$p \to q \equiv \neg q \to \neg p \equiv \neg p \lor q$

$\sum_{i=k}^{n} 1 = n + 1 - k$	$\sum_{i=1}^{n} i = \frac{1}{2} n(n+1)$	$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$
$\sum_{i=1}^{n} f(x) = n * f(x)$	$\sum_{i=1}^{n} i^2 = \frac{1}{6} n(n+1)(2n+1)$	$\sum_{i=0}^{n} \frac{1}{2^i} = 2 - \frac{1}{2^n}$
$\sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r} (r \neq 1)$	$\sum_{i=1}^{n} i^3 = \frac{1}{4} n^2 (n+1)^2$	$\sum_{i=1}^{n} \log i = \log n!$

For polynomials, growth rate is the highest order. For nested sums, growth rate is number of nesting plus order of summand. Analyze the largest order term to find behavior

$\int_{m-1}^{n} f(x) \le \sum_{i=m}^{n} f(i) \le \int_{m}^{n+1} f(x)$ Linear combination of m, n: Z=mx + ny >0

Any number Q = (quotient)(divisor) +(remainder)

$\gcd(m, n) = \gcd(m, rem(n, m))$

$$
\begin{aligned}
\gcd(42, 108) &= \gcd(24, 42) && 24 = \mathbf{108} - 2 \cdot \mathbf{42} \\
&= \gcd(18, 24) && 18 = 42 - 24 = 42 - \underbrace{(108 - 2 \cdot 42)}_{24} = 3 \cdot \mathbf{42} - \mathbf{108} \\
&= \gcd(6, 18) && 6 = 24 - 18 = \underbrace{(108 - 2 \cdot 42)}_{24} - \underbrace{(3 \cdot 42 - 108)}_{18} = 2 \cdot \mathbf{108} - 5 \cdot \mathbf{42} \\
&= \gcd(0, 6) && 0 = 18 - 3 \cdot 6 \\
&= 6 && \gcd(0, n) = n
\end{aligned}
$$

Remainders in Euclid's algorithm are integer linear combinations of 42 and 108.

In particular, $\gcd(42, 108) = 6 = 2 \times 108 - 5 \times 42$.

(A+B) mod C = (A mod C + B mod C) mod C If ac = bc (mod d), you can cancel c if gcd(c, d) = 1

Graphs are Isomorphic if the Vertices and Edges can be relabeled and be shown to be equal

Handshake Theorem: $\sum_{i=1}^{n} \delta_i = 2|E|$ Sum of degrees from all vertices is Twice # of edges

Sum Rule: N objects of two types: N1 of type1 and N2 of type2. Then, N = N1 + N2 .

Product Rule

Let N be the number of choices for a sequence X1, X2, X3…Xr

Let Nr be the number of choices for Xr after you choose X1 X2 X3 · · · Xr−1. N = N1 × N2 × N3 × N4 × · · · × Nr.

$\boxed{\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}}$ (sum rule) base cases: $\binom{n}{0} = 1; \binom{n}{n} = 1.$

Binomial Theorem:

$(x+y)^n = \sum_{i=1}^{n} \binom{n}{i} x^i y^{n-i}$

	No rep	rep
k-sequence	$\frac{n!}{(n-k)!}$	n^k
k-subset	$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$	$\binom{k+n-1}{n-1}$
(k1,k2,…kr)		$\frac{k!}{k1! + k2! \ldots kr!}$

$|A \cup B| = |A| + |B| - |A \cap B|$
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
Principle of Inclusion Exclusion