Circuits

Exam 2

COVID Spring 2020

1.	/25
2.	/25
3.	/25
4.	/25
Total	/100

Name Saaif Ahmed

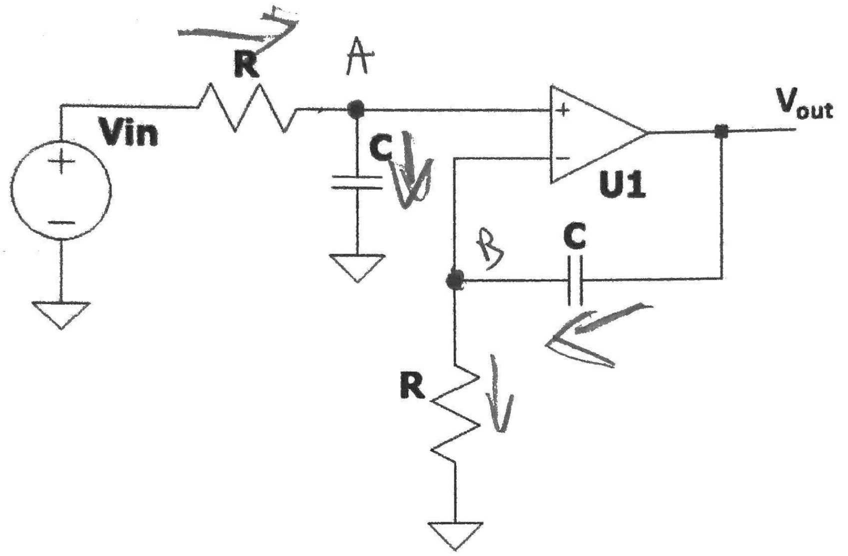
Notes:

1) If you are stuck on one part of the problem, choose 'reasonable' values on the following parts to receive partial credit.
2) You don't need to simplify all your numerical calculations. For example, you can leave square root terms in radical form.

Name Saqif Ahmed

1) Short Answer - (25 pts)

Derivations in the Time Domain



1.1: (10 pts) Derive the relationship between Vin and Vout. Hint: Use what you know about ideal op amps and KCL at nodes.

Do NOT use impedances, s-domain conversion to solve.

KCL at A: $\frac{V_{in}-V_a}{R} = I_{C_A}$

$= C\frac{dV_a}{dt}$

KCL at B:

$I_{C_B} = V_B/R$

$\frac{V_B}{R} = C\frac{d}{dt}(V_{out}-V_B)$

$= C\frac{dV_{out}}{dt} - C\frac{dV_B}{dt}$

knowing that $V_a = V_B$ $C\frac{dV_B}{dt} = \frac{V_{in}-V_a}{R}$

$\frac{V_a}{R} = C\frac{dV_{out}}{dt} - \frac{V_{in}-V_a}{R}$

$\frac{V_a}{R} = C\frac{dV_{out}}{dt} - \frac{V_{in}}{R} + \frac{V_a}{R}$

$\int\frac{V_{in}}{RC} = \int\frac{dV_{out}}{dt} \rightarrow \boxed{V_{out} = \frac{1}{RC}\int V_{in}\,dt}$

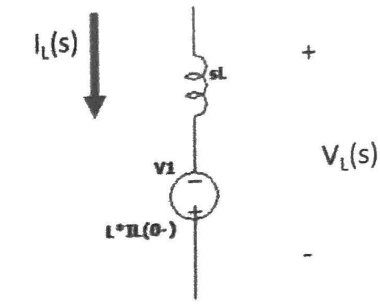
1.2: (15 pts) Circle all cases where the initial condition must be included in the definition in the s-domain. The diagram below is given as a reference for one of four examples of initial condition soruce configurations. ***Draw and label the other schematics then write equations below each for full credit.***

a. Definition of VL(s) for initial condition source in series with inductor

Equation: $V_L(s) = sL\, I_L(s) - L I(0^-)$

b. Definition of IL(s) for initial condition source in series with inductor

Equation:__________



c. Definition of VL(s) for initial condition source in parallel with inductor

Equation:__________

d. Defintion of IL(s) for intitial condition source in parallel with inductor

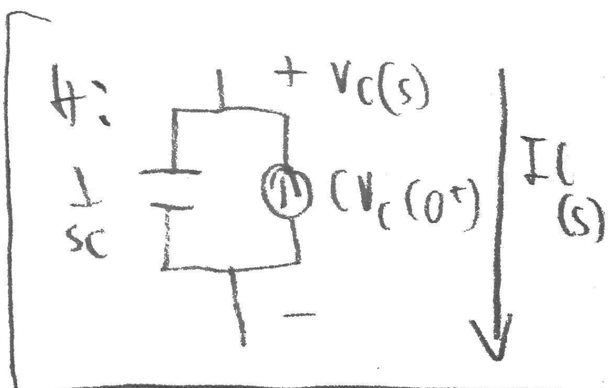
Equation: $I_L(s) = \frac{I_L(0^-)}{s} + \frac{V_L}{sL}$

e. Definition of VC(s) for initial condition source in series with capacitor

Equation: $V_C(s) = \frac{1}{sC} I_C(s) + \frac{V_C(0^-)}{s}$

f. Definition of IC(s) for initial condition source in series with capacitor

Equation:__________



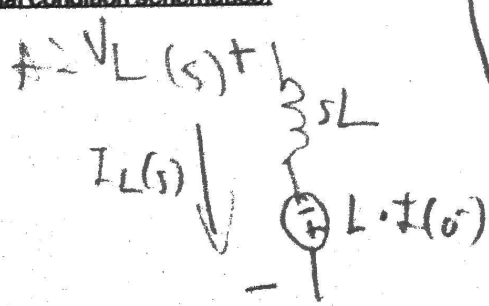
g. Definition of VC(s) for initial condition source in parallel with capacitor

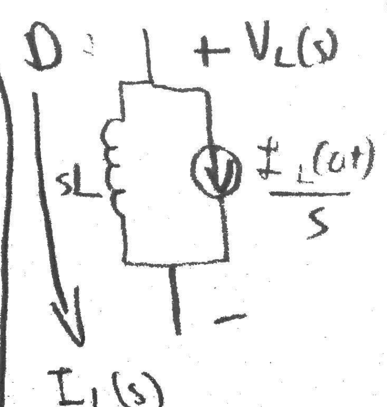
Equation:__________

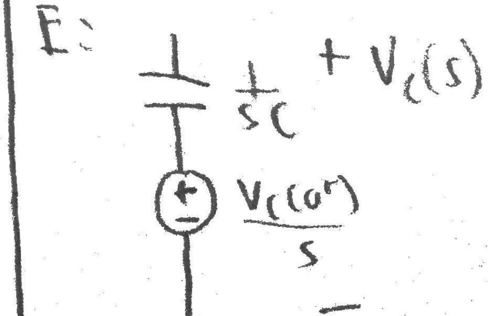
h. Definition of IC(s) for initial condition source in parallel with capacitor

Equation: $I_C(s) = \frac{V_C}{1} \cdot sC - C V_C(0^+)$

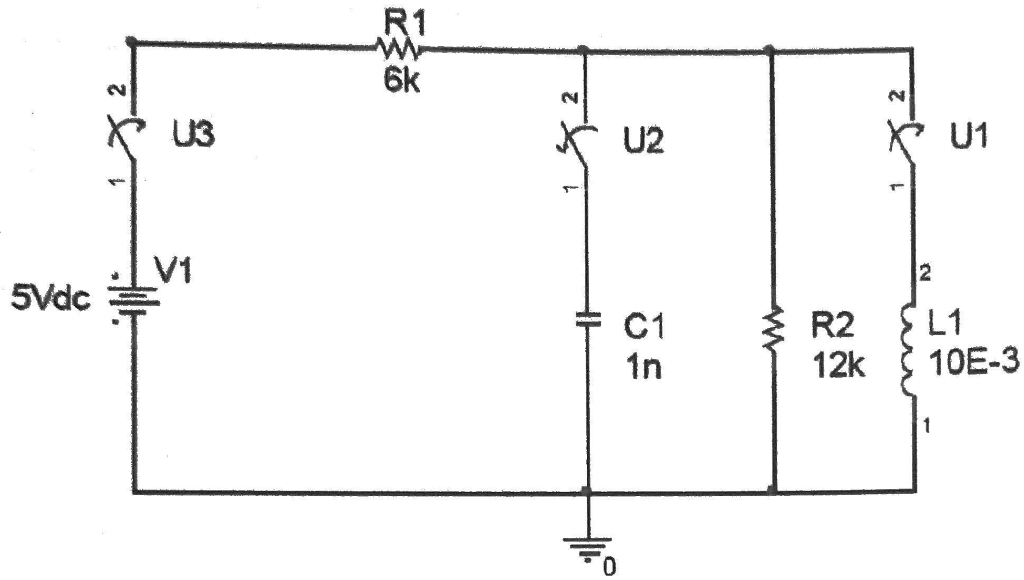
Initial condition schematics:







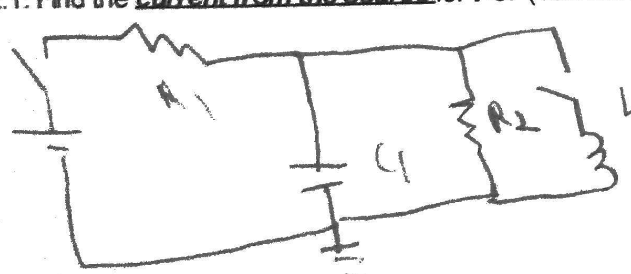
2) **First Order - Switching Differential Equations (25 pts)**



THIS PROBLEM MUST BE DONE BY <u>DIFFERENTIAL EQUATIONS!</u>

In the above circuit,
1. For t<0, switches U1 and U3 are open and U2 is closed.
2. At t=0, U3 closes
3. At t=3μs, U2 opens and U1 closes

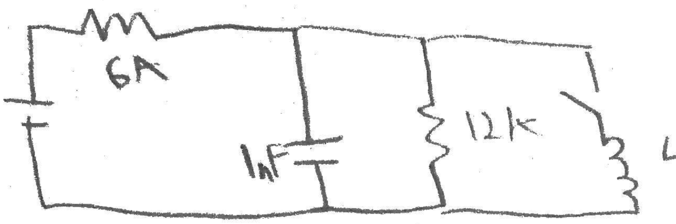
2.1: Find the ***current from the source*** for t<0. (You MUST draw the circuit for this time region!)



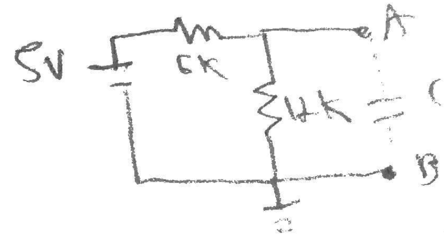
No current
open circuit

$I_s(t<0)$	0 A

2.2: Find the **current from the source** for 0<t<3μs. (You MUST draw the circuit for this time region!)



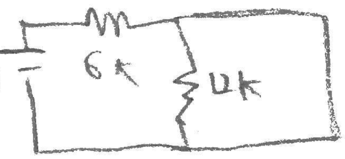
Thevenin



$V_{TH} = V_{AB}$

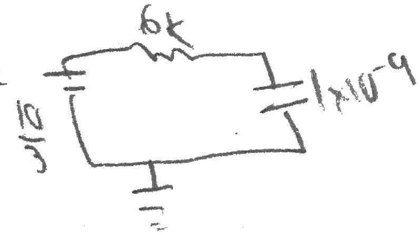
$= \frac{5(12K)}{18K} = \frac{10}{3}$

R_{TH}:



$R_{TH} = 6K$

Thevenin!



$V_c(0^+) = 0V$

$V_c(\infty) = \frac{10}{3}$

Guess $V_c(t) = Ae^{-\frac{t}{\tau}} + B$

$Ae^{-\frac{t}{\tau}} + B = 0 \mid t = 0^+$

$A + B = 0$

$V_c(t) = -\frac{10}{3} e^{-t/6\times10^{-6}} + \frac{10}{3}$

$B = \frac{10}{3} \mid t = \infty \quad A = -\frac{10}{3}$

$\frac{dV_c}{dt} C = I_s = \frac{-1}{6\times10^{-6}} \cdot \frac{-10}{3} e^{-\frac{t}{6\times10^{-6}}}$

$= \left(\frac{5000000}{9} e^{-\frac{1}{6\times10^{-6}}t}\right) 1\times10^{-9}$

Is(0<t<3μs)	$\left(\frac{5000000}{9} e^{-\frac{1}{6\times10^{-6}}t}\right) 10^{-9}$

2.3: Find the ***current from the source*** for t > 3μs. (You MUST draw the circuit for this time region!)

Thevenin

$V_{TH} = \frac{5(12k)}{18k}$ $R_{TH} = 6k$

$I_L = I_s$

Knowing that $I_L = Ae^{-t/\tau} + B$ $I_L(3\mu s) = 0A$ $I_L(\infty) = \frac{10}{3} \div 6$

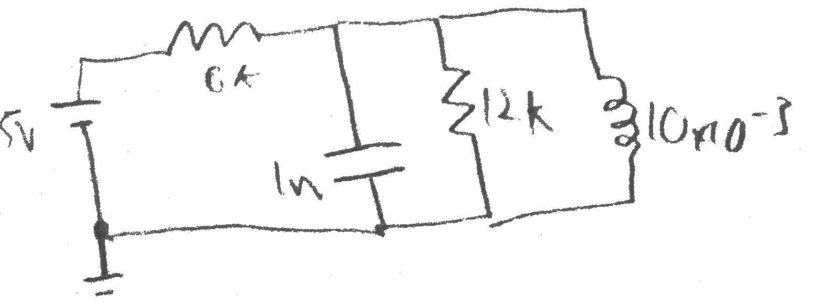
$= \frac{5}{9}A$

$I_L(0) = Ae^{-0} + B = 0$

$I_L(\infty) = B = \frac{5}{9}$ $I_L = I_s = -\frac{5}{9}e^{-t/1.66\times10^{-6}}$

Is(t>3μs)	$I_s = -\frac{5}{9}e^{-60000t} + \frac{5}{9}$

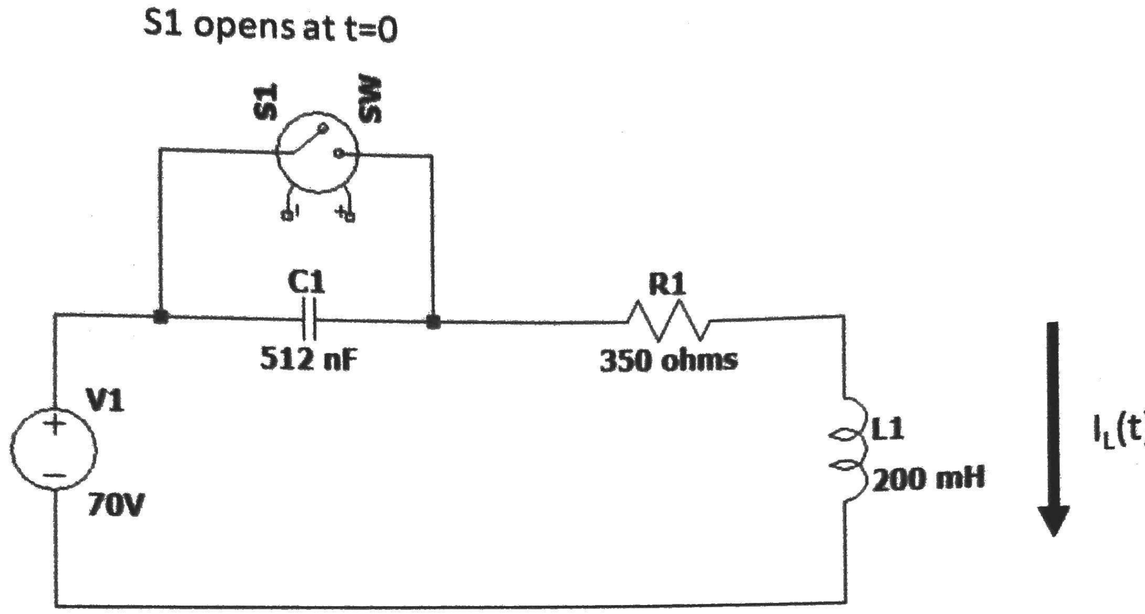
2.4: **1 pt** If U2 stays closed at t=3μs how would you approach finding the current through the inductor IL(t)? Draw the circuit and write ***the first two major steps*** to solve the problem quickly. (I understand this may be somewhat subjective to perspective but I am curious...)



Step 1. Source convert to get everything parrallel. Simplify

Step 2: Calculate new initial conditions.

3) Second Order - Differential Equations (25 pts)



THIS PROBLEM MUST BE DONE BY <u>DIFFERENTIAL EQUATIONS!</u>

3.1: Find IL(t) for t>0.

Cap is shorted across. Series RL for $t<0$.

$I_L(0^-) = .2\,A$ $\qquad \alpha = \frac{R}{2L} \qquad w_0 = \frac{1}{\sqrt{LC}}$

$V_c(0^-) = 0V$

$w_0 > \alpha$ Underdamped 2 comple roots

$V_c(\infty) = 70V$ $\qquad B = 3000$

Guess that $V_c(t) = e^{-875t}\left(A_1\cos(3000t) + A_2\sin(3000t)\right) + A_3$

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$$\frac{dv_c}{dt} \cdot C = I_c(0^-) = -3000e^{-875t} A_1 \sin(3000t) + 3000e^{-875t} A_2 \cos(3000t)$$
$$-875e^{-875t} A_1 \cos(3000t) - 875e^{-875t} A_2 \sin(3000t)$$
$$\cdot 512 \times 10^{-9}$$

$$V_c(\infty) = 70V = A_3$$

$$\cancel{-3000e^0 A_1 \sin(0)} + 3000e^0 A_2 \cos(0) - 875e^0 A_1 \cos(0) \cancel{-875e^0 A_2 \sin(0)}$$

$$V_c(0^-) = 0V = e^0(A_1 \cancel{\cos(0)} + \cancel{A_2 \sin(0)}) + 70$$

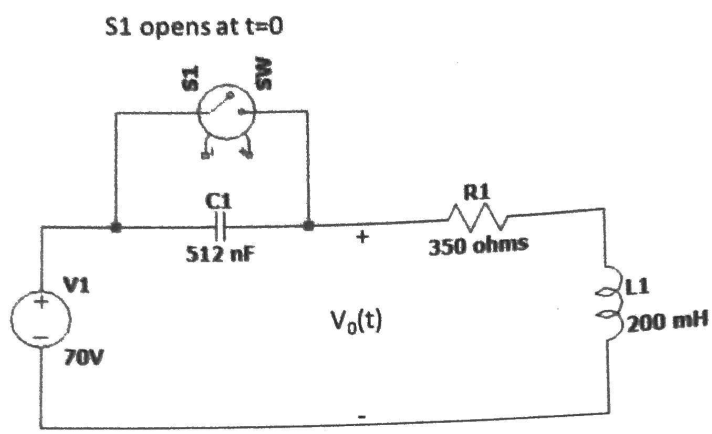
$$A_1 + 70 = 0V$$
$$(3000A_2 - 875A_1) \cdot 512 \times 10^{-9} = .2$$
$$A_1 = -70 \quad A_2 = \frac{2635}{24}$$

$$I_L(t) = I_c(t) = \frac{dv_c}{dt} \cdot C = (2.1 \times 10^5 e^{-875t} \sin(3000t) + 329375 e^{-875t} \cos(3000t)$$
$$+ 61250 e^{-875t} \cos(3000t) - 96067.7 e^{-875t} \sin(3000t))$$
$$\cdot 512 \times 10^{-9}$$

$$I_c(t) = (390625 e^{-875t} \cos(3000t) + 306067.7 e^{-875t} \sin(3000t)) \cdot 512 \times 10^{-9}$$

$I_L(t)$ for $t>0$	$\frac{1}{5}e^{-875t} \cos(3000t) + .16e^{-875t} \sin(3000t)$

4) Second Order Laplace (25 pts)

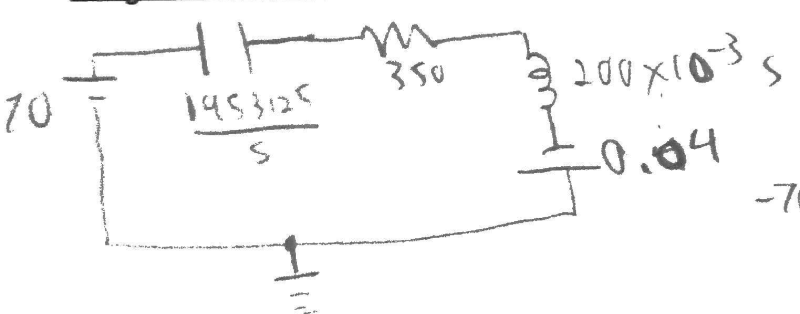


THIS PROBLEM MUST BE DONE BY LAPLACE
specifically conversion of components into an s-domain circuit!

4.1: Find Vo(t) for t>0 using ***Laplace from the s-domain equivalent circuit. Note: Vo(t) is the voltage across R1 AND L1.***

$L \cdot I(0^-) = 200\times10^{-3} \cdot \frac{70}{350}$

$= .04$



$-70V + i_1 \frac{1953125}{s} + i_1 350 + i_1 200\times10^{-3} s - .04 = 0$

$i_1\left(\frac{1953125}{s} + 350 + 200\times10^{-3} s\right) = 70.04$

$i_1 = 70.04 / \left(\frac{1953125}{s} + 350 + 200\times10^{-3} s\right)$

Vo(t) for t>0	On next page & boxed

More space on the next page!

$$i_1 = \frac{70.04}{\frac{1953125}{s} + \frac{350s}{s} + \frac{.2s^2}{s}} = \frac{70.04\,s}{.2s^2 + 350s + 1953125}$$

$$\frac{-350 \pm \sqrt{350^2 - 4(.2)(1953125)}}{2 \times .2} = -875 \pm 3000\,i$$

$$i_1 = \frac{70.04s}{(s+875+3000i)(s+875-3000i)} = \frac{A}{s+875+3000i} + \frac{B}{s+875-3000i}$$

$$70.04s = A(s+875-3000i) + B(s+875+3000i)$$

for $s = -875 + 3000i$ $\quad A = 70.04(-875+3000i)$

$B = 70.04(-875-3000i)$

For VL: $C = 70.04(-875+3000i)^2 \cdot .2$

$D = 70.04(-\cancel{885}875+3000i)^2 \cdot .2$

$$V_o(s) = i_1 R_1 + i_1 sL - 0.04$$

$$\boxed{\begin{aligned} V_o(t) = &-61285 + 210120i\, e^{-(875+3000i)} + 61285 - 210120i\, e^{-(875-3000i)} \\ &-115347125 - 73542000i\, e^{-(875+3000i)} \\ &+115347125 - 73542000i\, e^{-(875-3000i)} - 0.04 \end{aligned}}$$

4.2: How would you approach making sure that your answer from Problem 3.1 (IL(t)) corresponds with your answer in Problem 4.1 (Vo(t))?

take $V_o(t)$ and use that in a series RL circuit

$\frac{1}{L} \int V_o(t) = I_L(t)$ in problem 3.1

Extra credit (5 pts): Calculate whether your answers correspond to one another.