mean

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Unit 3
Probability and Random Variable

MCQ (1 Marks)

51. A random variable X has the following probability distribution:

Value of X, x:	0	1	2	3	4	5	6	7	8
P(x):	a	3a	5a	7a	9a	11a	13a	15a	17a

Determine the value of a. (CO3)
- A. 1/81
- B. 2/81
- C. 5/81
- D. 1

$\because \sum P_i = 1$
$\therefore a+3a+5a+7a+9a+11a+13a+15a+17a=1$
$81a=1, \quad a=1/81$

df or cdf

If we want to find pdf from df ⇒ differentiate the df

52. The $\boxed{\text{distribution function}}$ of a random variable X is given by $F(x) = e^x - e^{-x}(1+x)$. Find the corresponding density function of random variable X. (CO3)
- A. $e^x + xe^{-x}$
- B. $e^x - e^{-x}(1+x)$
- C. $e^x + e^{-x}$
- D. xe^{-x}

$pdf = \frac{d}{dx}[cdf]$
$= \frac{d}{dx}\{e^x - e^{-x} - xe^{-x}\}$
$= e^x + e^{x} - \{x\cdot -e^{-x} + e^{-x}\times 1\}$
$= e^x + \cancel{e^x} + xe^x - \cancel{e^x}$

53. If $f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & Otherwise \end{cases}$ find the probability $P\left(\frac{1}{4} \le x \le \frac{1}{2}\right)$. (CO3)
- A. 3/16
- B. 2/16
- C. 9/16
- D. N.O.T

$P(\frac{1}{4} \le x \le \frac{1}{2}) = \int_{1/4}^{1/2} 2x \, dx$
$= \left[2\cdot\frac{x^2}{2}\right]_{1/4}^{1/2} = \left[x^2\right]_{1/4}^{1/2} = \frac{1}{4} - \frac{1}{16} = \frac{4-1}{16} = \frac{3}{16}$

$\leftarrow 0 \le x \le 1$

54. The diameter, say X of an electric cable, is assumed to be a continuous random variable with p.d.f $f(x)=6x(1-x)$. Compute $P\left(x \le \frac{1}{2} | \frac{1}{3} \le x \le \frac{2}{3}\right)$. (CO3)
- A. 11/26
- B. 13/26
- C. 9/26
- D. 17/26.

55. The diameter, say X of an electric cable, is assumed to be a continuous random variable with p.d.f: $f(x) = 6x(1-x)$. Determine the number k such that $P(X < k) = P(X > k)$. (CO3)
- A. $\frac{1+\sqrt{3}}{2}$
- B. $\frac{1-\sqrt{3}}{2}$
- C. $\frac{1\pm\sqrt{3}}{2}$
- D. 1/2.

Given: $0 \le x \le 1$

$\int_0^k f(x)\,dx = \int_k^1 f(x)\,dx$
$\int_0^k (6x-6x^2)\,dx = \int_k^1 (6x-6x^2)\,dx$
$\left[6\frac{x^2}{2} - 6\frac{x^3}{3}\right]_0^k = \left[6\frac{x^2}{2} - 6\frac{x^3}{3}\right]_k^1$
$3k^2 - 2k^3 = (3-2) - \{3k^2 - 2k^3\}$
$3k^2 - 2k^3 = 1 - 3k^2 + 2k^3$
$6k^2 - 4k^3 - 1 = 0$
$4k^3 - 6k^2 + 1 = 0 \quad ; k = \frac{1}{2}$

63. A petrol pump is supplied with petrol once a day. If its daily volume of sales(x) in thousands of litters is distributed by $f(x) = 5(1-x)^4, 0 \le x \le 1$, what must be the capacity of its tank in order that the probability that its supply will be exhausted in a given day shall be 0.01? (CO3)

Solution: Let the capacity of tank is 'a'

$$p(x \ge a) = 0.01$$

$$\int_a^1 5(1-x)^4 \, dx = 0.01$$

$$\left[\frac{-5(1-x)^5}{5}\right]_a^1 = 0.01$$

$$1 - a = \left(\frac{1}{100}\right)^{\frac{1}{5}}$$

$$a = 1 - \left(\frac{1}{100}\right)^{\frac{1}{5}} = 0.60189$$

$$a = 0.60189 \times 1000 = 601.89 \text{ L}$$

64. If a random variable X has density function- (CO3)

$$f(x) = \begin{cases} 1/4 & -2 < x < 2 \\ 0 & elsewhere \end{cases}$$

Obtain- $P(|X| > 1)$ and $P(2X + 3 > 5)$

Solution: $p(|x|>1) = 1 - p(|x|) = 1 - \int_{-1}^{1} f(x)\, dx$

$$= 1 - \int_{-1}^{1} \frac{1}{4}\, dx = 1 - \frac{1}{4}[x]_{-1}^{1}$$

$$= 1 - \frac{1}{4}[1-(-1)] = 1 - \frac{1}{2} = \frac{1}{2}$$

$$2X + 3 > 5 \Rightarrow 2X > 2 \Rightarrow X > 1$$

$$p(2X+3>5) = \int_1^2 \frac{1}{4}\, dx = \frac{1}{4}[x]_1^2 = \frac{1}{4}$$

$\hookrightarrow p(x>1) =$

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Unit 4
Expectations and Probability Distribution

MCQ (1 Marks)

76. Find the variance for the following discrete distribution: (CO4)

X	0	1	2
P(X)	1/4	1/4	1/2

A. 10/16
B. 12/16
C. 11/16 ✓
D. 2/11

Variance $= \Sigma x_i^2 p_i - (\Sigma x_i p_i)^2$
$= \frac{9}{4} - \frac{25}{16}$
$= \frac{11}{16}$

77. If PMF of random variable X is given by $P(X = R) = {}^nC_r\, p^r q^{n-r}; r = 0,1,2 \dots n$; then Variance of $(2X + 3)$ is: (CO4)

A. $2npq + 3$
B. $4npq + 3$
C. $4npq - 3$
D. $4npq$ ✓

Variance $(2x+3) = 4V(x) + 0$
$= 4npq$

variance for B.D $V(X) = npq$

78. If the pdf of a random variable X is given by $f(x) = 2x$ for $0 < x < 1$. Then find $E(2x^2 - 2)$: (CO4)

A. 2
B. 1
C. −2
D. −1 ✓

$E(2x^2-2) = 2E(x^2) - E(2)$
$= 2\int_0^1 x^2\, 2x\, dx - 2$
$= 4\left[\frac{x^4}{4}\right]_0^1 - 2 = 1 - 2 = -1$

79. Suppose that X is a Poisson random variable, If $P(x = 2) = \frac{2}{3}P(X = 1)$, then $P(0)$ is: (CO4)

A. $e^{-4/3}$ ✓
B. $e^{-3/4}$
C. $e^{4/3}$
D. $e^{3/4}$

$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$

ATQ; $P(X=2) = \frac{2}{3} P(X=1)$

$\frac{e^{-\lambda} \cdot \lambda^2}{2!} = \frac{2}{3}\frac{e^{-\lambda} \cdot \lambda^1}{1!} \Rightarrow \frac{\lambda^2}{\lambda^1} = \frac{4}{3} \Rightarrow \lambda = \frac{4}{3}$

$\therefore P(0) = \frac{e^{-4/3} \cdot \lambda^0}{0!}$
$= e^{-4/3}$

80. For Binomial distribution sum of mean and variance is 6 and difference of mean and variance is 2 then the find probability of success. (CO4)

A. 1
B. 0.5 ✓
C. 0.25
D. 0.75

$np + npq = 6$
$np - npq = 2$
$2np = 8 \Rightarrow np = 4$

ii) $4 + npq = 6 \Rightarrow npq = 2$

$\frac{npq}{np} = \frac{2}{4} = \frac{1}{2} = 0.5$

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

81. If variance of Poisson distribution is 2. If $P(2) = 0.2706$. Find $P(4)$. (CO4)

In P.D ⇒ variance = mean = $\lambda = 2$

A. 0.2706
B. 0.1804
C. 0.1431
D. 0.0902

$$P(4) = \frac{e^{-\lambda} \cdot \lambda^4}{4!} = \frac{e^{-2} \times 2^4}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{e^{-2} \times \cancel{16}\, \cancel{8}\, 2}{\cancel{4 \times 3 \times 2}}$$

$$= \frac{2}{3} \times e^{-2}$$

$$= \frac{2}{3} \times 0.135$$

$$= 0.090$$

82. Normal Distribution is applied for ________ (CO4)

A. Continuous Random Variable
B. Discrete Random Variable
C. Irregular Random Variable
D. Uncertain Random Variable

Very Short Answer Type Questions (2 Marks)

83. Find the moment generating function of a random variable whose moments about origin are $\nu_r = (r+1)!\,2^r$. (CO4)

Solution: $M_X(t) = \sum e^{nt} p(X=n) = \sum \frac{t^n}{n!} \nu_n = \sum \frac{t^n}{n!} (n+1)!\, 2^n$

$$= \sum \frac{t^n}{n!} (n+1)!\ n!\ 2^n$$

$$= \sum t^n (n+1)\, 2^n$$

$$\sum_{n=0} (n+1)(2t)^n = 1 \cdot (2t)^0 + 2(2t)^1 + 3 \cdot (2t)^2 + \cdots$$

$$= 1 + 2 \cdot 2t + 3(2t)^2 + \cdots$$

$$= (1-2t)^{-2}$$

84. A Binomial random variable X satisfies the relation $9P(X=4) = P(X=2)$ When $n = 6$. Find value of $P(X=1)$. (CO4)

Solution: $9\,P(x=4) = P(x=2)$

$$9\ {}^6C_4\, p^4 q^2 = {}^6C_2 \cdot p^2 q^4$$

$9p^2 = q^2 \Rightarrow 3p = q$ as both are always positive

$$p + q = 1$$

$$p + 3p = 1 \Rightarrow 4p = 1 \Rightarrow p = \frac{1}{4}$$

$$3p = q \Rightarrow \frac{3}{4} = q$$

$$P(x=1) = {}^6C_1 \cdot p^1 q^5 = 6\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^5 = \frac{1458}{4096} = 0.356$$

90. It is given that 2% of the electric bulbs manufactured by a company are defective. Using Poisson distribution find the probability that a sample of 200 bulbs will contain (CO4)

I. No defective bulb
II. Two defective bulbs
III. Atmost three defective bulbs.

Solution : $p = 0.02$, $n = 200$, $\lambda = np = 200 \times 0.02 = 4$

$$P(X = x) = \frac{e^{-\lambda} \lambda^{x}}{x!} \quad , \lambda \text{ is mean} = np$$

(i) $P(X=0) = \dfrac{e^{-4}(4)^0}{0!} = 0.0183$

(ii) $P(X=2) = \dfrac{e^{-4}(4)^2}{2!} = 0.1465$

(iii) $P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$$= e^{-4}\left[1 + \frac{4}{1!} + \frac{16}{2!} + \frac{64}{3!}\right] = 0.4335$$

91. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours, estimate the number of bulbs likely to burn forMore than 2150 hours. (CO4)

Solution : $n = 2000$, $\mu = 2040$, $\sigma = 60$, $x = 2150$

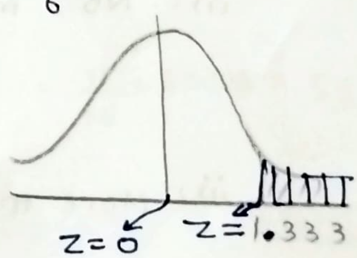
$$z = \frac{x-\mu}{\sigma} = \frac{2150-2040}{60} = \frac{110}{60} = \frac{11}{6} = 1.333$$

$P(X > 2150) = 0.5 - P(2040 < x < 2150)$

$= 0.5 - P\left(0 < Z < \frac{11}{6}\right)$

$= 0.5 - 0.4664$

$= 0.0336$



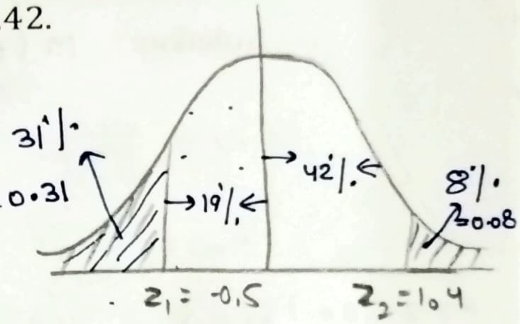
No. of bulbs $= 0.0336 \times 2000$

$= 67.2$

$\simeq 67$

96. In a distribution exactly Normal, 31% of the items are under 45 and 8% are over 64. What are the mean and Standard deviation of this Distribution? It is given that if (CO4)

$f(t) = \frac{1}{\sqrt{2\pi}}\int_0^t e^{\frac{-x^2}{2}}\,dx$ then $f(0.5) = 0.19, f(1.4) = 0.42$.



Solution: $P(x > 64) \Rightarrow P(\mu < x < 64)$

$= 0.5 - 0.08$

$= 0.42$

$P(0 < x < 45) = 0.3$

$0.5 - P(45 < x < \mu) = 0.31$

$P(45 < x < \mu) = 0.19$

$\boxed{z_2 \text{ for } 64 = 1.4}$

$\frac{64 - \mu}{\sigma} = 1.4$

$1.4\sigma + \mu = 64$ —— (i)

$\boxed{z_1 \text{ for } 45 = -0.5}$

$\frac{45 - \mu}{\sigma} = -0.5$

$\mu - 0.5\sigma = 45$

Subtract (ii) from (i)

$1.4\sigma + \mu - \mu + 0.5\sigma = 64 - 45$

$1.9\sigma = 19$

$\boxed{\sigma = 10}$

Put in (i)

$14 + \mu = 64 \Rightarrow \boxed{\mu = 50}$

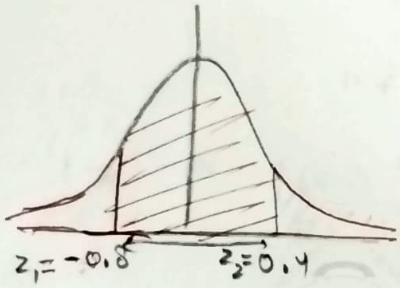
98. In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming distribution to be normal find (CO4)

I. How many students score between 12 and 15?
II. How many score above 18?
III. How many score below 8?
IV. How many score 16?

Solution: $n = 1000, \quad \mu = 14, \quad \sigma = 2.5$

(i) Scroe b/w 12 & 15 : $z_1 = \frac{12-14}{2.5} = -0.8$

$z_2 = \frac{15-14}{2.5} = 0.4$



$P(12 < x < 15) = P(z_1 < z < z_2) = 0.2881 + 0.1554$

$= 0.4436 \times 1000$

$= 443.6$ students

≈ 444 students

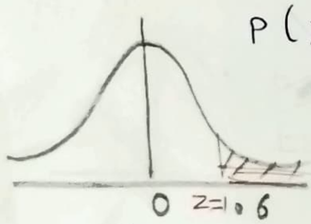
(ii) Score below 18 : $z = \frac{18-14}{2.5} = \frac{4}{2.5} = 1.6$

$P(x > 18) = P(z > 1.6) = 0.5 - P(0 < z < 1.6)$

$= 0.5 - 0.4452$

$= 0.0548 \; (\times 1000)$

≈ 55 ~~per~~ students



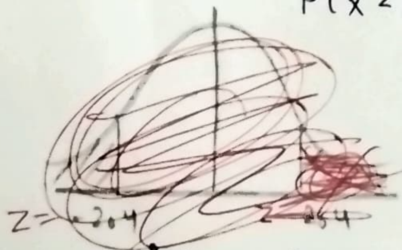
(iii) Score below 8 : $z = \frac{8-14}{2.5} = -\frac{6}{2.5} = -2.4$

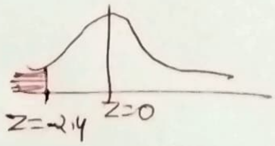
$P(x < 8) = P(z < -2.4) = 0.5 - P(0 < z < -2.4)$

$= 0.5 - 0.4918$

$= 0.0082 \; (\times 1000)$

≈ 8 students





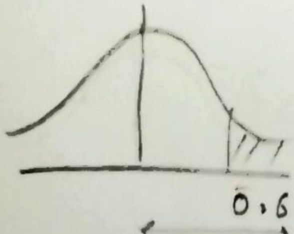
(iv) Score 16 : $P(15.5 < x < 16.5)$

$= P(0.6 < z < 1)$

$= 0.3413 - 0.2257$

$= \cancel{0.0886}$ $0.1156 \; (\times 1000)$

$=$ ~~8 students~~ 116 students



unit-2

workbook(MCQ)

$\boxed{\text{Ans 30}}$

$\bar{x}$, mean $= 50$

standard deviation $= 9$

$n = 200$

LOS $= 95\%$

Formula: for confidence interval (LOS - 95%) for large sample

$$= \bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

$$= 50 \pm 1.96 \times \frac{9}{\sqrt{200}}$$

$$= 50 - 1.96 \times \frac{9}{\sqrt{200}} \quad \& \quad 50 + 1.96 \times \frac{9}{\sqrt{200}}$$

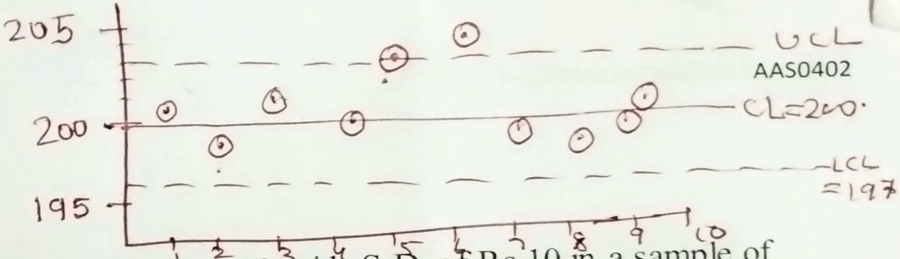
$$= 48.753 \quad \& \quad 51.247$$

General formula for confidence interval

$$= \bar{x} \pm Z_{\alpha}(\text{LOS}) \times \frac{\sigma}{\sqrt{n}}$$

this is according to LOS

Unit 2
Ans 42



z-test

41. The average income of persons was Rs 210 with S.D. of Rs 10 in a sample of 100 people of a city. For another sample of 150 persons, the average income was Rs 220 with S.D. of Rs 12. The S.D. of income of the people of the city
... difference between the average

24. Calculate the rank correlation coefficient between X and Y from the following data-(CO1)

X	15	20	27	13	45	60	20	75
Y	50	30	55	30	25	10	30	70

Solution:

X	Y	R_x / R_1	R_y / R_2	$D_i = R_x - R_y$	D_i^2
15	50	7	3	4	16
20	30	5.5	5	.5	.25
27	55	4	2	2	4
13	30	8	5	3	9
45	25	3	7	-4	16
60	10	2	8	-6	36
20	30	5.5	5	.5	.25
75	70	1	1	0	0
					81.5

$m_1 = 2, \quad m_2 = 3$

$$r = 1 - \frac{6\left[\Sigma D_i^2 + \frac{1}{12} m_1 (m_1^2 - 1) + \frac{1}{12} m_2 (m_2^2 - 1)\right]}{N(N^2-1)}$$

$$= 1 - \frac{6\left[81.5 + \frac{1}{12} \times 2 \times 3 + \frac{1}{12} \times 3 \times 8\right]}{8 \times 63}$$

$$= 1 - \frac{6\left[81.5 + 0.5 + 2\right]}{8 \times 63} = 1 - \frac{6[84]}{8 \times 63}$$

$$= 1 - \frac{504}{504} = 1 - 1 = 0$$

Conclusion: No correlation between marks of A & B.