Important Questions of Engineering Mathematics 4 (AAS0402)

UNIT-1

1. The following data to an economist for the purpose of economics analysis:

$$n = 1000, \sum fd = 50, \sum fd^2 = 1970, \sum fd^3 = 2948, \sum fd^4 = 86752$$

where $d = x - 48$. Do you think distribution is platykurtic?

2. Calculate the first four central moments for the following frequency distribution and hence find the coefficient of skewness and kurtosis and comment upon the nature of the distribution.

Class-interval	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	6	8	17	21	15	11	2

3. Obtain a relation of the form $y = ab^x$ for the following data by the method of least square:

x	2	3	4	5	6
y	8.3	15.4	33.1	65.2	127.4

4. Use the method of least squares to fit the curve $y = c_0 x + \frac{c_1}{\sqrt{x}}$ for the following data:

x	0.2	0.3	0.5	1	2
y	16	14	11	6	3

5. A person runs the same track for 5 consecutive days and is timed as follows:

Day (x)	1	2	3	4	5
Time (y)	15.3	15.1	15	14.5	14

Make a least square fit to the above data using a function $a + {}^{b}/_{x} + {}^{c}/_{x^2}$.

6. Find the coefficient of correlation from the following data:

X	65	63	67	64	68	62	70	66
Y	68	66	68	65	69	66	68	65

7. Find rank correlation for the following data:

X	90	30	82	45	32	65	40	88	73	66
Y	85	42	75	68	45	63	60	90	62	58

8. Find the regression of x on y from the following data: $n = 4, \sum x = 24, \sum y = 44, \sum xy = 306, \sum x^2 = 164, \sum y^2 = 574$. Find the estimated value of x, when $y = 6$.

9. The following results were obtained in the analysis of data on yield of dry bark in ounce (y) and age in years (x) of 200 cinchona plants:

		x	y
Average	:	9.2	16.5
Standard Deviation	:	2.1	4.2

Correlation coefficient $= 0.84$. Construct the two lines of regression and estimate the yield of dry bark of a plant of age 8 years.

10. Find the multiple linear regression of x on y and z from the data relating to three variables:

X	7	12	17	20
Y	4	7	9	12
Z	1	2	5	8

Unit 2

1. The mean weight obtained from a random sample of size 100 is 64gms. The S.D. of the weight distribution of the population is 3 gms. Test the statement that the mean weight of the population is 67 gms at 5% level of significance. Also set up 99% confidence limits of the mean weight of the population. (Given |Z|=1.96 at 5% level of significance)
2. The height of 6 randomly chosen sailors in inches are 63, 65, 68, 69, 71 and 72. Those of 9 randomly chosen soldiers are 61, 62, 65, 66, 69, 70, 71, 72 and 73. Test whether the sailors are on the average taller than soldiers. ((Given that value of ***t*** for 13 d.f. at 5% level of significance is 2.16)
3. A die is thrown 276 times and the results of these throws are given below:

No. appeared on the die	1	2	3	4	5	6
frequency	40	32	29	59	57	59

Test whether the die is biased or not. (Given $\chi^2_{0.05} = 11.07$ at 5 d.f.)

4. The following table gives the number of good and bad parts produced by each of the three shifts in a factory:

	Good parts	Bad Parts	Total
Day shift	960	40	1000
Evening shift	940	50	990
Night shift	950	45	995
Total	2850	135	2985

Test whether or not the production of bed parts is independent of the shift on which they were produced. (Given $\chi^2_{0.05} = 5.991$ at 2 d.f.)

5. Two independent sample of sizes 7 and 6 had the following values:

Sample A	28	30	32	33	31	29	34
Sample B	29	30	30	24	27	28	

Examine whether the samples have been drawn from normal populations having the same variance. ($F_{tab.} = 4.39$ for $v_1 = 5, v_2 = 6$ at 5% level of significance)

6. The following figures relates to the production in kg of three varieties I, II, III of wheat shown in 12 plots:

Variety I	14	16	18		
Variety II	14	13	15	22	
Variety III	18	16	19	19	20

Is there any significant difference in the production of three varieties? Given the tabulated value of F for $v_1 = 2$ and $v_2 = 9$ at 5% level of significance is 4.26.

7. The number of defective in 20 samples each of 2000 items are given below:

425	430	216	341	225	322	280	306	337	305
356	402	216	264	126	409	193	280	326	389

Calculate the values of the central line UCL and LCL.

8. Define np chart. In a blade manufacturing factory, 1000 blades are examined daily. Draw the np chart for the following table and examine whether the process is under control?

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of Defective blades	9	10	12	8	7	15	10	12	10	8	7	13	14	15	16

9. Ten pieces of cloth out of different rolls of equal length contained the following defects: 1,3,5,0,6,0,9,4,4,3
   Draw a control chart for the no. of defects and state whether the process is in state of statistical control.
10. In the experiments on pea breeding, the following frequencies of seeds were obtained:

Round and yellow	Wrinkled and yellow	Round and green	Wrinkled and green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9:3::3:1. Examine the correspondence between theory and experiment. If the tabulated value is 7.815 at 5% level of significance.

UNIT 3

Probability and Random Variables

1. A brokerage survey reports that 30 percent of individual investors have used a discount broker, that is, one which does not charge the full commission. In a random Sample of 9 individuals, what is the probability Distribution that have used a discount broker.
2. A random variable X has the following probability mass function:

x	0	1	2
$p(x)$	$3c^3$	$4c-10c^2$	$5c-1$

   i. Find c.
   ii. Evaluate $P(X<2), P(2X+3\geq 5)$ and $P(1<X\leq 2)$.
   iii. If $P(X\leq a)<\frac{1}{2}$ find the maximum value of a.
3. Find probability distribution of the number of doublets in four throws of a pair of dice.
4. Find the value of a so that the function f(x) defined as follows be a density function

$$f(x)=ae^{-\frac{x}{\sigma}}, 0\leq x\leq \infty$$

5. Find the mean and variance of the distribution:$dF=kx^2e^{-x}dx=1, 0<x<\infty$
6. If a random variable X has density function-

$$f(x)=\begin{cases} 1/4 & -2<x<2 \\ 0 & elsewhere \end{cases}$$

   Obtain- $P(|X|>1)$ and $P(2X+3>5)$
7. Given $f(x,y)=e^{-(x+y)}; x\geq 0, y\geq 0$.then find
   i. $P(X>1)$
   ii. $P(X<Y|X<2Y)$
   iii. $P(X+Y<1)$
8. The joint probability density function of two-dimensional random variable (X,Y) is given by-

$$f(x,y)=\begin{cases} \frac{8}{9}xy & 1\leq x\leq y\leq 2 \\ 0 & elsewhere \end{cases}$$

   i. Find marginal density function of X and Y.
   ii. Find the conditional density function of Y given $X=x$ and conditional distribution of X given $Y=y$.
9. If X and Y are two random variables having the joint pmf $p(x,y)=\frac{1}{27}(2x+y); x=0,1,2$; $y=0,1,2$. Find the conditional distribution for Y for $X=x$
10. A two-dimensional random variable (X, Y) have a bivariate distribution given by $P(x=x,Y=y)=\frac{x^2+y}{32}, for\ x=0,1,2,3\ and\ y=0,1$. Find the marginal distribution of X and Y.

UNIT-4

1. Fit a binomial distribution to the following data

X	0	1	2	3	4
F	30	62	46	10	2

2. If the probability of hitting a target is 10% and 10 shots are fired independently. What is the probability that the target will be hit at least once?
3. Out of 320 families with 5 children each, what percentage would be expected to have (i) 2 boys and 3 girls (ii) at least one boy? Assuming equal probability for boys and girls.
4. Suppose that a book of 600 pages contains 40 printing mistakes. Assuming that these errors are randomly distributed throughout the book and r, the number of errors per page has a Poisson distribution. What is the probability that 10 pages selected at random will be free from errors?
5. In a certain factory turning out razor blades, there is small chance of .002 for any blade to be defective. The blades are supplied in a packet of 10. Calculate the approximate number of packets containing no defective, one defective and two defective blades in a consignment of 10,000 packets.
6. Fit a Poisson distribution to the following data

Deaths	0	1	2	3	4
frequency	122	60	15	2	1

7. The income of a group of 10,000 persons was found to be normally distributed with mean rs 750 and standard deviation of Rs 50. Show that of this group 95% had income exceeding rs 668 and only 5% had income exceeding Rs 832. Also find the lowest income among the richest 100.
8. The daily wages of 1000 workers are distributed around a mean of Rs 140 and with a standard deviation of Rs 10. Estimate the number of workers whose daily wages will be
   (i) Between Rs 140 and Rs 144 (ii) less than Rs 126
9. A sample of 100 dry battery cells tested to find the length of life produced the following results:
   Mean=12 hrs., standard deviation= 3 hrs.
   Assuming the data to be normally distributed, what percentage of battery cells are expected to have life (i) More than 15 hrs. (ii) less than 6 hrs. (iii) between 10 and 14 hrs.
10. A monitor issues a warning signal when an action is needed as part of a production process. The interval, X hours, between successive signals follows an exponential distribution with parameter 0.08. (i) Find the probability that the interval between the next two signals is: (a) Between 10 and 20 hours; (b) Less than two hours; (c) Longer than 50 hours-

Unit 5

1. Solve the following
   a) 7^{126} is divided by 48, find the remainder.

b) If the number 3422213pq is divisible by 99, find the missing digit p and q.

2. A word consist of 9 letters; 5 consonants and 4 vowels. Three letters are chosen at random. What is the probability that more the one vowel will be selected?
3. Find the probability that when a hand of 7 cards is drawn from a well shuffled deck of 52 cards, it contains:
   i. All 4 kings
   ii. Exactly 3 kings
   iii. At least 3 kings.
4. A coin is tossed. If head comes up, a die is thrown but if tail comes up, the coin is tossed again. Find the probability of obtaining:
   i. Two tails
   ii. Head and number 6
   iii. Head and an even number.
5. a) How many words with or without meaning, each 2 of vowels and 3 consonants can be formed from the letter of word DAUGHTER?
   b) In how many ways can the letters of word PERMUTATIONS be arrange if
   i. The words start with P end with S
   ii. Vowels are all together.

6. The following table gives the percentage of marks obtained by seven students in six different subjects in an examination.
   The Numbers in the Brackets give the Maximum Marks in Each Subject.

Student	Subject (Max. Marks)					
	Maths	Chemistry	Physics	Geography	History	Computer Science
	(150)	(130)	(120)	(100)	(60)	(40)
Ayush	90	50	90	60	70	80
Aman	100	80	80	40	80	70
Sajal	90	60	70	70	90	70
Rohit	80	65	80	80	60	60
Muskan	80	65	85	95	50	90
Tanvi	70	75	65	85	40	60
Tarun	65	35	50	77	80	80

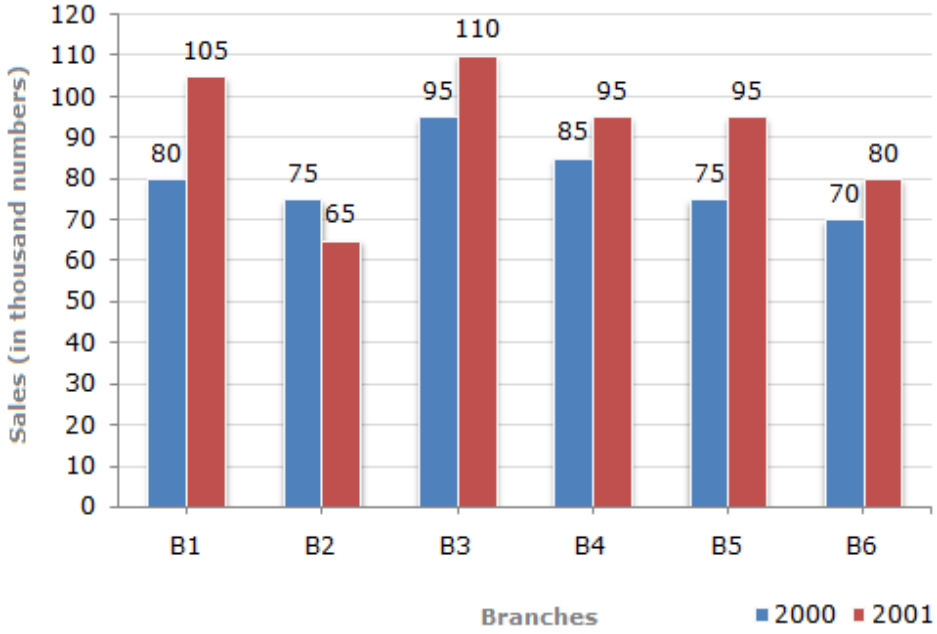
   A. What are the average marks obtained by all the seven students in Physics? (rounded off to two digit after decimal)
      a) 77.26
      b) 89.14
      c) 91.37
      d) 96.11

B. The number of students who obtained 60% and above marks in all subjects is?
   a) 1
   b) 2
   c) 3
   d) None

C. In which subject is the overall percentage the best?
   a) Maths
   b) Chemistry
   c) Physics
   d) History.

D. What is overall percentage of Tarun ?
   a) 52.5%
   b) 55%
   c) 60%
   d) 63%

7. The bar graph given below shows the sales of books (in thousand number) from six branches of a publishing company during two consecutive years 2000 and 2001.
Sales of Books (in thousand numbers) from Six Branches - B1, B2, B3, B4, B5 and B6 of a publishing Company in 2000 and 2001.



A. What is the ratio of the total sales of branch B2 for both years to the total sales of branch B4 for both years?
   a) 2:3
   b) 3:5
   c) 4:5
   d) 7:9

B. Total sales of branch B6 for both the years is what percent of the total sales of branches B3 for both the years?

a) 68.54%
b) 71.11%
c) 73.17%
d) 75.55%

C. What is the average sales of all the branches (in thousand numbers) for the year 2000?
   a) 73
   b) 80
   c) 83
   d) 88

D. Total sales of branches B1, B3 and B5 together for both the years (in thousand numbers) is?
   a) 250
   b) 310
   c) 435
   d) 560.

8. **Statements:** All the harmoniums are instruments. All the instruments are flutes.
   Conclusions: 1. All the flutes are instruments.
   2. All the harmoniums are flutes.
   a) Only (1) conclusion follows
   b) Only (2) conclusion follows
   c) Either (1) or (2) follows
   d) Neither (1) nor (2) follows
   e) Both (1) and (2) follow.
9. **Statements:** Some papers are pens. All the pencils are pens.
   Conclusions: 1. Some pens are pencils. 2. Some pens are papers.
   a) Only (1) conclusion follows
   b) Only (2) conclusion follows
   c) Either (1) or (2) follows
   d) Neither (1) nor (2) follows
   e) Both (1) and (2) follow.
10. Solve the following:
   i. If $f(x) + f(1-x) + f(1 + x) + f(2 + x) = 2x$ for all real value of x. If $f(0) = 1,\ f(-1) = f(1)$ then find the value of $f(4)$.
   ii. Given that $f(1) = 1\ and\ f(2) = 1$.
   $If\ f(n) = f(n+1) - f(n-1)$, then find the value of $\frac{f(8)-f(7)+f(5)}{f(7)-f(6)-f(4)}$