ IAdjusted R-squared: -0.018
## F-statistic: 0.75 on 1 and 13 DF, p-value: 0.4
```

## Is it tidy?

notiont	dorr trmo	mean behavior
patient	day_type	mean_benavior
1	moon_days	3.33
1	other_days	0.27
2	moon_days	3.67
2	other_days	0.59
3	moon_days	2.67
3	other_days	0.32
4	moon_days	3.33
4	other_days	0.19
5	moon_days	3.33
5	other_days	1.26

## Plotting with tidy data



## Regression with tidy data

```
fit <- lme4::lmer(mean_behavior ~ day_type + (1|patient), data = df_tidy)
summary(fit)
## Linear mixed model fit by REML ['lmerMod']
## Formula: mean_behavior ~ day_type + (1 | patient)
     Data: df_tidy
##
## REML criterion at convergence: 90.3
## Scaled residuals:
       Min 10 Median 30
                                         Max
## -2.27236 -0.30142 -0.04023 0.48540 2.44753
##
## Random effects:
## Groups Name
                 Variance Std.Dev.
## patient (Intercept) 0.1563 0.3954
## Residual
                       1.0659 1.0324
## Number of obs: 30, groups: patient, 15
## Fixed effects:
                    Estimate Std. Error t value
## (Intercept) 3.0220 0.2854 10.587
## day_typeother_days -2.4327 0.3770 -6.453
##
## Correlation of Fixed Effects:
              (Intr)
## dy_typthr_d -0.660
```

## Not tidy vs. tidy data

patient	moon_days	other_days
1	3.33	0.27
2	3.67	0.59
3	2.67	0.32
4	3.33	0.19
5	3.33	1.26

patient	day_type	mean_behavior
1	moon_days	3.33
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2	other_days	0.59
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3	other_days	0.32
4	moon_days	3.33
4	other_days	0.19
5	moon_days	3.33
5	other_days	1.26

## Not tidy vs. tidy data

patient	moon_days	other_days
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Not tidy

patient	day_type	mean_behavior
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3	moon_days	2.67
3	other_days	0.32
4	moon_days	3.33
4	other_days	0.19
5	moon_days	3.33
5	other_days	1.26

## tidyr::pivot\_longer()

patient	moon_days	other_days
1	3.33	0.27
2	3.67	0.59
3	2.67	0.32
4	3.33	0.19
5	3.33	1.26

	,	
patient	day_type	mean_behavior
1	moon_days	3.33
1	other_days	0.27
2	moon_days	3.67
2	other_days	0.59
3	moon_days	2.67
3	other_days	0.32
4	moon_days	3.33
4	other_days	0.19
5	moon_days	3.33
5	other_days	1.26

tidyr::pivot\_longer(data = df, cols = -patient, names\_to = "day\_type", values\_to = "mean\_behavior")

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Where does this course fit in my life?

Learn regression 22/39

### Traditional stats textbook

### CHAPTER 7

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- 7.2 General Concepts / 211
- 7.3 One-Sample Test for the Mean of a Normal Distribution: One-Sided Alternatives / 214
- 7.4 One-Sample Test for the Mean of a Normal Distribution: Two-Sided Alternatives / 222
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- 7.6 The Power of a Test / 232
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- 7.11 Case Study: Effects of Tobacco Use on Bone-Mineral Density in Middle-Aged Women / 265
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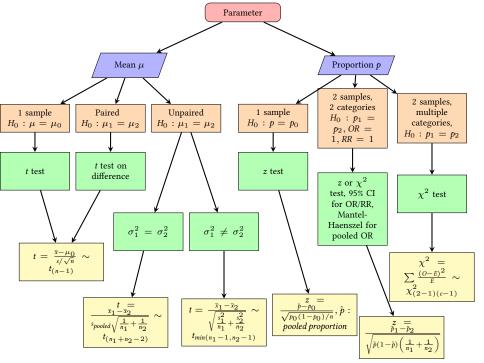
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- - 8.5 Interval Estimation for the Comparison of Means from Two Independent Samples (Equal Variance Case) / 290
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### Regression and Correlation Methods / 457

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- 11.3 Fitting Regression Lines-The Method of Least Squares / 461
- 11.4 Inferences About Parameters from Regression Lines / 465
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- 11.7 The Correlation Coefficient / 485
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### This course

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- Нур sis Testing: Two-Sample In 8.1 uction / 279
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Where does this course fit in my life?

## Statistical concepts

**RESULTS** The total populations were 462 445 in the lowarborder counties and 272 385 in the Illinois border counties. Population density was higher in the lowa counties (114.2 people per square mile) than in the Illinois counties (78.2 people per square mile). Trends of cumulative COVID-19 cases per 10 000 residents for the lowa and Illinois border counties were comparable before the Illinois stayat-home order, which went into effect at 5:00 PM on March 21 (March 15 to March 21: 0.024 per 10 000 residents vs 0.026 per 10 000 residents). After that, cases increased more quickly in Iowa and more slowly in Illinois. Within 10, 20, and 30 days after the enactment of the stay-at-home order in Illinois, the difference in cases was −0.51 per 10 000 residents (SE, 0.09; 95% CI, −0.69 to −0.32; P < .001), -1.15 per 10 000 residents (SE, 0.49; 95% CI, -2.12 to -0.18; P = .02), and -4.71 per 10 000 residents (SE, 1.99: 95% CI, -8.64 to -0.78: P = .02), respectively. The estimates indicate excess cases in the border Iowa counties by as many as 217 cases after 1 month without a stay-at-home order. This estimate of excess cases represents 30.4% of the 716 total cases in those lowa counties by that date. Sensitivity analyses addressing differences in timing of closing schools and nonessential businesses and differences in county population density and poverty rates between the 2 states supported these findings.

4

 $<sup>^{4}</sup>_{\rm https://jamanetwork.com/journals/jamanetworkopen/fullarticle/2766229}$ 

## Statistical concepts

Table 1. Difference-in-Differences Estimates of COVID-19 Cases Comparing Border Counties in Iowa With Those in Illinois Before and After the Stay-at-Home Order Was Issued in Illinois<sup>a</sup>

Period	Difference in COVID-19 cases per 10 000 residents <sup>b</sup>	Heteroskedasticity robust SE (95% CI) <sup>c</sup>	P value	Excess cases in lowa border counties	Excess cases as proportion of total cases, %
3/22-3/26	-0.14	0.04 (-0.23 to -0.06)	.001	6	32.4
3/27-3/31	-0.51	0.09 (-0.69 to -0.32)	<.001	24	38.0
4/01-4/05	-0.41	0.17 (-0.74 to -0.07)	.02	19	15.2
4/06-4/10	-1.15	0.49 (-2.12 to -0.18)	.02	53	17.8
4/11-4/15	-3.35	1.19 (-5.70 to -0.99)	.006	154	30.0
4/16-4/20	-4.71	1.99 (-8.64 to -0.78)	.02	217	30.4

Abbreviation: COVID-19, coronavirus disease 2019.

5

<sup>&</sup>lt;sup>a</sup> The regression model was estimated separately for each of 5-day period. The regression was estimated using least squares weighted by the 2019 county population. The regression adjusted for county and day fixed effects. The number of county-day observations was 180 for each regression.

<sup>&</sup>lt;sup>b</sup> This indicates the estimated difference-in-differences association of a stay-at-home order with COVID-19 cases in a given period relative to March 15 to March 21 (ie, the period before the stay-at-home order in Illinois was enacted).

<sup>&</sup>lt;sup>c</sup> Heteroskedasticity robust SEs were estimated because homoscedasticity is rejected for all post-period regressions.

<sup>5</sup> https://jamanetwork.com/journals/jamanetworkopen/fullarticle/2766229

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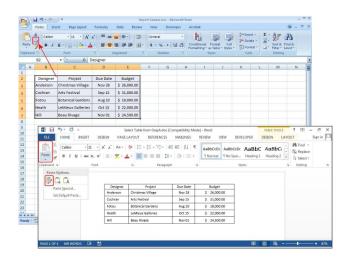
Learn regression

Understand the statistical results in a scientific pape.

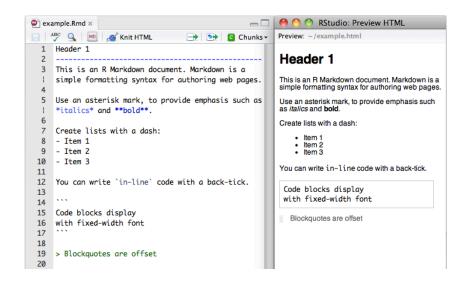
### Learn the tools for creating reproducible analyses

Where does this course fit in my life

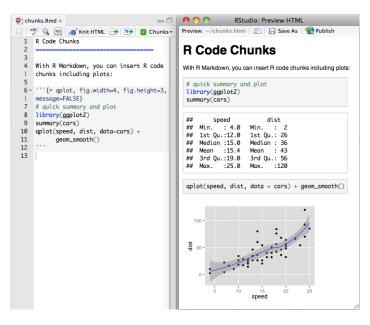
## Copy paste ad nauseam



## Markdown: HTML without knowing HTML

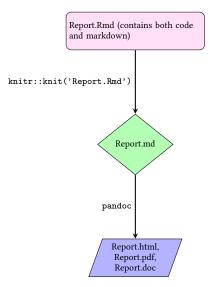


## R + Markdown = RMarkdown



### What rmarkdown does

### RMarkdown example:

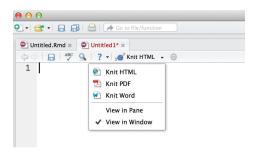


## Compiling a . Rmd document

The two steps on previous slide can be executed in one command:

rmarkdown::render()

or in RStudio:



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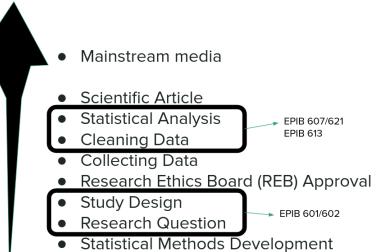
## Topics by level of exposure

Level of Exposure



- Mainstream media
- Scientific Article
- Statistical Analysis
- Cleaning Data
- Collecting Data
- Research Ethics Board (REB) Approval
- Study Design
- Research Question
- Statistical Methods Development

## First year courses





- Scientific Article
- Statistical Analysis
- Cleaning Data
- Collecting Data
- Research Ethics Board (REB) Approval
- Study Design
- Research Question

EPIB 601/602

EPIB 607/621 EPIB 613

Statistical Methods Development

What I do

## Session Info

```
R version 3.6.2 (2019-12-12)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Pop!_OS 19.10
Matrix products: default
BLAS: /usr/lib/x86_64-linux-gnu/openblas/libblas.so.3
LAPACK: /usr/lib/x86_64-linux-gnu/libopenblasp-r0.3.7.so
attached base packages:
[1] tools
              stats
                        graphics grDevices utils
                                                       datasets methods
[8] base
other attached packages:
 [1] NCStats_0.4.7
                        FSA_0.8.30
                                           forcats_0.5.0
                                                               stringr_1.4.0
 [5] dplvr 1.0.2
                        purrr 0.3.4
                                           readr 1.3.1
                                                               tidvr 1.1.2
 [9] tibble 3.0.3
                        ggplot2_3.3.2.9000 tidyverse_1.3.0
                                                               knitr 1.29
loaded via a namespace (and not attached):
 [1] Rcpp_1.0.4.6
                        lubridate_1.7.4
                                           lattice_0.20-38
                                                               assertthat_0.2.1
 [5] digest 0.6.25
                        R6 2.4.1
                                            cellranger 1.1.0
                                                               plyr_1.8.6
 [9] backports_1.1.9
                        reprex_0.3.0
                                            evaluate 0.14
                                                               httr 1.4.1
[13] highr 0.8
                        pillar 1.4.6
                                           TeachingDemos_2.12 rlang_0.4.7
[17] readxl 1.3.1
                        rstudioapi 0.11
                                           minga_1.2.4
                                                               nloptr_1.2.2.1
[21] Matrix 1.2-18
                        labeling 0.3
                                            splines 3.6.2
                                                               lme4 1.1-21
[25] munsell_0.5.0
                        broom_0.7.0
                                           compiler_3.6.2
                                                               modelr_0.1.5
[29] xfun 0.16
                        pkgconfig 2.0.3
                                            tidvselect 1.1.0
                                                               fansi 0.4.1
[33] crayon_1.3.4
                        dbplyr_1.4.2
                                           withr 2.2.0
                                                               ggpubr_0.2.5
[37] MASS 7.3-51.5
                        grid 3.6.2
                                           nlme 3.1-143
                                                               isonlite 1.7.0
[41] gtable_0.3.0
                        lifecvcle 0.2.0
                                           DBI 1.1.0
                                                               magrittr 1.5
[45] scales 1.1.1
                        cli_2.0.2
                                           stringi_1.4.6
                                                               farver_2.0.3
[49] ggsignif 0.6.0
                        fs 1.3.2
                                           xml2 1.3.0
                                                               ellipsis 0.3.1
[53] generics_0.0.2
                        vctrs 0.3.4
                                           boot 1.3-24
                                                               glue_1.4.2
[57] hms_0.5.3
                        colorspace_1.4-1
                                           rvest_0.3.5
                                                               haven_2.3.1
```