Multivar Exam #1 Saaif Ahmed PG 2

Honor Pledge:
   "I have neither given nor received any illegal aid on this exam"
         -Saaif Ahmed 9/30/20

Problem 7

Let $u = e^s \cos t$ and $v = e^s \sin t$, and let $s = x^2 - y^2$ and $t = 2xy$. Use the chain rule to calculate the expression $\dfrac{\partial u}{\partial x} - \dfrac{\partial v}{\partial y}$.

$u = e^{x^2-y^2} \cos(2xy) = e^{x^2} e^{-y^2} \cos(2xy)$
$v = e^{x^2-y^2} \sin(2xy) = e^{x^2} e^{-y^2} \cos(2xy)$

$\dfrac{\partial u}{\partial x} \left[e^{x^2} e^{-y^2} \cos(2xy) \right] = 2x e^{x^2} e^{-y^2} \cos(2xy) - e^{x^2} e^{-y^2} 2y \sin(2xy)$

$\dfrac{\partial v}{\partial y} \left[e^{x^2} e^{-y^2} \cos(2xy) \right] = -2y e^{-y^2} e^{x^2} \cos(2xy) - e^{x^2} e^{-y^2} 2x \sin(2xy)$

$\dfrac{\partial u}{\partial x} - \dfrac{\partial v}{\partial x} = 2x e^{x^2} e^{-y^2} \cos(2xy) - e^{x^2} e^{-y^2} 2y \sin(2xy) - (-2y e^{-y^2} e^{x^2} \cos(2xy) - e^{x^2} e^{-y^2} 2x \sin(2xy))$

$= 2x e^{x^2} e^{-y^2} \cos(2xy) - e^{x^2} e^{-y^2} 2y \sin(2xy) + 2y e^{-y^2} e^{x^2} \cos(2xy) + e^{x^2} e^{-y^2} 2x \sin(2xy)$

$= e^{x^2-y^2} (2x \cos(2xy) - 2y \sin(2xy) + 2y \cos(2xy) + 2x \sin(2xy))$
$= e^{x^2-y^2} (\cos(2xy)(2x + 2y) + \sin(2xy)(2x - 2y))$

Answer: $e^{x^2-y^2} (\cos(2xy)(2x + 2y) + \sin(2xy)(2x - 2y))$