Variable selection with the strong heredity constraint and its oracle property

Sahir Bhatnagar

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Trying to code the Strong Heredity Interaction Model (SHIM) in Choi et al. (2010).

1 Simulate the data

```
set.seed(123456)
p = 10 # number of predictors
n = 200 # number of test subjects
m = 200 # number of validation subjects
rho = 0.5
signal_to_noise_ratio = 4
main_effect_names <- paste0("x", 1:p) # names of the main effects
# names of the active set
true_var_names <- c("x1", "x2", "x3", "x4", "x1:x2", "x1:x3",
    "x1:x4", "x2:x3", "x2:x4", "x3:x4")
beta1 <- c(7, 2, 1, 1, 0, 0, 0, 0, 0, 0) %>% magrittr::set_names(true_var_names)
beta2 <- c(7, 2, 1, 1, 1, 0, 0, 0.5, 0.4, 0.1) %>% magrittr::set_names(true_var_names)
beta3 <- c(7, 2, 1, 1, 7, 7, 7, 2, 2, 1) %>% magrittr::set_names(true_var_names)
beta4 <- c(7, 2, 1, 1, 14, 14, 14, 4, 4, 2) %>% magrittr::set_names(true_var_names)
beta5 <- c(0, 0, 0, 0, 7, 7, 7, 2, 2, 1) %>% magrittr::set_names(true_var_names)
# simulate Toeplitz like structure
H <- abs(outer(1:p, 1:p, "-"))</pre>
cor <- rho^H
DT <- MASS::mvrnorm(n = n, mu = rep(0, p), Sigma = cor) %>% magrittr::set_colnames(paste0("x",
```

```
1:p)) %>% set_rownames(paste0("Subject", 1:n))
head(DT)
##
               x1
                     x2
                            xЗ
                                  x4
                                        x5
                                               x6
                                                      x7
## Subject1 -0.16 -0.92 -1.54 -0.79 -1.04 -0.148 0.257 -0.28
## Subject2 0.37 0.28 0.36 -0.69 0.60 -0.151 0.011
## Subject3 0.71 1.10 1.26 -0.19 -0.83 0.323 -0.223 1.16
## Subject4 -0.44 0.66 0.44 0.50 0.33 0.025 -1.229 -0.99
## Subject5 1.30 0.15 -0.82 -1.04 -1.32 -1.796 -0.782 -1.47
## Subject6 -0.82 -1.71 0.93 0.87 -0.09 -0.321 -0.616 -1.68
##
                x9
                     x10
## Subject1 -0.267
## Subject2 -0.098 0.32
## Subject3 -1.157 0.17
## Subject4 0.597 -0.48
## Subject5 -3.419 -2.32
## Subject6 -0.901 -1.24
# create X matrix which contains all main effects and
# interactions each column is standardized to mean O and sd 1
X <- model.matrix(as.formula(paste0("~(", paste0(main_effect_names,</pre>
    collapse = "+"), ")^2-1")), DT %>% as.data.frame()) %>% scale
crossprod(X) %>% diag
##
                                           x6
                                                         x8
       x1
              x2
                     xЗ
                            x4
                                   x5
                                                  x7
##
      199
             199
                    199
                            199
                                   199
                                          199
                                                 199
                                                        199
##
             x10 x1:x2
                         x1:x3 x1:x4
       x9
                                        x1:x5
                                              x1:x6
                                                      x1:x7
##
      199
             199
                    199
                            199
                                   199
                                          199
                                                 199
                                                        199
    x1:x8
           x1:x9 x1:x10
                         x2:x3
                                x2:x4
                                        x2:x5
                                              x2:x6
                                                      x2:x7
##
      199
             199
                    199
                            199
                                   199
                                          199
                                                 199
                                                        199
##
    x2:x8
           x2:x9 x2:x10
                                x3:x5
##
                         x3:x4
                                        x3:x6
                                              x3:x7
                                                      x3:x8
##
      199
             199
                    199
                            199
                                   199
                                          199
                                                 199
                                                        199
##
    x3:x9 x3:x10
                 x4:x5
                         x4:x6
                                x4:x7
                                        x4:x8
                                              x4:x9 x4:x10
             199
                            199
                                   199
##
      199
                    199
                                          199
                                                 199
                                                        199
    x5:x6
           x5:x7
                 x5:x8
                         x5:x9 x5:x10
                                        x6:x7
                                               x6:x8
                                                      x6:x9
##
##
      199
             199
                    199
                            199
                                   199
                                          199
                                                 199
                                                        199
## x6:x10
           x7:x8
                 x7:x9 x7:x10 x8:x9 x8:x10 x9:x10
##
      199
             199
                    199
                            199
                                   199
                                          199
                                                 199
apply(X, 2, mean) %>% sum
## [1] 1.6e-16
```

```
apply(X, 2, sd) %>% sum
## [1] 55
# # alternative, according to Bien et al 2013, they first
# standardize the main effects, # then take all pairwise
# products for interactions, then center those products #
# first scale the main effects X_temp <- DT %>% scale
# apply(X_temp, 2, mean) %>% sum <math>apply(X_temp, 2, sd) %>% sum
# # then create all pairwise interactions X_temp_2 <-
# model.matrix(as.formula(pasteO('~(',pasteO(main_effect_names,
# collapse = '+'), ') ^2-1')), X \%>\% as.data.frame())
# apply(X_{temp_2[,(p+1):ncol(X_{temp_2)]}, 2, sd) \% hist
# apply(X_temp_2[,(p+1):ncol(X_temp_2)], 2, mean) %>% hist #
# then center the product terms and combine with standardized
# main effects X \leftarrow cbind(X_temp_2[,1:p],
\# scale(X_temp_2[,(p+1):ncol(X_temp_2)], center = T, scale = T
# F)) dim(X) str(X) X %>% crossprod() %>% diag
y.star <- X[, names(beta4)] %*% beta4
error <- rnorm(n)</pre>
k <- sqrt(var(y.star)/(signal_to_noise_ratio * var(error)))</pre>
Y <- (y.star + k * error) %>% scale(center = TRUE, scale = FALSE)
colnames(Y) <- "Y"</pre>
(b0 <- mean(y.star + k * error))
## [1] 0.27
# names of interaction variables assuming interaction terms
# contain a ':'
interaction_names <- colnames(X) %>% grep(":", ., value = T)
```

REFERENCES

References

Nam Hee Choi, William Li, and Ji Zhu. Variable selection with the strong heredity constraint and its oracle property. *Journal of the American Statistical Association*, 105(489):354–364, 2010. 1

A R Code

```
sessionInfo()
getPckg <- function(pckg) install.packages(pckg, repos = "http://cran.r-project.org")</pre>
pckg = try(require(knitr))
if (!pckg) {
    cat("Installing 'knitr' from CRAN\n")
   getPckg("knitr")
   require(knitr)
pckg = try(require(data.table))
if (!pckg) {
   cat("Installing 'data.table' from CRAN\n")
   getPckg("data.table")
   require(data.table)
pckg = try(require(magrittr))
if (!pckg) {
   cat("Installing 'magrittr' from CRAN\n")
   getPckg("magrittr")
   require(magrittr)
pckg = try(require(glmnet))
if (!pckg) {
    cat("Installing 'glmnet' from CRAN\n")
   getPckg("glmnet")
   require(glmnet)
pckg = try(require(stringr))
if (!pckg) {
    cat("Installing 'stringr' from CRAN\n")
   getPckg("stringr")
   require(stringr)
```

```
pckg = try(require(plyr))
if (!pckg) {
   cat("Installing 'plyr' from CRAN\n")
   getPckg("plyr")
   require(plyr)
set.seed(123456)
p = 10 # number of predictors
n = 200 # number of test subjects
m = 200 # number of validation subjects
rho = 0.5
signal_to_noise_ratio = 4
main_effect_names <- pasteO("x", 1:p) # names of the main effects
# names of the active set
true_var_names <- c("x1", "x2", "x3", "x4", "x1:x2", "x1:x3",
    "x1:x4", "x2:x3", "x2:x4", "x3:x4")
beta1 <- c(7, 2, 1, 1, 0, 0, 0, 0, 0) %>% magrittr::set_names(true_var_names)
beta2 <- c(7, 2, 1, 1, 1, 0, 0, 0.5, 0.4, 0.1) %>% magrittr::set_names(true_var_names)
beta3 <- c(7, 2, 1, 1, 7, 7, 7, 2, 2, 1) %>% magrittr::set_names(true_var_names)
beta4 <- c(7, 2, 1, 1, 14, 14, 14, 4, 4, 2) %>% magrittr::set_names(true_var_names)
beta5 <- c(0, 0, 0, 0, 7, 7, 7, 2, 2, 1) %>% magrittr::set_names(true_var_names)
# simulate Toeplitz like structure
H <- abs(outer(1:p, 1:p, "-"))
cor <- rho^H
DT <- MASS::mvrnorm(n = n, mu = rep(0, p), Sigma = cor) %>% magrittr::set_colnames(paste0("x",
   1:p)) %>% set_rownames(paste0("Subject", 1:n))
head(DT)
# create X matrix which contains all main effects and
# interactions each column is standardized to mean 0 and sd 1
X <- model.matrix(as.formula(paste0("~(", paste0(main_effect_names,</pre>
   collapse = "+"), ")^2-1")), DT %>% as.data.frame()) %>% scale
crossprod(X) %>% diag
apply(X, 2, mean) %>% sum
```

```
apply(X, 2, sd) %>% sum
# # alternative, according to Bien et al 2013, they first
# standardize the main effects, # then take all pairwise
# products for interactions, then center those products #
# first scale the main effects X_temp <- DT %>% scale
# apply(X_temp, 2, mean) %>% sum <math>apply(X_temp, 2, sd) %>% sum
# # then create all pairwise interactions X_temp_2 <-
# model.matrix(as.formula(pasteO('~(',pasteO(main_effect_names,
# collapse = '+'), ') ^2-1')), X \%>\% as.data.frame())
# apply(X_{temp_2[,(p+1):ncol(X_{temp_2)]}, 2, sd) \% hist
# apply(X_temp_2[,(p+1):ncol(X_temp_2)], 2, mean) %>% hist #
# then center the product terms and combine with standardized
# main effects X \leftarrow cbind(X_temp_2[,1:p],
\# scale(X_temp_2[,(p+1):ncol(X_temp_2)], center = T, scale =
# F)) dim(X) str(X) X %>% crossprod() %>% diag
y.star <- X[, names(beta4)] %*% beta4
error <- rnorm(n)</pre>
k <- sqrt(var(y.star)/(signal_to_noise_ratio * var(error)))</pre>
Y <- (y.star + k * error) %>% scale(center = TRUE, scale = FALSE)
colnames(Y) <- "Y"</pre>
(b0 <- mean(y.star + k * error))
# names of interaction variables assuming interaction terms
# contain a ':'
interaction_names <- colnames(X) %>% grep(":", ., value = T)
#' variables: character vector of variable names for which you want the univariate regression est
\#' must be contained in the column names of x
#' x: matrix that includes all data corresponding to variables with corresponding column names
#' y: response (matrix form)
#' returns OLS coefficients as a p x 1 data.frame
uni_fun <- function(variables, x, y) {</pre>
   res <- plyr::ldply(variables, function(i) {</pre>
```

```
fit \leftarrow lm.fit(x = x[, i, drop = F], y = y) # dont need to add intercept because y has be
        fit$coefficients[1]
   }) %>% magrittr::set_rownames(variables) %>% magrittr::set_colnames("univariate_beta") %>%
        as.matrix
   return(res)
# function that takes a vector of betas (which are the main
# effects) and alphas which are the interaction effects, and
# outputs the main effects, and converts the alphas to gammas
# note that alpha_ij = qamma_ij * beta_i*beta_j, i < j
convert <- function(betas.and.alphas, main.effect.names, interaction.names,</pre>
    epsilon = 1e-04) {
    # betas.and.alphas is the result from uni_fun create output
    # matrix
   betas_and_gammas <- matrix(nrow = nrow(betas.and.alphas)) %>%
        magrittr::set_rownames(rownames(betas.and.alphas))
   for (k in interaction.names) {
        # get names of main effects corresponding to interaction
        (main <- betas.and.alphas[k, , drop = F] %>% rownames %>%
            stringr::str_split(":") %>% unlist)
        # convert alpha to gamma BUT NEED TO CHECK IF BETAS ARE O!!!!
        betas_and_gammas[k, ] <- if (any(abs(betas.and.alphas[main,</pre>
            ]) < epsilon))
            0 else betas.and.alphas[k, ]/prod(betas.and.alphas[main,
            1)
    }
    # add back the main effects which dont need to be transformed
   for (j in main.effect.names) {
        betas_and_gammas[j, ] <- betas.and.alphas[j, ]</pre>
   return(betas_and_gammas)
```

```
# function that converts gammas to alphas
convert2 <- function(betas.and.gammas, main.effect.names, interaction.names) {</pre>
    # betas.and.gammas = rbind2(beta_hat_next,
    # gamma_hat_previous) betas.and.gammas is the result from
    # shim function create output matrix
   betas.and.alphas <- matrix(nrow = nrow(betas.and.gammas)) %>%
        magrittr::set_rownames(rownames(betas.and.gammas))
   for (k in interaction.names) {
        # k='x1:x10' get names of main effects corresponding to
        # interaction
        (main <- betas.and.gammas[k, , drop = F] %>% rownames %>%
            stringr::str_split(":") %>% unlist)
        # convert alpha to gamma
        betas.and.alphas[k, ] <- betas.and.gammas[k, ] * prod(betas.and.gammas[main,
            ])
    }
    # add back the main effects which dont need to be transformed
   for (j in main.effect.names) {
        betas.and.alphas[j, ] <- betas.and.gammas[j, ]</pre>
   return(betas.and.alphas)
# function to calculate xtilde in step 3 of algorithm
xtilde <- function(interaction.names, data.main.effects, beta.main.effects) {</pre>
    #' interaction.names character vector of interaction names
    #' data.main.effects data frame or matrix containing the main effects data
    #' beta.main.effects data frame or matrix containing the coefficients of main effects
    #' Only the main effects are used in this step,
    #' however you can provide this function, either the betas.and.alphas or
    #' betas.and.gammas because only the betas (main effect) paramters
    #' are used in the calculation of xtilde
```

```
# create output matrix
   xtildas <- matrix(ncol = length(interaction.names), nrow = nrow(data.main.effects)) %>%
        magrittr::set_colnames(interaction.names)
   for (k in interaction.names) {
        # get names of main effects corresponding to interaction
        (main <- k %>% stringr::str_split(":") %>% unlist)
        # step 3 to calculate x tilda
        xtildas[, k] <- prod(beta.main.effects[main, ]) * data.main.effects[,</pre>
            main[1], drop = F] * data.main.effects[, main[2],
            drop = F
    }
   return(xtildas)
# adaptive weights used in fitting algorithm, intercept set
# to 0 because Y is centered
ridge_weights <- function(x, y, main.effect.names, interaction.names) {</pre>
    #' interaction.names character vector of interaction names
    #' main.effect.names character vector of main effect names
    \#' must be contained in the column names of x
    #' x: matrix that includes all data corresponding to variables with corresponding column name.
    #' all columns have mean O and variance 1
   #' y: centered response (matrix form)
    #' returns ridge weights as a p x 1 data.frame
    #' STILL NEED TO DECIDE IF I SHOULD MULTIPLY WEIGHTS BY LOG(N)/N AS DONE IN SECTION 3.1 AND I.
    #' PAPER BY WANG LI AND TSAI 2007 JRSSB
   n <- length(y)</pre>
    # fit the ridge to get betas and alphas
    (fit <- glmnet::cv.glmnet(x = x, y = y, alpha = 0, standardize = F,
        intercept = F))
    # remove intercept
    (betas.and.alphas <- coef(fit, s = "lambda.1se") %>% as.matrix() %>%
        magrittr::extract(-1, , drop = F))
```

```
# (fit \leftarrow lm.fit(x = x, y = y)) # remove intercept
    # (betas.and.alphas <- coef(fit) %>% as.matrix())
    # create output matrix
    weights <- matrix(nrow = nrow(betas.and.alphas)) %>% magrittr::set_rownames(rownames(betas.and.alphas))
    # main effects weights
   for (j in main.effect.names) {
        weights[j, ] \leftarrow abs(1/betas.and.alphas[j, ]) #* log(n)/n
   for (k in interaction.names) {
        # get names of main effects corresponding to interaction
        (main <- betas.and.alphas[k, , drop = F] %>% rownames %>%
            stringr::str_split(":") %>% unlist)
        weights[k, ] <- abs(prod(betas.and.alphas[main, ])/betas.and.alphas[k,</pre>
            ]) \#* log(n)/n
   return(weights)
soft <- function(x, y, beta, lambda, weight) {</pre>
    \# user must supply x AND y, or beta.. but not both i set it
    # up this way because to get the sequence of lambdas, I use
    # the beta argument so that I only compute this once. I use
    # the x, y argument for the CV folds lambda can be a vector
    # and this functions will return each thresholded beta for
    \# each lambda e.g. soft(0.25, lambda =
    \# seq(0.001, 0.65, length.out = 100), 1.5)
    if (missing(x) & missing(y) & missing(beta))
        stop("user must supply x AND y, or beta but not both")
    if (missing(x) & missing(y))
        return(list(beta = sign(beta) * pmax(0, abs(beta) - lambda *
            weight)))
    if (missing(beta)) {
        # (beta <- lm.fit(x = cbind2(rep(1, length(y)), x), y = y) %>%
```

```
# coef %>% magrittr::extract(2))
        (beta \leftarrow lm.fit(x = x[, 1, drop = F], y = y) %>% coef %>%
            magrittr::extract(1))
        \# lm.fit(x = cbind2(rep(1, length(y_tilde_2)), x_tilde_2), y =
        # y_tilde_2) %>% coef %>% magrittr::extract(2)
        b_lasso <- sign(beta) * pmax(0, abs(beta) - lambda *
            weight)
        # return(list('beta' = b_lasso, 'df' = nonzero(b_lasso)))
        return(b lasso)
    }
cv_lspath <- function(outlist, lambda, x, y, foldid) {</pre>
    # typenames <- c(misclass = 'Misclassification Error', loss =
    # 'Margin Based Loss') if (pred.loss == 'default') pred.loss
    # <- 'loss' if (!match(pred.loss, c('loss'), FALSE)) {</pre>
    # warning('Only 'loss' available for least squares
    # regression; 'loss' used') pred.loss <- 'loss' }</pre>
    y <- as.double(y)</pre>
    nfolds <- max(foldid)</pre>
    predmat <- matrix(NA, length(y), length(lambda))</pre>
    nlams <- double(nfolds)</pre>
    for (i in seq(nfolds)) {
        # i = 1
        which <- foldid == i
        # this gives be the fitted object for each CV fold
        fitobj <- outlist[[i]]</pre>
        # this gives the predicted responses for the subjects in the
        # held-out fold for each lambda so if each fold has 20
        # subjects, and there are 100 lambdas, then this will return
        # a 20 x 100 matrix
        preds <- x[which, , drop = FALSE] %*% t(fitobj$beta)</pre>
        # preds <- predict(fitobj, x[which, , drop = FALSE], type =</pre>
        # 'link')
        nlami <- length(fitobj$lambda)</pre>
        predmat[which, seq(nlami)] <- preds</pre>
```

```
nlams[i] <- nlami
    cvraw <- (y - predmat)^2</pre>
    cvob <- cvcompute(cvraw, foldid, nlams)</pre>
    cvraw <- cvob$cvraw
    N <- cvob$N
    cvm <- apply(cvraw, 2, mean, na.rm = TRUE)</pre>
    cvsd <- sqrt(apply(scale(cvraw, cvm, FALSE)^2, 2, mean, na.rm = TRUE)/(N -</pre>
         1))
    list(cvm = cvm, cvsd = cvsd, name = "MSE")
cvcompute <- function(mat, foldid, nlams) {</pre>
    nfolds <- max(foldid)</pre>
    outmat <- matrix(NA, nfolds, ncol(mat))</pre>
    good <- matrix(0, nfolds, ncol(mat))</pre>
    mat[is.infinite(mat)] <- NA</pre>
    for (i in seq(nfolds)) {
        mati <- mat[foldid == i, ]</pre>
         outmat[i, ] <- apply(mati, 2, mean, na.rm = TRUE)</pre>
         good[i, seq(nlams[i])] <- 1</pre>
    N <- apply(good, 2, sum)
    list(cvraw = outmat, N = N)
lamfix <- function(lam) {</pre>
    llam <- log(lam)</pre>
    lam[1] \leftarrow exp(2 * llam[2] - llam[3])
    lam
nonzero <- function(beta, bystep = FALSE) {</pre>
    beta <- as.matrix(beta)</pre>
    nr = nrow(beta)
    if (nr == 1) {
         if (bystep)
             apply(beta, 2, function(x) if (abs(x) > 0)
                  1 else NULL) else {
```

```
if (any(abs(beta) > 0))
                 1 else NULL
        }
    } else {
        beta = abs(beta) > 0
        which = seq(nr)
        ones = rep(1, ncol(beta))
        nz = as.vector((beta %*% ones) > 0)
        which = which[nz]
        if (bystep) {
            if (length(which) > 0) {
                beta = as.matrix(beta[which, , drop = FALSE])
                nzel = function(x, which) if (any(x))
                   which[x] else NULL
                 which = apply(beta, 2, nzel, which)
                 if (!is.list(which))
                   which = data.frame(which)
                which
            } else {
                dn = dimnames(beta)[[2]]
                 which = vector("list", length(dn))
                names(which) = dn
                 which
        } else which
getmin <- function(lambda, cvm, cvsd) {</pre>
    cvmin <- min(cvm)</pre>
    idmin <- cvm <= cvmin
    lambda.min <- max(lambda[idmin])</pre>
    idmin <- match(lambda.min, lambda)</pre>
    semin <- (cvm + cvsd)[idmin]</pre>
    idmin <- cvm <= semin
    lambda.1se <- max(lambda[idmin])</pre>
    list(lambda.min = lambda.min, lambda.1se = lambda.1se)
```

```
lambda.interp <- function(lambda, s) {</pre>
    if (length(lambda) == 1) {
        nums <- length(s)</pre>
        left <- rep(1, nums)</pre>
        right <- left
        sfrac <- rep(1, nums)
    } else {
        s[s > max(lambda)] <- max(lambda)
        s[s < min(lambda)] <- min(lambda)
        k <- length(lambda)</pre>
        sfrac <- (lambda[1] - s)/(lambda[1] - lambda[k])</pre>
        lambda <- (lambda[1] - lambda)/(lambda[1] - lambda[k])</pre>
        coord <- approx(lambda, seq(lambda), sfrac)$y</pre>
        left <- floor(coord)</pre>
        right <- ceiling(coord)</pre>
        sfrac <- (sfrac - lambda[right])/(lambda[left] - lambda[right])</pre>
        sfrac[left == right] <- 1</pre>
    list(left = left, right = right, frac = sfrac)
plot.cv.uninet <- function(x, sign.lambda = 1, ...) {</pre>
    cvobj <- x
    xlab <- "log(Lambda)"</pre>
    if (sign.lambda < 0)</pre>
        xlab <- paste("-", xlab, sep = "")</pre>
    plot.args <- list(x = sign.lambda * log(cvobj$lambda), y = cvobj$cvm,</pre>
        ylim = range(cvobj$cvupper, cvobj$cvlo), xlab = xlab,
        ylab = cvobj$name, type = "n")
    new.args <- list(...)</pre>
    if (length(new.args))
        plot.args[names(new.args)] <- new.args</pre>
    do.call("plot", plot.args)
    error.bars(sign.lambda * log(cvobj$lambda), cvobj$cvupper,
        cvobj$cvlo, width = 0.01, col = "darkgrey")
    points(sign.lambda * log(cvobj$lambda), cvobj$cvm, pch = 20,
        col = "red")
    axis(side = 3, at = sign.lambda * log(cvobj$lambda), labels = paste(cvobj$nz),
        tick = FALSE, line = 0)
```

```
abline(v = sign.lambda * log(cvobj$lambda.min), lty = 3)
    abline(v = sign.lambda * log(cvobj$lambda.1se), lty = 3)
    invisible()
coef.cv.uninet <- function(object, s = c("lambda.1se", "lambda.min"),</pre>
    ...) {
    if (is.numeric(s))
        lambda <- s else if (is.character(s)) {</pre>
        s <- match.arg(s)</pre>
        lambda <- object[[s]]</pre>
    } else stop("Invalid form for s")
    coef.uninet(object$uninet.fit, s = lambda, ...)
coef.uninet <- function(object, s = NULL, type = c("coefficients",</pre>
    "nonzero"), ...) {
    type <- match.arg(type)</pre>
    # b0 <- t(as.matrix(objectfb0)) rownames(b0) <- '(Intercept)'
    nbeta <- rbind2(object$beta)</pre>
    if (!is.null(s)) {
        vnames <- dimnames(nbeta)[[1]]</pre>
        # dimnames(nbeta) <- list(NULL, NULL)</pre>
        lambda <- object$lambda
        lamlist <- lambda.interp(lambda, s)</pre>
        nbeta <- nbeta[, lamlist$left, drop = FALSE] * lamlist$frac +</pre>
            nbeta[, lamlist$right, drop = FALSE] * (1 - lamlist$frac)
        # dimnames(nbeta) <- list(vnames, paste(seq(along = s)))</pre>
    if (type == "coefficients")
        return(nbeta)
    if (type == "nonzero")
        return(nonzero(nbeta[1, , drop = FALSE], bystep = TRUE))
# fits weighted lasso with single predictor and no intercept
uninet <- function(x, y, nlambda = 100, method = c("ls"), lambda.factor = ifelse(nobs <
    nvars, 0.01, 1e-04), lambda = NULL, penalty.factor = rep(1,
    nvars)) {
```

```
\# x=x\_tilde_2; y=y\_tilde_2; nlambda = 100; method = c('ls');
# lambda.factor = ifelse(nobs < nvars, 0.01, 1e-04); lambda =
# NULL; penalty.factor = rep(1, nvars); standardize = TRUE;
\# eps = 1e-08; maxit = 1e+06
method <- match.arg(method)</pre>
this.call <- match.call()</pre>
y <- drop(y)
x <- as.matrix(x)</pre>
np \leftarrow dim(x)
nobs <- as.integer(np[1])</pre>
nvars <- as.integer(np[2])</pre>
vnames <- colnames(x)</pre>
N = nrow(x)
if (is.null(vnames))
    vnames <- paste("V", seq(nvars), sep = "")</pre>
if (length(y) != nobs)
    stop("x and y have different number of observations")
if (length(penalty.factor) != nvars)
    stop("The size of L1 penalty factor must be same as the number of input variables")
# if (lambda2 < 0) stop('lambda2 must be non-negative') maxit
# <- as.integer(maxit) lam2 <- as.double(lambda2)
penalty.factor <- as.double(penalty.factor)</pre>
# pf2 <- as.double(pf2) isd <- as.integer(standardize) eps <-
# as.double(eps) dfmax <- as.integer(dfmax) pmax <-
# as.integer(pmax) if (!missing(exclude)) { jd <-</pre>
# match(exclude, seg(nvars), 0) if (!all(jd > 0)) stop('Some
# excluded variables out of range') jd <-
# as.integer(c(length(jd), jd)) } else jd <- as.integer(0)
if (is.null(lambda)) {
    if (lambda.factor >= 1)
        stop("lambda.factor should be less than 1")
    # flmin <- as.double(lambda.factor)</pre>
    max_lam <- double(1)</pre>
    # loop to figure out max lambda its the minimum lambda that
    # gives a beta of O
    beta_tilde \leftarrow lm.fit(x = x, y = y) \% coef
```

```
# ulam = 0
        b = 1
        while (b != 0) {
            max_lam \leftarrow max_lam + 0.001
            b <- soft(beta = beta_tilde, lambda = max_lam, weight = penalty.factor)
        }
        lambda <- seq(lambda.factor * max_lam, max_lam, length.out = 100)</pre>
    } else {
        flmin <- as.double(1)</pre>
        if (any(lambda < 0))</pre>
             stop("lambdas should be non-negative")
        ulam <- as.double(rev(sort(lambda)))</pre>
        nlam <- as.integer(length(lambda))</pre>
    }
    fit <- switch(method, ls = soft(x = x, y = y, lambda = lambda,
        weight = penalty.factor))
    fit$lambda <- if (is.null(lambda))</pre>
        lamfix(lambda) else lambda
    fit$call <- this.call</pre>
    # fitfdf <- nonzero(fitfbeta)</pre>
    class(fit) <- c("uninet", class(fit))</pre>
    fit
cv.uninet <- function(x, y, lambda = NULL, nfolds = 10, foldid,
    ...) {
    # penalty.factor = adaptive.weights[colnames(x),] lambda =
    # NULL x = x_tilde_2; y = y_tilde_2
    np \leftarrow dim(x)
    nobs <- as.integer(np[1])</pre>
    nvars <- as.integer(np[2])</pre>
    # lambda.factor = ifelse(nobs < nvars, 0.01, 1e-04) nfolds =
    # 10
    N = nrow(x)
```

```
y \leftarrow drop(y)
# if (is.null(lambda)) { if (lambda.factor >= 1)
# stop('lambda.factor should be less than 1') #flmin <-
# as.double(lambda.factor) ulam <- double(1) } # loop to
# figure out max lambda # its the minimum lambda that gives a
# beta of 0 beta_tilde <- lm.fit(x = x, y = y) %>% coef #ulam
 \# = 0 \ b = 1 \ while (b != 0) \{ ulam <- ulam + 0.001 \ b <- ulam <- ulam - 0.001 \ b <- ulam <- ulam - 0.001 \ b <- ulam - 0.00
 # soft(beta = beta_tilde, lambda = ulam, weight =
 # penalty.factor) } lambda <- seq(lambda.factor*ulam, ulam,</pre>
# length.out = 100)
# betas for each lambda sapply(lambda, function(i) soft(beta
# = beta_tilde, lambda = i, weight = penalty.factor))
uninet.object <- uninet(x, y, lambda = lambda, ...)</pre>
# uninet.object <- uninet(x, y, lambda = lambda,
# penalty.factor = adaptive.weights[colnames(x_tilde_2),] )
lambda <- uninet.object$lambda</pre>
nz <- sapply(coef(uninet.object, type = "nonzero"), length)</pre>
if (missing(foldid))
          foldid = sample(rep(seq(nfolds), length = N)) else nfolds = max(foldid)
if (nfolds < 3)
          stop("nfolds must be bigger than 3; nfolds=10 recommended")
outlist = as.list(seq(nfolds))
for (i in seq(nfolds)) {
          which = foldid == i
          if (is.matrix(v))
                     y_sub = y[!which, ] else y_sub = y[!which]
          outlist[[i]] = uninet(x = x[!which, , drop = FALSE],
                    y = y_sub, lambda = lambda, ...)
cvstuff <- do.call(cv_lspath, list(outlist, lambda, x, y,</pre>
```

```
foldid))
    cvm <- cvstuff$cvm</pre>
    cvsd <- cvstuff$cvsd</pre>
    cvname <- cvstuff$name
    out <- list(lambda = lambda, cvm = cvm, cvsd = cvsd, cvupper = cvm +
        cvsd, cvlo = cvm - cvsd, name = cvname, nzero = nz, name = cvname,
        uninet.fit = uninet.object)
    lamin <- getmin(lambda, cvm, cvsd)</pre>
    obj <- c(out, as.list(lamin))</pre>
    class(obj) <- "cv.uninet"</pre>
    obj
Q_theta <- function(x, y, beta, gamma, weights, lambda.beta,
    lambda.gamma, main.effect.names, interaction.names) {
    betas.and.alphas <- convert2(betas.and.gammas = rbind2(beta,</pre>
        gamma), main.effect.names = main.effect.names, interaction.names = interaction.names)
    crossprod(y - x %*% betas.and.alphas - b0) + lambda.beta *
        (crossprod(weights[main.effect.names, ], abs(beta))) +
        lambda.gamma * (crossprod(weights[interaction.names,
            ], abs(gamma)))
```

B Session Information

```
sessionInfo()
## R version 3.2.2 (2015-08-14)
## Platform: x86_64-pc-linux-gnu (64-bit)
## Running under: Ubuntu 14.04 LTS
##
## locale:
   [1] LC_CTYPE=en_CA.UTF-8
                                LC_NUMERIC=C
   [3] LC_TIME=en_CA.UTF-8
                                LC_COLLATE=en_CA.UTF-8
##
   [5] LC_MONETARY=en_CA.UTF-8 LC_MESSAGES=en_CA.UTF-8
   [7] LC_PAPER=en_CA.UTF-8
                                 LC_NAME=C
##
  [9] LC_ADDRESS=C
##
                                 LC_TELEPHONE=C
## [11] LC_MEASUREMENT=en_CA.UTF-8 LC_IDENTIFICATION=C
##
## attached base packages:
## [1] stats
                graphics grDevices utils
                                           datasets
## [6] base
##
## other attached packages:
## [1] plyr_1.8.3
                      stringr_1.0.0
                                       glmnet_2.0-2
## [4] foreach_1.4.3
                      Matrix_1.2-2
                                       magrittr_1.5
## [7] data.table_1.9.4 knitr_1.11
##
## loaded via a namespace (and not attached):
                        lattice_0.20-33 codetools_0.2-14
## [1] Rcpp_0.12.2
## [4] MASS_7.3-44
                        chron_2.3-45
                                        grid_3.2.2
## [7] formatR_1.2.1
                       evaluate_0.8
                                       highr_0.5.1
## [10] stringi_1.0-1
                       reshape2_1.4.1
                                        iterators_1.0.8
## [13] tools_3.2.2 methods_3.2.2
```