Points	30	28-29	26-27	24-25	22-23	20-21	18-19	16-17	14-15	12-13	< 12
Grade	1.0	1.3	1.7	2.0	2.3	2.7	3.0	3.3	3.7	4.0	5.0

9 December 2021

Final Examination in Stochastic Signals and Systems

Do NOT use red pens! Do NOT use pencils! Solutions written with pencil are VOID. Duration 120 minutes.
Allowed means: 1 page (size A4, single-sided), handwritten only. Participants at the university: Write your solutions in the answer form only. Online participants: Write your solutions on a blank sheet.

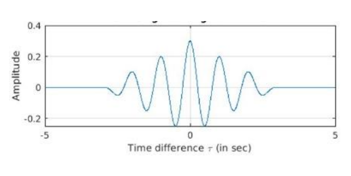
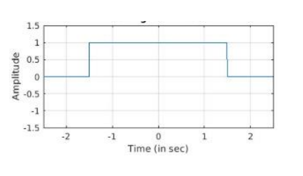
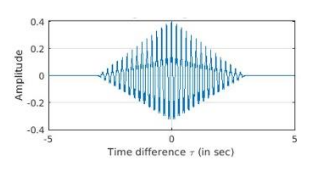
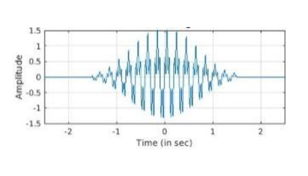
Problem #1: Power Spectral Density (10 points)

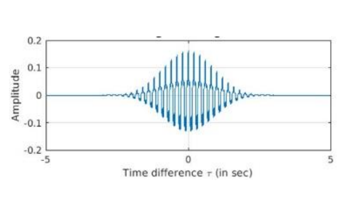
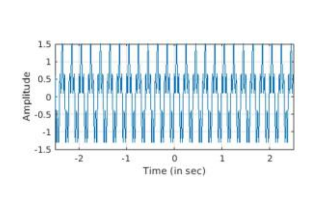
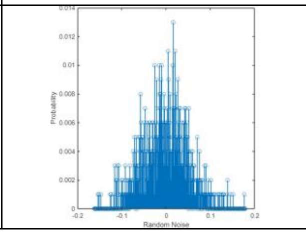
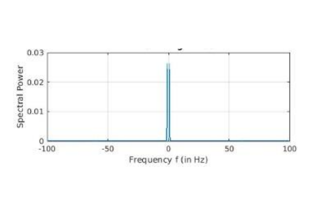
Analyze the MATLAB program and answer the following questions.

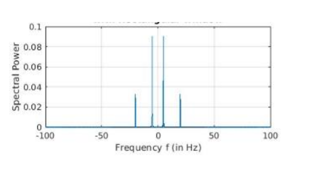
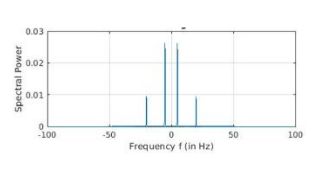
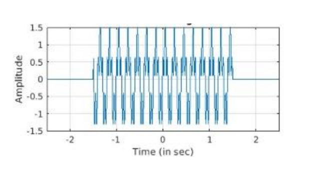
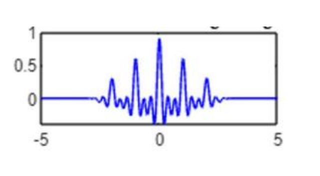
a) Match the output plots (see next page) to the corresponding program line number(s). If you detect the corresponding line number(s) then write it (them) in the appropriate box of the answer form (see below). Leave blank all other boxes of the answer form. (**4 points**)

```
123  clc;
124  clear all;
125  close all;
126  f1 = 1;
127  f2 = 20;
128  N  = 1000;
129  Fs = 200; % Sampling rate 200 Hz
130  t = ((-N/2):(N/2)-1)/Fs; % Time axis from -2.5 sec to 2.495  sec
131  y = 1*sin(2*pi*f1*t) + 0.6*cos(2*pi*f2*t);
132  windowLength = 3; % 3 seconds
133  rectWindow = [zeros(1, 1*Fs) rectwin(windowLength*Fs)'  zeros(1, 1*Fs)];
134  hammingWindow = [zeros(1, 1*Fs) hamming(windowLength*Fs)'  zeros(1, 1*Fs)];
135  
136  figure('Name','');
137  subplot(3,2,1)
138  plot(t,y),title(''),ylim([-1.5 1.5]), xlim([-2.5 2.5]), xlabel('Time (in sec)'), ylabel('Amplitude')
139  grid on
140  subplot(3,2,2)
141  plot(t,rectWindow),title(''),ylim([-1.5 1.5]), xlim([-2.5 2.5]), xlabel('Time (in sec)'), ylabel('Amplitude')
142  grid on
143  sig1 = y.*rectWindow;
144  subplot(3,2,3)
145  plot(t,sig1),title(''),ylim([-1.5 1.5]), xlim([-2.5 2.5]), xlabel('Time (in sec)'), ylabel('Amplitude')
146  grid on
147  [r1,lags1] = xcorr(sig1,'biased');
148  tau1 = lags1/Fs;
149  subplot(3,2,4)
150  plot(tau1,r1),title(''), xlabel('Time difference \tau (in sec)'), ylabel('Amplitude')
151  grid on
152  Rxxdft1 = abs(fftshift(fft(r1)))/N;
153  freq1 = -Fs/2:Fs/length(r1):Fs/2-(Fs/length(r1));
154  subplot(3,2,5)
155  plot(freq1,Rxxdft1),title(''), xlabel('Frequency f (in Hz)'),ylabel('Spectral Power')
156  grid on
157  sig2 = y.*hammingWindow;
158  subplot(3,2,3)
159  plot(t,sig2),title(''),ylim([-1.5 1.5]), xlim([-2.5 2.5]), xlabel('Time (in sec)'), ylabel('Amplitude')
160  grid on
161  [r2,lags2] = xcorr(sig2,'biased');
162  tau2 = lags2/Fs;
163  subplot(3,2,4)
164  plot(tau2,r2),title(''), xlabel('Time difference \tau (in sec)'), ylabel('Amplitude')
165  grid on
166  Rxxdft2 = abs(fftshift(fft(r2)))/N;
167  freq2 = -Fs/2:Fs/length(r2):Fs/2-(Fs/length(r2));
168  subplot(3,2,5)
169  plot(freq2,Rxxdft2),title(''), xlabel('Frequency f (in Hz)'),ylabel('Spectral Power')
170  grid on;
```

Final Examination in Stochastic Signals and Systems – 22 July 2021

I	II	III	IV
			

V	VI	VII	VIII
			

IX	X	XI	XII
			

b) Explain in one sentence the major advantage of the window used in line 157. (**2 points**)
c) Write down the mathematical equation corresponding to line 147 as detailed as possible. (**2 points**)
d) Calculate the alias frequency if the sampling rate would be changed to 25 Hz. (**2 points**)

Answer form for problem #1

Please fill in the right answer.

<table>
<tr><td rowspan="2">a)</td><td>I</td><td>II</td><td>III</td><td>IV</td><td>V</td><td>VI</td><td>VII</td><td>VIII</td><td>IX</td><td>X</td><td>XI</td><td>XII</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>b)</td><td colspan="12">The major advantage of using the Hamming window (as implemented in line 157) is that it helps reduce spectral leakage in the frequency domain when performing spectral analysis or computing the Fourier transform of a signal.</td></tr>
<tr><td>c)</td><td colspan="12">The autocorrelation of a signal sig1 is calculated using the 'biased' option, which means that the denominator in the calculation is normalized by the number of samples (N).
The mathematical equation for the 'biased' autocorrelation calculation can be written as follows:
$r_1[k] = \frac{1}{N} \sum_{n=-N+1}^{N-1} \mathrm{sig1}[n] \cdot \mathrm{sig1}[n-k]$
Where:
• $r_1[k]$ is the autocorrelation value at lag k (stored in the output vector $r1$).
• N is the number of samples in the input signal sig1.
• $\mathrm{sig1}[n]$ represents the nth sample of the input signal sig1.
• k is the lag index, and k varies from $-N+1$ to $N-1$ (stored in the vector lags1).
The result $r1$ will contain the autocorrelation values for all possible lags from $-N+1$ to $N-1$, representing how the signal is correlated with itself at different time shifts.</td></tr>
<tr><td>d)</td><td colspan="12">Given that the original sampling rate is 200 Hz, the Nyquist frequency is $\frac{200}{2} = 100$ Hz. This means that any frequency components above 100 Hz in the original signal would be aliased (folded) back into the frequency range from 0 to 100 Hz when sampled at 200 Hz.
Now, if we change the sampling rate to 25 Hz, we need to determine the alias frequency for the signal components above the new Nyquist frequency, which is $\frac{25}{2} = 12.5$ Hz.
So, any frequency component in the original signal above 12.5 Hz would be aliased (folded) back into the frequency range from 0 to 12.5 Hz when sampled at 25 Hz.
To summarize, the alias frequency when the sampling rate is changed to 25 Hz would be 12.5 Hz.</td></tr>
</table>

Problem #2: Random processes and correlation functions (10 points)
Let the statistically independent stationary random processes, x (ζ, t) and y (ζ, t). Their mean values are positive and are their autocorrelation functions are:

$$s_{xx}(\tau) = \frac{4+\frac{\tau^2}{T^2}}{2+\frac{\tau^2}{T^2}} \qquad T > 0$$

$$s_{yy}(\tau) = 4 + e^{-\frac{|\tau|}{T}} \qquad T > 0$$

Let z (ζ, t) be zero-mean random process defined as

$$z(\zeta, t) = a\, x(\zeta, t) + b\, y(\zeta, t)$$

The random process z (ζ, t) has the same variance as the random process x(ζ, t).

a) Determine all possible pairs of coefficients a and b. (**6 points**)
b) Determine the autocorrelation function $s_{zz}(\tau)$. (**4 points**)

Answer form for problem #2
Fill in the final results only. No calculation.

a)	
b)	

Problem #3: Correlation functions and Optimum Systems (10 points)

a) Explain the difference between a causal Wiener-Kolmogorov filter and a noncausal Wiener-Kolmogorov filter. **(4 points)**
b) Draw a simple block diagram illustrating the usage of an adaptive filter and a second microphone for noise suppression in order to improve the audibility of a telephone conversation. (**2 points**)
c) Maximum Likelihood estimation uses the so-called likelihood function $f_w(\underline{w}\,|x)$. Please explain this function and give a statement how it is used. (**2 points**)
d) Calculate the autocorrelation function of $x(e,t) = \check{x}\sin(\omega_0 t + \varphi(e))$ with $f_\varphi(\varphi) = {}^1\!/_{2\pi}$ for $0 < \varphi < 2\pi$ and $f_\varphi(\varphi) = 0$ elsewhere. (**2 points**)

Useful antiderivatives	Useful addition theorems
$\int sin^2\, a\, da = \frac{1}{2}a - \frac{1}{4}\sin 2a$ $\int cos^2\, a\, da = \frac{1}{2}a + \frac{1}{4}\sin 2a$ $\int sin\, a \cos a\, da = \frac{1}{2}\, sin^2 a$	$\sin(a+b) = \sin a \cos b + \cos a \sin b$ $\cos(a+b) = \cos a \cos \mathrm{b} - \sin a \sin b$

Answer form for problem #3

a)	
b)	

c)	
d)	**Only final result, no calculation:**