Points	30	28-29	26-27	24-25	22-23	20-21	18-19	16-17	14-15	12-13	< 12
Grade	1.0	1.3	1.7	2.0	2.3	2.7	3.0	3.3	3.7	4.0	5.0

22 July 2021

Final Examination in Stochastic Signals and Systems

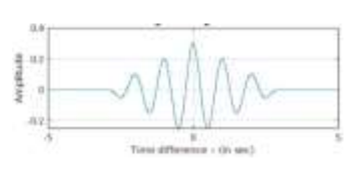
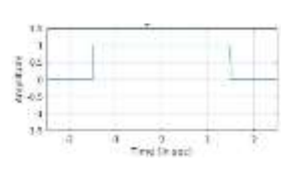
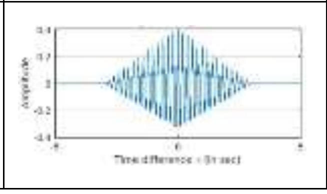
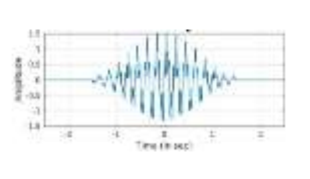
Do NOT use red pens! Do NOT use pencils! Solutions written with pencil are VOID. Duration 120 minutes.
Allowed means: 1 page (size A4, single-sided), handwritten only. Participants at Messe Frankfurt: Fill your solutions in the answer form only. Online participants: Write on a blank sheet.

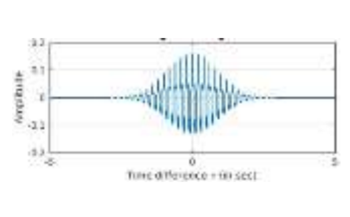
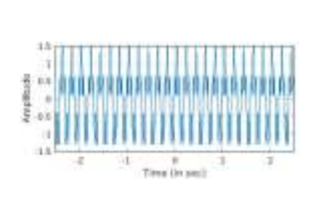
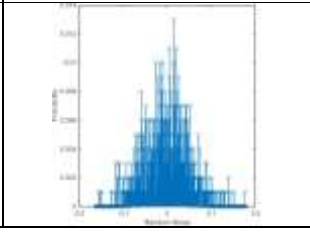
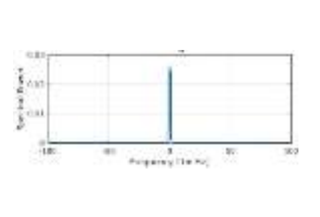
Problem #1: Power Spectral Density (10 points)

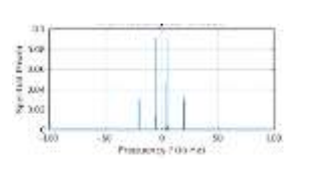
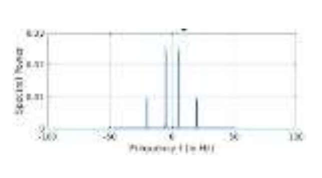
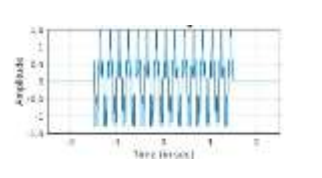
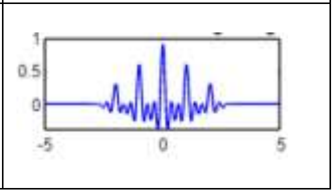
Analyze the MATLAB program and answer the following questions.

a) Match the output plots (see next page) to the corresponding program line number(s). If you detect the corresponding line number(s) then write it in the appropriate box of the answer form (see below). If there is a plot without any corresponding line number then simply tick the appropriate box in the answer form (or write in “not found”). (**5 points**)

```
1   clc;
2   clear all;
3   close all;
4   f1 = 1;
5   f2 = 20;
6   N  = 1000;
7   Fs = 200; % Sampling rate 200 Hz
8   t = ((-N/2):(N/2)-1)/Fs; % Time axis from -2.5 sec to 2.495  sec
9   y = 1*sin(2*pi*f1*t) + 0.6*cos(2*pi*f2*t);
10  windowLength = 3; % 3 seconds
11  rectWindow = [zeros(1, 1*Fs) rectwin(windowLength*Fs)'  zeros(1, 1*Fs)];
12  hammingWindow = [zeros(1, 1*Fs) hamming(windowLength*Fs)'  zeros(1, 1*Fs)];
13
14  figure('Name','');
15  subplot(3,2,1)
16  plot(t,y),title(''),ylim([-1.5 1.5]), xlim([-2.5 2.5]), xlabel('Time (in sec)'), ylabel('Amplitude')
17  grid on
18  subplot(3,2,2)
19  plot(t,rectWindow),title(''),ylim([-1.5 1.5]), xlim([-2.5 2.5]), xlabel('Time (in sec)'), ylabel('Amplitude')
20  grid on
21  sig1 = y.*rectWindow;
22  subplot(3,2,3)
23  plot(t,sig1),title(''),ylim([-1.5 1.5]), xlim([-2.5 2.5]), xlabel('Time (in sec)'), ylabel('Amplitude')
24  grid on
25  [r1,lags1] = xcorr(sig1,'biased');
26  tau1 = lags1/Fs;
27  subplot(3,2,4)
28  plot(tau1,r1),title(''), xlabel('Time difference \tau (in sec)'), ylabel('Amplitude')
29  grid on
30  Rxxdft1 = abs(fftshift(fft(r1)))/N;
31  freq1 = -Fs/2:Fs/length(r1):Fs/2-(Fs/length(r1));
32  subplot(3,2,5)
33  plot(freq1,Rxxdft1),title(''), xlabel('Frequency f (in Hz)'),ylabel('Spectral Power')
34  grid on
35  sig2 = y.*hammingWindow;
36  subplot(3,2,3)
37  plot(t,sig2),title(''),ylim([-1.5 1.5]), xlim([-2.5 2.5]), xlabel('Time (in sec)'), ylabel('Amplitude')
38  grid on
39  [r2,lags2] = xcorr(sig2,'biased');
40  tau2 = lags2/Fs;
41  subplot(3,2,4)
42  plot(tau2,r2),title(''), xlabel('Time difference \tau (in sec)'), ylabel('Amplitude')
43  grid on
44  Rxxdft2 = abs(fftshift(fft(r2)))/N;
45  freq2 = -Fs/2:Fs/length(r2):Fs/2-(Fs/length(r2));
46  subplot(3,2,5)
47  plot(freq2,Rxxdft2),title(''), xlabel('Frequency f (in Hz)'),ylabel('Spectral Power')
48  grid on;
```

I	II	III	IV
			

V	VI	VII	VIII
			

IX	X	XI	XII
			

b) Name the method of calculating the PSD used in the program. (**1 point**)
c) Name another method for PSD calculation. (**1 point**)
d) Write down the MATLAB code for the method named in part c). (**1 point**)
e) How would the plot of the PSD vary if you would change the sampling rate to 30 Hz? (**2 points**)

Answer form for problem #1

Please fill in the right answer.

a)	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**	**X**	**XI**	**XII**
b)												
c)												
d)												
e)												

Problem #2: Random processes and correlation functions (10 points)
Let

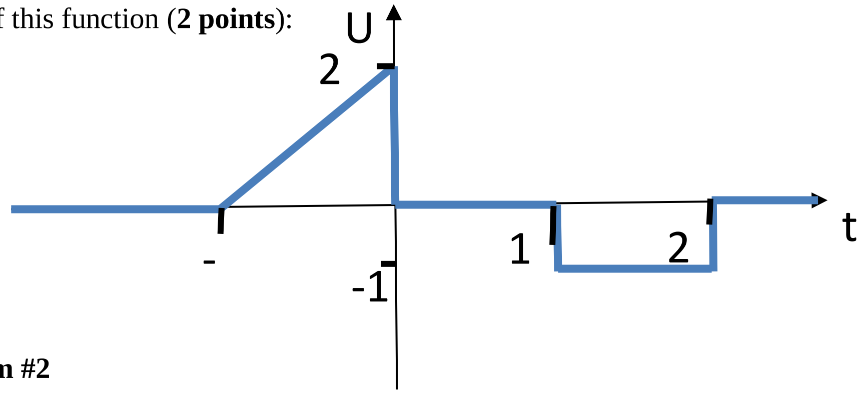
$$s_{xx}(\tau) = a^2 \cdot e^{-5\cdot|\tau|\cdot\cos(\omega\tau)}$$

the autocorrelation function (ACF) of the stationary random process x(ζ,t). From another stationary random process z(ζ,t) the probability density function (PDF) is known:

$$f_z(z,t) = \frac{1}{\sqrt{2\pi\sigma_z^2(t)}} e^{-\frac{(z-m_z^{(1)}(t))^2}{2\sigma_z^2(t)}}$$

Moreover let a stationary Gaussian random process $y(\zeta, t)$.

a) Calculate the first moment of the random process x(ζ,t). (**2 points**)
b) Calculate the auto-covariance of the random process x(ζ,t). (**2 points**)
c) Calculate the central moment $\mu_z^{(3)}$ of the random process z(ζ, t). (**2 points**)
d) Write down the PDF of the random process $y(\zeta, t)$. (**1 point**)
e) If your task would be to find the maximum value of the ACF of the stationary random process $y(\zeta, t)$, which value of τ would you choose for the calculation? (**1 point**)
f) Sketch the ACF of this function (**2 points**):



Answer form for problem #2
Fill in the final result only

a)	
b)	
c)	
d)	
e)	
f)	

Problem #3: Random Processes and Optimum Systems (10 points)

a) Assume that you would have to create a measuring system for the length x of a car passing by. You may use a high quality measuring system but Gaussian white noise superimposes the output of your measurement equipment. The known output of your system is a random measuring vector $\underline{w}(e)$ which is a function of a sequence of measuring values w_i. Its conditional probability density $f_w(\underline{w}|x)$ is known from former experiments. Shortly describe which strategy you could use to create an optimum estimator for the length of the car and name this approach. **(4 points)**

b) Assume a perfectly working Matched Filter. What would be the output of that filter if the shape of the transmitted signal would be a delta impulse? (**2 points**)

c) In which situations would you need an extended Kalman filter (EKF) instead of a classical Kalman filter? Please answer in only one sentence. (**2 points**)

d) Calculate the autocorrelation function of $x(e.t) = \check{x}\sin(\omega_0 t + \varphi(e))$ with $f_\varphi(\varphi) = {}^1/_{2\pi}$ for $0 < \varphi < 2\pi$ and $f_\varphi(\varphi) = 0$ elsewhere. (**2 points**)

Answer form for problem #3

Be brief!

a)	
b)	
c)	
d)	**Only final result:**