

HW 5

Thursday, October 15, 2020 1:36 PM

1.

a)

$$P\left(Z \leq \frac{x - \mu}{\sigma}\right) = 1 - Q(Z) \quad 1 - P\left(Z \leq \frac{x - \mu}{\sigma}\right) = Q(Z)$$
$$Z = \frac{x - \mu}{\sigma} = \frac{219 - 210}{15} = 0.6 \quad Q(Z) = 0.2743$$

Answer: 0.2743

b)

$$P(|X - 210| > 21) = P(X > 231) = Q\left(\frac{231 - 210}{15}\right) = 0.0808$$

Answer: 0.0808

c)

$$P(x \in [195, 216]) = P(X > 195) - P(X > 216) = Q\left(\frac{195 - 210}{15}\right) - Q\left(\frac{216 - 210}{15}\right)$$

Answer: 0.4967

2.

a)

$$P(Y > a) = P\left(Y > \frac{y - \mu}{\sigma}\right) = P\left(Y > \frac{y - 45}{4}\right) = 0.1$$

Thus need to find $Q\left(\frac{y - 45}{4}\right) = 0.1$

$$y \approx 50.126$$

Answer: $a = 50.126$

b)

$$P(|X - 45| < b) = 0.5 = 1 - Q(X < 45) = 0.5$$

$$P(X < b + 45) = 0.5$$

$$Q(0) = 0.5 \rightarrow b = -45$$

Answer: $b = -45$

3.

a)

For these to be equal the ratio must be the same. Thus

$$\frac{d - \mu_1}{\sigma_1} = \frac{d - \mu_2}{\sigma_2} ; \frac{d - 100}{5} = \frac{d - 90}{6}$$

Thus $d = 150$

Answer: $D = 150$

b)

$$P(X > 110 | X > 105) = \frac{P(X > 110)}{P(X > 105)} = \frac{Q(\frac{110 - 100}{5})}{Q(\frac{105 - 100}{5})} = \frac{0.02275}{0.15865}$$

$$0.02275/0.15865=0.1434$$

Answer: 0.1434

4.

$$P(X \geq c) = c$$

$$\text{Thus } Q\left(\frac{c-b}{b}\right) = c$$

$$Q\left(\frac{c}{b} - \frac{b}{b}\right) = c$$

I just spammed numbers into Wolfram Alpha and found out that 0.5 for b/c works.

Test:

$$Q\left(\frac{0.5-0.5}{0.5}\right) = 0.5 \rightarrow Q(0) = 0.5 \text{ which is true.}$$

As a matter of trying to reason something: $Q\left(\frac{c}{b} - \frac{b}{b}\right) = c$. Feels like you need to get something clean. The clean results for $Q(x)$ include 0.5 and 1. 1 would not be in since that's the entire integral.

Thus we need to get $Q\left(\frac{c-b}{b}\right) = Q(0)$. If $c = b$ then we have our answer.

Answer: $b = c = 0.5$