

1) Given $\mu = \begin{pmatrix} \mu_a \\ \mu_b \\ \mu_c \end{pmatrix} \quad \Sigma = \begin{pmatrix} \Sigma_{aa} & \Sigma_{ab} & \Sigma_{ac} \\ \Sigma_{ba} & \Sigma_{bb} & \Sigma_{bc} \\ \Sigma_{ca} & \Sigma_{cb} & \Sigma_{cc} \end{pmatrix}$

joint distribution of (x_a, x_b)
can be expressed as

$$\begin{pmatrix} x_a \\ x_b \end{pmatrix} \sim N \left[\begin{pmatrix} \mu_a \\ \mu_b \end{pmatrix}, \begin{pmatrix} \Sigma_{aa} & \Sigma_{ab} \\ \Sigma_{ba} & \Sigma_{bb} \end{pmatrix} \right]$$

Gaussian with vectors mean $(\mu_a, \mu_b)'$
Covariance matrix $\begin{bmatrix} \Sigma_{aa} & \Sigma_{ab} \\ \Sigma_{ba} & \Sigma_{bb} \end{bmatrix}$

We can partition the components
into 2 categories x_a and x_b

We can derive conditional
distribution

$$x_a / x_b \sim N [\mu', \Sigma']$$

Here $\mu' = \mu_a + \Sigma_{ab} \Sigma_{bb}^{-1} (x_b - \mu_b)$

and $\Sigma' = \Sigma_{aa} - \Sigma_{ab} \Sigma_{bb}^{-1} \Sigma_{ba}$

Since x_c has been marginalized out no need to consider it.

$$P(x_a/x_b) \propto P(N[\mu', \Sigma'])$$

2) Given $J(\omega) = f(\omega^T \phi(x_1), \dots, \omega^T \phi(x_N)) + g(\omega^T \omega)$

$g(\cdot)$ is a monotonically increasing

with $\omega = \sum_{n=1}^N \alpha_n \phi(x_n) + \omega_{\perp}$

$$J(\omega) = f(\omega^T \phi(x_1), \dots, \omega^T \phi(x_N))$$

$$\Rightarrow f(t) + g(\omega^T (z_1 \phi(x_1) + z_2 \phi(x_2) + \dots + z_n \phi(x_n) + \omega_{\perp}))$$

$$T = \omega^T \sum_{n=1}^N z_n \phi(x_n) + \omega_{\perp}$$

where $z_n = 1$

$$J(\omega) = f(\omega^T \phi(x_1) \dots \omega^T \phi(x_n)) + g(\omega^T (z_1 \phi(x_1) + \dots + z_n \phi(x_n) + \omega_{\perp}))$$

This signifies the linearity of the basis function

$\phi(x_n)$ for $n = 1, 2, \dots, N$

Each of these $J(w)$ takes a corresponding $\phi(x_n)$ form in a linear way.

Hence proved, that the value of w that minimizes $J(w)$ takes the form of a linear combination of the basis functions $\phi(x_n)$ for $n = 1 \dots N$ ✓