Introduction to Inferential Statistics

EPIB 607 - FALL 2020

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slides compiled on August 30, 2020

Objectives

Visualize/Analyze/Interpret data using statistical methods with a computer

Gather data into analysis ready format

Learn regression

Understand the statistical results in a scientific paper

Learn the tools for creating reproducible analyses

Where does this course fit in my life?

Objectives for this course

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Data is the new oil[1]

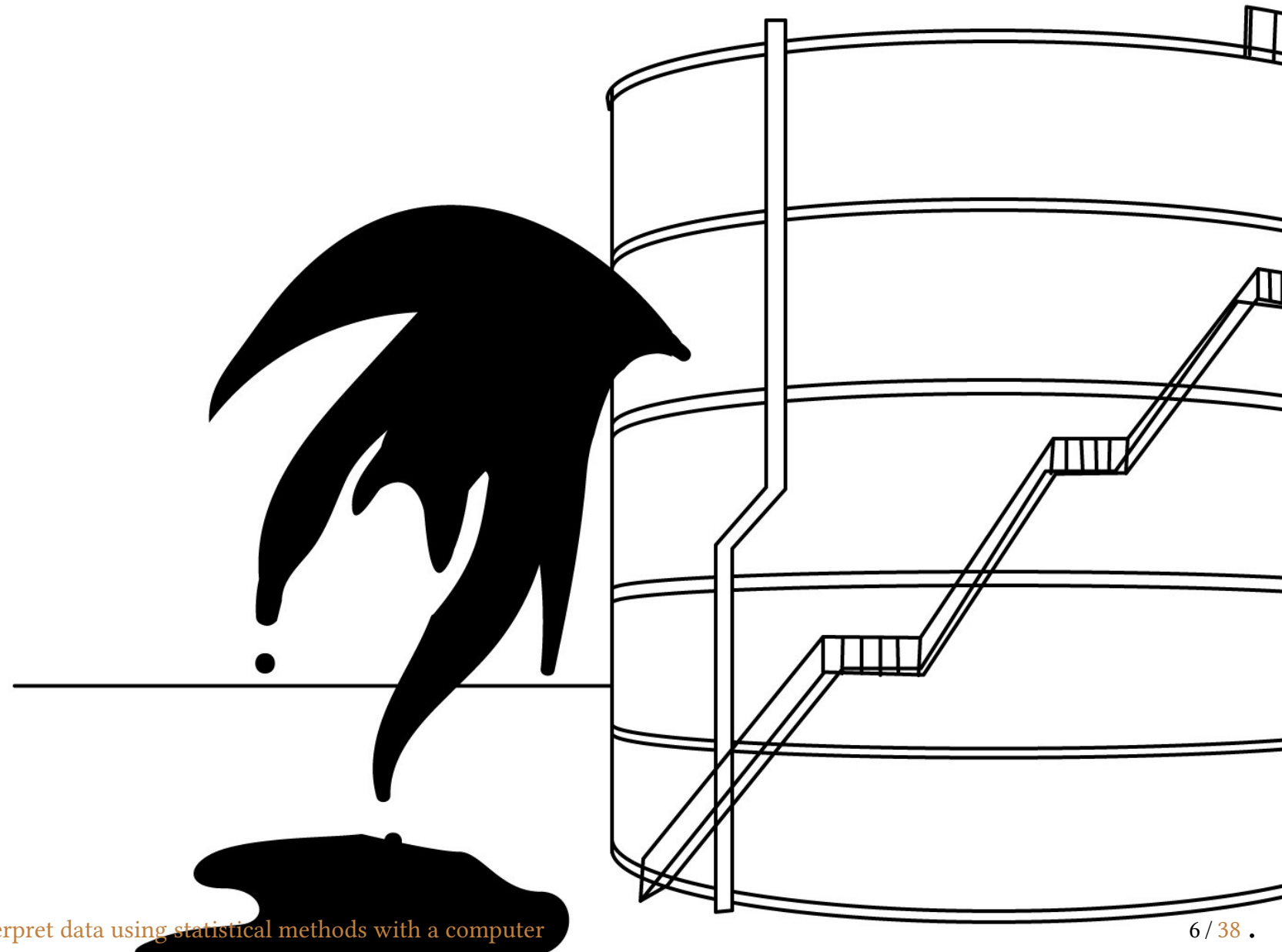
Fuel of the future

Data is giving rise to a new economy

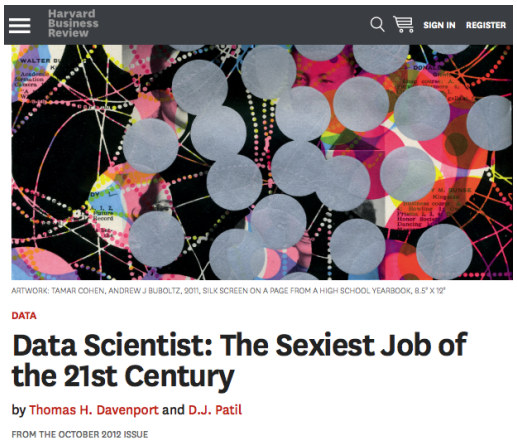
How is it shaping up?

[1] https://www.economist.com/briefing/2017/05/06/data-is-giving-rise-to-a-new-economy

Danger^2



Data science[3]



[3] https://hbr.org/2012/10/data-scientist-the-sexiest-job-of-the-21st-century

Why R ?

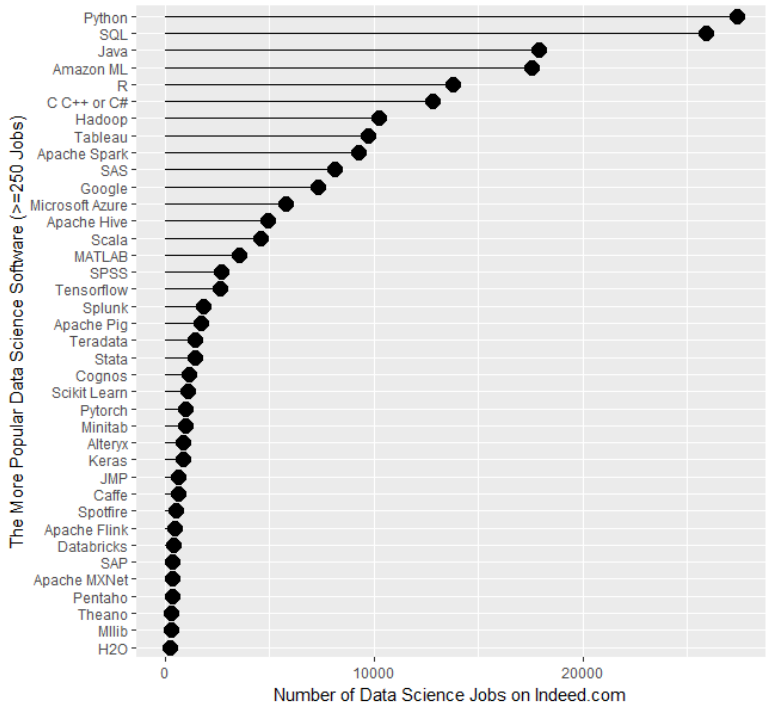


Fig. 1: Data as of May 2019

http://r4stats.com/articles/popularity/

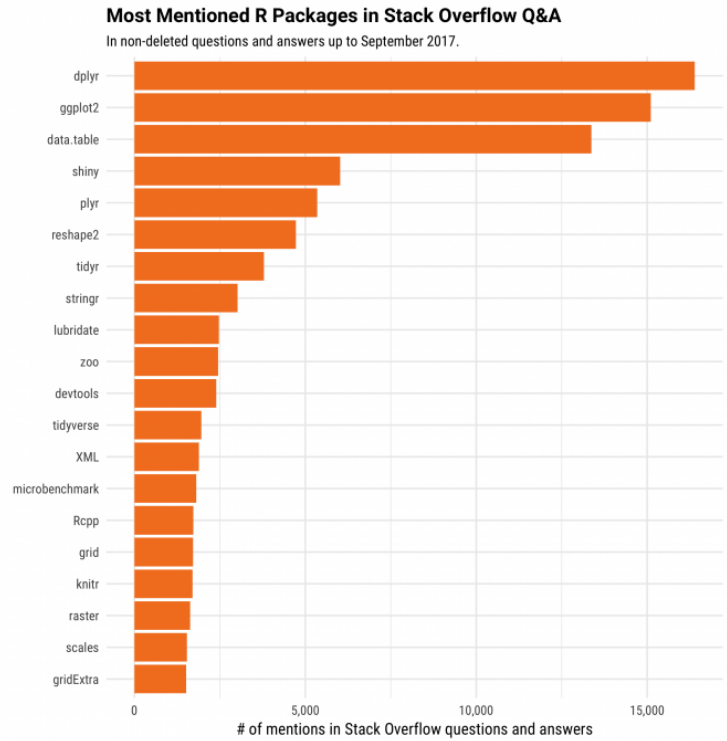


Fig. 2: Popular R packages

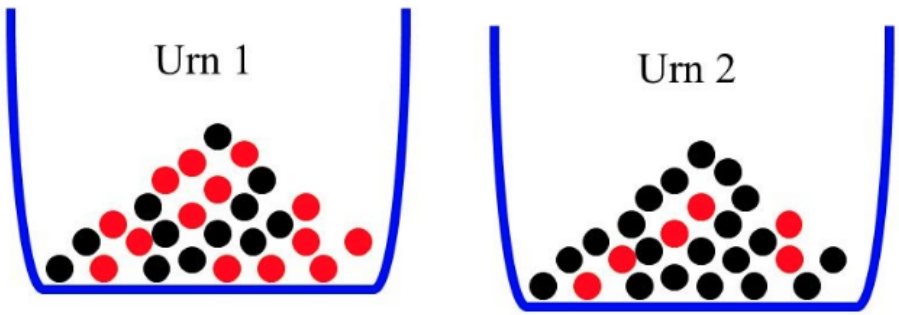
https://stackoverflow.blog/2017/10/10/impressive-growth-r/

First day in a statistics course

Example:

We have two urns. Urn 1 contains 14 red balls and 12 black balls. Urn 2 contains 6 red balls and 20 black balls.

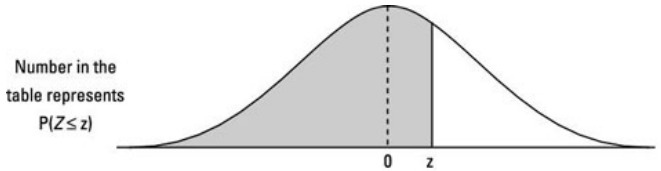
An Urn is selected at random and a ball is selected from that urn.



If the ball turns out to be red what is the probability that it came from the first urn?

	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
1	5.1	3.5	1.4	0.2	setosa
2	4.9	3.0	1.4	0.2	setosa
3	4.7	3.2	1.3	0.2	setosa
4	4.6	3.1	1.5	0.2	setosa
5	5.0	3.6	1.4	0.2	setosa
6	5.4	3.9	1.7	0.4	setosa
7	4.6	3.4	1.4	0.3	setosa
8	5.0	3.4	1.5	0.2	setosa
9	4.4	2.9	1.4	0.2	setosa
10	4.9	3.1	1.5	0.1	setosa
11	5.4	3.7	1.5	0.2	setosa
12	4.8	3.4	1.6	0.2	setosa
13	4.8	3.0	1.4	0.1	setosa
14	4.3	3.0	1.1	0.1	setosa
15	5.8	4.0	1.2	0.2	setosa

Second day in a statistics course



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

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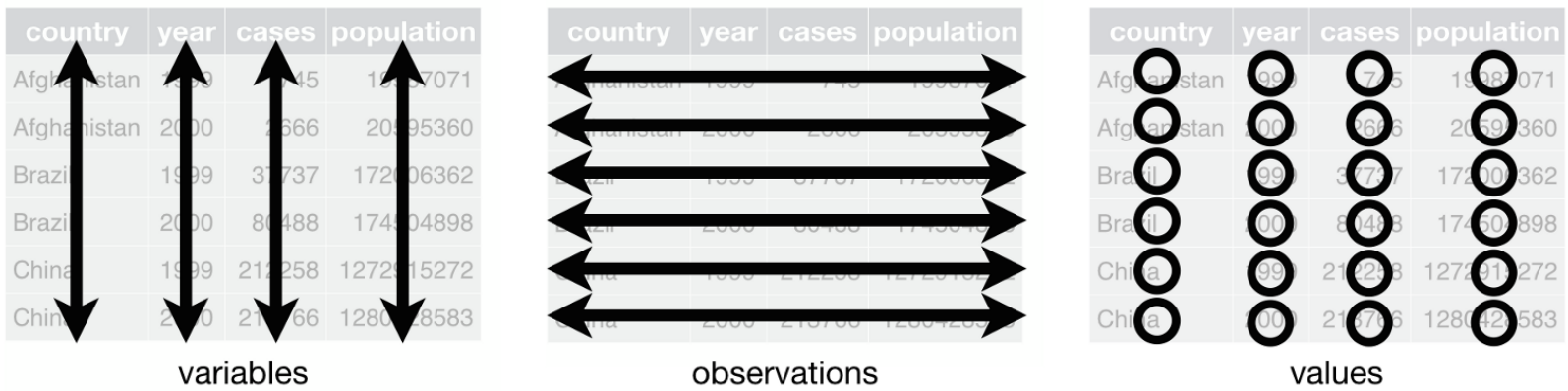
Where does this course fit in my life?

Tidy data

- Each variable forms a column.
- Each observation forms a row.
- Each type of observational units forms a table
- Tidy data is ready for regression routines and plotting



Example: Does a full moon affect behaviour?

- Many people believe that the moon influences the actions of some individuals.
- A study of dementia patients in nursing homes recorded various types of disruptive behaviors every day for 12 weeks.
- Days were classified as moon days if they were in a 3-day period centered at the day of the full moon.
- For each patient, the average number of disruptive behaviors was computed for moon days and for all otherdays.

patient	moon_days	other_days
1	3.33	0.27
2	3.67	0.59
3	2.67	0.32
4	3.33	0.19
5	3.33	1.26
6	3.67	0.11
7	4.67	0.30

Is it tidy?

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Question: Can I plot the data?

Is it tidy?

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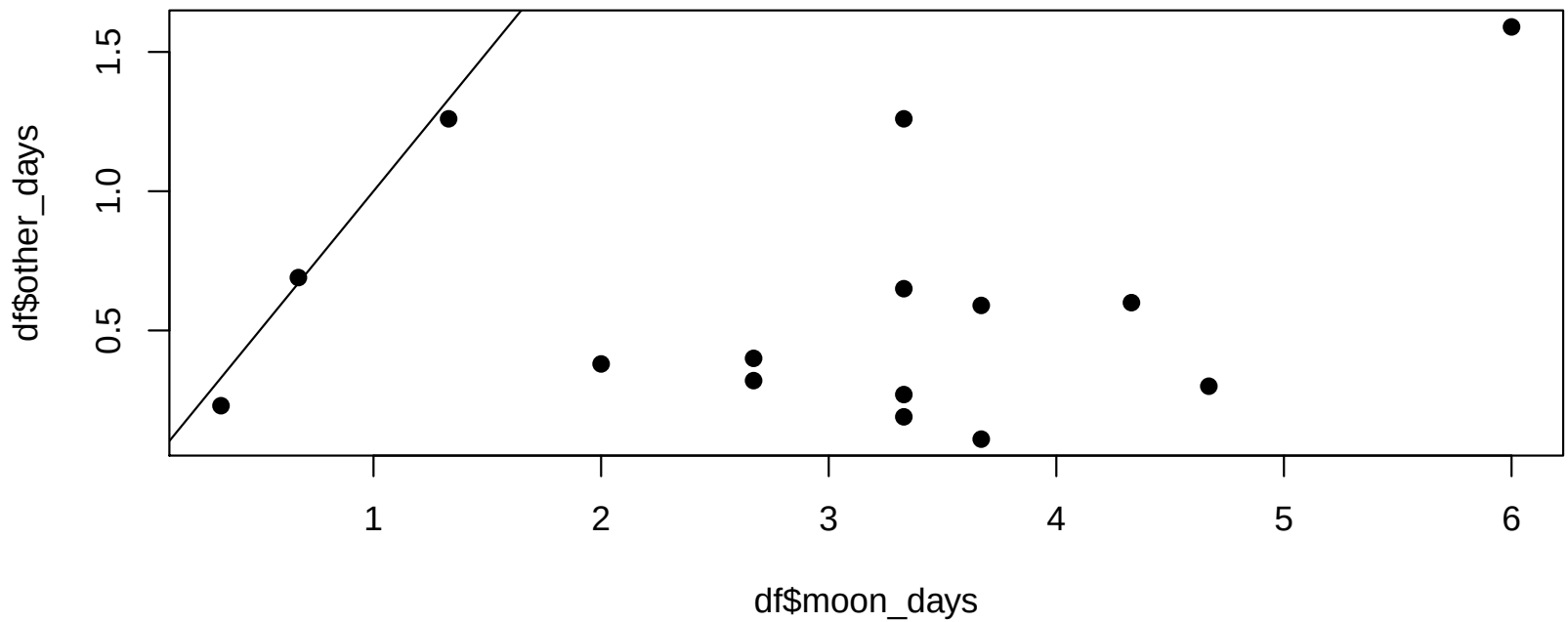
Question: Can I plot the data?

```
plot(df$moon_days, df$other_days, pch = 19)
abline(a=0,b=1)
```



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Question: Can I fit a meaningful regression model directly to the variables in the data?

Is it tidy?

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Question: Can I fit a meaningful regression model directly to the variables in the data?

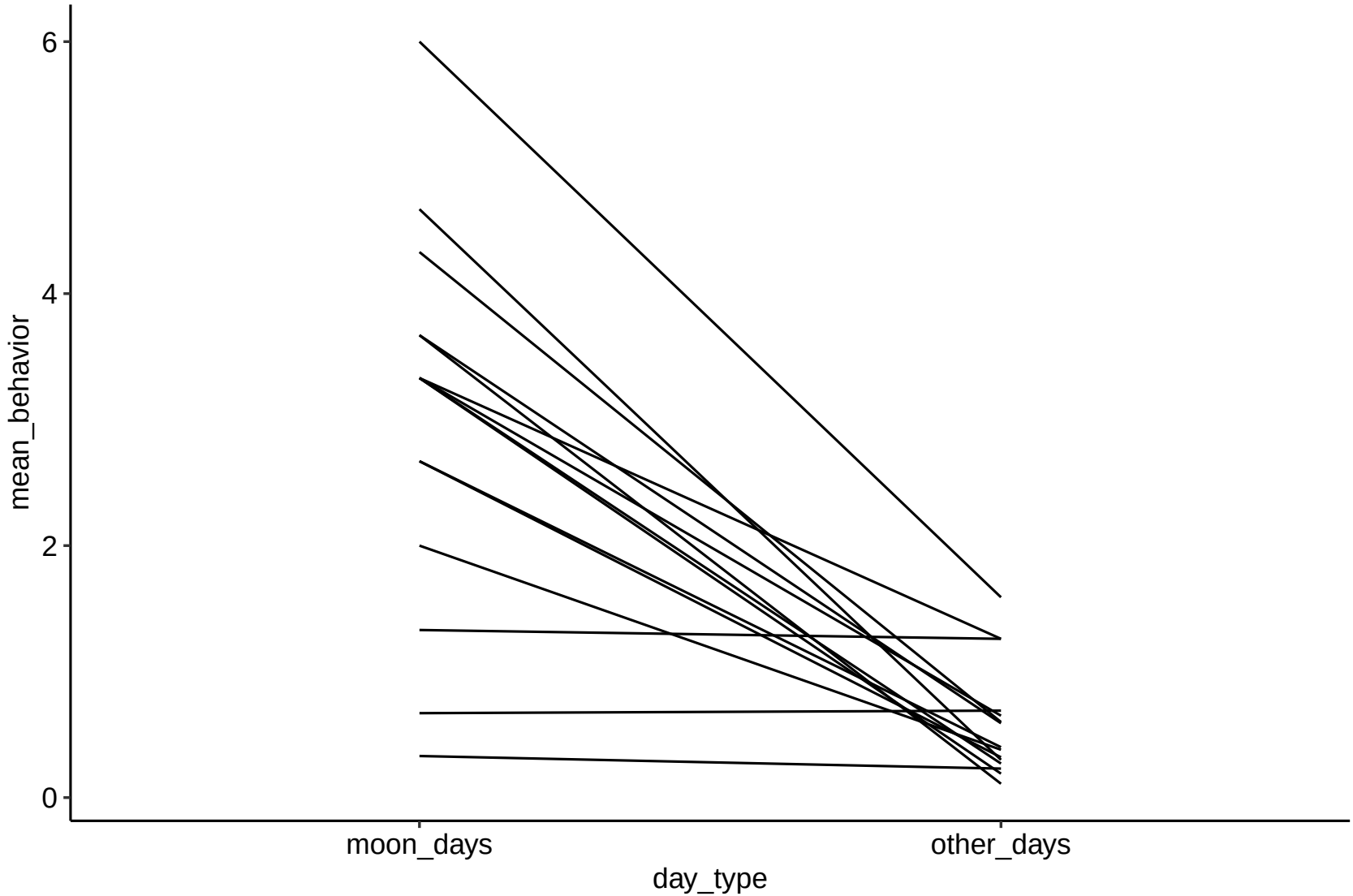
```
## Call: lm(formula = moon_days ~ other_days, data = df)
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)     2.56       0.66     3.9    0.002
## other_days      0.79       0.91     0.9    0.402
##
## Residual standard error: 1.5 on 13 degrees of freedom
## Multiple R-squared: 0.055,^^IAdjusted R-squared: -0.018
## F-statistic: 0.75 on 1 and 13 DF,  p-value: 0.4
```

Is it tidy?

patient	day_type	mean_behavior
1	moon_days	3.33
1	other_days	0.27
2	moon_days	3.67
2	other_days	0.59
3	moon_days	2.67
3	other_days	0.32
4	moon_days	3.33
4	other_days	0.19
5	moon_days	3.33
5	other_days	1.26

Plotting with tidy data

```
ggplot(data = df_tidy, mapping = aes(x = day_type, y = mean_behavior, group = patient)) + geom_line()
```



Regression with tidy data

```
fit <- lme4::lmer(mean_behavior ~ day_type + (1|patient), data = df_tidy)
summary(fit)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: mean_behavior ~ day_type + (1 | patient)
##    Data: df_tidy
##
## REML criterion at convergence: 90.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -2.27236 -0.30142 -0.04023  0.48540  2.44753
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  patient  (Intercept) 0.1563   0.3954
##  Residual             1.0659   1.0324
## Number of obs: 30, groups:  patient, 15
##
## Fixed effects:
##                   Estimate Std. Error t value
## (Intercept)         3.0220     0.2854  10.587
## day_typeother_days  -2.4327     0.3770  -6.453
##
## Correlation of Fixed Effects:
##             (Intr)
## dy_typthr_d -0.660
```

Not tidy vs. tidy data

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tidy

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4	other_days	0.19
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5	other_days	1.26

tidyr::pivot_longer()

patient	moon_days	other_days
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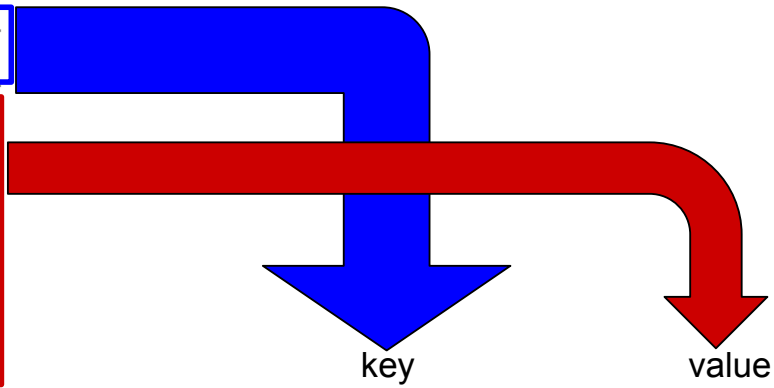
5	3.33	1.26



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3	other_days	0.32
4	moon_days	3.33
4	other_days	0.19
5	moon_days	3.33
5	other_days	1.26

```
tidyr::pivot_longer(data = df, cols = -patient, names_to = "day_type", values_to = "mean_behavior")
```

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Traditional stats textbook

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CHAPTER 11

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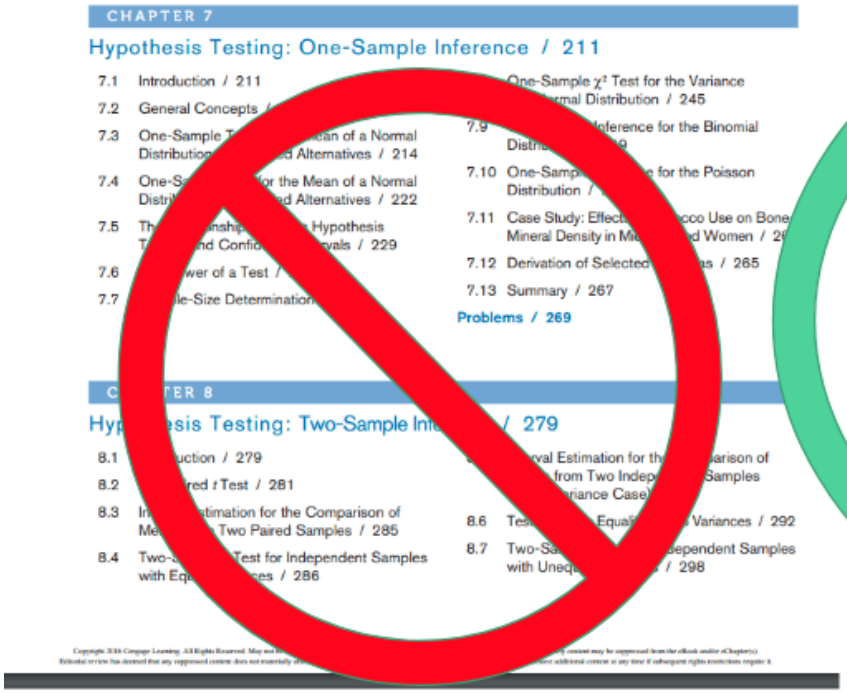
This course



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Statistical concepts

RESULTS The total populations were 462 445 in the Iowa border counties and 272 385 in the Illinois border counties. Population density was higher in the Iowa counties (114.2 people per square mile) than in the Illinois counties (78.2 people per square mile). Trends of cumulative COVID-19 cases per 10 000 residents for the Iowa and Illinois border counties were comparable before the Illinois stay-at-home order, which went into effect at 5:00 PM on March 21 (March 15 to March 21: 0.024 per 10 000 residents vs 0.026 per 10 000 residents). After that, cases increased more quickly in Iowa and more slowly in Illinois. Within 10, 20, and 30 days after the enactment of the stay-at-home order in Illinois, the difference in cases was −0.51 per 10 000 residents (SE, 0.09; 95% CI, −0.69 to −0.32; $P < .001$), −1.15 per 10 000 residents (SE, 0.49; 95% CI, −2.12 to −0.18; $P = .02$), and −4.71 per 10 000 residents (SE, 1.99; 95% CI, −8.64 to −0.78; $P = .02$), respectively. The estimates indicate excess cases in the border Iowa counties by as many as 217 cases after 1 month without a stay-at-home order. This estimate of excess cases represents 30.4% of the 716 total cases in those Iowa counties by that date. Sensitivity analyses addressing differences in timing of closing schools and nonessential businesses and differences in county population density and poverty rates between the 2 states supported these findings.

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[4] https://jamanetwork.com/journals/jamanetworkopen/fullarticle/2766229

Statistical concepts

Table 1. Difference-in-Differences Estimates of COVID-19 Cases Comparing Border Counties in Iowa With Those in Illinois Before and After the Stay-at-Home Order Was Issued in Illinois[a]

Period	Difference in COVID-19 cases per 10 000 residents[b]	Heteroskedasticity robust SE (95% CI)[c]	*P* value	Excess cases in Iowa border counties	Excess cases as proportion of total cases, %
3/22-3/26	−0.14	0.04 (−0.23 to −0.06)	.001	6	32.4
3/27-3/31	−0.51	0.09 (−0.69 to −0.32)	<.001	24	38.0
4/01-4/05	−0.41	0.17 (−0.74 to −0.07)	.02	19	15.2
4/06-4/10	−1.15	0.49 (−2.12 to −0.18)	.02	53	17.8
4/11-4/15	−3.35	1.19 (−5.70 to −0.99)	.006	154	30.0
4/16-4/20	−4.71	1.99 (−8.64 to −0.78)	.02	217	30.4

Abbreviation: COVID-19, coronavirus disease 2019.

[a] The regression model was estimated separately for each of 5-day period. The regression was estimated using least squares weighted by the 2019 county population. The regression adjusted for county and day fixed effects. The number of county-day observations was 180 for each regression.

[b] This indicates the estimated difference-in-differences association of a stay-at-home order with COVID-19 cases in a given period relative to March 15 to March 21 (ie, the period before the stay-at-home order in Illinois was enacted).

[c] Heteroskedasticity robust SEs were estimated because homoscedasticity is rejected for all post-period regressions.

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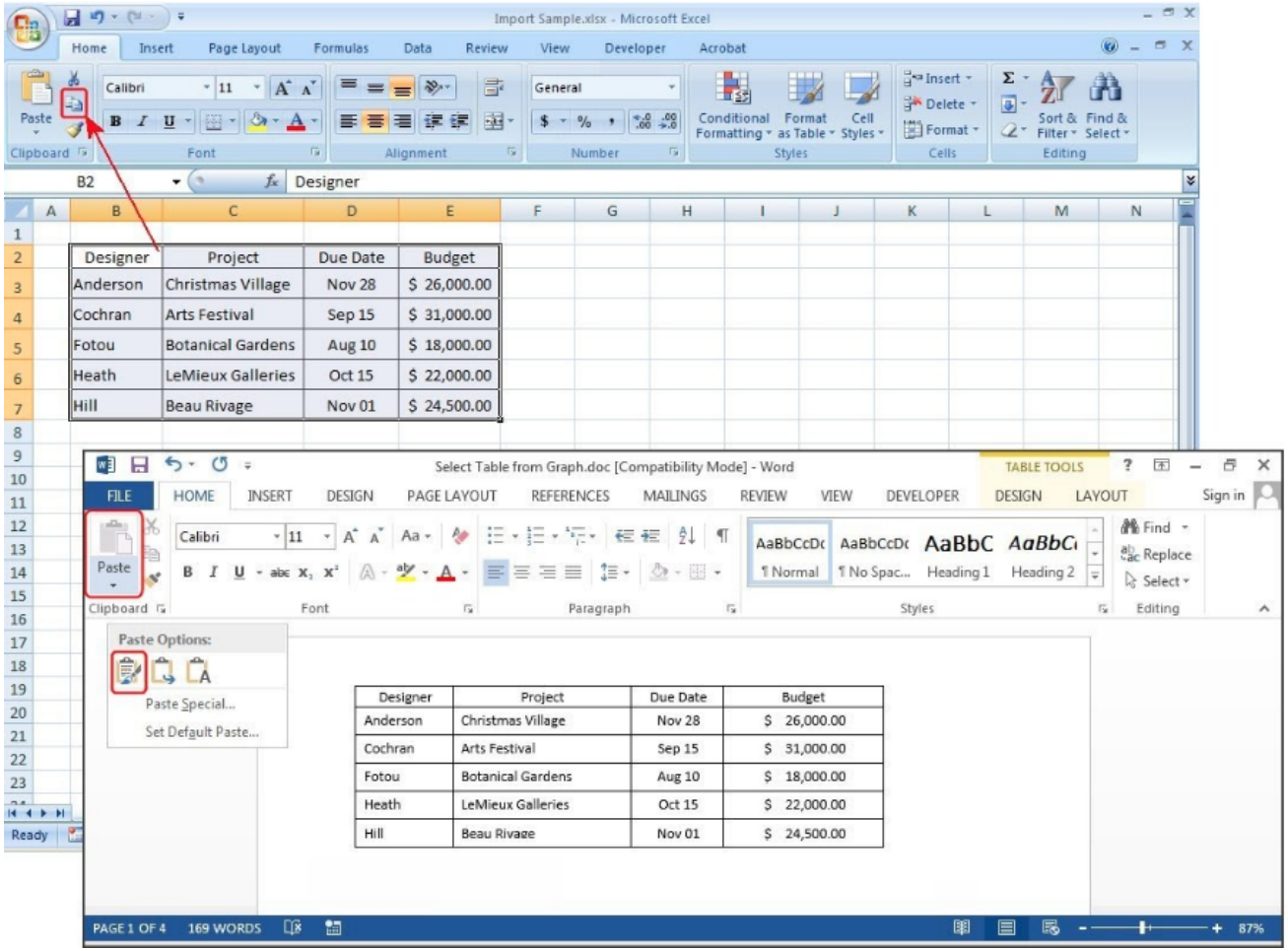
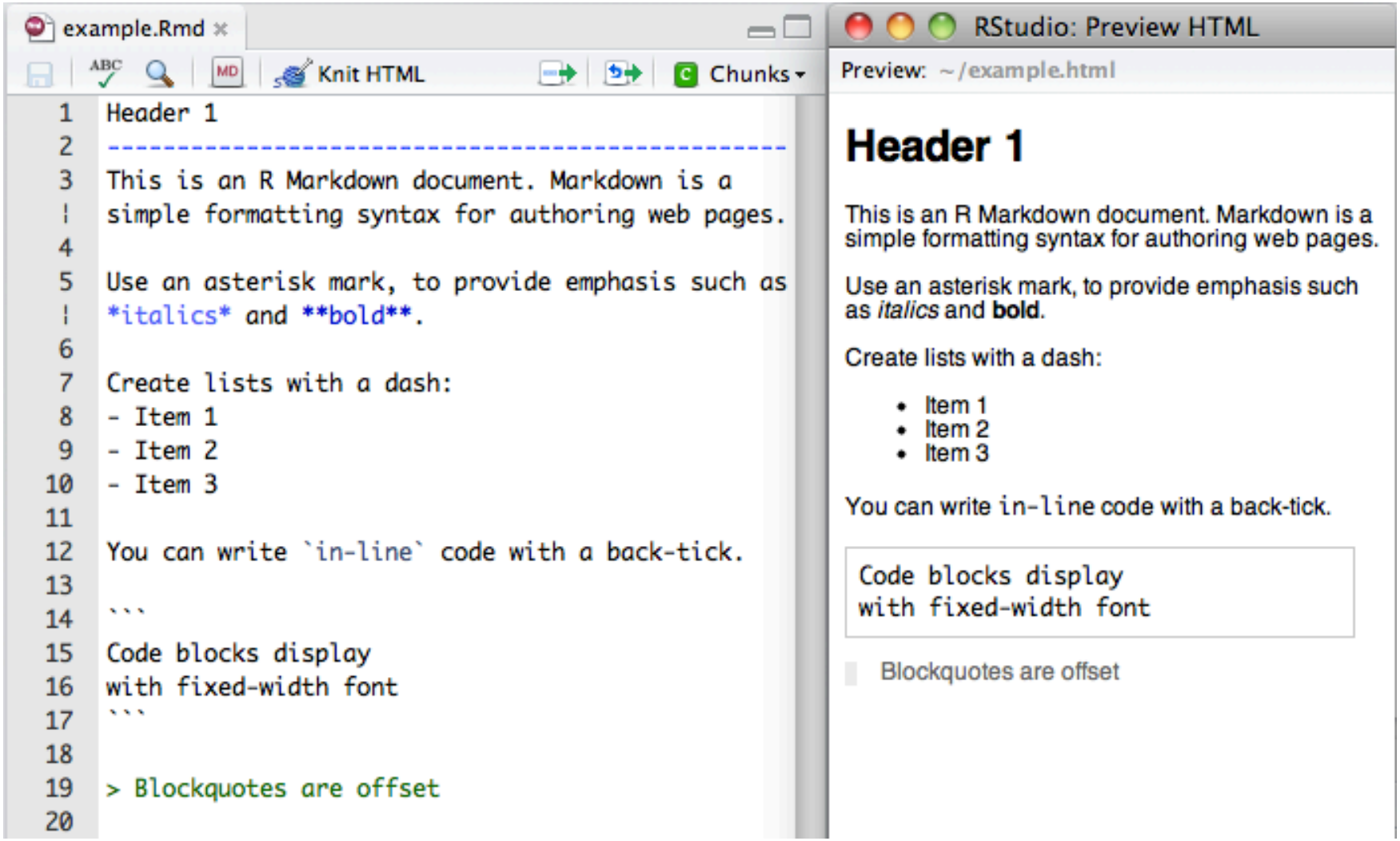
Where does this course fit in my life?

Copy paste ad nauseam



Markdown: HTML without knowing HTML



R + Markdown = RMarkdown

chunks.Rmd

Knit HTML | Chunks

```
1  R Code Chunks
2  ==========
3
4  With R Markdown, you can insert R code
   chunks including plots:
5
6  ```{r qplot, fig.width=4, fig.height=3,
   message=FALSE}
7  # quick summary and plot
8  library(ggplot2)
9  summary(cars)
10 qplot(speed, dist, data=cars) +
11       geom_smooth()
12 ```
13
```

RStudio: Preview HTML

Preview: ~/chunks.html | Save As | Publish

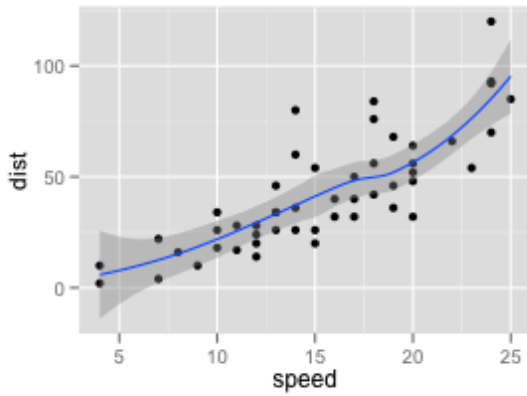
R Code Chunks

With R Markdown, you can insert R code chunks including plots:

```
# quick summary and plot
library(ggplot2)
summary(cars)
```

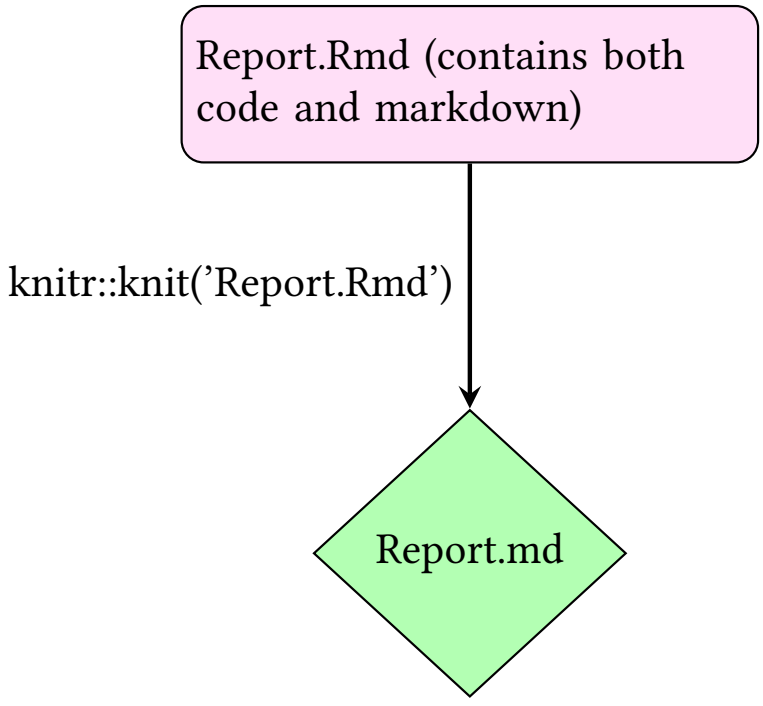
```
##      speed           dist
##  Min.   : 4.0   Min.   :  2
##  1st Qu.:12.0   1st Qu.: 26
##  Median :15.0   Median : 36
##  Mean   :15.4   Mean   : 43
##  3rd Qu.:19.0   3rd Qu.: 56
##  Max.   :25.0   Max.   :120
```

```
qplot(speed, dist, data = cars) + geom_smooth()
```



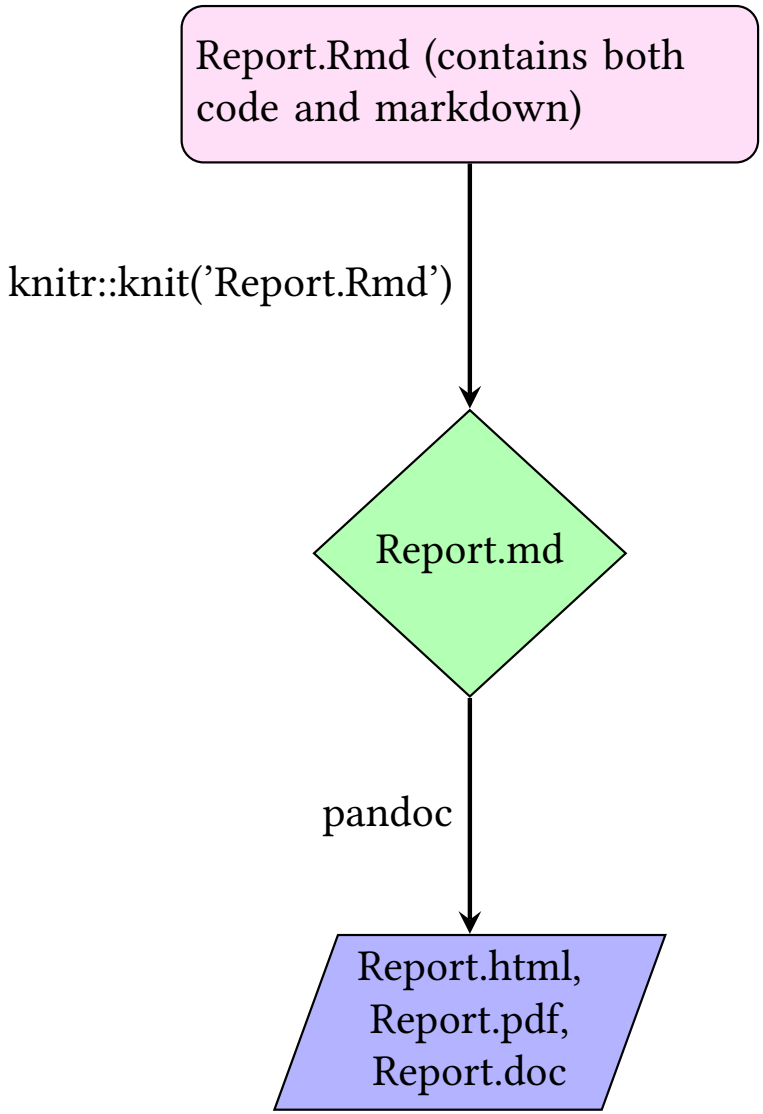
What rmarkdown does

RMarkdown example:



What rmarkdown does

RMarkdown example:

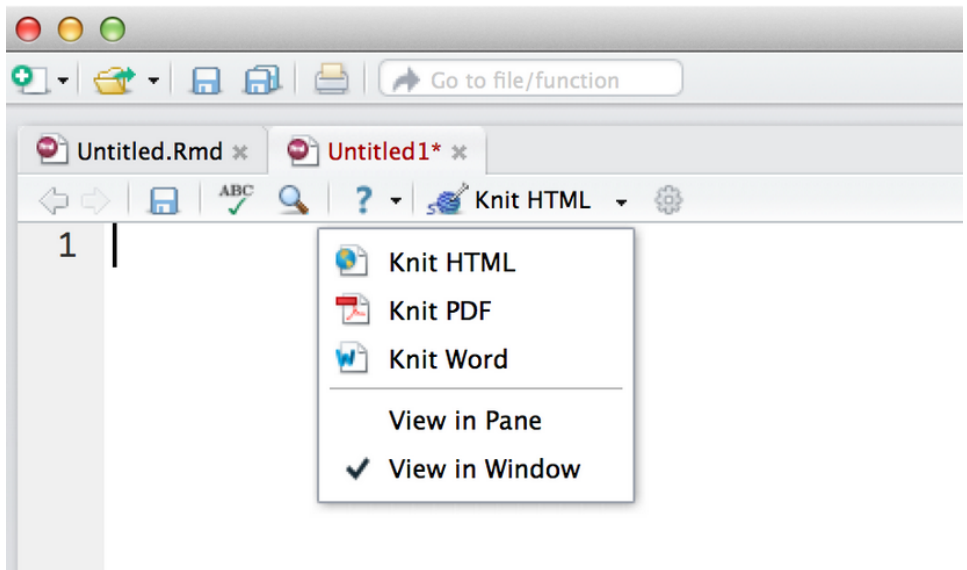


Compiling a .Rmd document

The two steps on previous slide can be executed in one command:

```
rmarkdown::render()
```

or in RStudio:



Objectives

Visualize/Analyze/Interpret data using statistical methods with a computer

Gather data into analysis ready format

Learn regression

Understand the statistical results in a scientific paper

Learn the tools for creating reproducible analyses

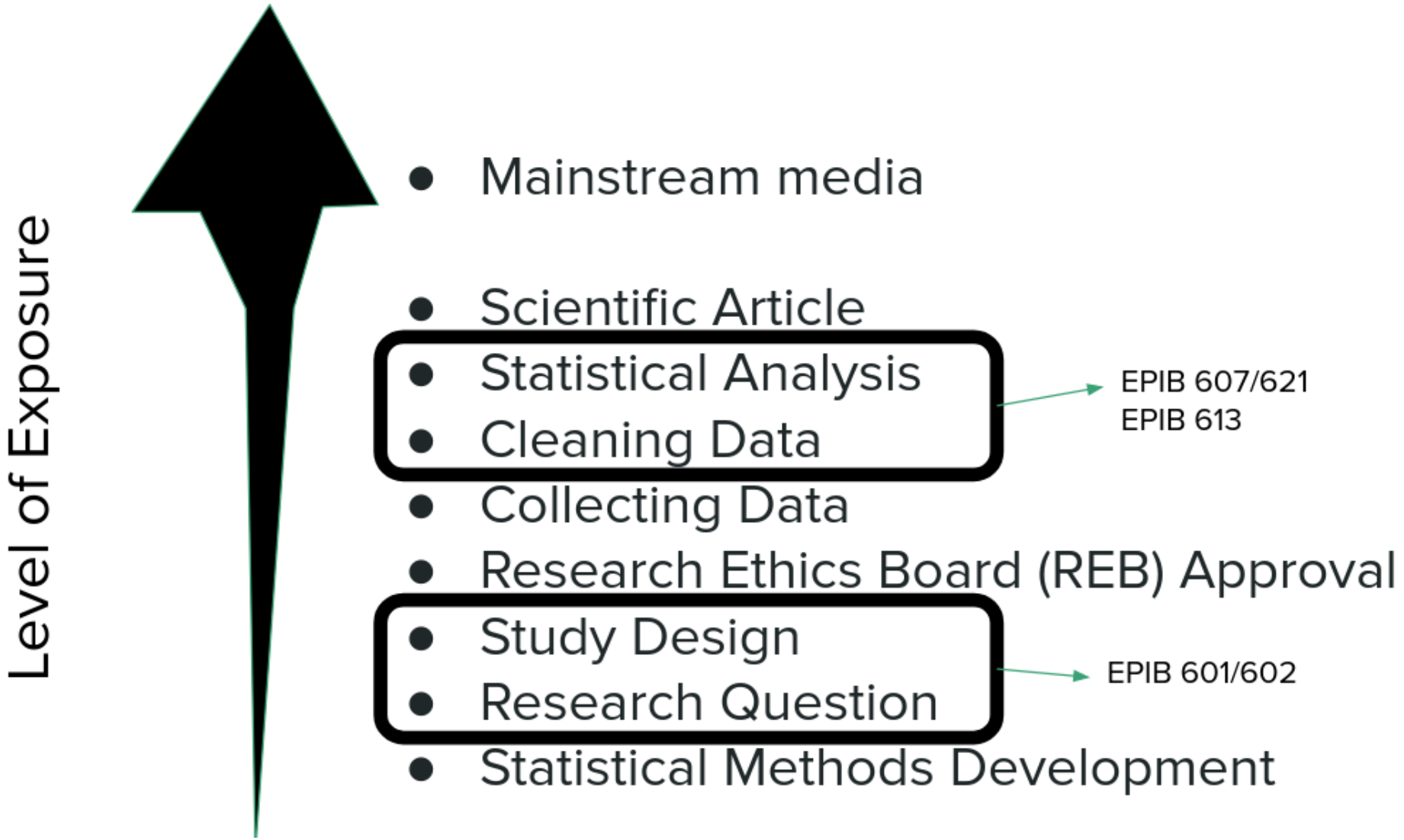
Where does this course fit in my life?

Topics by level of exposure

- Mainstream media

- Scientific Article
- Statistical Analysis
- Cleaning Data
- Collecting Data
- Research Ethics Board (REB) Approval
- Study Design
- Research Question
- Statistical Methods Development

First year courses



My area of research

Session Info

```
R version 3.6.2 (2019-12-12)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Pop!_OS 19.10

Matrix products: default
BLAS:   /usr/lib/x86_64-linux-gnu/openblas/libblas.so.3
LAPACK: /usr/lib/x86_64-linux-gnu/libopenblasp-r0.3.7.so

attached base packages:
[1] stats     graphics  grDevices utils     datasets  methods   base

other attached packages:
 [1] NCStats_0.4.7      FSA_0.8.30         forcats_0.5.0      stringr_1.4.0
 [5] dplyr_1.0.2        purrr_0.3.4        readr_1.3.1        tidyr_1.1.2
 [9] tibble_3.0.3       ggplot2_3.3.2.9000 tidyverse_1.3.0    knitr_1.29

loaded via a namespace (and not attached):
 [1] Rcpp_1.0.4.6       lubridate_1.7.4    lattice_0.20-38    assertthat_0.2.1
 [5] digest_0.6.25      R6_2.4.1           cellranger_1.1.0   plyr_1.8.6
 [9] backports_1.1.9    reprex_0.3.0       evaluate_0.14      httr_1.4.1
[13] highr_0.8          pillar_1.4.6       TeachingDemos_2.12 rlang_0.4.7
[17] readxl_1.3.1       rstudioapi_0.11    minqa_1.2.4        nloptr_1.2.2.1
[21] Matrix_1.2-18      labeling_0.3       splines_3.6.2      lme4_1.1-21
[25] munsell_0.5.0      broom_0.7.0        compiler_3.6.2     modelr_0.1.5
[29] xfun_0.16          pkgconfig_2.0.3    tidyselect_1.1.0   fansi_0.4.1
[33] crayon_1.3.4       dbplyr_1.4.2       withr_2.2.0        ggpubr_0.2.5
[37] MASS_7.3-51.5      grid_3.6.2         nlme_3.1-143       jsonlite_1.7.0
[41] gtable_0.3.0       lifecycle_0.2.0    DBI_1.1.0          magrittr_1.5
[45] scales_1.1.1       cli_2.0.2          stringi_1.4.6      farver_2.0.3
[49] ggsignif_0.6.0     fs_1.3.2           xml2_1.3.0         ellipsis_0.3.1
[53] generics_0.0.2     vctrs_0.3.4        boot_1.3-24        tools_3.6.2
[57] glue_1.4.2         hms_0.5.3          colorspace_1.4-1   rvest_0.3.5
[61] haven_2.3.1
```