

Assignment 1

Sahishnu (cs21btech11009)

Abstract—This document contains the solution for Assignment 1 (ICSE Class 10 Maths 2018 Q.2(c))

2(c) [ICSE 10 2018]:

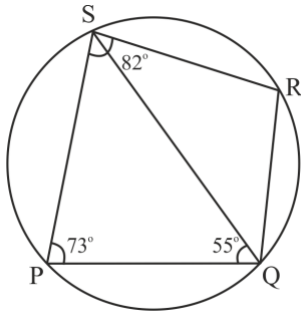


Fig. 1. Problem figure

PQRS is a cyclic quadrilateral. Given $\angle QPS = 73^\circ$, $\angle PQS = 55^\circ$ and $\angle PSR = 82^\circ$, calculate:

- (i) $\angle QRS$
- (ii) $\angle RQS$
- (iii) $\angle PRQ$

Solution:

(i) We know that, In a Cyclic quadrilateral, sum of a pair of opposite angles results in 180° . Hence,

$$\begin{aligned}\angle QPS + \angle QRS &= 180^\circ \\ \rightarrow 73^\circ + \angle QRS &= 180^\circ \\ \Rightarrow \angle QRS &= 107^\circ\end{aligned}\quad (1)$$

(ii) Again, from the fact that sum of a pair of opposite angles is 180° ,

$$\begin{aligned}\angle PSR + \angle PQR &= 180^\circ \\ \rightarrow 82^\circ + \angle PQS + \angle RQS &= 180^\circ \\ \rightarrow 82^\circ + 55^\circ + \angle RQS &= 180^\circ \\ \Rightarrow \angle RQS &= 43^\circ\end{aligned}\quad (2)$$

(iii) We know that in a circle, a chord always subtends equal angles at all the points on a particular arc. Consider the chord PQ,

$$\rightarrow \angle PSQ = \angle PRQ \quad (3)$$

We know that the sum of angles in a triangle equals to 180° , Consider the triangle $\triangle PQS$,

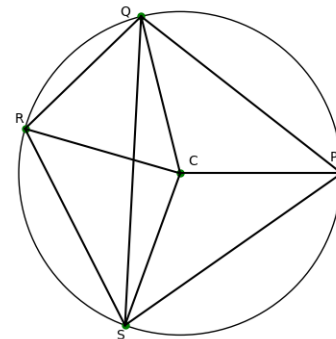
$$\begin{aligned}\rightarrow \angle PSQ + \angle SPQ + \angle PQS &= 180^\circ \\ \rightarrow \angle PSQ + 73^\circ + 55^\circ &= 180^\circ \\ \rightarrow \angle PSQ &= 52^\circ\end{aligned}$$

Substituting this result in the equation (3),

$$\Rightarrow \angle PRQ = 52^\circ \quad (4)$$

Before construction of the figure, let us try and find out some of the required angles in plotting it.

In the figure, label the centre of the circle as C, and join the points P, Q, R, S to it. Make a diagram such that the point P is at the rightmost point of the figure.



Let,

$\angle CPQ = x$, $\angle CSP = y$, $\angle CRS = z$, $\angle CQR = \lambda$
and $\angle CSQ = \theta$.

As the triangles, $\triangle CPS$, $\triangle CSR$, $\triangle CRQ$,
 $\triangle CQP$ and $\triangle CQS$ are isosceles,

$$\begin{aligned}\angle CPQ &= \angle CQP = x \\ \angle CSP &= \angle CPS = y \\ \angle CRS &= \angle CSR = z \\ \angle CQR &= \angle CRQ = \lambda \\ \angle CQS &= \angle CSQ = \theta\end{aligned}$$

We hope to find each of the angles that the
chords, PQ, QR, RS, SP make at the center, i.e.,
the angles, $\angle PCQ$, $\angle QCR$, $\angle RCS$, $\angle SCP$.

From the given information and the results
we previously obtained, we can see the following
equations being true.

$$x + y = 73^\circ \quad (5)$$

$$y + \theta = 52^\circ \quad (6)$$

$$x + \theta = 55^\circ \quad (7)$$

$$y + z = 82^\circ \quad (8)$$

$$z + \lambda = 107^\circ \quad (9)$$

$$\lambda + x = 98^\circ \quad (10)$$

On solving the above equations, we
can obtain the values of the variables as,
 $x = 38^\circ$, $y = 35^\circ$, $z = 47^\circ$, $\lambda = 60^\circ$.

From these values of the variables, we can
see that,

$$\begin{aligned}\angle PCQ &= 180^\circ - 2x = 104^\circ \\ \angle PCS &= 180^\circ - 2y = 110^\circ \\ \angle RCQ &= 180^\circ - 2\lambda = 60^\circ \\ \text{As } \angle PCR &= \angle PCQ + \angle QCR, \\ \angle PCR &= 104^\circ + 60^\circ = 164^\circ.\end{aligned}$$

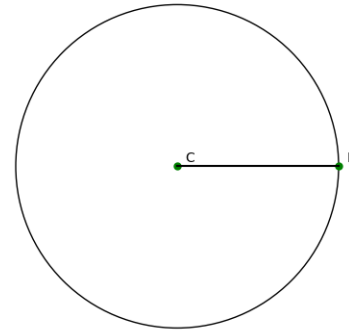
Variable	Variable type (input/output)	Value
$\angle PSR (\alpha)$	input	82°
$\angle QPS (\beta)$	input	73°
$\angle SQP (\gamma)$	input	55°
$\angle PCQ$	output	$360 - 2\beta - 2\gamma = 104^\circ$
$\angle PCR$	output	$2\alpha = 164^\circ$
$\angle PCS$	output	$2\gamma = 110^\circ$

TABLE I

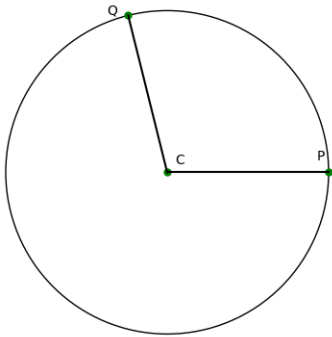
INPUT AND OUTPUT VARIABLES FOR DRAWING THE FIGURE

Construction of the given figure:

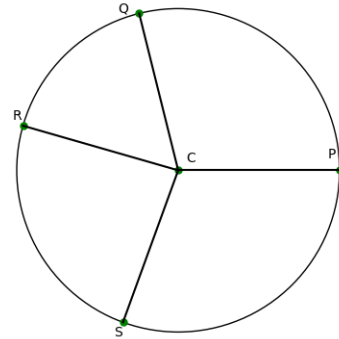
- 1) Draw a circle of radius 4 units with the centre C. (The radius mentioned here is just for scale). Now as a reference point, label the point P at the rightmost end of the circle and join them.



- 2) Now, by using a protractor with the line CP as the base, mark the point Q on the circle at an angle of $\angle 104^\circ$ anti clockwise and join it to the center of the circle.



- 4) Using a protractor with the line CP as base, mark the point S on the circle at an angle of $\angle 110^\circ$ clockwise and join it to the center of the circle.



- 5) Now, join the points P, Q, R and S to form the desired cyclic quadrilateral PQRS.

- 3) Now, again using a protractor with the line CP as the base, mark the point R on the circle at an angle of $\angle 164^\circ$ anti clockwise and join to the center of the circle.

