ECON 525: Homework 4

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1 Problem 1:

Consider the same Rust-type model as in the previous homework assignment. We estimate the structural model parameters using two-step methods and the data you generated. Recall the integrated value function formulation and our matrix notation,

$$\mathbf{V} = \Pi^{\mathcal{P}} + \beta G^{\mathcal{P},p}\mathbf{V}$$

which implies

$$\mathbf{V} = \left(I - \beta G^{\mathcal{P},p}\right)^{-1} \Pi^{\mathcal{P}}. \qquad (1)$$

(a) Compute estimates of $\mathcal{P}$ and p (denoted $\widehat{\mathcal{P}}$ and $\widehat{p}$) using a frequency estimator.

Solution:

The following is the estimate of $\mathcal{P}$ denoted by $\widehat{\mathcal{P}}$:

x_t	$\Pr(i_t = 1 \vert x_t, \theta)$
0	0.0182
1	0.0550
2	0.1267
3	0.2351
4	0.3607
5	0.4820
6	0.5921
7	0.6903
8	0.7782
9	0.8223
10	0.9268

We only observe $x_t = 10$ once in the data and $i_t = 1$ for $x_t = 10$. This means that the $\Pr(i_t = 1|x_t = 10)$ =1. However, to deal with issues about taking logs we change it to 0.99. We also provide the estimate of p, denoted by $\widehat{p}$. We provide two matrices $\widehat{p}_0$ and $\widehat{p}_1$ where $\widehat{p}_i$ consists of $\Pr(x'|x, i, \theta)$. So, $\widehat{p}_{i(jk)}$ =$\Pr(x_{t+1} = x_k|x_t = x_j, i_t = i, \theta)$.

$$
\widehat{p}_0 = \begin{bmatrix}
0.1951 & 0.8049 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.1978 & 0.8022 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.1999 & 0.8001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.2005 & 0.7995 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.2028 & 0.7972 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.1989 & 0.8011 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.2048 & 0.7952 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2139 & 0.7861 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2035 & 0.7965 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2549 & 0.7451 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
\widehat{p}_1 = \begin{bmatrix}
0.2034 & 0.7966 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.1950 & 0.8050 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.1970 & 0.8030 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.2007 & 0.7993 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.1976 & 0.8024 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.1988 & 0.8012 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.2058 & 0.7942 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.2055 & 0.7945 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.1856 & 0.8144 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 \\
0.1653 & 0.8347 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.2895 & 0.7105 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

(b) Using these, we can compute

$$
\widehat{\mathbf{V}} = \left(I - \beta G^{\widehat{\mathcal{P}}, \widehat{p}}\right)^{-1} \Pi^{\widehat{\mathcal{P}}}
$$

and

$$
\Pr(i_{nt} = 1 | x_{nt}, \theta) = \Pr\left(\widehat{V^1}(x_{nt}, \theta) + \epsilon_{1nt} \geq \widehat{V^0}(x_{nt}, \theta) + \epsilon_{0nt}\right) \qquad (2)
$$

Then, use (2) to form the likelihood for the choices in the data. Find a set of parameters that maximizes the likelihood function.

Solution:

We search over $\theta_1 \in [0, 0.1, 0.2, \ldots, 2]$, $\theta_2 \in [0, 0.01, 0.02, \ldots, 0.2]$ and $\theta_3 \in [0, 0.5, 1, 1.5, \ldots, 10]$. We find that the following set of parameters maximizes the likelihood:

$$
\boxed{\widehat{\theta} = [0.3, 0, 4]}.
$$

(c) Use forward simulation following pages 21-22 of Lecture 6 slides. Specifically, compute $\tilde{\tilde{V}}^1(x_t, \theta_1, R)$ and $\tilde{\tilde{V}}^0(x_t, \theta_1, R)$ by simulation. Then, you can use (2) to form the likelihood for the choices in the data. Find a set of parameters that maximizes the likelihood function.

Solution:

We again search over $\theta_1 \in [0, 0.1, 0.2, \ldots, 2]$, $\theta_2 \in [0, 0.01, 0.02, \ldots, 0.2]$ and $\theta_3 \in [0, 0.4, 0.8, 1.2, \ldots, 8]$. We find that the following set of parameters maximizes the likelihood:

$$\boxed{\widehat{\theta}_{fs} = [0.3, 0, 4]}.$$

(d) Compare your estimates from (b) and (c) with the full-solution estimates.

Solution:

The estimate from (b) matches the full solution estimate exactly. We get the same set of parameters in (b) that we got from the full solution estimates: $\widehat{\theta}_1 = 0.3, \widehat{\theta}_2 = 0, \widehat{\theta}_3 = 4$. Similarly, the estimate from (c) also matches the full solution estimates exactly with $\widehat{\theta}_{1,fs} = 0.3, \widehat{\theta}_{2,fs} = 0$ and $\widehat{\theta}_{3,fs} = 4$. However, the value function solution from forward simulation is slightly different from the full solution estimates. This is probably coming from simulation error and we do not get the property that V_1 is independent of x_t. The value functions from forward simulation are still close to the full solution value functions, so the difference is not troublesome. The value functions from forward simulation evaluated at the optimal parameters are given below:

x_t	$V_0(x_t)$	$V_1(x_t)$
0	-5.8144	-9.8078
1	-7.2124	-9.8144
2	-7.7686	-9.8129
3	-8.5460	-9.8100
4	-9.0096	-9.8124
5	-9.8109	-9.8115
6	-10.5793	-9.8059
7	-10.6477	-9.8061
8	-10.7383	-9.8219
9	-11.1925	-9.8380
10	-11.5559	-9.7396
