Assignment 6

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Question

Papoulis-Pillai Ch 6 Ex 4-16:

a)The function $g(x)$ is monotone increasing and $y = g(x)$. Show that

$$F_{xy}(x,y) = \begin{cases} F_x(x), & \text{if } y > g(x) \\ F_y(y), & \text{if } y < g(x) \end{cases}$$

b)Find $F_{xy}(x,y)$ if $g(x)$ is monotone decreasing.

Solution

a) Let us find out the function F(x,y) for the possible values y can take. As the first case,

$$y = y_1 < g(x) \tag{1}$$

then,

$$F(x,y) = P\{x \leq x, y \leq y_1\}$$

$$= P\{y \leq y_1\} \; [\because \text{ all probability masses are on the line y = g(x)}]$$

$$= F_y(y_1)$$

Solution

If,

$$y = y_2 > g(x) \tag{2}$$

then,

$$F(x, y) = P\{x \leq x, y \leq y_2\}$$

$$= P\{x \leq x\} \text{ [}\because \text{ all probability masses are on the line y = g(x)]}$$

$$= F_x(x)$$

Solution

b) Given, g(x) is monotone decreasing. Following the same procedure as before. If

$$y = y_1 < g(x), \tag{3}$$

then

$$F(x, y) = P\{x \leq x, y \leq y_1\}$$

$$= 0$$

Solution

If

$$y = y_2 > g(x), \tag{4}$$

For this situation, let $g^{-1}(y_2) = x'$ (The function is invertible because it is monotonic)

$$F(x, y) = P\{x \leq x, y \leq y_2\}$$

$$= P\{x \leq x'\} - P\{y > y_2\}$$

$$= F_x(x) - [1 - F_y(y_2)]$$