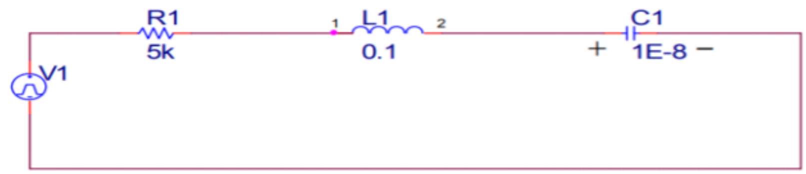
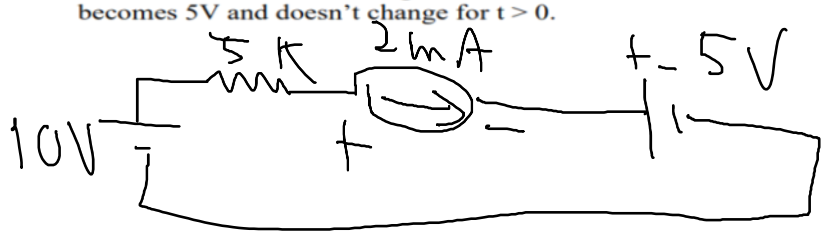
Circuits HW #5

Friday, February 21, 2020 11:59 AM

1 A:



At t = 0⁻, the voltage across the capacitor is 5V (polarity shown), the current through the inductor is 2mA to the 'right' and the source is 10V. At t = 0⁺, the voltage source becomes 5V and doesn't change for t > 0.



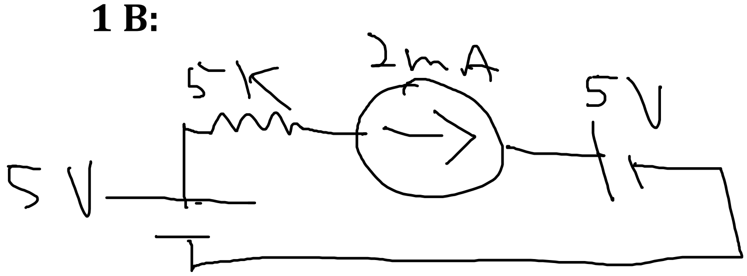
	Current	Voltage
R1	2mA	10V
C1	2mA	5V
L1	2mA	-5V
V1	2mA	10V

Mesh Analysis Constrained Node

Take KVL to solve for V_{L1}

$-V1 + V_{R1} + V_{L1} + V_{C1} = 0$
$-10V + 10V + V_{L1} + 5V = 0$
$V_{L1} = -5V$

1 B:



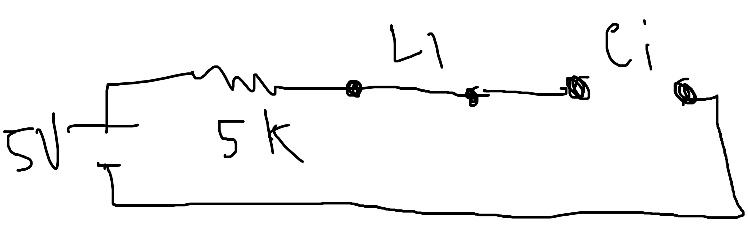
	Current	Voltage
R1	2mA	10V
C1	2mA	5V
L1	2mA	-10V
V1	2mA	5V

Mesh Analysis Constrained Node

Take KVL to solve for V_{L1}
$-5V + 10V + V_{L1} + 5V = 0$
$V_{L1} = -10V$

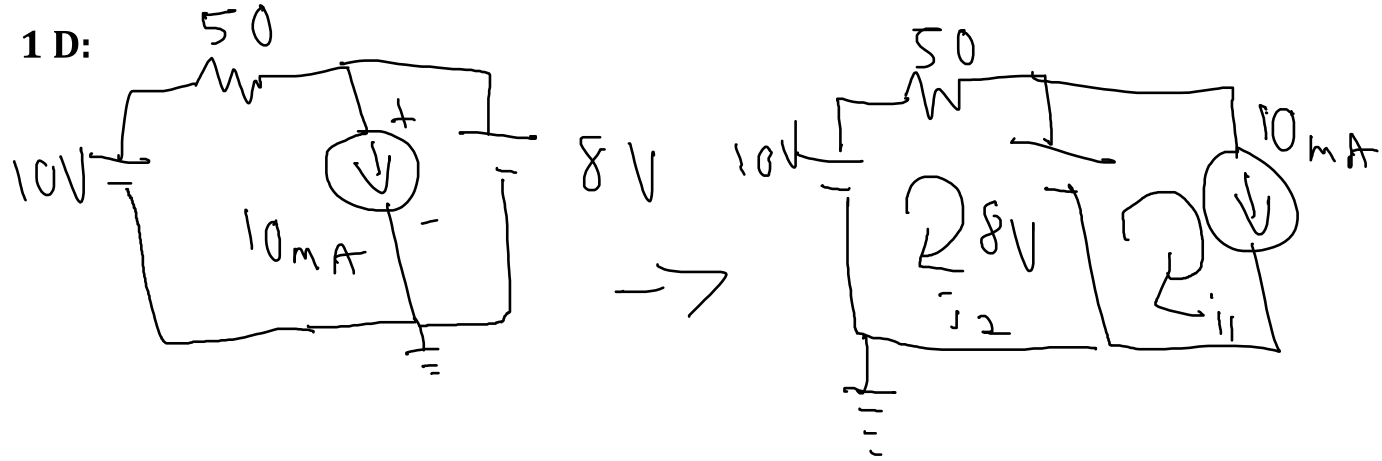
1 C:

DC steady state



	Current	Voltage
R1	0mA	0V
L1	0mA	0V
C1	0mA	5V
V1	0mA	5V

1 D:



Mesh Analysis Constrained Node:

Use KVL to solve for V_{R1}
$-10V + V_{R1} + 8V = 0$
$V_{R1} = 2V$

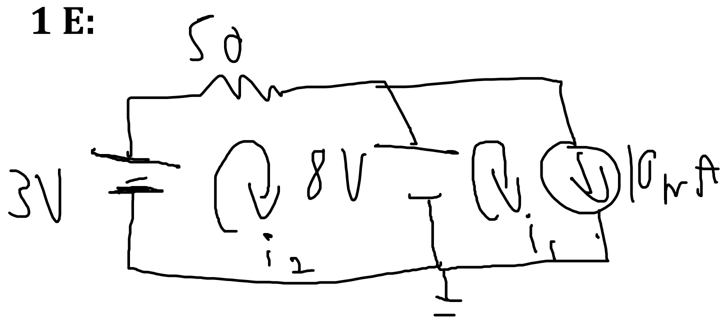
$i_2 = \frac{V_{R1}}{R_1} = 40mA$

$I_{C1} = i_2 - i_1 = 30mA$

Use KVL to solve V_{L1}
$-8V + V_{L1} = 0$
$V_{L1} = 8V$

	Current	Voltage
R1	40mA	2V
L1	10mA	8V
C1	30mA	8V
V1	40mA	10V

1 E:



Mesh Analysis Constrained Node:

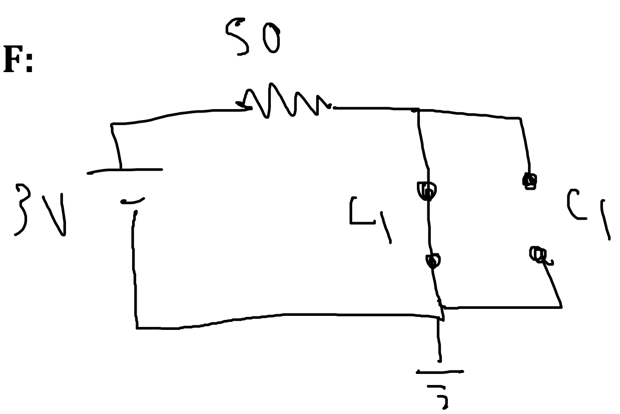
Use KVL to solve for V_{R1}
$-3V + V_{R1} + 8V = 0$
$V_{R1} = -5V$

$i_2 = \frac{V_{R1}}{R1} = -100mA$

$I_{C1} = i_2 - i_1 = -110mA$

	Current	Voltage
R1	-100mA	-5V
L1	10mA	8V
C1	-110mA	8V
V1	-100mA	3V

1 F:



	Current	Voltage
R1	60mA	3V
L1	60mA	0V
C1	0A	0V
V1	60mA	3V

2 A:

Take KCL at the node V_a

$$\frac{V_a - V_s}{R} + I_L + I_c = 0$$

$$\frac{V_a - V_s}{R} + \frac{1}{L}\int V_c(t) + C\frac{dV_c}{dt} = 0$$

$\int V_c(t) = L\left(-C\frac{dV_c}{dt} - \frac{V_C - V_s}{R}\right)$ take derivative of both sides

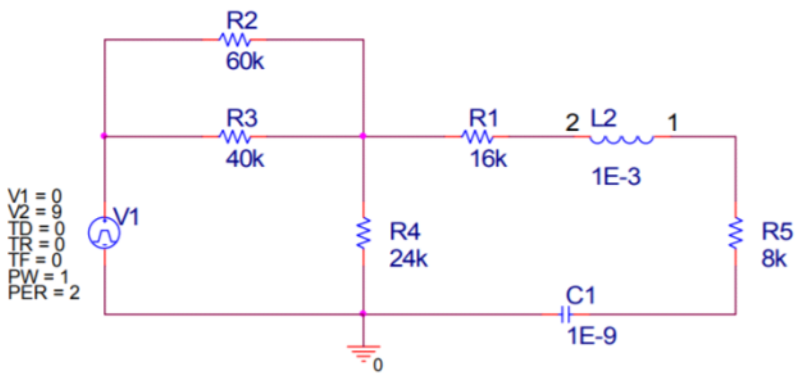
$$V_c(t) = -LC\frac{dV_c^2}{d^2t} - \frac{L}{R}\left(\frac{dV_c}{dt} - \frac{dV_s}{dt}\right)$$

2 B: From the equation solved in 2A we divide out $-LC$ from each term.

$$\alpha = \frac{1}{RC}\ ;\omega = \sqrt{\frac{1}{LC}}$$

2 C:

3 A:



In the above circuit, the initial conditions are zero and the source can be considered a step function, 9u(t).

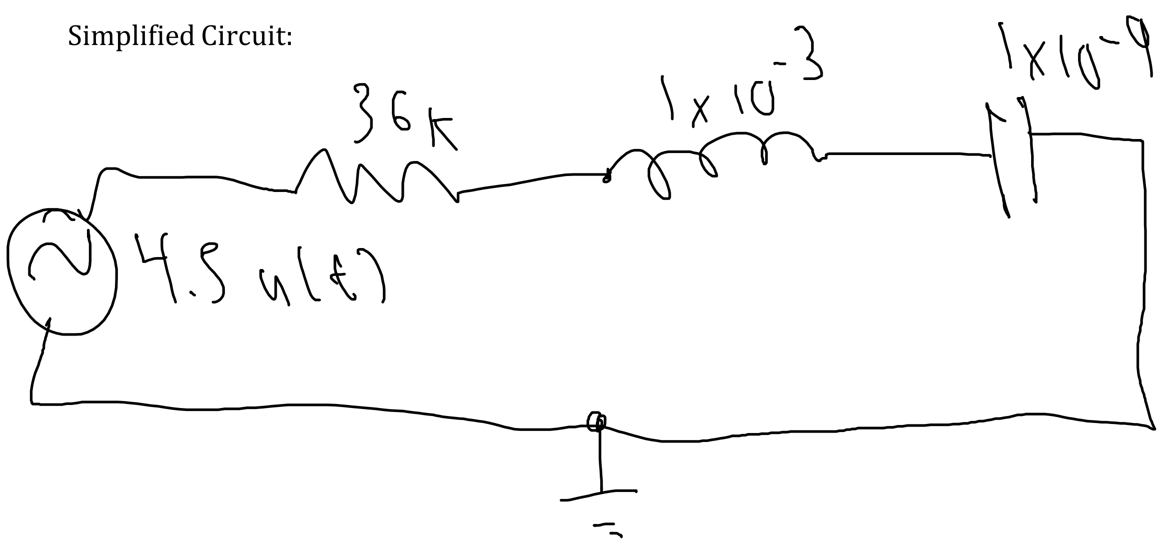
Find the Thevenin Circuit

$V_{TH} = V_{24K}$ Use Voltage Divider. $V_{24K} = 9u(t) * \frac{(24k)}{\left(\frac{1}{60k}+\frac{1}{40k}\right)^{-1}+24k} = 4.5u(t)$

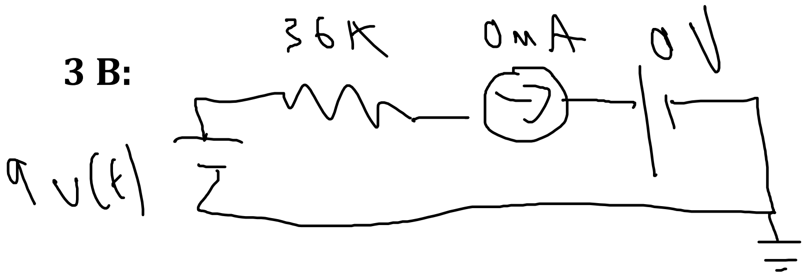
Short across open load and find R_{TH}

$$R_{TH} = \left(\frac{1}{60k}+\frac{1}{40k}\right)^{-1} + \left(\frac{1}{24k}+\frac{1}{16k+8k}\right)^{-1} = 36k$$

Simplified Circuit:

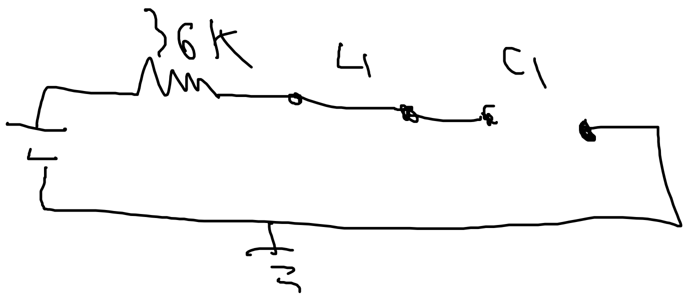


3 B:



Because the inductor had no current through it at $t(0^-)$, the cap will have current $0A$ at $t(0^+)$. Voltage of cap will also be $0V$.
At $t(0^+)$: $I_c = 0A$; $V_c = 0V$

3 C:



The DC SS of a cap is open circuit so the current is at $t(\infty) = 0A = I_c$

3 D:

$$V_c(t) = \frac{d^2v_{cn}}{dt^2} + 2\alpha\frac{dv_{cn}}{dt} + \omega^2 \mathrm{v_{cn}} = 0 \rightarrow \mathrm{s}^2 + 2\alpha \mathrm{s} + \omega^2 = 0 \rightarrow \mathrm{s}_1, \mathrm{s}_2 = -\alpha \pm \sqrt{\alpha^2 - \omega^2}$$

$s_1 = -27799.24$; $s_2 = -35972200.76$

$V_{cn} = A_1 e^{-27799.24t} + A_2 e^{-35972200.76t} + A_3$

$V_{cn}(0^-) = A_1 + A_2 + A_3 = 0$

$\frac{dv_{cn}}{dt} = \frac{i_l(0^-)}{c} = -27799.24A_1 - 35972200.76A_2 = 0$

$V(\infty) = 4.5V = A_3 \rightarrow A_1 + A_2 = -4.5V$

Solve the system: $A_1 = -4.503$; $A_2 = 0.00348$

$V_{cn}(t) = (-4.503)e^{-27799.24t} + (0.00348)e^{-35972200.76t} + 4.5V$

$$\frac{dv_{cn}(t)}{dt} * C = (125180e^{-27799.24t} - 125183.26e^{-35972200.76t})10^{-9} = \frac{di_c(t)}{dt}$$

3 E:

Characteristic S polynomial: $s^2 + 2\alpha s + \omega_0^2 = 0$ Our equation: $s^2 + 2(1.8 * 10^7) + (10^{12}) = 0$

3 F:

$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega^2} \rightarrow s_1 = -27799.24$, $s_2 = -35972200.76$

3 G:

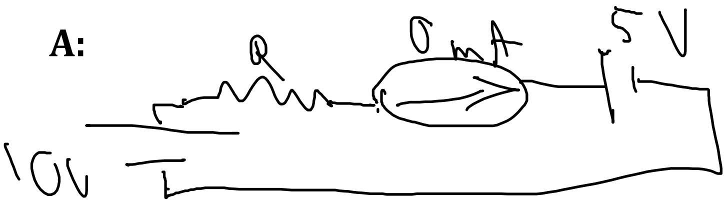
2 Real unequal roots: **System is Overdamped**

3 H:

For Overdamped systems : $V_c(t) = A_1 e^{-s_1 t} + A_2 e^{-s_2 t} + A_3$

Problem 4: RLC series Circuits

A:



$I_L = 0A$
Take KVL, no voltage across resistor: $V_L = 5V$

B: In DCC steady state the cap is open circuit so no current through loop $I_L = 0A$

C: $KVL: V_s = IR + L\frac{di}{dt} + V_{c_0} + \frac{1}{C}\int i\, dt$

$$V_s - \left(\frac{d^2V_c}{dt^2} + \frac{R}{L}\frac{dV_c}{dt} + v_c\right) - RC\frac{dV_c}{dt} = V_L$$

D:

We first solve for $V_c(t)$
$s^2 + 2\alpha s + \omega^2 = 0 \;\; ; \;\; \alpha^2 > \omega^2$
$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega^2} \quad ; s_1 = -399749.84 \;\; ; \;\; s_2 = -250.16$

$V_c(0^-) = A + B + C = 5$
$\frac{dV_c}{dt}(0^-) = -399749.84A - 250.16B = 0$
$V_c(\infty) = 10 = C$

Solving the system we get: $A = 0.00313 \;\; ; \;\; B = -5.003$

Thus
$V_L(t) = V_s u(t) - (0.00313e^{-399749.84t} - 5.003e^{-250.16t} + 10) - (0.004)(-1251e^{-399749t} + 1251.6e^{-250.16t})$

E:

$s^2 + 2\alpha s + \omega^2 = 0$ 1 real roots $s_1, s_2 = -\alpha = -10^4$
$V_c(0^-) = Ae^{-10^4 t} + tBe^{-10^4 t} + C = 5$
$\frac{dV_c}{dt}(0^-) = -10^4 e^{-10^4 t}A - 10^4 e^{-10^4 t}Bt + Be^{-10^4 t} = 0$
$V_c(\infty) = C = 10$

substitute in t and solve the system: $A = -5 \;\; ; B = -50000$
Thus
$V_L(t) = V_s u(t) - \left(-5e^{-10^4 t} - 50000te^{-10^4 t} + 10\right) - (2 * 10^{-4})(50000e^{10^4 t} + 5 * 10^8 te^{-10^4 t} - 50000e^{10^4 t})$

F:

2 complex roots $\beta = \sqrt{\omega^2 - \alpha^2} = 9683 \;\; ; \;\; s_1 = -2500 + j9683 \;\; ; \;\; s_2 = -2500 - j9683$
$V_c = e^{-2500t}(A\cos(9683t) + B\sin(9683t)) + C = 5$
$\frac{dV_c}{dt}: 9683e^{-2500t}A\sin(9683t) + 9683e^{-2500t}B\cos(9683t) - 2500e^{-2500t}A\cos(9683t) - 2500e^{-2500t}B\sin(9683t)$
$V_C(\infty) = C = 10$
Substitute in $t = 0^+$ and solve the system: $A = -5 \;\; ; B = -1.29$

Thus:

$V_L(t)$
$= V_s u(t) - \left(e^{-2500t}(-5\cos(9683t) - 1.29\sin(9683t) + 10)\right) - (-2.26e^{-2500t}\sin(9683t) + 4.47 * 10^{-4}\cos(9683t))$

Problem 5: RLC series design problem

Circuit is critically damped.

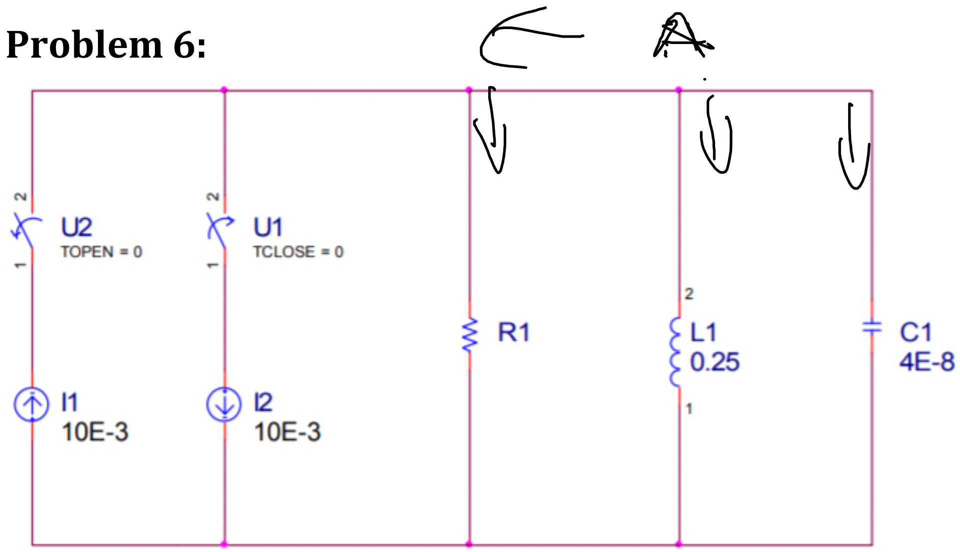
$\alpha^2 = \omega^2 = -2 * 10^6$

$$\omega^2 = 2 * 10^6 = \frac{1}{LC} = \frac{1}{L * 2 * 10^{-9}} \quad ; L = 250$$

$$\alpha^2 = \left(\frac{R}{2L}\right)^2 = 2 * 10^6 = \left(\frac{R}{2(250)}\right)^2 \quad ; R = 500000\sqrt{2}$$

The source function is equal to Cap voltage at $t = \infty$ thus source $= 10u(t)$

Problem 6:



At t =0, U1 closes and U2 opens.

A : Nodal at point A: $I_2 + I_R + I_L + I_c = 0$

$10mA + 0A + 10mA + I_c = 0 \quad ; I_c = -20mA$

B : $I_C(\infty) = 0A$

C : $I_R + I_L + I_C = -10mA$

$$\frac{1}{R}\frac{dV_c}{dt} + \frac{V}{L} + C\frac{d^2V_C}{dt^2} = 0 \qquad s^2 + 2\alpha s + \omega^2 \quad 2\ complex\ roots \quad \beta = \sqrt{\omega^2 - \alpha^2} = 9950$$

$$\frac{1}{RC}\frac{dV_c}{dt} + \frac{V}{LC} + \frac{d^2V_c}{dt^2} = 0$$

$V_c(t) = e^{-1000t}(A\cos(9950t) + B\sin(9950t)) \ ; V_c(0) = 0 = A$

$I_c(t) = (4 * 10^{-8})(9950e^{-1000t}B\cos(9950t) - 1000e^{-100t}B\sin(9950t)) = -20mA$

$B = -50.25$

$I_c(t) = (4 * 10^{-8})(-5 * 10^5 e^{-1000t}\cos(9950t) + 50250e^{-1000t}\sin(9950t))$

Problem 6

D :

$$\frac{1}{RC}\frac{dV_C}{dt}+\frac{V}{LC}+\frac{d^2V_c}{dt^2}=0$$

$s^2+2\alpha s+\omega^2=0$ $\quad 2\, real\, roots\;\; s_1, s_2=-\alpha\pm\sqrt{\alpha^2-\omega^2}\;\; s_1=-1010\;\; ; s_2=-98989$

$V_c(0)=A+B=0$
$I_c(0)=(4*10^{-8})(-1010Ae^{-1010t}-98989Be^{-98989t})=-20mA$

Solving the system we get $A=5.05\;\; ; B=-5.05$
$I_c(t)=(4*10^{-8})(-1010(5.05)e^{-1010t}+98989(5.05)e^{-98989t})$