Week 1: Introduction

Energy and power

$$E = mc_0^2 = qV = k_B T$$

$$E_{ph} = h\nu = \frac{hc}{\lambda}$$

$$E_{ph}(eV) = \frac{1240(eV \cdot nm)}{\lambda(nm)}$$

$$E = \int P(t)dt$$

$$P = VI = I^2 R$$

Planck's law

$$L_e^{BB}(T,\lambda) = \frac{2hc_0^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc_0}{\lambda k_B T}\right) - 1}$$

$$I_e^{BB}(T,\lambda) = L_e^{BB}(T,\lambda)\Omega_{sun}$$

$$\Omega_{sun} = \pi\left(\frac{R_{sun}}{AU - R_{Earth}}\right)^2$$

Air mass

$$AM = \frac{1}{\cos\theta}$$

Week 2: Semiconductor basics

Band gap

$$E_G = E_C - E_V$$

Fermi-Dirac function

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)}$$

Density of states in conduction and valence bands:

$$g_C(E) = \left(\frac{4\sqrt{2}\,\pi m_n^*}{h^3}\right)^{3/2} (E - E_C)^{1/2}$$

$$g_V(E) = \left(\frac{4\sqrt{2}\,\pi m_p^*}{h^3}\right)^{3/2} (E - E_V)^{1/2}$$

Charge carrier density in conduction and valence band

$$n_0 = \int_{E_C}^{E_{\text{top}}} g_C(E) f(E) dE$$

$$p_0 = \int_{E_{\text{bottom}}}^{E_V} g_V(E)\left[1 - f(E)\right] dE$$

Charge carrier density in conduction and valence band applying Boltzmann approximation

$$n = N_C \exp\left(\frac{E_F - E_C}{k_B T}\right) \quad \textbf{for } E_C - E_F \geq 3k_B T$$

$$p = N_V \exp\left(\frac{E_V - E_F}{k_B T}\right) \quad \textbf{for } E_F - E_V \geq 3k_B T$$

Effective densities of conduction and valence band states

$$N_C = 2 \cdot \left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{\frac{3}{2}}$$

$$N_V = 2 \cdot \left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{\frac{3}{2}}$$

Carrier concentration intrinsic semiconductor in equilibrium conditions

$$np = n_i^2 = N_C N_V \exp\left(\frac{E_V - E_C}{k_B T}\right) = N_C N_V \exp\left(\frac{-E_G}{k_B T}\right),$$

Carrier concentration in doped semiconductors in equilibrium conditions

N-type material

$$n = p + N_D \approx N_D$$

$$p = \frac{n_i^2}{n} \approx \frac{n_i^2}{N_D} << n$$

P-type material

$$p = n + N_A \approx N_A$$

$$n = \frac{n_i^2}{p} \approx \frac{n_i^2}{N_A} << p_0$$

Carrier concentration in semiconductors under illumination:

$$np \neq n_i^2$$

$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{k_B T}\right)$$

$$n = N_C \exp\left(\frac{E_{Fn} - E_C}{k_B T}\right)$$

$$p = N_V \exp\left(\frac{E_V - E_{Fp}}{k_B T}\right)$$

Carrier transport mechanisms

Charge transport: drift

$$\mathbf{v}_{dn} = -\mu_n \boldsymbol{\xi}$$

$$\mathbf{v}_{dp} = \mu_p \boldsymbol{\xi}$$

$$\mathbf{J}_{n,drift} = -q\, n\, \mathbf{v}_{dn} = q\, n\, \mu_n \boldsymbol{\xi}$$

$$\mathbf{J}_{p,drift} = q\, p\, \mathbf{v}_{dp} = q\, p\, \mu_p \boldsymbol{\xi}$$

$$\mathbf{J}_{drift} = q\,(p\,\mu_p + n\,\mu_n)\boldsymbol{\xi}$$

Charge transport: diffusion

$$\mathbf{J}_{n,diff} = q\, D_n \nabla n$$

$$\mathbf{J}_{p,diff} = -q\, D_p \nabla p$$

$$\mathbf{J}_{diff} = q\left(D_n \nabla n - D_p \nabla p\right)$$

Total current density:

$$\mathbf{J} = \mathbf{J}_{drift} + \mathbf{J}_{diff} = q\,(p\,\mu_p + n\,\mu_n)\boldsymbol{\xi} + q\left(D_n \nabla n - D_p \nabla p\right)$$

Einstein relations:

$$\frac{D_n}{\mu_n} = \frac{k_B T}{q}$$

$$\frac{D_p}{\mu_p} = \frac{k_B T}{q}$$

Week 3: Generation and recombination

Electrical conductivity

$$\sigma = q\mu_n n + q\mu_p p$$

Thermal recombination/generation rate

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} = -\frac{\Delta n}{\tau_n}$$ for electrons in a p-type material

$$\left.\frac{dp}{dt}\right|_{\substack{\text{thermal}\\ \text{R-G}}} = -\frac{\Delta p}{\tau_p}$$ for holes in an n-type material

Recombination under external generation

$$R_n^{'} = \alpha_r\ p\,n \approx \alpha_r\ p_0\ \delta n$$ for electrons in a p-type material

$$R_p^{'} = \alpha_r\ n\,p \approx \alpha_r\ n_0\ \delta p$$ for holes in an n-type material

Ambipolar transport equation

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \mathbf{E}\frac{\partial(\delta n)}{\partial x} + G - \frac{\delta n}{\tau_{n0}} = \frac{\partial(\delta n)}{\partial t}$$

$$D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \mathbf{E}\frac{\partial(\delta p)}{\partial x} + G - \frac{\delta p}{\tau_{p0}} = \frac{\partial(\delta p)}{\partial t}$$

Ambipolar transport equation 3D

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla\cdot\mathbf{J}_n - R_n + G_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla\cdot\mathbf{J}_p - R_p + G_p$$

Week 4: The P-N junction

Charge density in depletion region

$$\rho(x) = q N_D$$ for $-l_n \le x \le 0$

$$\rho(x) = -q N_A$$ for $0 \le x \le l_p$

$$N_A l_p = N_D l_n$$

Poisson equation

$$\frac{d^2\psi}{dx^2} = -\frac{d\,\xi}{dx} = -\frac{\rho}{\varepsilon_r\,\varepsilon_0}$$

$$\xi = \frac{1}{\varepsilon_r\,\varepsilon_0}\int \rho\, dx$$

$$\xi(-l_n) = \xi(l_p) = 0$$,

Built-in voltage

$$V_{bi} = E_G - E_{Fn} - E_{Fp} = \frac{k_B T}{q}\ln\left(\frac{N_A\,N_D}{n_i^2}\right)$$

$$V_{bi} = \frac{q}{2\varepsilon_r\,\varepsilon_0}\left(N_D\,l_n^2 + N_A\,l_p^2\right)$$

Depletion region width of p-n junction

$$W = l_n + l_p = \sqrt{\frac{2\varepsilon_r\,\varepsilon_0}{q}\psi_0\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

Current-voltage relationship

$$J_{rec}(V_a) = J_{rec}(V_a = 0)\exp\left(\frac{qV_a}{k_B T}\right)$$

$$J(V_a) = J_{rec}(V_a) - J_{gen}(V_a) - J_{ph} = J_0\left[\exp\left(\frac{qV_a}{k_B T}\right) - 1\right] - J_{ph}$$

$$J_0 = q\,n_i^2\left(\frac{D_n}{L_n\,N_A} + \frac{D_p}{L_p\,N_D}\right)$$

$$J_0 = \left[\frac{qD_n n_{p0}}{L_n} + \frac{qD_p p_{n0}}{L_p}\right]$$

$$J_{ph} = q\cdot G\left(L_n + W + L_p\right)$$

External parameters of solar cell

$$V_{oc} = \frac{k_B T}{q}\ln\left(\frac{J_{ph}}{J_0} + 1\right)$$

$FF = \dfrac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 1}$, where $v_{oc} = \dfrac{qV_{oc}}{k_B T}$

$$\eta = \frac{P_{\max}}{P_{in}} = \frac{J_{mp}\,V_{mp}}{P_{in}} = \frac{J_{sc}\,V_{oc}\,FF}{P_{in}}$$

Week 5: Advanced Concepts in Semiconductors

Schottky barrier (for an n-type semiconductor)

$$\phi_{Bn} = \phi_m - \chi$$

$$V_{bi} = \phi_{Bn} - \phi_n$$

Depletion region width Schottky barrier

$$W = \sqrt{\frac{2\varepsilon_s\left(V_{bi} + V_R\right)}{q}\left(\frac{1}{N_d} + \frac{1}{N_a}\right)}$$

Thermionic emission

$$J(V) = A^* T^2 \exp\left[-\frac{q\varphi_{Bn}}{k_B T}\right]$$

Tunneling

$$J_T \propto \exp\left[-\frac{q\varphi_{Bn}}{E_{00}}\right] \quad \text{, with} \quad E_{00} = \frac{q\hbar}{2}\sqrt{\frac{N_d}{\varepsilon m_n}}$$

Specific contact resistance

$$R_c = \frac{\frac{k_B T}{q}\exp\left(\frac{+q\phi_{Bn}}{k_B T}\right)}{A^* T^2}$$ Thermionic emission

$$R_c \propto \exp\left(\frac{+2\sqrt{\varepsilon_s m_n^*}}{\hbar} \cdot \frac{\phi_{Bn}}{\sqrt{N_d}}\right)$$ Tunneling

Electron affinity rule

$$\Delta E_C = q(\chi_n - \chi_p)$$

$$\Delta E_C + \Delta E_V = E_{Gp} - E_{Gn} = \Delta E_G$$

Heterojunction built-in voltage

$$qV_{bi} = q(\phi_{sp} - \phi_{sn})$$

$$qV_{bi} = -\Delta E_C + \Delta E_G + k_B T \ln\left(\frac{N_{Vn}}{p_{n0}}\right) - k_B T \ln\left(\frac{N_{Vp}}{p_{p0}}\right)$$

Week 6: Light management 1, Refraction/Dispersion/Diffraction

Electric and magnetic field strengths

$$E(r,t) = E_0 \exp(ik_y y - i\omega t) = E_0 \sin(k_y y - \omega t)$$

$$H(r,t) = H_0 \exp(ik_y y - i\omega t) = H_0 \sin(k_y y - \omega t)$$

$$k_y = \frac{n\,\omega}{c_0} = \frac{2\pi}{\lambda} = \frac{2\pi\, n\nu}{c_0}$$

Incident power

$$P = \int I\, dA$$

$$I = \int P(\lambda) d\lambda$$

$$\Phi = \int_0^{\lambda} \phi(\lambda) d\lambda = \int_0^{\lambda} \frac{P(\lambda)\lambda}{hc} d\lambda$$

Spectral utilization

$$\eta_{ult} = p_{abs} p_{use} = \frac{\int_0^{\lambda_G} \Phi(\lambda) \frac{hc}{\lambda} d\lambda}{\int_0^{\infty} \Phi(\lambda) \frac{hc}{\lambda} d\lambda} \cdot \frac{E_G \int_0^{\lambda_G} \Phi(\lambda) d\lambda}{\int_0^{\lambda_G} \Phi(\lambda) \frac{hc}{\lambda} d\lambda} = \frac{E_G \int_0^{\lambda_G} \Phi(\lambda) d\lambda}{\int_0^{\infty} \Phi(\lambda) \frac{hc}{\lambda} d\lambda}$$

Front grid

$$C_f = \frac{A_f}{A_{tot}}$$

$$R = \frac{\rho L}{WH}$$

Complex refractive index

$$\tilde{n} = \sqrt{\tilde{\varepsilon}} = n + i\kappa$$

Absorption coefficient

$$\alpha = \frac{4\pi\kappa}{\lambda}$$

Lambert-beer law

$$I_x = I_0 \exp(-\alpha x)$$

Snell's law

$$n_1(\lambda)\sin(\theta_i) = n_2(\lambda)\sin(\theta_t)$$

$$\theta_i = \theta_r$$

$$\theta_{\text{critical}} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\theta_{Brewster} = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

Fresnel coefficients

$$r_S = \frac{n_1\cos(\theta_i) - n_2\cos(\theta_t)}{n_1\cos(\theta_i) + n_2\cos(\theta_t)}$$

$$r_P = \frac{n_1\cos(\theta_t) - n_2\cos(\theta_i)}{n_1\cos(\theta_t) + n_2\cos(\theta_i)}$$

$$R = r^2 = 1 - T$$

$$R_P = R_S = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)$$ under normal incidence

$$R_P = \frac{1}{2}\left(r_S^2 + r_P^2\right)$$ for unpolarised light

Wave superimposition

$$C_0 = A_0 + B_0\exp\left(i\varphi\right)$$

Dielectric multi-layers

$$\frac{\lambda_B}{4} = n_H d_1 = n_L d_2$$

$$R = \left(\frac{n_0 n_L^{2p} - n_S n_H^{2p}}{n_0 n_L^{2p} + n_S n_H^{2p}}\right)^2$$

$$\Delta\lambda = \frac{4}{\pi}\lambda_B \sin^{-1}\left(\frac{n_H - n_L}{n_H + n_L}\right)$$

Diffraction grating

$$\frac{2\pi d\sin\theta}{\lambda} = m\varphi$$

Week 7. Light management II: light scattering

Rayleigh scattering

$$I(\theta) = I_0 \left(\frac{2\pi}{\lambda}\right)^4 \left(\frac{n_p^2 - n_0^2}{n_p^2 + 2n_0^2}\right)^2 \left(\frac{d}{2}\right)^6 \frac{1+\cos^2\theta}{2R^2}$$

Radiance

$$d\Omega = \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi$$

$$P = \int_S \int_{2\pi} L_e \cos\theta \, d\Omega \, dS$$

$$L_e = \frac{1}{\cos\theta} \frac{\partial^4 P}{\partial S \partial \Omega} \qquad L_e^* = \frac{L_e}{n^2}$$

Etendue

$$Etendue = \frac{P}{L_e^*} = n^2 dS \cos\theta \, d\theta$$

Week 8. Solar cell engineering

External quantum efficiency

$$EQE(\lambda) = \frac{I_{ph}(\lambda)}{\varphi(\lambda) q}$$

$$EQE(\lambda) = (1 - R(\lambda)) \cdot IQE_{op}(\lambda) \cdot \eta_g(\lambda) \cdot IQE_{el}(\lambda)$$

$$J_{\mathrm{sc}}(V = 0\,\mathrm{V}) = q \int_0^\lambda \Phi(\lambda) \cdot \mathrm{EQE}(\lambda) d\lambda$$

Solar cell efficiency

$$\eta = \frac{\int_0^{\lambda_G} \Phi(\lambda) \frac{hc}{\lambda} d\lambda}{\int_0^{\infty} \Phi(\lambda) \frac{hc}{\lambda} d\lambda} \cdot \frac{E_G \int_0^{\lambda_G} \Phi(\lambda) d\lambda}{\int_0^{\lambda_G} \Phi(\lambda) \frac{hc}{\lambda} d\lambda} \cdot (1 - R(\lambda)) \cdot IQE_{op}(\lambda) \cdot \eta_g(\lambda) \cdot IQE_{el}(\lambda) \cdot \frac{A_f}{A_{tot}} \cdot \frac{qV_{oc}}{E_G} FF$$