HW 1

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1:

An outcome of the survey is a given amount of people x is shown to have an MP3 Player. Such a sample space could look like

$S = \{1\ person, 2\ peoople, 3\ people, 4\ people, \ldots, 25\ people\}$

2:

A: The Sample Space is all points in a circle of radius 10km centered on the cell tower.

B: $\frac{(5*10^3)^2\pi - (2*10^3)^2\pi}{(10^4)^2\pi} = 0.21$

3:

A:

$P(A \cap B|C) = \frac{P(A \cap B \cap C)}{P(C)}$

From Bayes Rule: $P(A \cap B|C) = \frac{P(C|A \cap B)P(A \cap B)}{P(C)}$

$$P(C|A \cap B) = \frac{P(A \cap B \cap C)}{P(A \cap B)} = \frac{P(A \cap B|C)P(C)}{P(A \cap B)}$$

$$P(B|A \cap C) = \frac{P(A \cap B \cap C)}{P(A \cap C)} = \frac{P(A \cap C|B)P(B)}{P(A \cap C)}$$

$$P(A|C \cap B) = \frac{P(A \cap B \cap C)}{P(C \cap B)} = \frac{P(C \cap B|A)P(A)}{P(C \cap B)}$$

We can see that $P(A \cap B \cap C) = P(A \cap B|C)P(C) = P(A \cap C|B)P(B) = P(C \cap B|A)P(A)$

So $P(A \cap B|C) = \frac{P(C \cap B|A)P(A)}{P(C)} = \frac{P(A|C \cap B)P(C \cap B)}{P(C)}$

$P(B|C) = \frac{P(C \cap B)}{P(C)}$ substitute in above we see that

$P(A \cap B|C) = P(A|B \cap C)P(B|C)$

As desired

B:

$$P(A \cap B|C) = P(A|B \cap C)P(B|C) = \frac{P(A \cap B \cap C)}{P(C)}$$

Thus $P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C)$

As desired

4:

$A = ap\ fail; C = chip\ is\ fault$

$$P(A|C) = \frac{1}{3}\ ; P(A|\bar{C}) = \frac{1}{10}\ ; P(C) = \frac{1}{4}$$

$$P(C|A) = \frac{P(A|C)P(C)}{P(A)}$$

$$P(A) = P(A|C)P(C) + P(A|\bar{C})P(\bar{C})$$

$$P(A) = \frac{1}{3} * \frac{1}{4} + \frac{1}{10} * \frac{3}{4} = \frac{19}{120}$$

$$P(C|A) = \frac{\frac{1}{3} * \frac{1}{4}}{\frac{19}{120}} = \frac{10}{19}$$

5:

A:

For set A and B are disjoint they must satisfy $A \cap B = \theta$ where θ is the empty set

B:

For independent events A and B they must satisfy $P(A \cap B) = P(A)P(B)$

C:

Let $A = \theta$ where θ is the empty set and let $B = 日本$ where $日本$ is any non-empty set.
Let the $P(B) = c$ where $0 \leq c \leq 1$ and by definition $P(A) = 0$
Thus we have that $P(A \cap B) = P(\theta) = 0$ and $P(A)P(B) = 0 * c = c$
Therefore these two are disjoint and independent

6:

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \binom{n}{0} p^0(1-p)^n > \frac{1}{2}$$
$$\binom{n}{0} p^0(1-p)^n < \frac{1}{2}$$
$$n(1-p)^n < \frac{1}{2}$$
$$\mathrm{n} * \ln(1-p) < \ln\left(\frac{1}{2}\right) \rightarrow n < \log_{(1-p)}\left(\frac{1}{2}\right)$$

Answer: $n > \log_{(1-p)}\left(\frac{1}{2}\right)$

7:

$$P(A \cap C|B) = \frac{P(A \cap B \cap C)}{P(B)} = \frac{P(B|A \cap C)P(A)}{P(B)}$$
$$P(A \cap B \cap C) = P(C \cap B|A)P(A) = P(A|C \cap B)P(C \cap B)$$
$$P(A \cap C|B) = \frac{P(A|C \cap B)P(C \cap B)}{P(B)} = P(A|B \cap C)P(C|B)$$
Therefore if and only if $P(A|B \cap C) = P(A|B)$
Then
$P(A \cap C|B) = P(A|B)P(C|B)$

8:

A:

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

B:

$P(A) - P(A \cap B) = P(A \cap B^C)$
$P(A \cap B^C) = P(B) - P(A \cup B)$

C:

$P\left(B \cup (A \cap B^C)\right) = P(A \cup B) \cap P(B \cup B^C) = P(A \cup B)$

D:

$P(A^C \cap B^C) = P(A \cup B)^C = 1 - P(A \cup B)$

9:

Consider $B_1, \ldots B_n$ which are disjoint subsets of a probability space
We have that $P(\cup_i B_i) = \sum_i P(B_i)$ from the axioms of probability
If $B \subset F \rightarrow P(B) \leq P(F)$
We also have that $P(F) = P(B) + P(F - B)$
Let $B_i = F_i - \cup_{j=1}^{i-1} F_j$
Since we clarified that $B \subset F$
$\cup_i^\infty B_i = \cup_i^\infty F_i$

Therefore we can say that $P(\cup_i^\infty B_i) = P(\cup_i^\infty F_i)$
And thus we prove that $P(\cup_i F_i) \leq \sum_i P(F_i)$

10:

A:

$\sigma = \{\emptyset, \{1,2,3\}, \{3,4,5\}, \{4,5\}, \{1,2\}, \{1,2,3,4,5\}\}$

B:

$P(\emptyset) = 0.\ P(\{1,2,3\}) = \frac{5}{8}.\ P(\{3,4,5\}) = \frac{7}{8}.\ P(\{1,2\}) = \frac{1}{8}.\ P(\{4,5\}) = \frac{3}{8}$

C:

$P(\{1\}) = 0$ Because it is not in the sigma algebra

11:

$$p_X(x) = \left\{ \begin{matrix} \frac{1}{7}, x = 0 \\ \frac{2}{7}, x = 1, -1 \\ \frac{2}{7}, x = 2, -2 \\ \frac{1}{7}, x = 3 \\ \frac{1}{7}, x = 4 \end{matrix} \right\}$$

12:

$P(X = 1) = \binom{15}{1} (0.1)^1 (1 - 0.1)^{15-1} = \ 0.343$

13:

$Var(cX) = E[(cX)^2] + E[cX]^2$
$= c^2(E[X^2] - E[X]^2)$
$= Var(Y) = c^2 \sigma^2$

14:

Binomial Distribution : $P(k\ bit\ error) = \binom{n}{k} p^k (1-p)^{n-k}$

15:

A:

$0.9 \leq \frac{1}{0.25^2(0.1)} \rightarrow 0.1 > \frac{1}{0.1(0.25^2)} \rightarrow n > \frac{1}{0.1(0.25^2)}$
Thus $n > 160$
Measure more than 160 students

B:

$n > \frac{1}{0.1(1^2)}$
Measure more than 10 students

16:

$P(X=0,Y=0)=\binom{100}{0}(0.01)^0(.99)^{100}$
$=.99^{100}$
$P(X>1|max>1)=\frac{1-P(X\leq 1)}{1-P(X\leq 1,Y\leq 1)}$
$P(X\leq 1,Y\leq 1)=P(X\leq 1)P(Y\leq 1)\leftarrow X\ and\ Y\ independent$
$P(X>1|max>1)=\frac{1-.99^{100}}{1-(.99^{100})^2}$
Answer: 0.732

17:

It is not possible to say that these events are not mutually exclusive
$P(A)+P(B)>1$ Thus we cannot say that are mutallu exclusive due to axiom of probability

18:

Axiom 1: Non Negative. From the set definition we see that it is a collection of non-negative numbers. Axiom 1 approved

Axiom 2: Sum of all probabilities is 1 or $P(sample\ space)$. From the set definition we see that $\sum_{s\in F}p(s)=1$. So for a mapping of $P:F\to R$ the sum of all probabilites is clearly 1.

Axiom 3: Additivity: Since E is defined as a collection of subsets in F we know that the $P(E)+P(E^C)=1$ since $P(F)=1$. Because we can define E arbitraily this will work for any combination of set $s_1,\dots,s_m$. Thus additivity is true.

19:

A:

$p=1/6$ This is because when you integrate over this discrete Joint PDF, in order to get the CDF the CDF has to be 1. Because this is discrete we can add $\frac{1}{6}+\frac{1}{2}+\frac{1}{6}+p=1$ thus $p=\frac{1}{6}$

B:

Obtain marginal pmf by summing over all values of Y where Y=3.
$P(Y=3)=\frac{2}{3}$

20:

$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}II\{X_{2i-1}=X_i\}$. The probability of two X's being the same is $\frac{1}{9}+\frac{1}{36}+\frac{1}{4}=\frac{7}{18}$
So the indicator function will output 7 1's for every 18 trials based on the probability. If n=18. then the limit is $\frac{7}{18}$. $n\to\infty$ then this is n mulitples of 7 divided by n multiples of 18.

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}II\{X_{2i-1}=X_i\}=\frac{7}{18}$$