

Exercise #2

Task 2.1

Given the joint density function

$$f_{xy}(x, y) = \begin{cases} 2 & \text{for } x \geq 0 \text{ and } x + y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate $f_x(x)$, $f_y(y)$, $F_{xy}(x, y)$, $F_x(x)$, $F_y(y)$

Task 2.2

Let $a(\zeta)$ be a random variable taking the value 0 with probability $p_0 = \frac{1}{4}$ and the value 1 with probability $p_1 = \frac{3}{4}$. A random process $x(\zeta, t)$ is defined as:

$$x(\zeta, t) = \begin{cases} 1 - \frac{4}{T} t a(\zeta) & \text{for } 0 \leq t < \frac{T}{2} \\ -1 + \left(\frac{4}{T} t - 2\right) a(\zeta) & \text{for } \frac{T}{2} \leq t < T \\ 0 & \text{otherwise} \end{cases}$$

- Sketch all distinct pattern functions of the random process $x(\zeta, t)$.
- Calculate the mean $m_x^{(1)}(t)$.
- Calculate the autocorrelation function $s_{xx}(t_1, t_2)$.
- Calculate the variance $\sigma_x^2(t)$.

Task 2.3

Given the following signal

$$x(t) = \sin(2\pi f t)$$

with

$$f = 1\text{Hz}$$

Use Matlab for the calculation of

- The ACF of $x(t)$ using a rectangular time window
- The ACF of $x(t)$ using a Hamming window.

and plot both results

Hint: To generate Hamming Window in MATLAB, use "w = hamming(L)" where L is the length of the sequence over which you wish to apply the filtering window. For the generation of a rectangular window, use "w = rectpuls(x, a)" for a rectangular pulse of unit amplitude centered around $x = 0$ and width = a.

Explain the difference between your results and the result from the calculation done in the lecture.