

Computational Methodology for Image-based Outcomes Prediction in Liver Cancer

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Challenges for ML in Radiology

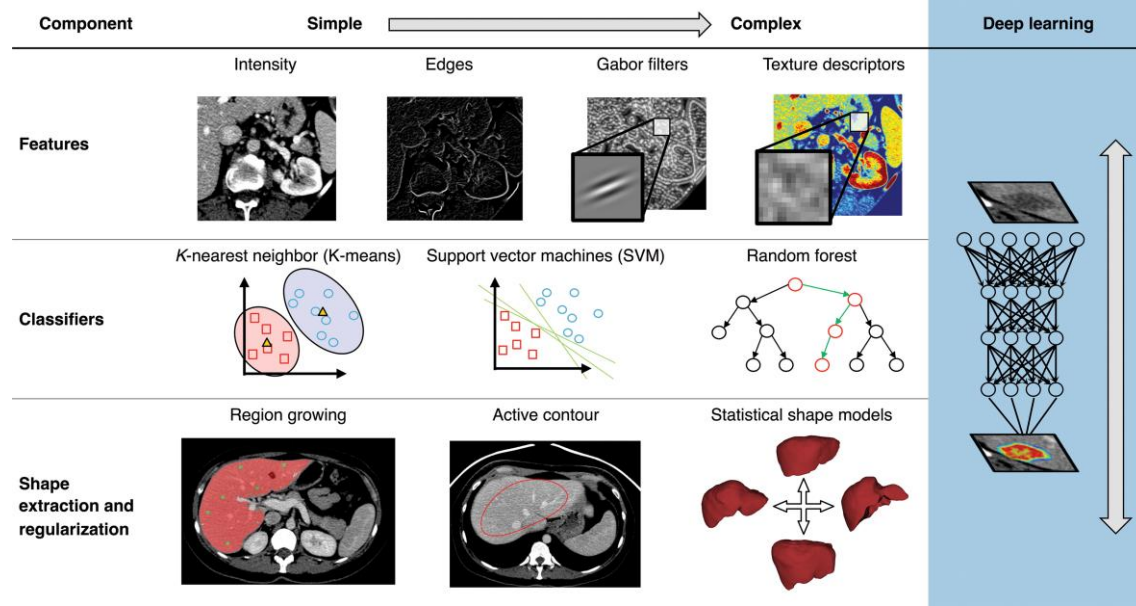
Outcomes Prediction after Therapy using Imaging

Radiomics:

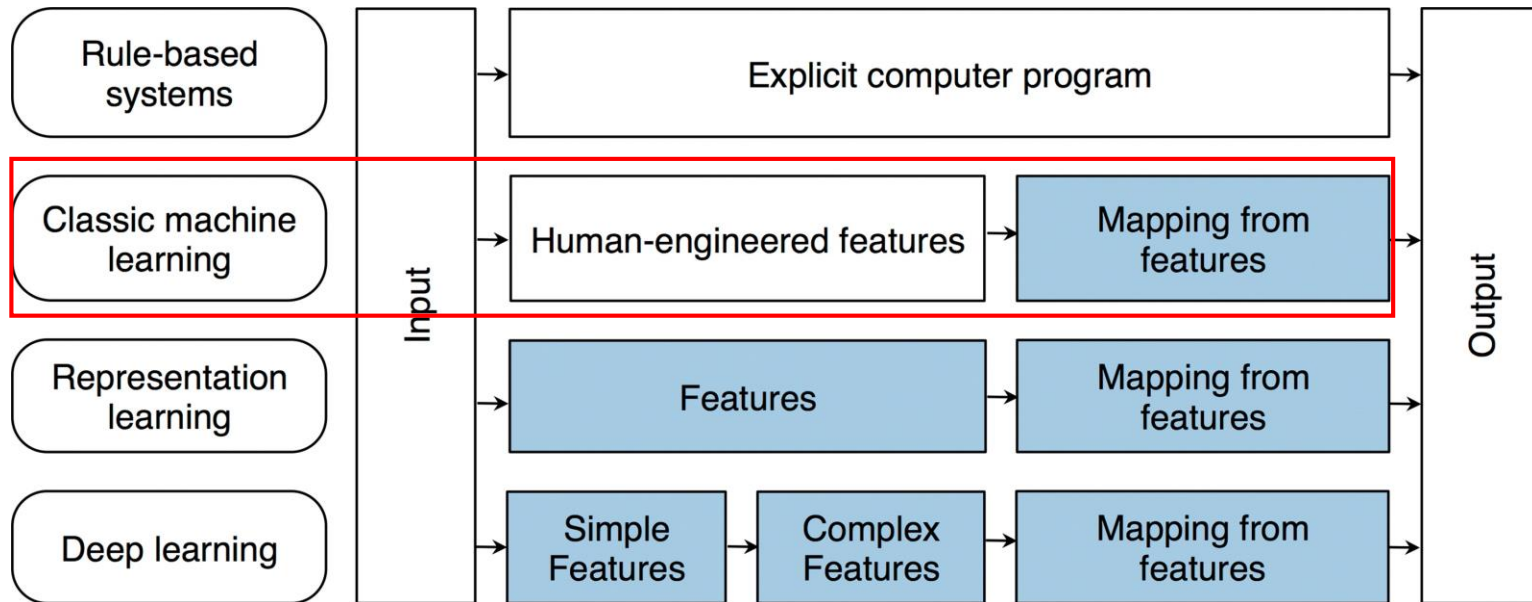
- Require segmentation
- Often done on one 2D slice
- Time consuming for a volume

Deep learning:

- Requires huge training set
- Sensitive to image orientation
- 3D can be really computationally heavy

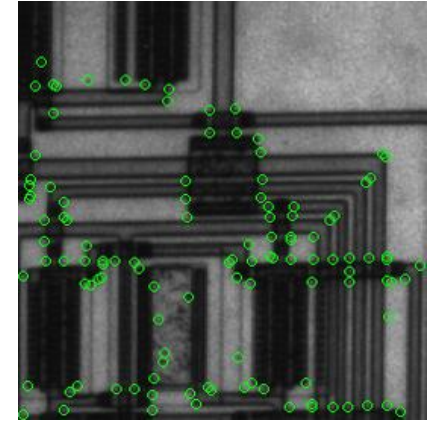
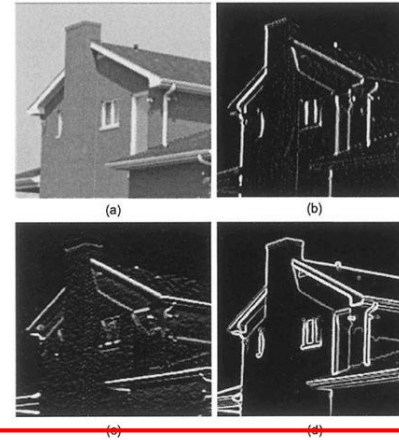


Feature-based Classification



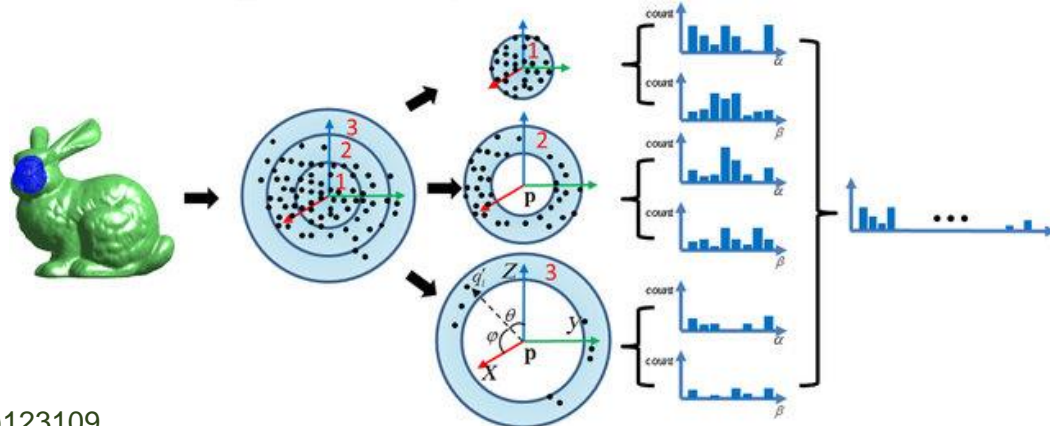
Local Image Descriptors

- Same size than the original image
- Enhance textures in images
- Can facilitate automated detection of features for image classification



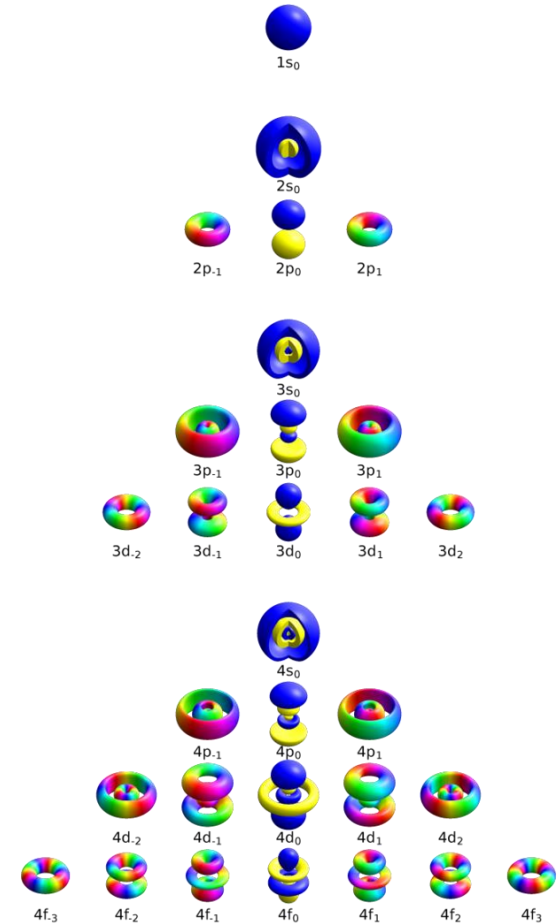
Non-Local Image Descriptors

- Compact description of a volume
- Allows faster image classification with minimal loss of information
- No need for explicit segmentation



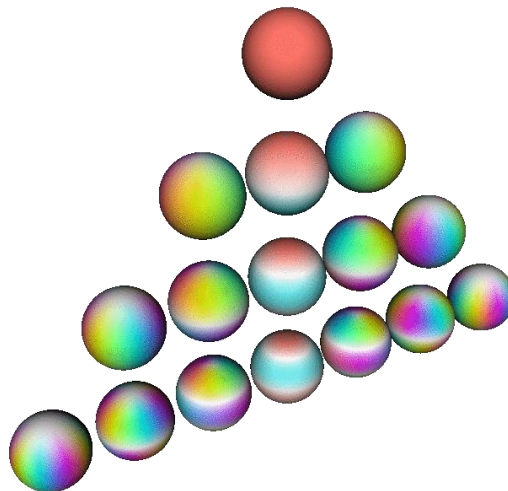
Schrödinger Equation Solutions

- Coefficients of the angular basis:
Spherical Harmonics
- Coefficients of the radial basis:
Associated Laguerre Polynomials



Laplace's Spherical Harmonics Functions $Y_l^m(\theta, \varphi)$

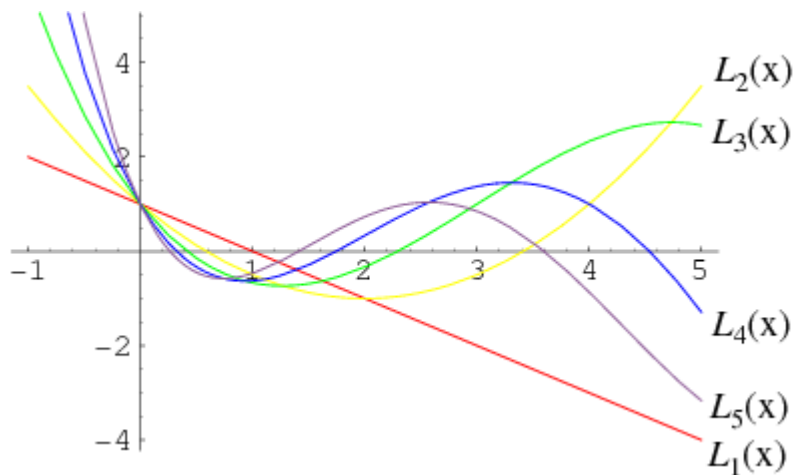
- Angular part of the complex electronic cloud structure of atoms
- Three quantum numbers n, l, m
- Combined with a radial basis, they can fully describe complex shapes



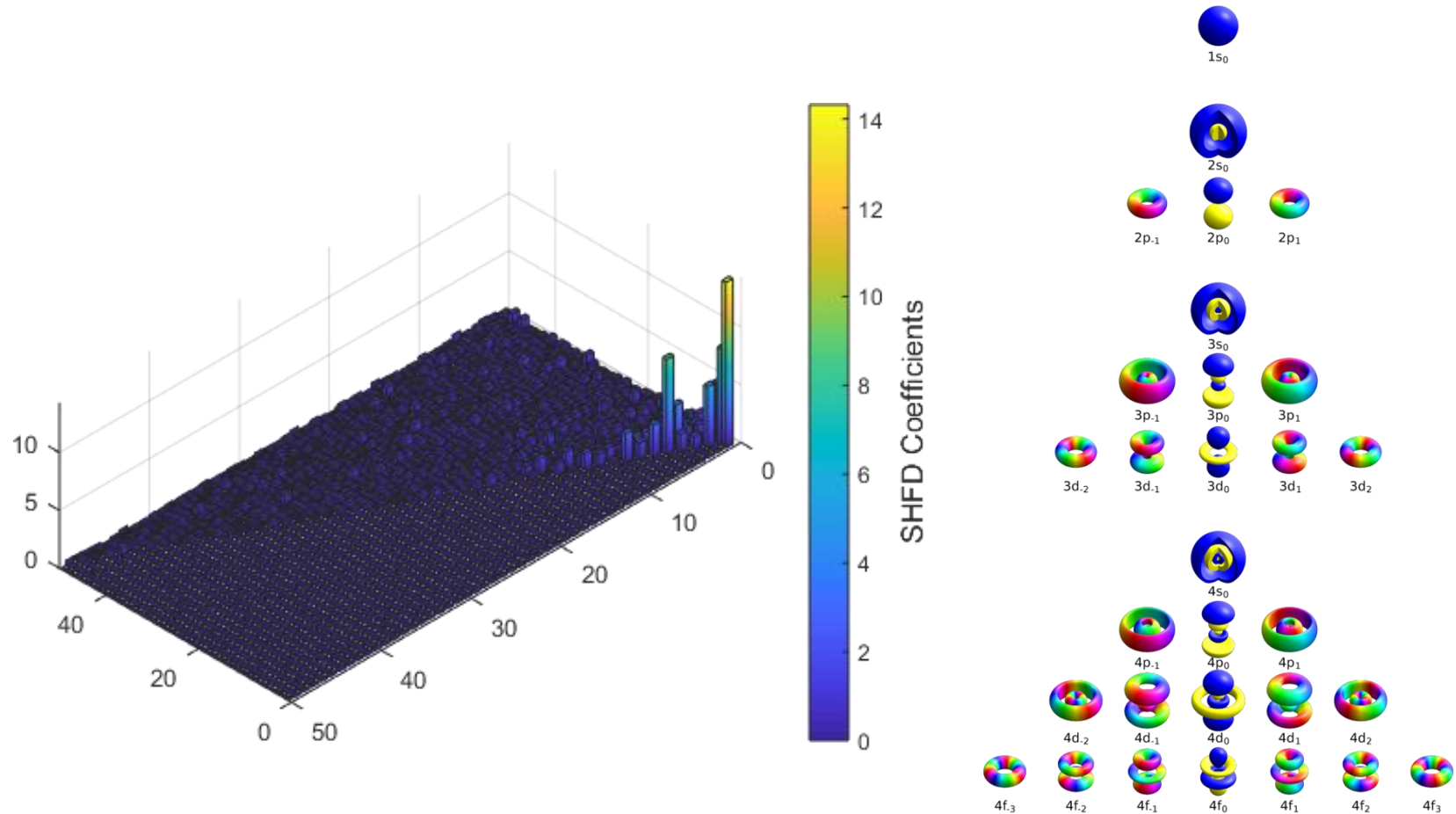
From H. Skibbe *et al*

- Damped Fourier expansion as radial basis $w_k^R(r)$
- Computationally efficient choice for replacing associated Laguerre polynomials (really expensive)

$$w_k^R(r) = \frac{1}{\sqrt{R}} e^{2\pi i k r \frac{1}{R}}$$

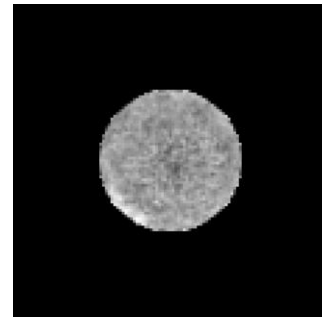
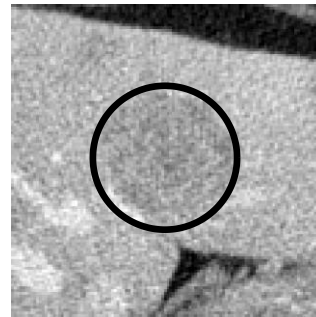
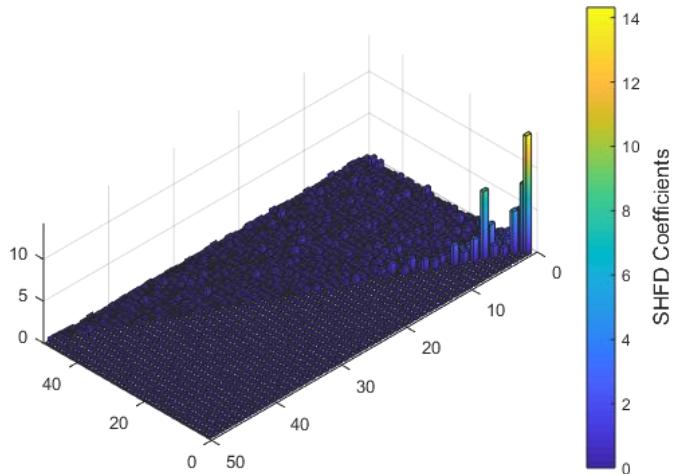


Spherical Harmonics Fourier Descriptors McGill



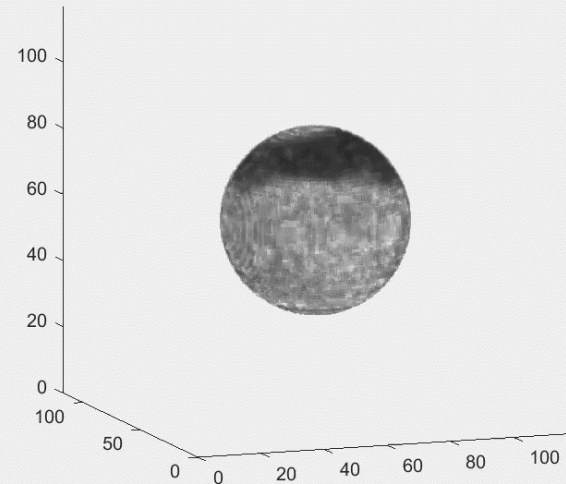
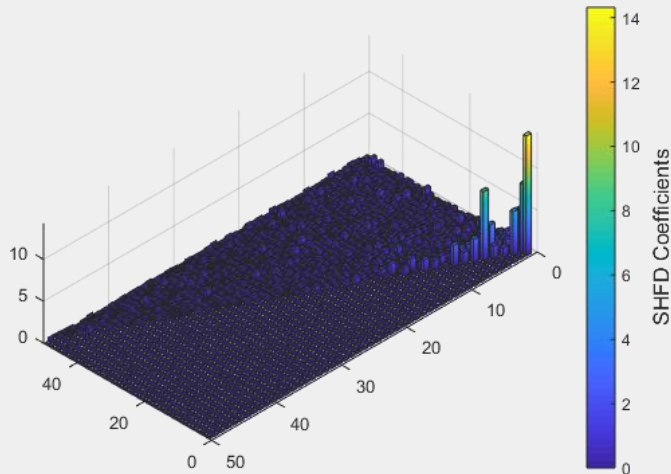
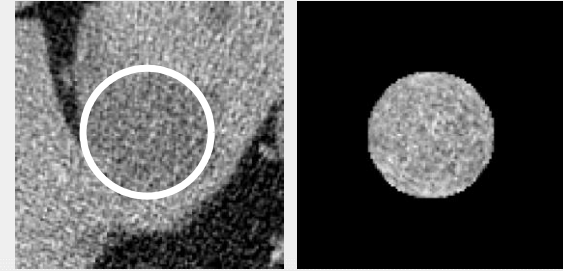
Hypothesis

- Underlying tumour information can be encoded in spherical harmonics Fourier decomposition coefficients
- No need to segment: just select the center of the tumour and expand to a radius that will englobe the whole tumour



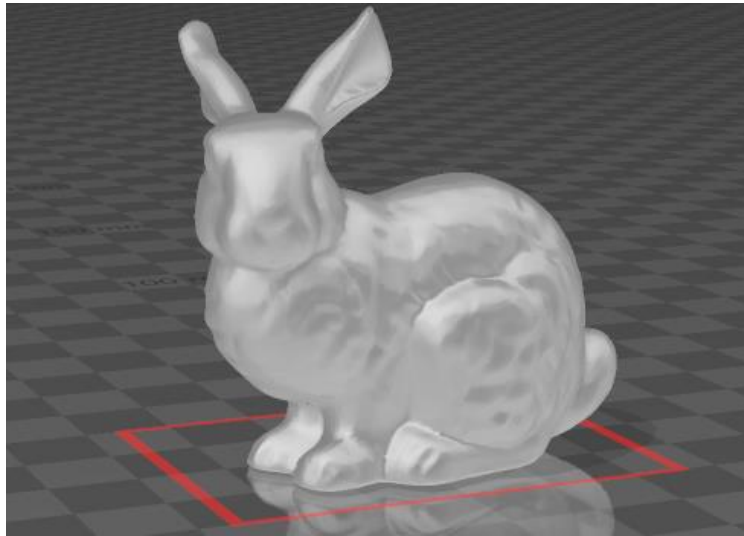
Example

- $\approx 100,000$ voxels in segmented tumour volume
- Could be represented by 50×50 matrix, 2,500
→ evaluated in seconds
- Computationally efficient for classification
(40 times smaller!)



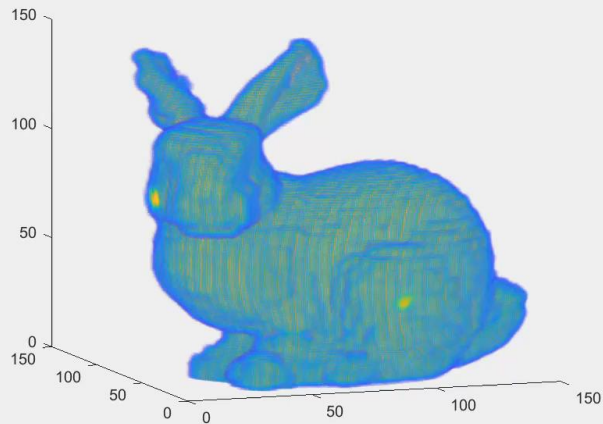
Rationale

- Already been shown that using only these coefficients with no other prior information on the volume, we can reconstruct complex structures with minimal loss of information

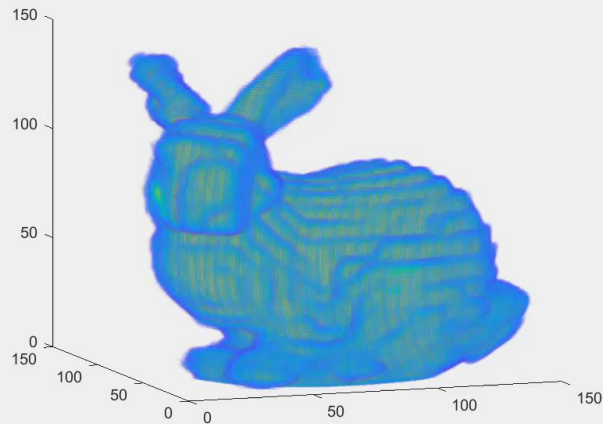


Population

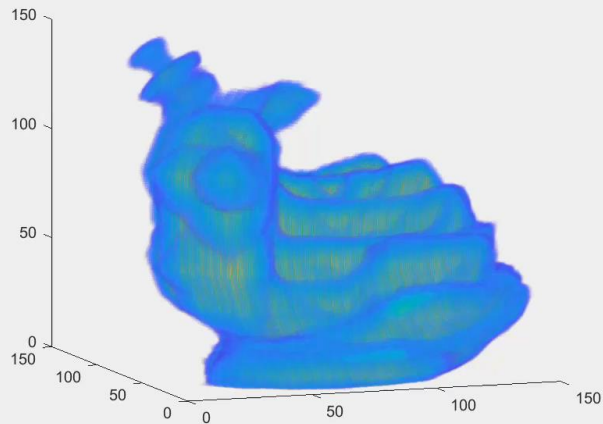
$L_{max} = 50$



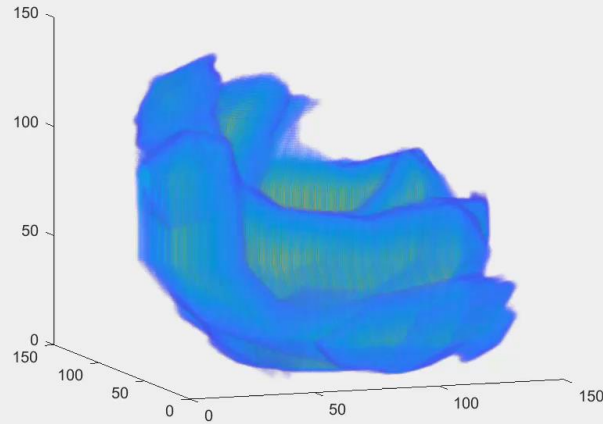
$L_{max} = 25$



$L_{max} = 10$

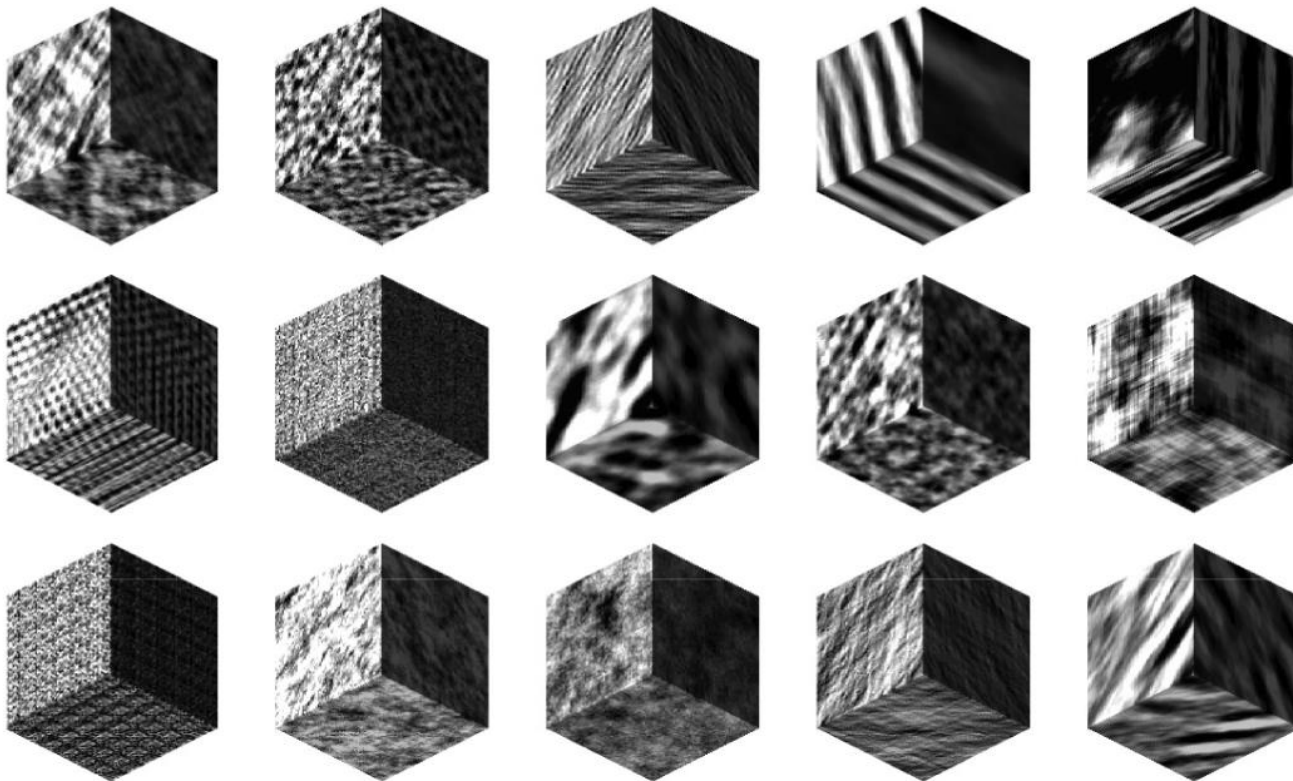


$L_{max} = 5$



RFAI Benchmark Dataset

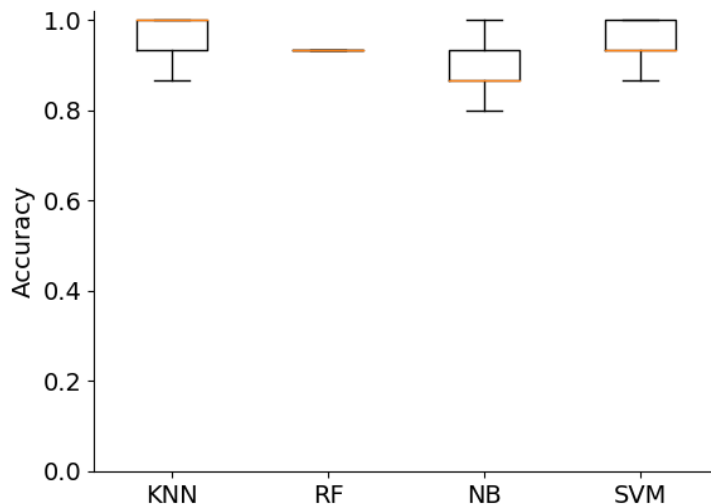
Fourier Texture Dataset



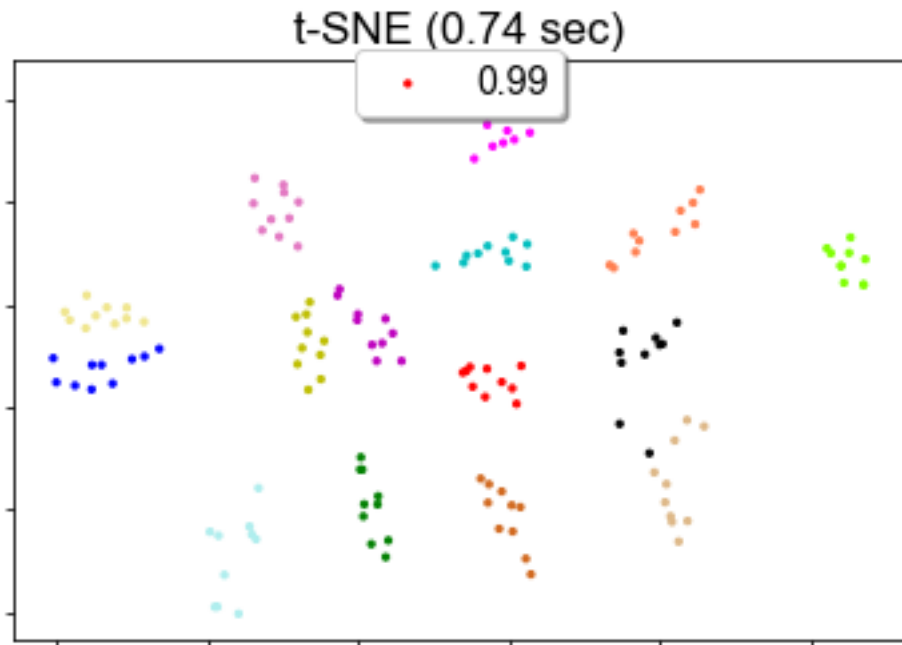
- Classification Performance Evaluated with 4 Machine Learning-based Classifiers:
 - *K-nearest Neighbors (KNN)*
 - *Random Forest (RF)*
 - *Gaussian Naive Bayes (NB)*
 - *Support Vector Machines (SVM)*
- Cross-validated Metrics with 95% Confidence Interval:
 - *Accuracy*

Preliminary Results (Fourier Texture)

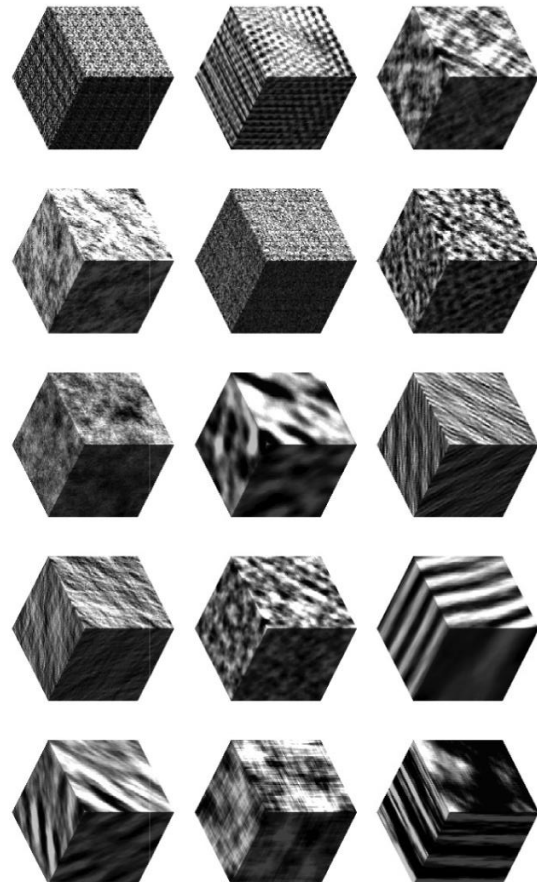
| Data | ML-Classifier | Accuracy |
|----------------------------|---------------|-----------------|
| Fourier ($L_{max} = 50$) | KNN | 0.97 ± 0.05 |
| | RF | 0.93 ± 0.04 |
| | NB | 0.86 ± 0.08 |
| | SVM | 0.94 ± 0.05 |



T-dist. Stochastic Neighbor Embedding

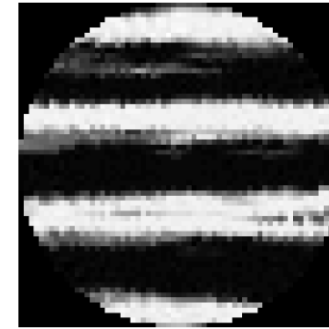
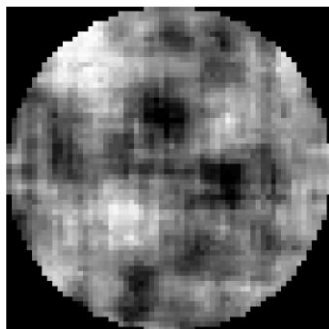
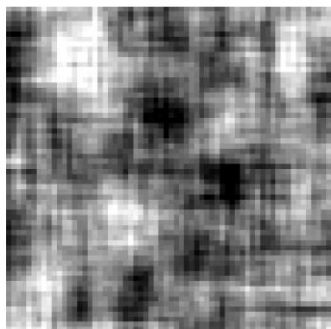
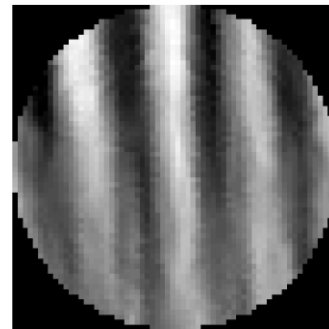
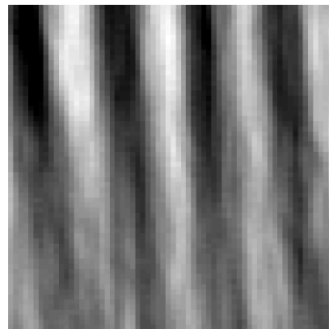
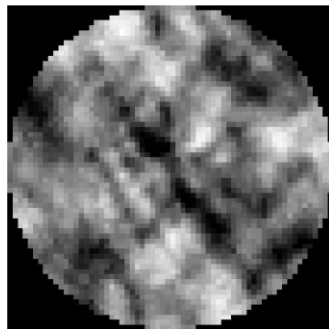
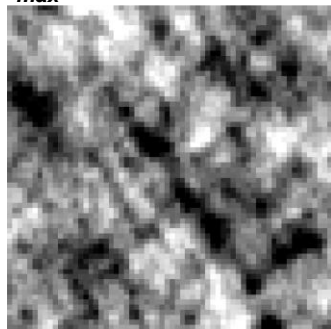


Accuracy of 0.99 with Linear Discriminant Analysis

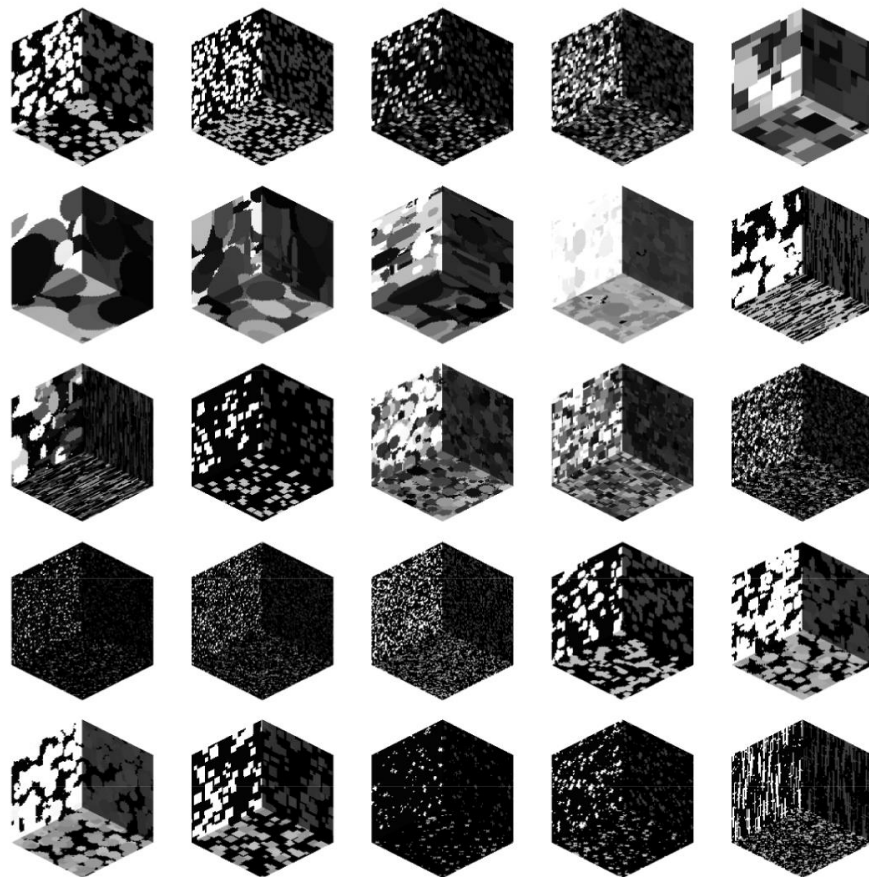


Fourier Texture Reconstructions

$L_{max} = 50$



Geometric Texture Dataset

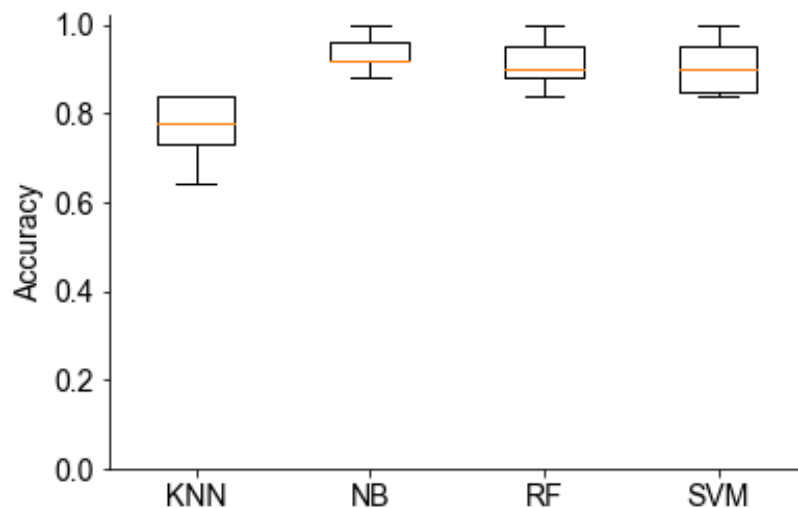


Preliminary Results (Geometric Texture)

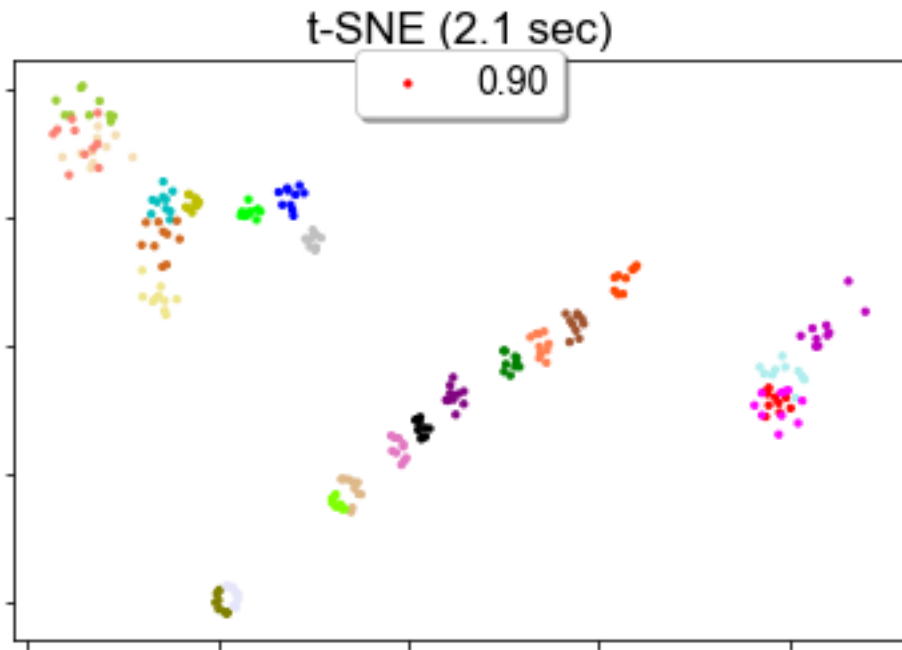


McGill

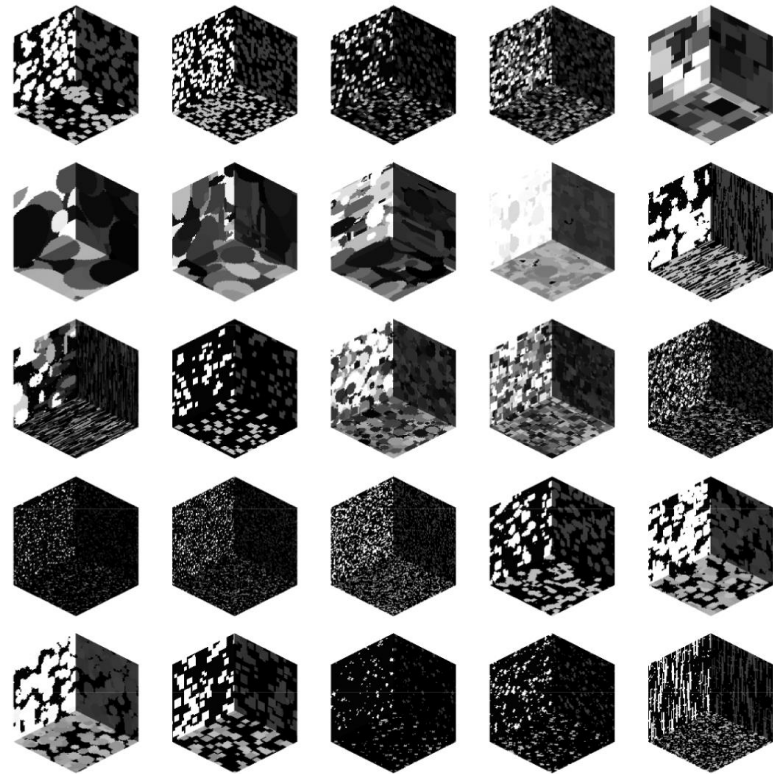
| Data | ML-Classifier | Accuracy |
|------------------------------|---------------|-----------------|
| Geometric ($L_{max} = 50$) | KNN | 0.77 ± 0.06 |
| | RF | 0.94 ± 0.04 |
| | NB | 0.92 ± 0.05 |
| | SVM | 0.91 ± 0.05 |



T-dist. Stochastic Neighbor Embedding

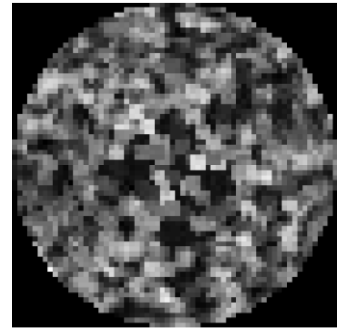
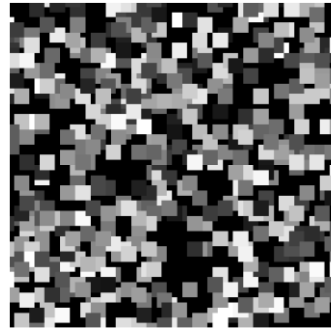
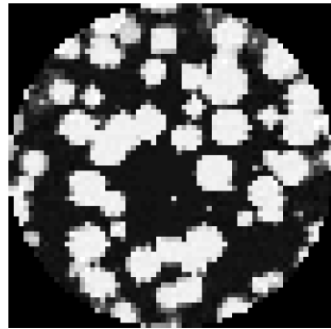
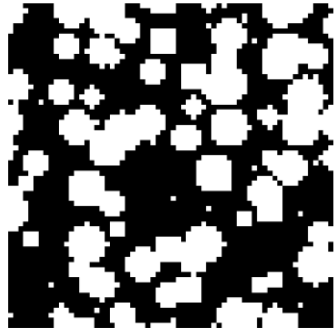


Accuracy of 0.90 with Linear Discriminant Analysis

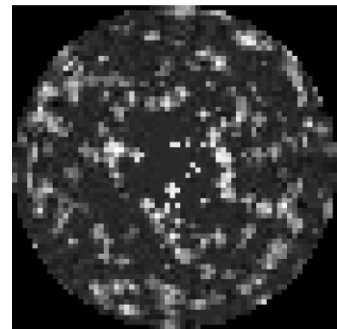
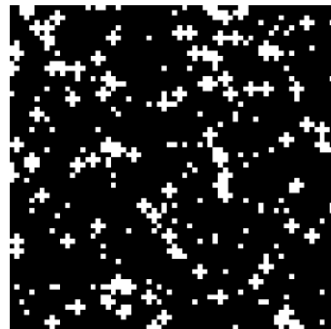
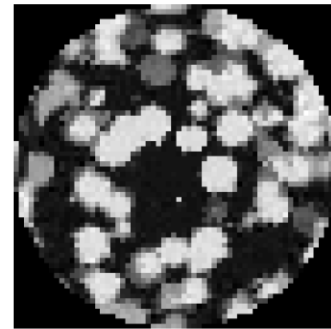
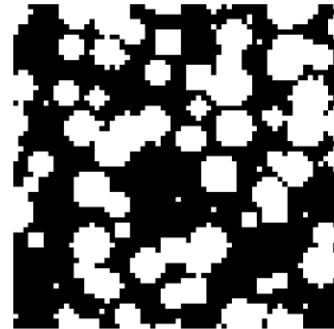


Geometric Texture Reconstructions

$L_{max} = 150$



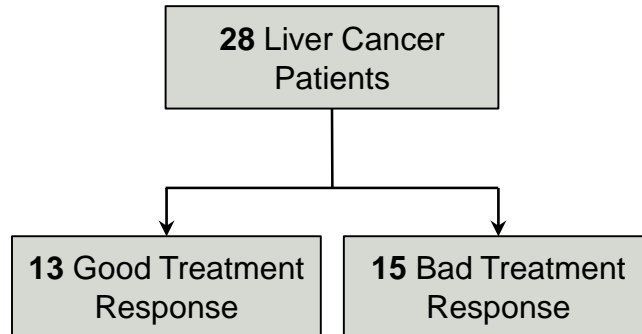
$L_{max} = 50$



Liver Tumours Dataset

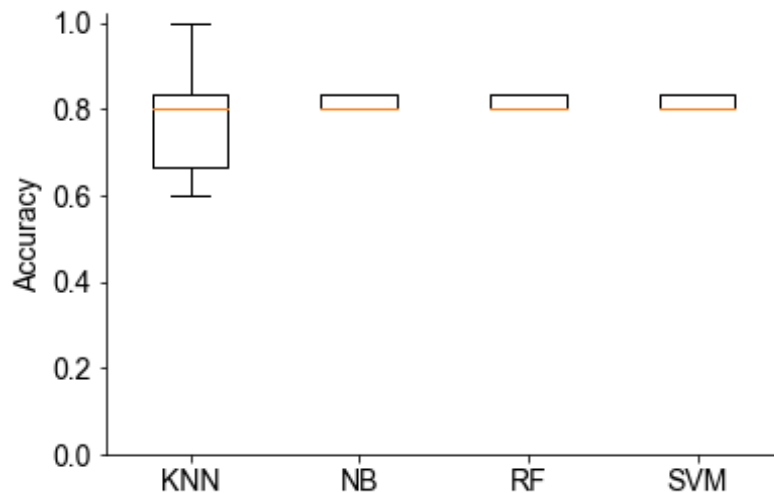
Liver Tumours

- Baseline CT scan prior to treatment
- Follow-up CT scan 2 months after treatment (outcomes 6 months after)
- SHFD evaluated on baseline data to see if it was enough to predict treatment response



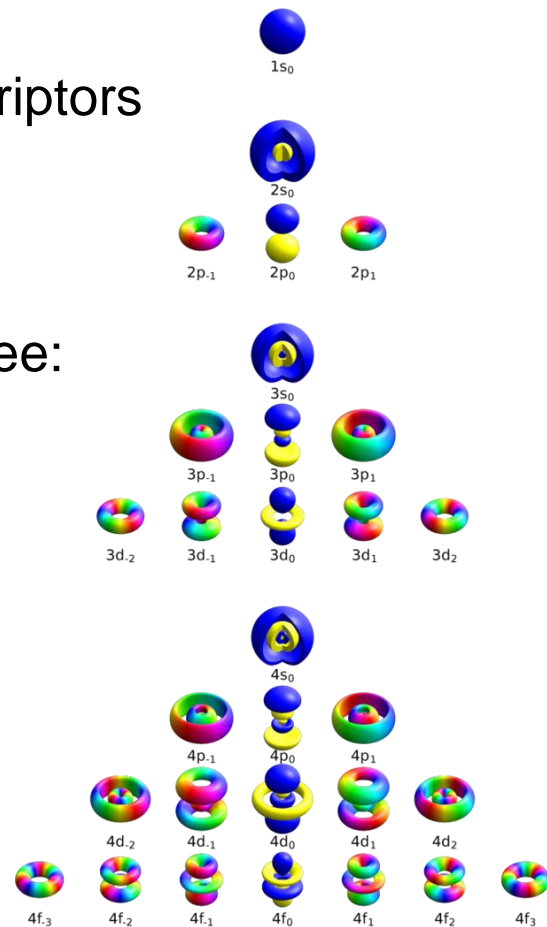
Preliminary Results (Fourier Texture)

| Data | ML-Classifier | Accuracy |
|--|---------------|-----------------|
| Liver with STA Toolbox ($L_{max} = 50$) | KNN | 0.78 ± 0.10 |
| | RF | 0.83 ± 0.06 |
| | NB | 0.83 ± 0.06 |
| | SVM | 0.83 ± 0.06 |



Conclusions

- Provided compact rotation-invariant image descriptors
 - fast classification
 - no need for segmentation
- Need to apply this tool to more tumour data to see:
 - How it performs in a larger dataset?
 - Can we get similar or better results than state-of-the-art radiomics (on segmented medical images)?



Acknowledgments

- Ozan Ciga
- Nikesh Muthukrishnan
- Peter Savadjiev
- Caroline Reinhold
- Reza Forgani

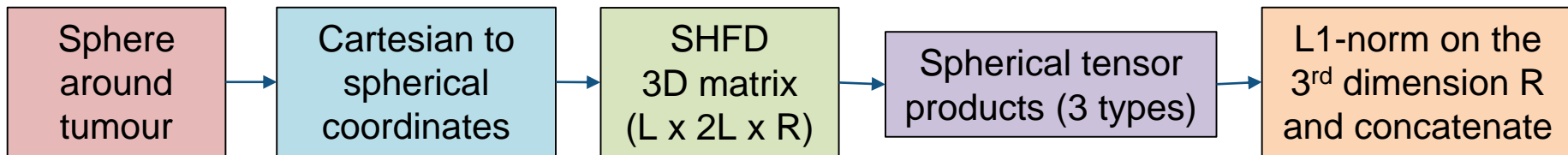


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Appendix

Spherical Harmonics Fourier Descriptors McGill



$$c_{\ell 0}^{Rk} = |\text{Real}((\mathbf{a}_{\ell}^{Rk} \bullet_0 \mathbf{a}_{\ell}^{Rk})^{\frac{1}{2}})|$$

$$c_{\ell 1}^{Rk} = |\text{Imag}((\mathbf{a}_{\ell}^{Rk} \bullet_0 \mathbf{a}_{\ell}^{Rk})^{\frac{1}{2}})|$$

$$\begin{aligned} c_{\ell 2}^{Rk} &= (\mathbf{a}_{\ell}^{Rk} \bullet_0 \mathbf{a}_{\ell}^{R,-k})^{\frac{1}{2}} = \langle \mathbf{a}_{\ell}^{Rk}, \mathbf{a}_{\ell}^{Rk} \rangle^{\frac{1}{2}} \\ &= \|\mathbf{a}_{\ell}^{Rk}\| \end{aligned}$$

Spherical Harmonics Fourier Descriptors (SHFD): \mathbf{F}_{KL}

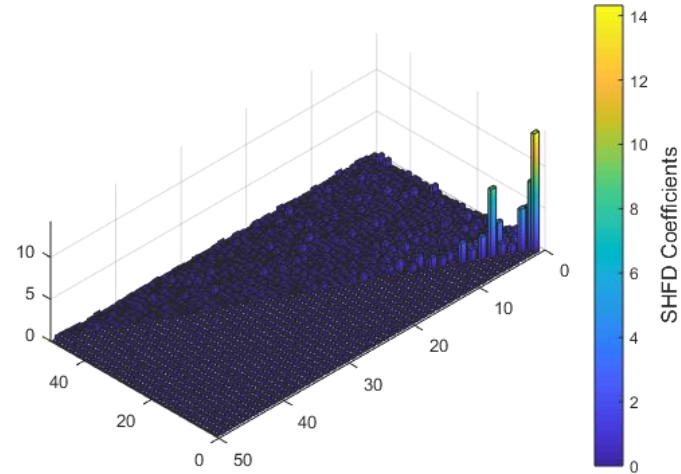
- *phase information* of the coefficient is preserved in c_{l0}^{Rk} and c_{l1}^{Rk}
- c_{l2}^{Rk} represents the *energy bands* for l and k

$$c_{l0}^{Rk} = |\text{Real}((\mathbf{a}_\ell^{Rk} \bullet_0 \mathbf{a}_\ell^{Rk})^{\frac{1}{2}})|$$

$$c_{l1}^{Rk} = |\text{Imag}((\mathbf{a}_\ell^{Rk} \bullet_0 \mathbf{a}_\ell^{Rk})^{\frac{1}{2}})|$$

$$\begin{aligned} c_{l2}^{Rk} &= (\mathbf{a}_\ell^{Rk} \bullet_0 \mathbf{a}_\ell^{R,-k})^{\frac{1}{2}} = \langle \mathbf{a}_\ell^{Rk}, \mathbf{a}_\ell^{Rk} \rangle^{\frac{1}{2}} \\ &= \|\mathbf{a}_\ell^{Rk}\| \end{aligned}$$

$$\mathbf{F}_{KL} = \{\mathbf{c}_0^{0T}, \dots, \mathbf{c}_\ell^{RkT}, \dots, \mathbf{c}_L^{RK T}\}$$



Laplace's Spherical Harmonics Functions $Y_l^m(\theta, \varphi)$

- Elevation angle θ
- Azimuthal angle φ
- Associate Legendre polynomials $P_l^m(\cos \theta)$
- Azimuthal quantum number l (from atomic physics)
- Magnetic quantum number m

$$Y_l^m(\theta, \phi) = \sqrt{\frac{2l+1(l-m)!}{4\pi(l+m)!}} P_l^m(\cos \theta) e^{im\phi} \quad P_l^m(x) = \frac{(-1)^m}{(2^l l!)} (1+x^2)^{\frac{m}{2}} \frac{(d^{l+m})}{(dx^{l+m})} (x^2-1)^l$$

H. Skibbe *et al*

- SPHARM as angular basis $f(\theta, \varphi)$ with coefficients b_{lm}
- Damped Fourier expansion as radial basis $w_k^R(r)$
 - Orthonormal basis for functions on interval $[0, R]$
- Expansion of function $f(\mathbf{r})$ defined on a sphere
 - with the expansion coefficients a_l^{Rk} and the orthonormal basis functions $E_{lm}^{Rk}(\mathbf{r})$

$$f(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} b_{\ell m} Y_m^{\ell}(\theta, \phi)$$

$$= \sum_{\ell=0}^{\infty} (\mathbf{b}_{\ell})^T \mathbf{Y}^{\ell}(\theta, \phi)$$

$$w_k^R(r) = \frac{1}{\sqrt{R}} e^{2\pi i k r \frac{1}{R}}$$

$$f(\mathbf{r}) = \sum_{k=-\infty}^{\infty} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m}^{Rk} E_{\ell m}^{Rk}(\mathbf{r})$$

$$= \sum_{k=-\infty}^{\infty} \sum_{\ell=0}^{\infty} (\mathbf{a}_{\ell}^{Rk})^T \mathbf{E}_{\ell}^{Rk}(\mathbf{r})$$

$$E_{\ell m}^{Rk}(\mathbf{r}) = Y_m^{\ell}(\theta, \phi) w_k^R(r) \frac{1}{r}$$

H. Skibbe *et al*

- Expansion separately performed
 - angular part $Y_m^l(\theta, \varphi)$ using a spherical harmonic transformation
 - radial part $w_k^R(r) \frac{1}{r}$ using an ordinary 1D Fourier transformation
- Expansion coefficients a_l^{Rk}
 - weighting $f(r)$ by distance from centre r

$$\begin{aligned}
 \langle f, E_{\ell m}^{Rk} \rangle &= \dots \\
 &= \int_0^R w_{-k}^R(r) b_{\ell m}(r) dr \\
 &= a_{\ell m}^{Rk}
 \end{aligned}$$

H. Skibbe *et al*

- Expansion separately performed
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$$f'(r, \theta, \phi) = r f(r, \theta, \phi)$$