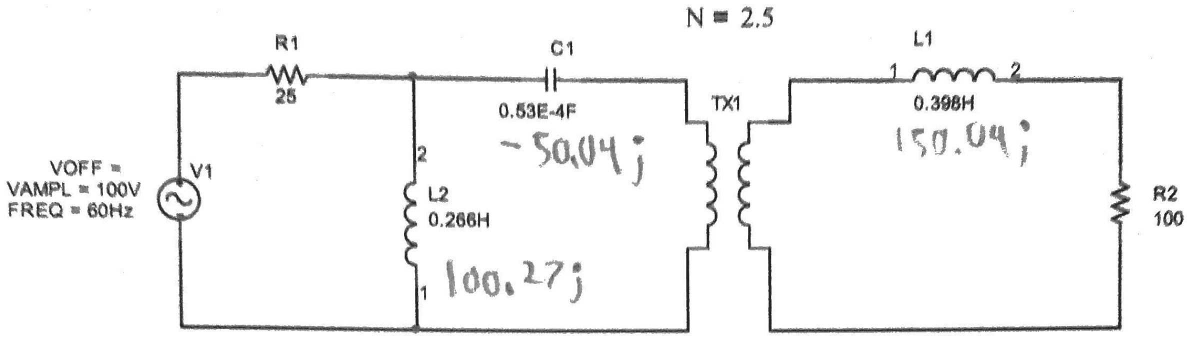
Electric Circuits
ECSE 2010

Prof. Shayla Sawyer
HW8

Spring 2020

Problem 1) Ideal Transformers



The above circuit has a 100 V, 60 Hz source.

Hint: Draw in impedance form, and find Thevenin Equivalent first

a. Determine the equivalent circuit when referring the secondary to the primary (and draw the circuit).

b. Determine the equivalent circuit when referring the primary to the secondary (and draw the circuit).

c. Determine the total complex power dissipated through the impedance in the primary.

d. Determine the total complex power dissipated through the impedance in the secondary.

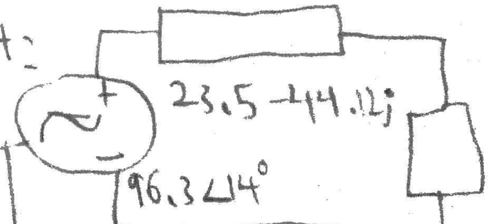
e. Verify that the power dissipated is equal to the power generated by the source.

A: $V_{TH} = \frac{100(100j)}{100j+25} = 94.12 + 23.53j$ $\quad Z_{th} = \frac{Z_{R_1} \cdot Z_{L2}}{Z_{R_1} + Z_{L2}} + Z_{C_1} =$ $23.53 - 44.12j$

$Z_{Load_{eq}} = \frac{Z_{Load}}{(2.5)^2} = \frac{100+150j}{(2.5)^2} = 16.24j$

$\tan^{-1}\left(\frac{23.53}{94}\right) = 14^\circ$ $\quad \sqrt{94.12^2 + 23.53^2} = 96.293$

TH:

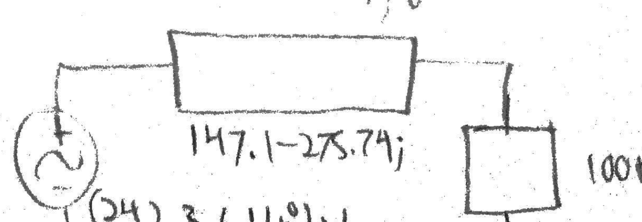


B: $Z_{eq_p} = (2.5)^2 \cdot Z_{TH} = (147.1 - 275.74j)$ $\quad Z_{Load} = 100 + 150j$

$V_{STH} = N V_{TH} = 2.5(94.12 + 23.5j) = 235.3 + 58.8j\ V$

$\sqrt{235.3^2 + 58.8^2} = 242.25$ TH:

$\tan^{-1}\left(\frac{58.8}{235.3}\right) = 14^\circ$



C: $Z_{eq} = Z_{s-p} + Z_{Leq} = 34.53 - 20.12j$ $\quad \sqrt{(20.12)^2 + (39.53)^2} = 44.3\angle -27^\circ$ $\quad \tan^{-1}\left(\frac{-20.12}{39.53}\right)$

$I_{s-p} = \frac{V_{s-p}}{Z_{eq}} = 2.19\angle 41^\circ$ $\quad Z = \sqrt{23.5^2 + (-44.1)^2}\angle \tan^{-1}\left(\frac{-44.1}{23.5}\right)$

$= 50\angle -62^\circ$

$S = \left(\frac{2.19}{\sqrt{2}}\right)^2 \cdot 50\cos(-62^\circ) + \left(\frac{2.19}{\sqrt{2}}\right)^2 \cdot 50\sin(-62^\circ)j = \boxed{56.14 - 105.58j}$

D: $Z_{eq} = Z_{p-s} + Z_{Leq} = 247 - 125.74j$ $\quad I_{p-s} = \frac{2453 + 588j}{247 - 125.74j}$

$\sqrt{150^2 + 100^2}\angle \tan^{-1}\left(\frac{156}{100}\right) = 180.3\angle 56.31^\circ$ $\quad = .875\angle 40.75^\circ$

$S = \left(\frac{.875}{\sqrt{2}}\right)^2 \cdot 180.3 \cdot \cos(56.31^\circ) + \left(\frac{.875}{\sqrt{2}}\right)^2 \cdot 180.3 \cdot \sin(56.31^\circ)$

$= \boxed{38.296 + 57.422j}$

E:

$Z_{eq} = 44.3\angle -27^\circ$

$V = 96.9\angle 14^\circ$

$S = \frac{\left(\frac{96.9}{\sqrt{2}}\right)^2}{44.3}\cos(-27) + \frac{\left(\frac{96.9}{\sqrt{2}}\right)^2}{44.3}\sin(-27) = 94.42 + (-48.11)j$

$$\begin{array}{r} 38.296 + 57.422j \\ + \; 56.14 + (-105.58)j \\ \hline 94.4 + (-49.2)j \end{array}$$ = S_{source} checks out.

C: $Z_{eq} = Z_{s\to p} + Z_{Leq} = 39.53 - 20.12j$ $\quad \sqrt{(20.12)^2 + (39.53)^2} = 44.3\angle -27^\circ$

$\tan^{-1}\left(\frac{-20.12}{39.53}\right)$

$I_{s-p} = \frac{V_{s\to p}}{Z_{Eq}} = 2.19\angle 41^\circ \quad Z = \sqrt{23.5^2 + (-44.1)^2} \angle \tan^{-1}\left(\frac{-44.1}{23.5}\right)$

$= 50\angle -62^\circ$

$S = \left(\frac{2.19}{\sqrt{2}}\right)^2 \cdot 50\cos(-62^\circ) + \left(\frac{2.19}{\sqrt{2}}\right)^2 \cdot 50\sin(-62^\circ)j = \boxed{56.14 - 105.58j}$

D: $Z_{eq} = Z_{p\to s} + Z_{Leq} = 247 - 125.74j \quad I_{p-s} = \frac{235.3 + 58.8j}{247 - 125.79j}$

$\sqrt{150^2 + 100^2} \angle \tan^{-1}\left(\frac{150}{100}\right) = 180.3\angle 56.31^\circ \qquad = .875\angle 40.75^\circ$

$S = \left(\frac{.875}{\sqrt{2}}\right)^2 \cdot 180.3 \cdot \cos(56.31^\circ) + \left(\frac{.875}{\sqrt{2}}\right)^2 \cdot 180.3 \cdot \sin(56.31^\circ)$

$= \boxed{38.296 + 57.422j}$

E:

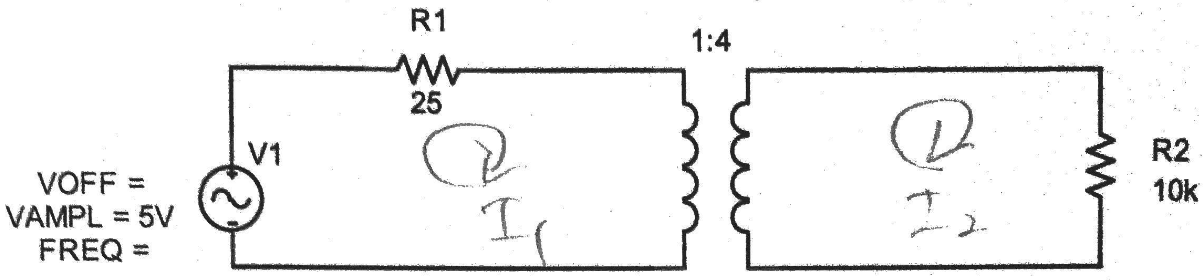
$Z_{eq} = 44.3\angle -27^\circ$

$V_s = 96.9\angle 14^\circ$

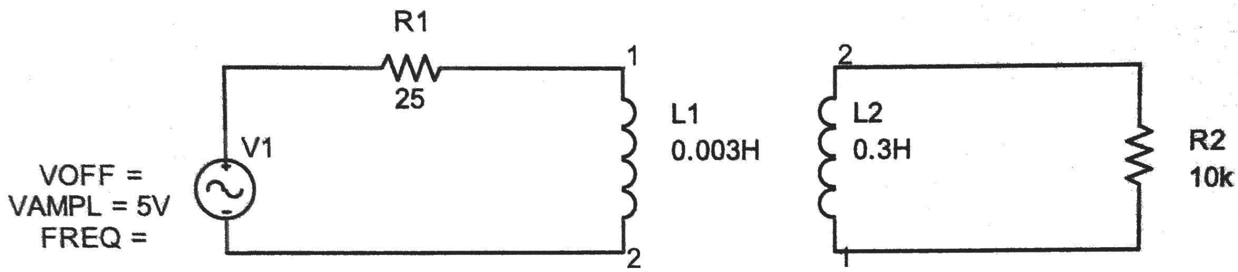
$S = \frac{\left(\frac{96.9}{\sqrt{2}}\right)^2}{44.3}\cos(-27) + \frac{\left(\frac{96.9}{\sqrt{2}}\right)^2}{44.3}\sin(-27) = 94.42 + (-48.11)j$

$38.296 + 57.422j$
$+\ 56.14 + (-105.58)j$
$\overline{94.4 + (-48.2)j}$ $= S_{source}$ checks out.

Problem 2) Real Transformers



a. Determine the voltage across R2 in phasor form



$$M = k \cdot \sqrt{L_1 \cdot L_2}$$

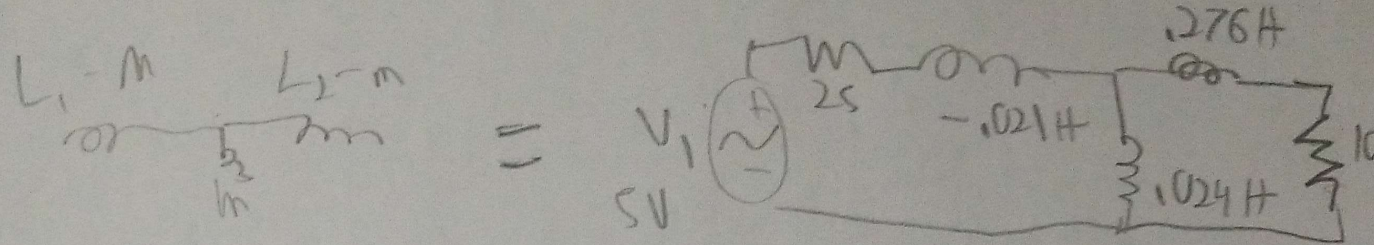
b. If the real transformer is constructed of two inductors (as show) and the coupling coefficient is k=0.8, determine the Tee model of the circuit. Draw your circuit.

A: $Z_{R2} = \frac{1}{N^2} \cdot R_2 = 625$ $\quad I_1 = \frac{V_1}{650} = 7.69\,mA$

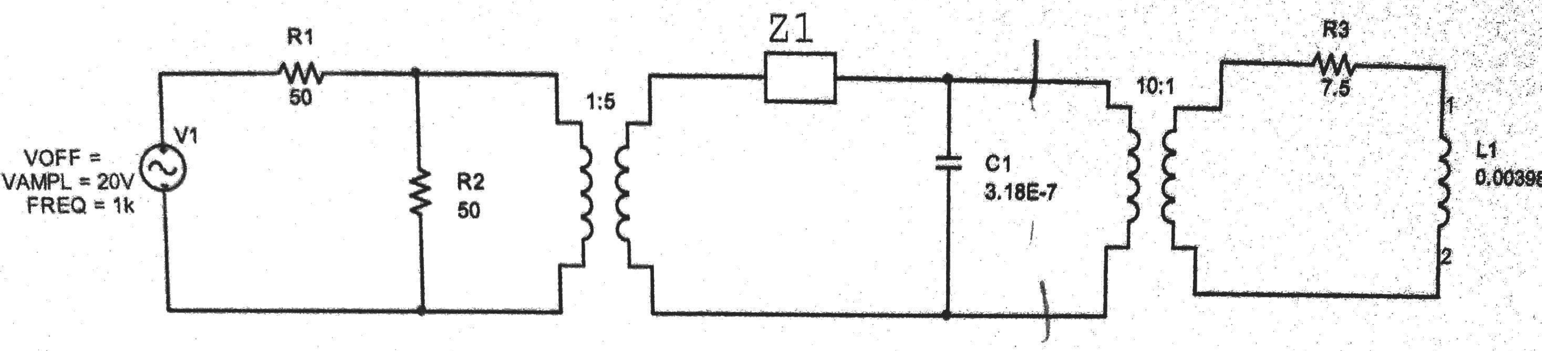
$I_2 = 7.69/4 = 1.923\,mA$ $\quad V_{R_2} = I_2 R_2 = 19.23\,V$

$\boxed{V_{R_2} = 19.23 \angle 0^\circ}$

$B = .8\sqrt{.003 \cdot .3} = .024$



Problem 3) Impedance matching



a. For the transformer circuit abive, design Z1 such that the power dissipated across the resistor R3 is maximized.

A: $\cancel{Z_1 \neq \overline{\frac{1}{3.18\cdot10^{-7}\cdot 2\pi\cdot 10^{3} j}} \cdot 10^{2} \cdot (7.5 + 0.00398)}$

$$Z_1 - \frac{1}{3.18\cdot 10^{-7}\, 2\pi \cdot 10^{3} j} = 750 + .398 i$$

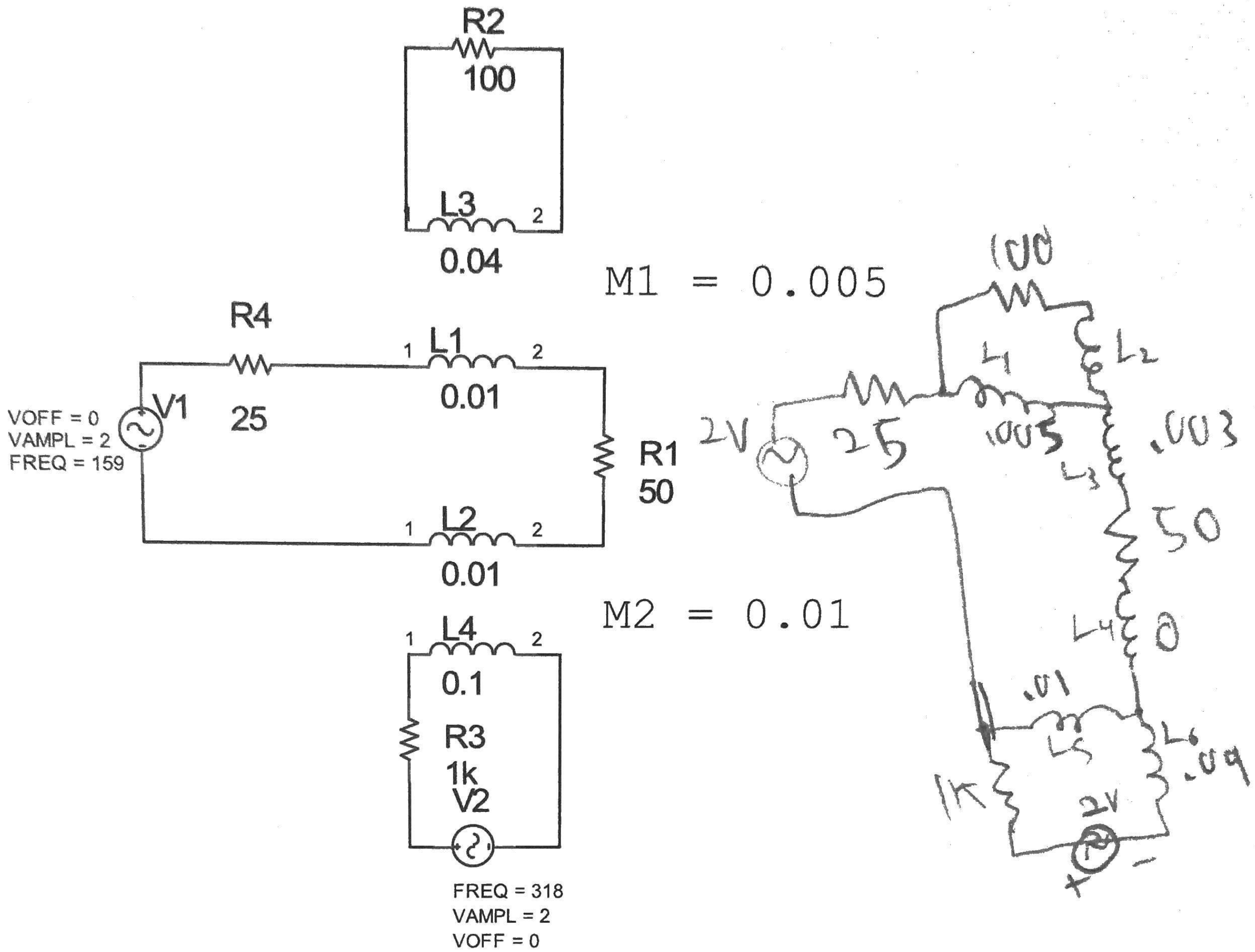
$$Z_1 = 750 + 2000.22 i$$

From reffering ~~to~~ primary $Z_1 \rightarrow \frac{Z_1}{5^2}$

So Z_1 will need to be

$$\boxed{Z_1 = 18750 + 50005.5 i}$$

Problem 4) Mutual Inductance

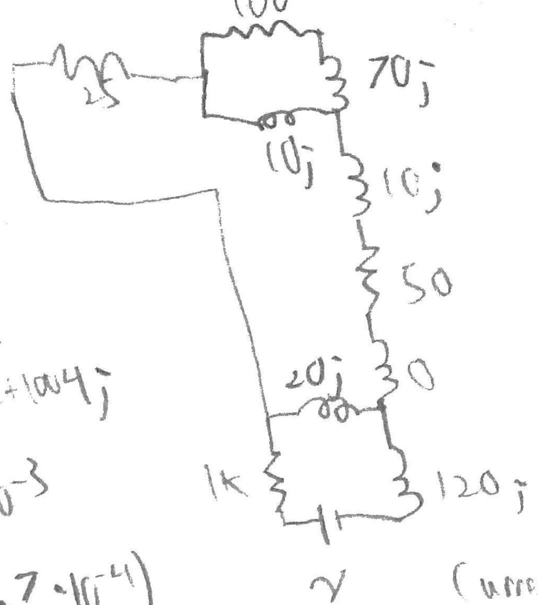


Find the current and voltage through R1 in the above couple circuits. There are two locations with inductive coupling and both sets have additive coupling. Additionally, there are two voltage sources with different excitation frequencies. Suggestion, use superposition in your analysis.

For V_1: $25 + \left(\frac{1}{5j} + \frac{1}{100+35j}\right)^{-1} + 5j + 50 + \left(\frac{1}{10j} + \frac{1}{1000+90j}\right)^{-1}$

$= 75.32 + 20j$ $\quad I_{R_1} = \frac{2}{75.32+20j} = .025 - .007j\ A$

For $V_2 =$



$$1000 + \left(\frac{1}{20j} + \frac{1}{25 + \left(\frac{1}{10j} + \frac{1}{100+70j}\right)^{-1} + 10j + 50}\right)^{-1}$$

$+ 120j$

$= 100.22 + 1004j$

$$I_{source} = \frac{2}{100.22 + 1004j}$$

$= 1.9 \times 10^{-3}$

$+ (2.7 \cdot 10^{-4})$

Current divider

$$I_{R_1} = \frac{1.9 \times 10^{-3} - 2.7 \times 10^{-4} (20j)}{20j + \left(25 + \left(\frac{1}{10j} + \frac{1}{100+70j}\right)^{-1} + 10j + 50\right)^{-1}}$$

$= 1.1 \times 10^{-4} + 3.3 \times 10^{-4} j$

$I_{R_1 V_1} + I_{R_1 V_2} = .02517 - .06667i \cdot R_1 = 1.26 - .33i$

$V_{R_1} = 1.30 \angle -14^\circ$

$I_{R_1} = .026 \angle -14^\circ$