Signals & Systems HW #8

1:

a)

$$H(s) = \left(\frac{1}{\frac{s}{\omega_c} + 1}\right)$$

$\omega_0 = 400\pi$

$\Delta\omega = 600\pi$

$Q = \dfrac{2}{3}$

$BPF \to LPF\,;\ \frac{s}{\omega_c} \to Q(\frac{s}{\omega_0} + \frac{\omega_0}{s})$

$$H(s) = \frac{1}{1 + \frac{Qs}{\omega_0} + \frac{Q\omega_0}{s}} = \frac{s\omega_0}{s\omega_0 + Qs^2 + Q\omega_0^2}$$

$$H(s) = \frac{1884.95s}{s^2 + 1884.95s + 1.58 * 10^6}$$

central is 200Hz

Cutoff is 1000Hz

b)

$$H(s) = \frac{Q\left(\frac{s}{\omega_0} + \frac{\omega_0}{s}\right)}{1 + \frac{Qs}{\omega_0} + \frac{\omega_0 Q}{s}}$$

$$= \frac{s\omega_0\left(\frac{s}{\omega_0} + \frac{\omega_0}{s}\right)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$= \frac{s^2 + \omega^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$H(s) = \frac{s^2 + 1.58 * 10^6}{s^2 + 1884.95s + 1.58 * 10^6}$$

central is 200Hz

2:

s^4	1	2	b
s^3	3	a	0
s^2	$\frac{6-a}{3}$	$\frac{3b}{3}$	0
s	$\frac{\frac{6-a}{3}a-3b}{\left(\frac{6-a}{3}\right)}$	0	0
1	b	0	0

$\frac{6-a}{3} > 0 \to a < 6$

$\frac{\frac{6-a}{3}a-3b}{\left(\frac{6-a}{3}\right)} > 0 \to \left(\frac{6-a}{3}\right)a > 3b \to 6a - a^2 > 9b$

Stable system:

$0 < a < 6$

$0 < b < \frac{6a-a^2}{9}$

3:

$\omega = 4\frac{rad}{s}$

$\omega^2 = 2b = 16 \to b = 8$

$2\zeta\omega = (a+b)$

$\zeta = \left(\frac{a+8}{8}\right)$

overdamped $\to b > 1$

$a + 8 < -8 \to a < -16$

$b = 8$ & $a < -16$

4:

a)

$$e^{-at}u(t) \to \frac{1}{s+a}$$

unstable past a
zero pole at $+a$
unstable

b)

$$e^{at}u(t) \to -\frac{1}{s-a}$$

unstable past a
zero pole at $+a$
unstable

c)

$$e^{-a|t|}u(t) \to \frac{1}{s+a} - \frac{1}{s-a}$$

$-a, a$ pole locations
unstable

d)

$$\cos(\omega t + b) = \cos(\omega t)\cos(b) - \sin(\omega t)\sin(b)$$
$$= \frac{s\cos b}{s^2+\omega^2} - \frac{\omega \sin b}{s^2+\omega^2}$$

Zero at -$j\omega$
stable

5:

$1 + AH(s)$

$= 1 + \frac{A}{(s+2)(s+3)(s+5)} = 0$

$= (s+2)(s+3)(s+5) + A = 0$

$s^3 + 10s^2 + 31s + 30 + A = 0$

RH

s^3	1	31
s^2	10	$30 + A$
s	$\frac{310 - 30 - A}{10}$	0
1	$30 + A$	

$\frac{310 - 30 - A}{10} > 0$

$280\ - A > 0$

$A < 280$

$30 + A > 0$

$A > 30$

Range: $-30 < A < 280$

6:

a)

$$\frac{Y(s)}{X(s)} = \frac{AH(s)}{1 + AH(s)G(s)}$$

$$= \frac{\frac{A}{2s+2}}{1 + A\left(\frac{1}{2s+2}\right)\left(\frac{1}{s-4}\right)}$$

$$= \frac{A(s-4)}{(2s+2)(s-4) + A}$$

$$\frac{Y(s)}{X(s)} = \frac{4s - 4A}{2s^2 - 6s - 8 + A}$$

b)

$$\frac{Y(s)}{E(s)} = \frac{Y(s)}{X(s)} * \frac{X(s)}{E(s)}$$

$$\frac{E(s)}{X(s)} = \frac{1}{1 + AH(s)}$$

$$= \frac{(2s+2)(s-4)}{2s^2 - 6s - 8 + A}$$

$$\frac{Y(s)}{E(s)} = \frac{4s - 4a}{2s^2 - 6s - 8 + A} * \frac{2s^2 - 6s - 8 + a}{(2s+2)(s-4)}$$

$$= \frac{4s - 4A}{2s^2 - 6s - 8}$$

7:

a)

$$H(s) = \frac{Y(s)}{X(s)} = \frac{KG(s)}{1 + KG(s)}$$

$$= \frac{K * \frac{1}{(s+1)(s+2)}}{1 + \frac{K}{(s+1)(s+2)}}$$

$$= \frac{K}{s^2 + 3s + 2 + K}$$

$$H(s) = \frac{K}{s^2 + 3s + 2 + K}$$

$$2\omega_n = 3\,;\,\omega_n^2 = 2 + K$$

$$\omega_n = \frac{3}{2}$$

$$K + 2 = \frac{9}{4} \rightarrow K = 0.25$$

b)

$$\frac{Y(s)}{X(s)} * \frac{X(s)}{E(s)}$$

$$\frac{E(s)}{X(s)} = 1 - \frac{K}{s^2 + 3s + 2 + K}$$

$$= \frac{s^2 + 3s + 2}{s^2 + 3s + 2 + K}$$

$$\frac{Y(s)}{E(s)} = \frac{s^2 + 3s + 2 + K}{s^2 + 3s + 2} * \frac{K}{s^2 + 3s + 2 + K}$$

$$= \frac{K}{s^2 + 3s + 2}$$

c)

$$\frac{1}{s} * \frac{1}{(s+1)(s+2)}$$

$$L^{-1}\left(\frac{1}{s} * \frac{1}{(s+1)(s+2)}\right) = \frac{1}{2s} - \frac{1}{s+1} + \frac{1}{2(s+2)}$$

$$e(t) = \frac{1}{2}u(t) - e^{-t} + \frac{1}{2}e^{-2t}$$