Tanking from the Start: Dynamic Moral Hazard with Change in Preference

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Abstract

This paper extends the traditional two-period dynamic moral hazard model, where a risk-neutral principal hires a wealth-constrained but risk-neutral agent, by introducing an exogenous probability of a change in the principal's preferences in the second period. This change, modeled as indifference between outcomes, occurs only after a first-period failure. Using the NFL draft system as a case study, where team incentives shift after elimination from playoff contention and losing can improve draft position, the analysis demonstrates that a higher probability of change in preference reduces first-period effort. This dynamic creates 'vicious' and 'virtuous' cycles, where weaker teams exert less effort and stronger teams exert more. These cycles can be eliminated, and first-period effort increased, if the change in preference ensures no positive outcome for the principal, such as no improvement in draft picks from losing.

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1. Introduction

In this paper, I analyze a novel dynamic moral hazard problem. I consider a risk-neutral principal, who can hire a risk-neutral but wealth-constrained agent for a project. The project can either succeed or fail, and the agent can exert unobservable effort to increase the probability of success. I extend this standard problem to incorporate a change in preference for the principal in the second period. Specifically, I model a scenario where, from the principal's perspective, the difference between success and failure shrinks in the second period. I examine the effect this change in preference has on the induced effort in the first period even though the two periods are technologically independent and the preferences in the first period remain unchanged.

The paper is motivated by the fact that long-term contracts are often established by businesses for their employees. However, conditions may change during the length of the contract, which may affect the principal's preferences. The NFL and its draft system serve as the primary example in this paper, where season-long or multi-season contracts resemble long-term agreements. In the NFL, a team may find itself out of playoff contention during the season, leading to a shift in preference and potentially incentivizing losing to secure a better draft pick under the NFL's draft rule.[1] This paper specifically examines the scenario where a change in conditions leads to the principal's indifference between success and failure in the second period. This problem is particularly interesting because, in a long-term contract, even though preferences may change at any point, the incentives have to be in place, affecting all periods.

In 2022, the NFL generated \$11.9 billion in national revenue, increasing to \$12.9 billion in 2023, highlighting the industry's vast scale.[2] The prevalence of changing conditions, such as teams missing the playoffs and the observed behavior of 'tanking' – where a team intentionally tries to lose games to improve draft pick prospects – makes this setting particularly relevant.[3] However, this model extends beyond the NFL and can be applied

[1] For a complete guide on the rules of the NFL draft, please refer to NFL Operations: The Rules of the Draft.

[2] See the CBS Sports report, "NFL breaks the bank: Here's how much each team made in national revenue in 2023" (2024).

[3] For example, see Bassinger, 2015 for a report on how the Tampa Bay Buccaneers blew a lead in the fourth quarter by benching some of their starters in the second half. See also Inabinett, 2022 for an article on the term 'Tanking for Tua', which describes the Miami Dolphins' strategy for the 2019 NFL

to other contexts with slight adjustments. For example, debt-ridden businesses may opt for further losses to enter bankruptcy and avoid payments to creditors. Similarly, firms might intentionally deliver poor returns to avoid becoming too large and triggering regulatory oversight. In patent races, a company might shift strategy after falling behind. In all these cases, conditions change during contract's duration, but a long-term contract requires incentives to already be in place. While the model and its interpretation in this paper are specific to the NFL, they can be adapted to fit these other contexts.

Following the NFL application, the incentive to lose after being knocked out of the playoff race is clear and has been empirically documented in similar leagues.[4] Tanking incentives are undesirable from the regulator's (league's) perspective, as seen with the Dallas Mavericks being fined for tanking.[5] Tanking can reduce gate revenue, as fans may lose interest in attending less competitive games where teams put forth less effort.[6] It can also harm players' careers, as their value is often tied to team performance.

While tanking has been empirically documented, the severity of tanking incentives before being knocked out of playoff contention has received little attention. Moreover, although empirical evidence exists, there has been limited theoretical work investigating the issue. Most prior research on tanking has relied on tournament theory as the primary framework, which suggests that varying levels of effort may be displayed based on the potential incentives or rewards.[7] This paper contributes to the literature by using a principal-agent framework to explore tanking incentives more comprehensively, including the incentives to tank from the start of the season, not just after playoff elimination, making the analysis applicable to a wider range of contexts. This theoretical analysis could provide valuable insights for regulators to better understand these incentives and inform policy measures to mitigate the adverse effects of tanking.

In the static one-period problem, there is the standard trade-off between rent ex-

season to secure the No. 1 selection in the 2020 NFL Draft.

[4] For instance, Fornwagner (2019) analyzes NHL data, showing that teams lose more games after being eliminated from playoff contention, and this behavior reflects a deliberate strategy rather than a lack of motivation or disappointment. See also Taylor & Trogdon (2002) and Gong et al. (2022) for similar studies in the NBA.

[5] Refer to the Forbes report, "NBA Fines Dallas Mavericks $750,000 For Tanking" for more coverage on the story.

[6] For instance, Price et al. (2010) estimated that losing games could reduce gate revenue of tanking teams by approximately $2 million under the assumption that they would purposely lose 10 games towards the end of the regular season in order to compete for the top pick.

[7] See Connelly et al. (2014).

traction and inducing effort, leading to a downward distortion of effort compared to the first-best solution. When the standard model is extended to include a second period, the optimal second-best contract demonstrates memory. Even though a success in the first period has no technological effect on the likelihood of success in the second period, the principal implements higher second-period effort following a success in the first period. The principal provides the agent with strong incentives in the second period after a first-period success (and weak incentives after a failure). This approach motivates the agent to work hard in the first period, not only for the direct reward of first-period success but also to enjoy higher second-period rent. By using the second period to provide first-period incentives, it becomes cheaper to induce effort in the first period than in a static problem. Consequently, the principal implements higher than the static second-best effort in the first period.

As mentioned above, I extend the standard two-period model by introducing the possibility of a change in preference in the second period, where the principal becomes indifferent between the project outcomes. In line with the NFL draft rule, I model this indifference by increasing the payoff from losing in the second period, mirroring the fact that a team out of playoff contention can benefit from losing by securing a better draft pick. Under the assumption that a change in preference is only possible after a first-period failure, the paper shows that teams with a higher ex-ante probability of experiencing the change in preference induce lower first-period effort. In other words, teams more likely to miss the playoffs after an early loss try less hard, increasing their chances of losing. This creates 'vicious' and 'virtuous' cycles, where weaker teams exert less effort and stronger teams try harder, worsening competitive imbalance – an issue for regulators aiming to promote competitive balance.

This result arises because the first-period outcome impacts the second-period expected payoff and this effect outweighs the positive carrot-and-stick effect on first-period effort. A team can improve its expected second-period payoff by intentionally losing early, signaling that tanking may begin from the start of the season, not just after being eliminated from playoff considerations. I believe this finding represents a novel contribution to the theoretical and empirical literature on tanking. Specifically, if a team expects low playoff chances, it will exert lower effort from the outset reinforcing those low expectations. However, these cycles can be eliminated by removing any positive returns after a team is out of playoff contention. If a team knows it cannot gain anything after a preference change, it will work harder to avoid this scenario. This models a situation where the draft rule is unrelated to season performance, encouraging teams to exert consistent effort throughout the season.

The literature on repeated moral hazard encompasses various strands that often lack a common framework, making it challenging to relate different papers to one another. In particular, different assumptions about when the agent consumes, access to credit, and control of savings are prevalent. Most papers consider repeated versions of the traditional moral hazard setting where the agent is risk-averse and the trade-off is between insurance and incentives.[8] The initial studies of dynamic agency were by Rubinstein (1979) and Radner (1981) demonstrated that in an infinitely repeated version of the static one-period model, the first best can be achieved if there is no discounting.[9] The pioneering result in this strand was established by Rogerson (1985), who showed that optimal second-period incentives depend on the first-period outcome, indicating that the contract exhibits memory, even though the periods are technologically independent.[10] This result is driven by the consumption-smoothing of the risk-averse agent.[11] However, I consider a risk-neutral but wealth-constrained agent, and following Ohlendorf & Schmitz (2012), I identify this memory property despite the absence of a consumption-smoothing motive in my analysis.

In recent years, there has been growing interest in repeated moral hazard models involving risk neutrality and limited liability. For instance, Clementi & Hopenhayn (2006), DeMarzo & Fishman (2007a, 2007b), and Biais et al. (2010) focus on the long-run dynamics of firm size and survival rates.[12] In particular, Biais et al. (2010) studied a

[8] For more detailed surveys, see Chiappori et al. (1994) and Bolton & Dewatripont (2004).

[9] These authors assumed that long-term relationships can better address incentive problems because time allows sharper inferences about the performance. However, Fudenberg et al. (1986) disputed this by arguing that repetition alters the agent's preferences rather than improving monitoring accuracy.

[10] Lambert (1983) and Townsend (1982) also highlighted the significance of memory in long-term relationships.

[11] Rogerson (1985) assumed that the agent cannot borrow and save. Allen (1985), Malcomson & Spinnewyn (1988) and Fudenberg et al. (1986) argue that the benefits of long-term contracting are a result of the restrictions on borrowing and saving.

[12] The authors examine how an entrepreneur can be optimally incentivized to avoid large risks or to reveal private information about the cash flow, and whether optimal investment pattern can be implemented

continuous-time model with a risk-neutral agent protected by limited liability, where the agent exerted unobservable effort to reduce the probability of large but infrequent losses. However, these dynamic models assumed the entrepreneur's incentive problem as a binary choice for tractability. Bierbaum (2002) also compared short-term and long-term contracts using a binary-effort model. In contrast, my model, while limited to two periods, finds optimal effort levels across periods when effort can be adjusted continuously

Crémer (1995), Baliga & Sjöström (1998), Che & Yoo (2001) and Schmitz (2005) studied dynamic models with risk-neutral agents, hidden actions, and wealth constraints. However, these papers relied on different features such as private information about productivity, observable but unverifiable effort, common shocks, and technological links between the periods. I assume no private information or common shocks, and that the periods are technologically independent. Finally, Ohlendorf & Schmitz (2012) considered a pure two-period repeated moral hazard model with continuous effort and a risk-neutral, wealth constrained agent. Their findings showed that the optimal dynamic contract under full commitment exhibits memory, with induced first-period effort being higher than the static second-best.[13] As a novel contribution, I introduce the possibility of a change in the principal's preference in the second period to their model.[14] I demonstrate that this change in preference affects first-period effort, despite the fact that the change in preference occurs only in the second period, and that induced first-period effort may be lower than the static second-best, depending on how the preference change is modeled.

The remainder of the paper is organized as follows. Section 2 introduces the benchmark model, followed by a discussion of the the change in preference in Section 2.1. In Section 3, I solve the optimal contracts for both the benchmark model and for change in preference under full commitment. Section 4 explores variations of the change in preference, and

using standard financial contracts.

[13] While renegotiation is shut down in this paper to simplify the exposition and align with the application, the dynamic moral hazard literature has extensively explored renegotiation. In fact, renegotiation is relevant even in static moral hazard models with risk-averse agents, as exposing the agent to additional risk may become unnecessary after effort is exerted. Fudenberg & Tirole (1990) and Ma (1991, 1994) showed that whether effort incentives are reduced depends on who makes the renegotiation offer. In contrast, Ohlendorf & Schmitz (2012) found, with risk-neutral agents protected by limited liability, that renegotiation matters only in repeated moral hazard, and the specifics of the renegotiation game do not affect the results.

[14] Strack & Taubinsky (2021) is the only other work I have found addressing preference chances, but their study focuses on time inconsistency, where the agent's rather than the principal's preferences can change.

finally, Section 5 concludes.

2. Benchmark Model

The paper explores an adaptation of the dynamic moral hazard model introduced by Ohlendorf & Schmitz (2012), excluding the option for the principal to invest in or terminate the project. There are two parties, a principal (she) and an agent (he), both of whom are risk neutral. In the context of the paper, the principal is likened to a team owner/manager in the NFL, while the agent represents a player. The team owner/manager represents the interests of the team, hence we will frequently refer to them as the 'team' when discussing the principal. The agent is protected by limited liability arising from liquidity constraints, ensuring non-negative payments. The parties' reservation utilities are assumed to be zero. At some initial date 0, the principal makes a take-it-or-leave it offer to the agent. If the agent rejects the contract, the game ends and the parties receive their reservation utility. Upon accepting the contract, the agent exerts unobservable first-period effort $e_1 \in [0, 1]$ at date 1 incurring disutility from exerting effort given by $c(e_1)$. While the paper initially presents a general function for the disutility of effort satisfying Assumption 1, I will later employ a specific cost function ($c(e) = e^2$) to solve the model and derive results. The utilization of the specific cost function will provide insights into the problem without adding analytical complexity.

Assumption 1. *The effort cost function satisfies*

(a) $c'(\cdot) \geq 0, c'(\cdot) \geq 0, c'''(\cdot) \geq 0,$ *and* $c''(e) > 0$ *for all* $e > 0$

(b) $c(0) = 0, c'(0) = 0,$ *and* $c'(1) \geq 1$

At date 2, either a success $(y = 1)$ or a failure $(y = 0)$ is realized, where the probability of success is normalized to equal the effort level, that is, $P(y_1 = 1|e_1) = e_1$. The principal's verifiable return is simply given by the outcome, y_1. The transfers $t_1(y_1)$ is paid out to the agent. Then, the agent chooses an unboservable second-period effort level $e_2(y_1) \in [0, 1]$ at date 3 incurring a cost of effort $c(e_2(y_1))$. Finally, at date 4 the verifiable second-period return y_2 is realized where $y_2 \in \{0,1\}$ and $P(y_2 = 1|e_2(y_1)) = e_2(y_1)$. The transfer $t_2(y_1, y_2)$ is paid out to the agent.[15]

The paper assumes that the two periods are independent. In particular, a first period success (failure) does not make a second-period success (failure) more probable. In other words, the paper assumes away any technological spillovers. The paper further assumes that the principal can commit not to renegotiate the contract that is written at date 0.[16] The timing of events is illustrated below:

$T = 0$	$T = 1$	$T = 2$	$T = 3$	$T = 4$
The principal makes a take-it-or-leave-it contract offer to the agent	The agent chooses an unobservable first-period effort level $e_1 \in [0,1]$	Outcome y_1 is realized and $t_1(y_1)$ is paid out	The agent chooses an unobservable second-period effort level $e_2(y_1) \in [0,1]$	Outcome y_2 is realized and $t_2(y_1, y_2)$ is paid out
first period			second period	

2.1 Change in Preference

The paper extends the Ohlendorf & Schmitz (2012) model described above by incorporating the potential for a change in the principal's preferences in the second period. As mentioned in the introduction, this change in preference is meant to capture the idea that the difference in payoff between winning ($y = 1$) and losing ($y = 0$) shrinks in the second period for the principal. In particular, I will examine the scenario where the principal is indifferent between winning and losing in the second period, meaning that the returns from $y_2 = 0$ and $y_2 = 1$ are equal. The diminished disparity in payoffs can be modeled in two distinct manners: **(1)** increase payoff from losing and **(2)** decrease payoff from winning while keeping the other payoff fixed. I will primarily focus on case **(1)** and refer to this framework as '***Win from Losing***'. The emphasis on this particular scenario follows from the application to the NBA and its associated draft system. A team may benefit

[15] I assume that the agent cannot be replaced at date 2. It might be too costly for the principal to replace the agent because, for example, hiring a new agent might require costly training for the job.

[16] Using the NFL as the application to my paper, there are significant restrictions on contract renegotiations. For instance, renegotiations for drafted rookie players cannot occur until the end of the final regular season game. Similar restrictions exist for veteran contracts as well. For more details on restrictions on contract renegotiations, please refer to NFL-NFLPA, 2020.

from losing by obtaining a higher draft pick, while winning entitles them to a bonus. In such a case, the payoff from winning remains unchanged compared to the benchmark case, while the payoff from losing is increased. However, I will assume that an NFL team only considers this potential benefit of losing once she is no longer in playoff contention. So, the change in preference only happens after the team is eliminated from playoff contention. I will refer to framework **(2)** as '***Lose from Winning***' and will discuss it as an extension.[17]

The change in preference depends on first-period output and will only occur after a failure in the first period.[18] The dependence on first-period output can be justified by considering that a team remains in playoff contention after a win, while only a loss can take them out of contention. Specifically, the principal experiences a change in preference in the second period following a first period failure ($y_1 = 0$) with an exogenous probability p. Given that the change in preference occurs solely after elimination from playoff contention, I can consider p as the ex-ante probability of not making the playoffs after a loss. Notice that poorer-performing teams are going to have a higher p signifying a greater probability of experiencing a change in preference. Poor-performing teams are going to be lower-ranked and lower ranked teams will have a higher p since they are more susceptible to elimination after a loss compared to higher-ranked teams.[19]

2.1.1 *Win from Losing*

The *Win from Losing* model reflects the diminishing difference in payoffs between success and failure for the principal by increasing the rewards for losing. This model is inspired by the concept that once a team is out of playoff contention, she gains from losing through obtaining a better draft pick.[20] In this case, following a first-period success ($y_1 =$

[17] This framework envisions a scenario where the draft rule is unrelated to a team's performance. A team cannot obtain a higher draft pick by worsening their rankings through a loss. However, once she is out of the playoff race, she will not receive any bonus for winning either. In this case, the payoff from loss remains unchanged compared to the benchmark, while the payoff from winning is decreased.

[18] I consider an extension where the principal can experience a change in preference (with some exogenous probability), independent of first-period output. In the NFL setting, this means that a team may be out of playoff contention even after a win. This extension can be motivated by the fact that whether a team is still in playoff contention depends not only on that team's results but also on the results of other teams. For instance, a team may remain in the playoff race after losing if their rivals also lose. Conversely, a team may be eliminated from the playoff race after winning if their rivals also win.

[19] I assume that p is common knowledge as rankings are observable.

[20] The benefit from losing in the form of a better draft pick is also present when a team is still in playoff contention. I assume that the reward for making the playoffs is large enough that a team prioritizes

1), the principal's return in the second-period is simply the return from the benchmark model. Therefore, the principal's return in the second-period following $y_1 = 1$ is given by y_2. Following a failure in the first-period ($y_1 = 0$), the principal's return in the second-period is characterized below:

$$\text{Principal's return} = \begin{cases} y_2 & \text{with prob } (1-p) \\ 1 & \text{with prob } p \end{cases}$$

The principal does not experience a change in preference in the second-period even after a first-period failure with probability $(1-p)$. In that case, the principal's return in the second period is simply the outcome, that is, y_2. With probability p, the principal experiences a change in preference following $y_1 = 0$. The *Win from Losing* framework increases the return from losing such that the principal is indifferent between winning and losing. Given that the payoff from winning remains unchanged from the benchmark case at 1, this entails that the payoff from losing must also be increased to 1 in this case. Figure 1 provides a graphical summary of the return to the principal across the two periods.

To simplify, the principal's return is 1 with certainty in the event of a change in preference (which occurs with probability p) in the *Win from Losing* framework, as she will receive 1 regardless of whether she wins or lose. In the context of the NFL, when a team wins after being eliminated from playoff contention, she still receive a winning bonus. However, if she loses after dropping out of playoff contention, she now gains the advantage of securing a better draft pick. The teams are guaranteed a certain payoff once she drops out of the playoff race.

It is important to note that the principal will induce zero effort in the second period after experiencing a change in preference. If the change in preference occurs, the principal is guaranteed a payoff of 1 and she is indifferent between winning and losing. Consequently, the principal will not want to induce any effort as inducing effort will not affect the payoff but only increase the cost to the principal. This makes it easy to observe that teams will opt to '*tank*' once she is out of playoff contention as she experiences the change in preference. While this particular result may seem obvious, the impact, if any, of the

securing a playoff sport first. A team will only consider the possibility of a better draft pick once she is no longer in playoff contention.

possibility of a change in preference in the second-period on the induced effort level in the first-period remains unclear. In the subsequent section, I investigate this issue and discuss how induced effort in the first-period fluctuates with the probability of change in preference (p).

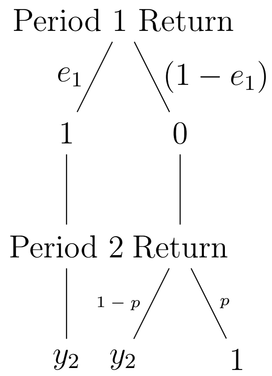

Figure 1: Principal's return in *Win from Losing*

3. Optimal Contracting

3.1 One-Shot Contracting Benchmark Model

The principal solves for the first-best static effort level e_S^{FB} by maximizing the expected total surplus:

$$S(e) := e - c(e) \tag{1}$$

and is thus characterized by:

$$S'(e_S^{FB}) = 1 - c'(e_S^{FB}) = 0 \tag{2}$$

Equation (2) is the standard marginal benefit (of effort) equal to marginal cost (of effort) result. The principal can implement the static first-best effort level, but would have to leave all of her returns to the agent. Therefore, the principal faces a trade-off between expanding the overall size of the pie and retaining a larger portion for herself. In situations where efforts are unobservable, the principal will not provide any compensation in the event of failure ($y = 0$).[21] Let t denote the principal's transfer payment to the agent in case of a success ($y = 1$). The principal solves for the second-best effort level when effort levels are not observable by maximizing her expected payoff subject to the usual incentive-compatibility and participation constraints. The agent's expected payoff from exerting effort e is $et - c(e)$, the agent's maximization has an interior solution characterized by $t = c'(e)$.[22] The key takeaway is this one-to-one relationship between effort and the transfers set by the principal which is due to continuous effort levels. Hence, setting transfers is equivalent to setting effort levels and I will think of the principal's problem as one of directly setting the effort levels. The principal maximizes her expected profit

$$P(e) := e\left(1 - c'(e)\right); \tag{3}$$

hence the first-order condition that characterizes the second-best effort level e_S^{SB} is

$$P'(e_S^{SB}) = 1 - c'(e_S^{SB}) - e_S^{SB} c''(e_S^{SB}) = 0. \tag{4}$$

Assumption 1 guarantees that the function P is concave. Following Ohlendorf and Schmitz, I also define

$$A(e) := ec'(e) - c(e) \tag{5}$$

the agent's rent from a contract that leads him to choose effort e. A is a strictly increasing, convex, and non-negative function which are evident from its derivative: $A'(e) = ec''(e)$. As the principal increases the induced effort levels, the agent will receive higher rents. Consequently, the principal induces a downward distortion of the effort level, such that, $e_S^{SB} < e_S^{FB}$. The following proposition summarizes the above results for the cost function given by $c(e) = e^2$.[23]

[21] This is a standard result. See, for example, Bolton & Dewatripont (2004) (Section 4.1.2) for an illustration of the one-shot moral hazard with risk-neutrality and resource constraints.

[22] When an individual agent's output can take only two values, then the agent's incentive constraint can be replaced without loss of generality by the first-order condition of the agent's problem. See Bolton & Dewatripont (2004) for a discussion.

[23] It can be easily verified that $c(e) = e^2$ satisfies Assumption 1.

Proposition 1. *For cost of effort function, $c(e) = e^2$, the principal's expected profit is*

$$P(e) = e\,(1 - 2e)\,; \tag{6}$$

the agent's rent associated with induced effort level e is

$$A(e) = e^2. \tag{7}$$

The first-best effort level induced by the principal in the static problem is

$$e_S^{FB} = \frac{1}{2}, \tag{8}$$

and the second-best effort level induced is

$$e_S^{SB} = \frac{1}{4}. \tag{9}$$

Proof. Verify by plugging $c(e) = e^2$ into (3), (5), (2) and (4) respectively.

3.2 Two-period Benchmark Model

We will now solve for the optimal contract in the two-period benchmark model without a change in preference.[24] This is equivalent to solving the change in preference model with $p = 0$ and is a standard two-period dynamic moral hazard model with a risk neutral agent and continuous effort. If efforts were observable, the principal would implement the effort levels $e_{1D}^{FB} = e_{2D}^{FB}(0) = e_{2D}^{FB}(1) = e_S^{FB}$[25] with a forcing contract that leaves no rent to the agent.

For the rest of the paper, I will assume that effort levels are unobservable. I look for the optimal contract offered by the principal in response to the presence of moral hazard. Following Ohelndorf and Schmitz, we do not impose any ad hoc restrictions on the class of feasible contracts, that is, there is complete contracting in the sense of Tirole (1999).

[24] For ease of exposition, we disregard discounting.
[25] The subscript 'D' refers to the dynamic problem and the subscript 'S' refers to the static problem.The number in the subscript is used to denote the period in the dynamic problem.

The second best contract specifies transfer payments from the principal to the agent (which can depend on first and second period outcomes). Therefore, the contract will specify first-period transfers $t_1(y_1)$ to be made at date 2 and the second-period transfers $t_2(y_1, y_2)$ to be made at date 4. The transfer payments satisfy the usual incentive compatibility constraints. The incentive compatibility constraints for the second period are

$$e_2(y_1) \in \arg\max_{e\in[0,1]} et_2(y_1, 1) + (1 - e)t_2(y_1, 0) - c(e) \tag{IC_2}$$

The continuation payoff of the agent once the first period outcome is realized is given by

$$a(y_1) = t_1(y_1) + e_2(y_1)t_2(y_1, 1) + (1 - e_2(y_1))t_2(y_1, 0) - c(e) \tag{10}$$

Then, the first-period incentive compatibility constraint is given by

$$e_1 \in \arg\max_{e\in[0,1]} ea(1) + (1 - e)a(0) - c(e) \tag{IC_1}$$

The transfer payments pin down recommended effort levels from the first-order condition of the agent's incentive compatibility constraints. The transfers also have to satisfy the limited liability constraints of the agent. The limited liability constraints are given by

$$t_1(y_1) \geq 0 \tag{LL_1}$$

for the first period and by

$$t_2(y_1, y_2) \geq 0 \tag{LL_2}$$

for the second period. (LL_2) reflects the assumption that the agent cannot be forced to pay back payments that he received in the past.[26]

Further, following Ohlendorf and Schmitz, I simplify the class of contracts that need to be considered. Agent's effort choice in the second period is solely determined by the

[26] Otherwise, the limited liability constraint would read $t_2(y_1, y_2) \geq -t_1(y_1)$.

difference between $t_2(y_2,1)$ and $t_2(y_2,0)$, hence, any contract with a reward for failure in the second period ($t_2(y_1,0) > 0$) can be replaced by contracts that provide sufficiently large rewards at date 2 ($t_1(y_1)$). We can modify any given incentive scheme (t_1, t_2) to set $t_2(y_1, 0) = 0$ by defining the following:

$$\begin{aligned}
\tilde{t}_1(y_1) &= t_1(y_1) + t_2(y_1, 0) \\
\tilde{t}_2(y_1, 0) &= 0, \text{ and} \\
\tilde{t}_2(y_1, 1) &= \max\{t_2(y_1, 1) - t_2(y_1, 0), 0\}
\end{aligned}$$

The new transfer scheme $(\tilde{t}_1, \tilde{t}_2)$ induces the same second period effort levels as (t_1, t_2), the same continuation payoffs $a(1)$ and $a(0)$, and therefore also the the same first period effort levels. The new transfer scheme also satisfies the limited liability constraints and lead to the same expected payoffs. Thus, without loss of generality, I only focus on the set of contracts with $t_2(y_1, 0) = 0$ for the principal's optimization problem. I ignore (LL_2) in the optimization problem because the second-period incentive compatibility constraint, combined with Assumption 1, ensures that the limited liability constraint $t_2(y_1, 1) \geq 0$ is satisfied. Since the agent can always choose not to exert any effort at all, the limited liability constraint along with the incentive compatibility constraint also guarantees the agent's participation. However, the optimal long-term contract needs to satisfy an additional constraint,

$$t_1(y_1) + A(e_2(y_1)) = a(y_1) \text{ for } y_1 \in \{0, 1\} \tag{11}$$

which emerges from the fact the agent's transfers in the second-period must match what was promised in the contract. The transfers are promised because the contract specifies $e_2(1)$ and $e_2(0)$, and, thus the principal implicitly chooses the transfers in the second-period. The constraint defined by (11) ensures that the principal does not renege on the transfers implicitly promised to the agent based on the effort levels chosen for the second-period. Therefore, the principal's problem is to maximize

$$e_1 \left(1 - t_1(1) + e_2(1) \left(1 - t_2(1,1)\right)\right) + (1 - e_1) \left(-t_1(0) + e_2(0) \left(1 - t_2(0,1)\right)\right)$$

$$\text{subject to } (IC_1), (IC_2), (LL_1), \text{ and } (11)$$

The following proposition characterizes the second-best solution of the two-period benchmark model without a change in preference.

Proposition 2. *For any cost of effort function, $c(e)$, satisfying* Assumption 1, *the induced effort levels in the principal's optimal contract satisfy*

$$e_S^{FB} \geq e_{2D}^{SB}(1) > e_{1D}^{SB} > e_S^{SB} > e_{2D}^{SB}(0) > 0. \tag{12}$$

Proof. See the Appendix.

The optimal second best contract demonstrates memory: $e_{2D}^{SB}(1) > e_S^{SB} > e_{2D}^{SB}(0)$[27] The principal implements higher (lower) effort in the second period following a success (failure) in the first period despite the fact that the two periods are technologically independent. The resulting larger or smaller second-period rent acts as an indirect reward or punishment for the wealth-constrained agent based on the first-period outcome. The incentives provided in the second period act as carrot and stick for the first period: $e_{2D}^{SB}(1)$ serves as the carrot and $e_{2D}^{SB}(0)$ as the stick.

In the one-shot interaction, the most severe punishment available to the principal for failure is not to pay anything to the agent. With a two-period contract, stronger incentives can be provided. The second period can now be used to incentivize the first period through $e_{2D}^{SB}(1)$ and $e_{2D}^{SB}(0)$, making first-period incentives 'cheaper' to provide. This leads the principal to implement a higher first-period effort than in the static second

[27] If an outsider observed today a principal-agent pair that was successful and another identical pair that was not successful, they would be right to predict that the first pair also is more likely to succeed tomorrow. In other words, there is a serial correlation across periods generated endogenously which Ohlendorf & Schmitz (2012) refer to as the 'hot hand' effect.

best ($e_{1D}^{SB} > e_S^{SB}$). These insights are easily illustrated by considering a specific cost function, $c(e) = e^2$.[28]

Proposition 3. *For cost of effort function, $c(e) = e^2$, the induced effort levels in the principal's optimal contract satisfy*

$$e_S^{FB} = e_{2D}^{SB}(1) > e_{1D}^{SB} > e_S^{SB} > e_{2D}^{SB}(0) > 0. \tag{13}$$

Proof. See the Appendix.

The first-period effort in the second-best problem, e_{1D}^{SB}, must satisfy (A.22). This particular first-order condition demonstrates that e_{1D}^{SB} increases with $e_{2D}^{SB}(1)$ and decreases with $e_{2D}^{SB}(0)$ as $e_{2D}^{SB}(y_1) \leq e_S^{FB} = \frac{1}{2}$ for $y_1 \in \{0, 1\}$. A higher $e_{2D}^{SB}(1)$ implies a larger carrot, meaning a greater reward for success in the first period—while a lower $e_{2D}^{SB}(0)$ signifies a harsher stick, or a stronger punishment for failure in the first period. Alternatively, combining (A.19) with (A.17) and (A.18), along with the fact that $t_1(0) = 0$, we have:

$$e_{1D}^{SB} = \frac{t_1(1) + e_{2D}^{SB}(1)^2 - e_{2D}^{SB}(0)^2}{2} \tag{14}$$

This equation demonstrates that the principal can motivate the agent to work hard not only through a direct transfer ($t_1(1)$) in the first period but also by strategically using the carrot and the stick. The prospect of a higher second period-rent following a first period success motivates the agent to exert higher effort in the first-period compared to the static second-best.

3.3 Two-period Model with *Win from Losing*

Now, we want to solve for the optimal contract under the *Win from Losing* framework. In this setting, the principal may experience a change in preference in the second-period with probability p only following a failure in the first period ($y_1 = 0$). Under this framework, when the principal experiences a change in preference, the payoff from

[28] For the remainder of the paper, we will continue to assume that the cost of effort function is given by $c(e) = e^2$ to simplify the exposition.

losing increases and yields a guaranteed payoff of 1 in the second-period, regardless of the outcome. As previously mentioned, the probability of a change in preference is analogous to the probability of an NFL team not making the playoffs. Following the NFL analogy, the *Win from Losing* considers the scenario where, after dropping out of playoff contention, a team may benefit from losing by securing a better draft pick while still having the potential to earn a bonus from winning.

In this context, the principal would 'tank' if the change in preference occurs, that is, the principal would induce no effort in the second period.[29] However, it is not immediately clear how the induced effort level in the first period will be affected by this probability of a change in preference, given that the two periods are technologically independent. The following proposition summarizes my findings:

Proposition 4. *Assume that the principal cannot renegotiate. Under the Win from Losing framework, the principal implements:*

(1) the static second best in the second-period following a first-period success, that is, $e_{2D}^{SB}(1) = e_S^{FB}$ *is invariant of* p

(2) lower second-period effort following a first-period failure as the probability of the change in preference increases, that is, $e_{2D}^{SB}(0)$ *is decreasing in* p

(3) lower first-period effort as the probability of the change in preference increases, that is, e_{1D}^{SB} *is decreasing in* p.

Proof. See the Appendix.

The proposition states that the induced effort level in the second period following a first-period success ($y_1 = 1$) is the static first-best level. Since we are considering the possibility of a change in preference only after a failure in the first period, there is no chance of such a change following a success. Consequently, the principal's expected

[29] This could be the case if the principal had the ability to renegotiate the contract written at date 0. Alternatively, this could occur if the principal could offer one-period contracts after observing the realized state, meaning she could offer a second-period contract based on whether the change in preference has occurred. I assume that the principal offers a long-term contract at date 0, before observing the realized state and cannot renegotiate. However, I will consider the case where the principal can offer one-period contracts having observed the realized state to compare the outcomes for both the principal and the agent relative to the optimal long-term contract.

payoff in the second period after a first-period success is equivalent to that in the two-period benchmark model, and it remains independent of the probability of the change in preference, p. As a result, the principal implements the same effort level as in the standard dynamic case, (e_S^{FB}), which remains invariant with respect to p. There are no distortions to the effort level from the standard case following a success because principal's payoff is unchanged from the standard case once she has secured a first-period win.

On the other hand, the principal's induced effort level in the second period following a first-period failure ($y_1 = 0$) decreases as p increase. To understand this, consider a static version of the game following a first-period failure. As p increases, the difference in the principal's payoff between winning and losing diminishes. Consequently, the principal does not want to pay the agent as much for a success ($y = 1$). As the wedge between $t(1)$ and $t(0)$ – the payments tied to success and failure – shrinks, the incentives provided to the agent weaken. This wedge is critical to satisfying the agent's incentive compatibility constraint. A smaller gap between $t(1)$ and $t(0)$ results in reduced incentives, leading to lower induced effort from the agent. Therefore, as p increases, the gap between $t(1)$ and $t(0)$ decreases, which in turn reduces the agent's effort level, explaining why e_{2D}^{SB} decreases with p.

The main result of the proposition is that the induced effort level in the first-period is decreasing in p. If the first-period outcome is a success, the principal receives a payoff of 1 with probability $e_{2D}^{SB}(1)$. In contrast, if the first-period outcome is a failure, the principal's payoff in the second period is 1 with probability $e_{2D}^{SB}(0) + (1 - e_{2D}^{SB}(0))p$. This is because the principal can receive a payoff of 1 in the second period following a first-period failure in two ways: either by winning, which occurs with with probability $e_{2D}^{SB}(0)$, or by losing (with probability $1 - e_{2D}^{SB}$) and experiencing a change in preference, which happens with probability p. The larger the p, the higher the expected payoff in the second-period following a first-period failure, since the principal becomes more likely to receive a payoff of 1 even if the second period outcome is a loss. While the expected second-period payoff after a first-period success remains constant as p changes, the expected second-period payoff after a first-period failure increases with p. Thus, as p increases, the principal becomes more likely to accept a loss in the first period. This growing incentive to 'lose' in the first period results in a reduction in the effort induced in the first period. In essence, the difference in principal's overall expected payoff from winning and losing in the first-period diminishes as p increases. Following the argument with respect to $e_{2D}^{SB}(0)$ above, the principal is less inclined to demand costly high effort and prioritize success in the first-period. These insights are illustrated in Figure 2.

Following the carrot-and-stick analogy from the benchmark case, in this scenario, the carrot $(e_{2D}^{SB}(1))$ remains fixed as p varies, while the stick $(e_{2D}^{SB}(0))$ falls as p increases. However, despite the fact that the stick is becoming more punishing while the carrot stays constant, the first-period effort, e_{1D}^{SB}, still decreases. This occurs because, even though the two periods are technologically independent, the change in preference (driven by p) depends on the first-period outcome, introducing some interdependence between the two periods. As a result, the effort induced in the first period, e_{1D}^{SB} influences the second-period expected payoff. If p were independent of the first-period outcome, the e_{1D}^{SB} would not have any impact on the second-period payoff, and the standard carrot-and-stick analogy would apply. In the benchmark case, e_{1D}^{SB} increases with $e_{2D}^{SB}(1)$ and decreases with $e_{2D}^{SB}(0)$. This relationship still holds in this new framework. However, winning in the first period now reduces the second-period expected payoff by $(1 - e_{2D}^{SB}(0))(p)$, which is the additional payoff available to the principal under the *Win from Losing* framework. Therefore, a higher e_{1D}^{SB} increases the chances of winning in the first period, which in turn reduces the likelihood of potentially obtaining the additional payoff in the second period. As p increases, the second-period expected payoff also increases and exerts a negative influence on e_{1D}^{SB}. This negative effect on the second-period expected payoff dominates the positive effect from the strengthening of the stick, leading to a reduction in the induced first-period effort.[30]

To further clarify the proposition, let us consider a case where $p = 1$. If the outcome in the first period is a loss, the principal becomes indifferent between winning and losing

[30] To illustrate this point, I refer to (A.33) in the Appendix. As p increases, the effect from the harsher stick is captured by $\left(2e_{2D}^{SB}(0) - 1\right)\frac{\partial e_{2D}^{SB}(0)}{\partial p}$ which is positive for the relevant range of $e_{2D}^{SB}(0)$. If this were the only effect in play, then e_{1D}^{SB} would increase as a result of the harsher stick. However, there is an additional effect stemming from the impact on the second-period expected payoff, given by $p\frac{\partial e_{2D}^{SB}(0)}{\partial p} + e_{2D}^{SB}(0) - 1 < 0$, which ultimately dominates. This negative effect on the second-period expected payoff offsets the influence of the harsher stick, leading to an overall decrease in e_{1D}^{SB} as p increases.

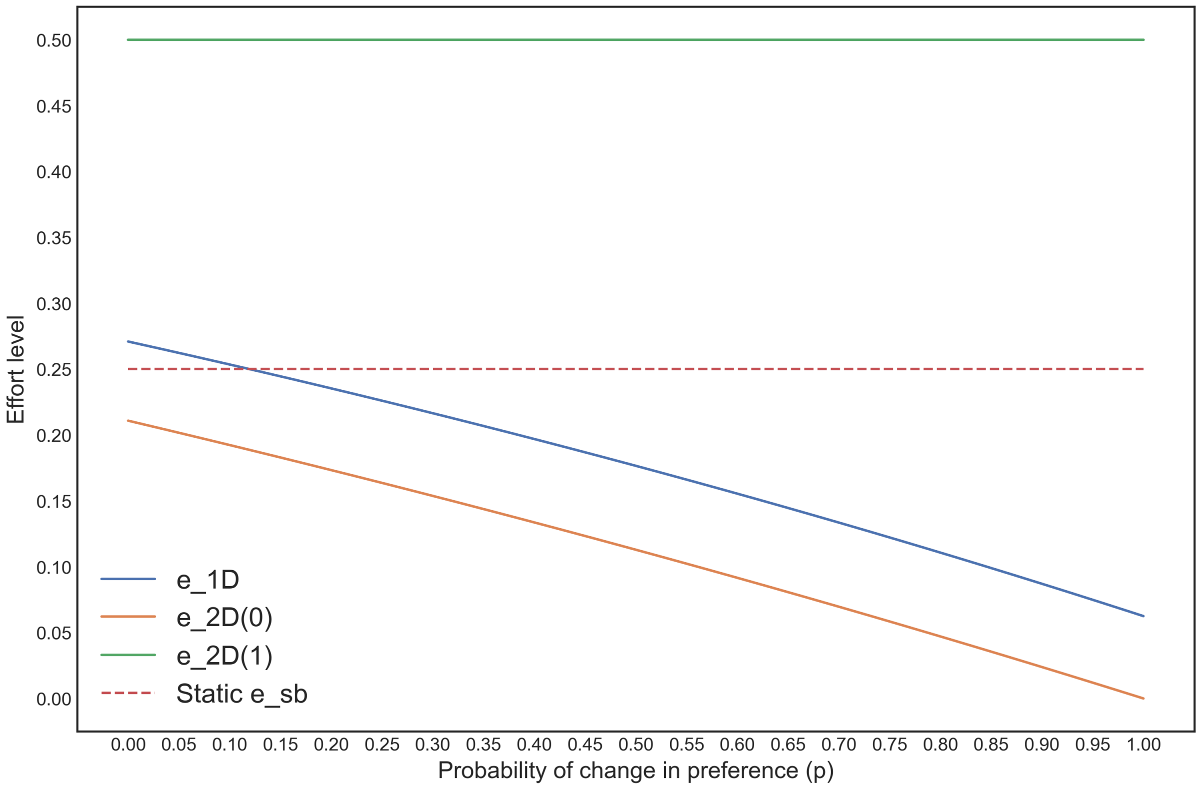

Figure 2: Principal's implemented effort levels as a function of p

The principal's implemented effort level in the second period after a first-period success (green line) is invariant with respect to the probability of change in preference. However, the effort level implemented in the second period after a first-period failure (orange line), as well as the effort level in the first period (blue line), both decrease as the probability of the change in preference increases.

in the second period, as she will receive the same payoff regardless of the outcome. In this case, inducing costly effort simply adds to the principal's costs without increasing her payoff. Consequently, the principal would offer no payment to the agent in the second period, leading the agent to exert no second period effort. In our NFL context, $p = 1$ can be interpreted as an NFL team being definitively out of playoff contention after a loss in the first period. In this scenario, the team would have no incentive to induce effort in the second period, knowing that whether they lose (and secure a better draft pick) or win (and receive the winning bonus), they are guaranteed a positive payoff. Furthermore, given the team is guaranteed a positive payoff after a first-period failure and only has a probabilistic chance of receiving a positive payoff following a first-period success, the team would actually prefer to lose in the first period. This strategic incentive to lose in order to guarantee a positive second-period payoff leads them to reduce effort in the first period.

The question then arises as to why, at $p = 1$, the principal induces any positive first-period effort at all. In this case, the principal could guarantee a payoff of 1 and save on effort costs in the second period by simply losing in the first period. So, why does the principal not set $e_{1D}^{SB} = 0$, save on effort costs in both periods, and guarantee a loss in the first period, thus ensuring a positive payoff in the second period. The reason lies in capturing the potential payoff from success in the first period. While losing in the first period guarantees a positive payoff in the second period, winning in the first period offers an immediate reward that the principal may still value. Even at $p = 1$, the principal may prefer to maintain some first-period effort to capture the payoff associated with a first-period success. I verify this by introducing a discount factor, $\delta \in (0, 1)$. The analysis shows that the induced first-period effort level increases as δ decreases. In other words, as the first period becomes more valuable relative to the second period, the principal's incentive to induce higher effort in the first period increases.[31] This highlights that the non-zero first-period effort is driven by the potential to earn the winning bonus in the first period, even when the team could otherwise guarantee a positive second-period outcome (whether through a better draft pick by losing, or a bonus by winning).

Proposition 4 illustrates the existence of both a 'vicious' and a 'virtuous' cycle. The intuition behind this is that weaker teams—those that are already lower in the rankings—tend to have higher values of p, reflecting the fact that they are more likely to be knocked out of playoff contention after a loss. At the beginning of the season, these are the teams predicted to have lower rankings by the end of the season. Teams with higher values of p induce less effort in the first period, which increases their chances of experiencing the change in preference, meaning they are more likely to be eliminated from playoff contention. The team with $p = 1$ – one that knows for sure it will be out of the playoffs race following a loss – induces the lowest first-period effort, making it the most likely to fail and fall out of playoff contention. This creates a vicious cycle: weaker teams exert less effort in the first period, increasing the likelihood of failure, which in turn raises the probability of experiencing the change in preference (that is, missing the playoffs). This also diminishes their incentives to induce effort during the second period. Conversely, better teams – those with lower p values – induce higher effort in the first period. These

[31] The details of this result is provided in the Appendix **A.4**.

teams are more likely to win and stay in playoff contention creating a virtuous cycle where success breeds more success. Stronger teams try harder, increasing their chances of winning, while weaker teams exert lower effort and are more likely to lose, reinforcing their lower ranking. In this way, the *Win from Losing* framework results in a self-reinforcing process where initial differences in team ranking become amplified during a season.

3.3.1 Comparison of Contracts

In this subsection, I compare the optimal long-term contract derived above, focusing on the agent's rent and principal's profit, with alternative contract structures. Specifically, I examine (i) one-period contracts with an interim individual rationality constraint,[32] and (ii) one-period contracts where the principal can observe the realized state (that is, whether the change in preference has occurred) in the second period before offering a second-period contract.

I first determine the effort levels induced by the principal in each period across the three different contract structures. As illustrated above, in the optimal long-term contract, the principal induces e_{1D}^{SB} in the first period. In the second period, following a first-period failure, the principal induces $e_{2D}^{SB}(0)$, while after a first-period success, the principal induces $e_{2D}^{SB}(1)$. For contract structure (i) – one-period contracts with an interim individual rationality constraint – the principal induces e_S^{SB} in the first period. If there is a first-period success, there is no possibility of a change in preference. In this case, the principal faces the same problem in the second period and induces e_S^{SB}. After a first-period failure, the principal now faces a modified maximization problem due to the potential change in preference. Here, the principal induces $e_{Sp}^{SB} = \frac{1-p}{4}$, which is the static second-best effort of this new problem, in the second period. In contract structure (ii), the principal induces e_S^{SB} in the first period. In the second period, the principal can observe whether a change in preference has occurred before offering the second-period contract. If no change in preference occurs, the principal induces the static second-best effort, e_S^{SB}. However, if a change in preference occurs, the principal induces zero effort because she receives the same payoff regardless of the outcome, and inducing effort is costly. Since the change in preference is only possible after a first-period failure, in the second period, the

[32] This is simply the static second-best contract repeated in each period.

principal induces zero effort with probability $(1-e_S^{SB})p$ and induces e_S^{SB} with probability $1-\left(1-e_S^{SB}\right)p$.

Rent to the Agent

To compare the rent to the agent across the three different contract structures, I calculate the expected rent in each case:

(i) One-period contracts with an interim individual rationality constraint:

$$\text{Expected Rent} = A(e_S^{SB}) + e_S^{SB}A(e_S^{SB}) + \left(1-e_S^{SB}\right)A(e_{Sp}^{SB})$$

The principal induces e_S^{SB} in the second period following a first-period success, which occurs with probability e_S^{SB}. After a first-period failure, which happens with probability $(1-e_S^{SB})$,The principal induces e_{Sp}^{SB}, which is the static second-best effort of the new static problem under the change in preference.

(ii) One-period contracts where the principal can observe the realized state:

$$\text{Expected Rent} = A(e_S^{SB}) + \left(1-p\left(1-e_S^{SB}\right)\right)A(e_S^{SB})$$

In this case, the second-period rent depends on whether a change in preference occurs. With probability $p(1-e_S^{SB})$, the principal experiences a change in preference, inducing zero effort, and the agent earns no rent in the second period.

(iii) Optimal long-term contract:

$$\text{Expected Rent} = A(e_{1D}^{SB}) + A(e_{2D}^{SB}(0))$$

Here, the agent receives an additional rent of $A(e_S^{FB})$ in the second period following a first-period success, but this rent is effectively extracted from the agent in the first period.

Figure 3 compares the agent's rent across three different contract structures as the probability of a change in preference, p increases. Across all these contracts, the agent's rent decreases as p increases, but the decline is the steepest under the optimal long-term contract. This implies that the agent's rent is significantly reduced under this contract as the probability of a change in preference rises. Conversely, the agent retains more rent as p increases under contract structure (ii), where the principal can observe the realized state, as the decline is more gradual. The principal's ability to observe the state seems to benefit the agent compared to the other two contracts when there is the possibility of a change in preference.

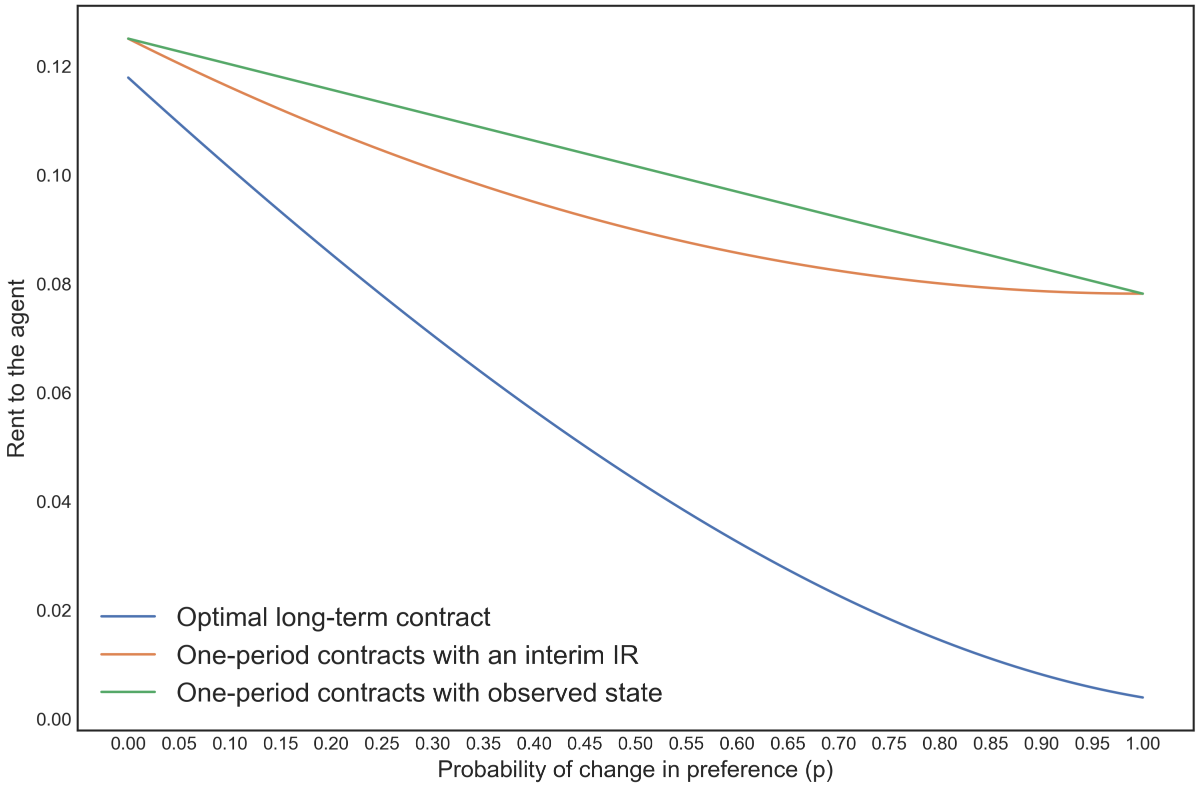

Figure 3

The rent to the agent decreases as p increases for all contracts. The effect on the agent's rent is the largest under the optimal long-term contract (blue curve). Consequently, the agent receives the least rent under the optimal long-term contract and the most rent under the contract where the principal can observe the realized state (green line), regardless of the value of p.

For all values of p, the agent earns the least rent under the optimal long-term contract, reflecting the principal's ability to extract more rent in this scenario. The agent prefers the contract structure (ii) since it provides the highest rent across all values of p. Interestingly, even though the first-period effort level is higher in the optimal long-term contract compared to the other contracts, the agent still receives less rent. This is because the second-period effort, following a first-period failure, is substantially lower than the expected second-period effort levels in the other two contracts. This negative difference in second-period effort dominates the positive difference in the first-period effort levels. Since the agent's rent in the long-term contract depends on the second-period effort level induced after a first-period failure, the agent ends up worse off in this contract.

Profit for the Principal

To compare the principal's expected profit across the three different contracts, I first calculate the expected profit in each case:

(i) One-period contracts with an interim individual rationality constraint:

$$\begin{aligned}\text{Expected Profit} = e_S^{SB}\left[1 - c'\left(e_S^{SB}\right) + e_S^{SB}\left(1 - c'\left(e_S^{SB}\right)\right)\right] + \\ \left(1 - e_S^{SB}\right)\left[e_{Sp}^{SB}\left(1 - c'\left(e_{Sp}^{SB}\right)\right) + p\left(1 - e_{Sp}^{SB}\right)\right]\end{aligned}$$

(ii) One-period contracts where the principal can observe the realized state:

$$\begin{aligned}\text{Expected Profit} = e_S^{SB}\left[1 - c'\left(e_S^{SB}\right) + e_S^{SB}\left(1 - c'\left(e_S^{SB}\right)\right)\right] + \\ \left(1 - e_S^{SB}\right)\left[(1-p)\left(e_S^{SB}\left(1 - c'\left(e_S^{SB}\right)\right)\right) + p\right]\end{aligned}$$

(iii) Optimal long-term contract:

$$\begin{aligned}\text{Expected Profit} = e_{1D}^{SB}\left[1 - t_1(1) + e_{2D}^{SB}(1)\left(1 - t_2(1,1)\right)\right] + \\ \left(1 - e_{1D}^{SB}\right)\left[e_{2D}^{SB}(0)\left(1 - t_2(0,1)\right) + p\left(1 - e_{2D}^{SB}(0)\right)\right]\end{aligned}$$

where $t_1(1) = c'(e_{1D}^{SB}) - A(e_{2D}^{SB}(1)) + A(e_{2D}^{SB}(0))$, $t_2(1,1) = c'\left(e_{2D}^{SB}(1)\right)$ and $t_2(0,1) = c'\left(e_{2D}^{SB}(0)\right)$.

Figure 4 illustrates how the principal's profit varies with different contract structures as p increases. The principal's profit rises with p across all contract types, which is expected because a higher p increases the probability of guaranteeing a payoff of 1 in the second period, even in the event of a failure. In other words, as p increases, the expected payoff in the second period grows, resulting in an overall increase in profit for all contract structures.

The principal prefers the optimal long-term contract over the one-period contracts with an interim individual rationality constraint. This follows from revealed preference – although the repeated static second-best contract is available in the principal's long-term optimization problem, the principal chooses the optimal long-term contract. Surprisingly, this long-term contract yields higher profit than the one-period contracts where the principal can observe the state. The knowledge of the state and the ability to tailor the second-period contract based on the state does not benefit the principal in terms of higher profit. In fact, the principal would prefer not to know the state and to offer long-term contracts. The rationale is that the change in preference, and hence the expected second-period profit, depends on the outcome of the first period. As p increases, the expected second-period payoff rises, but this increase occurs only after a failure in the first period. With a long-term contract, the principal can strategically reduce first-period effort as p increases, thereby increasing the chance of realizing this higher expected second-period payoff and improving overall expected profit.

To further illustrate, consider the case where $p = 1$, meaning the principal will certainly experience a change in preference after a failure in the first period. In this scenario, the principal can secure a guaranteed payoff of 1 in the second period if she loses in the first period. Under a contract where the principal observes the state before offering the second-period contract, the principal will implement the static second-best effort in the first period and then induce zero effort in the second period. In this case, the principal does not use the first period to enhance the chances of guaranteeing a second-period payoff of 1. In contrast, under the optimal long-term contract, the principal induces zero effort in the second period but strategically lowers first-period effort to increase the probability of a first-period failure, thereby improving the chance of obtaining the guaranteed payoff of 1 in the second period. The ability to influence the probability of achieving this guaranteed payoff with the long-term contract explains why it is more profitable and preferred by the principal.

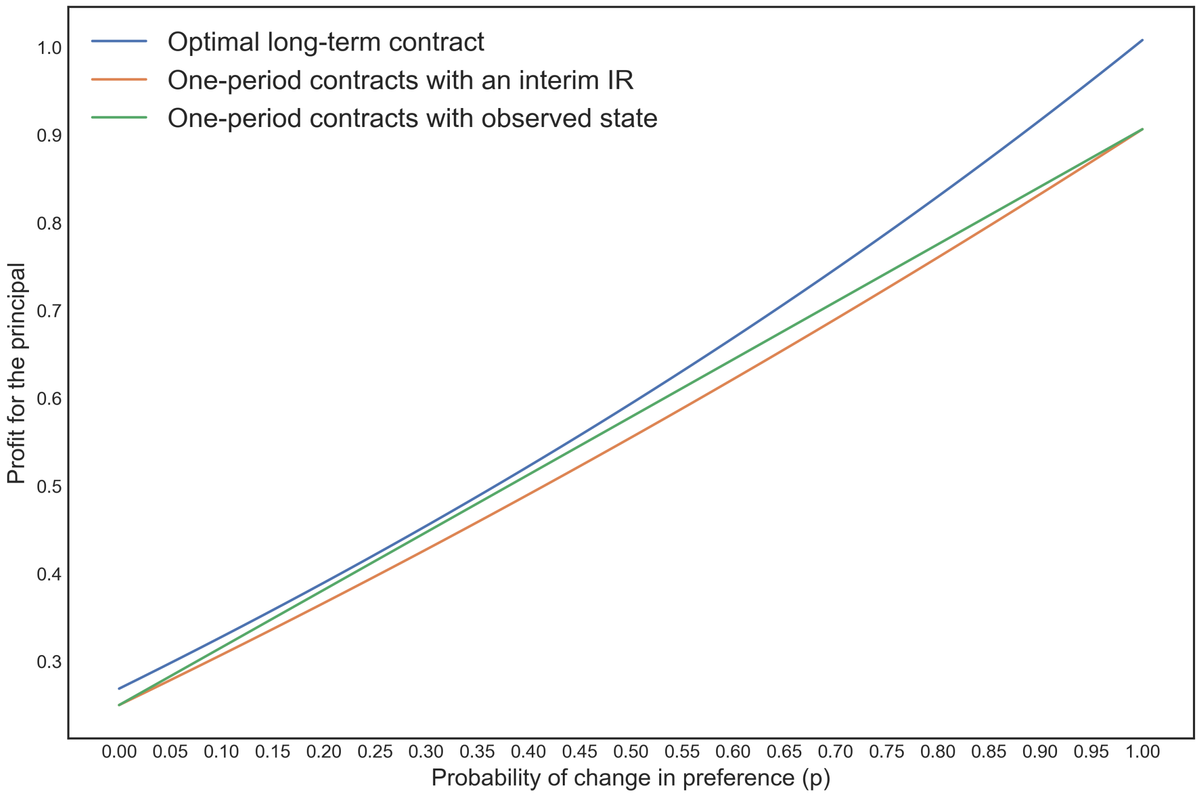

Figure 4

The principal's profit increases with p for all contract types. The principal's profit is the highest under the optimal long-term contract (blue curve) for all values of p.

4. Extensions

In this section, I will explore variations in the principal's change of preference. First, I explore the case of a dependent change in preference, where the payoff from success decreases with an exogenous probability p following a first-period failure, which I refer to as the '*Lose from Winning*' framework. Next, I analyze a case where the change in preference is independent of the first-period outcome, occurring with equal probability after both success and failure, and demonstrate the equivalence between '*Win from Losing*' and '*Lose from Winning*' in this context. I will also consider a scenario where a change in preference can occur after both a first-period success and failure but with different probabilities. Finally, I explore a situation where the principal prefers losing to winning, rather than being indifferent, by reversing the payoffs for success and failure. For the upcoming analysis, I will use numerical results rather than formal proofs, as the numerical analysis is sufficient for presenting and discussing the findings.[33]

4.1 *Lose from Winning*

In this framework, the principal may experience a change in preference with probability p, but only after a first-period failure ($y_1 = 0$). This change in preference reduces the payoff from success (from 1 to 0) while keeping the payoff from failure fixed at 0. If the change in preference occurs, the principal becomes indifferent between winning and losing in the second period, as the payoff is zero in both outcomes. In this scenario, the principal earns a positive payoff in the second period if a second-period success follows a first-period success. Additionally, the principal can still earn a positive payoff if a second-period success follows a first-period failure, but only if the change in preference does not occur. The probability of this latter outcome is given by $e_{2D}^{SB}(0)(1-p)$, where $e_{2D}^{SB}(0)$ is the probability of success in the second-period following a first-period failure, and $(1-p)$ is the probability that the principal does not experience the change in preference. The principal's objective is to maximize their overall expected payoff,

$$e_1 \left[1 - t_1(1) + e_2(1)\left(1 - t_2(1,1)\right)\right] + (1 - e_1)\left[-t_1(0) + e_2(0)\left(1 - p - t_2(0,1)\right)\right]$$

subject to (IC_1), (IC_2), (LL_1), and (11).

Figure 5 depicts how effort levels in both periods vary with p. The second-period effort level following a first-period success, $e_{2D}^{SB}(1)$, remains constant at the static second-best level (e_S^{SB}) for all values of p. This is because there is no change in preference after a first-period success, keeping the principal's preferences unchanged from the benchmark case. In contrast, the second-period effort level following a first-period failure, $e_{2D}^{SB}(0)$ decreases as p increases. As p increases, the expected payoff from winning in the second period (after a first-period failure), $e_{2D}^{SB}(0)\,(1-p)$, approaches zero, which is also the payoff from losing in the second period. As the payoff difference between winning and losing narrows, the principal becomes more indifferent between the two outcomes. As a result, the principal's incentive to induce higher effort decreases, since inducing effort is costly and the potential gain from a higher payoff diminishes.

[33] The optimization problems along with the code for the numerical analyses is provided in Appendix **B**.

Figure 5 shows that e_{1D}^{SB} increases with p. When the principal wins in the first period ($y_1 = 1$), her expected second-period payoff is $e_{2D}^{SB}(1)$. However, after a first-period loss ($y_1 = 0$), the expected second-period payoff becomes $e_{2D}^{SB}(0)(1 - p)$. As p increases, this expected payoff following a loss decreases, while the expected payoff after a success remains fixed. This provides incentive for the principal to avoid the lower payoff after a loss and to seek the higher expected second-period payoff that follows a first-period success. To increase the chances of obtaining this higher payoff, the principal induces a higher first-period effort to induce a success. In other words, the principal wants to ensure success in the first-period and avoid the risk of experiencing a change in preference that results in a guaranteed payoff of 0 in the second period.

To clarify, consider the case where $p = 1$. In this scenario, if the principal loses in the first period ($y_1 = 0$), she will receive a payoff of zero in the second period with certainty. On the other hand, if $y_1 = 1$, the principal has a probability of $e_{2D}^{SB}(1)$ of earning a payoff of 1 in the second period. Thus, the principal aims to avoid the scenario where she is guaranteed to earn zero in the second period, explaining why she induces a higher first-period effort compared to the benchmark case.

Recalling the carrot-and-stick analogy from the benchmark case, we observe that in this framework, the carrot ($e_{2D}^{SB}(1)$) remains fixed as p varies, while the stick ($e_{2D}^{SB}(0)$) falls as p increases. This mirrors the *Win from Losing* framework; however, in this case, e_{1D}^{SB} increases with p. The difference lies in the how the effect on the expected second-period payoff interacts with the stick effect. In this framework, the effect on the second-period expected payoff reinforces the stick effect, with both increasing the first-period effort level. Like in *Win from Losing*, e_{1D}^{SB}, impacts the expected second-period payoff. However, unlike in *Win from Losing*, the first-period effort positively influences the expected second-period payoff. A higher first-period effort reduces the likelihood of the change in preference and, therefore, the risk of receiving the guaranteed zero payoff in the second period. In *Win from Losing*, an additional expected payoff of p follows a first-period failure. This payoff opposes (and dominates) the stick effect, ultimately reducing first-period effort as p increases. This opposing mechanism is absent in the current scenario, which explains why e_{1D}^{SB} rises with p.[34]

[34] Under the *Win from Losing* framework, the expected second-period payoff following a first period

This framework represents a scenario where a team's performance does not influence their draft pick position. Once a team is out of playoff contention, they neither receive a bonus for winning nor any advantage from losing in terms of securing a better draft pick. In this case, once out of the playoffs, the team gains no benefit from either result in the second-period. For instance, consider a team that knows it will be eliminated from playoff contention after a first-period loss. In this situation, the team is incentivized to exert higher effort and win in the first period, as they cannot earn a positive payoff in the second period after a first-period loss. Therefore, this framework encourages higher effort in the first-period, as teams aim to keep their playoff chances alive and maintain the possibility of a positive return in the second period.

This framework has significant implications for NFL draft policy, especially if the goal is to promote more competition. A draft rule that is independent of a team's performance would reduce the incentive for teams to tank early in the season in hopes of securing a better draft position. Instead, teams would be more likely to put in higher effort to remain in playoff contention. Moreover, this framework eliminates the 'vicious' and 'virtuous' cycles present in the current draft rule system as illustrated in Proposition 4. Under *Lose from Winning*, lower-ranked teams (those with a higher probability p of missing the playoffs) try harder and induce a higher first-period effort, fostering greater competition.

4.2 Change in Preference Independent of First-period Outcome

In this framework, the principal faces a probability p of experiencing a change in preference in the second period, independent of the first-period outcome. Thus, the principal could encounter a change in preference following either a first-period success ($y_1 = 1$) or a first-period failure ($y_1 = 0$). Applied to the NFL context, this framework reflects a scenario where a team's playoff contention is influenced not only by their own performance but also by the performance of other teams. For example, a team could still be eliminated from playoff contention after a win if their rivals also win.

In this independent setting, under the *Win from Losing* framework, the principal's

failure is given by $e_{2D}^{SB}(0)(1-p)+p$. In contrast, in the *Lose from Winning* framework, the expected second-period payoff following a first-period failure is simply $e_{2D}^{SB}(0)(1-p)$.

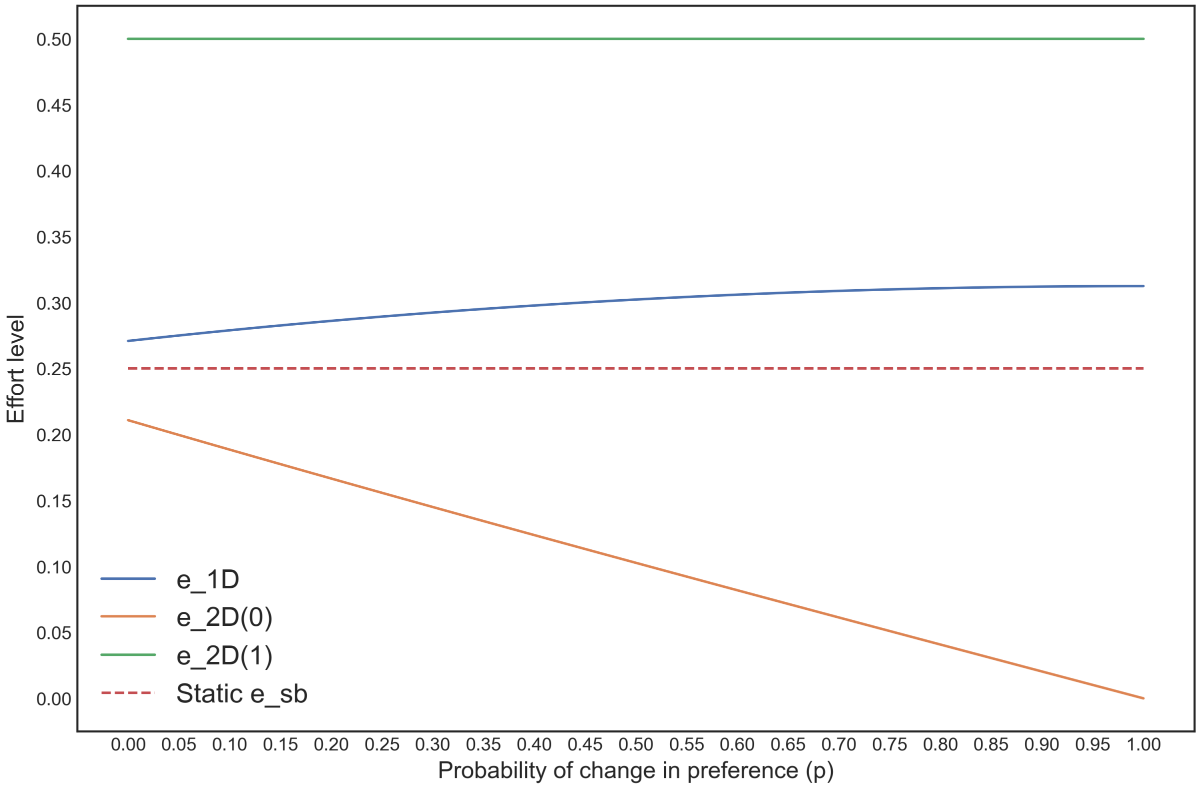

Figure 5

The principal's implemented effort level in the second period after a first-period success (green line) is invariant with respect to the probability of change in preference. However, the effort level implemented in the second period after a first-period failure (orange line), as well as the effort level in the first period (blue curve), both decrease as the probability of the change in preference increases.

expected second-period payoff after a first-period outcome, $y_1 \in \{0, 1\}$, is:

$$E\pi_2^{WFL}(y_1) = e_2(y_1)\,(1 - p - t_2(y_1, 1)) + p \tag{15}$$

In this case, since the principal receives an additional fixed payoff of p after both a first-period success and failure, this benefit does not play a role in determining the optimal effort levels. As a result, the principal's optimization problem in the *Lose from Winning* framework is effectively the same, where this additional fixed benefit p is absent and the principal's expected second-period payoff for any first-period output, $y_1 \in \{0, 1\}$, is given by:

$$E\pi_2^{LFW}(y_1) = e_2(y_1)(1 - p - t_2(y_1, 1)) \tag{16}$$

Therefore, the two frameworks are equivalent when the change in preference can occur independently of the first-period outcome. In this scenario, the principal's problem can be simplified to maximizing

$$e_1 \left[1 - t_1(1) + e_2(1)\left(1 - p - t_2(1,1)\right)\right] + (1 - e_1)\left[-t_1(0) + e_2(0)\left(1 - p - t_2(0,1)\right)\right]$$

subject to (IC_1), (IC_2), (LL_1), and (11).

Figure 6 illustrates the optimal effort levels as a function of p. The second-period effort level following a first-period failure, $e^{SB}_{2D}(0)$, decreases as p increases. This is because, as p rises, the difference between the returns from winning and losing in the second period diminishes. Consequently, the principal induces lower effort in the second period as the potential gain from winning shrinks and the cost of inducing effort remains. Similarly, the second-period effort level following a first-period success, $e^{SB}_{2D}(1)$, also decreases with p. In the independent setting, the change in preference can occur even after a first-period success. Therefore, the decrease in the second-period effort level following a first-period success is due to the same reason as for after a first-period failure: the diminishing difference in returns between winning and losing in the second period. Thus, the principal's incentive to induce high effort diminishes with p after both first-period outcomes.

The induced effort level in the first-period, e^{SB}_{1D}, also decreases with p. The decline in effort is driven by the carrot-and-stick effect. As p increases, the carrot ($e^{SB}_{2D}(1)$) decreases faster than the stick ($e^{SB}_{2D}(0)$). Since, the reward diminishes more rapidly than the punishment intensifies, the overall incentive for first-period effort is reduced. The expected return in the second period is the same regardless of whether the first-period is a success or a failure, because the change in preference is independent of first-period outcome. As a result, the first-period effort does not influence the principal's expected second-period return. Thus, the only factor influencing first-period effort is the carrot-and-stick effect. Even though the principal's preferences for the first period do not change, the potential change in preference in the second period affects impacts the choice of first-period effort.

The induced effort level in the first-period is bounded below by the static second-best level of effort, e^{SB}_{S}. When $p = 1$, the principal faces a guaranteed change in preference in the second period. This implies that regardless of the first-period outcome, the second-period returns from success and failure are identical, making the principal indifferent between the two outcomes in the second period. Therefore, the principal will not compensate the agent in the second period and induce zero effort, regardless of the first-period outcome. Consequently, the second period can no longer be used as a tool to provide incentives for first-period effort, as there is no longer any carrot or stick effect from the second period. The principal is forced to rely solely on the first period to provide incentives for the first-period effort. This simplifies the the problem of finding the optimal first-period effort as a static problem, leading the principal to induce the static second-best effort level. The first-period effort has no impact on the second-period expected return, leaving the principal with no incentive to adjust first-period effort levels beyond what is optimal in a static scenario.

In the independent setting, the results for first-period effort is consistent with *Win from Losing*. The 'vicious' and 'virtuous' cycles still occur, with lower-ranked teams exerting less effort than higher-ranked teams. However, due to the independent nature of this framework, the first-period effort is never lower than the static second-best level, limiting the extent of the cycles. Thus, a team will never exert less effort than they would in a one-shot static game.

4.3 Varying p with First-period Outcome

In this extension, I explore the *Lose from Winning* framework while allowing for the possibility of a change in preference even after a first-period success. However, the probability of the change in preference can vary based on the first-period outcome. Let p_1 denote the probability of change in preference in the second period after a first-period success ($y_1 = 1$) and p_0 denote the probability of change in preference in the second period after a first-period failure ($y_1 = 0$). Further, I assume that $p_1 < p_0$, meaning the change in preference is more likely after a failure in the first period. This assumption can be motivated through the NFL example, where a team is more likely to be out of the playoff race after a loss, even if other results also play a role.

In this scenario, the principal's problem is to maximize the following

$$e_1 \left[1 - t_1(1) + e_2(1)(1 - p_1 - t_2(1,1))\right] + (1 - e_1)\left[-t_1(0) + e_2(0)(1 - p_0 - t_2(0,1))\right]$$

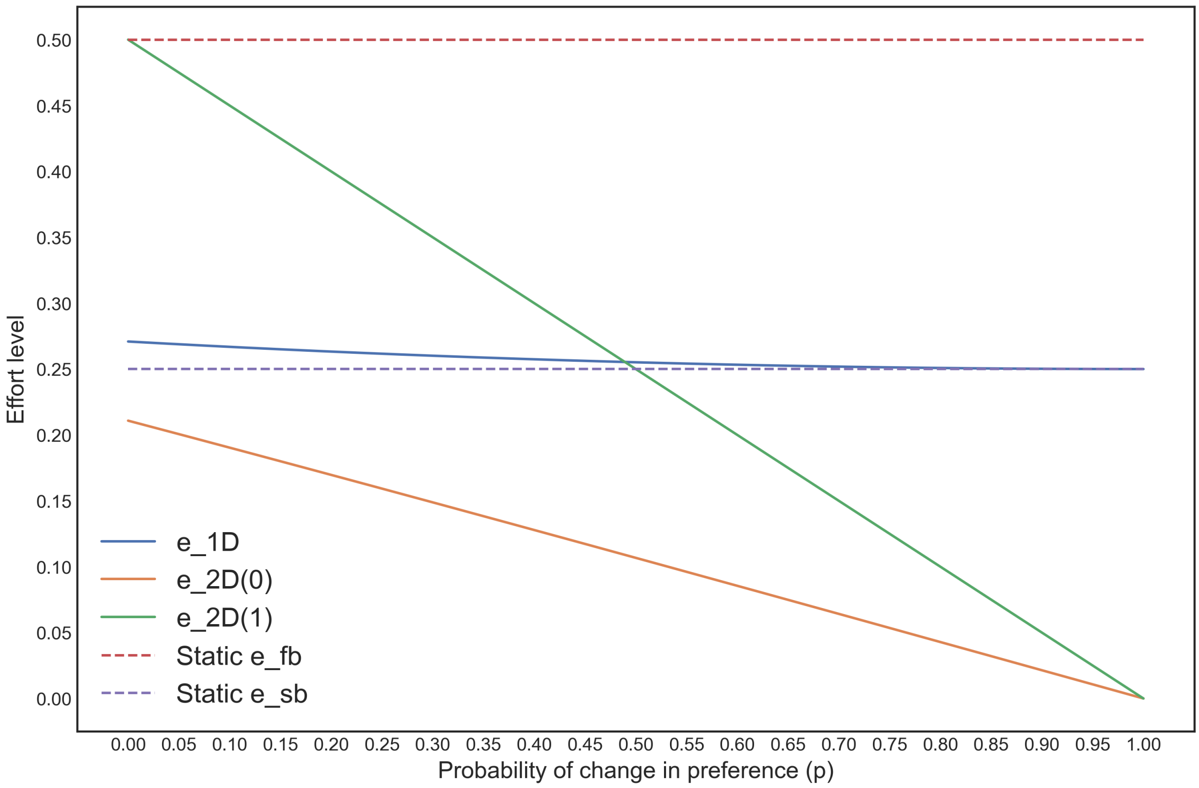

Figure 6

The principal's implemented effort levels in the second period, following both a first-period success (green line) and a first-period failure (orange line), decreases as the probability of change in preference, p, increases. When $p = 1$, the principal induces zero effort in the second period regardless of the first period outcome. The induced first-period effort level (blue curve) also decreases with p, and at $p = 1$, the principal simply induces the static second-best effort level in the first-period.

subject to the usual constraints. The goal is to verify whether the positive effect on first-period effort observed in the *Lose from Winning* framework holds in this more general case as well. Figure 7 illustrates how effort levels vary with the probabilities p_0 and p_1. To interpret the results, I analyze how the induced first-period effort level, e_{1D}^{SB}, changes as one of these probabilities varies while the other is held constant.[35]

When the probability of a change in preference after a first-period failure, p_0, is held constant, e_{1D}^{SB} decreases as the probability of change in preference after a first-period success, p_1, increases. As p_1 increases while p_0 remains constant, $e_{2D}^{SB}(0)$ stays fixed while $e_{2D}^{SB}(1)$ decreases. This occurs because a higher p_1 reduces the difference in payoffs between winning and losing in the second period, but only after a first-period success, making the

[35] An analysis of the *Win from Losing* framework under this setting is provided in Appendix **B**. The finding that e_{1D}^{SB} decreases with the probability of change in preference after a first-period failure, p_0, remains valid when the probability of a change in preference after a first-period success, p_1, is held constant.

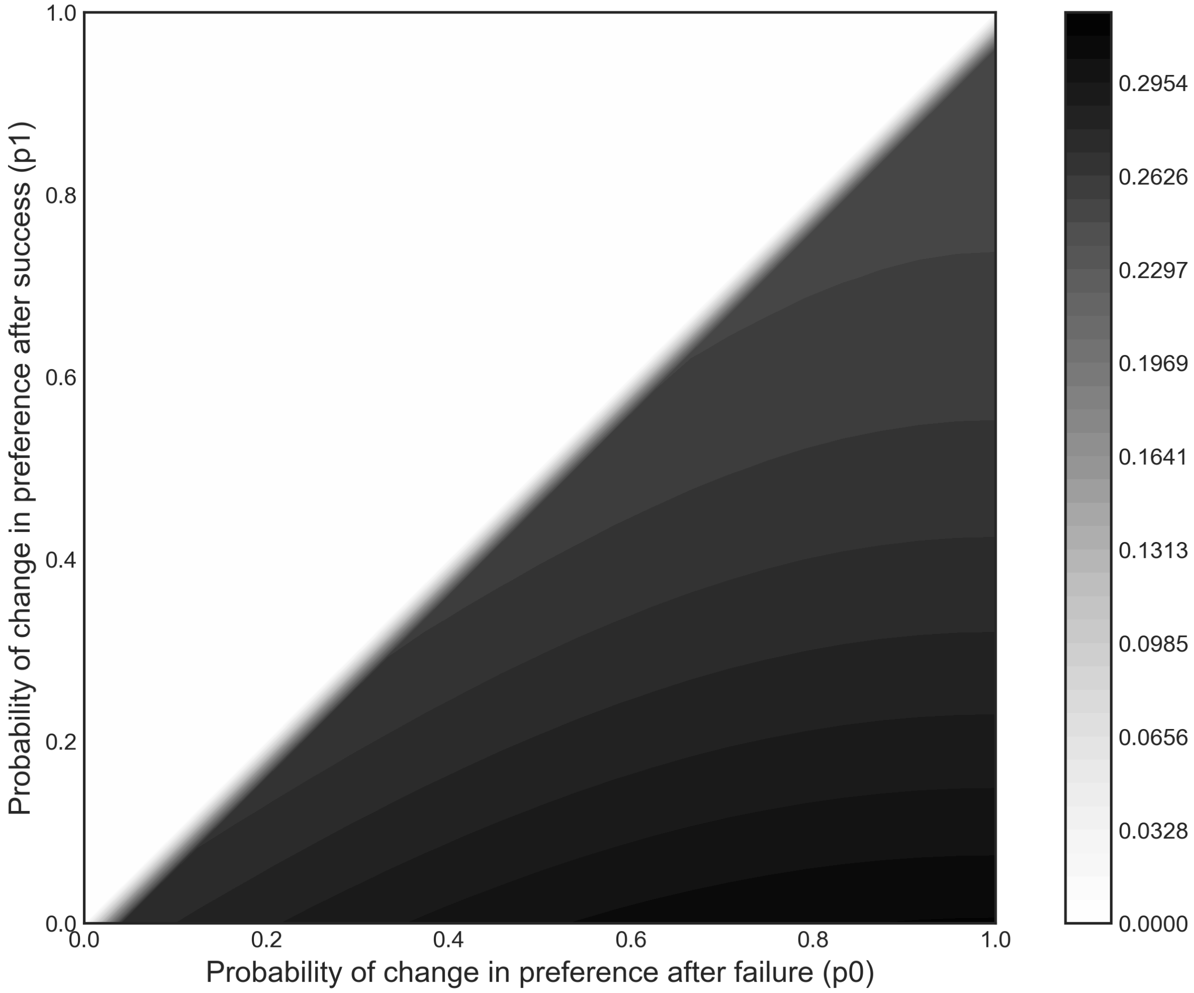

Figure 7

Holding p_0 constant, the induced first-period effort level, e_{1D}^{SB}, decreases as p_1 increases. Conversely, when p_1 is held fixed, e_{1D}^{SB} increases as p_0 increases.

principal more indifferent between the two outcomes and leading to lower second-period effort. In the *Lose from Winning* framework, the effect of the carrot-and-stick on e_{1D}^{SB} aligns with the impact of the expected second-period payoff. In this setting, the expected second-period payoff after a first-period success decreases, while the expected second-period payoff after a first-period failure remains constant. This creates a negative effect on e_{1D}^{SB} because higher first-period effort increases the chance of obtaining the decreasing expected second-period payoff; compounding the negative effect from the diminishing size of the carrot.

Conversely, when p_1 is held constant, e_{1D}^{SB} increases as p_0 increases. In this case, as p_0 rises, $e_{2D}^{SB}(1)$ remains constant, while $e_{2D}^{SB}(0)$ decreases as the principal becomes more indifferent between winning and losing after a first-period failure. Here, the stick becomes harsher, inducing a positive effect on e_{1D}^{SB}. As the expected second-period payoff after a first-period failure decreases with p_0, while the expected second-period payoff after a first-period success stays constant, this also positively affects e_{1D}^{SB} because a higher first-period effort reduces the chance of ending up in the scenario with a diminishing expected second-period payoff.

Therefore, the results from *Lose from Wining* also hold in this extension. This extension can be viewed as a more general version of the different settings considered in this paper. Specifically, the scenario where the change in preference is independent of the first-period outcome is a special case of this extension with $p_0 = p_1$, and the *Lose from Winning* framework is simply a special case with $p_1 = 0$.[36]

4.4 Reversed Payoffs for Success And Failure

In this extension, I examine whether the 'vicious' and 'virtuous' cycles demonstrated in *Win from Losing* are exacerbated when the principal not only becomes indifferent but actually prefers losing in the second period. I focus on the case where the change in preference depends on the first-period outcome and occurs only after a first-period failure. Suppose that after the change in preference, the payoffs are reversed: success in the second period yields 0, and failure yields 1.

Figure 8 depicts how effort levels vary with the probability of a change in preference, and how the induced first-period effort level compares to *Win from Losing* framework. The second-period effort levels, both following a first-period success and failure, behave similarly to those in *Win from Losing*. The first-period effort level, e_{1D}^{SB}, also follows a similar trend, decreasing as p increases. However, for every p, the new e_{1D}^{SB} is higher than in *Win from Losing*. The key reason is that in the second period, the principal already induces zero effort when indifferent between the two outcomes, so the second-period effort remains zero when they prefer losing. However, in this case, a change in preference no longer guarantees a payoff of 1 in the second period, reducing the incentive to shirk in the first period.

To better understand this, consider the different effects on e_{1D}^{SB}. Using the carrot-and-

[36] The same is true the for *Win from Losing* framework.

stick analogy, as p increases, the stick becomes harsher while the carrot stays fixed. If this were the only factor, e_{1D}^{SB} would increase. However, winning in the first period now reduces the second period expected payoff by $(1 - 2e_2(0))\, p$, which is a smaller reduction compared to $(1 - e_2(0))\, p$ in *Win from Losing*. This opposing effect on e_{1D}^{SB}, while still dominant, is weaker compared to the *Win from Losing* framework. As a result, although the overall effect on e_{1D}^{SB} remains negative, the stronger stick effect combined with the weaker negative impact from the expected second-period payoff leads to a smaller decrease in e_{1D}^{SB}.

This extension demonstrates that the negative effect on the induced first-period effort is now weaker. Interestingly, this outcome is somewhat counter intuitive, as the change in preference is stronger in this case. In the context of our NFL application, this envisions a situation where a team, once out of playoff contention, no longer receives a bonus for winning but can still benefit from securing a better draft pick by losing. Since a team can no longer earn a positive payoff from winning after being eliminated from playoff contention, they exert more effort compared to the *Win from Losing* framework, where a positive payoff is guaranteed after being knocked out of playoff contention.

5. Conclusion

In this paper, I extended the literature on dynamic moral hazard problem with a wealth-constrained, risk-neutral agent by exploring how moral hazard interacts with the potential for changes in the principal's preferences over time. A key novel contribution to literature is the introduction of a possible change in preference in the second period, which occurs with some exogenous probability. This change in preference is modeled as indifference between the two outcomes, success and failure. I examined how this potential change in preference affects the first-period effort level, even though the change in preference can only occur in the second period, and the two periods are technologically independent. The findings highlight the added complexity the principal faces when strategically using second-period incentives to induce first-period effort, given the potential for preference changes.

I used NFL as the main application, where the change in preference reflects a scenario in which a team is no longer in the playoff race. The main model for the paper reflects the

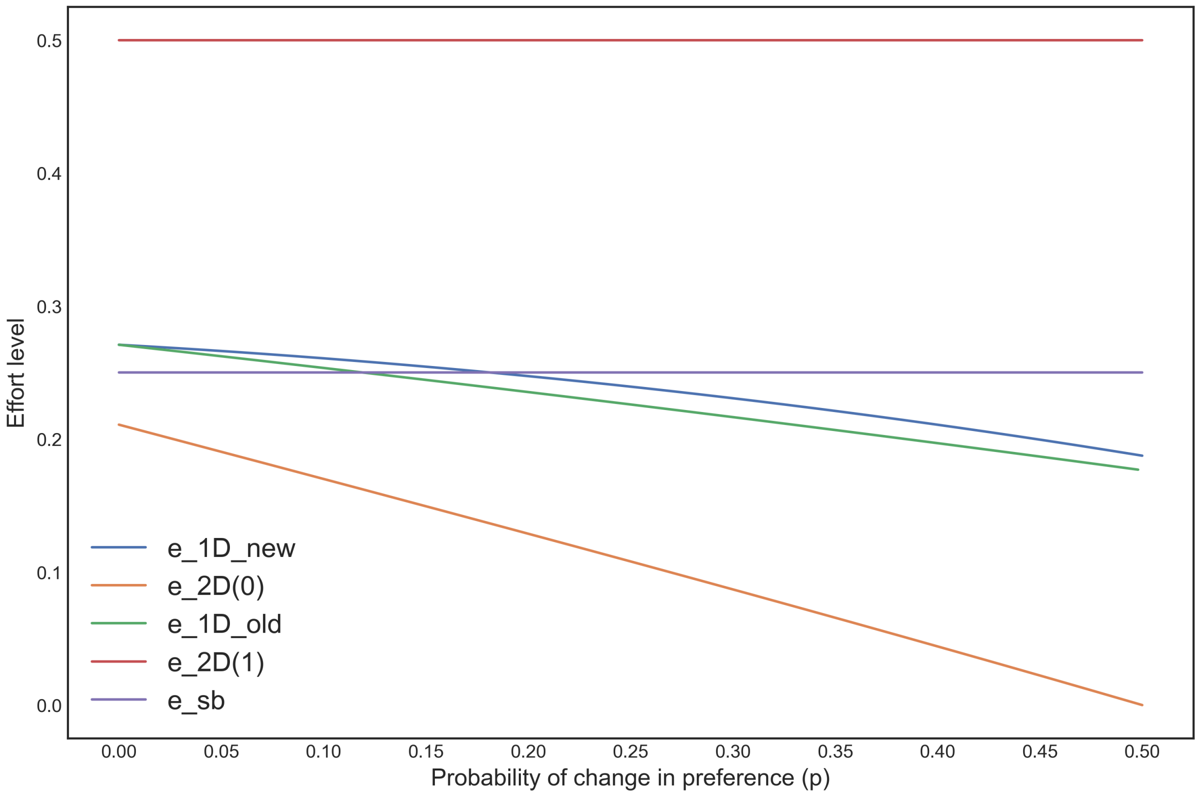

Figure 8

The principal's implemented effort level in the second period after a first-period success (red line) is constant with respect to the probability of change in preference. However, the effort level implemented in the second period after a first-period failure (orange line), as well as the effort level in the first period (blue line), both decrease as the probability of the change in preference increases. Notably, for every value of p, the first-period effort level in this extension is higher than the corresponding effort level in the *Win from Losing* framework (green line).

current NFL system, where once a team is out of playoff contention, winning guarantees a bonus, and losing improves their draft position. The main finding of the paper is that an ex-ante higher probability of a change in preference reduces first-period effort. This leads to weaker teams exerting less effort and increasing their chances of failure, while stronger teams exert more effort, enhancing their likelihood of success. This creates 'vicious' and 'virtuous' cycles, which, from the perspective of the league - concerned with competitive balance - are undesirable. The analysis suggests that the current draft system encourages 'tanking' behavior not only after a team is eliminated from playoff contention but also earlier in the season, as teams that expect to miss the playoffs after a loss may prefer to lose early on to secure the positive outcome guaranteed by missing the playoffs.

While this paper does not directly model the effects of a draft lottery system, it suggests that even a lottery system where the probability of securing a higher draft pick is linked to season performance may not entirely eliminate these cycles. However, it could mitigate their severity by reducing the guaranteed positive expected payoff for teams out of playoff contention. Furthermore, I demonstrate that a draft rule unrelated to performance (and with no winning bonus after being eliminated from the playoff race) would encourage teams to try harder from the beginning, as they would to avoid a guaranteed non-positive outcome.

Future research could focus on endogenizing the probability of a change in preference. An important extension would involve exploring a contest design framework that endogenizes this probability, considering a regulator's problem of maximizing effort across both periods. This paper can serve as building block for that purpose and for other future work. The model presented in this paper provides a general framework that can be adapted to accommodate various scenarios involving change in preference. For instance, one might easily generalize the model by allowing different cost functions or returns across the periods, depending on the application. A more generalized version might allow the the principal to receive a payoff of α for success and β for failure after a change in preference. This paper specifically examined cases where $\alpha = \beta \in \{0, 1\}$ and found that the trend in the first-period effort level shifts as $\alpha = \beta = 1$ moves from 0 to 1. There may be a critical payoff value of $\alpha = \beta$ where the first-period effort begins increasing with the probability of change in preference. This general framework, though beyond the scope of this paper, could provide valuable insights into draft rule design, such as optimizing draft lottery probabilities. Future research could also consider more complex cases where α and β are not equal.

Although this paper focused on the NFL and its draft rule, the framework can be adapted to other applications, as mentioned in the introduction. The concept of changing preferences could apply to bankruptcy, where a heavily indebted firm might prefer to take on additional debt to declare bankruptcy and avoid repaying creditors. Another application could involve firms avoiding regulation by strategically limiting growth, akin to 'losing' in the first period to avoid becoming too large and facing regulatory scrutiny. Similarly, the model could be used to analyze research and development (R&D) in patent races, where a firm will reduce R&D efforts after losing a race, affecting initial investment decisions.

In conclusion, this paper offers important policy implications for inducing effort in dynamic moral hazard settings where the principal's preferences may shift over time. It also provides a foundation for exploring other applications where changes in the principal's preferences can play a critical role.

Appendix

A.1

Proof of Proposition 2. In order to prove the result, we will first solve the one-period problem of finding the optimal continuation contract that leaves the agent with a certain payoff. The result is then used to find the optimal continuation payoff in the two-period problem. Let $\pi(a)$ be the principal's maximum continuation payoff when she implements the expected second-period payoff a of the agent. The principal can implement any second-period effort level e_2 by setting $t_2(y_1, 1) = c'(e_2)$ such that the agent gets $A(e_2) = e_2c'(e_2) - c(e_2)$ and the principal gets $P(e_2) = e_2 - e_2c'(e_2)$. In order to characterize the function π, we have to find the continuation contract (t_1, t_2, e_2) with $t_2 = c'(e_2)$ that maximizes the principal's payoff among those that implement a given expected payoff a of the agent.

The principal in period 1 solves

$$\max_{t_1, e_2} P(e_2) - t_1$$

subject to $t_1 \geq 0$ and $t_1 + A(e_2) = a$. Transform the problem by replacing t_1 with $a - A(e_2)$:

$$\max_{e_2} P(e_2) + A(e_2) - a$$

subject to $a \geq A(e_2)$. We will consider two cases for a:

(Case 1: $\boldsymbol{a \geq A(e_S^{FB})}$): We can note that $A(e_S^{FB}) = e_S^{FB}c'(e_S^{FB}) - c(e_S^{FB}) = e_S^{FB} -$

$c(e_S^{FB}) = S(e_S^{FB})$ and $P(e_S^{FB}) = 0$. The required payoff to the agent is greater than the possible total surplus $S(e^{FB})$ and can therefore only be achieved with a non-negative transfer $t_1 = a - A(e_2) > 0$. The limited liability constraint cannot be binding and we can perform the maximization above without the constraint:

$$\max_{e_2} P(e_2) + A(e_2) - a = \max_{e_2} e_2 - c(e_2) - a$$

The first-order condition implies $c'(e_2) = 1$. Therefore, $e_2 = e_S^{FB}$ when the agent is promised a second-period payoff of $a \geq A(e_S^{FB})$.

(Case 2: $\boldsymbol{a < A(e_S^{FB})}$): The limited liability constraint is binding ($t_1 = 0$) which yields $a = A(e_2)$, implying $e_2 = A^{-1}(a)$. The principal receives $P(e_2)$ where $e_2 = A^{-1}(a)$. The following table summarizes the results:

Table 1: Continuation contract that optimally implements a given continuation payoff a to the agent

	t_1	e_2	$\pi(a)$
if $a \geq A(e_S^{FB})$	$a - A(e_S^{FB})$	e_S^{FB}	$S(e_S^{FB}) - a$ <0
if $a < A(e_S^{FB})$	0	$A^{-1}(a)$	$P(A^{-1}(a))$

Next, we will argue that $\pi(a)$ is concave. To that end, we make the following claim:

$$\pi'(a) = \frac{P'(e_2)}{A'(e_2)} \tag{A.1}$$

(Case 1: $\boldsymbol{a \geq A(e_S^{FB})}$): The derivative of $\pi(a)$ for this range of values of a yields $\pi'(a) = -1$ since $\pi(a) = S(e_S^{FB}) - a$. Using (4) and $c'(e_S^{FB}) = 1$:

$$P'(e_S^{FB}) = -e_S^{FB} c''(e_S^{FB}) \tag{A.2}$$

Further, taking the derivative of (5) and evaluating at $e = e_S^{FB}$ yields:

$$A'(e_S^{FB}) = e_S^{FB} c''(e_S^{FB}) \tag{A.3}$$

Therefore, we can rewrite $\pi'(a)$ as the following:

$$\pi'(a) = \frac{P'(e_S^{FB})}{A'(e_S^{FB})} = \frac{P'(e_2)}{A'(e_2)}$$

(Case 2: $a < A(e_S^{FB})$): The maximum continuation payoff to the principal for this range of values of a is $\pi(a) = P(A^{-1}(a))$ which yields:

$$\pi'(a) = \frac{P'(A^{-1}(a))}{A'(A^{-1}(a))} \tag{A.4}$$

Given the implemented second-period effort level is $e_2 = A^{-1}(a)$, we confirm our claim:

$$\pi'(a) = \frac{P'(e_2)}{A'(e_2)}$$

For $\pi(a)$ to be concave, $\pi''(a)$ needs to be negative. From (A.1), we easily calculate the second derivative of $\pi(a)$ to be

$$\pi''(a) = \frac{P''(e_2(a)) - A'(e_2(a)) - A''(e_2(a))P'(e_2(a))}{(A'(e_2(a)))^2} < 0 \tag{A.5}$$

verifying that $\pi(a)$ is concave.

Now, we move onto establishing the result in the proposition. From Table 1, we can easily see that no effort level greater than e_S^{FB} will be implemented and hence $e_{2D}^{SB}(1) \leq e_S^{FB}$. To show how the first-period effort level compares, we will solve the principal's maximization problem. In the first period, the agent chooses an effort level

$$e_1 = \arg\max_e ea(1) + (1-e)a(0) - c(e) \tag{A.6}$$

where $a(1)$ denotes the agent's continuation payoff in case of a success and $a(0)$ in case of a failure. The principal can chose any pair of non-negative continuation payoffs $a(0)$ and $a(1)$ to get the payoff

$$e_1\left(1 + \pi(a(1))\right) + (1-e_1)\pi(a(0)) \tag{A.7}$$

for any effort level e_1. Now, if the optimal one-period contract was repeated unconditionally it would yield $2P(e_S^{SB})$ whereas if we had $a(1) \leq a(0)$ with $e_1 = 0$, the principal would receive $P(e_S^{SB})$ at best. Thus, we can ignore the $a(1) \geq 0$ constraint. From (A.6), we obtain

$$c'(e_1) = a(1) - a(0) \tag{A.8}$$

which characterizes the incentive compatible first-period effort level. We can state the principal's optimization problem in terms of e_1 and $a(0)$ as

$$\max_{e_1} e_1(1 + \pi(c'(e_1) + a(0)) + (1 - e_1)\pi(a(0))$$

$$\text{subject to } a(0) \geq 0$$

with

$$e_{1D}^{SB} = \arg\max_{e_1} e_1(1 + \pi(c'(e_1) + a(0)) + (1 - e_1)\pi(a(0)). \tag{A.9}$$

The Lagrangian for this problem is

$$\mathscr{L}(e_1, a(0), \lambda) = e_1(1 + \pi(c'(e_1) + a(0)) + (1 - e_1)\pi(a(0)) + \lambda a(0) \tag{A.10}$$

with $\lambda \geq 0$. Using $A'(e) = ec''(e)$, we notice that in the optimum it must hold that:

$$\frac{\partial \mathscr{L}}{\partial e_1} = 1 + \pi(a(1)) - \pi(a(0)) + A'(e_{1D}^{SB})\pi'(a(1)) = 0 \tag{A.11}$$

$$\frac{\partial \mathscr{L}}{\partial a(0)} = 0 \implies -\lambda = e_{1D}^{SB}\pi'(a(1)) + (1 - e_{1D}^{SB})\pi'(a(0)) \tag{A.12}$$

with either $a(0) = 0$ or $a(0) > 0$ and $\lambda = 0$. We will use the first order conditions to show that in the optimum $e_{1D}^{SB} < e_{2D}^{SB}(1)$. First, suppose that $a(1) < A(e_S^{FB})$ to see from Table 1,

$$c'(e_{1D}^{SB}) = a(1) - a(0) \leq a(1) = A(e_{2D}^{SB}(1)) < c'(e_{2D}^{SB}(1)) \tag{A.13}$$

This yields $e_{1D}^{SB} < e_{2D}^{SB}(1)$ since $c''(\cdot) \geq 0$. Second, suppose that $a(1) \geq A(e_S^{FB})$ to get $\pi'(a(1)) = -1$ and $\pi(a(1)) < 0$ from Table 1. Then, the first order condition (A.11) gives us that $A'(e_{1D}^{SB}) < 1$. On the other hand, $A'(e_1) = e_1c''(e_1) \geq c'(e_1)$ for any e_1 because

$c''(\cdot)$ is weakly increasing. This implies that $c'(e_{1D}^{SB}) < 1 = c'(e_S^{FB})$ and since $c'(\cdot)$ is weakly increasing, we must have that $e_{1D}^{SB} < e_S^{FB} = e_{2D}^{SB}(1)$.

Next, we show that $e_{1D}^{SB} > e_S^{SB}$. Using (A.8) and the derivative of (3) we can rewrite the first order condition (A.11) as

$$P'(e_{1D}^{SB}) = \pi(a(0)) + a(0) - (\pi(a(1)) + a(1)) - A'(e_{1D}^{SB})(\pi'(a(1)) + 1) \tag{A.14}$$

Noting that $\pi'(a(1)) \geq -1$ and the that the second period surplus increases in the agent's implemented payoff, we can conclude that $P'(e_{1D}^{SB}) \leq 0$. Hence, $e_{1D}^{SB} \geq e_S^{SB}$ and $e_{1D}^{SB} = e_S^{SB}$ can only hold if $\pi'(a(1)) = -1$ with $a(0) = 0$ and $a(1) + \pi(a(1)) = 0$.

Finally, we will show $e_{2D}^{SB}(0) < e_S^{SB}$. We can observe that $\pi'(a(1)) \leq \pi'(a(0))$ since $\pi(a)$ is concave. In the case of $a(0) > 0$ and $\lambda = 0$, it must be true that $\pi'(a(1))$ and $\pi'(a(0))$ have opposite signs for (A.12) to be fulfilled. Thus, from (A.1), $P'(e_{2D}^{SB}(1)) < 0 < P'(e_{2D}^{SB}(0))$ implying $e_{2D}^{SB}(0) < e_S^{SB} < e_{2D}^{SB}(1)$. Therefore, considering all of the above leads to the desired result:

$$e_S^{FB} \geq e_{2D}^{SB}(1) > e_{1D}^{SB} > e_S^{SB} > e_{2D}^{SB}(0) > 0.$$

A.2

Proof of Proposition 3. The principal's problem is to maximize

$$e_1\left(1 - t_1(1) + e_2(1)\left(1 - t_2(1,1)\right)\right) + (1 - e_1)\left(-t_1(0) + e_2(0)\left(1 - t_2(0,1)\right)\right)$$

$$\text{subject to } (IC_1), (IC_2), (LL_1), \text{ and } (11).$$

Using $c(e) = e^2$, the principal's problem can be rewritten as:

$$\max_{t_1(0),t_1(1),t_2(1,1),t_2(0,1)} e_1\left(1 - t_1(1) + e_2(1)\left(1 - t_2(1,1)\right)\right) + (1 - e_1)\left(-t_1(0) + e_2(0)\left(1 - t_2(0,1)\right)\right)$$

subject to

$$t_1(0) \geq 0 \tag{A.15}$$

$$t_1(1) \geq 0 \tag{A.16}$$

$$a(0) = t_1(0) + e_2^2(0) \tag{A.17}$$

$$a(1) = t_1(1) + e_2^2(1) \tag{A.18}$$

$$a(1) - a(0) = 2e_1 \tag{A.19}$$

$$t_2(1,1) = 2e_2(1) \tag{A.20}$$

$$t_2(0,1) = 2e_2(0) \tag{A.21}$$

We substitute the equality constraints into the objective function to simplify the principal's problem as follows:

$$\max_{e_1, e_2(0), e_2(1), t_1(0)} e_1 - 2e_1^2 + e_1(e_2(1) - e_2(1)^2) + (1 - e_1)(e_2(0) - e_2(0)^2) - t_1(0) - e_2^2(0)$$

subject to (A.15) and (A.16)

We can also observe that (A.15) implies (A.16), allowing us to ignore (A.16), which represents the limited liability constraint following success in the first period. The Lagrangian is then given by

$$\mathscr{L} = e_1 - 2e_1^2 + e_1(e_2(1) - e_2(1)^2) + (1 - e_1)(e_2(0) - e_2(0)^2) - t_1(0) - e_2^2(0) + \lambda(t_1(0)).$$

From the first-order conditions, the following must hold at the optimum:

$$[e_{1D}^{SB}]: \ 1 - 4e_{1D}^{SB} + e_{2D}^{SB}(1) - e_{2D}^{SB}(1)^2 - e_{2D}^{SB}(0) + e_{2D}^{SB}(0)^2 = 0 \tag{A.22}$$

$$[e_{2D}^{SB}(0)]: \ (1 - e_{1D}^{SB}) - 2(1 - e_{1D}^{SB})e_{2D}^{SB}(0) - 2e_{2D}^{SB}(0) = 0 \tag{A.23}$$

$$[e_{2D}^{SB}(1)]: \ e_{1D}^{SB} - 2e_{1D}^{SB}e_{2D}^{SB}(1) = 0 \tag{A.24}$$

$$[t_1(0)]: \; -1 + \lambda = 0 \tag{A.25}$$

From (A.24), we find that $e_{2D}^{SB}(1) = \frac{1}{2} = e_S^{FB}$. Additionally, (A.25) implies that $\lambda > 0$ and hence $t_1(0) = 0$. Solving the two equations ((A.22) and (A.23)) with two unknowns yields $e_{1D}^{SB} = 0.27$ and $e_{2D}^{SB}(0) = 0.21$.[37] Therefore, we have our desired result:[38]

$$e_S^{FB} = e_{2D}^{SB}(1) > e_{1D}^{SB} > \frac{1}{4} = e_S^{SB} > e_{2D}^{SB}(0) > 0.$$

A.3

Proof of Proposition 4. The principal maximizes

$$e_1(1 - t_1(1) + e_2(1)(1 - t_2(1,1)) + (1 - e_1)(-t_1(0) + e_2(0)(1 - p - t_2(0,1)) + p)$$

subject to (A.15) – (A.21)

As in the proof of Proposition 3, we can ignore (A.16). Using the equality constraints, the principal's problem reduces to the following:

$$\max_{e_1, e_2(0), e_2(1), t_1(0)} e_1(1 - e_2(1)^2 + e_2(1)) + (1 - e_1)(-e_2(0)^2 + e_2(0)(1-p) + p) - 2e_1^2 - t_1(0) - e_2(0)^2$$

subject to (A.15).

The Lagrangian is given by:

$$\mathscr{L} = e_1(1 - e_2(1)^2 + e_2(1)) + (1 - e_1)(-e_2(0)^2 + e_2(0)(1-p) + p) - 2e_1^2 - t_1(0) - e_2(0)^2 + \lambda(t_1(0))$$

From the first-order conditions, the following must hold at the optimum:

[37] The solution is obtained using computational software. The code can be found in Appendix **B**.
[38] The result is consistent with Table 1 and Proposition 2.

$$[e_{2D}^{SB}(1)]: \ -2e_{2D}^{SB}(1)e_{1D}^{SB} + e_{1D}^{SB} = 0 \tag{A.26}$$

$$[e_{2D}^{SB}(0)]: \ -2(1-e_{1D}^{SB})e_{2D}^{SB}(0) + (1-e_{1D}^{SB})(1-p) - 2e_{2D}^{SB}(0) = 0 \tag{A.27}$$

$$[e_{1D}^{SB}]: \ (1+\frac{1}{4}) - 4e_{1D}^{SB} + e_{2D}^{SB}(0)^2 - e_{2D}^{SB}(0)(1-p) - p = 0 \tag{A.28}$$

$$[t_1(0)]: \ -1+\lambda = 0 \tag{A.29}$$

Similar to the proof of Proposition 3, we observe that $t_1(0) = 0$ due to (A.29). Additionally, from (A.26), we find that $e_{2D}^{SB}(1) = \frac{1}{2} = e_S^{FB}$ which establishes the first part of the proposition. To prove the second part of the proposition, we first combine (A.27) and (A.28) and then implicitly differentiate the resulting equation to obtain:

$$\frac{\partial e_{2D}^{SB}(0)}{\partial p} = \frac{3e_{2D}^{SB}(0)^2 - 2e_{2D}^{SB}(0) - 2(1-p)e_{2D}^{SB}(0) - 2p - \frac{7}{4}}{-6e_{2D}^{SB}(0)^2 + 6(1-p)e_{2D}^{SB}(0) + 2p - (1-p)^2 + \frac{27}{2}} \tag{A.30}$$

To proceed, I fix $p \in [0,1]$ and for this fixed p, I define the following:

$$f(e_{2D}^{SB}(0)) := 3e_{2D}^{SB}(0)^2 - 2e_{2D}^{SB}(0) - 2(1-p)e_{2D}^{SB}(0) - 2p - \frac{7}{4} \tag{A.31}$$

$$g(e_{2D}^{SB}(0)) := -6e_{2D}^{SB}(0)^2 + 6(1-p)e_{2D}^{SB}(0) + 2p - (1-p)^2 + \frac{27}{2} \tag{A.32}$$

The critical point of $g(e_{2D}^{SB}(0))$ occurs $e_{2D}^{SB}(0) = \frac{1-p}{2}$. For the fixed p, $g''(e_{2D}^{SB}(0)) < 0$ establishing that the critical point indeed corresponds to a maximum. To show that $g(e_{2D}^{SB}(0)) > 0$, it suffices to show that the maximum value of $g(e_{2D}^{SB}(0))$ is positive, along with its values at the endpoints $e_{2D}^{SB}(0) = 0$ and $e_{2D}^{SB}(0) = 1$. At $e_{2D}^{SB}(0) = 0$, we have $g(0) = \frac{27}{2} + 2p - (1-p)^2$. Since $g(0) > \frac{25}{2}$, it is clearly positive. Similarly, we can check that $g(e_{2D}^{SB}(0) = 1) = -6 + 6(1-p) + \frac{27}{2} + 2p - (1-p)^2 > \frac{19}{2}$ is also positive. Finally, evaluating $g(e_{2D}^{SB}(0))$ at the critical point $e_{2D}^{SB}(0) = \frac{1-p}{2}$ yields $g(\frac{1-p}{2}) = \frac{27}{2} + 2p + \frac{1}{2}(1-p)^2 > 0$. Therefore, this confirms that $g(e_{2D}^{SB}(0))$ is positive for all $e_{2D}^{SB}(0) \in [0,1]$ holding $p \in [0,1]$ fixed. Since p was arbitrary, this implies that $g(e_{2D}^{SB}(0))$ is positive for all $e_{2D}^{SB}(0) \in [0,1]$ and $p \in [0,1]$ establishing that the denominator of (A.30) is positive everywhere.

Similarly, the critical point of $f(e_{2D}^{SB}(0))$ occurs at $e_{2D}^{SB}(0) = \frac{2-p}{3}$. Given that $f''(e_{2D}^{SB}(0)) >$

0, the critical point corresponds to a minimum. We can easily verify that $f(0) < 0$ and $f(1) < 0$ and the minimum value of $f(e_{2D}^{SB}(0))$ is given by $f(\frac{2-p}{3}) = -\left(\frac{1}{3}x^2 + \frac{2}{3}x + \frac{37}{12}\right) < 0$. Since, for any fixed p, the minimum value of $f(e_{2D}^{SB}(0)$ is negative, and the function values at the endpoints are also negative, it follows that $f(e_{2D}^{SB}(0)) < 0$ for all $e_{2D}^{SB}(0) \in [0,1]$. Following the same reasoning as above, the numerator of (A.30) is negative everywhere. Since the denominator of (A.30) is positive and the numerator is negative for all $p \in [0,1]$ and $e_{2D}^{SB}(0) \in [0,1]$, we conclude that $\frac{\partial e_{2D}^{SB}(0)}{\partial p} < 0$, thus establishing the second result of the proposition.

Finally, to prove the last part of the proposition, we implicitly differentiate (A.28) to obtain:

$$4\frac{\partial e_{1D}^{SB}}{\partial p} = \left[2e_{2D}^{SB}(0) - (1-p)\right]\frac{\partial e_{2D}^{SB}(0)}{\partial p} + e_{2D}^{SB}(0) - 1 \tag{A.33}$$

Note that $-1 < 2e_{2D}^{SB}(0) - (1-p) < 2$. It is easy to observe that for $0 < 2e_{2D}^{SB}(0) - (1-p) < 2$, we have $\frac{\partial e_{1D}^{SB}}{\partial p} < 0$ since $\frac{\partial e_{2D}^{SB}(0)}{\partial p} < 0$, as shown earlier, and $e_{2D}^{SB}(0) < 1$. All that remains is to show that $\frac{\partial e_{1D}^{SB}}{\partial p} < 0$ for $-1 < 2e_{2D}^{SB}(0) - (1-p) < 0$. In this case, it must be that $e_{2D}^{SB}(0) < \frac{1}{2}$. For such values of $e_{2D}^{SB}(0)$, we find that $\frac{\partial e_{2D}^{SB}(0)}{\partial p} > -\frac{7}{25}$ which implies that:

$$\left[2e_{2D}^{SB}(0) - (1-p)\right]\frac{\partial e_{2D}^{SB}(0)}{\partial p} + e_{2D}^{SB}(0) < \frac{7}{25} + \frac{1}{2} < 1.$$

Therefore, $\frac{\partial e_{1D}^{SB}}{\partial p} < 0$, which gives us the desired result that the first-period effort level decreases as p increases.

A.4

Adding discounting to 'Win from Losing' for $p = 1$. Let $\delta \in (0,1)$ be the discount factor. The principal maximizes

$$e_1\left[1 - t_1(1) + \delta(e_2(1)(1 - t_2(1,1)))\right] + (1 - e_1)\left[-t_1(0) + \delta(e_2(0)(1 - p - t_2(0,1)) + p)\right]$$

subject to

$$a(0) = t_1(0) + \delta e_2^2(0) \tag{A.34}$$

$$a(1) = t_1(1) + \delta e_2^2(1) \tag{A.35}$$

and (A.15), (A.16), (A.19)-(A.21). The principal's problem reduces to the following:

$$\max_{e_1, e_2(0), e_2(1), t_1(0)} e_1(1-\delta e_2(1)^2+\delta e_2(1))+(1-e_1)(-\delta e_2(0)^2+\delta e_2(0)(1-p)+\delta p)-2e_1^2-t_1(0)-\delta e_2(0)^2$$

subject to (A.15)

The Lagrangian is given by:

$$\mathscr{L} = e_1(1-\delta e_2(1)^2+\delta e_2(1))+(1-e_1)(-\delta e_2(0)^2+\delta e_2(0)(1-p)+\delta p)-2e_1^2-t_1(0)-\delta e_2(0)^2+\lambda(t_1(0))$$

From the first-order conditions, the following must hold at the optimum:

$$[e_{2D}^{SB}(1)] : \; -\delta 2 e_{2D}^{SB}(1) e_{1D}^{SB} + \delta e_{1D}^{SB} = 0 \tag{A.36}$$

$$[e_{2D}^{SB}(0)] : \; \delta(-2(1-e_{1D}^{SB})e_{2D}^{SB}(0) + (1-e_{1D}^{SB})(1-p) - 2e_{2D}^{SB}(0)) = 0 \tag{A.37}$$

$$[e_1] : \; -4e_{1D}^{SB} + 1 + \delta\left(\frac{1}{4} + e_{2D}^{SB}(0)^2 - e_{2D}^{SB}(0)(1-p) - p\right) = 0 \tag{A.38}$$

$$[t_1(0)] : \; -1 + \lambda = 0$$

Since we are only concerned with the first-period effort level, e_{1D}^{SB}, at $p = 1$, we can simplify equations (A.37) and (A.38) to obtain the following:

$$(1 - e_{1D}^{SB}) e_{2D}^{SB}(0) = e_{2D}^{SB}(0) \tag{A.39}$$

$$4e_{1D}^{SB} = 1 + \delta\left(\frac{1}{4} + e_{2D}^{SB}(0)^2 - 1\right) \tag{A.40}$$

From (A.39), we have two possible solutions: either $e_{2D}^{SB}(0) = 0$ or $e_{1D}^{SB} = 0$. If $e_{1D}^{SB} = 0$, then substituting into (A.40) leads to $e_{2D}^{SB}(0)^2 = \frac{3}{4} - \frac{1}{\delta} < 0$, which is a contradiction given $\delta \in (0, 1)$. Therefore, $e_{2D}^{SB}(0) = 0$, and substituting again into (A.40) yields:

$$e_{1D}^{SB} = \frac{1}{4} - \frac{3}{16}\delta \tag{A.41}$$

Taking the derivative of (A.41) with respect to δ, we get: $\frac{de_{1D}^{SB}}{d\delta} < 0$. Since this derivative is negative, it follows that the induced first-period effort level increases as δ decreases, which is the desired result.

B

The Jupyter Notebook containing the code for the results and the plots presented in this paper can be found here.

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