Strict submission deadline: 5 June 2023 at 11:30 am.
Submit a PDF or Word document for calculations and plots. Submit Matlab source code for Matlab programs. Use the submit button.

Exercise #3

Task 3.1

Let a random process

$$x(\zeta, t) = \sin(2\pi ft) + \sin(3\pi ft) + \alpha \cdot n(\zeta, t)$$

The frequency f is 300 Hz, α is 0.1, and n(ζ, t) is normally distributed random noise. An A/D converter takes samples of a pattern function of the process with a sampling frequency of 3 kHz. The length of the buffer of the A/D converter is 2048.

a) Write a Matlab program that calculates and plots the PSD (power spectral density) of the sampled pattern function random process x(ζ, t) using the Wiener-Khintchine theorem. Don't use the Matlab function for direct PSD calculation. Use the Matlab function "randn" for the noise. Plot the sampled time signal in the timeframe from +0.00s to +0.02s, and plot the PSD (positive frequencies only). Don't forget the axis labels.

b) Increase α to 0.3 and run your program again. What do you observe? Submit the plots.

c) Increase α to 2.0 and run your program again. What do you observe? Submit the plots.

d) Run your program with α set to 0.3 and the sampling frequency to 900 Hz. Submit the plots.

e) Run your program with α set to 0.3 and the sampling frequency to 450 Hz. Submit the plots. Explain the results.

f) Run your program with α set to 0.3 and a sampling frequency of 3 kHz. In contrast to b) the buffer of the A/D converter now should have a length of 8192. Submit the plots.

g) Take the settings of f) but instead of normally distributed random noise add uniformly distributed noise.

Task 3.2

Let a discrete stationary random process x (ζ, t).
The outcomes of the process are the values $x_1 = -1, x_2 = 0 \ and \ x_3 = 1$
The probabilities of the occurrence of those outcomes are

$$P(\{x(\zeta, t + \tau) = x_i\}| \{x(\zeta, t) = x_j\}) = \begin{cases} \frac{1}{3}\left(1 + 2\,e^{-|\tau|}\right) & for \ i = j \\ \frac{1}{3}\left(1 - e^{-|\tau|}\right) & for \ i \neq j \end{cases} \qquad i, j = 1,2,3$$

a) Calculate the probabilities

$$P(\{x(\zeta, t) = \ x_i\}) \quad for \ i = 1,2,3.$$

b) Calculate the ACF $s_{xx}(\tau)$