Saart Ahmed

"On my honor, I have neither given nor recieved unauthorized aid on this exam"

-Saart Ahmed

#1 A:

$S = \{0, 1, 2, 3, 4 \ldots \infty\}$

all ~~positive~~ Natural numbers

B: $P(Y > X)$

$P(Y=1)(P(X=0)) + P(Y=2)(P(X=0) + P(X=1))$

$= \frac{1}{3} \cdot \frac{1^0}{0!} e^{-1} + \frac{1}{3}\left(\frac{1^0}{0!} + \frac{1^1}{1!}\right) e^{-1}$

$= .3679$

C: $P(Y=1 \mid Y > X) = \frac{P(Y=1 \cap Y > X)}{P(Y > X)} = \frac{P(Y > X \mid Y=1)\, P(Y=1)}{P(Y > X)}$

$= \frac{P(Y > X \mid Y=1) \cdot \frac{1}{3}}{0.3679}$

$= e^{-1} \cdot \frac{1}{3} / 0.3679$

$= 0.0451$

D: $P(Y=1) \cdot P(Y > X) = \frac{1}{3} \cdot 0.3679 = .1226$

$P(Y=1 \cap Y > X) = P(Y=1 \mid Y > X) \cdot P(Y > X) = 0.01659$

$P(Y=1) \cdot P(Y > X) \neq P(Y=1 \cap Y > X)$

not independent

$+ \; + \; + \; + \; + \; + \; \underline{\times} \; \underline{\times}$

#2 A: 2 bad 6 good. 6 failures needed

$\frac{6}{8} \cdot \frac{5}{7} \cdot \frac{4}{6} \cdot \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} \quad \frac{2 \cdot 1}{8 \cdot 7} = \frac{2}{56} = \frac{1}{28}$

B: More than 4 needs 4 failures

$+ \; + \; + \; + \; \_ \; \_ \; \underline{\times} \; \underline{P}$

$\frac{6}{8} \cdot \frac{5}{7} \cdot \frac{4}{6} \cdot \frac{3}{5} \cdot$

Probability of impostors going out at , 5, 6 / ~~8,7~~ / ~~8~~, 8 /

~~6, 7 / 6, 8 / 7, 8 /~~

_ _ x x   _ x _ x   x _ _ x

_ x x _   x x _ _   x _ x _

$\frac{\#\text{ of ways to choose 4 failures}}{\#\text{ of ways to get}} \rightarrow \frac{\binom{6}{4}}{7!}$

$\frac{\#\text{ of ways to fail } > 4}{\#\text{ of ways to eject to end game}} = \frac{\binom{6}{4} + \binom{6}{5} + \binom{6}{6}}{\binom{8}{2} + \binom{6}{2} + \binom{4}{2} + \binom{2}{2}}$

$$\frac{15 + 6 + 1}{28 + 15 + 6 + 1} = \frac{11}{25}$$

3 A) $P(1) + P(2) + P(3) + \cancel{P(4)}$

$\frac{2}{3} + \frac{1}{3} \cdot \frac{2}{3} + \left(\frac{1}{3}\right)^2 \frac{2}{3} = \frac{26}{27}$

B) Binomial

$\binom{7}{5} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^2 = \frac{224}{729}$

$\frac{7!}{5!(2!)} \cdot \left(\frac{2}{3}\right)^5 \cdot \frac{1}{9}$

C) Binomial

$$\boxed{PMF_X = p(x) = \binom{6}{x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{6-x}}$$

4 A)

$$c(-2-1)^2 + c((-1-1)^2 + c((0-1)^2 + c((1-1)^2 + c((2-1)^2 = 1$$

$$9c + 4c + c + 0 + c = 1$$

$$9c + 6c = 1 \qquad 15c = 1 \qquad \boxed{c = \frac{1}{15}}$$

$$p_x(-2) = \frac{1}{15}(-2-1)^2 = \frac{3}{5}$$

$$p_x(-1) = \frac{4}{15}$$

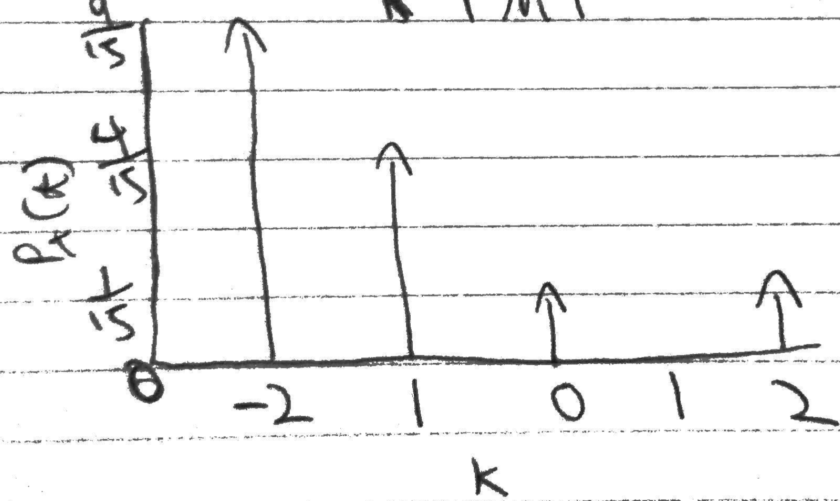
$$p_x(0) = \frac{1}{15}$$

$$p_x(1) = 0$$

$$p_x(2) = \frac{1}{15}$$

Verify $\frac{3}{5} + \frac{1}{15} + \frac{1}{15} + \frac{4}{15} = 1$ ✓

PMF



B) direct formula: $(-2)\left(\frac{3}{5}\right) + (-1)\left(\frac{4}{15}\right) + (0)\left(\frac{1}{15}\right) + 0 + 2\left(\frac{1}{15}\right)$

$$= -\frac{4}{3} = E(X)$$

C) $Var(X) = E(X^2) - E(X)^2$

$$(-2)^2\left(\frac{3}{5}\right) + (-1)^2\left(\frac{4}{15}\right) + (2)^2\left(\frac{1}{15}\right) - \left(-\frac{4}{3}\right)^2$$

$$= \frac{44}{15} - \frac{16}{9} = \frac{52}{45} = Var(X)$$

5 A) $E(x) = E(\text{commercial}) + E(\text{shows}) - 300$

$E(\text{commercial}) = E(\text{poisson}) = \lambda = 4$

$E(\text{commercial}) = \lambda \cdot \$100 = \$400$

$E(\text{shows}) = E(\text{uniform}) = \frac{a+b}{2}$

$E(\text{shows}) = \frac{0+\cancel{6}}{2} \cdot \$500 = \cancel{\$1750}\ \$1500$

$E(x) = \$400 + \cancel{\$1750}^{\$1500} - \$300$

$= \cancel{\$1850} = \boxed{\$1600}$

B) $Var(x) = Var(\text{commercial}) + Var(\text{shows}) - 300$

$= \lambda \cdot \$100 + \$500 \cdot \frac{(6+1)^2-1}{12} - 300$

$= 400 + 2000 - 300$

$\boxed{Var(x) = \$2100}$