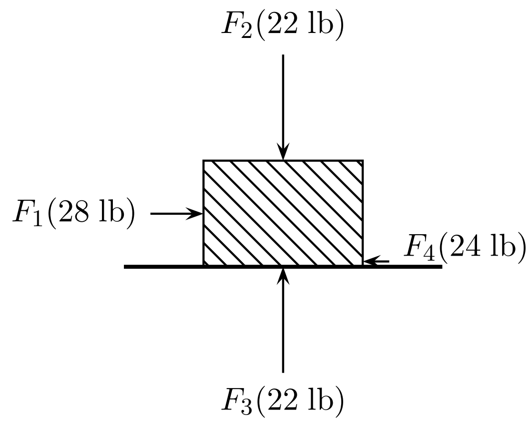
1.

A 22-pound crate is pushed across the floor by a 28-pound horizontal force F_1. Aside from the pushing force and gravity F_2, there is also a 22-pound force F_3 exerted upward on the crate and a 24-pound frictional force F_4, as shown in the figure.



What kinds of work do the force F_1, F_2, F_3, F_4 do, respectively?

1. $+,\ +,\ 0,\ 0$

2. $+,\ 0,\ 0,\ -$

3. $-,\ +,\ -,\ +$

4. $+,\ 0,\ +,\ 0$

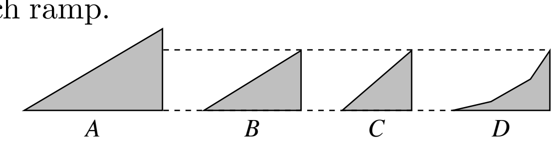
5. $+,\ -,\ +,\ -$

6. $+,\ 0,\ 0,\ 0$

---

2 (a).

Consider pushing a cart up each of the four frictionless ramps. The cart begins at rest to the left of each ramp and then ends at rest at the top. Let W_A, W_B, W_C and W_D be the amount of work needed to push the cart up each ramp.



What describes the relationship between the work required in each case?

choices on next page →

1. $W_A = W_B = W_C < W_D$

2. $W_A > W_B = W_C = W_D$

3. $W_A > W_B > W_C > W_D$

4. $W_A = W_B = W_C = W_D$

5. $W_A < W_B < W_C < W_D$

6. $W_A < W_B = W_C = W_D$

2(b).Use set-up from 2(a).

Just as each cart reaches the top of each ramp, it is released and rolls back down to the left. Just as it reaches the floor, its velocity on each ramp is v_A, v_B, v_C and v_D.

Which of the following describes the relationship between the final velocities in each case?

1. $v_A > v_B = v_C = v_D$

2. $v_A < v_B < v_C < v_D$

3. $v_A > v_B > v_C > v_D$

4. $v_A = v_B = v_C = v_D$

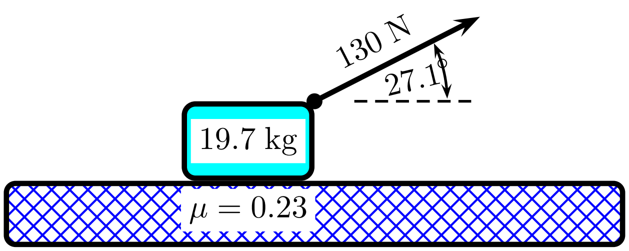
5. $v_A < v_B = v_C = v_D$

6. $v_A = v_B = v_C < v_D$

3.

A 19.7 kg block is dragged over a rough, horizontal surface by a constant force of 130 N acting at an angle of 27.1° above the horizontal. The block is displaced 64.3 m, and the coefficient of kinetic friction is 0.23.

The acceleration of gravity is 9.8 m/s^2 .



Find the work done by the force of friction.

4.

A man lifts a 30.6 kg bucket from a well and does 7.94 kJ of work.

The acceleration of gravity is 9.8 m/s^2 .

How deep is the well? Assume that the speed of the bucket remains constant as it is lifted.

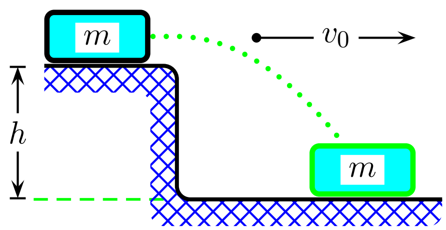
5.

An outfielder throws a 1.48 kg baseball at a speed of 82 m/s and an initial angle of 37.6°.

What is the kinetic energy of the ball at the highest point of its motion?

6.

A rock of mass m is thrown horizontally off a building from a height h. The speed of the rock as it leaves the thrower's hand at the edge of the building is v_0, as shown.



What is the kinetic energy of the rock just before it hits the ground?

1. $K_f = \dfrac{1}{2}\, m\, v_0^2 - m\, g\, h$

2. $K_f = m\, g\, h$

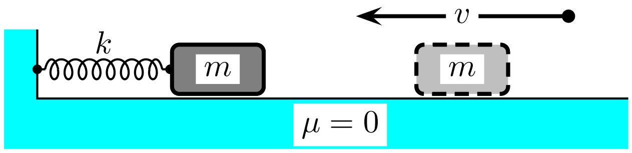
3. $K_f = \dfrac{1}{2}\, m\, v_0^2 + m\, g\, h$

4. $K_f = \dfrac{1}{2}\, m\, v_0^2$

5. $K_f = m\, g\, h - \dfrac{1}{2}\, m\, v_0^2$

7.

A block of mass m slides on a horizontal frictionless table with an initial speed v_0. It then compresses a spring of force constant k and is brought to rest.



How much is the spring compressed x from its natural length?

1. $x = v_0 \sqrt{\dfrac{m}{k}}$

2. $x = \dfrac{v_0^2}{2\,g}$

3. $x = v_0\,\dfrac{m\,g}{k}$

4. $x = v_0 \sqrt{\dfrac{m\,g}{k}}$

5. $x = \dfrac{v_0^2}{2\,m}$

6. $x = v_0\,\dfrac{m}{k\,g}$

7. $x = v_0 \sqrt{\dfrac{k}{m\,g}}$

8. $x = v_0 \sqrt{\dfrac{k}{m}}$

9. $x = v_0\,\dfrac{m\,k}{g}$

10. $x = v_0\,\dfrac{k}{g\,m}$

8(a).

A frictionless roller coaster is given an initial velocity of v_0 at height $h = 29$ m. The radius of curvature of the track at point A is $R = 28$ m.

The acceleration of gravity is $9.8\ \mathrm{m/s^2}$.

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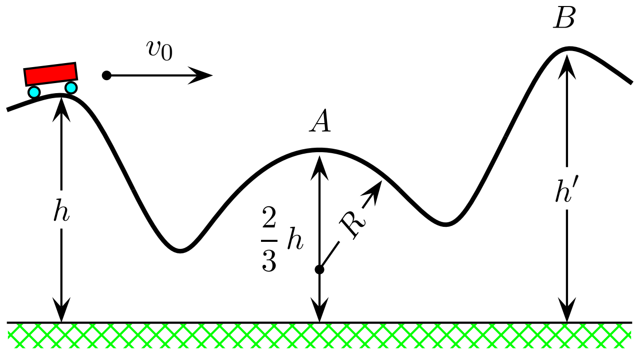


Find the maximum value of v_0 so that the roller coaster stays on the track at A solely because of gravity.

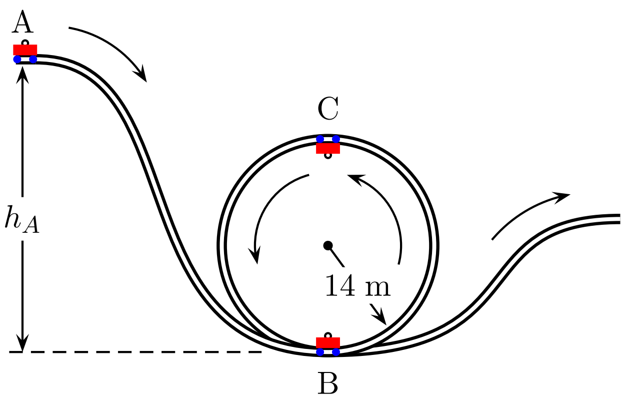
8(b). For this, use the set-up and answer from 8(a).

Using the value of v_0 calculated in question 1, determine the value of h' that is necessary if the roller coaster just makes it to point B.

9.

Consider a frictionless roller coaster such as depicted below.

The acceleration of gravity is 9.8 m/s^2 .



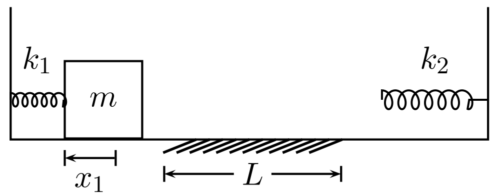
Passenger cars start at point A with zero initial speed, accelerate as they go down to point B, swing around the circular vertical loop B $\to$ C $\to$ B of radius 14 m, then go on towards further adventures (not shown). When a car goes through the top of the loop (point C), the passengers feel weightless (for just a moment).

What is the height h_A of the starting point A above the loop's bottom B?

10.

A spring of constant 20 N/cm is compressed a distance 8 cm by a(n) 0.3 kg mass, then released. It skids over a frictional surface of length 2 m with coefficient of friction 0.17, then compresses the second spring of constant 5 N/cm.

The acceleration of gravity is 9.8 m/s^2.



How far will the second spring compress in order to bring the mass to a stop?

11.

The potential energy of a two-particle system separated by a distance r is $U(r) = \dfrac{A}{r}$, where $A = 115$ J $\cdot$ m is a constant.

Find the radial force F_r if $r = 66.6$ m.

ANSWERS

1. Choice 2 is correct.

Explanation:

The crate moves in the direction of F_1.

Gravity and the normal force do no work since they do not act in the direction of the motion.

The pushing force does positive work, and the frictional force does negative work.

2(a). Choice 2 is correct.

Explanation:

The work done to the block is equal to the change of the potential energy, $m\,g\,h$, with h the height of the ramp, since the cart is at rest at both the bottom and the top. According to the figure, B, C and D have the same height h and only A has a larger h, the relationship between the work done should be $W_A > W_B = W_C = W_D$.

2(b). Choice 1 is correct.

Explanation:

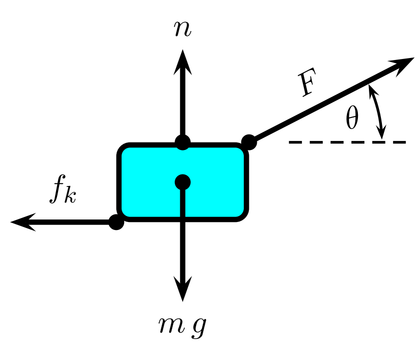
After the cart is released and rolls down to the bottom, the potential energy changes into kinetic energy. Only A has a higher potential, so its speed at the bottom is also larger than the others while B, C and D have the same speed at the bottom.

3.

Correct answer: -1979.35 J.

Explanation:

Consider the force diagram



To find the frictional force, $F_{friction} = \mu\,\mathcal{N}$, we need to find $\mathcal{N}$ from vertical force balance. Note that $\mathcal{N}$ is in the same direction as the y component of F and opposite the force of gravity. Thus

$$F\,\sin\theta + \mathcal{N} = m\,g$$

so that

$$\mathcal{N} = m\,g - F\,\sin\theta\,.$$

Thus the friction force is

$$\vec{F}_{friction} = -\mu\,\mathcal{N}\,\hat{\imath} = -\mu\,(m\,g - F\,\sin\theta)\,\hat{\imath}\,.$$

The work done by friction is then

$$\begin{aligned} W_\mu &= \vec{F}_{friction}\cdot\vec{s} = -|f_\mu|\,|s| \\ &= -\mu\,(m\,g - f_\mu\,\sin\theta)\,s_x \\ &= -0.23\,[(19.7\text{ kg})\,(9.8\text{ m/s}^2) \\ &\quad - (130\text{ N})\,\sin 27.1^\circ]\,(64.3\text{ m}) \\ &= -1979.35\text{ J}\,. \end{aligned}$$

4.

Correct answer: 26.4773 m.

Explanation:

$$\begin{aligned} \text{Given :}\quad W &= 7.94\text{ kJ}\,, \\ g &= 9.8\text{ m/s}^2\,,\quad \text{and} \\ m &= 30.6\text{ kg}\,. \end{aligned}$$

The weight mg is lifted a distance h, so the the work is

$$\begin{aligned} W &= m\,g\,h \\ h &= \frac{W}{m\,g} \\ &= \frac{7.94\text{ kJ}}{(30.6\text{ kg})\,(9.8\text{ m/s}^2)} \\ &= \boxed{26.4773\text{ m}}\,. \end{aligned}$$

5.

Correct answer: 3123.4 J.

Explanation:

$$\text{Given: } \quad m = 1.48 \text{ kg}\,, \quad \theta = 37.6^\circ\,, \quad \text{and} \quad v = 82 \text{ m/s}\,.$$

At its highest point, the baseball has only a horizontal velocity $v_x = v \cos\theta$ contributing to its kinetic energy, so the kinetic energy is

$$\begin{aligned} K &= \frac{1}{2}\, m\, v_x^2 \\ &= \frac{1}{2}\, m\, (v\, \cos\theta)^2 \\ &= \frac{1}{2}(1.48 \text{ kg})\,[(82 \text{ m/s})\, \cos 37.6^\circ]^2 \\ &= \boxed{3123.4 \text{ J}}\,. \end{aligned}$$

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6. Choice 3 is correct.

Explanation:

We can use work-energy theorem, $K_f - K_i = W$, where in the present problem, $W = (m\,g)\,h$, and $K_i = \dfrac{1}{2}\, m\, v_0^2$.

Thus, $K_f = K_i + W = \boxed{\dfrac{1}{2}\, m\, v_0^2 + m\, g\, h}$.

Comments for Matter Interaction readers: The energy principle in MI-text is referred to as the work-energy theorem in traditional text. Here the work done by the gravitational force near the surface of the earth is given by: $W = (-m\,g)(-h) = m\,g\,h$.

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7. Choice 1 is correct.

Explanation:

Total energy is conserved (no friction). The spring is compressed by a distance x from its natural length, so

$$\frac{1}{2}\, m\, v_0^2 = E_i = E_f = \frac{1}{2}\, k\, x^2\,, \quad \text{or}$$

$$x^2 = \frac{m}{k}\, v_0^2\,, \quad \text{therefore}$$

$$x = v_0 \sqrt{\frac{m}{k}}\,.$$

8(a).

Correct answer: 9.21593 m/s.

Explanation:

$$\text{let}: \quad h = 29 \text{ m} \quad \text{and}$$
$$R = 28 \text{ m}\,.$$

At point A, the weight of coaster must be just large enough to supply the centripetal acceleration. Thus

$$m\left(\frac{v_A^2}{R}\right) = m\,g\,,$$

or

$$v_A^2 = R\,g\,.$$

Applying conservation of mechanical energy from the start to point A,

$$\frac{1}{2}m\,v_0^2 + m\,g\,h = \frac{1}{2}m\,v_A^2 + m\,g\left(\frac{2\,h}{3}\right)$$
$$v_0^2 = v_A^2 - \frac{2\,g\,h}{3}$$
$$= R^2\,g^2 - \frac{2\,g\,h}{3}$$

$$v_0 = \sqrt{R\,g - \frac{2\,g\,h}{3}} = \sqrt{g\left(R - \frac{2\,h}{3}\right)}$$
$$= \sqrt{(9.8 \text{ m/s}^2)\left[28 \text{ m} - \frac{2\,(29 \text{ m})}{3}\right]}$$
$$= \boxed{9.21593 \text{ m/s}}\,.$$

8(b).

Correct answer: 33.3333 m.

Explanation:

If the speed of the coaster is to be zero at point B, conservation of mechanical energy from the start to point B gives

$$0 + m\,g\,h' = \frac{1}{2}m\,v_0^2 + m\,g\,h$$
$$= \frac{1}{2}m\,g\left(R - \frac{2\,h}{3}\right) + m\,g\,h$$
$$h' = \frac{R}{2} + \frac{2\,h}{3}$$
$$= \frac{28 \text{ m}}{2} + \frac{2\,(29 \text{ m})}{3}$$
$$= \boxed{33.3333 \text{ m}}\,.$$

9.

Correct answer: 35 m.

Explanation:

$$\text{Let :} \quad R = 14 \text{ m} \quad \text{and}$$
$$g = 9.8 \text{ m/s}^2 \,.$$

A passenger feels weightless when his acceleration $\vec{a}$ is precisely equal to the free-fall acceleration $\vec{g}$. At the top of the loop (point C), passengers have no tangential acceleration while their normal acceleration is directed straight down and has magnitude

$$a_N = \frac{v^2}{R} \,.$$

Weightlessness happens when this acceleration equals g, therefore the cars should go through point C at speed

$$v_C = \sqrt{R\,g} \,.$$

In the absence of friction, the mechanical energy of a car is conserved as it follows the coaster's route, thus

$$K_A + U_A = K_C + U_C$$
$$\Downarrow$$
$$\frac{m}{2}\,v_A^2 + m\,g\,h_A = \frac{m}{2}\,v_C^2 + m\,g\,2\,R.$$

Therefore, in light of the initial condition $v_A = 0$, we have

$$\begin{aligned}
m\,g\,h_A &= m\,g\,2\,R + \frac{m}{2}\,v_C^2 \\
&= m\,g\,2\,R + \frac{m}{2}\,R\,g \\
&= \frac{5}{2}\,m\,g\,R \\
h_A &= \frac{5}{2}\,R \\
&= \frac{5}{2}\,(14 \text{ m}) \\
&= \boxed{35 \text{ m}}\,.
\end{aligned}$$

10.

Correct answer: 14.6975 cm.

Explanation:

Consider the kinetic energy of the mass. The mass receives its initial kinetic energy from the potential energy of the spring

$$K_o = U_{s1} = \frac{1}{2} k_1 x_1^2$$

It loses kinetic energy by doing work on the frictional surface

$$K_{lost} = W_{fr} = \mu m g L$$

and loses its remaining kinetic energy by compressing the second spring to its maximum

$$K_{lost} = U_{s2} = \frac{1}{2} k_2 x_2^2$$

Thus

$$U_{s1} - W_{fr} - \frac{1}{2} k_2 x_2^2 = 0$$

$$U_{s1} - W_{fr} = \frac{1}{2} k_2 x_2^2$$

Multiplying by 2 and dividing by k_2 gives us

$$\frac{2(U_{s1} - W_{fr})}{k_2} = x_2^2$$

Thus

$$\begin{aligned} x_2 &= \sqrt{\frac{2(U_{s1} - W_{fr})}{k_2}} \\ &= \sqrt{\frac{2(6.4 \text{ J} - 0.9996 \text{ J})}{5 \text{ N/cm}}} \\ &= 14.6975 \text{ cm} \end{aligned}$$

11.

Correct answer: 0.0259268 N.

Explanation:

From the connection between potential energy and the force:

$$F = -\frac{\partial U}{\partial r} \quad \text{we obtain} \quad F_r = \frac{A}{r^2}$$

$$= \frac{115 \text{ J} \cdot \text{m}}{(66.6 \text{ m})^2}$$

$$= 0.0259268 \text{ N}$$