ECON 525: Homework 6

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1 Problem 1:

Use Chebyshev interpolation of degree 5 to approximate the following functions over the interval $[-1, 1]$. For both functions plot the true function and its approximation.

(a) $f(x) = (1 + 25x^2)^{-1}$

Solution:

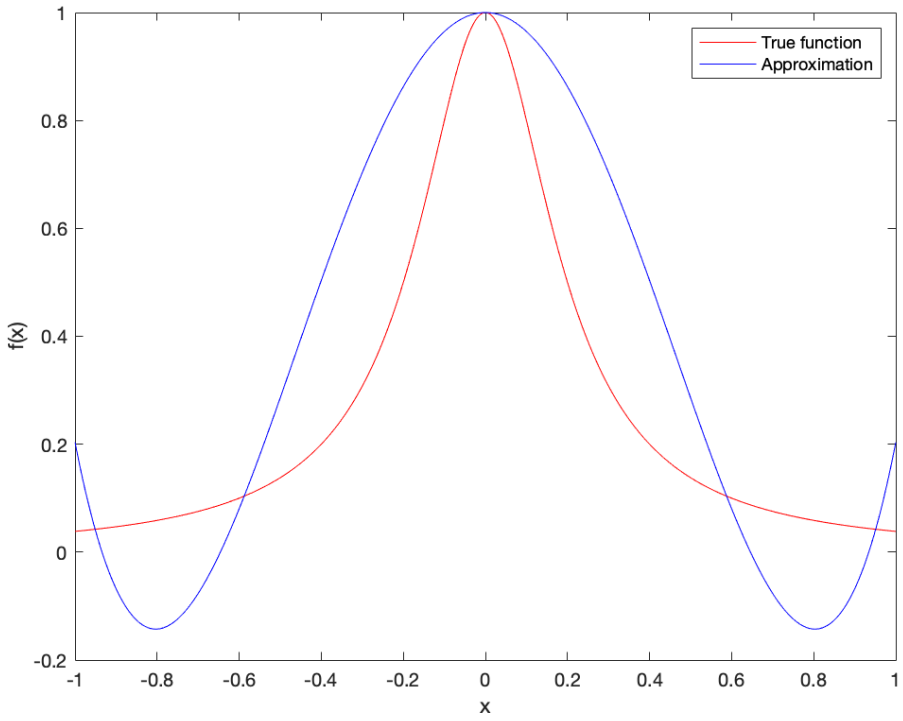

Figure 1: Approximation using Chebhysev interpolation of degree 5

We can observe that the general shape of the approximation matches the shape of the true function. However, the approximation is not good enough for this small degree of interpolation. The approximated function matches the true function for only certain values of x. To improve our approximation, we need to increase the degree of interpolation. We should expect to see that the approximation should be getting close to the true function. This is made clear in the following figure:

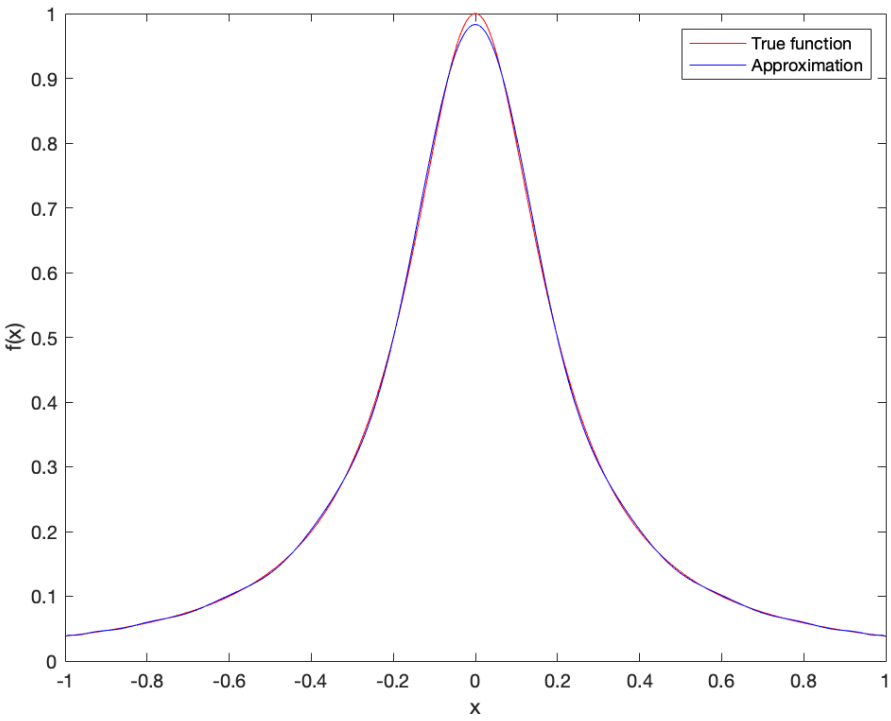

Figure 2: Approximation using Chebhysev interpolation of degree 24

The approximation is now much closer to the true function. The approximation matches the true function at a greater number of x values and it seems like there is a lot of overlap. We should expect the functions to completely overlap as we increase the degree of interpolation.

(b) $f(x) = \min\left[\max\left[-1, 4(x-0.2)\right], 1\right]$

Solution:

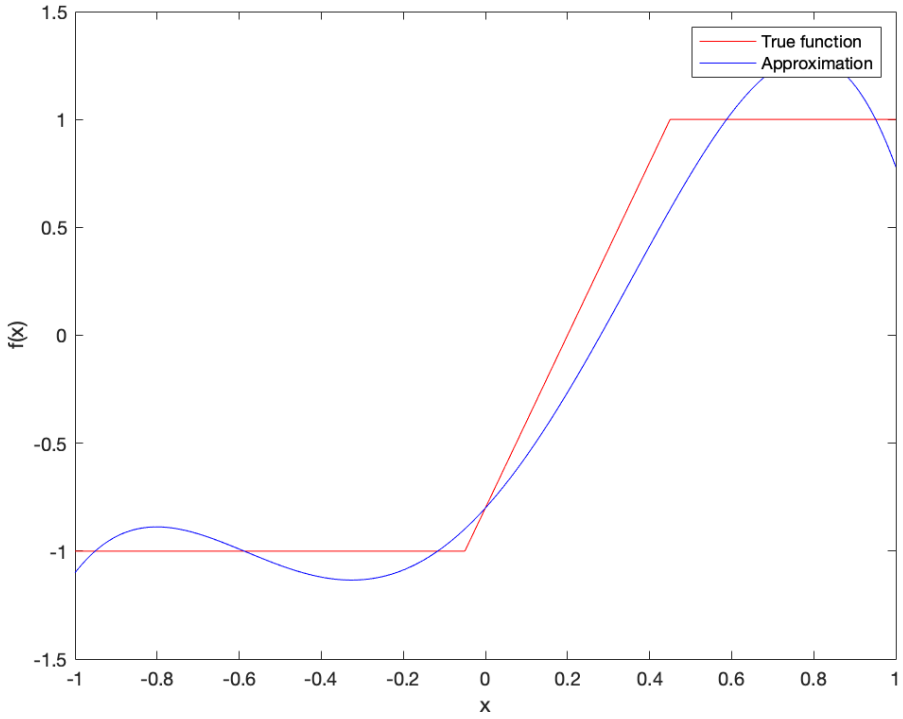

Figure 3: Approximation using Chebyshev interpolation of degree 5

We have the same observations as Problem 1. The approximation has the same general shape as the true function but there is discrepancy at most x values. We should expect the approximation function to 'converge' to the true function as we increase the degree of interpolation. This is evident in the following figure:

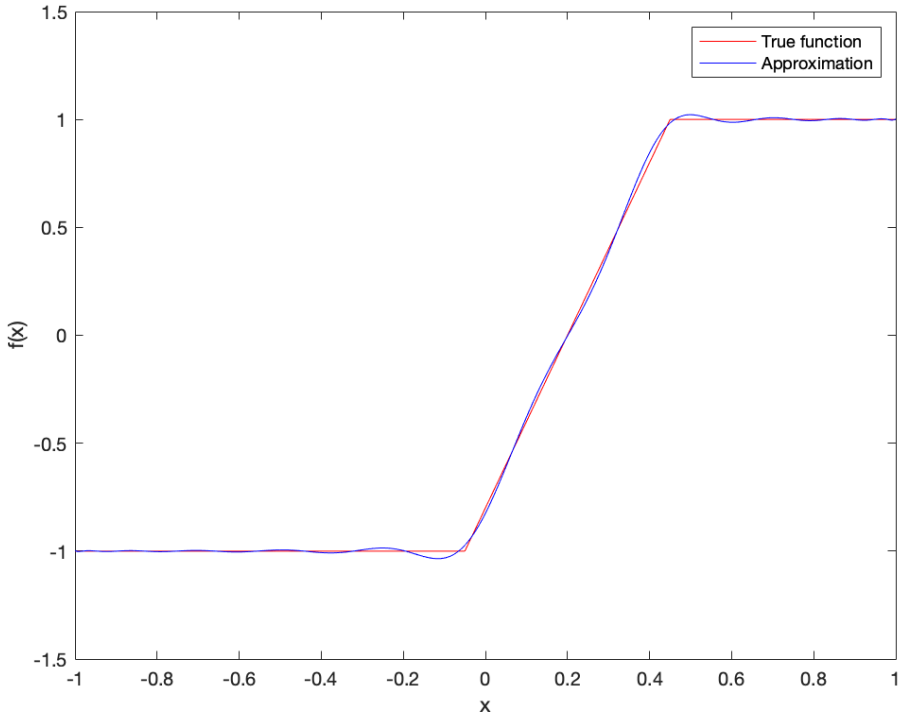

Figure 4: Approximation using Chebyshev interpolation of degree 24

The approximated function overlaps the true function for most values of x. The approximated function is not exactly equal to the true function as is evident by looking at values around $x = -0.1$. However, this does much better than the one with fewer degrees of interpolation and again, we should expect the approximated function to overlap the true function more and more as we increase the degree of interpolation.
