

MATH 680 - Assignment 2 - March 30, 2015

Sahir Bhatnagar & Maxime Turgeon

Department of Epidemiology, Biostatistics and Occupational Health
McGill University

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1 Numerical Results

1.1 Introduction

1.2 Nonparametric Bootstrap Smoothing

model	m	Cp	Bootstrap
Linear	2	1151	20
Quadratic	3	1434	13
Cubic	4	636	35
Quartic	5	1579	8
Quintic	6	1776	19
Sextic	7	2758	5

Table 1: C_p model selection for the Cholesterol data. $\sigma = 22$ was used in all bootstrap replications. Last column shows percentage each model was selected as the C_p minimizer, among $B = 4000$ bootstrap replications

	m1	m2	m3	m4	m5	m6
Mean	-13.87	-3.51	5.13	-1.68	-4.64	-12.01
St.dev.	3.45	3.40	5.87	5.80	11.90	45.35

Table 2: Mean and standard deviation of $\hat{\mu}_1^*$ as a function of the selected model, 4000 nonparametric bootstrap replications.

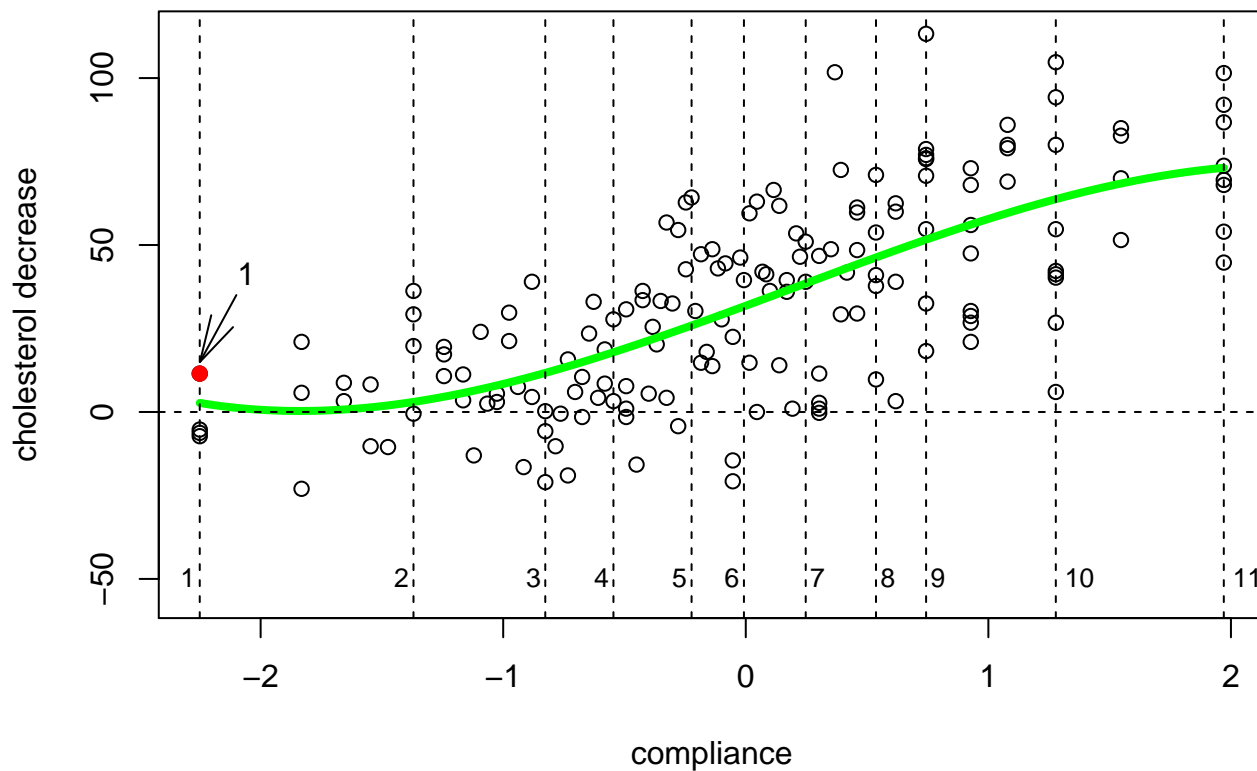
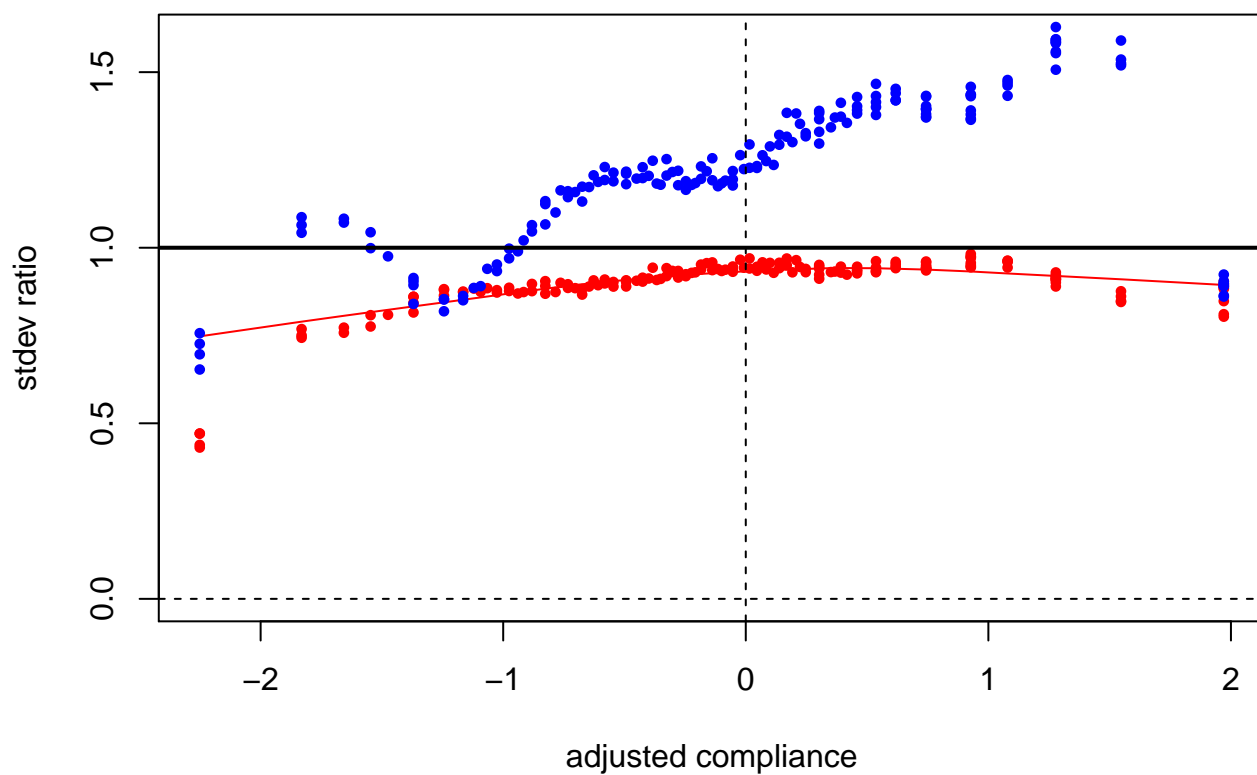


Figure 1: Cholesterol decrease vs. compliance with cubic regression fitted curve

Figure 2: Solid points: ratio of standard deviations, taking account of model selection or not, for the 164 values $\hat{\mu}_j$ from the regression curve in Figure 1. Dashed line: ratio of $\tilde{s}d_B$ to $\hat{s}d_B$

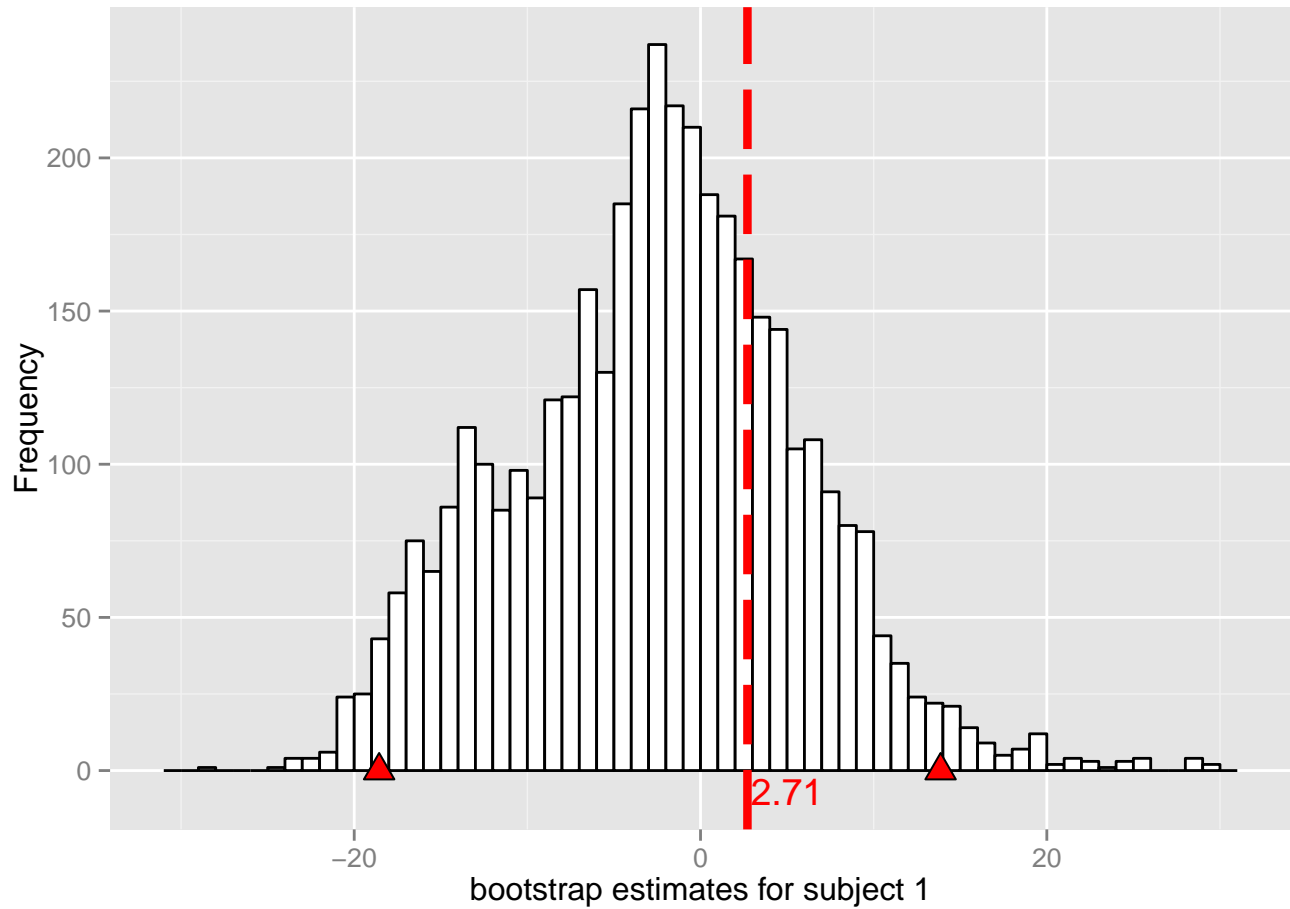


Figure 3: $B=4000$ bootstrap replications of the Cp-OLS regression estimate for Subject 1. Triangles indicate 2.5th and 97.5th percentiles of the histogram

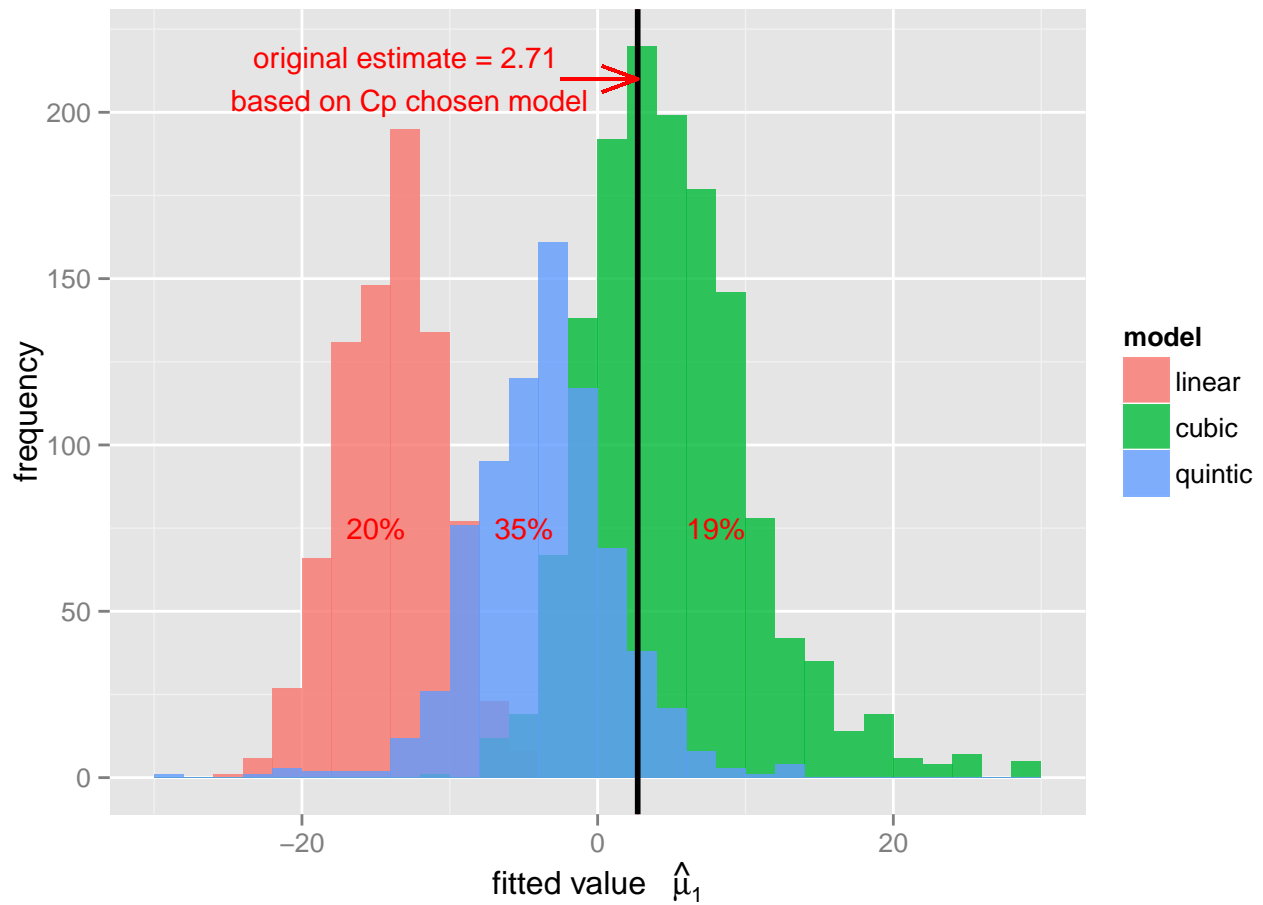


Figure 4: Fitted values for subject 1, from $B=4000$ nonparametric bootstrap replications separated by three most frequently chosen models by C_p

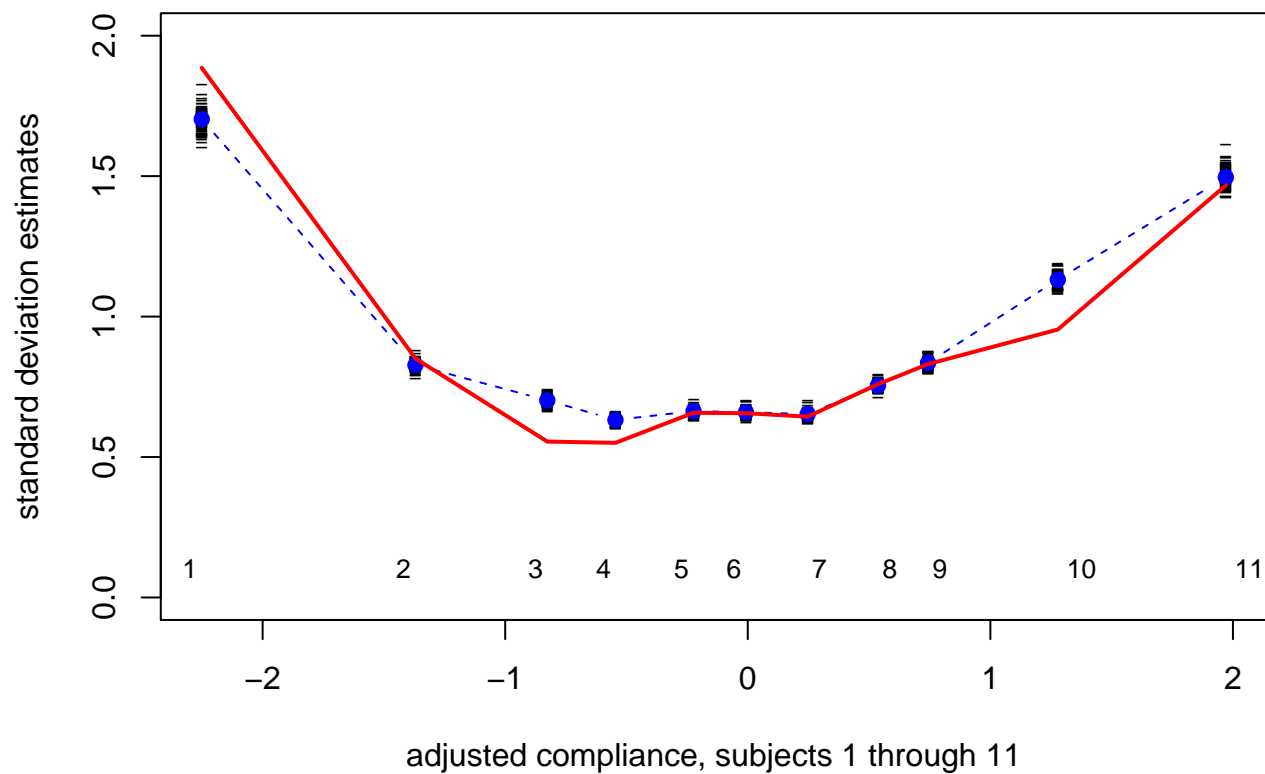


Figure 5: Simulation test of Theorem 2. Cholesterol data; 100 simulations, 1000 parametric bootstraps each, for the 11 subjects indicated at the bottom of Figure 1

	m1	m2	m3	m4	m5	m6
Mean	-13.87	-3.51	5.13	-1.68	-3.83	-3.54
St.dev.	3.45	3.40	5.87	5.80	5.61	8.83

Table 3: Mean and standard deviation of $\hat{\mu}_1^*$ as a function of the selected model. Some bootstrap samples that led to the quintic and sextic model being selected give a bad fit. Removing them and recomputing table 2 shows that the discrepancy between our table 2 and Efron’s is due to bootstrap variability.

type	Interval	Length	Center.point
Standard	(-23.52, 28.94)	52.47	2.71
Percentile	(-18.33, 13.93)	32.26	-2.20
Smoothed	(-15.13, 9.29)	24.42	-2.92

Table 4: Three approximate 95% bootstrap confidence intervals for μ_1 , the response for Subject 1, Cholesterol data

1.3 Accuracy of the Smoothed Bootstrap Estimates

1.4 Parametric Bootstrap Smoothing

2 Discussion

The main example in this paper was based on all subset selection in a linear regression context. The discussion by Wang, Sherwood and Li investigate the proposed method in a regularization procedure, a GLM, quantile and nonparametric regression. Overall, their numerical examples show that Efron’s proposed smoothed estimator results in more accurate confidence intervals with good coverage probabilities. The rationale behind using L_1 -norm type penalty functions for model selection as opposed to the all subset method used by Efron, is due to the well known result that all subset selection methods are unstable (Breiman, 1996a). Since these model selection procedures are driven by the data, it has been shown that even changing one point from the learning set, can result in a completely different chosen model (Breiman, 1996b).

We performed a similar analysis on the `prostate` data set (Stamey et al., 1989); a study that examined the relation between level of prostate specific antigen (PSA) and eight clinical measures e.g. Gleason score, cancer volume, prostate weight in 97 men who were about to receive prostatectomy. We produced $B = 4000$ bootstrap samples of the `prostate` data set. For each of the bootstrap samples, we performed model selection using all subset selection as well as LASSO (Tibshirani, 1994), SCAD Fan and Li (2001) and MCP Zhang (2010). For each model selection procedure, we calculated the fitted value for observation 95 based on the chosen model (resulting in 4000 fitted values for each of the 5 procedures). All the analysis was performed in R (R Core Team, 2013). The LASSO was implemented using the `glmnet` package (Friedman et al., 2010). SCAD and MCP were implemented using the coordinate descent algorithm in the `ncvreg` package (Breheny and Huang, 2011). The tuning parameter λ was chosen using 5 fold cross validation. Note, for the SCAD and MCP penalties, which have an additional tuning parameter, we used the suggested

value of 3.7 and 3.0, respectively. All subset selection via the BIC and Cp criterion were implemented using the `leaps` package (Lumley and Miller, 2009). The histograms for the fitted values are given in Figure 6.

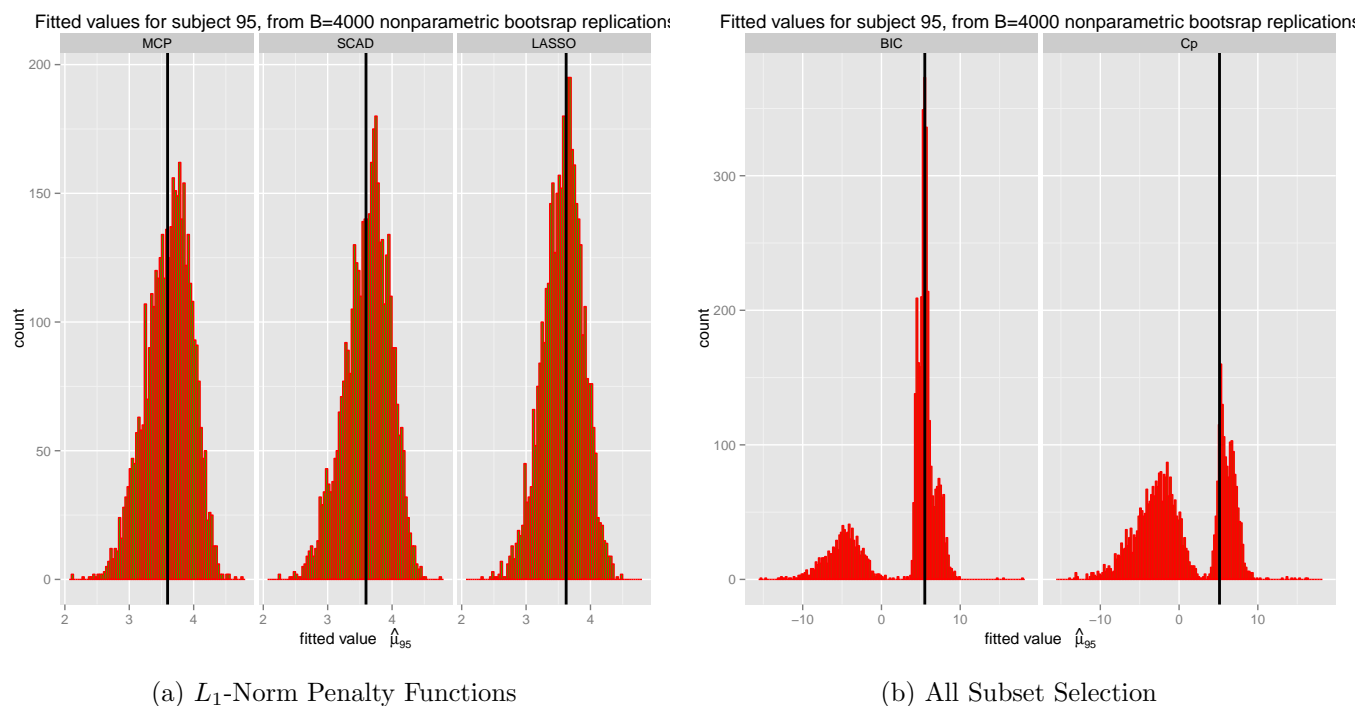
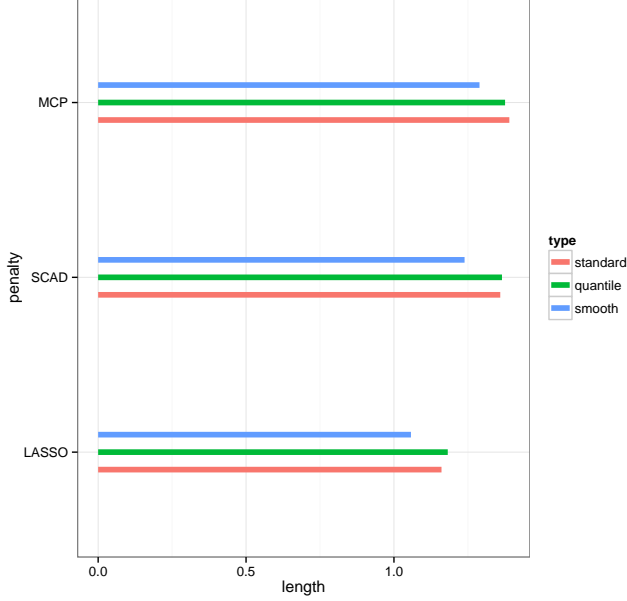
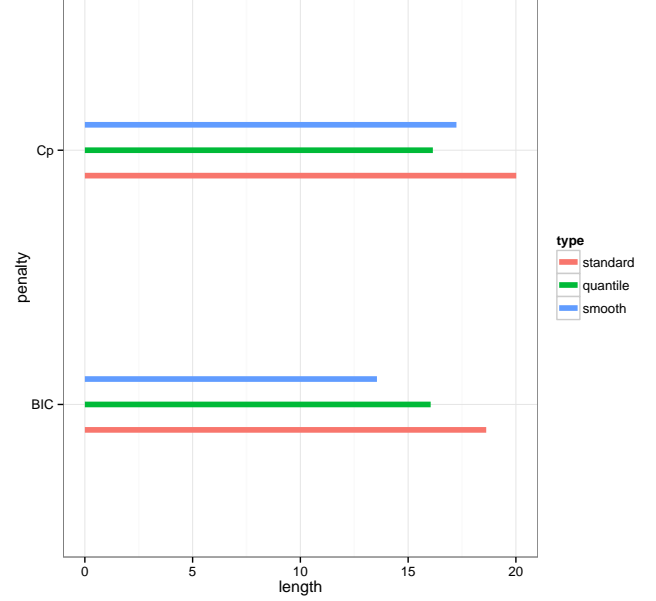


Figure 6: Histogram of fitted values for subject 95 based on 4000 Bootstrap samples for the `prostate` data set

Figure 6b confirms the work of Breiman (1996a), i.e., the all subset selection procedure is very sensitive to changes in the data. The LASSO, SCAD and MCP all produce much more stable estimates, with the distributions of the fitted values looking normal and centred around their mean (Figure 6a).

Figure 7 compares the lengths of the three confidence intervals types for each procedure. We see that the new smoothed confidence intervals, based on the proposed smoothed standard deviation \hat{sd}_B , outperforms (is shorter) the standard and quantile confidence intervals across all model selection procedures, with the LASSO providing the smallest intervals. Table 5 provides the numerical values of our analysis. Again, we see that the L_1 -Norm penalties all perform similarly, with good coverage probabilities. Although the coverage probabilities are reasonable for the all subset methods, the length of the interval is much wider.

95% Confidence Intervals for fitted value of Subject 95 based
B=4000 nonparametric bootstrap samples for MCP, SCAD and LASSO penalties(a) L_1 -Norm Penalty FunctionsLength of 95% Confidence Intervals for fitted value of Subject 95 based
on B=4000 nonparametric bootstrap samples for Cp and BIC

(b) All Subset Selection

Figure 7: Length of confidence intervals for $\hat{\mu}_{95}$ based on 4000 Bootstrap samples for the **prostate** data set

Table 5: Prostate data, B=4000, Observation 95

model	type	fitted value	sd	length	coverage
LASSO	standard	3.62	0.31	1.21	0.94
	quantile			1.20	0.95
	smooth	3.57	0.29	1.14	0.93
SCAD	standard	3.60	0.35	1.37	0.95
	quantile			1.33	0.95
	smooth	3.62	0.33	1.28	0.93
MCP	standard	3.60	0.35	1.38	0.96
	quantile			1.35	0.95
	smooth	3.61	0.33	1.29	0.94
BIC	standard	5.50	4.75	18.62	0.84
	quantile			16.05	0.95
	smooth	3.22	3.46	13.55	0.83
Cp	standard	5.13	5.11	20.02	0.86
	quantile			16.15	0.95
	smooth	0.64	4.40	17.24	0.97

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