

Sheela Ahmed

Professor Coppenbarger

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Two Romanian mathematicians, Ioan Casu and Arpad Benyl, redirected back to a discovery in 1936 by another Romanian mathematician, Dimitre Pompeiu. The theorem is known as Pompeiu's Theorem and states that:

*Given: an equilateral triangle  $\triangle ABC$ , where  $M$  is a point in the plane determined by it. The lengths  $MA$ ,  $MB$ , and  $MC$  all form the sides of the triangle. The triangle formed by these sides is degenerate if and only if  $M$  lies on the circumcircle of  $\triangle ABC$ .*

Casy and Benyl mention that most students will encounter this theorem as a corollary to the Ptolemy inequality, which refers to a quadrilateral with all four vertices on a circle. After this, they mention Pompeiu's Area Formula, which states that:

*Let  $\triangle ABC$  be acute with side-lengths  $a$ ,  $b$ , and  $c$  and area  $S_0$ , and let  $M$  be a point in its interior. Let  $S^*$  denote the area of the triangle having side-lengths  $MA$ ,  $MB$ , and  $MC$ , and let  $S_1$ ,  $S_2$ ,  $S_3$  denote the areas of  $\triangle BCM$ ,  $\triangle CAM$ , and  $\triangle ABM$ . Then*

$$S^* S_0 = S_1 S_2 + S_2 S_3 + S_3 S_1.$$

Ioan Casu and Arpad Benyl revisited this old theorem to prove it in multiple ways because the original as through complex numbers. So they included a proof using inversion as well.