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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: IV - THEORY EXAMINATION (2023 - 2024)

Subject: Engineering Mathematics- IV

Time: 3 Hours

Max. Marks: 100

General Instructions:**IMP:** Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of **three Sections -A, B, & C.** It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A**20****1. Attempt all parts:-**

- 1-a. Which of the following is type of correlation.... (CO1) 1
- (a) Positive Correlation
 - (b) Perfect Correlation
 - (c) Negative Correlation
 - (d) All the above
- 1-b. Which is the types of regression (CO1) 1
- (a) Linear regression
 - (b) Non-linear regression
 - (c) Multiple linear regression
 - (d) All of above
- 1-c. Which of the following distribution is used to compare two variances? (CO2) 1
- (a) t-Test
 - (b) F -Test
 - (c) Normal Distribution

(d) Poisson Distribution

- 1-d. Control limits are set in the interval: (CO2) 1
- (a) $[\mu - 3\sigma, \mu + 3\sigma]$
 - (b) $[\mu - 2\sigma, \mu + 2\sigma]$
 - (c) $[\mu - \sigma, \mu + \sigma]$
 - (d) None of these
- 1-e. Suppose, four coins are tossed, the value of a random variable H (No. of heads) is: (CO3) 1
- (a) 1, 2, 3, 4
 - (b) 0,1,2,3
 - (c) 0,1,2
 - (d) 0,1,2,3,4
- 1-f. The sum of probabilities of a discrete random variable is (CO3) 1
- (a) Zero
 - (b) Four
 - (c) One
 - (d) Three
- 1-g. In Standard normal distribution, the value of mode is _____ (CO4) 1
- (a) 2
 - (b) 1
 - (c) 0
 - (d) any value
- 1-h. If 'm' is the mean of a Poisson Distribution, then variance is given by _____ (CO4) 1
- (a) m^2
 - (b) $m^{1/2}$
 - (c) m
 - (d) $m/2$
- 1-i. If the sum of two numbers is 31 and their product is 240, then find the absolute difference between the numbers. (CO5) 1
- (a) 3
 - (b) 1
 - (c) 4

(d) 5

1-j. Find the unit digit of 4137^{754} (CO5)

1

(a) 9

(b) 7

(c) 3

(d) 1

2. Attempt all parts:-

2.a. If $\beta_2 = 3$ then the distribution is called _____(CO1) 2

2.b. Write the test statistics for the difference of two samples means for small samples. (CO2) 2

2.c. A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and the variance of the number of successes. (CO3) 2

2.d. Differentiate between Binomial distribution and Poisson distribution. (CO4) 2

2.e. How many words can be formed by using all letters of the word "MATHEMATICS"? (CO5) 2

SECTION B

30

3. Answer any five of the following:-

3-a. Calculate the rank correlation coefficient between x and y from the following data – (CO1) 6

x	15	20	27	13	45	60	20	75
y	50	30	55	30	25	10	30	70

3-b. If for two random variables x and y with same mean, the two line of regression are $y = ax + b$, $x = \alpha y + \beta$ then show that $\frac{b}{\beta} = \frac{1-\alpha}{1-\alpha}$. (CO1) 6

3-c. A die is thrown 276 times and the results of these throws are given below: (CO2) 6

No. appeared on die	1	2	3	4	5	6
Frequency	40	32	29	59	57	59

Test whether the die is biased or not. Given that tabular value of Chi-Square at 5% LOS for 5 degree of freedom is 11.09.

3-d. Samples of sizes 10 and 14 were taken from two normal populations with SD 3.5 and 5.2. The sample means were found to be 20.3 and 18.6. Test whether the means of the two populations are the same at 5% LOS. The tabulated value 6

is 2.07 at 5% LOS for 22 d.f. (CO2)

- 3.f. Define exponential distribution. Find the moment generating function for exponential distributions. (CO4) 6
- 3.e. The diameter, say X , of a electric cable, is assumed to be continuous random variable with pdf $f(x) = 6x(1-x)$, $0 \leq x \leq 1$ (CO3) 6
- i) Obtain an expression for the c.d.f. of X .
- ii) Determine the number k such that $P(X < k) = P(X > k)$.
- 3.g. How many words can be formed by using all letters of the word "DAUGHTER" so that the vowels always come together? (CO5) 6

SECTION C

50

4. Answer any one of the following:-

- 4-a. Calculate first four central moment for the following data: (CO1) 10

Class interval	5-15	15-25	25-35	35-45	45-55	55-65
Frequency	10	20	25	20	15	10

- 4-b. Find the rank correlation coefficient for the following data: (CO1) 10

x	23	27	28	28	29	30	31	33	35	36
y	18	20	22	27	21	29	27	29	28	29

5. Answer any one of the following:-

- 5-a. The following figures relate to the production in kg of three varieties I, II, III of wheat shown in 12 plots: (CO2) 10

Variety I	14	16	18	-	-
Variety II	14	13	15	22	-
Variety III	18	16	19	19	20

Is there any significant difference in the production of three varieties? Given the tabulated value for d.f. (2,9) is 4.26 at 5% level of significance.

- 5-b. An inspection of 10 samples of size 100 each from 10 lots reveal the following number of defectives: 10

No. of defectives: 16 18 11 18 21 10 20 18 17 21

Construct the np-chart. Do these indicate that the quality characteristics inspected is under statistical control? State your recommendation. (CO2)

6. Answer any one of the following:-

- 6-a. The joint probability density function of two-dimensional random variable (X,Y) is given by- 10

$$f(x,y) = \begin{cases} 6x^2y, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Verify that $\int_0^1 \int_0^1 f(x,y) dx dy = 1$

ii) Find

$$P\left(0 < X < \frac{3}{4}, \frac{1}{3} < Y < 2\right), P(X+Y < 1), P(X > Y) \text{ and } P(X < 1 \mid Y < 2). \text{ (CO3)}$$

6-b.

Let X be a continuous RV with pdf is define as:

$$f_X(x) = \begin{cases} kx, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

i) Determine the value of k

ii) Find cdf

iii) Find $P(1/4 < x < 2)$ (CO3)

10

7. Answer any one of the following:-

7-a. State the Poisson Probability distribution. Prove that Poisson distribution is limiting case of Binomial distribution. (CO4) 10

7-b. Define the mathematical expectations and write the laws of expectations. Prove that mathematical expectation of the sum of two discrete variables is equal to the sum of their mathematical expectations. (CO4) 10

8. Answer any one of the following:-

8-a. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5? (CO5) 10

8-b. If the number 3422213pq is divisible by 99, find the missing digits p and q. (CO5) 10