Saart Ahmed 66/925946

"I have not witnessed any wrongdoing nor have I personally violated any conditions of the Honor Code, while taking this examination."

- Saart Ahmed

12/16/20

1. $\frac{-a \pm \sqrt{a^2-100}}{2}$

$(s+2.5+aj)(s+2.5-aj)$

$s^2+2.5s-saj+2.5s+6.25$

$-2.5aj+saj+2.5aj+-a^2j^2$

$s^2+5s+6.25+a^2$

$(s-(-2.5+aj))(s-(-2.5-aj))$

$s^2-s(-2.5-aj)+s(-2.5+aj)$

$+(-2.5-aj)(-2.5+aj)$

$s^2+2.5s+saj-2.5s+saj+6.25$

$-a^2j^2$

$\cancel{s^2+2ajs+6.25+a^2} \quad \cancel{s^2+bs+25}$

$s^2+5s+6.25+a^2 = s^2+as+25$

$a=4.33$

$s^2+s(B+k)+2k$

$s^2+5s+25$

$B+k=5$

$2k=25 \quad k=12.5$

$B=-7.5$

$2k=\omega_n^2$

$\boxed{\begin{array}{l}\omega_n = 5 \text{ rad/sec} \\ \text{damping factor} = \frac{1}{2}\end{array}}$

$2\zeta\omega_n = B+k$

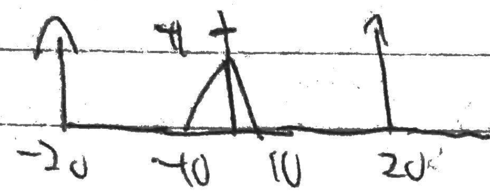
$= 5$

$\zeta = \frac{1}{2}$

2. $\cos(20t) \Rightarrow FT = \pi[\delta(w-20) + \delta(w+20)]$

$w_1 = x(t) + c(t) \rightarrow$ linearity of FT

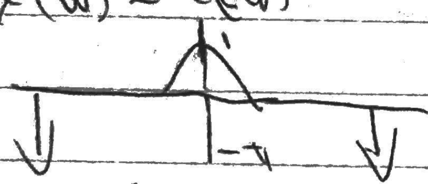
$= X(w) + C(w)$



$W_1(W)$

likewise

$X(w) - C(w)$

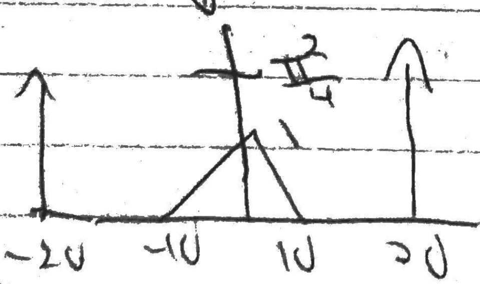


$W_2(W)$

$\Downarrow$

$\frac{1}{4}(\text{input})^2$

$\Downarrow$



$V_1(w)$

$(x+c)^2$

$x^2 + 2cx + c^2$

$(x-c)^2 (x-c)$

$x^2 - 2cy + c^2$

$x^2 + 2cx + c^2$

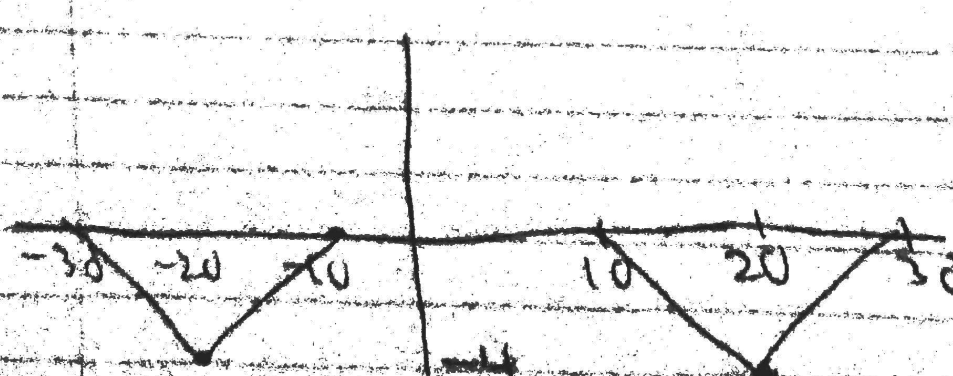
$\cancel{x^2 - 2cx}$

$x^2 - 2cx + c^2 - \cancel{x^2} - \cancel{2cx} - \cancel{c^2}$

$y(t) = -4\, c(t) \cdot x(t)$

$Y(w) = -4\,(C(w) * X(w))$

$Y(w)$



3. ~~use~~ laplae

$$h(s) = \frac{1}{s+2}$$

$$x(s) = \frac{1}{s} - \frac{e^{-s2}}{s}$$

$$Y(s) = \left(\frac{1}{s+2}\right)\left(\frac{1}{s} - \frac{e^{-2s}}{s}\right)$$

$$= \frac{1-e^{-2s}}{s(s+2)}$$

$$\frac{A}{s} + \frac{B}{s+2} = \frac{1-e^{-2s}}{s(s+2)}$$

$$A(s+2) + B(s) = 1 - e^{-2s}$$

$$s=-2 \qquad -2B = 1-e^{4}$$

$$B = \frac{1-e^{4}}{-2} \qquad s=0$$

$$\cancel{A=} \qquad 2A = 1-e^{-4}$$

$$A = \frac{1-e^{-4}}{2}$$

$$Y(s) = \frac{1-e^{4}}{2s} + \frac{1-e^{-4}}{2s}$$

$$Y(t) = \mathcal{L}^{-1}(Y(s))$$

$$\boxed{Y(t) = \frac{1-e^{4}}{2}u(t) + \frac{1-e^{-4}}{2}u(t)}$$

~~th~~ 5. A: $\frac{-2 \pm \sqrt{2^2-4(2)}}{2}$ $\qquad s^2+2s+2$

$$\frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm 2i}{2}$$
$$= -1 \pm i$$

[Poles are: $(-1+i), (-1-i)$]

s^2	1	2
s^1	2	0
s_0	$\frac{1}{2}(4-1)$	

system is stable because first colum is all same sign.

B: $T(s) = \dfrac{\frac{s}{s^2+2s+2}}{1+\frac{ks}{s^2+2s+2}}$

$$T(s) = \frac{s}{\cancel{s^2+2s+2}} \cdot \frac{\cancel{s^2+2s+2}}{(s^2+2s+2+ks)}$$
$$= s^2+(2+k)s+2$$

s^2	1	2
s^1	$2+k$	0
s^0		

$$\frac{1}{2+k}((2+k)2-1)$$
$$(2+k)2-1>0$$
$$4+2k-1$$
$$2k+3>0$$
$$k>-\frac{3}{2}$$

$$k > -\frac{3}{2}$$

6. $X(s) = \frac{s+3}{(s+1)(s^2+2s+5)}$ $\frac{A}{(s+1)} + \frac{B}{s^2+2s+5} = X(s)$

$A = \frac{1}{2}$ $B = -\frac{1}{2}(s-1)$

$\frac{1}{2(s+1)} + \frac{-s+1}{2(s^2+2s+5)}$

$-\frac{1}{2}\frac{(s+1)}{(s+1)^2+4} + \frac{1}{(s+1)^2+4}$

$x(t) = \frac{1}{2}e^{-t}u(t) + (e^{-t}\cos(2t))u(t)$
$+ (\frac{1}{2}e^{-t}\sin(2t))u(t)$

$Y(s) = \frac{(s+3)e^{-2s}}{s+2}$ $e^{-2s} \rightarrow$ time shift t

$\frac{(s+3)}{(s+2)} \rightarrow \delta(t) + e^{-2t}$

$Y(t) = u(t-2)(\delta(t-2) + e^{-2(t-2)})$

$Z(s) = \frac{A}{(s+2)} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2} = \frac{s+3}{(s+2)(s+1)^2}$

$A = 1,\ B = -1,\ C = 2$

$\mathcal{L}^{-1}(Z(s)) =$

$z(t) = e^{-2t}u(t) + e^{-t}(u(t)) + 2e^{-t}(u(t))t$

8. $X(s) = \cancel{\frac{e^{-s}}{s}} + \frac{e^{-4\pi}}{s-2} + \cancel{\frac{e^{-4\pi s}\cdot e^{-8\pi}}{s-2}}$

$$\int_1^{\infty} e^{-2t} \cdot e^{-st}\, dt = \int_1^{\infty} e^{-2t-st}\, dt$$

$$\int_1^{\infty} e^{(-s-2)t}\, dt$$

$$= \frac{1}{-s-2} e^{-t(s+2)} \Big|_1^{\infty}$$

$$= \frac{e^{-s-2}}{-s-2}$$

$$\int_{4\pi}^{\infty} e^{2t} e^{-st}\, dt = \int_{4\pi}^{\infty} e^{t(2-s)}\, dt$$

$$\frac{1}{(2-s)} e^{t(2-s)} \Big|_{4\pi}^{\infty}$$

$$X(s) = \left[\frac{e^{-s-2}}{-s-2} + \frac{e^{-4\pi}}{s-2} + \frac{e^{-4\pi s}\cdot e^{-8\pi}}{2-s}\right]$$

10. $a_0 = \frac{1}{T}\int_{-2}^{2} x(t)\,dt$

$= \frac{1}{4} \cdot 6 \qquad a_0 = \left(\frac{6}{4}\right)$

$a_0 = \frac{6}{4}$

$a_k = \frac{1}{4}\int_{-2}^{2} x(t) e^{-jk\omega_0 t}\,dt \qquad \omega_0 = \frac{\pi}{2}$

$a_k = \frac{1}{4}\left(\int_{-2}^{-1} e^{-jk\frac{\pi}{2}t}\,dt + 2\int_{-1}^{1} e^{-jk\frac{\pi}{2}t} + \int_{1}^{2} e^{-jk\frac{\pi}{2}t}\,dt\right)$

$\frac{2e^{-jk\frac{\pi}{2}t}}{-jk\pi}\Big|_{-2}^{-1} + 2\left(\frac{2e^{-jk\frac{\pi}{2}t}}{-jk\pi}\Big|_{-1}^{1}\right) + \Big|_{1}^{2}$

$\frac{2e^{+jk\frac{\pi}{2}}}{-jk\pi} - \frac{2e^{jk\pi}}{-jk\pi} + 2\left(\frac{2e^{-jk\frac{\pi}{2}}}{-jk\pi} - \frac{2e^{jk\frac{\pi}{2}}}{-jk\pi}\right)$

$+ \frac{2e^{-jk\pi}}{-jk\pi} - \frac{2e^{-jk\frac{\pi}{2}}}{-jk\pi}$

$\frac{2e^{jk\frac{\pi}{2}} - 2e^{jk\pi}}{-jk\pi} + \frac{2e^{-jk\pi} - 2e^{-jk\frac{\pi}{2}}}{-jk\pi}$

$+ \frac{4e^{-jk\frac{\pi}{2}} - 4e^{jk\frac{\pi}{2}}}{-jk\pi}$

$\frac{-2e^{jk\frac{\pi}{2}} - 2e^{jk\pi} + 2e^{-jk\pi} - e^{-jk\frac{\pi}{2}}}{-j4\pi}$

$\cos($

$-2jk\pi\cos\left(-\frac{k\pi}{2}\right)$

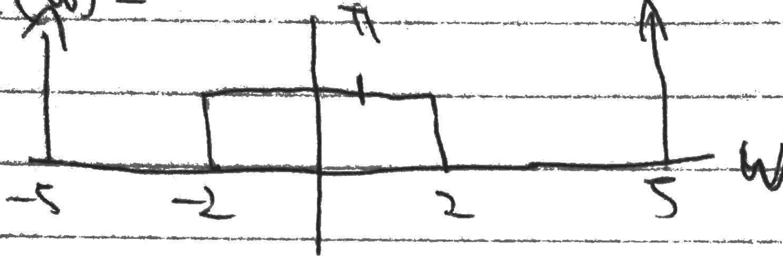
$\left[a_k = -4jk\pi\cos\left(-\frac{k\pi}{2}\right) + 4k\pi\sin(k\pi)\right]$

11. A: $x(t) \Rightarrow FT =$

$$\frac{\sin(2t)}{(2t)} + \cos(5t) \rightarrow \pi(\delta(w-5) + \delta(w+5))$$

$W = 2$

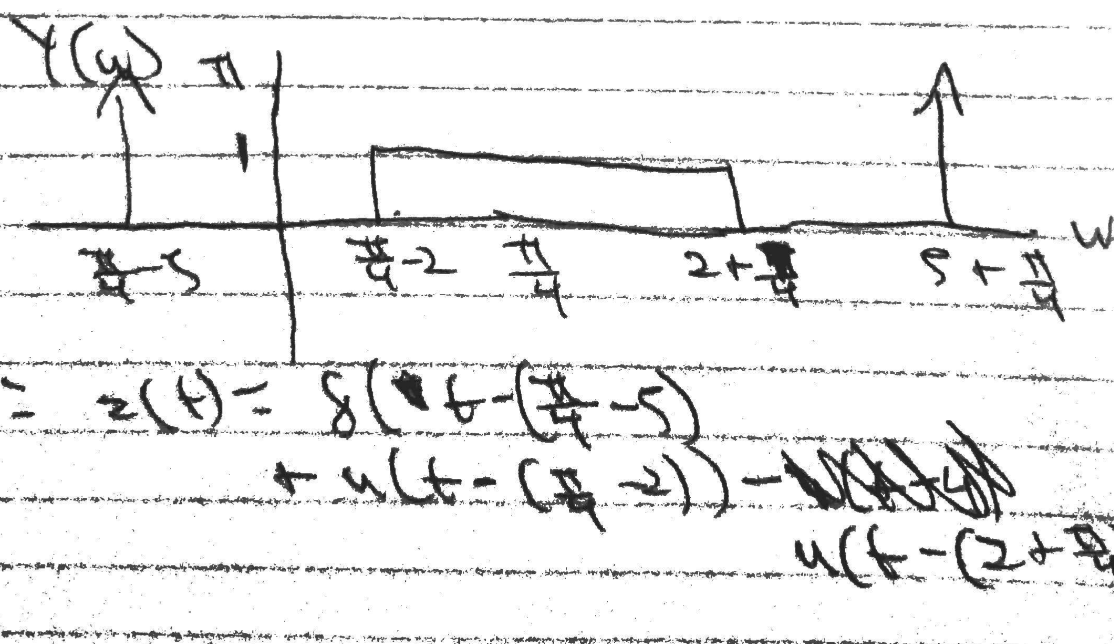
$X(w) =$



B: highest freq = 5 $w = 5$

Nyquist rate = 10

C: $Y(w)$ = repeats $X(w)$ every $\frac{\pi}{4}$



D: $z(t) = \delta(t - (\frac{\pi}{4} - 5)) + u(t - (\frac{\pi}{4} - 2)) - u(t - (2 + \frac{\pi}{4}))$