Assignment 4

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Question

Papoulis-Pillai Ch 2 Ex 2-22:
A biased coin is tossed till a head appears for the first time. What is the probability that the number of tosses required is odd?

Solution

Let,

$$P(H) = p \tag{1}$$

$$P(T) = q = 1 - p \tag{2}$$

Also let the event 'A_i' denote,

$$A_i = \text{" Head appears for the first time at the } i^{th} \text{ trial."} \tag{3}$$

$$i.e., \ A_i = \{T, T, T, T, \ldots\ldots T, H\} \ (i - 1 \text{ tails and a head}) \tag{4}$$

Solution

Assuming that each coin toss is independent of the other,

$$P(A_i) = P(\{T, T, \ldots\ldots, T, H\}) = P(T)P(T)\ldots\ldots P(T)P(H) = q^{i-1}p \quad (5)$$

$$P("\text{ Head appears on an odd toss}") = P(A_1 + A_3 + A_5 + \ldots\ldots)$$

$$= \sum_{i=0}^{\infty} P(A_{2i+1}) = \sum_{i=0}^{\infty} q^{2i}p = p\sum_{i=0}^{\infty} q^{2i}$$

$$= \frac{p}{1-q^2} = \frac{p}{(1-q)(1+q)}$$

$$= \frac{1}{1+q} = \frac{1}{2-p}$$

$$(6)$$

Solution

$\therefore$ The Probability that the first head appears on an odd toss is $\boxed{\dfrac{1}{2-p}}$