Multivar Quiz #8 Saaif Ahmed

Honor Pledge:
"I have neither given nor received any illegal aid on this exam"
-Saaif Ahmed 11/11/20

Problem 6

Let $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$.   Compute the products $A^{-1}B$ and

$B^T A^{-1}$.

$Det(A)$
$0(1^2 - 1^2) + 0(1^2 - 0) + 1(0 - 1^2) = -1$

$A^{-1} = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$

$R1' = r1 - r2$

$= \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$

$R2' = r2 - r3$

$= \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$

Thus

$A^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = A^{-1}B$

$B^T = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$

$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} = B^T A^{-1}$

Answer: $\begin{bmatrix} 0 & -1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = A^{-1}B$; $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} = B^T A^{-1}$