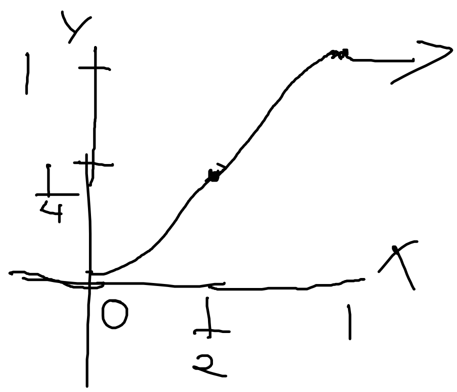
HW 4

Friday, October 9, 2020 1:09 PM

1:

a)



b)

Since $F_x(x) = \int_0^x P(x)$ and for $x \in \left[0, \frac{1}{2}\right] \to F_x(x) = x^2$

$P\left(X < \frac{1}{4}\right) = F_x\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^2$

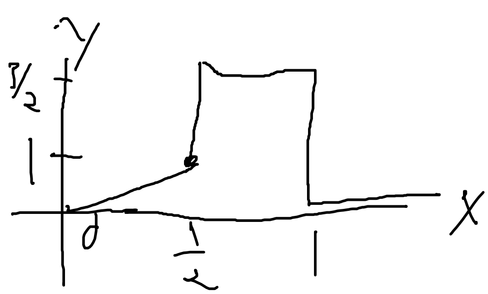
Answer: $\frac{1}{16}$

c)

$P\left(X \in \left[\frac{3}{8}, \frac{3}{4}\right]\right) = P\left(X \leq \frac{3}{4}\right) - P\left(X \leq \frac{3}{8}\right) = \left(\frac{3}{2} * \frac{3}{4} - \frac{1}{2}\right) - \left(\frac{3}{8}\right)^2 = \frac{31}{64}$

Answer: $\frac{31}{64}$

d)

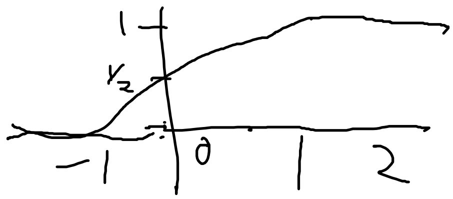


2:

a)

$$F_X(-\infty) = \frac{1}{1+e^{-2(-\infty)}} = \frac{1}{\infty} = 0\,; F_X(\infty) = \frac{1}{1+e^{-\infty}} = \frac{1}{1+\frac{1}{e\wedge\infty}} = \frac{1}{1+0} = 1$$

b)



c)

$$P(-1 < X < 2) = P(X < 2) - P(X < -1) = \frac{1}{1+e^{-4}} - \frac{1}{1+e^{2}} \approx 0.9$$

Answer: 0.9

d)

For a continuous random variable the probability the random variable is any discrete value is 0.

e)

The PDF is the derivative of the CDF thus

$$\frac{dF}{dx}\left[\frac{1}{1+e^{-2x}}\right] = \frac{2e^{-2x}}{(1+e^{-2x})^2}$$

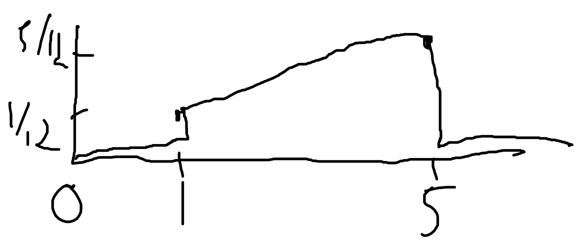
3:

a)

$$\int_1^5 cx\,dx = \frac{1}{2}cx^2\Big|_1^5 = \frac{25}{2}c - \frac{1}{2}c = 1$$

Answer: $c = \frac{1}{12}$

b)



c)

$$\int_2^3 \frac{1}{12}x\,dx = \frac{1}{12}\left(\frac{1}{2}(3^2) - \frac{1}{2}(2^2)\right) = \frac{5}{24}$$

d)

$$F_x(x) = \int f_x(x) = \frac{1}{12}\left(\frac{1}{2}x\right) = \frac{1}{24}x$$

4:

a)

0 lol

$1 - F_X(x) = e^{-\lambda * 5} = 0.082$

b)

$\alpha = \lambda T = 10\lambda$

$P(7\ in\ 10\ minutes) = \dfrac{10\lambda^7 e^{-10\lambda}}{7!} = 0.1044$

c)

$\int_3^\infty F_x(x) = 1 - F_x(x) = 0.5 = e^{-\lambda 3}$

Answer: $\lambda = 0.231$