Assignment #8 - Saaif Ahmed

Sunday, November 17, 2019 9:28 PM

Problem 18.20

(a) $P[\min(\mathbf{X}, \mathbf{Y}) \leq m]$.

$\mathbf{X}$ and $\mathbf{Y}$ are independent so the P of $\boldsymbol{X} \wedge \boldsymbol{Y}$ is equal to $\boldsymbol{X} * \boldsymbol{Y}$.

Calculate $P[\min(\boldsymbol{X}, \boldsymbol{Y}) \geq m]$ and the answer is $1 - P[\min(\boldsymbol{X}, \boldsymbol{Y}) \geq m]$

PDF of $\mathbf{X}$ is $\frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots$

Sum them all together: $P[\boldsymbol{X} > m] = \sum_{i=0}^{m-1} \frac{1}{2^i} = 1 - \frac{1}{2^{m-1}}$

So $P[\boldsymbol{X} \wedge \boldsymbol{Y}] = \left(1 - \frac{1}{2^{m-1}}\right)^2$

Thus $1 - \left(1 - \frac{1}{2^{m-1}}\right)^2 = P[\min(\boldsymbol{X}, \boldsymbol{Y}) \leq m]$

Answer: $1 - \left(1 - \frac{1}{2^{m-1}}\right)^2$

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Problem 18.33

(l) Draw 10 cards from a shuffled deck and count the number of aces.
Drawing affects the probability

Answer: Not Binomial

(m) You have 10 shuffled decks. Draw one card from each deck and count the number of aces.
Drawing 1 card from each deck does not affect the next draw

Answer: Binomial

(o) Toss 20 fair coins and re-toss (just once) all coins which flipped H. Count the number of:

(i) Coins showing heads at the end.
Number of tosses depends on the first toss so not fixed

Answer: Not binomial

(ii) Heads tossed in the experiment.
Not binary so not binomial. Can get 0 heads, 1 heads, or 2 heads.

Answer: Not binomial

(p) Your total winnings in n fair coin flips when you win $2 per H and lose $1 per T.
You can count the number of successes k binomially but the answer is keeping track of winnings which goes between H and T.

Answer: Not binomial

(q) A box has 50 bulbs in a random order, with 5 being defective. Of the first 5 bulbs, count the number defective.
Trials depend on each other. Not like choosing questions on a test.

Answer: Not binomial

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Problem 19.11

A game costs $x to play. You toss 4 fair coins. If you get more heads than tails, you win $10 + x for a profit of $10. Otherwise, you lose and get nothing back, so your loss is $x. What is your expected profit?

Use Law of Total Expectation:

Let $X =$ "profit"

$$E[X] = E[X|more\ H] * P[more\ H] + E[X|less\ H] * P[less\ H]$$

$$= 10 * \frac{5}{16} + \frac{(-x)11}{16}$$

$$= \frac{50 - 11x}{16}$$

Answer: $\frac{50-11x}{16}$

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Problem 19.35

A box has 1024 fair and 1 two-headed coin. You pick a coin randomly, make 10 flips and get all H.

(a) You flip the same coin you picked 100 times. What is the expected number of H?
Construct Law of Total Expectation expression

$E[H] = E[H \mid fair] * P[fair] + E[H \mid not\ fair] * P[not\ fair]$
$P[fair]$ is influenced by given information

$$P[fair|10H] = \frac{P[fair \cap 10H]}{P[10H]} = \frac{P[10H|fair]*P[fair]}{P[10H]}$$

$$P[10H] = P[10H \mid fair] * P[fair] + P[10|not\ fair] * P[not\ fair]$$

$$= \frac{1}{2^{10}} * \frac{1024}{1025} + 1 * \frac{1}{1025} = \frac{2}{1025}$$

$$P[fair\ |10H] = \frac{P[10|fair]*P[fair]}{P[10H]} = \frac{\frac{1}{2^{10}}*\frac{1024}{1025}}{\frac{2}{1025}} = \frac{1}{2}$$

$$E[H] = 50 * \frac{1}{2} + 100 * \frac{1}{2} = 75$$

Answer: 75

(b) You flip the same coin you picked until you get H. What is the expected number of flips you make?

$$E[toss] = E[toss \mid fair] * P[fair] + E[toss\ |not\ fair] * P[not\ fair]$$

$$= 2 * \frac{1}{2} + 1 * \frac{1}{2} = 1\frac{1}{2}$$

Answer: $1\frac{1}{2}$ or $\frac{3}{2}$

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Problem 19.54

A Martian couple has children until they have 2 males (sexes of children are independent). Compute the expected number of children the couple will have if, on Mars, males are:

(a) Half as likely as females.

Geometric distribution. In this case the chance for male is $\frac{1}{3}$

Expected value is $\frac{1}{p}$ and events are independent

$E[kids\ for\ two\ boys] = \frac{1}{p} + \frac{1}{p} = 3 + 3 = 6$

Answer: 6

(b) Just as likely as females.

Geometric distribution. In this case the chance for male is $\frac{1}{2}$

Expected value is $\frac{1}{p}$ and independent

$E[kids\ for\ two\ boys] = \frac{1}{p} + \frac{1}{p} = 2 + 2 = 4$

Answer: 4

(c) Twice as likely as females.

Geometric distribution. In this case the chance for male is $\frac{2}{3}$

Expected value is $\frac{1}{p}$ and independent

$E[kids\ for\ two\ boys] = \frac{1}{p} + \frac{1}{p} = \frac{3}{2} + \frac{3}{2} = 3$

Answer: 3

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Problem 20.11

Ten sailors have a night out on shore. They return drunk and sleep in random bunks. Compute:

(a) The probability that all sailors sleep in their own bunks.

Permutation of 10 sailors and only 1 is correct so

$\frac{1}{10!}$

Answer: $\frac{1}{10!}$

(b) The probability that 1 sailor sleeps in the wrong bunk.

This is clearly 0. If one sailor has the wrong bunk that means another one will have to have the wrong bunk.

Answer: 0

(c) The probability that 2 sailors sleep in the wrong bunk.

Same as calculating probability that 8 sailors get the correct bed.

From 10 sailors pick 8 : $\binom{10}{8}$

Then multiply by probability of all being correct : $\frac{1}{10!} * \binom{10}{8} = \frac{45}{10!}$

Answer: $\frac{45}{10!}$

(d) The expected number of sailors that sleep in their own bunk.

Each sailor has a $\frac{1}{10}$ chance to get their own bed.

So $E[X_i] = \frac{1}{10}$

For all 10 sailors $E[X] = \frac{1}{10} * 10 = 1$

Answer: 1 sailor