School of Physics and Astronomy

Mphys project
Log book

Sahl Rowther
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Abstract

Supervisor: Prof. Ken Rice   22 Weeks

Contents

Week 1: September 11^{th}–17^{th} 2017

1.1 Storing planet data

Aim: To write a class that can store various properties of a planet. The class can take in the following properties:

- *Name* - the planets name.
- *mass* - the mass of the planet ($M_{\oplus}$).
- a - the orbital radius (AU).
- n - the orbital frequency ($^{\circ}\,yr^{-1}$).
- e - the eccentricity.
- i - the inclination (degrees).
- Ω - the longitude of ascending node (degrees).
- ϖ - the longitude of pericentre (degrees).

Code:

```python
class planet():
    def __init__(self, Name="", Period=None, e=None, a=None, i=None, Omega=None, omega_bar=None, Mass=None, n=None):
        self.name = Name
        self.period = Period
        self.e = e
        self.a = a
        self.i = i
        self.omega = Omega
        self.omega_bar = omega_bar
        self.mass = Mass
        self.n = n
        self.units = {'a' : 'AU', 'mass' : 'M_EARTH', 'period' : 'days', 'i' : 'degrees', 'omega' : 'degrees', 'omega_bar' : 'degrees', 'n' : 'degrees yr^(-1)'}

    def toString(self):
        unit_keys = list(self.units.keys())
        for attr in self.__dict__:
            if attr is not 'units':
                if self.__dict__[attr] is not None:
                    if attr in unit_keys:
                        print('{} : {} {}'.format(attr, self.__dict__[attr], self.units[attr]))
                    else:
                        print('{} : {}'.format(attr, self.__dict__[attr]))
        print()
```

Code 1: Planet object

Test code:

```
import pandas as pd

planets = pd.read_csv('solar_system.csv')
planet_b = planet(**planets.ix[2])
planet_b.toString()
```

Code 2: Test of planet object

Output:

```
name : Earth
e : 0.01671022
a : 1.00000011 AU
i : 5e-05 degrees
omega : 348.73936000000003 degrees
omega_bar : 102.94719 degrees
mass : 1.000167431 M_EARTH
n : 359.7480668 degrees yr^(-1)
```

Vedict: Test successful

1.2 Storing star system data

Aim: Create a class that stores the mass and radius of the central body. And also stores all the planets as a list. The class takes the following arguments:

- starMass - the mass of the star.
- starRadius - the radius of the star.
- planet_data_file - a file containing a list of planets with properties described in Section 1.1.

Code:

```
from planet import planet

class starSystem():

    def __init__(self, starMass, starRadius, planet_data_file):
        self.star_mass = starMass
        self.star_radius = starRadius
        self.planets = self.addPlanets(planet_data_file)

    def addPlanets(self, planet_data_file):
        planets = pd.read_csv(planet_data_file)
        planet_list = []
        for p in range(len(planets)):
            planet_list.append(planet(**planets.ix[p]))
```

```
        return planet_list

    def print_planets(self):
        print('Star mass =', self.star_mass, 'Msun')
        print('Star radius = ', self.star_radius, 'Rsun\n')
        for p in self.planets:
            p.toString()
```

Code 3: Star system object

Data: For testing, data from the HD3167 system were used.

Table 1: HD3167 planet data. Period is in days, a is in AU, Mass is in $M_\oplus$, i and Ω are in degrees.

Name	Period	a	Mass	i	e	Ω
b	0.959641	0.01815	5.02	0	0	0
c	29.8454	0.1795	9.8	0	0.267	0
d	8.509	0.07757	6.9	20	0.36	0

The mass and radius of the star is $0.86\,M_\odot$ and $0.86\,R_\odot$.

Test code:

```
import pandas as pd

star_system = starSystem(0.86, 0.86, 'Planets.csv')
star_system.print_planets()
```

Code 4: Test of star system object

Output:

```
Star mass = 0.86 Msun
Star radius =  0.86 Rsun

name : b
period : 0.959641 days
e : 0.0
a : 0.01815 AU
i : 0 degrees
omega : 0 degrees
mass : 5.02 M_EARTH

name : c
period : 29.8454 days
e : 0.267
```

```
a : 0.1795 AU
i : 0 degrees
omega : 0 degrees
mass : 9.8 M_EARTH

name : d
period : 8.509 days
e : 0.36
a : 0.07757 AU
i : 20 degrees
omega : 0 degrees
mass : 6.9 M_EARTH
```

Verdict: Test successful. All planetary data and star data stored successful.

1.3 Replicating inclination output

```
def get_property_all_planets(self, property_name, data_type="float"):
    property_list = np.zeros(len(self.planets), dtype=data_type)
    for idx, p in enumerate(self.planets):
        property_list[idx] = p.__dict__[property_name]

    return property_list
```

Code 5: Helper function to get a property value of all planets

Using Laplace-Lagrange secular theory, the equations of motion for the complex inclination vector, $z = i\exp(\imath\Omega)$, where i is the inclination and Ω is the ascending node, can be simplified to a linear eigenvalue problem:

$$\frac{dz_j}{dt} = i\sum_{k=1}^{N-1} B_{jk} z_k. \tag{1}$$

The frequency matrix $\mathbf{B}$ is only dependent on the mass and semi-major axis ratios of the planets, and is given by

$$B_{jj} = -\frac{n_j}{4}\sum_{k=0,\, k\neq j}^{N-1} \frac{m_k}{M_\star}\alpha_{jk}\bar{\alpha}_{jk} b_{3/2}^{(1)}(\alpha_{jk}), \tag{2a}$$

$$B_{jk} = -\frac{n_j}{4}\frac{m_k}{M_\star}\alpha_{jk}\bar{\alpha}_{jk} b_{3/2}^{(1)}(\alpha_{jk}). \tag{2b}$$

Where $n = \sqrt{GM_\star/a^3}$ is the mean orbital frequency, α_{jk} is the semi-major axis ratio given by

$$\alpha_{jk} = \begin{cases} a_j/a_k; & \text{if } a_j < a_k \\ a_k/a_j; & \text{if } a_k < a_j \end{cases} \tag{3}$$

and $b_{3/2}^{(1)}(\alpha)$ is the Laplace coefficient given by

$$b_{3/2}^{(1)}(\alpha) = \frac{1}{\pi}\int_0^{2\pi}\left[\frac{\cos\psi}{(1+\alpha^2-2\alpha\cos\psi)^{3/2}}\right]d\psi \tag{4}$$

```python
import numpy as np
from scipy import integrate

M_SUN = 1.9885*10**30
R_SUN = 6.9551*10**8
M_EARTH = 5.9726*10**24
AU = 149597870700

    def laplace_coefficient(self, alpha):
        integral_func = lambda psi, alpha: np.cos(psi)/(1+alpha**2-(2*alpha*np.cos(psi)))**(3./2.)
        return 1/np.pi*integrate.quad(integral_func, 0, 2*np.pi, args=(alpha,))[0]

    def matrix_B_eigenmodes(self):
        G_const = 6.6738*10**(-11)
        a = AU*self.get_property_all_planets('a')
        M_star_kg = M_SUN*self.star_mass
        n = np.sqrt(G_const*M_star_kg/a**3)

        m = M_EARTH*self.get_property_all_planets('mass')

        n_planets = len(self.planets)
        B = np.zeros([n_planets, n_planets])

        for j in range(n_planets):
            for k in range(n_planets):
                if j != k:
                    alpha_jk = a[j]/a[k]
                    if alpha_jk > 1:
                        alpha_jk = alpha_jk**(-1)
                    laplace_coeff = self.laplace_coefficient(alpha_jk)
                    alpha_jk_bar = np.where(a[k] < a[j], 1, alpha_jk)
                    B[j, k] = (n[j]/4)*(m[k]/M_star_kg)*alpha_jk*alpha_jk_bar*laplace_coeff
                else:
                    for kk in range(n_planets):
                        if kk != j:
                            alpha_jj = a[j]/a[kk]
                            if alpha_jj > 1:
                                alpha_jj = alpha_jj**(-1)
                            laplace_coeff = self.laplace_coefficient(alpha_jj)
                            alpha_jj_bar = np.where(a[kk] < a[j], 1, alpha_jj)
                            B[j, k] += (m[kk]/M_star_kg)*alpha_jj*alpha_jj_bar*laplace_coeff
                    B[j, k] *= -(n[j]/4)
        eigenvalues, eigenvectors = np.linalg.eig(B)
        return B, eigenvalues, eigenvectors
```

Code 6: Calculate the frequency matrix, **B**

Week 2: September 18^{th}–24^{th} 2017

Week 3: September 25^{th}–31^{st} 2017

3.1 Simulation of solar system

Storing the data

The planetary data for simulating the Solar System is given below.

Table 2: Solar System data. The mass in terms of $M_\oplus$ is given by m. The mean orbital frequency in degrees per year is given by n. The value of the semi-major axis in AU is given by a. The eccentricity of the orbit is given by e. The inclination of the orbit in degrees is given by i. The longitudes of pericentre and ascending node are given in degrees by ϖ and Ω respectively.

Name	m	n	a	e	i	ϖ	Ω
Mercury	0.055	1493.708	0.387	0.206	7.005	77.456	48.332
Venus	0.815	584.779	0.723	0.007	3.395	131.533	76.681
Earth	1.000	359.748	1.000	0.017	0.000	102.947	348.739
Mars	0.107	191.278	1.524	0.093	1.851	336.041	49.579
Jupiter	317.885	30.309	5.203	0.048	1.305	14.754	100.556
Saturn	95.178	12.215	9.537	0.054	2.484	92.432	113.715
Uranus	14.538	4.279	19.191	0.047	0.770	170.964	74.230
Neptune	17.150	2.182	30.069	0.009	1.769	44.971	131.722
Pluto	0.002	1.450	39.482	0.249	17.142	224.067	110.303

```
import numpy as np
import numpy.ma as ma
from scipy import integrate
import scipy.linalg
from scipy.optimize import fsolve
from sympy import symbols, Matrix, linsolve, diag
import matplotlib.pyplot as plt
from planet import planet

class solar_System():

    def __init__(self, starMass, starRadius, planet_data_file):
        self.star_mass = starMass
        self.star_radius = starRadius
        self.planets = self.addPlanets(planet_data_file)

    def addPlanets(self, planet_data_file):
        planets = pd.read_csv(planet_data_file)
        planet_list = []
        for p in range(len(planets)):
            planet_list.append(planet(**planets.ix[p]))
        return planet_list

```

```
    def get_property_all_planets(self, property_name, data_type="float"):
        property_list = np.zeros(len(self.planets), dtype=data_type)
        for idx, p in enumerate(self.planets):
            property_list[idx] = p.__dict__[property_name]
        return property_list
```

Code 7: Object for storing the data

Solving the equations of motion

The expression for the disturbing function, $\mathcal{R}_j$ is given by:

$$\begin{aligned}\mathcal{R}_j = n_j a_j^2 \Bigg[&\frac{1}{2} A_{jj} \left(h_j^2 + k_j^2\right) + \frac{1}{2} B_{jj} \left(p_j^2 + q_j^2\right) \\ &+ \sum_{i \neq j} A_{ji} \left(h_j h_i + k_j k_i\right) + \sum_{i \neq j} B_{ji} \left(p_j p_i + q_j q_i\right) \Bigg]\end{aligned} \tag{5}$$

Where n_j is the mean orbital frequency, a_j is the semi-major axis, and **A** and **B** are the frequency matrices defined as:

$$\begin{aligned}A_{jj} = n_j \Bigg[&\frac{3}{2} J_2 \left(\frac{R_\star}{a_j}\right)^2 - \frac{9}{8} J_2^2 \left(\frac{R_\star}{a_j}\right)^4 - \frac{15}{4} J_4^2 \left(\frac{R_\star}{a_j}\right)^4 \\ &+ \frac{1}{4} \sum_{k \neq j} \frac{m_k}{m_\star + m_j} \alpha_{jk} \bar{\alpha}_{jk} b_{3/2}^{(1)}(\alpha_{jk}) \Bigg]\end{aligned} \tag{6a}$$

$$A_{jk} = -\frac{n_j}{4} \frac{m_k}{m_\star + m_j} \alpha_{jk} \bar{\alpha}_{jk} b_{3/2}^{(2)}(\alpha_{jk}) \qquad (j \neq k) \tag{6b}$$

$$\begin{aligned}B_{jj} = -n_j \Bigg[&\frac{3}{2} J_2 \left(\frac{R_\star}{a_j}\right)^2 - \frac{27}{8} J_2^2 \left(\frac{R_\star}{a_j}\right)^4 - \frac{15}{4} J_4^2 \left(\frac{R_\star}{a_j}\right)^4 \\ &+ \frac{1}{4} \sum_{k \neq j} \frac{m_k}{m_\star + m_j} \alpha_{jk} \bar{\alpha}_{jk} b_{3/2}^{(1)}(\alpha_{jk}) \Bigg]\end{aligned} \tag{6c}$$

$$B_{jk} = \frac{n_j}{4} \frac{m_k}{m_\star + m_j} \alpha_{jk} \bar{\alpha}_{jk} b_{3/2}^{(1)}(\alpha_{jk}) \qquad (j \neq k). \tag{6d}$$

Where m is the mass, $\alpha < 1$ is the semi-major axis ratio, $\bar{\alpha} = 1$ if $a_k < a_j$, $\bar{\alpha} = \alpha$ if $a_j < a_k$, J_2 and J_4 are the first two zonal gravity coefficients, and the laplace coefficients are defined by:

$$b_s^{(j)}(\alpha) = \frac{1}{\pi} \int_0^{2\pi} \left[\frac{cos(j\psi)}{(1 + \alpha^2 - 2\alpha \cos\, \psi)^s} \right] d\psi. \tag{7}$$

Where s is a positive half integer, and j is an integer.

```
def calculate_laplace_coeff(alpha, j, s):
    return integrate.quad(lambda psi, alpha, j, s: np.cos(j*psi)/(1-2*alpha*np.cos(psi)+alpha**2)**s,
                          0, 2*np.pi, args=(alpha, j, s,))[0]/np.pi
```

Code 8: Calculating the laplace coefficient

And the vertical and horizontal components of the eccentricity and inclination are given by:

$$h_j = e_j \cos \varpi_j \tag{8a}$$

$$k_j = e_j \sin \varpi_j \tag{8b}$$

$$p_j = i_j \cos \Omega j \tag{8c}$$

$$q_j = i_j \sin \Omega j \tag{8d}$$

Where e_j is the eccentricity, i_j is the inclination, and ϖ_j and Ω_j are the longitude of pericentre and ascending node respectively.

Code:

```
    def frequency_matrix(self, matrix_id, J2=0, J4=0):
        M_star_kg = M_SUN*self.star_mass
        R = R_SUN*self.star_radius
        m = M_EARTH*self.get_property_all_planets('mass')
        n = self.get_property_all_planets('n')
        a = AU*self.get_property_all_planets('a')
        n_planets = len(self.planets)
        f_mat = np.zeros([n_planets, n_planets])

        if matrix_id == 'A':
            j_laplace_coeff_jk, j_laplace_coeff_jj = 2, 1
            front_factor = -1
            J2_correction = (((3/2)*J2*(R/a)**2)-((9/8)*(J2**2)*(R/a)**4)-((15/4)*J4*(R/a)**4))

        if matrix_id == 'B':
            j_laplace_coeff_jk = j_laplace_coeff_jj = 1
            front_factor = 1
            J2_correction = (((3/2)*J2*(R/a)**2)-((27/8)*(J2**2)*(R/a)**4)-((15/4)*J4*(R/a)**4))

        for j in range(n_planets):
            for k in range(n_planets):
                if j != k:
                    alpha_jk = a[j]/a[k]
                    if alpha_jk > 1:
                        alpha_jk = alpha_jk**(-1)
                    laplace_coeff = calculate_laplace_coeff(alpha_jk, j_laplace_coeff_jk, 3/2)
                    alpha_jk_bar = np.where(a[k] < a[j], 1, alpha_jk)
                    f_mat[j, k] = front_factor*(n[j]/4)*(m[k]/(M_star_kg+m[j]))*alpha_jk*alpha_jk_bar*laplace_coeff

                else:
                    for kk in range(n_planets):
```

```python
                    if kk != j:
                        alpha_jj = a[j]/a[kk]
                        if alpha_jj > 1:
                            alpha_jj = alpha_jj**(-1)
                        laplace_coeff = calculate_laplace_coeff(alpha_jj, j_laplace_coeff_jj, 3/2)
                        alpha_jj_bar = np.where(a[kk] < a[j], 1, alpha_jj)
                        f_mat[j, k] += (1/4)*(m[kk]/(M_star_kg+m[j]))*alpha_jj*alpha_jj_bar*laplace_coeff
                f_mat[j, k] += J2_correction[j]
                f_mat[j, k] *= -front_factor*(n[j])
        return f_mat
```

Code 9: Calculating **A** and **B**

Using **A** and **B**, the equations of motion in equations 8a to 8d can be reduced to two sets of eigenvalue problems, whose solutions are given by:

$$h_j = \sum_{i=0}^{N-1} e_{ji} \sin(g_i t + \beta_i), \qquad k_j = \sum_{i=0}^{N-1} e_{ji} \cos(g_i t + \beta_i) \tag{9a}$$

and

$$p_j = \sum_{i=0}^{N-1} I_{ji} \sin(f_i t + \gamma_i), \qquad q_j = \sum_{i=0}^{N-1} I_{ji} \cos(f_i t + \gamma_i). \tag{9b}$$

Where e_{ji} and I_{ji} are the scaled components of the eigenvectors of **A** and **B**. The frequencies g_i and f_i are the eigenvalues of **A** and **B**. The scaled eigenvectors can be expressed as:

$$S_i \bar{e}_{ji} = e_{ji} \qquad \text{and} \qquad T_i \bar{I}_{ji} = I_{ji}. \tag{10}$$

Where $\bar{e}_{ji}$ and $\bar{I}_{ji}$ are the normalised eigenvectors of **A** and **B**. The phases β_i and γ_i, as well as the scaling factors of the eigenvectors S_i and T_i are determined by the initial conditions.

Using the data in Table 2 and equations 8a to 8d, the initial conditions can be calculated.

```python
    def initial_conditions(self):
        e = self.get_property_all_planets('e')
        omega_bar = self.get_property_all_planets('omega_bar')*np.pi/180
        i = self.get_property_all_planets('i')*np.pi/180
        omega = self.get_property_all_planets('omega')*np.pi/180

        h = e*np.sin(omega_bar)
        k = e*np.cos(omega_bar)
        p = i*np.sin(omega)
        q = i*np.cos(omega)

        return h, k, p, q
```

Code 10: Calculating initial conditions

Using the calculated values of $\bar{e}_{ji}$ and by evaluating h_j in equation 9a at $t = 0$ and equating it to h_j from equation 8a, an augmented matrix can be created to solve for $S_i \sin\beta_i$, as shown below.

$$\left[\begin{array}{cccc|c} S_0\sin(\beta_0)\,\bar{e}_{00} & S_1\sin(\beta_1)\,\bar{e}_{01} & \cdots & S_{N-1}\sin(\beta_{N-1})\,\bar{e}_{0,N-1} & h_0 \\ S_1\sin(\beta_0)\,\bar{e}_{10} & S_1\sin(\beta_1)\,\bar{e}_{11} & \cdots & S_{N-1}\sin(\beta_{N-1})\,\bar{e}_{1,N-1} & h_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ S_{N-1}\sin(\beta_0)\,\bar{e}_{N-1,0} & S_{N-1}\sin(\beta_1)\,\bar{e}_{N-1,1} & \cdots & S_{N-1}\sin(\beta_{N-1})\,\bar{e}_{N-1,N-1} & h_{N-1} \end{array}\right] \tag{11}$$

A similar process can be done with k_j to solve for $S_i \cos\beta_i$:

$$\left[\begin{array}{cccc|c} S_0\cos(\beta_0)\,\bar{e}_{00} & S_1\cos(\beta_1)\,\bar{e}_{01} & \cdots & S_{N-1}\cos(\beta_{N-1})\,\bar{e}_{0,N-1} & h_0 \\ S_1\cos(\beta_0)\,\bar{e}_{10} & S_1\cos(\beta_1)\,\bar{e}_{11} & \cdots & S_{N-1}\cos(\beta_{N-1})\,\bar{e}_{1,N-1} & h_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ S_{N-1}\cos(\beta_0)\,\bar{e}_{N-1,0} & S_{N-1}\cos(\beta_1)\,\bar{e}_{N-1,1} & \cdots & S_{N-1}\cos(\beta_{N-1})\,\bar{e}_{N-1,N-1} & h_{N-1} \end{array}\right] \tag{12}$$

Solving the above two matrices gives one set of equations in terms of $S_i \sin\beta_i$ and another set of equations in terms of $S_i \cos\beta_i$. Solving them simultaneously results in values for S_i and β_i. A similar process can be done to solve for T_i and γ_i.

```
def scaling_factor_and_phase(p, *boundaries):
    s, phase = p
    return (s*np.sin(phase)-boundaries[0], s*np.cos(phase)-boundaries[1])
```

Code 11: Equations for simultaneously solving for the scale factor and phase in Code 12

```
    def solve_property(self, eigenvectors, initial_conditions):
        n = len(self.planets)
        aug = Matrix(np.zeros([n, n+1]))
        aug[:, :n] = eigenvectors
        aug[:, n] = initial_conditions

        result = linsolve(aug, *symbols('x0:'+str(n)))
        answers = np.zeros(n)
        for ans in result:
            for a, answer in enumerate(ans):
                answers[a] = answer
        return answers

    def find_all_scaling_factor_and_phase(self, eigenvectors_of_A, eigenvectors_of_B):
        x, y = eigenvectors_of_A, eigenvectors_of_B

        init_conditions = np.array(star_system.initial_conditions())
        h_solved = self.solve_property(x, init_conditions[0, :])
        k_solved = self.solve_property(x, init_conditions[1, :])
        p_solved = self.solve_property(y, init_conditions[2, :])
        q_solved = self.solve_property(y, init_conditions[3, :])

        n = len(self.planets)
```

```
        S, beta = np.zeros(n), np.zeros(n)
        T, gamma = np.zeros(n), np.zeros(n)

        for i in range(n):
            S[i], beta[i] = fsolve(scaling_factor_and_phase, (1, -1), args=(h_solved[i], k_solved[i],))
            T[i], gamma[i] = fsolve(scaling_factor_and_phase, (-1, 1), args=(p_solved[i], q_solved[i],))
        return S, beta, T, gamma
```

Code 12: Calculating the scale factors and phases

Once the scale factors and phases have been found, equations 9a and 9b can now be solved at any time t.

```
    def components_of_ecc_inc(self, scaled_eigenvector, eigenvalue, phase, t, eq_id):
        # eq_id = 'h', 'k', 'p', 'q'
        kwargs = {'scaled_eigenvector' : scaled_eigenvector, 'eigenvalue' : eigenvalue, 'phase' : phase, 't' :
    ↪  t}
        if eq_id == 'h' or eq_id == 'p':
            return self.get_h_or_p(**kwargs)
        if eq_id == 'k' or eq_id == 'q':
            return self.get_k_or_q(**kwargs)

    def get_h_or_p(self, scaled_eigenvector, eigenvalue, phase, t):
        n = len(self.planets)
        h_list = []
        for j in range(n):
            h = np.zeros_like(t)
            for i in range(n):
                h += scaled_eigenvector[j, i]*np.sin((eigenvalue[i]*t+phase[i])*np.pi/180)
            h_list.append(h)
        return np.array(h_list)

    def get_k_or_q(self, scaled_eigenvector, eigenvalue, phase, t):
        n = len(self.planets)
        k_list = []
        for j in range(n):
            k = np.zeros_like(t)
            for i in range(n):
                k += scaled_eigenvector[j, i]*np.cos((eigenvalue[i]*t+phase[i])*np.pi/180)
            k_list.append(k)
        return np.array(k_list)
```

Code 13: Calculating the vertical and horizontal components of the eccentricity and inclination

Finally, the eccentricity and inclination at any time t can be calculated using:

$$e_j(t) = \left(h_j^2 + k_j^2\right)^{1/2} \tag{13a}$$

$$i_j(t) = \left(p_j^2 + q_j^2\right)^{1/2} \tag{13b}$$

```
    def get_eccentricity(self, h_arr, k_arr):
        n = len(self.planets)
```

```
        h, k = h_arr, k_arr
        eccentricities = []
        for j in range(n):
            eccentricities.append(np.real(np.sqrt(h[j]*np.conjugate(h[j])+k[j]*np.conjugate(k[j]))))
        return np.array(eccentricities)

    def get_inclination(self, p_arr, q_arr):
        n = len(self.planets)
        p, q = p_arr, q_arr
        inclinations = []
        for j in range(n):
            inclinations.append(np.real(np.sqrt(p[j]*np.conjugate(p[j])+q[j]*np.conjugate(q[j]))))
        return np.array(inclinations)
```

Code 14: Calculating the eccentricity and inclination

The perihelion precession rate, $\dot{\varpi}$ can be found as follows. First equations 8a and 8b can be rearranged for ϖ as,

$$\tan\varpi = \frac{h_j}{k_j}. \tag{14}$$

Differentiating, using the chain rule, with respect to time gives,

$$\begin{aligned} \frac{1}{\cos^2\varpi}\frac{d\varpi}{dt} &= \frac{\frac{dh_j}{dt}k_j - \frac{dk_j}{dt}h_j}{k_j^2} \\ \frac{k_j^2}{\cos^2\varpi}\dot{\varpi} &= \dot{h}_j k_j - \dot{k}_j h_j \\ \dot{\varpi} &= \frac{\dot{h}_j k_j - \dot{k}_j h_j}{e_j^2}. \end{aligned} \tag{15}$$

Where in the last step, the substitution $e_j = h_j/\cos\varpi$ (from equation 8b) was used. The time derivatives of h_j and k_j can be found using the disturbing function:

$$\dot{h}_j = \frac{1}{n_j a_j^2}\frac{\partial\mathcal{R}_j}{\partial k_j}, \qquad \dot{k}_j = -\frac{1}{n_j a_j^2}\frac{\partial\mathcal{R}_j}{\partial h_j}. \tag{16}$$

Which then become:

$$\dot{h}_j = \sum_{i=0}^{N-1} A_{ji}k_i, \qquad \dot{k}_j = -\sum_{i=0}^{N-1} A_{ji}h_i. \tag{17}$$

Where the components of A_{ji} are described in equations 6a and 6b.

```
    def get_perihelion_precession_rates(self, A, eccentricities, h_list, k_list):
        n = len(self.planets)
        d_pidot_dt_list = []
        masks = []

        for j in range(n):
```

```
        h_dot_j, k_dot_j = 0, 0
        for i in range(n):
            h_dot_j += A[j, i]*k_list[i]
            k_dot_j -= A[j, i]*h_list[i]
        pidot_j = 3600*(k_list[j]*h_dot_j - h_list[j]*k_dot_j)/(eccentricities[j])**2
        d_pidot_dt_list.append(pidot_j)
    return d_pidot_dt_list
```

Code 15: Calculating precession rate, $\dot{\varpi}$ of Mercury

3.2 Tests of simulation

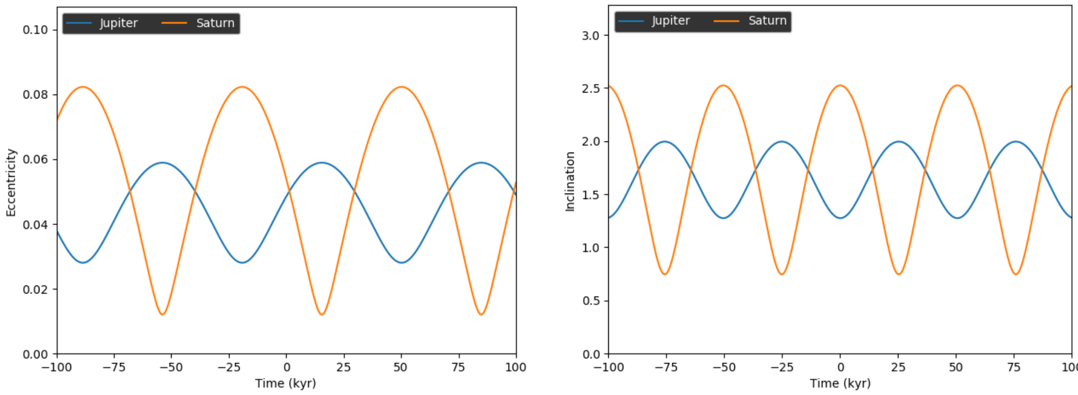
The following code was used to test the simulation.

```
def simulate(self, t, plot=False, separate=True):
    A, B = [star_system.frequency_matrix(matrix_id=mat_id, J2=-6.84*10**(-7), J4
    =2.8*10**(-12)) for mat_id in ['A', 'B']]
    g, x, f, y = *np.linalg.eig(A), *np.linalg.eig(B)
    S, beta, T, gamma = self.find_all_scaling_factor_and_phase(x, y)

    eccentricities = self.get_eccentricity(S*x, g, beta, t)
    inclinations = self.get_inclination(T*y, f, gamma, t)*180/np.pi
    names = [self.planets[p].name for p in range(len(self.planets))]
    if plot:
        if separate:
            plot_simulation_separate(t/10**6, eccentricities, 'Time (Myr)', 'Eccentricity', names)
            plot_simulation_separate(t/10**6, inclinations, 'Time (Myr)', 'Inclination', names)
        else:
            plot_simulation_all(t/10**6, eccentricities, 'Time (Myr)', 'Eccentricity', names)
            plot_simulation_all(t/10**6, inclinations, 'Time (Myr)', 'Inclination', names)

    kwargs = {'scaled_eigenvector' : S*x, 'eigenvalue' : g, 'phase' : beta,
              't' : t}
    h_list = self.eq_of_motion(**kwargs, eq_id='h')
    k_list = self.eq_of_motion(**kwargs, eq_id='k')
    kwargs = {'scaled_eigenvector' : S*x, 'eigenvalue' : f, 'phase' : gamma,
              't' : t}
    p_list = self.eq_of_motion(**kwargs, eq_id='p')
    q_list = self.eq_of_motion(**kwargs, eq_id='q')

    precession_rates = self.get_perihelion_precession_rates(A, eccentricities, h_list, k_list)

    idx = 0
    plot_precession_rate(t, precession_rates[idx], 'Mercury')
    plot_eccentricity(t, eccentricities[idx], 'Mercury')
```

Code 16: Test code for simulation

3.2.1 Jupiter and Saturn

The first test is to replicate the eccentricity and inclination outputs in Figure 7.1 of Murray & Dermott (1999) [1].



(a) Evolution of eccentricity of Jupiter and Saturn.

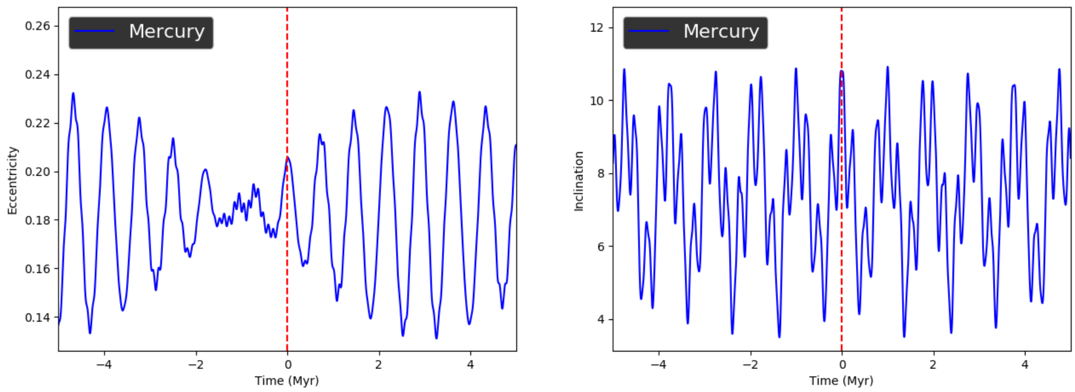
(b) Evolution of inclination of Jupiter and Saturn.

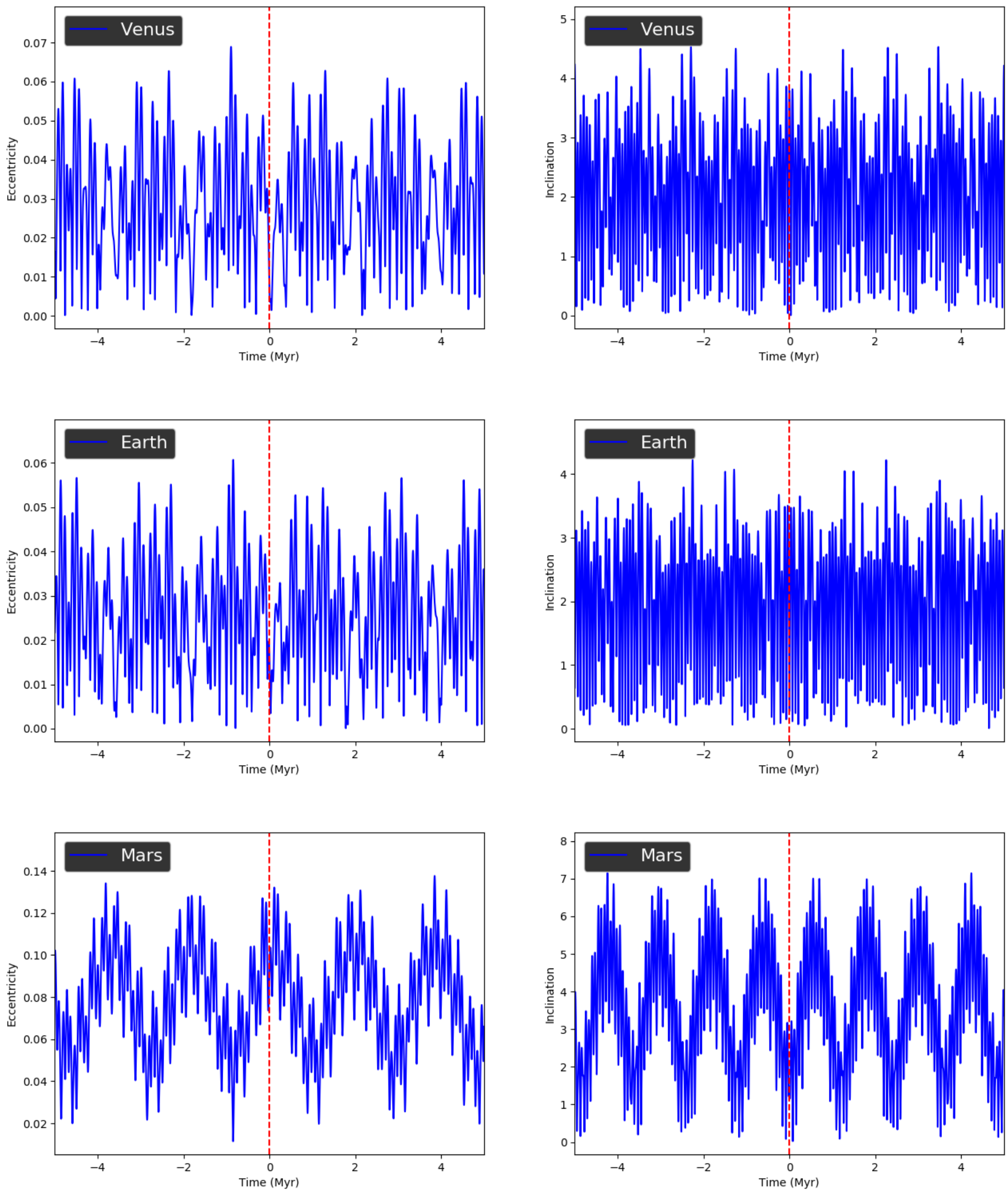
Figure 1

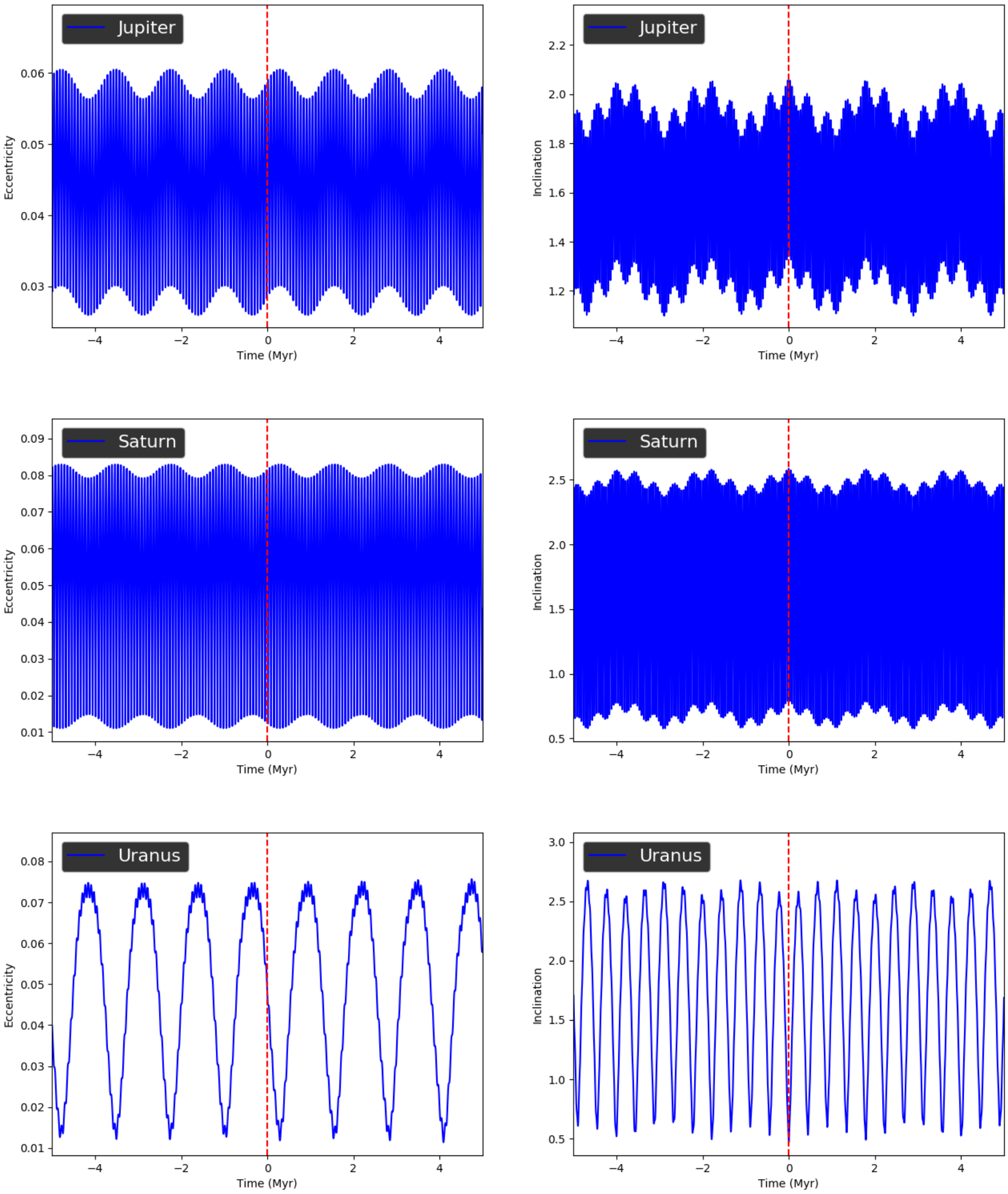
Verdict: Figure 1 is a very good match to that of the output in Murray & Dermott (1999) [1].

3.2.2 Whole Solar System

The plots for the eccentricity and inclination of each planet are as shown:




Venus
Eccentricity
Time (Myr)
Venus
Inclination
Time (Myr)
Earth
Eccentricity
Time (Myr)
Earth
Inclination
Time (Myr)
Mars
Eccentricity
Time (Myr)
Mars
Inclination
Time (Myr)


Jupiter
Eccentricity
Time (Myr)
Jupiter
Inclination
Time (Myr)
Saturn
Eccentricity
Time (Myr)
Saturn
Inclination
Time (Myr)
Uranus
Eccentricity
Time (Myr)
Uranus
Inclination
Time (Myr)

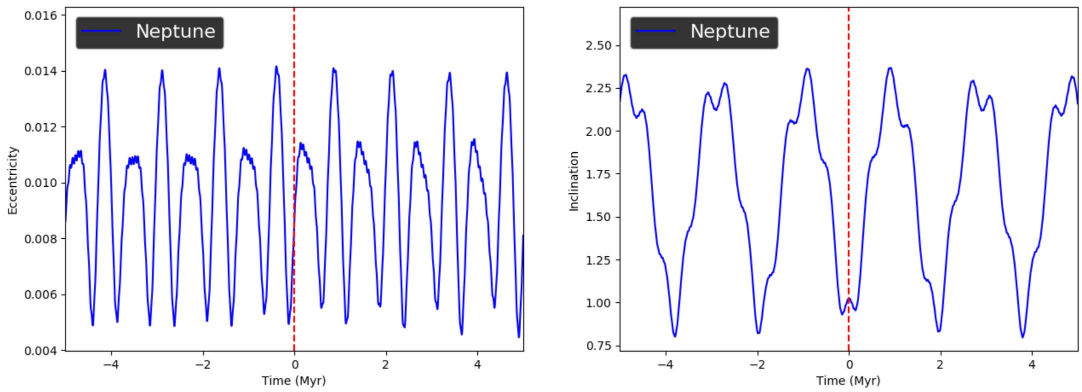


Figure 2: Eccentricity and inclination (in degrees) of each planet in the Solar System using J2000 data.

Verdict

By comparing these plots with existing results[1,2], it can be seen that the results are consistent with what is expected. It should be noted that the eccentricity plots do not match with Murray & Dermott as well as the inclinations. However when comparing the max and min eccentricity (especially Mercury) to the other data[2], the eccentricities do match well. Oddly, the inclinations in the other data[2] do not match as well; has lower I_{min} and higher I_{max}.

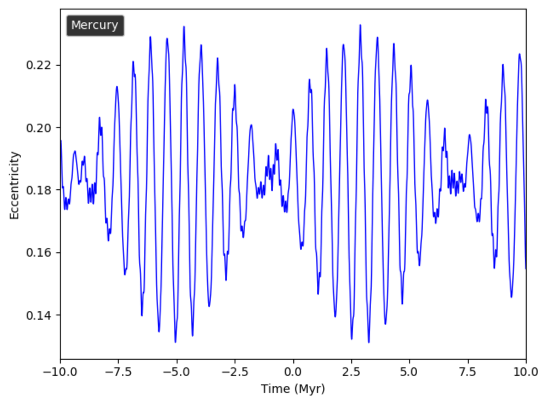
3.2.3 Precession of Mercury

Another test that serves as a good indicator of the accuracy of the simulation is determining the precession of Mercury. Applying Laplace-Lagrange secular theory is expected to yield a precession rate, $\dot{\varpi}$ of $544''yr^{-1}$[2,3].

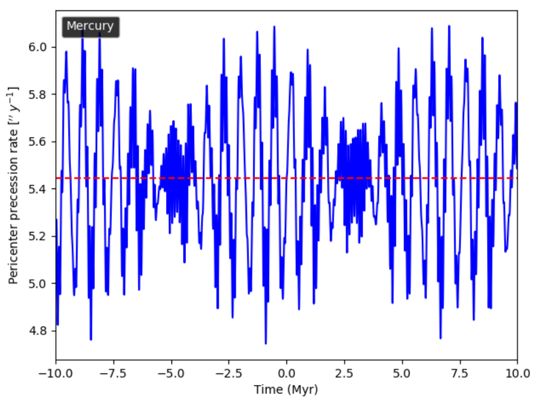
Verdict

From Figure 5b, it can be seen the mean precession, the red dashed line is equal to 544.86″ per century; consistent with expectations and without the addition of General Relativity. The plot of the precession rate also matched with expected results[2].

The effect of the oblateness of the Sun on the precession rate of Mercury was also found to be $\sim 0.08''$ per century. The small magnitude of the change is as expected.



(a) Evolution of eccentricity of Mercury.



(b) Precession rate, $\dot{\varpi}$ of Mercury.

Figure 3

Week 4: October 1^{st}–8^{th} 2017

4.1 Keplerian to Cartesian coordinates

The next test to check accuracy is to convert the output of the equations of motion to cartesian coordinates in order to plot the Solar System for visual inspection. For this we use:

- h, k, p, q - the equations of motion.
- e - the eccentricity.
- i - the inclination (in radians).
- a - the semi-major axis (in AU).
- n - the orbital frequency (in radians per year)

First the mean anomaly, $M(t)$ is found using

$$M(t) = n(t - t_0). \tag{18}$$

Then the eccentric anomaly $E \equiv E(t)$ is calculated by solving

$$M(t) = E(t) - e \sin E(t) \tag{19}$$

using the Newton-Raphson method:

$$E_{j+1} = E_j - \frac{f(E_j)}{\frac{d}{dE_j} f(E_j)} = E_j - \frac{E_j - e \sin E_j - M}{1 - e \cos E_j}, \qquad E_0 = M \tag{20}$$

The above equation was iterated until $E_{j+1} = E_j$. Then the true anomaly $\nu(t)$ was calculated using

$$\nu(t) = 2 \operatorname{arctan2} \left(\sqrt{1+e} \sin \frac{E(t)}{2}, \sqrt{1-e} \cos \frac{E(t)}{2} \right). \tag{21}$$

Where arctan2 is the two argument arctangent function. The distance r_c from the central body was then calculated using

$$r_c = a(1 - e \cos E(t)). \tag{22}$$

Then the position vector in the orbital frame was found using

$$\mathbf{o}(t) = \begin{pmatrix} o_x(t) \\ o_y(t) \\ o_z(t) \end{pmatrix} = r_c(t) \begin{pmatrix} \cos \nu(t) \\ \sin \nu(t) \\ 0 \end{pmatrix} \tag{23}$$

Then Ω, the longitude of the ascending node, and ω, the argument of the periapsis was found using

$$\Omega = \arctan2(p, q), \tag{24a}$$

$$\omega = \Omega - \arctan2(h, k). \tag{24b}$$

Finally the cartesian coordinates could be found using

$$\mathbf{r}(t) = \begin{pmatrix} o_x(t)(\cos(\omega)\cos(\Omega) - \sin(\omega)\cos(i)\sin(\Omega)) - o_y(t)(\sin(\omega)\cos(\Omega) + \cos(\omega)\cos(i)\sin(\Omega)) \\ o_x(t)(\cos(\omega)\sin(\Omega) + \sin(\omega)\cos(i)\cos(\Omega)) + o_y(t)(\cos(\omega)\cos(i)\cos(\Omega) - \sin(\omega)\sin(\Omega)) \\ o_x(t)(\sin(\omega)\sin(i)) + o_y(t)(\cos(\omega)\sin(i)) \end{pmatrix} \tag{25}$$

Code:

```python
def get_pi_or_omega(self, hp, kq):
    pi_om = []
    for i in range(len(self.planets)):
        pi_om.append(np.arctan2(hp[i], kq[i]))
    return np.array(pi_om)

def kep2cart(self, ecc, inc, h_list, k_list, p_list, q_list, time, t0, idx):
    O_list = self.get_pi_or_omega(p_list, q_list)
    w_list = O_list-self.get_pi_or_omega(h_list, k_list)
    n = self.get_property_all_planets('n')
    a = self.get_property_all_planets('a')

    Mt = n[idx]*np.pi/180*(time-t0)
    EA = []
    e, w, O, i = ecc[idx], w_list[idx], O_list[idx], inc[idx]
    for t in range(len(time)):
        E = Mt[t]
        f_by_dfdE = (E-e[t]*np.sin(E)-Mt[t])/(1-e[t]*np.cos(E))
        j, maxIter, delta = 0, 30, 0.0000000001
        while (j < maxIter)*(np.abs(f_by_dfdE) > delta):
            E = E-f_by_dfdE
            f_by_dfdE = (E-e[t]*np.sin(E)-Mt[t])/(1-e[t]*np.cos(E))
            j += 1
        EA.append(E)
    EA = np.array(EA)
    nu = 2*np.arctan2(np.sqrt(1+e)*np.sin(EA/2), np.sqrt(1-e)*np.cos(EA/2))

    rc = a[idx]*(1-e*np.cos(EA))
    o_vec = np.array([rc*np.cos(nu), rc*np.sin(nu), 0])

    rx = (o_vec[0]*(np.cos(w)*np.cos(O) - np.sin(w)*np.cos(i)*np.sin(O)) - o_vec[1]*(np.sin(w)*np.cos(O) + np.cos(w)*np.cos(i)*np.sin(O)))
    ry = (o_vec[0]*(np.cos(w)*np.sin(O) + np.sin(w)*np.cos(i)*np.cos(O)) + o_vec[1]*(np.cos(w)*np.cos(i)*np.cos(O) - np.sin(w)*np.sin(O)))
    rz = (o_vec[0]*(np.sin(w)*np.sin(i)) + o_vec[1]*(np.cos(w)*np.sin(i)))

    return rx, ry, rz
```

Code 17: Converting from Keplerian to Cartesian coordinates

4.2 Test of conversion

To test the conversion and the accuracy of the simulation, the Solar System was simulated for 1000 years and plotted below in Cartesian coordinates.

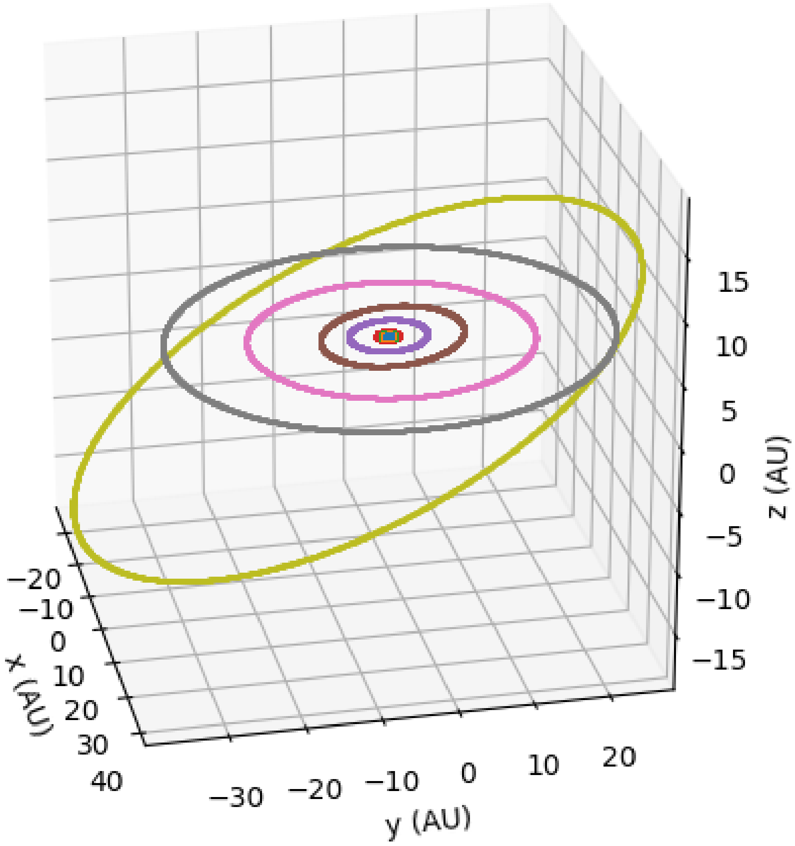


Figure 4: Plot of the Solar System simulated for a 1000 years from J2000 data.

Verdict: The plot is as expected. The plot was also used to ensure the time period of orbits were as expected, which they were.

4.3 General Relativity corrections

General Relativity (GR) corrections add an additional term to $\dot{\varpi}_j$. This additional term is given by [4]

$$\dot{\varpi}_j^{GR} = 3\frac{a_j^2 n_j^3}{c^2}. \tag{26}$$

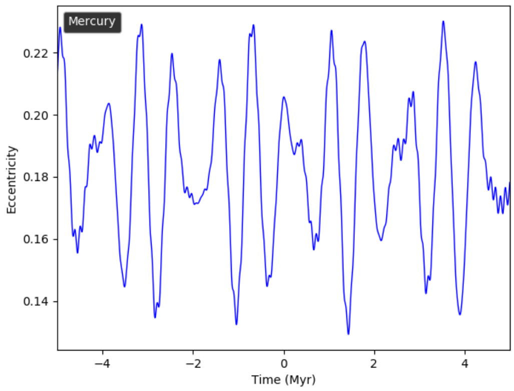
This term is added to the diagonal elements of **A** to accounts for the effects of GR.

4.4 Test of GR correction

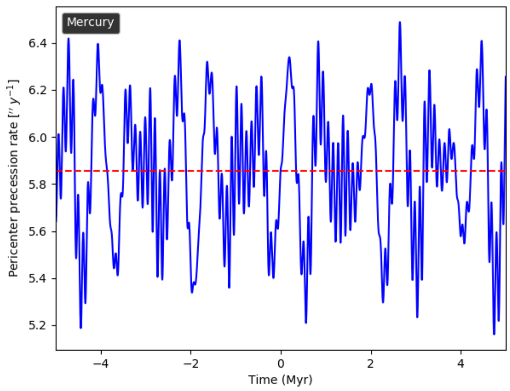
To test the effects of GR on the precession rates $\dot{\varpi}$, each planet was simulated by itself and its precession rate was calculated. The effect of GR on $\dot{\varpi}$, in arc seconds per century, of each planet is shown below

```
Mercury : 42.8928
Venus : 8.6069
Earth : 3.8309
Mars : 1.3483
Jupiter : 0.0621
Saturn : 0.0137
Uranus : 0.0024
Neptune : 0.0008
Pluto : 0.0004
```

The inclusion of GR had the largest effect on Mercury, as expected. This can be seen in the difference of the eccentricity plots in Figure 5 and previously in Figure 3.



(a) Evolution of eccentricity of Mercury.



(b) Precession rate, $\dot{\varpi}$ of Mercury.

Figure 5: Effect of GR on the orbit of Mercury.

Verdict: These values match very well to calculated values of GR's affect [5]. Additionally, including the effect of GR has also resulted in the eccentricity plot of Mercury becoming a closer match to Murray & Dermott (1999) [1].

Week 5: October 9^{th}–15^{th} 2017

5.1 Eccentricity damping corrections

We assume that the tides raised by planets are more dominant relative to tides raised by the star. Thus the eccentricity damping rate is given by [1,6]

$$\lambda = -\frac{\dot{e}}{e} = \frac{63}{4}\frac{1}{Q'_p}\frac{m_\star}{m_p}\left(\frac{R_p}{a_p}\right)^5 n_p. \tag{27}$$

Where R is the radius of the planet, $Q' \equiv 1.5Q/k_2$ is the modified tidal quality factor. Q is the tidal quality factor and k_2 is the Love number of degree 2 of the planet. All other terms are the same as before. The values of Q and k_2 are taken from Goldreich & Sotter (1966) [7], Zhang (1992) [8], and Gavrilov & Kharkov (1977) [9]. Similarly to the GR correction, $\sqrt{-1}\lambda$ is added to diagonal elements of $\mathbf{A}$ to account for the effect of tides.

Since the correction involved complex numbers, the code had to be adapted to work with complex numbers as opposed to real numbers. To do this, all data containers now store `dtype='complex128'`. The only major change involved altering Code 11 for solving for the scale factors and phase and is outline below.

```
def f(x, *boundaries):
    s_factor, phase = x
    return [s_factor*np.sin(phase)-boundaries[0], s_factor*np.cos(phase)-boundaries[1]]

def real_scaling_factor_and_phase(x1, *boundaries):
    s_factor, phase = x1[0]+1j*x1[1], x1[2]+1j*x1[3]
    x = [s_factor, phase]
    actual_f = f(x, *boundaries)
    return [np.real(actual_f[0]), np.imag(actual_f[0]), np.real(actual_f[1]), np.imag(actual_f[1])]
```

Code 18: Change to Code 11

5.2 Test of eccentricity damping

As before with GR, the effect of eccentricity damping is tested on the precession rate of the planets. It was found that this additional had no effect, at least on timescales of a few million years. However, for the Solar System, only a small change is expected, on timescales of billions of years.

5.3 Animating the Solar System

To be able to perform quick visual tests of the simulation (a few years at a time), code was written to create an animation of the solar system.

```
mport numpy as np
import pylab
import glob
import os
from matplotlib import pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import matplotlib.animation
import pandas as pd

files = glob.glob('Animate_solar_system/*.csv')
files.sort(key=os.path.getmtime)
files = "\n".join(files).split('\n')

n_planets = 4
colours = ['r', 'orange', 'b', 'g', 'brown', 'r', 'orange', 'b', 'g']

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ttl = ax.text(0, 1.05, 0, 'Time = years', transform = ax.transAxes, va='center')

all_x, all_y, all_z = [], [], []
points = []
for idx in range(n_planets):
    points.append(None)

    df = pd.read_csv(files[idx])
    x, y, z = np.array(df.x), np.array(df.y), np.array(df.z)
    all_x.append(x)
    all_y.append(y)
    all_z.append(z)
xyz = np.array([all_x, all_y, all_z])

def update_graph(num):
    global points
    global ttl
    data=df

    for idx in range(n_planets):
        if points[idx] is not None:
            points[idx].set_color(colours[idx])
            points[idx].set_markersize(1)
        points[idx], = ax.plot(all_x[idx][num:num+1], all_y[idx][num:num+1], all_z[idx][num:num+1], linestyle="", marker="o", color=colours[idx])
    ttl.set_text('Time = {:.2f} years'.format(df.time[num]))
    if num%5 == 0 and num > 10:
        if num%25 == 0:
            plt.cla()
            ttl = ax.text(0, 1.05, 0, '', transform = ax.transAxes, va='center')
            ax.plot(x0, y0, z0, 'b*', markersize=3, zorder=-999)
            ax.set_zlabel('z (AU)')
            ax.set_xlabel('x (AU)')
            ax.set_ylabel('y (AU)')

```

```
        ttl.set_text('Time = {:.2f} years'.format(df.time[num]))
        for idx in range(n_planets):
            ax.plot(all_x[idx][:num-1], all_y[idx][:num-1], all_z[idx][:num-1], linestyle="", marker="o",
        ↪ color=colours[idx], markersize=1)
            max_axis = np.max([np.abs(np.min(xyz)), np.max(xyz)])
            ax.set_zlim(-max_axis, max_axis)
            ax.set_ylim(-max_axis, max_axis)
            ax.set_xlim(-max_axis, max_axis)
    return graph,

x0, y0, z0 = np.zeros(2), np.zeros(2), np.zeros(2)
graph, = ax.plot(x0, y0, z0, 'b*', markersize=3, zorder=-999)

ax.view_init(45, 45)
max_axis = np.max([np.abs(np.min(xyz)), np.max(xyz)])
ax.set_zlim(-max_axis, max_axis)
ax.set_ylim(-max_axis, max_axis)
ax.set_xlim(-max_axis, max_axis)
ax.set_zlabel('z (AU)')
ax.set_xlabel('x (AU)')
ax.set_ylabel('y (AU)')

ani = matplotlib.animation.FuncAnimation(fig, update_graph, len(x),
                              interval=1, save_count=50, repeat=False)
ani.save('Animate_solar_system/Plots/rocky_planets.mp4', fps=24)
```

Code 19: Animating the Solar System

After viewing the animation it was found that all the planets behaved normally with 1 exception; Earth. The behaves normally at all times $t < 0$ years. However during $0 \leq t \leq 1$ years, the Earth behaves in an unexpected manner. During this time, the Earth slows down and then reverses at any extremely quickly before stopping again, at which point it goes back to its normal speed. At $t > 1$ years, the behaviour of the Earth returns to normal. This behaviour is also only seen if both Venus and Jupiter are present in the simulation.

The cause of this bug is unknown for now. However it does not have any effect on long term simulations (as seen by results in §4.4) due to the very short duration of the bug. Hence, the simulation can now be tested on other star systems.

5.4 Simulating other planet systems

5.4.1 Extracting data

The data of other star systems were taken from Nasa Exoplanet Archive. To aid in the extraction of data, a web crawler that takes the star alias as the argument was written.

```
import glob
```

```
import os
import sys
from bs4 import BeautifulSoup
from unidecode import unidecode
import requests
import numpy as np
import pandas as pd
from scipy import stats
import numpy.ma as ma

def mean_data(obj_id):
  """
  Extracts the mean data of each planet in the star system.
  """
    obj_id_split = obj_id.split(' ')
    obj_id = obj_id_split[0]
    output_id = obj_id_split[0]
    for i in range(1, len(obj_id_split)):
        obj_id += '+'+obj_id_split[i]
        output_id += '_'+obj_id_split[i]

    url = "https://exoplanetarchive.ipac.caltech.edu/cgi-bin/ExoOverview/nph-ExoOverview?
      ↪ objname={}&type=&label&aliases&exo&iden&orb&ppar&tran&note&disc&ospar&ts&nalc&
      ↪ force=&dhxr1507830887922".format(obj_id)
    response = requests.get(url)

    bs = BeautifulSoup(response.content, "html.parser")

    for idx, title in enumerate(bs.findAll('div', {'class': 'data'})):
        name = title.find('th').text
        if name == 'Planet Orbital Properties':
            index = idx

    planet_props = bs.findAll('div', {'class': 'data'})

    column_names = []
    planets = []
    p_idx = 0

    for idx, text in enumerate(planet_props[index].findAll()):
        text = unidecode(str(text))

        if 'th' in text:
            if 'class' not in text:
                if 'Reference' not in text:
                    if 'href' not in text:
                        column_names.append(text[4:-5])

        if idx > 1:
            text_split = text.split('\n')
            for t in text_split:
                if 'td' in t:
                    if 'href' not in t:
```

```
                        t = t[4:-5]
                        if '+-' in t:
                            t = t.split('+-')[0].split(' ')[-1]
                            planets[p_idx-1].append(t)
                        elif 'lt' in t or 'gt' in t:
                            t = t.split(';')[-1]
                            planets[p_idx-1].append(t)
                        else:
                            t = t.split('span')
                            if len(t) == 1:
                                t = t[0].split(' ')[-1]
                                if 'null' not in t:
                                    if t.isdigit() or '.' in t:
                                        planets[p_idx-1].append(t)
                                    else:
                                        if len(t) == 1:
                                            planets.append([])
                                            planets[p_idx].append(t)
                                            p_idx += 1
                                else:
                                    planets[p_idx-1].append(t)
                            else:
                                t = t[1].split('>')[1].split('<')[0]
                                planets[p_idx-1].append(t)
    planets = planets[::2]

    column_names_2 = []
    planets_2 = []
    p_idx = 0

    for idx, title in enumerate(bs.findAll('div', {'class': 'data'})):
        name = title.find('th').text
        if name == 'Planet Parameters':
            index = idx

    found_highlight = False
    for idx, text in enumerate(planet_props[index].findAll()):
        text = unidecode(str(text))

        if 'th' in text:
            if 'tr' not in text:
                if 'class' not in text:
                    if 'span' not in text:
                        if '(' in text:
                            t = text[5:-6]
                            if 'sup' in t:
                                t = t.split('<')[0]
                            column_names_2.append(t)

        if idx > 1:
            text_split = text.split('\n')
            for t in text_split:
                if 'td' in t:
```

```
                    if 'href' not in t:
                        t = t[4:-5]
                        if '+-' in t:
                            t = t.split('+-')[0].split(' ')[-1]
                            planets_2[p_idx-1].append(t)
                        elif 'lt' in t or 'gt' in t:
                            t = t.split(';')[-1]
                            planets_2[p_idx-1].append(t)
                        else:
                            t = t.split('span')
                            if len(t) == 1:
                                t = t[0].split(' ')[-1]
                                if 'null' not in t:
                                    if t.isdigit() or '.' in t:
                                        planets_2[p_idx-1].append(t)
                                    else:
                                        if len(t) == 1:
                                            planets_2.append([])
                                            # planets_2[p_idx].append(t)
                                            p_idx += 1
                                else:
                                    if len(t) == 1:
                                        planets_2[p_idx-1].append(t)
                                    elif 'null' in t:
                                        planets_2[p_idx-1].append(t)
                            else:
                                t = t[1].split('>')[1].split('<')[0]
                                planets_2[p_idx-1].append(t)

    planets_2 = planets_2[::2]

    for i in range(len(planets)):
        planets[i].extend(planets_2[i])

    for i in column_names_2:
        column_names.append(i)

    column_names = np.array(column_names)
    for c, col in enumerate(column_names):
        if col == 'Planet':
            column_names[c] = 'Name'
        if col == 'Period (days)':
            column_names[c] = 'n'
        if col == 'Semi-Major Axis (AU)':
            column_names[c] = 'a'
        if col == 'Inclination (deg)':
            column_names[c] = 'i'
        if col == 'Eccentricity':
            column_names[c] = 'e'
        if col == 'Longitude of Periastron (deg)':
            column_names[c] = 'pi'
        if col == 'Earth Mass':
            column_names[c] = 'Mass'
```

```python
        if col == 'Jupiter Mass':
            column_names[c] = 'Mj'

    mass_idx = np.where(['Mass' in x for x in column_names])[0]
    column_names[mass_idx[-1]] = 'Mass_2'

    planets = np.array(planets)

    labels, n_data = np.unique(planets[:, 0], return_counts=True)
    start_idx = np.zeros_like(labels)
    for s, l in enumerate(labels):
        start_idx[s] = np.where(planets[:, 0] == l)[0][0]
    start_idx = np.array(start_idx, dtype='int')

    planet_means = []

    for l in range(len(labels)):
        planet_means.append([])
        planet_means[l].append(labels[l])
        for col in range(1, planets.shape[1]):
            col_data = planets[:, col][start_idx[l]:start_idx[l]+n_data[l]]
            planet_l_data = ma.masked_array(col_data, col_data == 'null').compressed()
            try:
                compressed_array = ma.array(planet_l_data, dtype=float)
                if len(compressed_array) > 0:
                    planet_means[l].append(ma.mean(compressed_array))
                else:
                    planet_means[l].append(np.nan)
            except:
                print('null')

    planet_means = np.array(planet_means)

    columns_to_ignore = ['Passage', 'Date', 'Mj', 'Radii', 'g/cm', 'K']
    idxs = []
    for word in columns_to_ignore:
        idx = np.where([word in x for x in column_names])[0]
        for i in idx:
            idxs.append(i)

    df = pd.DataFrame(columns=column_names, index=range(0, len(planet_means)))
    for row in range(len(planet_means)):
        for col in range(len(column_names)):
            df.ix[row, col] = planet_means[row, col]
            if planet_means[row, col] == 'null':
                df.ix[row, col] = np.nan
            if col in idxs:
                df.ix[row, col] = np.nan

    return df

def read_data(obj_id):
  """
```

```
Extracts the data from the highlighted row of each planet in the star system.
"""
  obj_id_split = obj_id.split(' ')
  obj_id = obj_id_split[0]
  output_id = obj_id_split[0]
  for i in range(1, len(obj_id_split)):
      obj_id += '+'+obj_id_split[i]
      output_id += '_'+obj_id_split[i]

  url = "https://exoplanetarchive.ipac.caltech.edu/cgi-bin/ExoOverview/nph-ExoOverview?
    ↪ objname={}&type=&label&aliases&exo&iden&orb&ppar&tran&note&disc&ospar&ts&nalc&
    ↪ force=&dhxr1507830887922".format(obj_id)
  print('\nExtracting data of {} from:\n{}\n'.format(output_id, url))
  response = requests.get(url)

  bs = BeautifulSoup(response.content, "html.parser")

  for idx, title in enumerate(bs.findAll('div', {'class': 'data'})):
      name = title.find('th').text
      if name == 'Planet Orbital Properties':
          index = idx

  planet_props = bs.findAll('div', {'class': 'data'})

  planets = []
  column_names = []
  p_idx = 0

  found_highlight = False
  for idx, text in enumerate(planet_props[index].findAll()):
      text = unidecode(str(text))

      found_highlight = 'class="overview_highlight"' in text
      if 'th' in text:
          if 'class' not in text:
              if 'Reference' not in text:
                  if 'href' not in text:
                      column_names.append(text[4:-5])

      if idx > 1:
          if found_highlight:
              text_split = text.split('\n')
              for t in text_split:
                  if 'tr' not in t:
                      if 'href' not in t:
                          t = t[4:-5]
                          if '+-' in t:
                              t = t.split('+-')[0].split(' ')[-1]
                              planets[p_idx-1].append(t)
                          elif 'lt' in t or 'gt' in t:
                              t = t.split(';')[-1]
                              planets[p_idx-1].append(t)
                          else:
```

```
                                t = t.split('span')
                                if len(t) == 1:
                                    t = t[0].split(' ')[-1]
                                    if 'null' not in t:
                                        if t.isdigit() or '.' in t:
                                            planets[p_idx-1].append(t)
                                        else:
                                            planets.append([])
                                            planets[p_idx].append(t)
                                            p_idx += 1
                                    else:
                                        planets[p_idx-1].append(t)
                                else:
                                    t = t[1].split('>')[1].split('<')[0]
                                    planets[p_idx-1].append(t)
                found_highlight=False

    p_idx = 0

    for idx, title in enumerate(bs.findAll('div', {'class': 'data'})):
        name = title.find('th').text
        if name == 'Planet Parameters':
            index = idx

    found_highlight = False
    for idx, text in enumerate(planet_props[index].findAll()):
        text = unidecode(str(text))

        found_highlight = 'class="overview_highlight"' in text
        if 'th' in text:
            if 'tr' not in text:
                if 'class' not in text:
                    if 'span' not in text:
                        if '(' in text:
                            t = text[5:-6]
                            if 'sup' in t:
                                t = t.split('<')[0]
                            column_names.append(t)

        if idx > 1:
            if found_highlight:
                text_split = text.split('\n')
                for t in text_split:
                    try:
                        if 'tr' not in t:
                            if 'href' not in t:
                                t = t[4:-5]
                                if '+-' in t:
                                    t = t.split('+-')[0].split(' ')[-1]
                                    planets[p_idx-1].append(t)
                                elif 'lt' in t or 'gt' in t:
                                    t = t.split(';')[-1]
                                    planets[p_idx-1].append(t)
```

```
                                else:
                                    t = t.split('span')
                                    if len(t) == 1:
                                        t = t[0].split(' ')[-1]
                                        if 'null' not in t:
                                            if t.isdigit() or '.' in t:
                                                planets[p_idx-1].append(t)
                                            else:
                                                p_idx += 1
                                        else:
                                            planets[p_idx-1].append(t)
                                    else:
                                        t = t[1].split('>')[1].split('<')[0]
                                        planets[p_idx-1].append(t)
                        except:
                            print(t)
                found_highlight=False

    column_names = column_names[:]
    column_names = np.array(column_names)
    for c, col in enumerate(column_names):
        if col == 'Planet':
            column_names[c] = 'Name'
        if col == 'Period (days)':
            column_names[c] = 'n'
        if col == 'Semi-Major Axis (AU)':
            column_names[c] = 'a'
        if col == 'Inclination (deg)':
            column_names[c] = 'i'
        if col == 'Eccentricity':
            column_names[c] = 'e'
        if col == 'Longitude of Periastron (deg)':
            column_names[c] = 'pi'
        if col == 'Earth Mass':
            column_names[c] = 'Mass'
        if col == 'Jupiter Mass':
            column_names[c] = 'Mj'

    mass_idx = np.where(['Mass' in x for x in column_names])[0]
    column_names[mass_idx[-1]] = 'Mass_2'

    planets = np.array(planets)

    columns_to_ignore = ['Passage', 'Date', 'Mj', 'Radii', 'g/cm', 'K']
    idxs = []
    for word in columns_to_ignore:
        idx = np.where([word in x for x in column_names])[0]
        for i in idx:
            idxs.append(i)

    df = pd.DataFrame(columns=column_names, index=range(0, np.shape(planets)[0]))
    for row in range(np.shape(planets)[0]):
        for col in range(np.shape(planets)[1]):
```

```
            df.ix[row, col] = planets[row, col]
            if planets[row, col] == 'null':
                df.ix[row, col] = np.nan
            if col in idxs:
                df.ix[row, col] = np.nan

    for idx, title in enumerate(bs.findAll('div', {'class': 'data'})):
        name = title.find('th').text
        if name == 'Summary of Stellar Information':
            index = idx

    star_prop = []
    found_highlight = False
    for idx, text in enumerate(planet_props[index].findAll()):
        text = unidecode(str(text))

        found_highlight = 'Mass' in text
        if idx > 1:
            if found_highlight:
                text_split = text.split('\n')
                for t in text_split:
                    if 'tr' not in t:
                        if 'null' not in t:
                            if 'class' not in t:
                                t = t[4:-5].split('+-')
                                star_prop.append(float(t[0]))
                found_highlight = False

    df1 = pd.DataFrame(columns=['star_mass', 'star_radius'], index=range(0, 1))
    df1.ix[0, 0] = star_prop[0]
    if len(star_prop) == 1:
        df1.ix[0, 1] = np.nan
    else:
        df1.ix[0, 1] = star_prop[1]

    return df, df1

def compare_data(df_highlighted, df_average, output_id):
    rows, cols = df_highlighted.shape

    mass_idx = df_highlighted.columns.get_loc("Mass")
    n_idx = df_highlighted.columns.get_loc("n")
    for row in range(rows):
        for col in range(cols):
            if pd.isnull(df_highlighted.ix[row, col]) and not pd.isnull(df_average.ix[row, col]):
                df_highlighted.ix[row, col] = df_average.ix[row, col]

            if col == mass_idx:
                if str(df_highlighted.ix[row, col+2]) != 'nan':
                    df_highlighted.ix[row, col] = df_highlighted.ix[row, col+2]
            if col == n_idx:
                df_highlighted.ix[row, col] = 2*np.pi/(float(df_highlighted.ix[row, col])/365)*180/np.pi

```

```
422     cols_to_keep = ['Name', 'n', 'a', 'e', 'i', 'pi', 'Mass']
423     for pi in df_highlighted['pi']:
424         if pi == 'nan':
425             print('WARNING: nans exist for values of pi')
426             break
427     df_highlighted[cols_to_keep].to_csv('Exoplanets_data/'+output_id+'/'+'planets.csv', index=False)
428 
429 if __name__ == '__main__':
430     # obj_id = '55 Cnc'
431 
432     if len(sys.argv) > 1:
433         obj_id = sys.argv[1]
434         args = sys.argv[2:]
435         for a in args:
436             obj_id += ' '+a
437         obj_id = list(obj_id)
438         for index, s in enumerate(obj_id):
439             if s == '+':
440                 obj_id[index] = '%2B'
441         obj_id = ''.join(obj_id)
442 
443     df_highlight, df_star = read_data(obj_id)
444     df_mean = mean_data(obj_id)
445 
446     obj_id_split = obj_id.split(' ')
447     output_id = obj_id_split[0]
448     for i in range(1, len(obj_id_split)):
449         output_id += '_'+obj_id_split[i]
450 
451     folder = glob.glob('Exoplanets_data/'+output_id)
452     if len(folder) == 0:
453         os.system('mkdir '+'Exoplanets_data/'+output_id)
454 
455     compare_data(df_highlight, df_mean, output_id)
456     df_star.to_csv('Exoplanets_data/'+output_id+'/'+'star.csv', index=False)
```

Code 20: Extracting data from Nasa Exoplanet Archive

To use the code uncomment line 430 and replace the string with the star id to extract data from, or in the terminal type: `python planet_data_crawler_v2.py <star_id>`.

5.4.2 Simulation results

Table 3: Eccentricity results of various star systems

Planet	m ($M_\oplus$)	a (AU)	e_{obs}	$\bar{e}$	σ_e	e_{max}	e_{min}
24 Sex b	632.46	1.333	0.090	0.129	0.039	0.179	0.067
24 Sex c	273.32	2.080	0.290	0.254	0.036	0.301	0.200

Continued on next page

Table 3 – continued from previous page

Planet	m ($M_\oplus$)	a (AU)	e_{obs}	$\bar{e}$	σ_e	e_{max}	e_{min}
61 Vir b	5.10	0.050	0.120	0.202	0.045	0.284	0.114
61 Vir c	18.20	0.218	0.140	0.232	0.072	0.335	0.113
61 Vir d	22.90	0.476	0.350	0.314	0.030	0.355	0.266
BD+20 2457 b	6807.63	1.450	0.150	0.140	0.058	0.211	0.039
BD+20 2457 c	3963.17	2.010	0.180	0.161	0.076	0.251	0.010
CoRoT-7 b	5.74	0.017	0.000	0.000	0.000	0.000	0.000
CoRoT-7 c	8.40	0.046	0.000	0.000	0.000	0.000	0.000
GJ 163 b	10.60	0.061	0.073	0.081	0.013	0.100	0.060
GJ 163 c	6.80	0.125	0.099	0.090	0.013	0.109	0.069
GJ 163 d	29.40	1.030	0.373	0.373	0.000	0.373	0.373
GJ 876 b	635.63	0.208	0.032	0.071	0.025	0.104	0.027
GJ 876 c	177.98	0.130	0.256	0.203	0.037	0.256	0.143
GJ 876 d	6.03	0.021	0.207	0.190	0.012	0.207	0.172
GJ 876 e	14.60	0.334	0.055	0.134	0.043	0.199	0.054
HAT-P-13 b	270.46	0.043	0.013	0.010	0.005	0.016	0.001
HAT-P-13 c	4538.42	1.223	0.662	0.662	0.000	0.662	0.662
HD 11964 b	198.00	3.160	0.041	0.041	0.001	0.041	0.040
HD 11964 c	25.00	0.229	0.300	0.300	0.000	0.300	0.300
HD 12661 b	691.57	0.808	0.377	0.309	0.052	0.377	0.230
HD 12661 c	575.88	2.815	0.031	0.156	0.071	0.241	0.028
HD 128311 b	562.20	1.084	0.303	0.212	0.103	0.334	0.004
HD 128311 c	993.20	1.740	0.159	0.198	0.045	0.257	0.129
HD 133131 A b	451.00	1.440	0.330	0.332	0.097	0.457	0.174
HD 133131 A c	133.00	4.490	0.490	0.448	0.139	0.626	0.218
HD 134987 b	505.00	0.810	0.233	0.229	0.002	0.233	0.226
HD 134987 c	260.00	5.800	0.120	0.125	0.003	0.129	0.120
HD 142 b	397.27	1.020	0.170	0.165	0.031	0.208	0.122
HD 142 c	1684.40	6.800	0.210	0.210	0.002	0.213	0.207
HD 160691 b	343.24	1.497	0.128	0.084	0.031	0.129	0.025
HD 160691 c	576.52	5.235	0.099	0.100	0.002	0.103	0.098
HD 160691 d	10.55	0.091	0.172	0.170	0.002	0.174	0.167
HD 160691 e	165.87	0.921	0.067	0.146	0.046	0.210	0.065
HD 190360 b	495.79	4.010	0.313	0.313	0.000	0.313	0.313
HD 190360 c	19.07	0.130	0.237	0.235	0.001	0.237	0.234
HD 202206 b	5530.00	0.830	0.435	0.416	0.026	0.452	0.379
HD 202206 c	776.00	2.550	0.267	0.340	0.139	0.509	0.095
HD 217107 b	441.77	0.075	0.127	0.127	0.000	0.127	0.127
HD 217107 c	826.32	5.320	0.517	0.517	0.000	0.517	0.517
HD 3167 b	5.02	0.018	0.000	0.024	0.010	0.041	0.000
HD 3167 c	9.80	0.180	0.267	0.316	0.033	0.362	0.267

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Table 3 – continued from previous page

Planet	m ($M_\oplus$)	a (AU)	e_{obs}	$\bar{e}$	σ_e	e_{max}	e_{min}
HD 3167 d	6.90	0.078	0.360	0.232	0.106	0.360	0.033
HD 37124 b	215.00	0.534	0.054	0.145	0.070	0.245	0.023
HD 37124 c	207.00	1.710	0.125	0.113	0.050	0.192	0.002
HD 37124 d	221.00	2.807	0.160	0.120	0.041	0.191	0.026
HD 38529 b	266.65	0.131	0.257	0.254	0.004	0.262	0.249
HD 38529 c	4252.39	3.712	0.341	0.341	0.000	0.341	0.341
HD 73526 b	715.09	0.650	0.290	0.263	0.049	0.329	0.188
HD 73526 c	715.09	1.030	0.280	0.312	0.142	0.483	0.047
HD 74156 b	572.07	0.292	0.627	0.631	0.026	0.662	0.583
HD 74156 c	2561.60	3.850	0.432	0.432	0.002	0.436	0.429
HD 80606 b	1252.20	0.449	0.933	0.933	0.000	0.933	0.933
Kepler-30 b	11.30	0.180	0.042	0.034	0.006	0.043	0.025
Kepler-30 c	640.00	0.300	0.011	0.011	0.001	0.012	0.009
Kepler-30 d	23.10	0.500	0.022	0.027	0.006	0.034	0.018
ups And b	218.53	0.059	0.022	0.022	0.000	0.022	0.021
ups And c	629.60	0.828	0.260	0.269	0.008	0.280	0.258
ups And d	1313.22	2.513	0.299	0.206	0.081	0.306	0.067

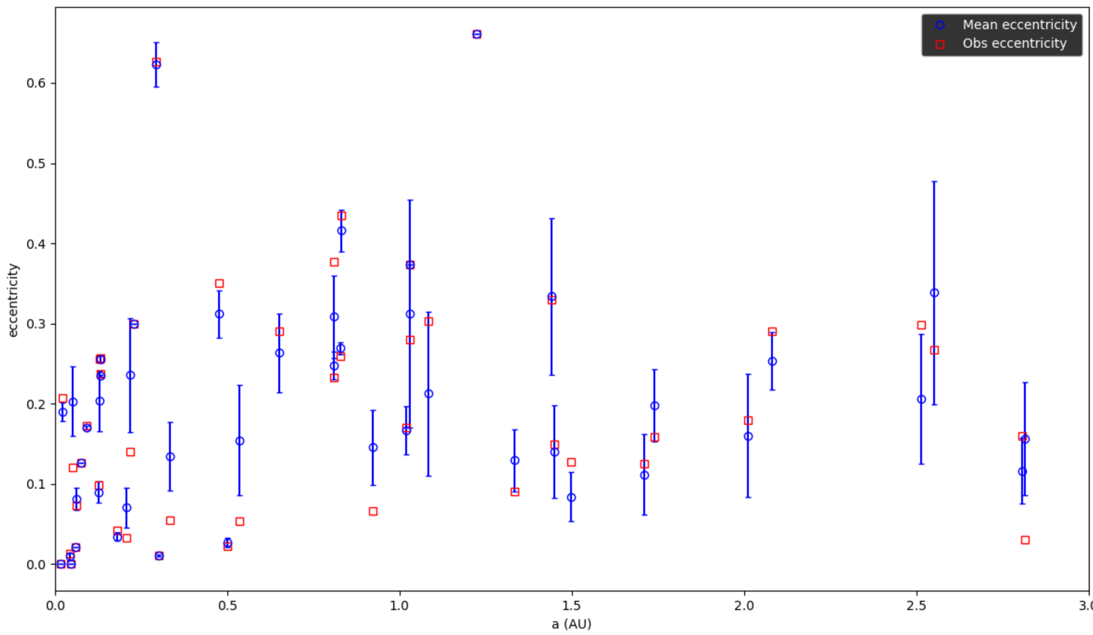

Figure 6

Week 6: October 16^{th}–22^{nd} 2017

6.1 Simulating tests in Rice 2015

6.2 Wacky earth bug fix

Week 7: October 23^{rd}–29^{th} 2017

7.1 Comparison with n-body simulations

Due to the failure to replicate the output of the 1st test from KR, the output from the secular theory described here was compared to output from n-body simulations in order to determine how well the code for secular theory matched with n-body simulations.

To use the n-body simulations, a new function, similar to the one in §4.1 was written to calculate the the initial position and velocity. This was done by extending the equations in §4.1 to also find the velocities.

$$\dot{\mathbf{o}}(t) = \begin{pmatrix} \dot{o}_x(t) \\ \dot{o}_y(t) \\ \dot{o}_z(t) \end{pmatrix} = \frac{\sqrt{\mu a}}{r_c(t)} \begin{pmatrix} -\sin E \\ \sqrt{1-e^2}\cos E \\ 0 \end{pmatrix} \tag{28}$$

where $\mu = GM$. G is the gravitational constant and M is the mass of the central body. The velocity is then given by:

$$\mathbf{v}(t) = \begin{pmatrix} \dot{o}_x(t)(\cos(\omega)\cos(\Omega) - \sin(\omega)\cos(i)\sin(\Omega)) - \dot{o}_y(t)(\sin(\omega)\cos(\Omega) + \cos(\omega)\cos(i)\sin(\Omega)) \\ \dot{o}_x(t)(\cos(\omega)\sin(\Omega) + \sin(\omega)\cos(i)\cos(\Omega)) + \dot{o}_y(t)(\cos(\omega)\cos(i)\cos(\Omega) - \sin(\omega)\sin(\Omega)) \\ \dot{o}_x(t)(\sin(\omega)\sin(i)) + \dot{o}_y(t)(\cos(\omega)\sin(i)) \end{pmatrix} \tag{29}$$

```python
def initial_pos_vel(self, idx):
    n = self.get_property_all_planets('n')
    a = self.get_property_all_planets('a')
    ecc = self.get_property_all_planets('e')
    inc = self.get_property_all_planets('i')*np.pi/180
    O_list = self.get_property_all_planets('Omega')*np.pi/180
    w_list = self.get_property_all_planets('pi')*np.pi/180-O_list

    M = 0
    e, w, O, i = ecc[idx], w_list[idx], O_list[idx], inc[idx]

    E = M
    f_by_dfdE = (E-e*np.sin(E)-M)/(1-e*np.cos(E))
    j, maxIter, delta = 0, 30, 0.0000000001
    while (j < maxIter)*(np.abs(f_by_dfdE) > delta):
        E = E-f_by_dfdE
        f_by_dfdE = (E-e*np.sin(E)-M)/(1-e*np.cos(E))
        j += 1
    EA = E
    nu = 2*np.arctan2(np.sqrt(1+e)*np.sin(EA/2), np.sqrt(1-e)*np.cos(EA/2))

    rc = a[idx]*(1-e*np.cos(EA))
    o_vec = np.array([rc*np.cos(nu), rc*np.sin(nu), 0])
```

```

        rx = (o_vec[0]*(np.cos(w)*np.cos(O) - np.sin(w)*np.cos(i)*np.sin(O)) - o_vec[1]*(np.sin(w)*np.cos(O) + np.cos(w)*np.cos(i)*np.sin(O)))
        ry = (o_vec[0]*(np.cos(w)*np.sin(O) + np.sin(w)*np.cos(i)*np.cos(O)) + o_vec[1]*(np.cos(w)*np.cos(i)*np.cos(O) - np.sin(w)*np.sin(O)))
        rz = (o_vec[0]*(np.sin(w)*np.sin(i)) + o_vec[1]*(np.cos(w)*np.sin(i)))

        mu = G_CONST*self.star_mass*M_SUN
        o_dot_vec = (np.sqrt(mu*a[idx]*AU)/(rc*AU))*np.array([-np.sin(EA), np.sqrt(1-e**2)*np.cos(EA), 0])

        vx = (o_dot_vec[0]*(np.cos(w)*np.cos(O) - np.sin(w)*np.cos(i)*np.sin(O)) - o_dot_vec[1]*(np.sin(w)*np.cos(O) + np.cos(w)*np.cos(i)*np.sin(O)))
        vy = (o_dot_vec[0]*(np.cos(w)*np.sin(O) + np.sin(w)*np.cos(i)*np.cos(O)) + o_dot_vec[1]*(np.cos(w)*np.cos(i)*np.cos(O) - np.sin(w)*np.sin(O)))
        vz = (o_dot_vec[0]*(np.sin(w)*np.sin(i)) + o_dot_vec[1]*(np.cos(w)*np.sin(i)))

        return np.array([rx, ry, rz]), np.array([vx, vy, vz])*(86400/AU)
```

Code 21: Calculating initial conditions for n-body simulation

For all the star systems below, the value of the ascending node, Ω of every planet was set to 0, unless specified. For some star systems, the inclinations, i of the planets are not known. In these cases, the inclination is set to 0.

7.1.1 HD 37124

Planet	a (AU)	e	i	ϖ	m $(M_\oplus)$
HD 37124 b	0.53364	0.054	0.0	130.0	215
HD 37124 c	1.7100	0.125	0.0	53.0	207
HD 37124 d	2.807	0.16	0.0	0.0	221

Table 4: Planet data for HD 37124

Using the above data for HD 37124, the eccentricity evolution was calculated for each planet and was compared to the output from n-body simulations.

Although the eccentricity evolution from the n-body calculation oscillate much more frequently than from secular theory for HD 37124 b and d, it can be seen that the results are in good qualitative agreement with each other.

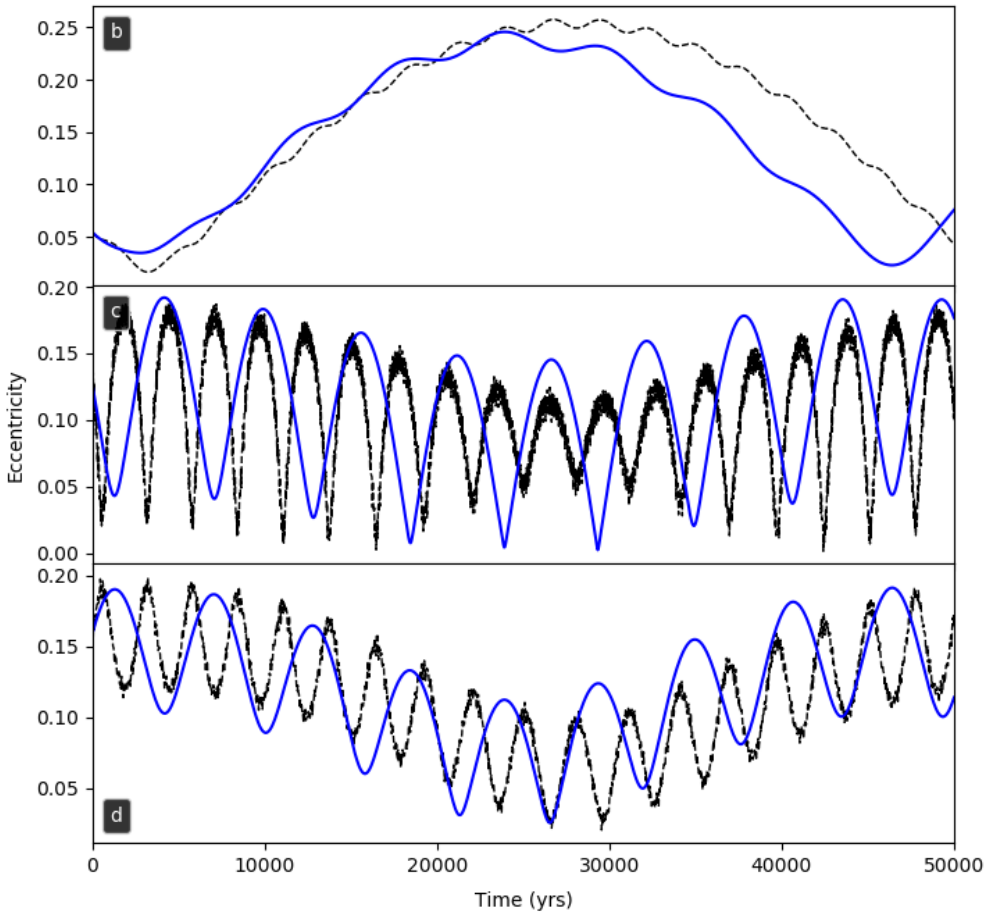

Figure 7: Comparison of secular theory (blue) for HD 37124 with n-body simulations (black).

7.1.2 HD 3167

Planet	a (AU)	e	i	ϖ	m $(M_\oplus)$
HD 3167 b	0.01815	0	0.0	0.0	5.02
HD 3167 c	0.1795	0.267	0.0	0.0	9.8
HD 3167 d	0.07757	0.36	0.0	0.0	6.9

Table 5: Planet data for HD 3167

Using the above data for HD 3167, the eccentricity evolution was calculated for each planet and was compared to the output from n-body simulations.

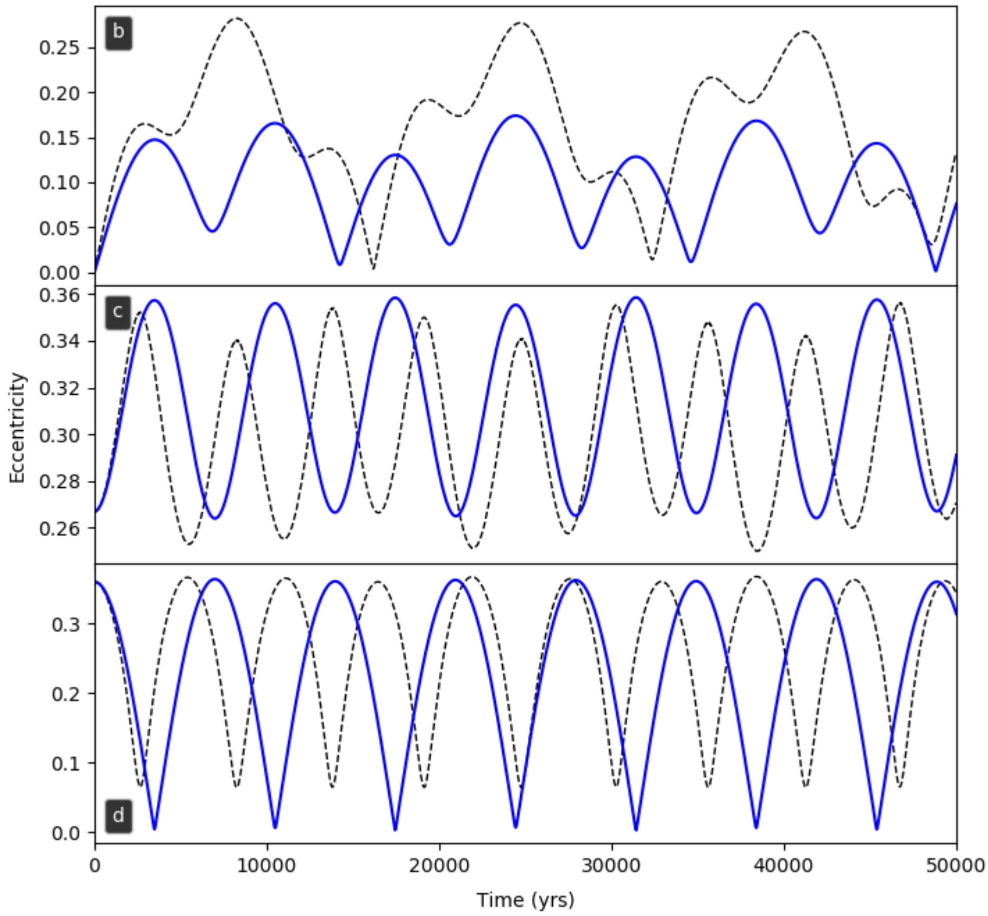

Figure 8: Comparison of secular theory (blue) for HD 3167 with n-body simulations (black).

7.1.3 HD 12661

Planet	a (AU)	e	i	ϖ	m $(M_\oplus)$
HD 12661 b	0.8079	0.3768	0.0	305.1	691.569
HD 12661 c	2.8145	0.031	0.0	101.5	575.884

Table 6: Planet data for HD 12661

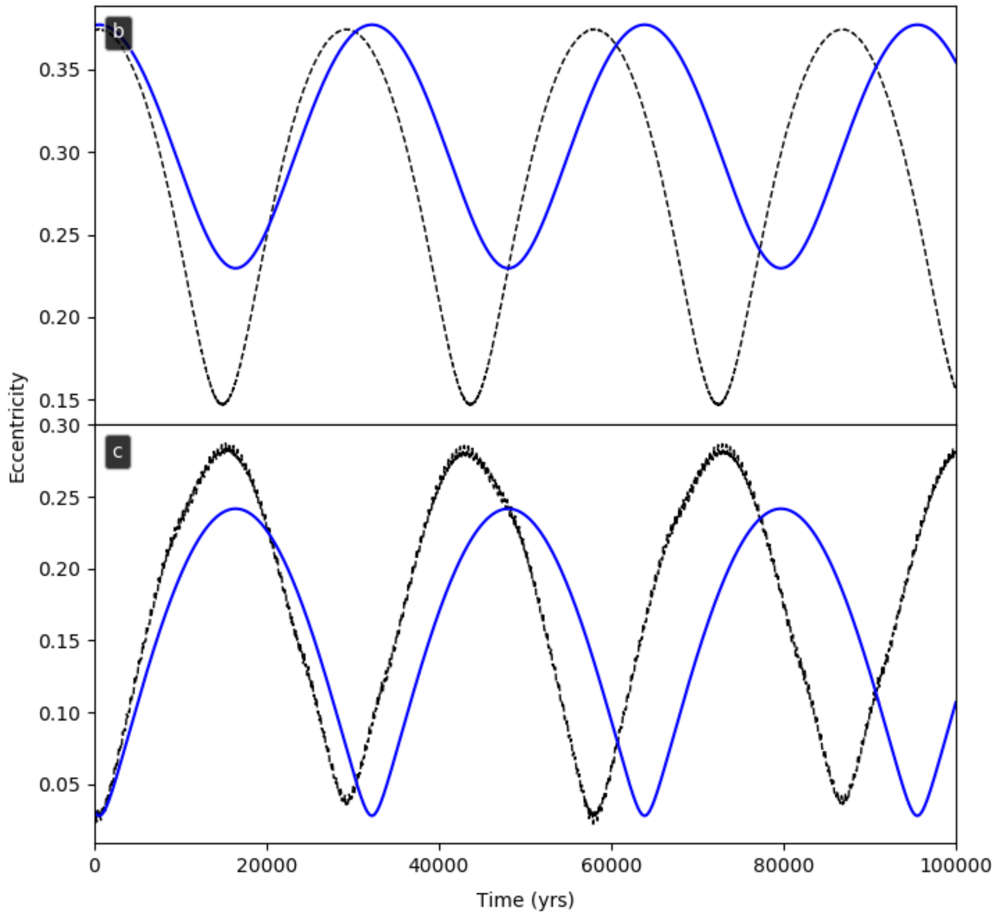

Figure 9: Comparison of secular theory (blue) for HD 12661 with n-body simulations (black).

7.1.4 61 Vir

Planet	a (AU)	e	i	ϖ	m $(M_{\oplus})$
61 Vir b	0.050201	0.12	0.0	105	5.1
61 Vir c	0.2175	0.14	0.0	341	18.2
61 Vir d	0.476	0.35	0.0	314	22.9

Table 7: Planet data for 61 Vir

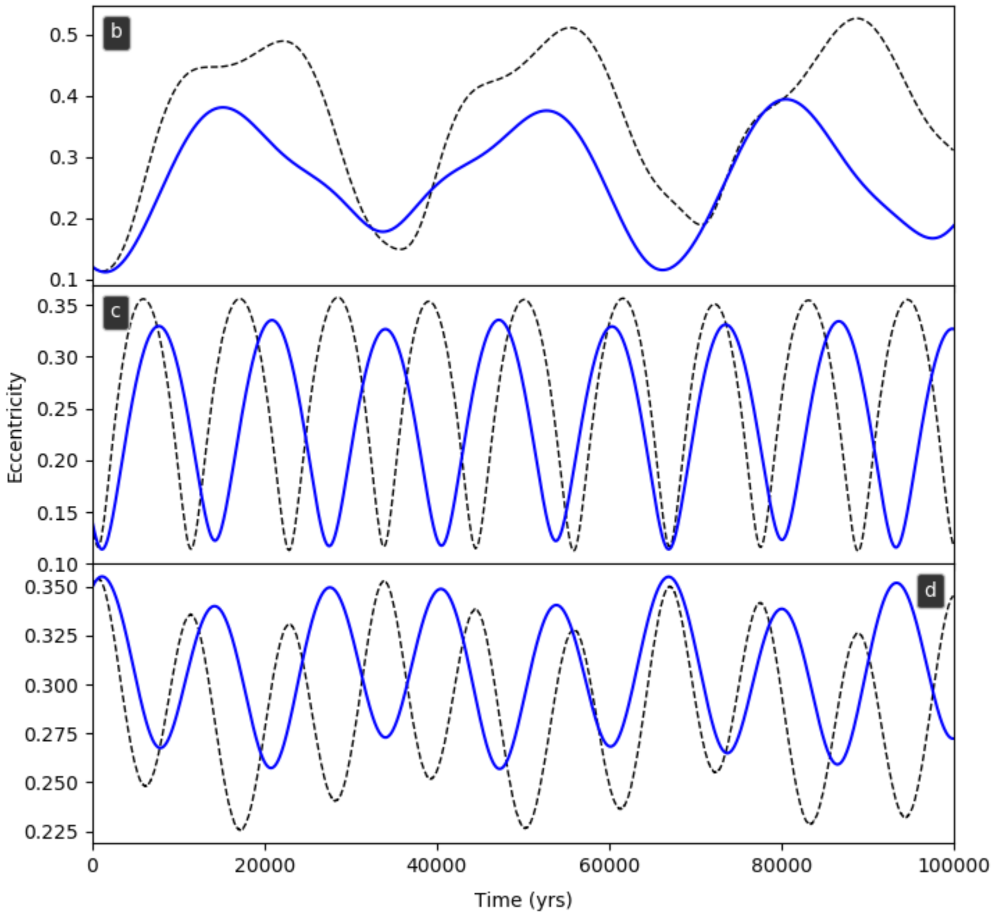

Figure 10: Comparison of secular theory (blue) for 61 Vir with n-body simulations (black).

7.1.5 24 Sex

Planet	a (AU)	e	i	ϖ	m $(M_\oplus)$
24 Sex b	1.333	0.09	0.0	9.2	632.46
24 Sex c	2.08	0.29	0.0	220.5	273.32

Table 8: Planet data for 24 Sex

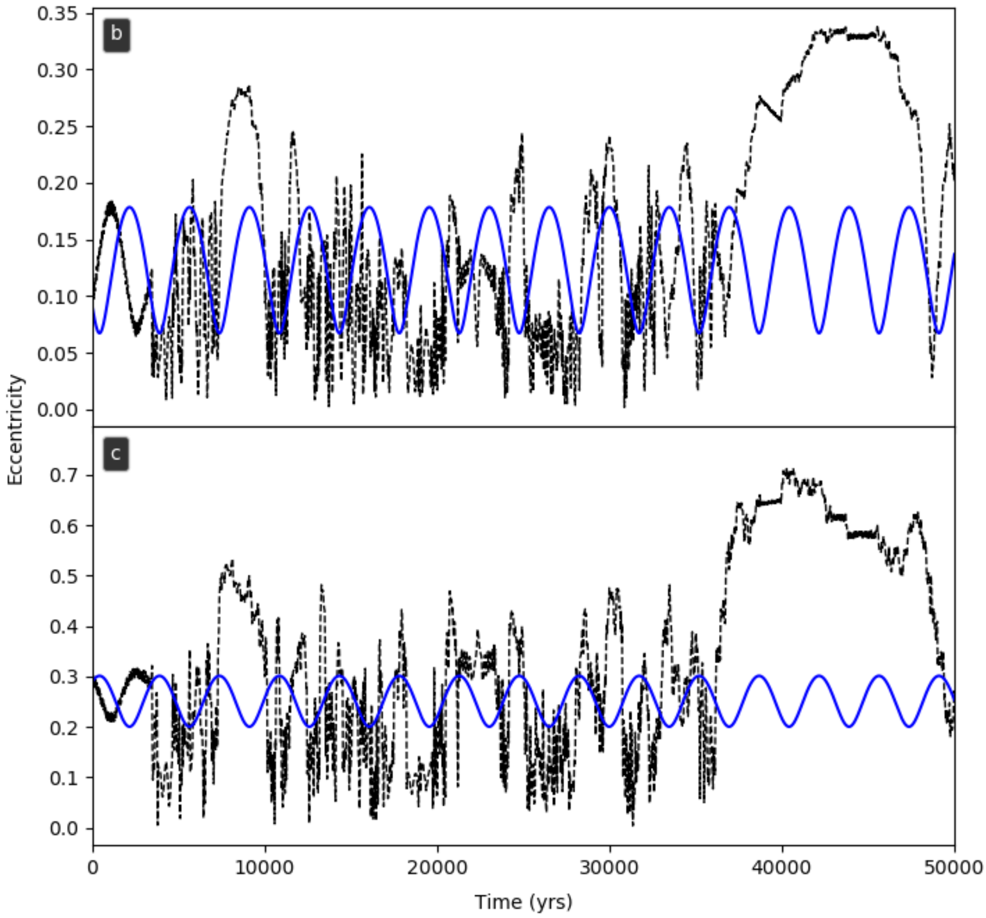

Figure 11: Comparison of secular theory (blue) for 24 Sex with n-body simulations (black).

7.1.6 CoRoT-7

Planet	a (AU)	e	i	ϖ	m $(M_{\oplus})$
CoRoT-7 b	0.017016	0.0	80.78	170.0	5.74
CoRoT-7 c	0.046	0.0	80	180	8.4

Table 9: Planet data for CoRoT-7

Frequency of oscillation for b is similar, but amplitude of oscillations is much higher for nbody.

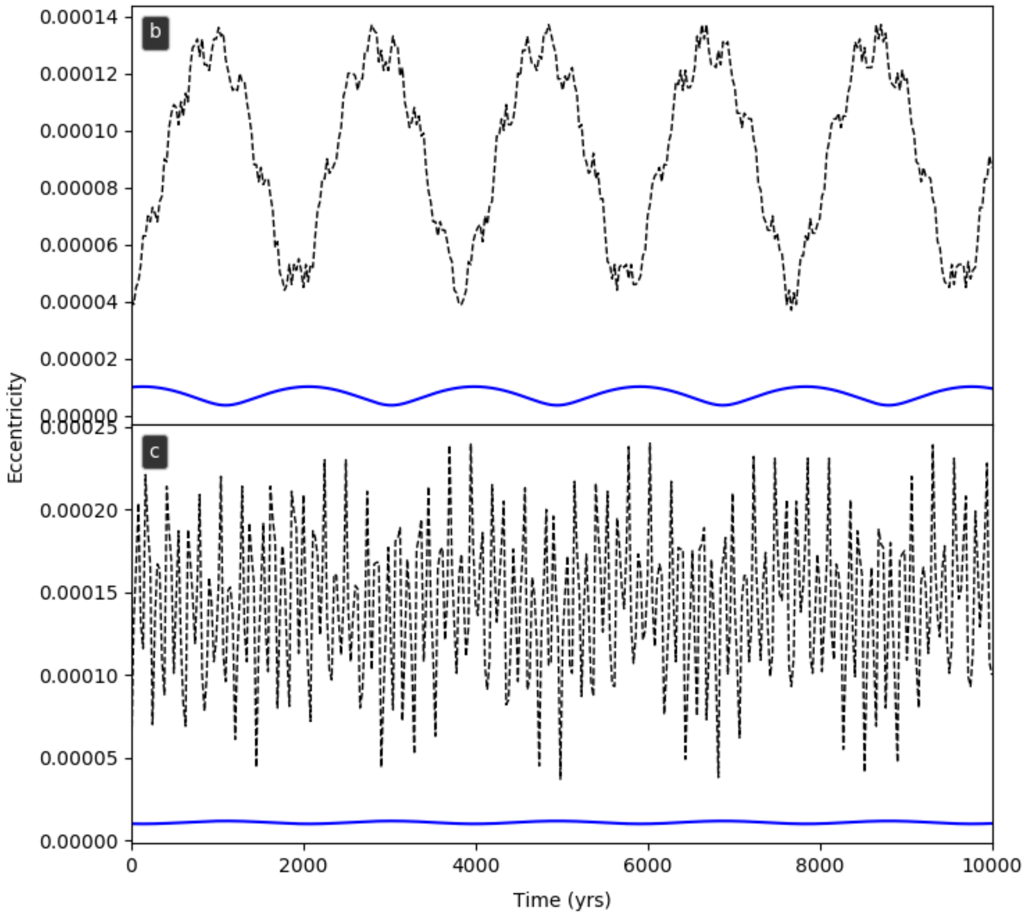

Figure 12: Comparison of secular theory (blue) for CoRoT-7with n-body simulations (black).

7.1.7 HD 142

Planet	a (AU)	e	i	ϖ	m $(M_\oplus)$
HD 142 b	1.02	0.17	0.0	327.0	397.27
HD 142 c	6.8	0.21	0.0	250.0	1684.4

Table 10: Planet data for HD 142

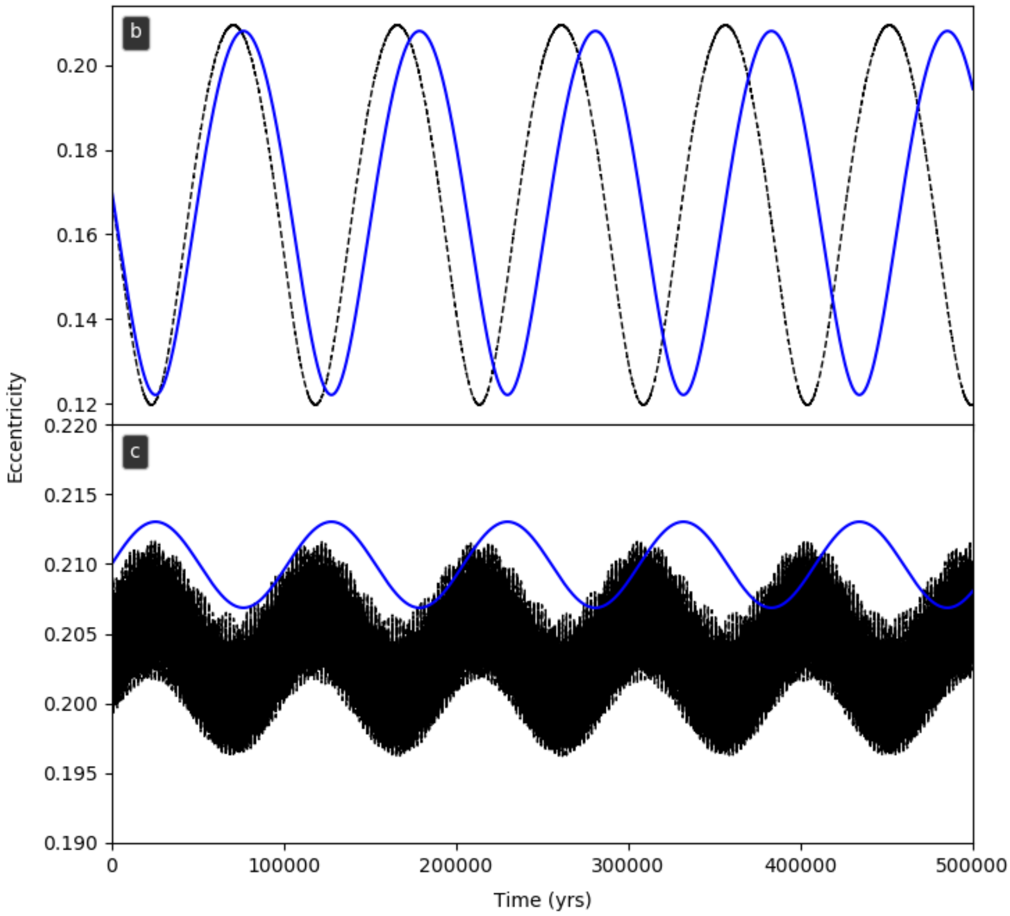

Figure 13: Comparison of secular theory (blue) for HD 142 with n-body simulations (black).

7.1.8 HD 168443

Planet	a (AU)	e	i	ϖ	m $(M_\oplus)$
HD 168443 b	0.2931	0.52883	0.0	172.923	2434.157
HD 168443 c	2.8373	0.2113	0.0	64.87	5464.221

Table 11: Planet data for HD 168443

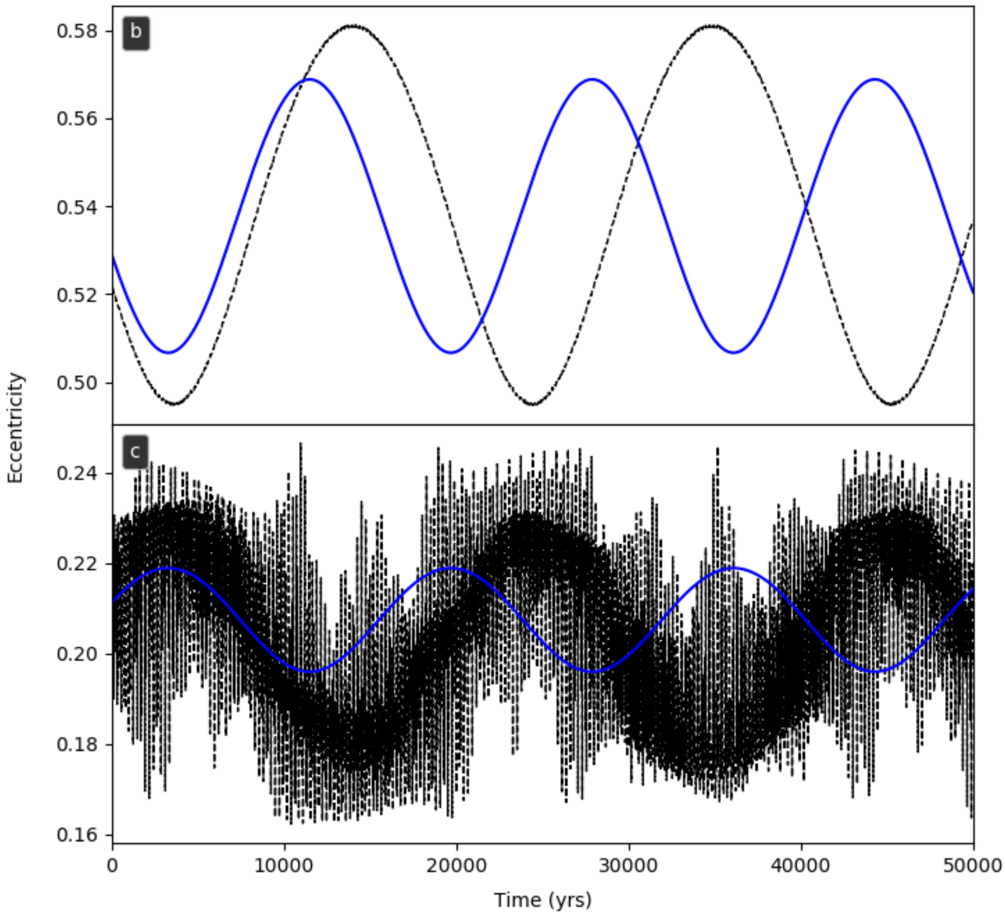

Figure 14: Comparison of secular theory (blue) for HD 168443 with n-body simulations (black).

7.1.9 HD 160691

Planet	a (AU)	e	i	ϖ	m $(M_{\oplus})$
HD 160691 b	1.497	0.128	0.0	22.0	343.24
HD 160691 c	5.235	0.0985	0.0	57.6	576.519
HD 160691 d	0.09094	0.172	0.0	212.7	10.5547
HD 160691 e	0.9210	0.0666	0.0	189.6	165.8685

Table 12: Planet data for HD 160691

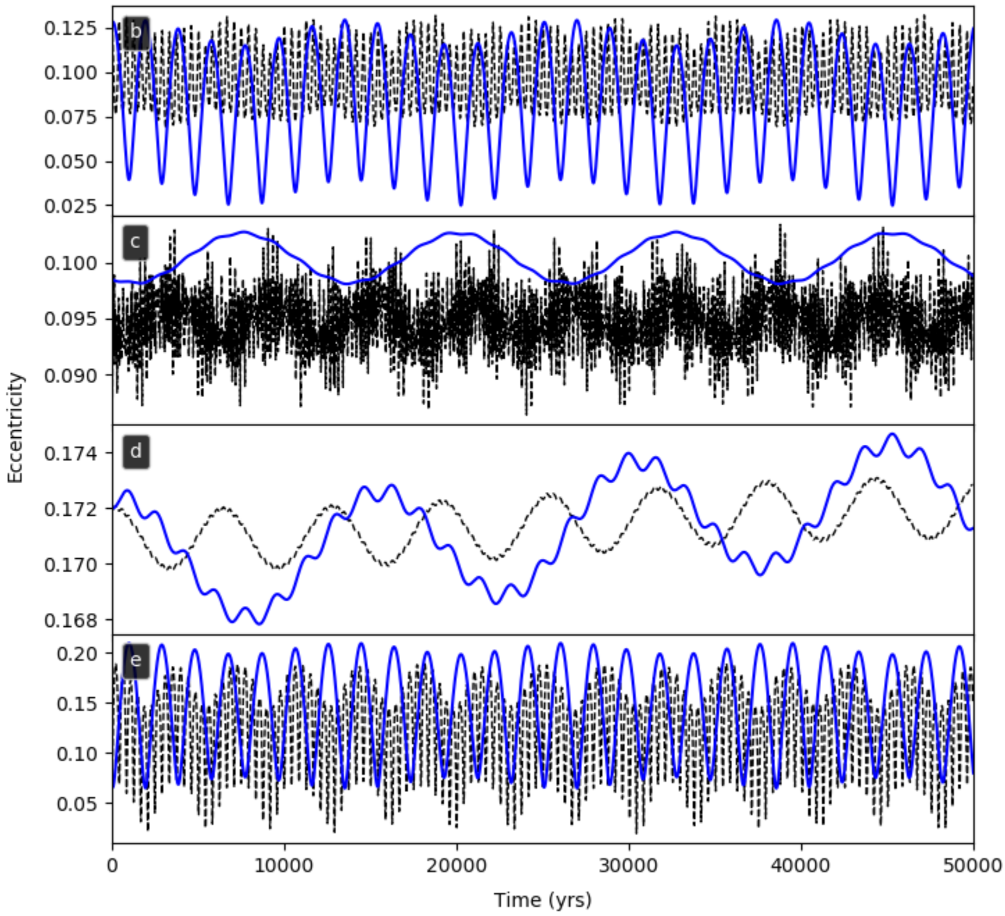

Figure 15: Comparison of secular theory (blue) for HD 160691 with n-body simulations (black).

7.1.10 Kepler-30

Planet	a (AU)	e	i	ϖ	m $(M_\oplus)$
Kepler-30 b	0.18	0.042	90.179	-31	11.3
Kepler-30 c	0.30	0.0111	90.3227	-49	640
Kepler-30 d	0.5	0.022	89.8406	-163	23.1

Table 13: Planet data for Kepler-30

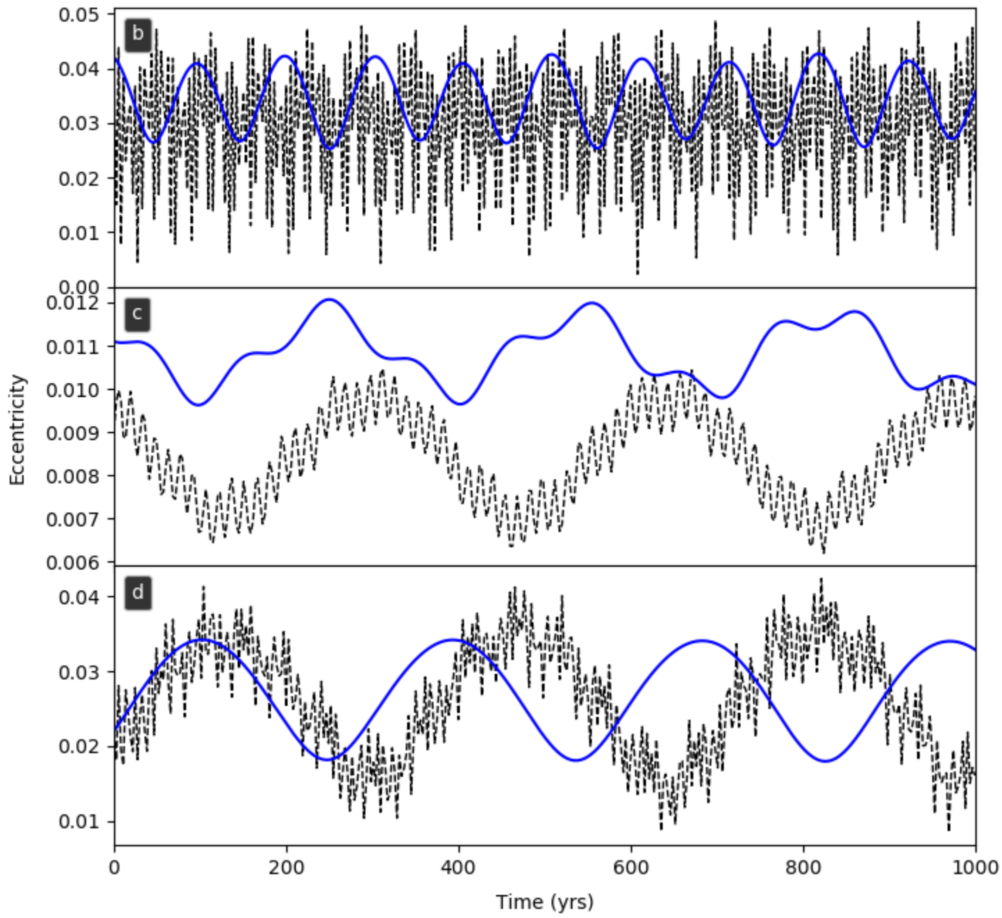

Figure 16: Comparison of secular theory (blue) for Kepler-30 with n-body simulations (black).

7.1.11 HD 10180

Planet	a (AU)	e	i	ϖ	m $(M_\oplus)$
HD 10180 c	0.06412	0.0730	90.0	328	13.2
HD 10180 d	0.12859	0.131	90.0	325	12.0
HD 10180 e	0.2699	0.051	90.0	147	25.6
HD 10180 f	0.4929	0.119	90.0	327	22.9
HD 10180 g	1.427	0.263	90.0	327	23.3
HD 10180 h	3.381	0.0950	90.0	142	65.66

Table 14: Planet data for Kepler-30

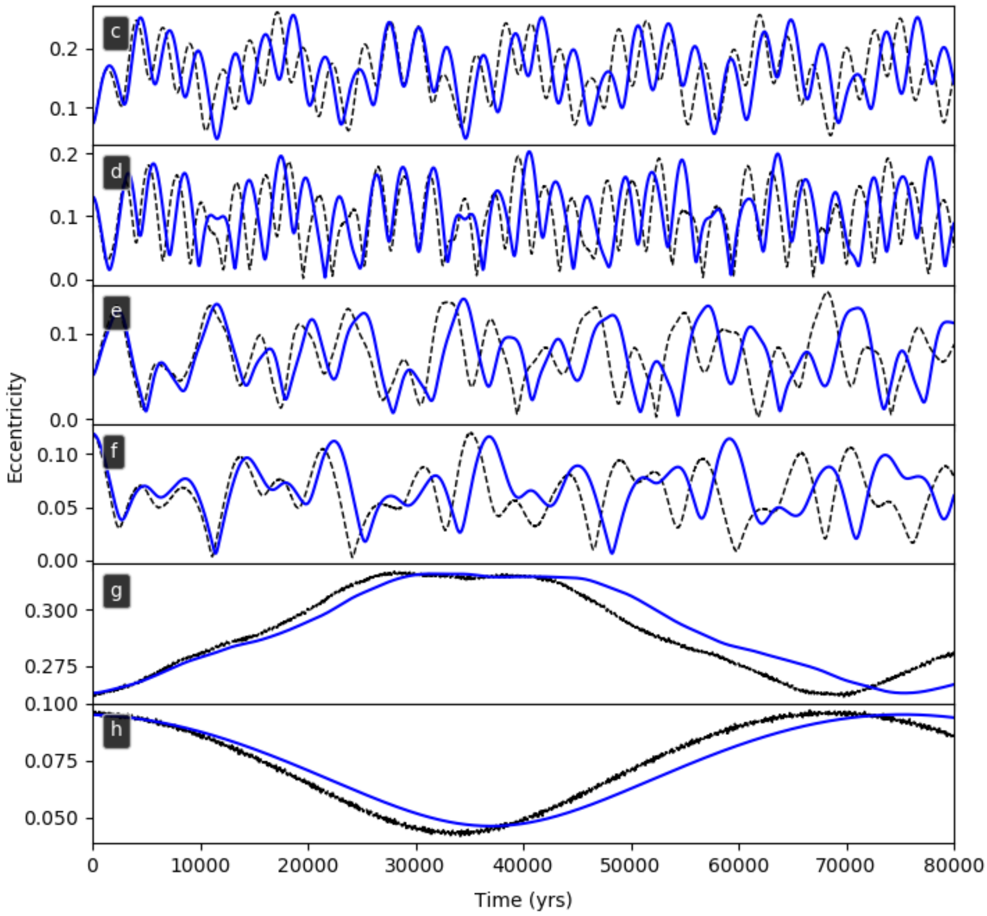

Figure 17: Comparison of secular theory (blue) for HD 10180 with n-body simulations (black).

7.1.12 HD 141399

Planet	a (AU)	e	i	ϖ	m $(M_{\oplus})$
HD 141399 b	0.415	0.04	0.0	-90	143
HD 141399 c	0.689	0.048	0.0	-140	423
HD 141399 d	2.09	0.074	0.0	-140	375
HD 141399 e	5.0	0.26	0.0	-10	210

Table 15: Planet data for Kepler-30

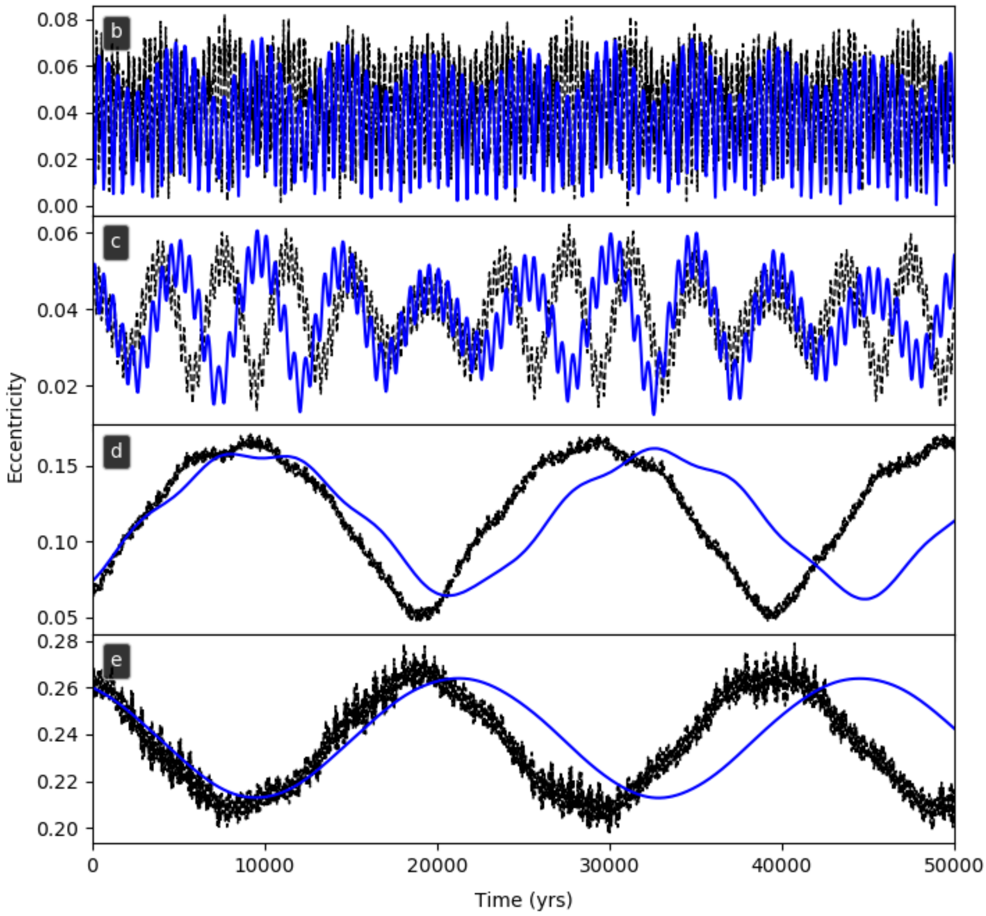

Figure 18: Comparison of secular theory (blue) for HD 141399 with n-body simulations (black).

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