Multivar Exam #3 Saaif Ahmed PG 1

Honor Pledge:

"I have neither given nor received any illegal aid on this exam" -Saaif Ahmed 12/9/20

Problem 2

Determine whether the matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ can be diagonalized. If yes, find the diagonal matrix D and an invertible matrix S such that $D = S^{-1}AS$. Then

compute A^4 .

$$(1 - \lambda)(4 - \lambda) + 2$$
$$\lambda^2 - 5\lambda + 6$$

$$\lambda = 3\lambda + 1$$

it can be diagonalized

Construct the eigenvectors

$$\begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$=\begin{bmatrix}1\\1\end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$
$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$
$$S = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$A^{4} = S^{-1}D^{4}S$$

$$D^{4} = \begin{bmatrix} 81 & 0 \\ 0 & 16 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} * \begin{bmatrix} 81 & 0 \\ 0 & 16 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -81 & 32 \\ 81 & -16 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} -49 & -130 \\ 65 & 146 \end{bmatrix}$$