# Linear Algebra HW#6 - Saaif Ahmed - 661925946

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52) Use Cramer's rule to solve the system

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= 8 \\ 4x_1 + 7x_2 + 5x_3 &= 20 \\ -2x_2 + 2x_3 &= 0 \end{aligned}$$

Question 52:

$$|A| = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 7 & 5 \\ 0 & -2 & 2 \end{bmatrix} \det A = 2*24 - 3*8 + 1(-8) = 16$$

$$|A_1| = \begin{bmatrix} 8 & 3 & 1 \\ 20 & 7 & 5 \\ 0 & -2 & 2 \end{bmatrix} \det A_1 = 8(24) - 3(40) + 1(-40) = 32$$

$$|A_2| = \begin{bmatrix} 2 & 8 & 1 \\ 4 & 20 & 5 \\ 0 & 0 & 2 \end{bmatrix} \text{bottom row is has 1 non zero} \det A_2 = 16$$

$$|A_3| = \begin{bmatrix} 2 & 3 & 8 \\ 4 & 7 & 20 \\ 0 & -2 & 0 \end{bmatrix} \text{bottom row has 1 non zero} \det A_3 = 16$$

**Answer:** $x = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} : x_1 = 2, x_2 = 1, x_3, = 1$

54) Given $A = \begin{bmatrix} 2 & -4 & 1 & 3 \\ 3 & -6 & 0 & -3 \\ -1 & 2 & 1 & 0 \\ 4 & -8 & -1 & 3 \end{bmatrix}$ compute adj$(A)$ and use it to find bases for $\mathcal{N}(A)$ and $\mathcal{N}(A^T)$

Question 54:

Col1 and Col2 are dependent so the minor matrix consisting of these columns will all be singular thus the determinant will be 0. Thus row 3 and 4 of the adjunct matrix are all 0. In addition $r_1 - 2r_3 = r_4$

Thus column will also be 0.

$$\det \begin{bmatrix} -6 & 0 & -3 \\ 2 & 1 & 0 \\ -8 & -1 & 3 \end{bmatrix} = -36 \quad \det \begin{bmatrix} 2 & 1 & 3 \\ 3 & 0 & -3 \\ 4 & -1 & 3 \end{bmatrix} = 36$$

$$\det \begin{bmatrix} 3 & 0 & 3 \\ -1 & 1 & 0 \\ 4 & -1 & 3 \end{bmatrix} = 18 \quad \det \begin{bmatrix} -4 & 1 & 3 \\ -6 & 0 & -3 \\ 2 & 1 & 0 \end{bmatrix} = -18$$

$$\det \begin{bmatrix} -4 & 1 & 3 \\ -6 & 0 & -3 \\ -8 & -1 & 3 \end{bmatrix} = 72 \quad \det \begin{bmatrix} 2 & 1 & 3 \\ 3 & 0 & -3 \\ -1 & 1 & 0 \end{bmatrix} = 18$$

$$adj\ (A) = \begin{bmatrix} -36 & 0 & 72 & 36 \\ -18 & 0 & 36 & 18 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_2$ is free $N(A) = span \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{Bmatrix}$ Of the transpose $x_3$ and $x_4$ are free so $N(A^T) = span \left( \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix}, \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix} \right)$

triangular form and deduce that $\det(M) = \det(A)\det(D - CA^{-1}B)$

56) Use the result in 55) (without proof) to compute the determinant of

$$M = \begin{bmatrix} 1 & 3 & -2 & 1 \\ 4 & 5 & 3 & -2 \\ -1 & 4 & 1 & 3 \\ 2 & -1 & 4 & -5 \end{bmatrix}$$

Question 56:

$\det(A) = -7$

$$A^{-1} = \begin{bmatrix} -\frac{5}{7} & \frac{3}{7} \\ \frac{4}{7} & -\frac{1}{7} \end{bmatrix} \quad D - \begin{bmatrix} -13 & \frac{57}{7} \\ 9 & -\frac{40}{7} \end{bmatrix} = \begin{bmatrix} 14 & -\frac{36}{7} \\ -5 & \frac{5}{7} \end{bmatrix}$$

$$CA^{-1} = \begin{bmatrix} \frac{11}{7} & -\frac{23}{7} \\ -\frac{6}{7} & \frac{17}{7} \end{bmatrix} \quad \det \begin{bmatrix} 14 & -\frac{36}{7} \\ -5 & \frac{5}{7} \end{bmatrix} = -\frac{100}{7}$$

$$CA^{-1}B = \begin{bmatrix} -13 & \frac{57}{7} \\ 9 & -\frac{40}{7} \end{bmatrix}$$

**Answer:** 100

59) Let $x_1, x_2, x_3 \in \mathbb{R}$ and denote by $\mathbb{P}_2$ the space of all real polynomials of degree at most 2. Let $T : \mathbb{P}_2 \to \mathbb{R}^3$ be the evaluation operator defined by $T : p(x) \to (p(x_1), p(x_2), p(x_3))$. Find the matrix representation of $T$ with respect to the standard basis, $\{e_i = x^i : i = 0, 1, 2\}$ and show that its determinant is equal to $(x_3 - x_2)(x_3 - x_1)(x_2 - x_1)$.

Question 59:

$$T(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} ; T(x) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} ; T(x^2) = \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \end{bmatrix}$$

Matrix of T $\begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix}$

$$x_2 x_3^2 - x_2^2 x_3 - x_1(x_3^2 - x_2^2) + x_1^2(x_3 - x_2)$$

which simplifies to $(x_3 - x_2)(x_3 - x_1)(x_2 - x_1)$

61) Find the eigenvalues and eigenvectors given the following matrices:

a. $A = \begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}$

b. $B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 4 \\ 0 & 2 & -2 \end{bmatrix}$

Question 61:

A: $\begin{bmatrix} 4-\lambda & 3 \\ 5 & 6-\lambda \end{bmatrix} (4-\lambda)(6-\lambda) - 15 = 0$

$\lambda_1 = 9, \lambda_2 = 1$ are the eigenvalues

$\begin{bmatrix} -5 & 3 & 0 \\ 5 & -3 & 0 \end{bmatrix}$ row reduce $\begin{bmatrix} 1 & -\frac{3}{5} & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\overrightarrow{v_{\lambda_1}} = \begin{bmatrix} 1 \\ -\frac{3}{5} \end{bmatrix}$

$\begin{bmatrix} 3 & 3 & 0 \\ 5 & 5 & 0 \end{bmatrix}$ row reduce $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\overrightarrow{v_{\lambda_2}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

B: $\begin{bmatrix} 1-\lambda & 2 & -1 \\ 0 & 0-\lambda & 4 \\ 0 & 2 & -2-\lambda \end{bmatrix} \rightarrow -\lambda^3 - \lambda^2 + 10\lambda - 8 = 0$

$\lambda_1 = 1$

$\lambda_2 = 2$

$\lambda_3 = -4$

$\begin{bmatrix} 0 & 2 & -1 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 2 & -3 & 0 \end{bmatrix}$ row reduce $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\overrightarrow{v_{\lambda_1}} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} -1 & 2 & -1 & 0 \\ 0 & -2 & 4 & 0 \\ 0 & 2 & -4 & 0 \end{bmatrix}$ row reduce $\begin{bmatrix} 1 & \square & -3 & \square \\ \square & 1 & -2 & \square \\ \square & \square & \square & \square \end{bmatrix}$ $\overrightarrow{v_{\lambda_3}} = \begin{bmatrix} -3 \\ -2 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 5 & 2 & -1 & 0 \\ 0 & 4 & 4 & 0 \\ 0 & 2 & 2 & 0 \end{bmatrix}$ row reduce $\begin{bmatrix} 1 & \square & -\frac{3}{5} & \square \\ \square & 1 & 1 & \square \\ \square & \square & \square & \square \end{bmatrix}$ $\overrightarrow{v_{\lambda_3}} = \begin{bmatrix} -\frac{3}{5} \\ 1 \\ 0 \end{bmatrix}$