HW 7

Sunday, November 1, 2020 9:21 PM

1:

a)

$X \downarrow vs\ H \rightarrow$	0	1	2	3
4	$40/512$	$48/512$	$32/512$	$8/512$
5	$64/512$	$40/512$	$20/512$	$4/512$
6	$84/512$	$30/512$	$12/512$	$2/512$
7	$99/512$	$21/512$	$7/512$	$1/512$

b)

Everything in the table is divided by 512. Numbers represent height of apartment buildings.

$X \downarrow vs\ H \rightarrow$	0	1	2	3
4	40	88	120	128
5	104	192	244	256
6	188	306	370	384
7	287	426	497	512

c)

H	0	1	2	3
	$287/512$	$139/512$	$71/512$	$15/512$

2:

a)

Given the PDF integrate across the whole thing to get the CDF equal to 1

$$\int_0^1 \int_0^2 (cx^2y^3 + cx^2y)dydx$$

$$\int_0^1 \left(\frac{1}{4}cx^2y^4 + \frac{1}{2}cx^2y^2\right)\Bigg|_0^2 dx$$

$$\int_0^1 4cx^2 + 2cx^2\ dx$$

$$\frac{4}{3}cx^3 + \frac{2}{3}cx^3\Bigg|_0^1 = \frac{4}{3}c + \frac{2}{3}c$$

Thus $\frac{6}{3}c = 1 \rightarrow c = \frac{1}{2}$

Answer: $c = \frac{1}{2}$

2 (continued):

b)

The CDF is the double integral of the PDF

$$\int_0^1 \int_0^2 \left(\frac{1}{2}x^2y^3 + \frac{1}{2}x^2y\right) dydx$$

because we are interested in the region given by the bounds we get

$$\int \int \left(\frac{1}{2}x^2y^3 + \frac{1}{2}x^2y\right) dydx$$

$$\int \frac{1}{8}x^2y^4 + \frac{1}{4}x^2y^2\, dx$$

Answer: $\frac{1}{24}x^3y^4 + \frac{1}{12}x^3y^2 : x \in [0,1], y \in [0,2]$

c)

$$\int_0^2 \frac{1}{2}x^2y^3 + \frac{1}{2}x^2y \;\; dy = f_x(x)$$

$$\frac{1}{8}x^2y^4 + \frac{1}{4}x^2y^2 \Big|_0^2 = 2x^2$$

Answer: $2x^2 : x \in [0,1]$

d)

$$\int_0^1 \frac{1}{2}x^2y^3 + \frac{1}{2}x^2y \;\; dx = f_y(y)$$

$$\frac{1}{6}x^3y^3 + \frac{1}{6}x^3y \Big|_0^1 = \frac{2}{6}y$$

Answer: $\frac{2}{6}y : y \in [0,2]$

e)

$$\frac{2}{6}y(2x^2) = \frac{4}{6}x^2y = f_x(x) * f_y(y) \neq F_{xy}(x,y)$$

Answer: Not independent

f)

$$\frac{2}{6}y \leq x\sqrt{2} = area\ shaded\ in\ from\ [0,1]\ and\ [0,2]$$

$1 - .47 = 0.53$

Answer: 0.53

3:

a)

looking at the coefficients (Var(X)). $-\frac{9x^2}{54} \rightarrow -\frac{1x^2}{6}$; $\sigma_x^2 = 6$

for Var(Y): $-\frac{4y^2}{54} = -\frac{y^2}{13.5}$; $\sigma_y^2 = 13.5$

Answer: $Var(X) = 6\, , Var(Y) = 13.5$

b)

$12x$. From $(x - \mu_x)^2 = x^2 + 2\mu_x + \mu_x^2 \rightarrow \mu_x = 6$

$20y$. From $\left(x - \mu_y\right)^2 = x^2 + 2\mu_y + \mu_y^2 \rightarrow \mu_y = 10$

Answer: $E(X) = 6\, , E(Y) = 10$

4:

a)

$$\int_0^1 \int_{-1}^1 \frac{3}{8} xy(x^2 + y^3 + 1)\ dy\, dx$$
$$\int_0^1 \frac{3x}{20}\ dx$$
$$\frac{1}{40} 3(1)^2$$
$$E(XY) = \frac{3}{40}$$

b)

$$\int_0^1 x \int_{-1}^1 \frac{3}{8}(x^2 + y^3 + 1) dy\, dx$$
$$\int_0^1 \frac{3x(x^2+1)}{4}\ dx = \mu_x = \frac{9}{16}$$

$$\int_{-1}^1 y \int_0^1 \frac{3}{8}(x^2 + y^3 + 1) dx\, dy$$

$$\int_{-1}^1 \frac{y(3y^3 + 4}{8} = \mu_y = \frac{3}{20}$$

$$COV(X, Y) = \frac{3}{40} - \frac{9}{16} * \frac{3}{20} = -\frac{3}{320}$$

c)

$$E\Big(320\big(2X(1+Y) + Y(1-X)\big)\Big) = E(320(XY) + 640X + 320) =$$
$$320\left(\frac{3}{40}\right) + 640\left(\frac{9}{16}\right) + 320\left(\frac{3}{20}\right)$$
Answer: $E(whatever\ was\ in\ the\ problem) = 432$

d)

Answer: They are correlated because the COV is not 0.