HW 2

Tuesday, September 15, 2020 1:53 PM

1:

a)

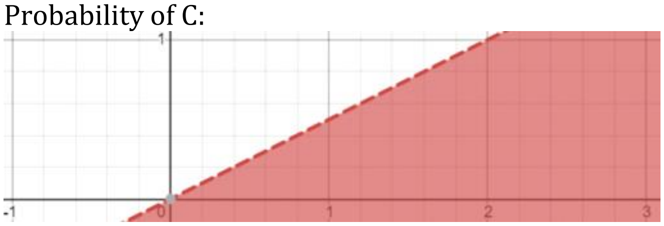
Probability of $X \in [0,2]$ in the range $[-1,3]$ or $P(X\ |X \in [0,2]) = \frac{1}{2}$

Probability of $X \in (0,3]$ in the range $[-1,3]$ or $P(X\ |X > 0) = \frac{3}{4}$

Probability of both or $P(X|\ X \in (0,2]) = 1/2$
This is because the areas overlap
They are not independent

b)

Probability of $A = \frac{3}{4}$
Probability of C:



We see that the inequality is true for the shaded area.
Thus calculating that shaded area is $(3+1) * \frac{h}{2} = 2$
So $P(C) = \frac{2}{4} = \frac{1}{2}$
The areas overlap
They are not independent

c)

Probability of $B = \frac{1}{2}$
Probability of $C = \frac{1}{2}$
The areas overlap
They are not independent

2:

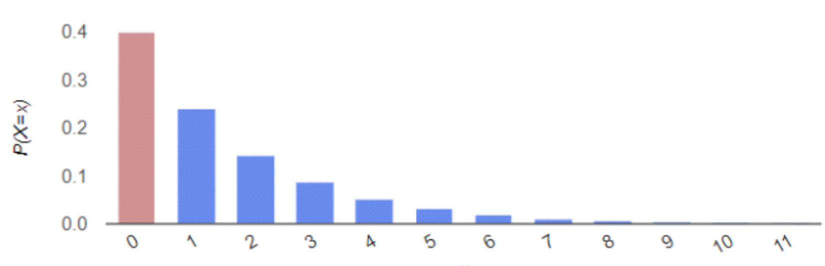
a)

F is a geometric random variable

b)

For $p = \frac{2}{5} \rightarrow \sum_{k=1}^{\infty}(1-p)^{k-1}p$
The first 4 values would be
$P(0) = 0.4$
$P(1) = 0.24$
$P(2) = 0.144$
$P(3) = 0.0864$

The sketch of the PMF is:



2 (continued):

c)

$P(F \leq 10) = P(0) + P(1) + \cdots + P(10)$

Thus we have

$$\sum_{k=0}^{11}\left(1-\left(\frac{2}{5}\right)\right)^{k}\left(\frac{2}{5}\right)$$

$$Sum = p\frac{1-\frac{3^{11}}{5}}{\left(1-\frac{3}{5}\right)} = \frac{48650978}{48828125} = 0.9964$$

3:

a)

C is a binomial random variable

b)

$$P(C=4) = \binom{6}{4}\left(1-\frac{2}{5}\right)^{2}\left(\frac{2}{5}\right)^{4} = 0.1382$$

c)

$$P(C \leq 2) = P(0) + P(1) + P(2) = \sum_{k=0}^{2}\binom{6}{k}\left(1-\frac{2}{5}\right)^{6-k}\left(\frac{2}{5}\right)^{k} = 0.5443$$

d)

$P(trail\ 1\ \&\ 2\ success \mid C=4) = P(C=4\ | trial\ 1\ \&\ 2\ sucess) \ * P(1\ \&\ 2\ success)/P(C=4)$

$P(C=4) = 0.1382$

$P(1\ \&\ 2\ sucess) = \frac{2}{5} * \frac{2}{5} = 0.16$

$P(C=4\ | trial\ 1\ \&\ 2\ sucess) = P(C=2\ from\ 4\ remaining\ trials) = 0.3456$

$$P(trail\ 1\ \&\ 2\ success \mid C=4) = 0.3456 * \frac{0.16}{0.1382} = 0.4001$$

4:

$$\frac{p(1-p)}{n\epsilon^2} = \frac{\frac{2}{5}\left(\frac{3}{5}\right)}{7500(100)^2} = 3.2 * 10^{-9}$$

5:

a)

$$P_X(0) = 2^0 * \frac{e^{-2}}{1} = e^{-2}$$

b)

$$P_X(2) = 1.5^2 * \frac{e^{-1.5}}{2} = 0.251$$