ECON 532: Homework 2

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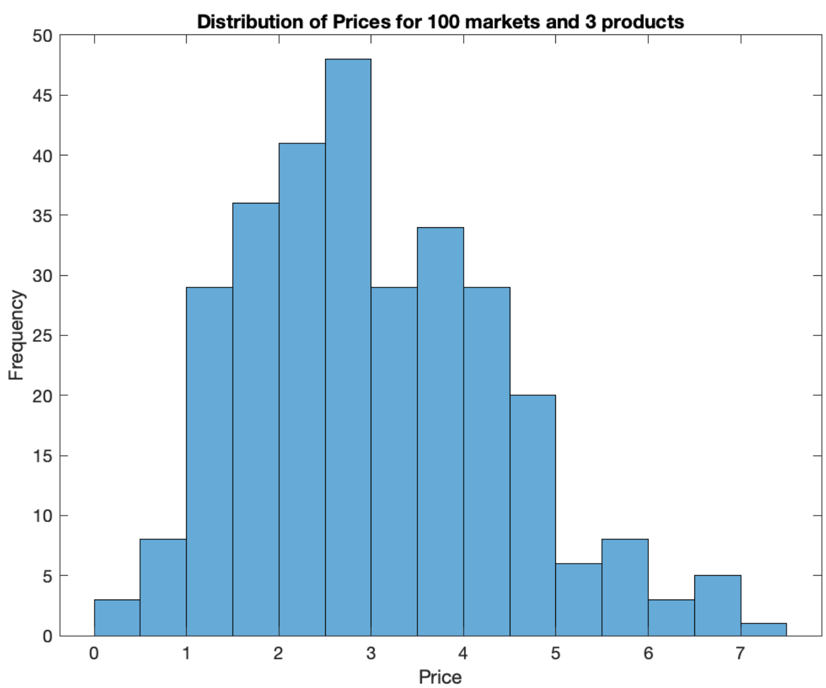
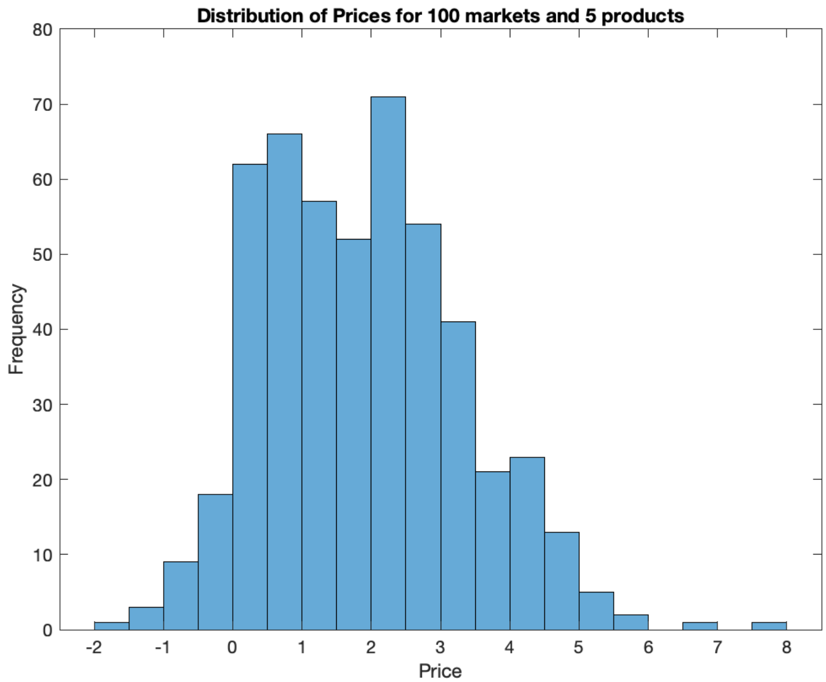
1 Problem 1:

In this problem, we are asked to compare the distribution of prices, profits and consumer surplus for the two different simulations.

1.1 Distribution of prices:

For the price distribution, we just reshape the P_{opt} variable to a vector and plot the histogram.





The prices in the 3 product market seem to be higher than the prices in the 5 product market. The median price in the 3 product market is 2.8279 and the mean price in the 3 product market is 3.0265. The median price in the 5 product market is 1.8159 and the mean price in the 5 product is 1.8911. Therefore, we can see that prices in the 3 product market are higher and this makes sense as more products brings greater competition in the 5 product market bringing down price.

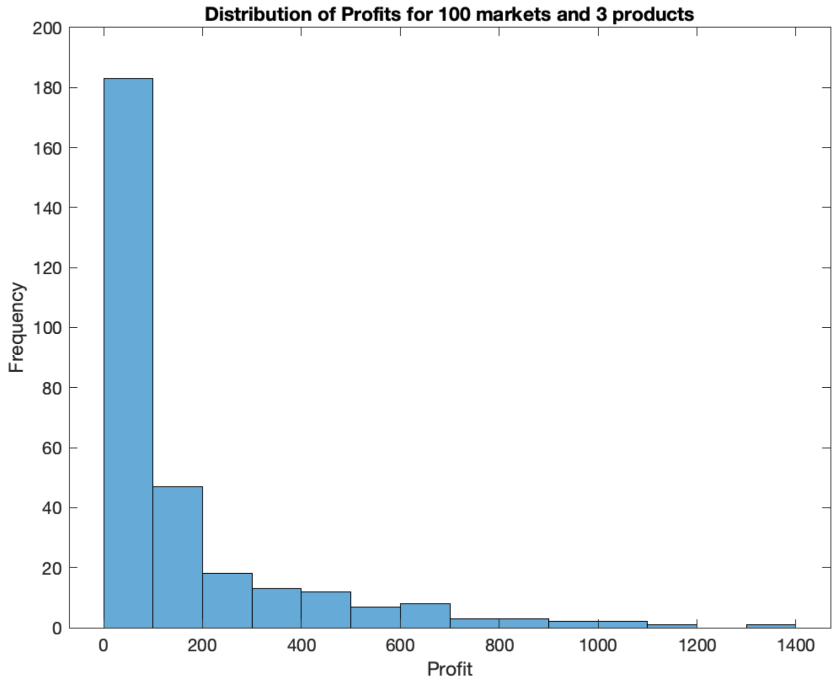
1.2 Distribution of profits:

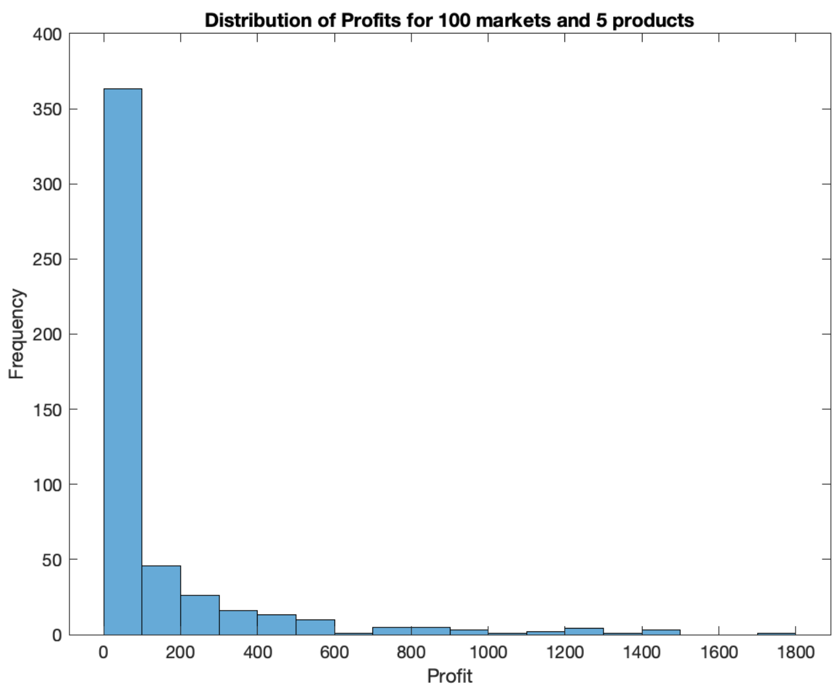
For the profit distribution, we first calculate the profit of the j^{th} firm in the m^{th} market: π_{jm}. To this end, we first calculate the marginal cost of j^{th} firm in the m^{th} market. This is given by:

$$MC_{jm} = \gamma_0 + \gamma_1 W_j + \gamma_2 Z_{jm} + \omega_{jm}$$

We can then calculate the markup using $P_{jm} - MC_{jm}$. Finally, the profit is given by:

$$\pi_{jm} = \text{market share} \times \text{number of consumers} \times \text{markup}$$





The median profit in the 3 product market is 52.6504 and the mean profit in the 3 product market is 154.3349. The median profit in the 5 product market is 20.3690 and the mean profit in the 5 product market is 127.1025. We can observe that the profits in the 3 product market are higher and again this makes sense due to the increase in competition from the increase in products.

1.3 Distribution of consumer surplus:

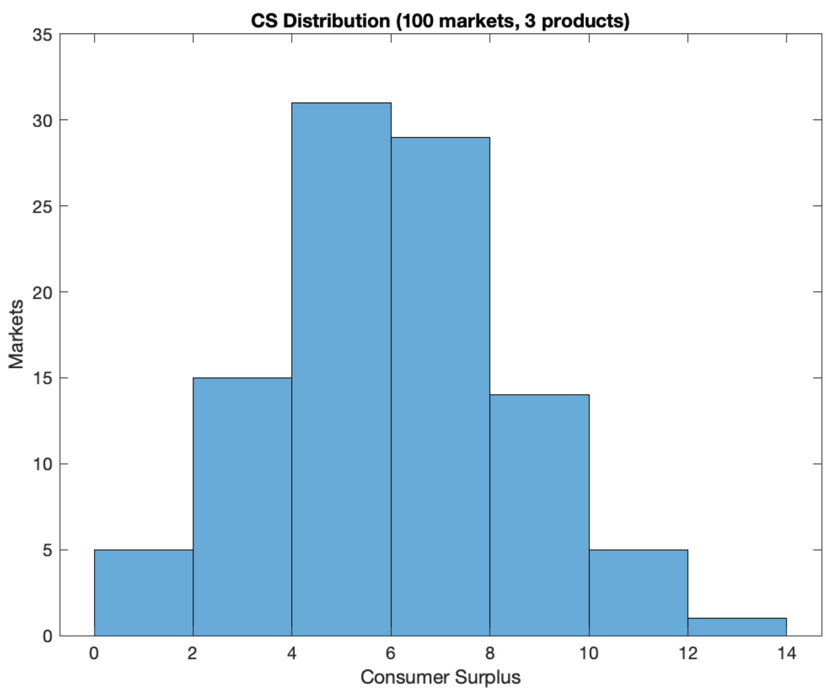
We have to construct the distribution of consumers. We will first construct the utility of the i^{th} consumer in consuming the j^{th} product in the m^{th} market: $U_{ijm} = X_{jm}\beta - p_{jm}\alpha_i + \epsilon_{ijm} + \xi_{jm}$. Since ϵ_{ijm} follows the standard Gumbel distribution we utilize the following result: if $\epsilon_i \sim_{iid}$ Gumbel(0,1) then

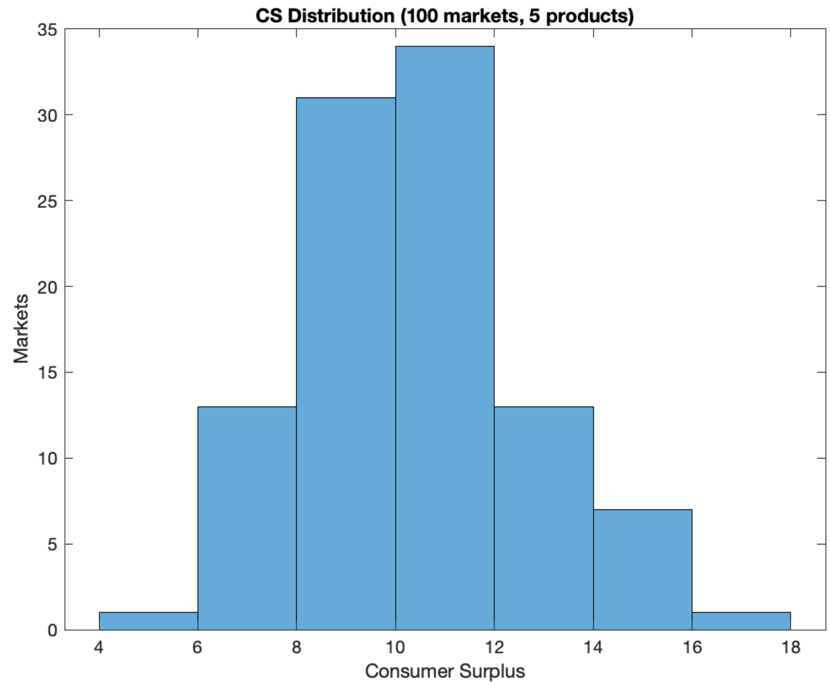
$$\mathbb{E}\left[\max_i(\delta_i + \epsilon_i)\right] = \gamma + \ln\left(\sum_i \exp(\delta_i)\right)$$

where $\gamma = 0.5$. Then, the consumer surplus for the i^{th} consumer in the m^{th} market is given by the following:

$$CS_{im} = \frac{1}{\alpha_i}\max_j U_{ijm}$$

where we use the result above to calculate the maximum of U_{ijm}. We then take the average over all the markets for the i^{th} consumer to plot the histogram.





In the 3 product case, we can actually see that some of the consumers actually avail the outside option giving them zero utility. This does not happen in the 5 product case. We can also see that the consumer surplus is significantly higher in the 5 product case as the consumers have more options and the firms are able to charge too high a price.

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2 Problem 2:

Problem 2.1 (a):

$$\mathbb{E}\left[\xi_{jm} X_{jm}\right] = \begin{bmatrix} 0.0435 \\ 0.0207 \\ 0.0355 \end{bmatrix}$$
$$\mathbb{E}\left[\xi_{jm} p_{jm}\right] = 0.2950$$
$$\mathbb{E}\left[\xi_{jm} \bar{p}_{jm}\right] = 0.1437$$

Problem 2.1 (b):

We can notice that $\mathbb{E}\left[\xi_{jm} X_{jm}\right] \approx 0$, so it is likely that this set of moment conditions is valid. This will be valid, in particular, if all observed product characteristics other than prices are exogenous. This will be relevant if the observed product characteristics are correlated with price. The second moment condition is small but not as close to zero as the first one. For this moment condition to be valid, we require that the firms do not know about consumer's values for the unobserved characteristics because otherwise they would use it to price their product and then we would have a correlation with price.

We can see that the third moment condition is not as close to zero as the first moment condition. The third moment condition can be thought of as a Hausman-type IV, and will be valid ξ_{jm} is independent across markets for each product j. If there are common cost shifters for the same product across markets, the moment condition will be relevant.

Problem 2.1 (c) I think we can use both BLP (demand side) and Nevo instruments under the correct assumptions. We already explained above certain assumptions required for the moment conditions to be valid. We need that the observed characteristics (other than price) to be exogenous. We also need ξ_{jm} to be independent across markets for each product j. We can notice that the observed characteristics are indeed generated independently of the unobserved characteristics. This satisfies our first assumption and so the BLP instruments should be valid. The unobserved characteristics are also generated identically and independently across markets for each j and the cost shifter is also generated independently of the unobserved characteristics. Therefore, Nevo instruments could also be used.

Problem 2.2 (a):

The instruments are given by

$$Z_j = \begin{bmatrix} X_j \\ \sum_{r \neq j, r \in \mathcal{F}_j} X_r \\ \sum_{r \notin \mathcal{F}_j} X_r \end{bmatrix}$$

The BLP moments are given by:

$$\begin{bmatrix} \mathbb{E}\left[\xi_{jm}x_{1jm}\right] = \mathbb{E}\left[\xi_{jm}\right] = 0 \\ \mathbb{E}\left[\xi_{jm}x_{2jm}\right] = 0 \\ \mathbb{E}\left[\xi_{jm}x_{3jm}\right] = 0 \\ \mathbb{E}\left[\xi_{jm}\left(\sum_{i\neq j}^{J} x_{2im}\right)\right] = 0 \\ \mathbb{E}\left[\xi_{jm}\left(\sum_{i\neq j}^{J} x_{3im}\right)\right] = 0 \end{bmatrix}$$

where $\xi_{jm} = \delta_{jm} - X_{jm}\beta + \alpha p_{jm}$, where δ_{jm} is the mean utility for the j^{th} product in m^{th} market. Since one of the characteristics is constant at we cannot include this when we consider other products' characteristics within a product's market and for product characteristics in all other markets due to collinearity issues.

Problem 2.2 (b):

Let us denote Z to be the instruments. Let X include the observed characteristics and the price. We can write the moment conditions as the following:

$$\mathbb{E}\left[\xi_{jm}Z_{jm}\right] = 0$$

Recall: $\xi_{jm} = \delta_{jm} - X_{jm}\beta + \alpha p_{jm}$ and so we can rewrite the above as a function of δ. Let $\theta' = [\beta_1, \beta_2, \beta_3, \alpha]'$ and $\theta = [\theta', \sigma_\alpha]$. The objective function will be given by:

$$M(\theta) = (\delta(\theta) - X\theta')'ZWZ'(\delta(\theta) - X\theta')$$

where $\delta(\theta)$ is 300×1, X is 300×4, Z is 300×5 and W is a 5×5 positive definite weighting matrix. For our purposes, we use $W = (Z'Z)^{-1}$. Alternatively, we can write the objective function as the following:

$$M(\theta) = \xi(\theta)'ZWZ'\xi(\theta).$$

Finally,

$$\widehat{\theta} = \arg\min_{\theta} M(\theta).$$

In our algorithm, we will in fact minimize over σ_α. Once we have found σ_α we can find the other parameters. Therefore, in fact we have:

$$\widehat{\sigma}_\alpha = \arg\min_{\sigma_\alpha}(\delta(\sigma_\alpha) - X\theta'(\sigma_\alpha))'ZWZ'(\delta(\sigma_\alpha) - X\theta'(\sigma_\alpha))$$

We can calculate θ' from the following:

$$(X'ZWZ'X)^{-1}(X'ZWZ'\delta(\widehat{\sigma}_\alpha))$$

Problem 2.2 (c):

We estimate the parameters using the following algorithm.

1. We simulate consumers by drawing $v_{ip} \sim LN(0, 1)$. We keep the this draw fixed.

2. We fix a value of σ_α.

3. Using the simulated consumers (v_{ip} and σ_α), we can get the aggregate market shares:

$$s_j \approx \frac{1}{n}\sum_{i=1}^{n}\frac{e^{\delta_j+\mu_{ij}}}{1+\sum_s e^{\delta_s+\mu_{is}}}$$

   where $\delta_{jm} = X_{jm}\widehat{\beta} - \widehat{\alpha}p_{jm} + \xi_{jm}$ is the mean utility and $\mu_{ij} = -\widehat{\sigma}_\alpha v_i p_{jm}$.

4. We solve for δ as a fixed point by having an initial guess of δ and the can calculating the simulated shares using the formula. We update δ and therefore the shares using the contraction mapping until we find the fixed point for δ. Essentially, we have to find a fixed point such that $s_j = s$, that is, model market shares is equal to the observed market shares.

5. Using the fact that $\delta_{jm} = X_{jm}\beta - \alpha p_{jm} + \xi_{jm}$, we run an IV regression on the estimated δ to estimate ξ_{jm}, α_{jm} and β_{jm}.

6. Finally, we calculate the GMM objective function using the instruments and the estimated ξ_{jm}.

7. Return the value of σ_α that minimizes the GMM objective functino and therefore the values of α and β

I used *MATLAB*'s *fminsearch* function for minimizing the GMM objective function for estimation. The estimates using demand-side moments only is provided below:

Parameter	True value	Estimate	Bias	Standard error
β_1	5	5.4935	0.4935	0.1432
β_2	1	1.1286	0.1286	1.12417
β_3	1	1.0575	0.0575	0.2480
α	-1	-1.0401	-0.0401	0.0897
σ_α	1	1.1155	0.1155	0.2885

Stability: (different initial guesses)

The following table contains the parameter estimates at different initial guesses.

Initial guess	0	0.5	1	1.5	2	2.5	3	4	5	SE
β_1	5.9680	5.9681	5.9681	5.9681	5.9681	5.9677	5.9681	5.9681	5.9677	1.422e-4
β_2	1.1980	1.1980	1.1980	1.1980	1.1980	1.1979	1.1980	1.1980	1.1979	2.851e-5
β_3	1.094	1.094	1.094	1.094	1.094	1.094	1.094	1.094	1.094	1.029e-5
α	-1.2609	-1.2609	-1.2609	-1.2609	-1.2609	-1.2608	-1.2609	-1.2609	-1.2608	3.346e-5
σ_α	1.01611	1.01611	1.01611	1.01611	1.01611	1.01611	1.01611	1.01611	1.01611	1.586e-5

We also report the standard errors and the parameter with the smallest standard error of the different run should be the most stable. We can see that the estimates do not change much depending on the initial guesses. We have to remember that there is a source of randomness here, namely, the ν_{im}. We get this particular result holding a set of ν_{im} values fixed. It is possible that we do not get this level of stability if we let ν_{im} to vary or even if we hold a different set of ν_{im} fixed. The standard errors are extremely small as a result because the parameter values do not change. From the standard errors, we find that β_3 seems to be most stable over the different runs.

Problem 2.2 (d):

Elasticity:

We will first describe how we calculate the elasticities. The price elasticitiy (own and cross) is given by:

$$\epsilon_{jk} = \frac{\frac{\partial s_j}{s_j}}{\frac{\partial p_k}{p_k}} = \frac{\partial s_j}{\partial p_k} \times \frac{p_k}{s_j}$$

We have given the formula on how to calculate shares above. Using that formula, we can show the following:

$$\frac{\partial s_j}{\partial p_k} = \begin{cases} \frac{1}{500}\sum_{i=1}^{500}(\alpha + \sigma_\alpha \nu_i)(s_{ij}^2 - s_{ij}) & \text{if } j = k \\ \frac{1}{500}\sum_{i=1}^{500}(\alpha + \sigma_\alpha \nu_i)(s_{ij}s_{ik}) & \text{otherwise} \end{cases}$$

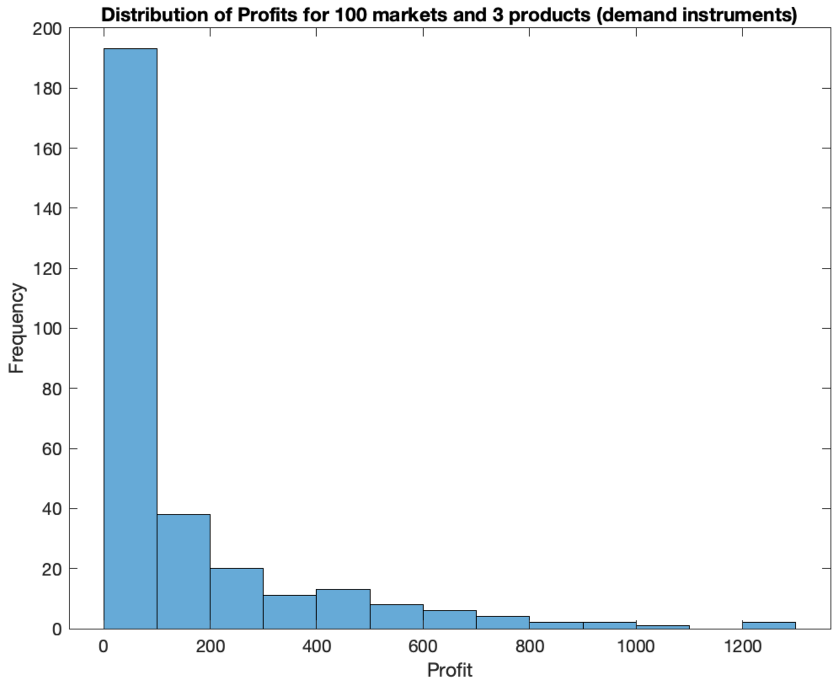
We will report two statistics here. We will first report own-price inelasticities. In particular, we will report the fraction of firms that have inelastic demand. We find that 8% of firms have products with inelastic demand whereas the true fraction is 8.33%. We can see that our estimation is pretty close.

Secondly, we report the median price elasticities. The true median price elasticities are included in parenthesis.

	Product 1	Product 2	Product 3
Product 1	-5.5993 (-5.008)	1.4282 (1.237)	1.0176 (0.896)
Product 2	0.3773 (0.327)	-2.4497 (-2.280)	0.8753 (0.768)
Product 3	0.3621 (0.324)	1.4626 (1.253)	-3.8665 (-3.488)

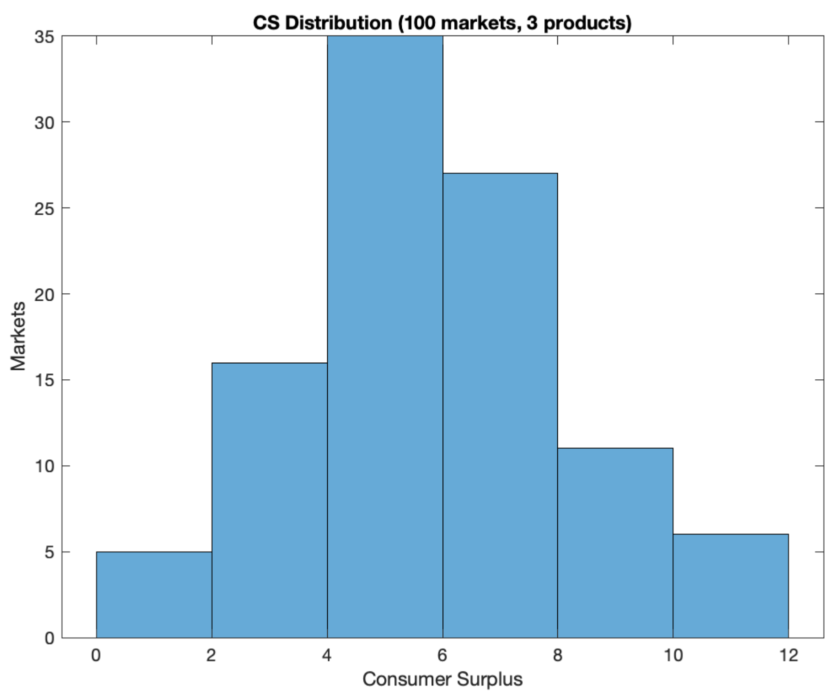
We can see that our estimated median elasticities is pretty close to the true median elasticities.

Profits: We used oligopoly assumption to compute the marginal costs.



The median profit is 47.2317 and the mean profit is 142.7213. The true median profit in the 3 product market is 52.6504 and the true mean profit is 154.3349. Therefore, we seem to be have underestimated the profits.

Consumer Surplus:



The distribution of estimated consumers surplus looks extremely similar to the true distribution. The median consumer surplus is 5.6877 and the mean consumer surplus is 5.8378. The true median consumer surplus is 5.9667 and the mean value is 5.9301. This confirms our thoughts from the inspection of the distribution that our estimated surplus is reasonable. We still see that some consumers avail the outside option.

Problem 2.2 (e): 10 Markets

Parameter	True value	Estimate	Bias	Standard error
β_1	5	1.5291	-3.4709	0.3688
β_2	1	-0.0065	-1.0065	9.8524
β_3	1	1.5984	0.5984	2.6949
α	-1	-1.1885	-0.1885	2.5722
σ_α	1	-0.1311	-1.1311	3.3703

The estimate for β_3 and α can be thought of as being reasonable. However, the estimates of the other variables are extremely far off. The bias in this case is much higher and so are the standard errors. The estimates are not reasonable at all and this makes sense because our sample size is significantly smaller. We only have 10% of the sample size compared to the 100 markets sample.

Stability: (different initial guesses)

The following table contains the parameter estimates at different initial guesses.

Initial guess	0	0.5	1	1.5	2	2.5	3	4	5	SE
β_1	3.1896	1.5291	1.5291	1.5277	1.5291	1.5277	1.5277	1.5291	1.5277	0.5537
β_2	0.4227	-0.0065	-0.0065	-0.0069	-0.0065	-0.0069	-0.0069	-0.0065	-0.0069	0.1432
β_3	1.1595	1.5984	1.5984	1.5988	1.5984	1.5988	1.5988	1.5984	1.5988	0.1464
α	-1.6550	-1.1885	-1.1885	-1.1879	-1.1885	-1.1879	-1.1879	-1.1885	-1.1879	0.1556
σ_α	-0.0106	-0.1311	-0.1311	-0.1311	-0.1311	-0.1311	-0.1311	-0.1311	-0.1311	0.0402

We can see that for our starting guess for $\sigma_\alpha = 0$ the estimates are not close compared to other guesses for σ_α. Other than this starting value, we can see that the estimates are pretty stable across different initial guesses. Looking at the standard errors, we can see that σ_α seems to be the most stable parameter. However, β_3 seems to be the most reasonable parameter across all runs as it has the smallest bias. The stability changes from the 100 market case in the sense that a particular initial guess gives a significantly different result.

Problem 2.3

Parameter	True value	Estimate	Bias	Standard error
β_1	5	5.7682	0.7682	0.0695
β_2	1	1.1271	0.1271	0.5993
β_3	1	1.0928	0.0928	0.1222
α	-1	-1.1641	-0.1641	0.0433
σ_α	1	1.0794	0.0794	0.1394

The estimates seem to be slightly more biased in this incorrect specification case. The bias is in the same direction as the correct specification case. I believe that the performance will depend on the seed and given some values of ν we will see better performance in the incorrect specification case. The standard errors are, however, smaller in the incorrect specification case.

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3 Problem 3:

Problem 3.1 (a)

The new moments are as follows:

$$\begin{bmatrix} \mathbb{E}\left[\xi_{jm} x_{1jm}\right] = \mathbb{E}\left[\xi_{jm}\right] = 0 \\ \mathbb{E}\left[\xi_{jm} x_{2jm}\right] = 0 \\ \mathbb{E}\left[\xi_{jm} x_{3jm}\right] = 0 \\ \mathbb{E}\left[\xi_{jm}\left(\sum_{i\neq j}^{J} x_{2im}\right)\right] = 0 \\ \mathbb{E}\left[\xi_{jm}\left(\sum_{i\neq j}^{J} x_{3im}\right)\right] = 0 \\ \mathbb{E}\left[\xi_{jm} W_j\right] = 0 \end{bmatrix}$$

Problem 3.1 (b):

Parameter	True value	Estimate	Bias	Standard error
β_1	5	5.0250	0.0250	0.0078
β_2	1	0.9795	-0.0205	0.0297
β_3	1	1.0392	0.0392	0.0137
α	-1	-0.9927	0.0073	0.0047
σ_α	1	0.9927	-0.0073	0.0054

Adding the supply side instruments improves our estimates considerably. The biases are much smaller and so are the standard errors.

Elasticity:

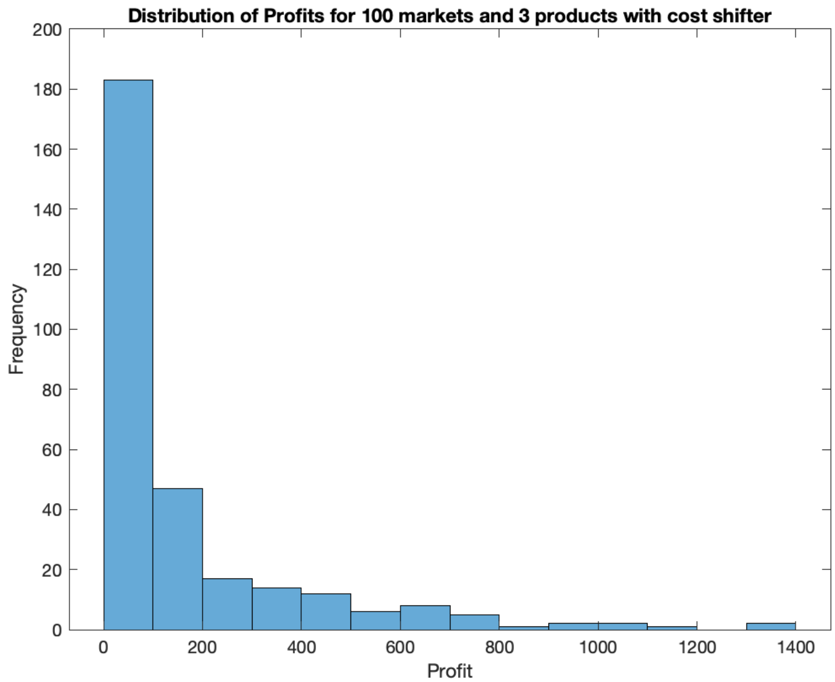
We will report two statistics here. We will first report own-price inelasticities. In particular, we will report the fraction of firms that have inelastic demand. We find that 8.3333% of firms have products with inelastic demand whereas the true fraction is 8.33%. We can see that our estimation is almost exact.

Secondly, we report the median price elasticities. The true median price elasticities are included in parenthesis.

	Product 1	Product 2	Product 3
Product 1	-4.950 (-5.008)	1.245 (1.237)	0.873 (0.896)
Product 2	0.325 (0.327)	-2.221 (-2.280)	0.764 (0.768)
Product 3	0.318 (0.324)	1.251 (1.253)	-3.464 (-3.488)

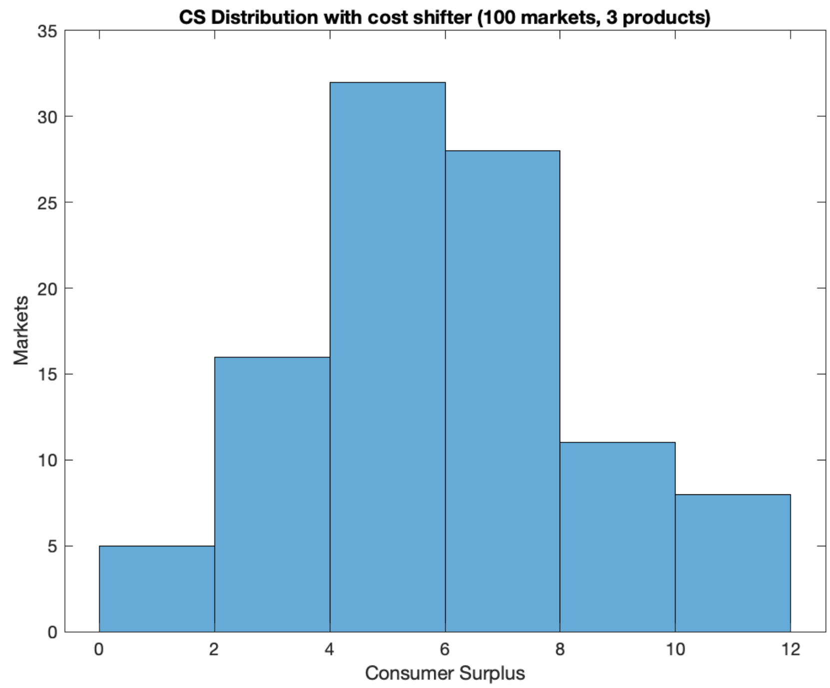
The estimated median elasticities are extremely close to the true median elasticities. In fact, the estimated median elasticities computed with the cost shifter is much closer to the true values compared to the median elasticities computed without the cost shifters.

Profits:



The median estimated profit is 53.1771 and the mean estimated profit is 156.3179. The true median profit in the 3 product market is 52.6504 and the true mean profit is 154.3349. Therefore, we seem to be have overestimated the profits. However, we are pretty close to the true median and mean values and much closer than the estimated profits computed without the cost shifter.

Consumer Surplus:



The median consumer surplus is 5.8938 and the mean consumer surplus is 6.0412. The true median consumer surplus is 5.9667 and the mean value is 5.9301. Again, we seem to be doing much better when we include the cost shifter.

Problem 3.1 (c): 10 Markets

Parameter	True value	Estimate	Bias	Standard error
β_1	5	4.2040	-0.7960	0.7392
β_2	1	0.6011	-0.3989	1.3866
β_3	1	1.6494	0.6494	0.3357
α	-1	0.6795	1.6795	0.1372
σ_α	1	1.8207	0.8207	0.6307

The bias and the standard errors are higher than the 100 markets case. This makes sense as the sample size is much smaller in the 10 market case. However, we can see that the cost shifter as an instrument makes a huge difference in the quality of the estimates. If we consider the 10 market cases with with and without the supply side instrument, we can observe that we do exceedingly better with the supply side instrument. Without the supply side instrument, the estimates were extremely off but we can see that with the supply side instrument the estimates are much closer to true value.

Stability: (different initial guesses)

The following table contains the parameter estimates at different initial guesses.

Initial guess	0	0.5	1	1.5	2	2.5	3	4	5	SE
β_1	4.2041	4.2040	4.2040	4.2040	4.2040	4.2040	4.2040	4.2040	4.2040	3.097e-5
β_2	0.6011	0.6011	0.6011	0.6011	0.6011	0.6011	0.6011	0.6011	0.6011	6.063e-6
β_3	1.6494	1.6494	1.6494	1.6494	1.6494	1.6494	1.6494	1.6494	1.6494	2.792e-6
α	0.6796	0.6795	0.6795	0.6796	0.6795	0.6796	0.6796	0.6795	0.6796	1.578e-5
σ_α	1.8208	1.8207	1.8207	1.8207	1.8207	1.8207	1.8207	1.8207	1.8207	1.904e-5

The estimates seem to be extremely stable. We see that the estimates from a starting value of 0 is different at the fourth decimal place. Again, this stability result may be due to the specific draw of ν. We also see that β_3 is the most stable parameter. We also see that the stability is better than in the case without the supply side instrument.

Problem 3.1 (d):

Adding the supply side instrument improves the estimates in both M=10 and the M=100 cases. The bias is lower in both the cases and so are the standard errors. We see that the stability also improves after adding the supply side instrument. The improvement of results can also be seen in the case of profits, consumer surplus and elasticities. The estimated median profits, consumer surplus distribution and profit distribution are also much closer to the true values. The estimates using BLP instruments only is reasonable but we can do significantly better by adding the supply side instrument because we can get closer to the true values.

Problem 3.2 (a):

We will write down marginal costs under different pricing assumptions.

(1) Marginal costs under perfect competition:

$$\text{MC} = p$$

where p is the price.

(2) Marginal costs under perfect collusion:

$$\text{MC} = p - \Delta^{-1} s(p)$$

where p is the price and the $(j,r)^{th}$ element of Δ is given by:

$$\Delta_{jr} = -\frac{\partial s_r}{\partial p_j}$$

(3) Marginal costs under oligopoly:

$$\text{MC} = p - \Delta^{-1} s(p)$$

where p is the price and the $(j,r)^{th}$ element of Δ is given by:

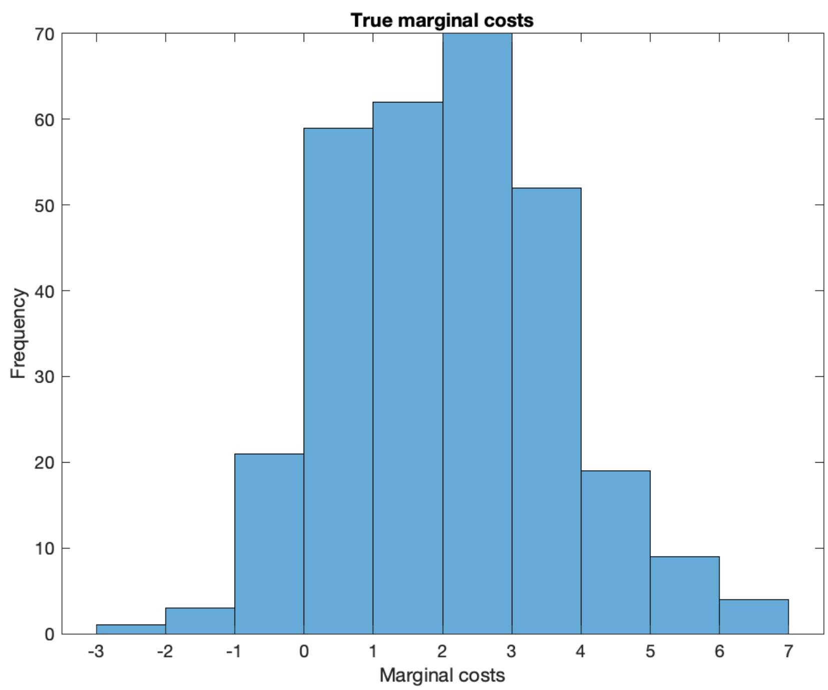
$$\Delta_{jr} = \begin{cases} -\frac{\partial s_r}{\partial p_j} & \text{if } r = j \\ 0 & \text{otherwise} \end{cases}$$

Therefore, we are only concerned about own price elasticities under this pricing rule.

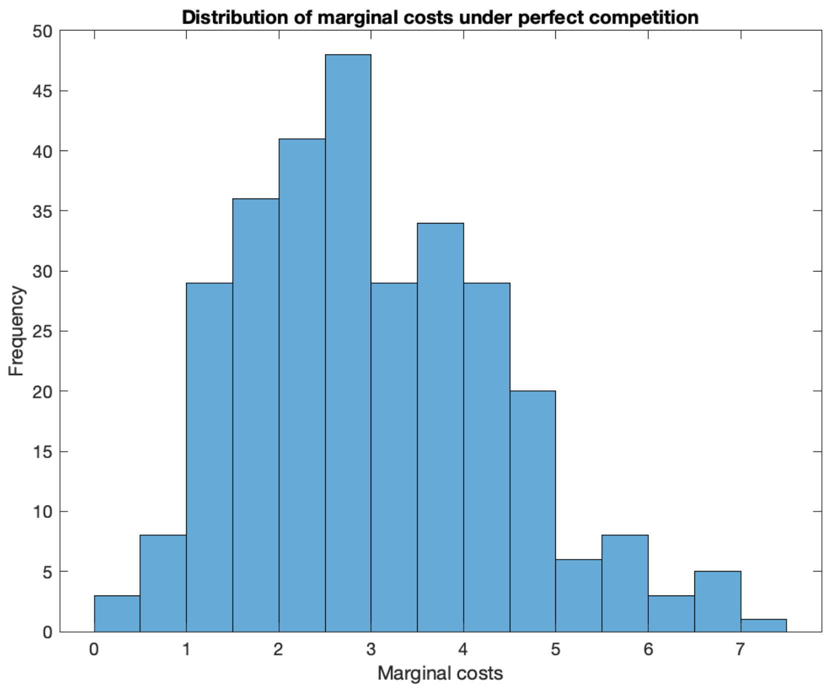
Problem 3.2 (b):

We will use the estimates we obtained from the $M = 100$ case with the supply side and the BLP instruments. The reason I pick these estimates is because the estimates had the smallest bias and the smallest standard errors. Therefore, these are the most reasonable set of estimates we have.

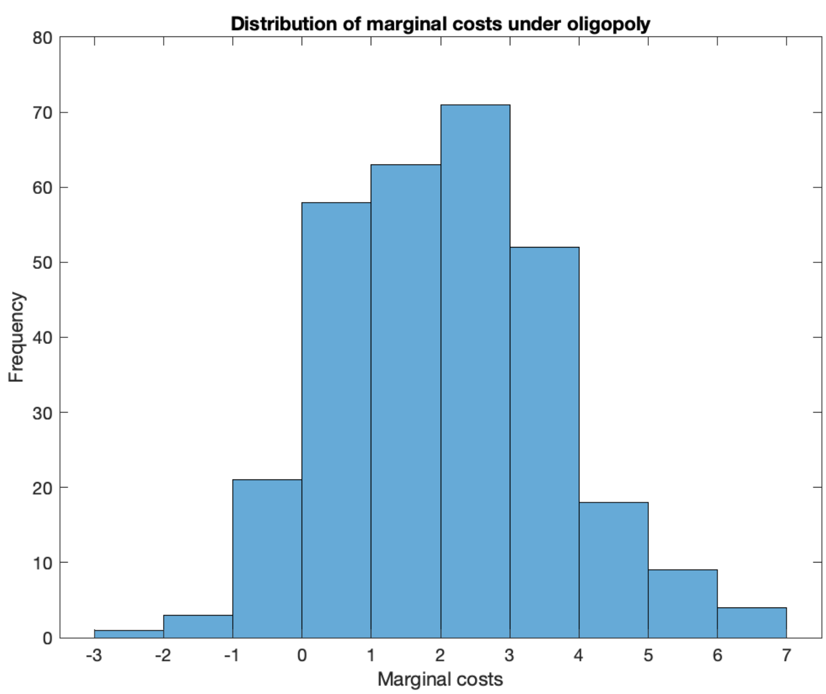
True marginal costs:



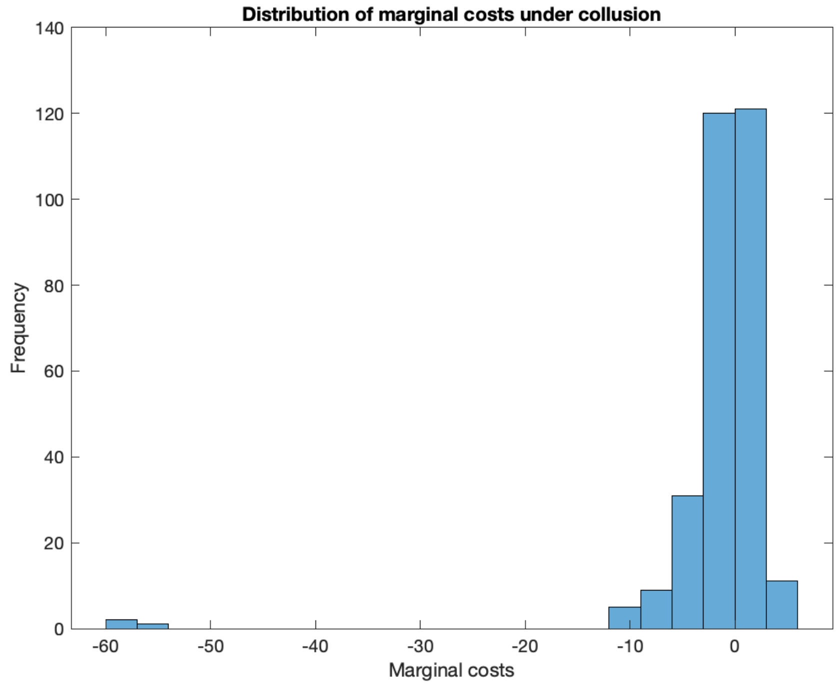
Marginal costs under perfect competition:



Marginal costs under oligopoly:



Marginal costs under collusion:



The following table contains the median and mean values of the marginal costs under the different pricing assumptions:

Marginal Costs	Median	Mean
True	2.0474	2.0512
Perfect competition	2.8279	3.0265
Oligopoly	2.0381	2.0419
Collusion	-0.4013	-1.1780

Using both the table and the distributions above, we can see that oligopoly assumption matches our true data the best. The distribution of marginal costs under oligopoly assumption almost exactly matches the true distribution and we can see that median and mean values under this assumption is also close to the true values. Collusion pricing is extremely far off and perfect competition pricing does not perform nearly as well as the oligopoly pricing.

Problem 3.2 (c):

Perfect competition:

Parameter	True value	Estimate	Bias	Standard error
β_1	5	4.8788	-0.1212	0.0074
β_2	1	0.9793	-0.0207	0.0284
β_3	1	1.0307	0.0307	0.0134
α	-1	-1.0055	-0.0055	0.0045
σ_α	1	0.9380	-0.0620	0.005
γ_1	2	2.9422	0.9422	-
γ_2	1	0.7677	-0.2323	-
γ_3	1	0.5583	-0.4417	-

The demand estimates have the smallest standard errors so far. However, the $M = 100$ case with supply side instrument has lower bias than the estimates under perfect competition assumption. This makes sense because perfect competition is not the correct assumption in our case.

Elasticity:

We will report two statistics here. We will first report own-price inelasticities. In particular, we will report the fraction of firms that have inelastic demand. We find that 8% of firms have products with inelastic demand whereas the true fraction is 8.33%. We can see that our estimation is extremely close but we did better with $M = 100$ with supply side instrument.

Secondly, we report the median price elasticities. The true median price elasticities are included in parenthesis.

	Product 1	Product 2	Product 3
Product 1	-5.0371 (-5.008)	1.2367 (1.237)	0.8457 (0.896)
Product 2	0.3174 (0.327)	-2.2165 (-2.280)	0.7388 (0.768)
Product 3	0.3179 (0.324)	1.2148 (1.253)	-3.4817 (-3.488)

We see that the median elasticities are really close to the true values.

Oligopoly:

Parameter	True value	Estimate	Bias	Standard error
β_1	5	4.9057	-0.0943	0.0074
β_2	1	0.9844	-0.0156	0.0284
β_3	1	1.0346	0.0346	0.0134
α	-1	-1.0040	-0.0040	0.004
σ_α	1	0.9478	- 0.0522	0.005
γ_1	2	1.9457	-0.0543	-
γ_2	1	0.9775	-0.0225	-
γ_3	1	-0.8971	-0.1029	-

These are our best estimates so far. We have the lowest bias on some of the demand parameters but definitely the lowest bias on the supply parameters. We also have the lowest standard errors here compared to the supply side instrument case. This makes sense because oligopoly is the correct market structure assumption in our case.

Elasticity:

We will report two statistics here. We will first report own-price inelasticities. In particular, we will report the fraction of firms that have inelastic demand. We find that 8% of firms have products with inelastic demand whereas the true fraction is 8.33%. We can see that our estimation is extremely close but we did better with $M = 100$ with supply side instrument.

Secondly, we report the median price elasticities. The true median price elasticities are included in parenthesis. I am not quite sure why I get the same median elasticities as the perfect competition case. This could be due to the fact the demand estimates are close to each other and the elasticities demand on these parameters.

	Product 1	Product 2	Product 3
Product 1	-5.0371 (-5.008)	1.2367 (1.237)	0.8457 (0.896)
Product 2	0.3174 (0.327)	-2.2165 (-2.280)	0.7388 (0.768)
Product 3	0.3179 (0.324)	1.2148 (1.253)	-3.4817 (-3.488)

Collusion:

Parameter	True value	Estimate	Bias	Standard error
β_1	5	5.1704	0.1704	0.0087
β_2	1	0.9361	-0.0639	0.0324
β_3	1	1.0410	0.0410	0.0143
α	-1	-0.9575	0.0425	0.0047
σ_α	1	1.0560	0.0560	0.0055
γ_1	2	-0.8324	-2.8324	-
γ_2	1	0.7332	-0.2668	-
γ_3	1	4.8557	3.8557	-

We can see that this gives us the worst performance out of all the assumptions. This does give a better result for the demand parameters compared to the demand side instruments only case. However, the supply side parameters are heavily biased. This makes sense because the collusion assumption is not appropriate in our case.

Elasticity:

We will report two statistics here. We will first report own-price inelasticities. In particular, we will report the fraction of firms that have inelastic demand. We find that 9% of firms have products with inelastic demand whereas the true fraction is 8.33%. We can see that this is the worst estimate of this statistic that we have computed so far.

Secondly, we report the median price elasticities. The true median price elasticities are included in parenthesis.

	Product 1	Product 2	Product 3
Product 1	-4.9834 (-5.008)	1.2226 (1.237)	0.8684 (0.896)
Product 2	0.3212 (0.327)	-2.1945 (-2.280)	0.7498 (0.768)
Product 3	0.3171 (0.324)	1.2235 (1.253)	-3.3913 (-3.488)

The estimated median elasticities are reasonably close to the true values. However, we achieved closer estimates with the other two pricing assumptions.

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4 Problem 4

Problem 4.1:

The set of parameter estimates that I trust the most for the following exercises is the ones recovered from the oligopoly assumption. These gives the closest supply side parameter estimates while giving extremely close demand side parameter estimates as well. These estimates also have the lowest standard error. Moreover, oligopoly assumption is the correct assumption in our case and will the closest estimated marginal cost distribution to the true distribution.

Problem 4.2 (a):

The merged firms pricing problem is:

$$\max_{p_{1t},p_{2t}} (p_{1t} - mc_{1t})s_{1t} + (p_{2t} - mc_{2t})s_{2t}$$

where t is the index for the market. For the following analysis, we will suppress the market index. The first order condition is given by:

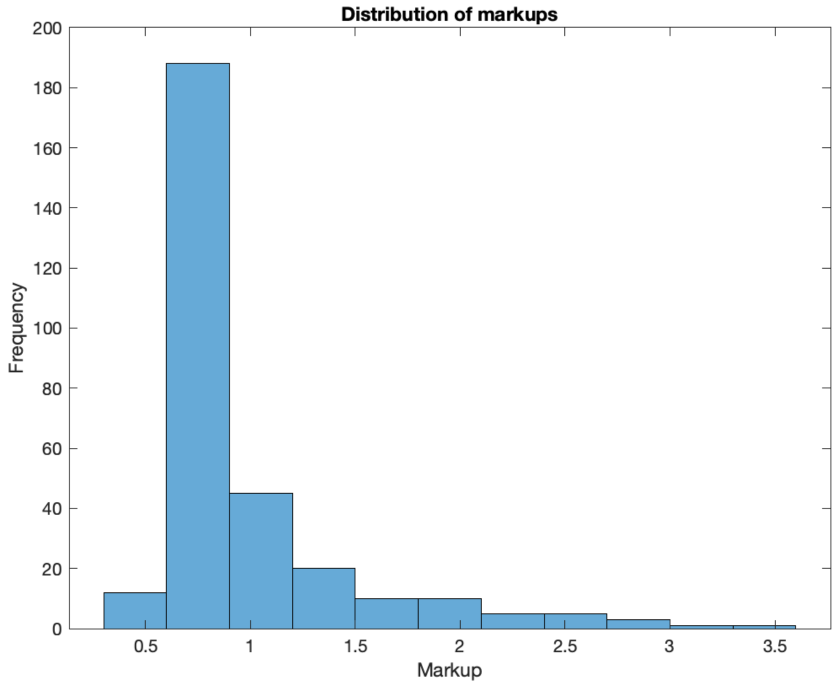
$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} mc_1 \\ mc_2 \end{bmatrix} - \Delta^{-1} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

where

$$\Delta_{ij} = \frac{\partial s_j}{\partial p_i} \text{ for } i, j = 1, 2$$

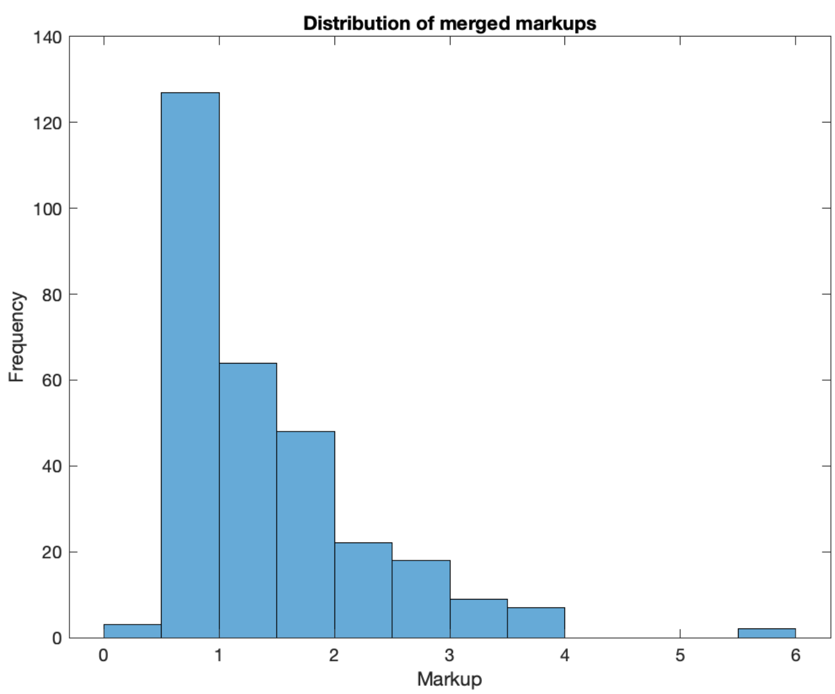
Problem 4.2 (b):

True markup:



The true median markup is 0.7865 and the true mean markup is 0.9814.

Merged markup:



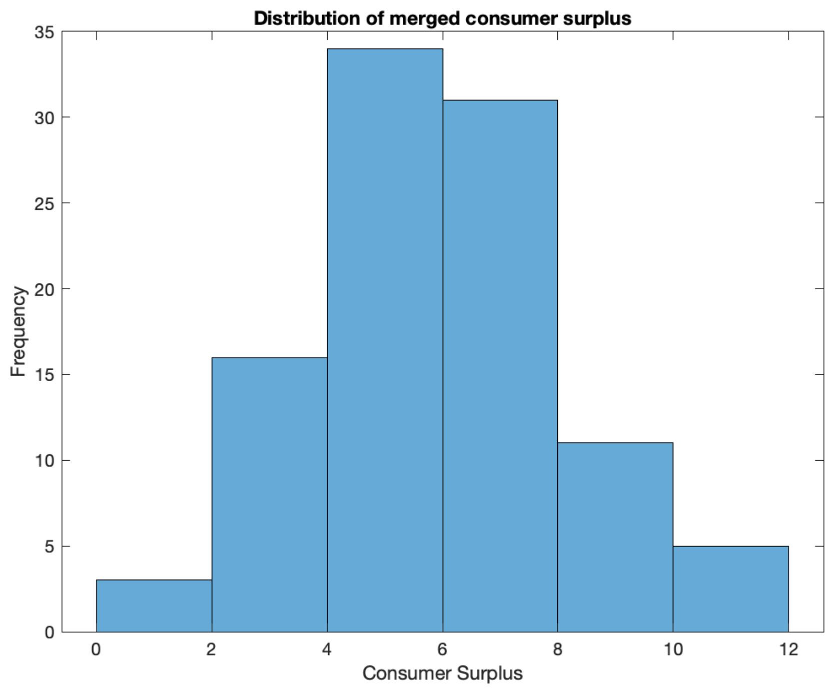
The merged median markup is 1.1150 and the merged mean markup is 1.4131.

From the distribution presented above along with the median and mean values, we can see that the merged markups are higher. However, we can do better in the sense that we can attribute the price changes to the respective products. Since firms 1 and 2 merged we should not expect to see a change in the markups of product 3. The following three presents the average difference in markups for the three products:

	Average increase in markup
Product 1	0.9326
Product 2	0.3625
Product 3	0

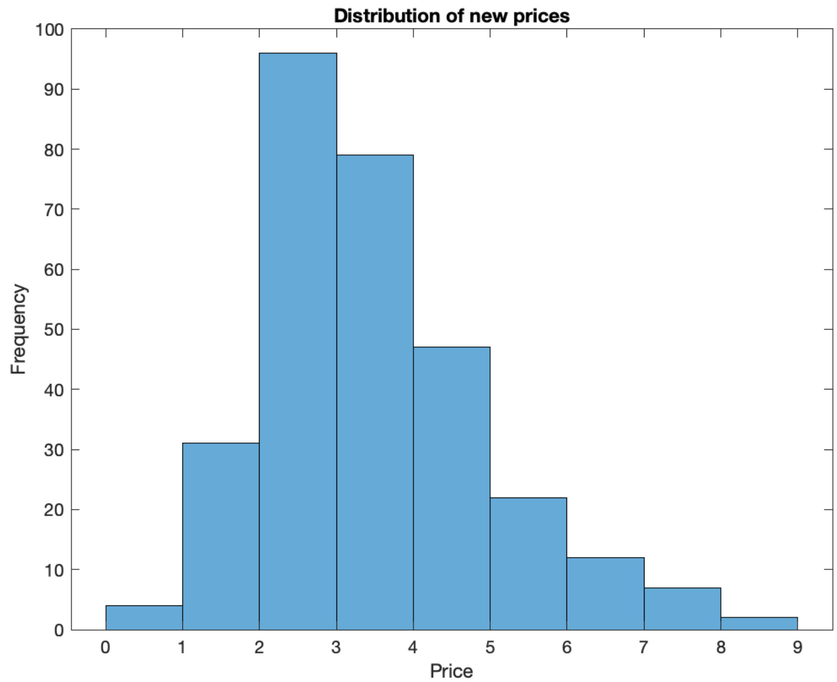
Problem 4.2 (c):

Consumer surplus:



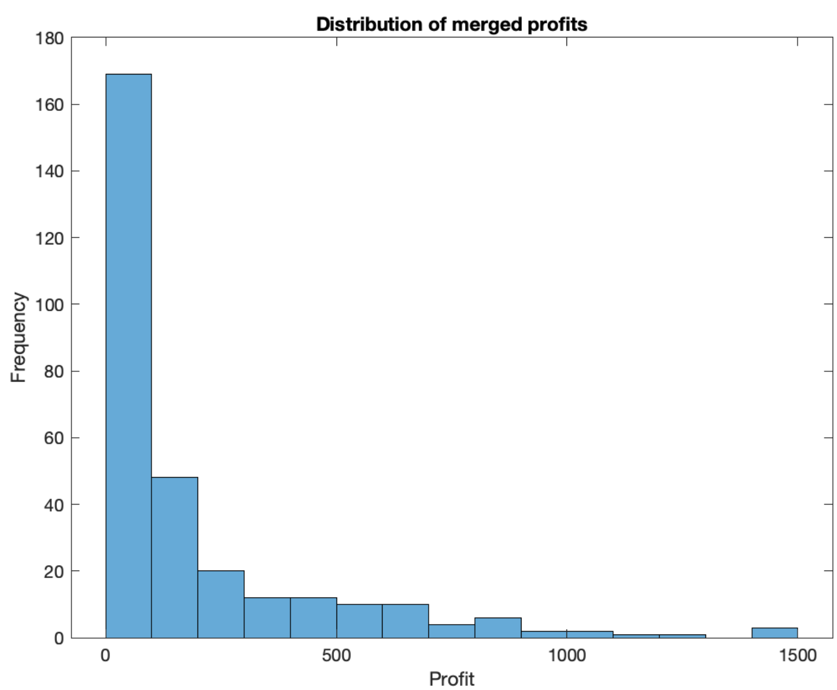
The mean surplus is 5.8984 and the median surplus is 5.8652. This is lower than the true surplus values and it should make sense that surplus decreases and firms are able to charge higher prices.

Prices:



Since the markup is higher, it is obvious that the prices are going to be higher. However, only the prices of firms 1 and 2 are going to be higher.

Profits:



The mean profit is 190.2729 and the median profit is 71.8560. The profit is higher than true profit and this makes sense because the firms can now charge higher price. Again, the profits should only be higher for the 2 merged firms and the same for the third firm.
