MA-221(Numerical Analysis)
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Lab Assignment-7
Date: 25/03/2025

Question 1: Supply Chain Optimization (Jacobi Method)

Background: A logistics company manages the distribution of essential goods to three warehouses while minimizing transportation costs. The transportation model leads to a system of linear equations representing supply constraints at different warehouses. The goal is to determine how much stock should be sent to each warehouse while satisfying demand constraints.

Problem Statement: Solve the following system of equations using the **Jacobi iteration method** with an absolute error tolerance of 10^{-4}:

$$8x + 3y - 2z = 130 \tag{1}$$
$$2x + 10y + 5z = 250 \tag{2}$$
$$-3x + 6y + 15z = 190 \tag{3}$$

Question 2: Electrical Circuit Analysis (Gauss-Seidel Method)

Background: An electrical circuit consists of three loops with resistors and voltage sources. By applying Kirchhoff's Voltage Law (KVL), the current flow through each loop can be modeled as a system of equations. The objective is to compute the current flowing through each loop in the circuit.

Problem Statement: Solve for i_1, i_2, i_3 using the **Gauss-Seidel method** until the successive approximations differ by less than 10^{-5}:

$$12i_1 - 4i_2 + 3i_3 = 10 \tag{4}$$
$$-3i_1 + 14i_2 - 6i_3 = 25 \tag{5}$$
$$5i_1 - 2i_2 + 17i_3 = 40 \tag{6}$$

Question 3: Financial Portfolio Balancing (Gauss-Seidel Method)

Background: An investment firm wants to allocate funds across three asset classes such that the expected returns match investor requirements. The portfolio is modeled as a system of linear equations where each equation represents return constraints. The objective is to determine the optimal allocation for each asset.

Problem Statement: Determine the values of x, y, z using the **Gauss-Seidel method** with at least **10 iterations** and analyze convergence:

$$0.08x + 0.04y + 0.03z = 0.07 \tag{7}$$
$$0.03x + 0.09y + 0.02z = 0.06 \tag{8}$$
$$0.05x + 0.02y + 0.07z = 0.08 \tag{9}$$

Question 4: Image Processing - Noise Reduction (Jacobi Method)

Background: In **image processing**, noise reduction is often performed using Gaussian blurring, which requires solving a system of equations representing pixel transformations.

Problem Statement: Solve for the pixel values $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9$ using the **Jacobi method**:

$$0.2P_1 + 0.1P_2 + 0.2P_3 = 50 \tag{10}$$
$$0.1P_1 + 0.4P_2 + 0.1P_3 + 0.1P_5 = 75 \tag{11}$$
$$0.2P_2 + 0.1P_3 + 0.2P_6 = 100 \tag{12}$$
$$0.1P_1 + 0.3P_4 + 0.1P_5 + 0.1P_7 = 125 \tag{13}$$
$$0.2P_2 + 0.1P_4 + 0.3P_5 + 0.2P_6 + 0.1P_8 = 150 \tag{14}$$
$$0.1P_3 + 0.2P_5 + 0.1P_6 + 0.1P_9 = 175 \tag{15}$$
$$0.2P_4 + 0.1P_5 + 0.2P_7 = 200 \tag{16}$$
$$0.1P_5 + 0.4P_8 + 0.1P_9 = 225 \tag{17}$$
$$0.2P_6 + 0.1P_8 + 0.2P_9 = 250 \tag{18}$$

Question 5: Heat Transfer in a Rod (SOR Method)

Background: The heat distribution along a rod follows a discretized model where the temperature at each segment is governed by neighboring temperatures. The system of equations below models the temperature at three points along the rod.

Problem Statement: Solve for T_1, T_2, T_3 using the **Successive Over-Relaxation (SOR) method** with a relaxation parameter of $r = 1.3$. Compare its performance against the Gauss-Seidel method:

$$3T_1 - T_2 = 150 \tag{19}$$
$$-2T_1 + 4T_2 - T_3 = 200 \tag{20}$$
$$-3T_2 + 5T_3 = 250 \tag{21}$$

Question 6: Economic Equilibrium Model (SOR Method)

Background: A simplified economic model involves three industries where the production of each industry depends on inputs from the other industries. This relationship is modeled as a system of equations where each variable represents the equilibrium price of goods in a given industry.

Problem Statement: Solve for x, y, z using the **SOR method** with an optimal relaxation factor:

$$1.2x - 0.4y + 0.3z = 90 \tag{22}$$
$$-0.5x + 1.8y - 0.7z = 120 \tag{23}$$
$$0.2x - 0.6y + 2.2z = 150 \tag{24}$$

Analyze stability and convergence of your solution.

Instructions for Students

1. Implement each problem using **Python, MATLAB, and C++**.
2. Use appropriate stopping criteria for iterative methods.