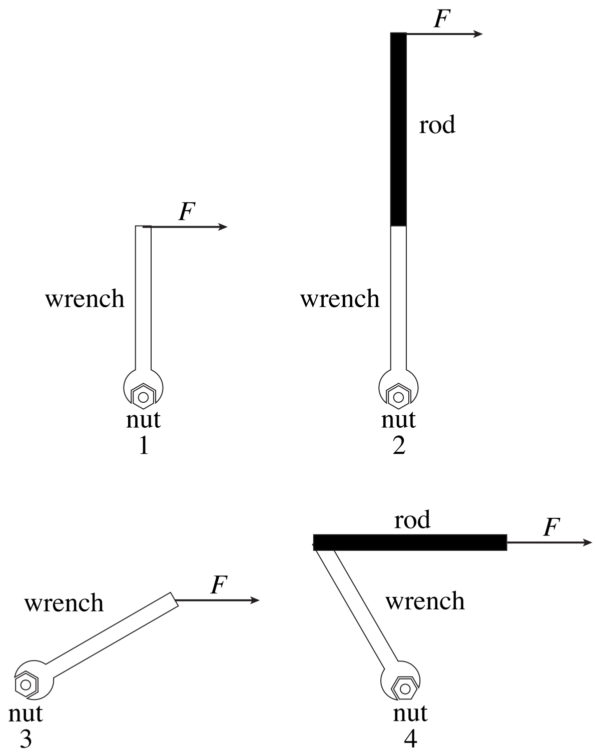
1.
You are using a wrench to tighten a nut.



Which of the following correctly lists the arrangements (above) in *descending* order of torque magnitude?

1. $\tau_1 \geq \tau_2 \geq \tau_3 \geq \tau_4$

2. $\tau_2 \geq \tau_4 \geq \tau_3 \geq \tau_1$

3. $\tau_3 \geq \tau_2 \geq \tau_4 \geq \tau_1$

4. $\tau_3 \geq \tau_4 \geq \tau_2 \geq \tau_1$

5. $\tau_2 \geq \tau_1 \geq \tau_4 \geq \tau_3$

6. $\tau_2 \geq \tau_4 \geq \tau_1 \geq \tau_3$

7. $\tau_3 \geq \tau_1 \geq \tau_2 \geq \tau_4$

8. $\tau_2 \geq \tau_1 \geq \tau_3 \geq \tau_4$

9. $\tau_1 \geq \tau_3 \geq \tau_2 \geq \tau_4$

10. $\tau_4 \geq \tau_2 \geq \tau_3 \geq \tau_1$

2.

For this problem, use table 10.2 to determine the moment of inertia for the long rod.

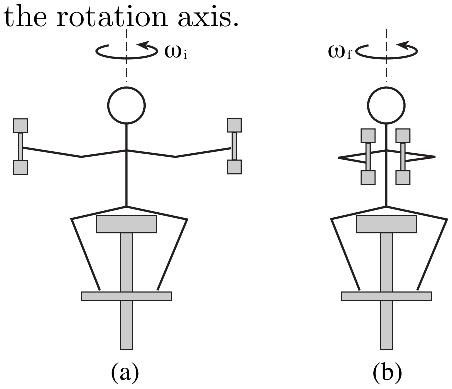
A uniform 250 g rod with length 41 cm rotates in a horizontal plane about a fixed, vertical, frictionless pin through its center. Two small 44 g beads are mounted on the rod such that they are able to slide without friction along its length. Initially the beads are held by catches at positions 13 cm on each sides of the center, at which time the system rotates at an angular speed of 29 rad/s . Suddenly, the catches are released and the small beads slide outward along the rod.

Find the angular speed of the system at the instant the beads reach the ends of the rod.

---

3 (a).

A student sits on a rotating stool holding two 5 kg objects. When his arms are extended horizontally, the objects are 0.9 m from the axis of rotation, and he rotates with angular speed of 0.7 rad/sec. The moment of inertia of the student plus the stool is 3 $\text{kg}\,\text{m}^2$ and is assumed to be constant. The student then pulls the objects horizontally to a radius 0.27 m from the rotation axis.



Calculate the final angular speed of the student.

3(b). Use the set-up from 3(a).

Calculate the change in kinetic energy of the system.

4.

A bullet of mass $3\,m$ moving with velocity v strikes tangentially the edge of a spoked wheel of radius R, and the bullet sticks to the edge of the wheel. The spoked wheel has a mass $5\,m$ concentrated on its rim (neglect the mass of the spokes). The wheel, initially at rest, begins to rotate about its center, which remains fixed on a frictionless axle.

What is the angular velocity of the spoked wheel after the collision?

1. $\dfrac{4}{11}\,\dfrac{v}{R}$

2. $\dfrac{1}{3}\,\dfrac{v}{R}$

3. $\dfrac{2}{5}\,\dfrac{v}{R}$

4. $\dfrac{3}{10}\,\dfrac{v}{R}$

5. $\dfrac{5}{7}\,\dfrac{v}{R}$

6. $\dfrac{3}{8}\,\dfrac{v}{R}$

7. $\dfrac{1}{4}\,\dfrac{v}{R}$

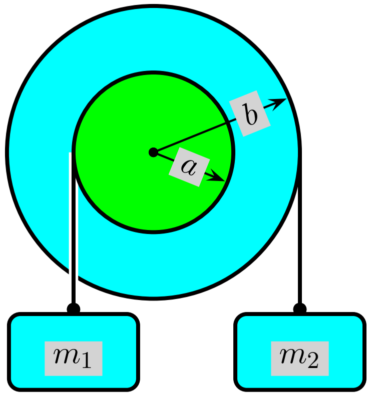
8. $\dfrac{2}{9}\,\dfrac{v}{R}$

9. $\dfrac{4}{9}\,\dfrac{v}{R}$

10. $\dfrac{2}{7}\,\dfrac{v}{R}$

5.

Consider the wheel-and-axle system shown below.



Which of the following expresses the condition required for the system to be in static equilibrium?

1. $a\, m_1 = b\, m_2$

2. $m_1 = m_2$

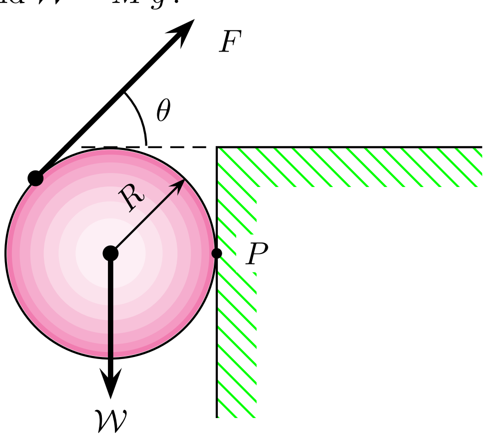
3. $a^2\, m_1 = b^2\, m_2$

4. $b^2\, m_1 = a^2\, m_2$

5. $a\, m_2 = b\, m_1$

6(a).

A solid sphere of radius R and mass M is held against a wall by a string being pulled at an angle θ . f is the magnitude of the frictional force and $\mathcal{W} = M\, g$.



To what does the torque equation $\sum_i \vec{\tau}_i = 0$ about point O (the center of the sphere) lead?

choices on next page →

1. $F = f$

2. $\mathcal{W} = f$

3. $F \cos^2\theta = f$

4. $F \sin\theta = f$

5. $F + \mathcal{W} = f$

6. $F \sin\theta\cos\theta = f$

6(b).

To what does the vertical component of the force equation lead?

1. $F \sin\theta = f + \mathcal{W}$

2. $F \sin\theta = \mathcal{W}$

3. $F \cos\theta + \mathcal{W} = f$

4. $F \sin\theta + f = \mathcal{W}$

5. $F \sin\theta = f$

6(c).

Find the smallest coefficient of friction μ needed for the wall to keep the sphere from slipping.

1. $\mu = \cos\theta$

2. $\mu = \dfrac{1}{\cos\theta}$

3. $\mu = \tan\theta$

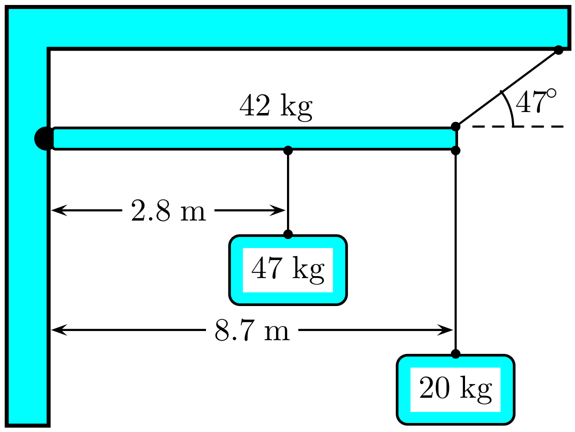
4. $\mu = \dfrac{1}{\tan\theta}$

5. $\mu = \dfrac{1}{\sin\theta}$

6. $\mu = \sin\theta$

7.

Two weights attached to a uniform beam of mass 42 kg are supported in a horizontal position by a pin and cable as shown in the figure.



What is the tension in the cable which supports the beam? The acceleration of gravity is 9.8 m/s^2 .

ANSWERS

1. Choice 5 is correct

Explanation:

$$\tau \equiv \vec{r} \times \vec{F}$$

The force as shown is in the x direction, so the perpendicular radius arm is always in the y direction. For a given force, a longer radius arm defines the greater torque τ, so

$$\tau_2 \geq \tau_1 \geq \tau_4 \geq \tau_3 .$$

2.

Correct answer: 20.0949 rad/s.

Explanation:

Basic Concepts:

$$\sum \vec{L} = \text{const} .$$

$$\begin{aligned} M &= 250 \text{ g} , \\ m &= 44 \text{ g} , \\ \ell &= 41 \text{ cm} , \\ h &= 13 \text{ cm} , \quad \text{and} \\ \omega_1 &= 29 \text{ rad/s} , \end{aligned}$$

Solution: From conservation of the angular momentum of the system we have

$$I_1 \, \omega_1 = I_2 \, \omega_2 ,$$

where

$$\begin{aligned} I_1 &= \frac{1}{12} \, M \, \ell^2 + 2 \, m \, h^2 \\ &= \frac{1}{12} \, (250 \text{ g}) \, (41 \text{ cm})^2 + 2 \, (44 \text{ g}) \, (13 \text{ cm})^2 \\ &= 49892.8 \text{ g cm}^2 . \end{aligned}$$

$$\begin{aligned} I_2 &= \frac{1}{12} \, M \, \ell^2 + 2 \, m \, \frac{\ell}{2}^2 \\ &= \frac{1}{12} \, (250 \text{ g}) \, (41 \text{ cm})^2 + 2 \, (44 \text{ g}) \, (20.5 \text{ cm})^2 \\ &= 72002.8 \text{ g cm}^2 . \end{aligned}$$

where $\frac{\ell}{2}$ is distance from the center to an end of the rod. Finally,

$$\begin{aligned} \omega_2 &= \omega_1 \, \frac{I_1}{I_2} \\ &= (29 \text{ rad/s}) \, \frac{(49892.8 \text{ g cm}^2)}{(72002.8 \text{ g cm}^2)} \\ &= 20.0949 \text{ rad/s} . \end{aligned}$$

3(a).
Correct answer: 2.08367 rad/s.

Explanation:

$$\begin{aligned}\text{Let :}\quad m &= 5\text{ kg}\,,\\ R &= 0.9\text{ m}\,,\\ r &= 0.27\text{ m}\,,\\ \omega &= 0.7\text{ rad/sec}\,,\quad\text{and}\\ I_s &= 3\text{ kg m}^2\,.\end{aligned}$$

$$\sum \vec{L} = \text{Constant.}$$

$$K_{rot} = \frac{1}{2}\, I\, \omega^2$$

The initial moment of inertia of the system is

$$\begin{aligned}I_i &= I_s + 2\, m\, R^2\\ &= (3\text{ kg m}^2) + 2\,(5\text{ kg})\,(0.9\text{ m})^2\\ &= 11.1\text{ kg m}^2\,.\end{aligned}$$

The final moment of inertia of the system is

$$\begin{aligned}I_f &= I_s + 2\, m\, r^2\\ &= \left(3\text{ kg m}^2\right) + 2\,(5\text{ kg})\,(0.27\text{ m})^2\\ &= 3.729\text{ kg m}^2\,.\end{aligned}$$

From conservation of the angular momentum it follows that

$$\begin{aligned}I_i\,\omega_i &= I_f\,\omega_f\\ \omega_f &= \omega_i\,\frac{I_i}{I_f}\\ &= (0.7\text{ rad/sec})\left(\frac{11.1\text{ kg m}^2}{3.729\text{ kg m}^2}\right)\\ &= \boxed{2.08367\text{ rad/s}}\,.\end{aligned}$$

3(b).

Correct answer: 5.37555 J.

Explanation:

$$\begin{aligned}
\Delta K &= K_f - K_i \\
&= \frac{1}{2} I_f \, \omega_f^2 - \frac{1}{2} I_i \, \omega_i^2 \\
&= \frac{1}{2} \left(3.729 \text{ kg m}^2\right) (2.08367 \text{ rad/s})^2 \\
&\qquad - \frac{1}{2} \left(11.1 \text{ kg m}^2\right) (0.7 \text{ rad/sec})^2 \\
&= (8.09505 \text{ J}) - (2.7195 \text{ J}) \\
&= \boxed{5.37555 \text{ J}} .
\end{aligned}$$

---

4. Choice 6 is correct.

Explanation:

The total angular momentum of the system is conserved since there is no external force. The total angular momentum before the collision is

$$L_{before} = R \, (3 \, m) \, v$$

and after the collision

$$\begin{aligned}
L_{after} &= L_{bullet} + L_{wheel} \\
&= (3 \, m) \, R^2 \, \omega + (5 \, m) \, R^2 \, \omega \\
&= (8 \, m) \, R^2 \, \omega \, ,
\end{aligned}$$

so from conservation of angular momentum,

$$\begin{aligned}
L_{before} &= L_{after} \\
R \, (3 \, m) \, v &= (8 \, m) \, R^2 \, \omega \\
\omega &= \frac{3}{8} \, \frac{v}{R} \, .
\end{aligned}$$

The total kinetic energy is not conserved because the collision is not elastic.

---

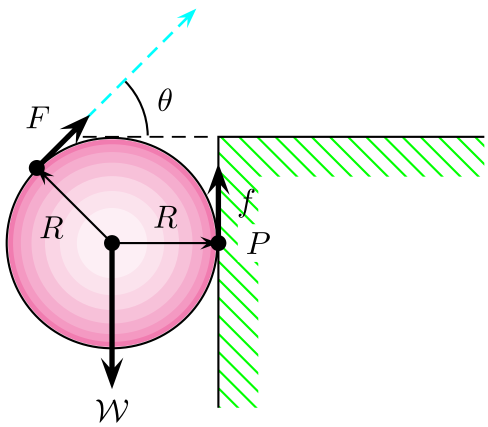
5. Choice 1 is correct.

Explanation:

In equilibrium, the total torque is zero, which gives

$$\boxed{a \, m_1 = b \, m_2} .$$

6(a). Choice 1 is correct.



Applying rotational equilibrium about O, the center of the sphere, $\sum_i \vec{\tau}_i = 0$, so

$$\begin{aligned}\tau_{CW} &= \tau_{CCW}\\ F\,R &= f\,R\\ F &= f\,.\end{aligned}$$

6(b). Choice 4 is correct.

Explanation:

Applying translational equilibrium vertically,

$$\sum_i F_{yi} = F\,\sin\theta + f - \mathcal{W} = 0$$

$$F\,\sin\theta + f = \mathcal{W}\,.$$

6(c). Choice 2 is correct.

Explanation:

Let N be the normal force. $f \le \mu\,N$; when μ is minimal, $f = \mu\,N$. Applying translational equilibrium horizontally,

$$\sum_i F_{xi} = F\,\cos\theta - N = 0$$

$$\begin{aligned}\mu\,N\,\cos\theta - N &= 0\\ N\,(\mu\,\cos\theta - 1) &= 0\end{aligned}$$

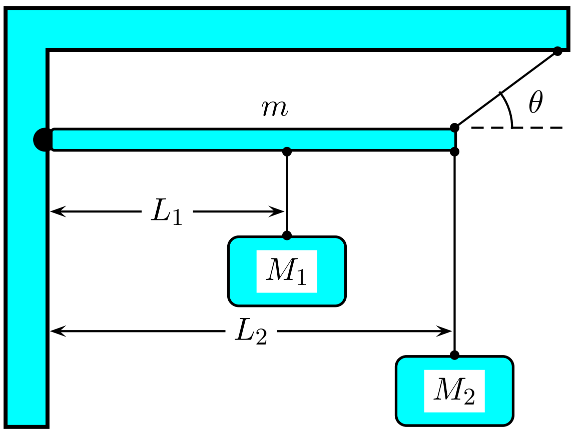
$$\mu = \frac{1}{\cos\theta}\,.$$

7.

Correct answer: 0.752084 kN.

Explanation:

$$\begin{aligned} \text{Let :} \quad m &= 42\ \text{kg}\,, \\ M_1 &= 47\ \text{kg}\,, \\ M_2 &= 20\ \text{kg}\,, \\ L_1 &= 2.8\ \text{m}\,, \\ L_2 &= 8.7\ \text{m}\,, \quad \text{and} \\ \theta &= 47^\circ\,. \end{aligned}$$



For equilibrium,

$$\sum \vec{F} = 0 \quad \text{and} \quad \sum \vec{\tau} = 0\,.$$

The sum of the torques about the pivot is

$$T\,L_2\,\sin\theta - m\,g\,\frac{L_2}{2} - M_1\,g\,L_1 - M_2\,g\,L_2 = 0$$

$$\begin{aligned} T &= \frac{m\,g}{2\,\sin\theta} + \frac{M_1\,g\,L_1}{L_2\,\sin\theta} + \frac{M_2\,g}{\sin\theta} \\ &= \frac{(42\ \text{kg})(9.8\ \text{m/s}^2)}{2\,\sin 47^\circ} \\ &\quad + \frac{(47\ \text{kg})\,(9.8\ \text{m/s}^2)\,(2.8\ \text{m})}{(8.7\ \text{m})\,\sin 47^\circ} \\ &\quad + \frac{(20\ \text{kg})(9.8\ \text{m/s}^2)}{\sin 47^\circ} \\ &= 752.084\ \text{N} = \boxed{0.752084\ \text{kN}}\,. \end{aligned}$$