1) Laplace transforms/Transfer functions

Use Laplace transform tables!!!!

1.1: Find the Laplace transform of

$$f(t) = \left(\cos(2t) + e^{-4t}\right)\cdot u(t) \qquad \text{(simplify into one ratio)}$$

$f(t) = \cos(2t)u(t) + e^{-4t}u(t)$ -> Use a table $\dfrac{s}{s^2+2^2} + (s+-4)^{-1}$

$$F(s) = \frac{2(s^2-2s+2)}{(s-4)(s^2+4)}$$

1.2: Find the poles and zeros of the following functions. Indicate any repearted poles and complex conjugate poles. Expand the transforms using partial fraction expansion.

1.2.1 $\quad F(s) = \dfrac{20}{(s+3)\cdot\left(s^2+8s+25\right)}$

Zeros: none

poles: -3, $-4+3i$, $-4-3i$ complex

$\dfrac{-8 \pm \sqrt{64-100}}{2} = -4+3i$, $-4-3i$

$$\frac{A}{(s+3)} + \frac{B}{(s-(-4+3i))} + \frac{C}{(s-(-4-3i))} = \text{pfe } F(s)$$

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1.2.2 $\quad F(s) = \dfrac{2s^2+18s+12}{s^4+9\cdot s^3+34\cdot s^2+90\cdot s+100}$

Zeros: $\dfrac{-9+\sqrt{33}}{2}$, $\dfrac{-9-\sqrt{33}}{2}$

poles: $-2, -5, -1+3i, -1-3i$

$\dfrac{-18 \pm \sqrt{18^2-4(24)}}{4} = \dfrac{-9+\sqrt{33}}{2}$, $\dfrac{-9-\sqrt{33}}{2}$

$s^4+9s^3+34s^2+90s+100$

$\downarrow$

$(s+2)(s+5)(s^2+2s+10)$

$-2 \quad -5 \quad \dfrac{-2 \pm \sqrt{2^2-40}}{2} = -1+3i$, $-1-3i$

$$\frac{A}{s+2} + \frac{B}{s+5} + \frac{C}{s+1-3i} + \frac{D}{s+1+3i}$$

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Problem 1.2.1

$$\frac{20}{(s+3)(s+4-3i)(s+4+3i)} = \frac{A}{(s+3)} + \frac{B}{(s+4-3i)} + \frac{C}{(s+4+3i)}$$

$$20 = A(s+4-3i)(s+4+3i) + B(s+3)(s+4+3i) + C(s+3)(s+4-3i)$$

For $s=-3$

$$20 = A(1-3i)(1+3i)$$

$$A = 2$$

For $s = -4-3i$

$$20 = C(-1-3i)(-6i)$$

$$C = -1 - \frac{1}{3}i$$

For $s = -4+3i$

$$20 = B(-1+3i)(6i)$$

$$B = -1 + \frac{1}{3}i$$

$$F(s) = \frac{2}{s+3} + \frac{-1+\frac{1}{3}i}{s+4-3i} + \frac{-1-\frac{1}{3}i}{s+4+3i}$$

$$\cancel{F(t) = 2e^{-3t} + (-1+\tfrac{1}{3}i)e^{(-4+3i)(t)} + (-1-\tfrac{1}{3}}$$

$$\boxed{f(t) = 2e^{-3t} + (-1+\tfrac{1}{3}i)e^{(-4+3i)t} + (-1-\tfrac{1}{3}i)e^{(-4-3i)t}}$$

Problem 1.2.2

$$2s^2+18s+12 = A(s+5)(s+1-3i)(s+1+3i) + B(s+2)(s+1-3i)(s+1+3i) + C(s+2)(s+5)(s+1+3i) + D(s+2)(s+5)(s+1-3i)$$

For $s=-5$

$-28 = B(-75)$ $\quad B=\frac{28}{75}$

For $s=-2$

$-16 = A(30)$ $\quad A=\frac{-8}{15}$

For $s=-1+3i$

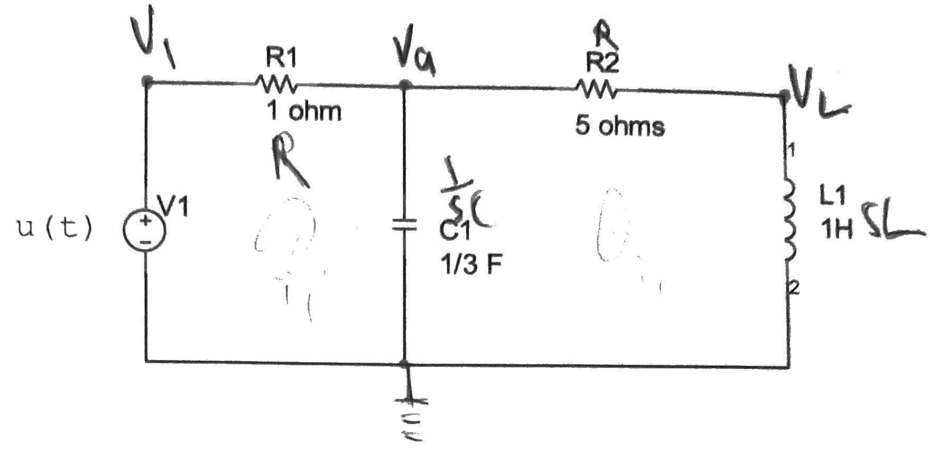
$-22+42j = (-90-30i)C$ $\quad C=0.08-\frac{37}{75}j$

For $s=-1-3i$

$-22+42j = (-90+30j)D$ $\quad D=0.08+\frac{37}{75}j$

$$F(t) = \frac{-8}{15}e^{-2t} + \frac{28}{75}e^{-5t} + \left(0.08-\frac{37}{75}j\right)e^{-(1-3i)t} + \left(0.08+\frac{37}{75}j\right)e^{-(1+3i)t}$$

2) S-domain equivalent



2.1: In the above circuit, the intial conditions are zero. Using s-transforms, find the voltage across the inductor for t>0.

$i_C = 0$ $V_C = 0$ $i_L = 0$, $V_L = 0$

$\frac{V_a - \frac{1}{s}}{R_1} + V_a sC + \frac{V_a - V_L}{R_2} = 0$ $\quad \frac{-V_L}{sL} = \frac{V_L - V_a}{R_2}$ $\quad V_a = +\frac{R_2 V_L}{sL}$

$\frac{V_L}{sL} + \frac{V_L - V_a}{R_2} = 0$ $\quad +\frac{R_2 V_L}{sL} + V_L = -V_a$

$\frac{5V_L}{s} + V_L - \frac{1}{s} + \left(\frac{5V_L}{s} + V_L\right)\frac{s}{3} + \frac{1}{\cancel{5}}\left(\frac{\cancel{5}V_L}{s} + \cancel{V_L} - \cancel{V_L}\right) = 0$

$\frac{5V_L}{s} + V_L - \frac{1}{s} + \frac{5}{3}V_L + \frac{V_L s}{3} + \frac{V_L}{s} = 0$ $\quad V_L\left(\frac{5}{s} + 1 + \frac{5}{3} + \frac{s}{3} + \frac{1}{s}\right) = \frac{1}{s}$

$V_L\left(\frac{s^2+8s+18}{3s}\right) = \frac{1}{s}$ $\quad V_L = \frac{1}{s} \div \frac{s^2+8s+18}{3s}$ $\quad V_L = \frac{3}{s^2+8s+18}$

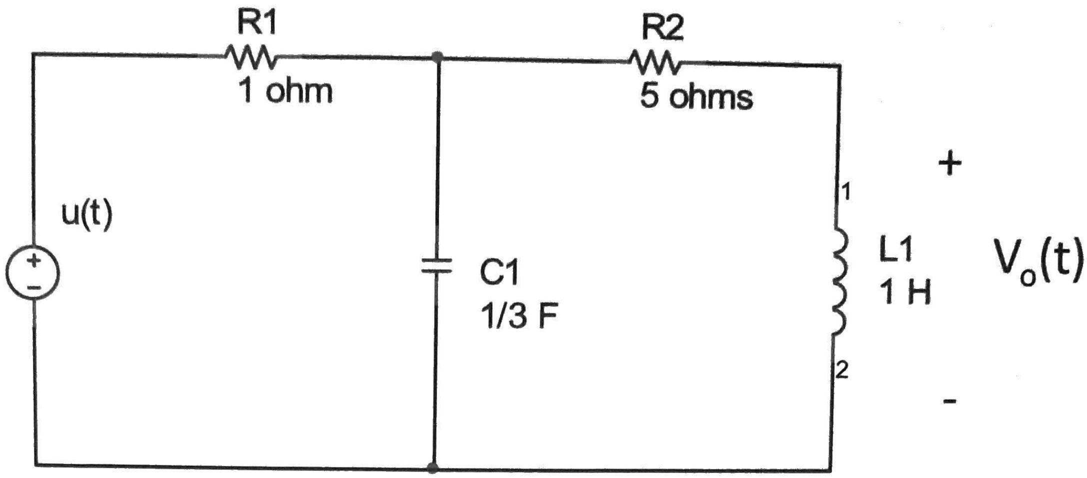
$V_L = \frac{3}{(s+4+j\sqrt{2})(s+4-j\sqrt{2})} = \frac{A}{s+4+j\sqrt{2}} + \frac{B}{s+4-j\sqrt{2}}$

$3 = A(s+4-j\sqrt{2}) + B(s+4+j\sqrt{2})\big|_{s=-4+j\sqrt{2}}$ $\quad 3 = B(2j\sqrt{2})$ $\quad B = \frac{3}{2j\sqrt{2}}$

$3 = A(-2j\sqrt{2})\big|_{s=-4-j\sqrt{2}}$ $\quad A = -\frac{3}{2j\sqrt{2}}$

$$\boxed{V_L(t) = \frac{-3}{2j\sqrt{2}}e^{-(4+j\sqrt{2})t} + \frac{3}{2j\sqrt{2}}e^{-(4-j\sqrt{2})t}}$$

3) Circuits and Differential Equations



3.1: Draw the s-domain equivalent circuit. Assume all intial conditions are zero and the source is an arbitrary source.

3.2 Using impedances, determine the expression for Vo(t). Consider using mesh analysis Make one ratio.

3.1

3.2 : $-\frac{1}{s} + i_1 + i_1\frac{3}{s} - i_2\frac{3}{s} = 0$

$\frac{3}{s}i_2 - \frac{3}{s}i_1 + 5i_2 + i_2 s = 0$

$\frac{1}{s} + i_2\frac{3}{s} = i_1 + i_1\frac{3}{s} = i_1\left(1+\frac{3}{s}\right)$

$i_1 = \left(\frac{1}{s} + i_2\frac{3}{s}\right) / \left(1+\frac{3}{s}\right)$

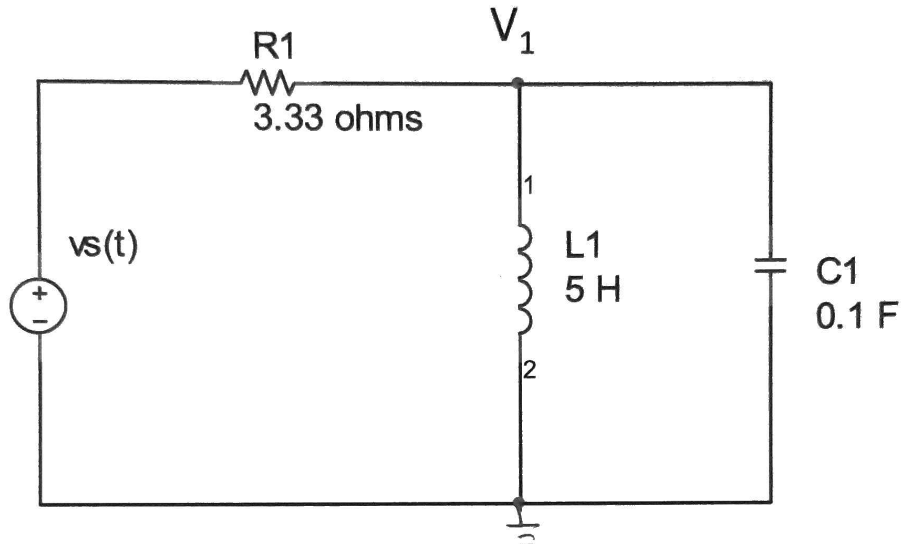
$\frac{3}{s}i_2 - \frac{3(1+3i_2)}{s(s+3)} + 5i_2 + i_2 s = 0$

$i_2 = \frac{3}{s(18+8s+s^2)}$; $V_L = i_2 \cdot s$ $\quad V_L = \frac{3\cancel{s}}{\cancel{s}(18+8s+s^2)} = \frac{3}{(s+4-j\sqrt{2})(s+4+j\sqrt{2})}$

$V_L = \frac{A}{s+4+j\sqrt{2}} + \frac{B}{s+4-j\sqrt{2}}$ | $s = -4+j\sqrt{2}$ $\quad B = \frac{3}{2j\sqrt{2}}$ $\quad$ A = complex conjugate

$$\boxed{V_L(t) = \frac{-3}{2j\sqrt{2}}e^{-(4+j\sqrt{2})t} + \frac{3}{2j\sqrt{3}}e^{-(4-j\sqrt{2})t}}$$

4) RLC and initial conditions



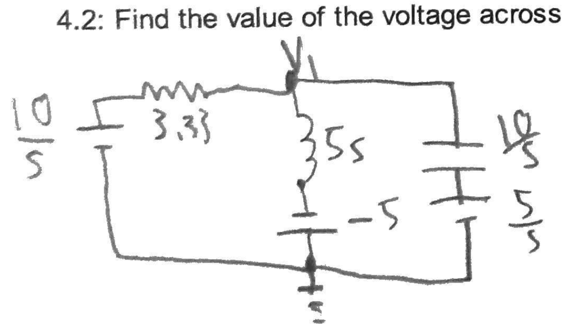
$v_S(t) = 10u(t)$

AND assume that -1A flows through the inductor and +5V is across the capacitor at t=0....i.e. vc(0)=5 and iL(0)= -1

4.1: Draw the s-domain equivalent with initial conditions.

4.2: Find the value of the voltage across the capacitor, vc(t), using nodal analysis (at node V1) and laplace.

4.1:



$$\frac{V_1 - \frac{10}{s}}{3.33} + \frac{V_1 + 5}{5s} + \frac{V_1 - \frac{5}{s}}{\frac{10}{s}} = 0$$

$$V_c = V_1 - \frac{5}{s}$$

4.2: $\frac{20 + 5s}{s^2 + 3s + 2} = V_1 = \frac{A}{(s+1)} + \frac{B}{(s+2)}$ $\quad 20 + 5s = A(s+2) + B(s+1$

$s = -2 \,|\, 10 = B(-1) \quad B = -10$ $\qquad s = -1 \,|\, 15 = A$

$V_c = \frac{15}{(s+1)} + \frac{-10}{s+2} - \frac{5}{s}$ $\qquad \boxed{V_c(t) = 15e^{-t} - 10e^{-2t} - 5}$