Assignment #6 - Saaif Ahmed

Sunday, October 27, 2019 4:17 PM

Problem 13.8

(a)

Possible words would be 26 letters for 5 "slots" so the total number with repetition of letters is 26^5

Answer: 26^5

(b)

Want sequences where there is no repetition so $\frac{26!}{(26-5)!} = \frac{26!}{21!} = \quad 7893600$

Answer: 7893600

(c)

Beginning with *abc* means that 3 slots are locked off which means 26 letters can go into the remaining 2 slots.

Answer: $26^2 = 676$

(d)

Union of sets. The first set |A| will be for *abc* which we calculated as 26^2. The set |B| for *xyz* follows the same logic as *abc* so that is also 26^2. The value of $|A \cap B|$ is 0 because the minimal size of the string would have to 6 for there to be overlap which is not the case here.

Answer: $26^2 + 26^2 = 1352$

(e)

Union of sets. The first set |A| will be for *abc* which we calculated as 26^2. The set |B| for *cde* follows the same logic as *abc* so that is also 26^2. The value of $|A \cap B|$ is 1 because the only overlap happens is the case of *abcde* which only happens once.

Answer: $26^2 + 26^2 - 1 = 1351$

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Problem 13.44

(a)

10 toppings and distribute amongst 4; order is irrelevant
k subsets from n : $\binom{10}{4} = \frac{10!}{6!4!} = 210$

Answer: $\binom{10}{4} = 210$

(b)

10 choosing 4 where order matters
k sequences from n : $\frac{10!}{(10-4)!} = 5040$

Answer: 5040

(c)

Repeating: 10 picking 4 order does not matter
k subsets from n with repetition: $\binom{10+4-1}{10-1} = \binom{13}{9}$

Answer: $\binom{13}{9}$

(d)

Repeating: 10 picking 4 order matters
k sequences from n with repetition: $10^4 = 10{,}000$

Answer: 10,000

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Problem 13.50

(a)

Can represent as series of 1's : 1_1_1_1_1_1_1_1_1_1 = 10
We have 3 + signs which means that we can make 4 groupings of 1's by choosing where to put the + signs.
9 gaps, 3+ signs
k subsets from n: $\binom{9}{3}$

Answer: $\binom{9}{3}$

(b)

Can represent as series of 1's : 1_1_1_1_1_1_1_1_1_1 = 10
We have 3 + signs which means that we can make 4 groupings of 1's by choosing where to put the + signs. This time we can repeat the + signs so 10 total options.

k subsets from n with repetition: $\binom{10+4-1}{4-1} = \binom{13}{3}$

Answer: $\binom{13}{3}$

(c)

Same logic as above but this time the options for each x value is increased or decreased.
x_1: $+3$
x_2: $+2$
x_3: -1
x_4: -2
Total amount changed: $+2$
Can represent this as 12 options per x.
k subsets from n with repetition: $\binom{12+4-1}{4-1} = \binom{15}{3}$

Answer: $\binom{15}{3}$

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Problem 13.51

(a) How many 20-bit binary strings contain 00.

Solve by finding how many strings do not contain 00

$Q(n) = n$-bit strings that do not contain 00
$Q(1) = 2$
$Q(2) = 3$
$Q(n) = Q(n-1) + Q(n-2)$

This recursion is the same as the Fibonacci series with different starting series. So analyze the Fibonacci and find the 22nd number. This is because this recursion defined here skips the first 2 (counting from 0) Fibonacci numbers.

$Q(20) = 17711$
$2^{20} - 17711 = 1030865$

Answer: 1030865

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Problem 13.61

What are the coefficients of x^3, x^4, x^5, x^6, x^7 of $(\sqrt{x} + 2x)^{10}$

Expand using the Binomial theorem

$$\sum_{k=0}^{10} \binom{10}{k} (\sqrt{x})^{10-k} \, (2x)^i$$

Try $k = 0$

$$\binom{10}{0} (\sqrt{x})^{10} (2x)^0 = x^5 * 1$$

This means that the smallest exponent of x in this expanision is 5.
Furthermore we know that any value of k that is odd will always leave a $\sqrt{x}$ term so we can ignore all values of k that are odd.

Try $k = 2$

$$\binom{10}{2} (\sqrt{x})^8 (2x)^2 = 4x^2 * 45x^4 = 180x^6$$

Try $k = 4$

$$\binom{10}{4} (\sqrt{x})^6 (2x)^4 = 16x^4 * 210x^3 = 3360x^7$$

Answer:

x^3: 0
x^4: 0
x^5: 1
x^6: 180
x^7: 3360

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Problem 14.5

(a) 10 identical candies distributed among 4 children.

k subsets with repetition: $\binom{10+4-1}{4-1} = \binom{13}{3}$

Answer: $\binom{13}{3}$

(b) A 15 letter Sequence must be made up of 5 A's, 5 B's, 5 C's

First A: $\binom{15}{5} = \frac{15!}{10!\,5!}$

Second B: $\binom{10}{5} = \frac{10!}{5!5!}$

Third C: $\binom{5}{5} = \frac{5!}{5!\,0!}$

Multiply all together : $\frac{15!}{5!*5!*5!} = 756756$

Answer: 756756

(c) 10 identical rings must be placed on your 10 fingers

k subsets with repetition: $\binom{10+10-1}{10-1} = \binom{19}{9}$

Answer: $\binom{19}{9}$

(d) 3 red, 3 green, 3 blue flags are to be arranged along the street for the parade

First A: $\binom{9}{3} = \frac{9!}{3!6!}$

Second B: $\binom{6}{3} = \frac{6!}{3!3!}$

Third C: $\binom{3}{3} = \frac{3!}{3!\,0!}$

Multiply all together : $\frac{9!}{3!*3!*3!} = 1680$

Answer: 1680

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Problem 14.14

(a) How many contain fewer 1's than 0's?

At most you can 4 1's. So that means you can do:

$$\binom{10}{4} + \binom{10}{3} + \binom{10}{2} + \binom{10}{1} + \binom{10}{0} = 386$$

Answer: 386

(b) How many contain 5 or more consecutive 1's?

If you section off 5 bits from 10 bits that means you have 2^5 bits combinations remaining.
How many ways can section off 5 bits from 10 bits?

You can section off 5 bits from 10 bits a total of 6 times

Let $k = \{1,2,3,4,5,6\}$ where k represents the starting position of the first 1.

For $k = 0$ you have 2^5 strings.
For $k > 1$ you have 2^4 strings because there is some repititions from $k = 0$ so the total number is 2^4 instead of 2^5

Total: $2^5 + 5(2^4) = 112$

Answer: 112

(c) How many contain 5 or more consecutive 0's?

Same logic as above except with 0 instead of 1. This is because in a binary string, the possible values are either 0 or 1.

Total: $2^5 + 5(2^4) = 112$

Answer: 112

(d) How many contain 5 or more consecutive 0's or 5 consecutive 1's?

Union of sets: $|A \cup B| = |A| + |B| - |A \cap B|$
$|A| =$ # of consecuive 0
$|B| =$ # of consecuive 1
$|A \cap B| = 2$: for the scenario of 1111100000 and 0000011111

Answer: 112 +112 -2 = 222