Saaif Ahmed

"On my honor, I have neither given nor recieved unauthorized aid on this exam"

\- [signature]
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1. A) $P(x \leq \frac{9}{4}) - P(x < 1)$.

$\frac{1}{2}\sqrt{2.25} - \frac{1}{2}\sqrt{1} = \boxed{0.25}$

B) $\frac{d}{dx} F(x) = f(x)$

$\frac{d}{dx}(0) = 0 \quad \frac{d}{dx}(\frac{1}{2}\sqrt{x}) = \frac{1}{4\sqrt{x}} \quad \frac{d}{dx}(1) = 0$

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4\sqrt{x}} & x \in [0,4] \\ 0 & x > 4 \end{cases}$$

C) $E(x) = \int x \cdot f(x) = \int_0^4 \frac{x}{4\sqrt{x}} dx$

$\int_0^4 \frac{x}{4\sqrt{x}} = \frac{1}{4}\int_0^4 x^{\frac{1}{2}} =$

$= \frac{1}{6} x^{3/2} \Big|_0^4$

$= \boxed{\frac{4}{3}}$

D) $Var(x) = E(x^2) - (E(x))^2$

$= \int_0^4 \frac{x^2}{4\sqrt{x}} - (\frac{4}{3})^2$

$= \frac{1}{4}\int_0^4 x^{\frac{3}{2}}$

$= \frac{1}{4}(\frac{2x^{\frac{5}{2}}}{5}) \Big|_0^4$

$= \frac{16}{5} - (\frac{4}{3})^2$

$= \boxed{\frac{64}{45}}$

2. A) 40 → 52

40 − 50 $\quad$ y axis $= \frac{1}{12}$

$P(x \in [40,50]) = \boxed{\frac{10}{12}}$

B) $P(Y \in [40,50]) = P(Y \le 50) - P(Y \le 40)$

$= (1 - e^{-46(50)}) - (1 - e^{-46(40)})$

$\cancel{1 - e^{-46(50)}}$

$= 1 - e^{-2300} - 1 + e^{-1840}$

$= \boxed{e^{-1840} - e^{-2300}}$

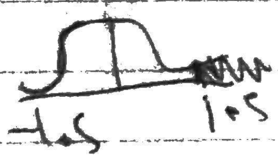
C) $P(Z \in [40,50]) = P(Z \ge 40) - P(Z \ge 50)$

$Q\left(\frac{40-46}{4}\right) - Q\left(\frac{50-46}{4}\right)$

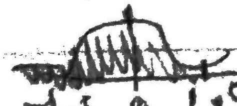
$Q\left(-\frac{3}{2}\right) - Q(1)$

$= \cancel{6.681 \times 10^{-2} - 1.587 \times 10^{-1}}$

$Q(-1.5)$



$1 - Q(1.5)$



$Q(0) + Q(0) - Q(1.5)$

$.5 + .5 - 6.681 \times 10^{-2}$

$.93319 - Q(1)$

$= 0.77449$

$P(Z \in [40,50]) = \boxed{0.77449}$

3. A) $X \to \lambda = 4$

$Y \to [10, 16]$ $Y \sim \mathrm{Unif} = \frac{1}{6}$

$E(V) = E(Y - 4X) = E(Y) - 4E(X)$

$= \frac{10+16}{2} - 4\left(\frac{1}{\lambda}\right)$

$= 13 - 1$

$= \boxed{12}$

B) $Var(V) = \cancel{Var(\ )}\ \cancel{E(V^2) - E(V)^2}$

$Var(V) = Var(Y - 4X)$

$= Var(Y) - 4^2 Var(X)$

$= \frac{(16-10)^2}{12} - 16\left(\frac{1}{\lambda^2}\right)$

$= 3 - 1$

$= \boxed{2}$

4. A) $5(0)^3 = 0$ $\boxed{\text{range} = [0, 5000]}$

$5(10^3) = 5000$

B) $P(W \leq w)$

$P(5D^3 \leq w)$

$P\left(D \leq \sqrt[3]{\frac{w}{5}}\right)$

$f\left(\frac{1}{50}\sqrt[3]{\frac{w}{5}}\right) \cdot \frac{d}{dw}\left(\sqrt[3]{\frac{w}{5}}\right.$

$\frac{1}{50}\sqrt[3]{\frac{w}{5}} \cdot \frac{1}{3\sqrt[3]{5}\, w^{\frac{2}{3}}}$

$= \frac{1}{150\sqrt[3]{5^2}\, w^{\frac{1}{3}}}$

$\int_0^{5000} \frac{w}{150\sqrt[3]{25}\sqrt[3]{w}}\, dw$

$= \frac{1}{150 \cdot \sqrt[3]{25}} \int_0^{5000} \frac{w}{\sqrt[3]{w}}$

$= \frac{1}{150\sqrt[3]{25}}\left(\frac{3}{5} w^{\frac{5}{3}}\right)\Big|_0^{5000}$

$= \boxed{2000}$

C) $\frac{1}{150\sqrt[3]{25}} \int \frac{1}{\sqrt[3]{w}} = \frac{1}{150\sqrt[3]{25}}\left(\frac{3}{2} w^{\frac{2}{3}}\right) = F_W(x)$

D) derived above

$f_W(x) = \frac{1}{150\sqrt[3]{25}} \cdot \frac{1}{\sqrt[3]{w}}$

* C, and D derived in B

5. A) integrat out the Y. aka add down the ~~scale~~ graph for values of X.

$$\boxed{f_X(x) = \frac{1}{2}x \;,\; x \in [0,2] \qquad 0, \text{ elsewhere}}$$

B) integrate out X, add across

$$f_Y(y) = \frac{1}{4}y \;,\; y \in [0,4)$$
$$0, \text{ elsewhere}$$

Uniform distro. $E(Y) = \frac{a+b}{2}$

$$= \frac{0+4}{2}$$
$$\boxed{= 2}$$

c) $\iint a(x+y)$

$a \iint (x+y)\,dx\,dy$

$a = \frac{1}{4}$

$f_{X,Y}(x,y) = f_X(x)\, f_Y(y)$

$\frac{1}{4}(x+y) \neq \frac{1}{2}x \cdot \frac{1}{4}y$

$\boxed{\text{Not independent because } f(x,y) \neq f_X(x) f_Y(y)}$

D) $\rho = \frac{\operatorname{Cov}(X,Y)}{\sigma_X \sigma_Y}$

$\operatorname{Cov}(X,Y) = \frac{7}{4} - \mu_X \mu_Y$
$= \frac{7}{4} - 1(2)$
$= -0.25$

The correlation is coefficient is $(-1,0)$ because the $\operatorname{Cov}(X,Y)$ is negative. Also it is not fully dependent