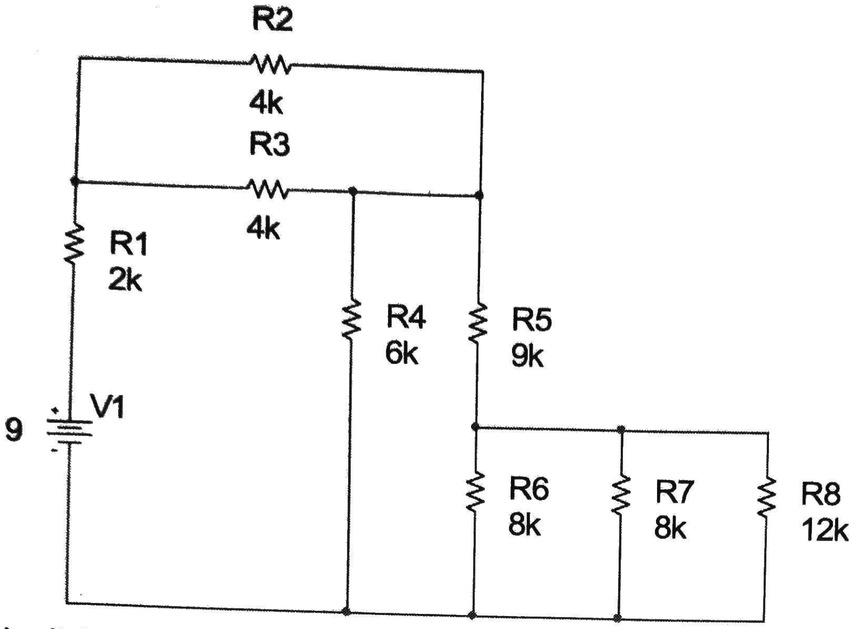
1) Equivalent Circuits/Circuit Reduction

Saaif Ahmed

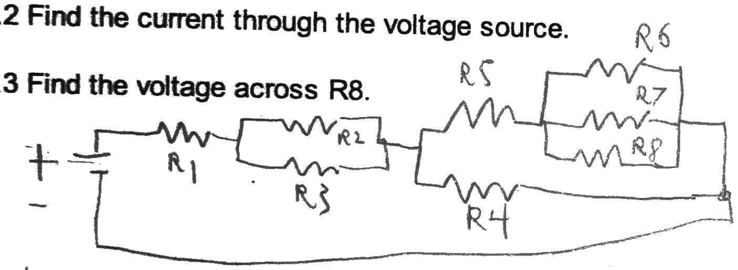


In the above circuit determine:

1.1: The equivalent resisance seen by the voltage source.

1.2 Find the current through the voltage source.

1.3 Find the voltage across R8.



$$\frac{1}{R_{678}} = \frac{1}{R_6} + \frac{1}{R_7} + \frac{1}{R_8} = \frac{1}{8k} + \frac{1}{8k} + \frac{1}{12k} \; ; \; R_{678} = 3k\,\Omega$$

$$R_5 + R_{678} = 12k\,\Omega \qquad \frac{1}{R_{45678}} = \frac{1}{R_{5678}} + \frac{1}{R_4} \; ; \; R_{45678} = 4000$$

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} \; ; \; R_{23} = 2000\Omega \; ; \; R_{12345678} = R_1 + R_{23} + R_{45678} = 8000\Omega$$

$$\boxed{1.1: 8000\Omega}$$

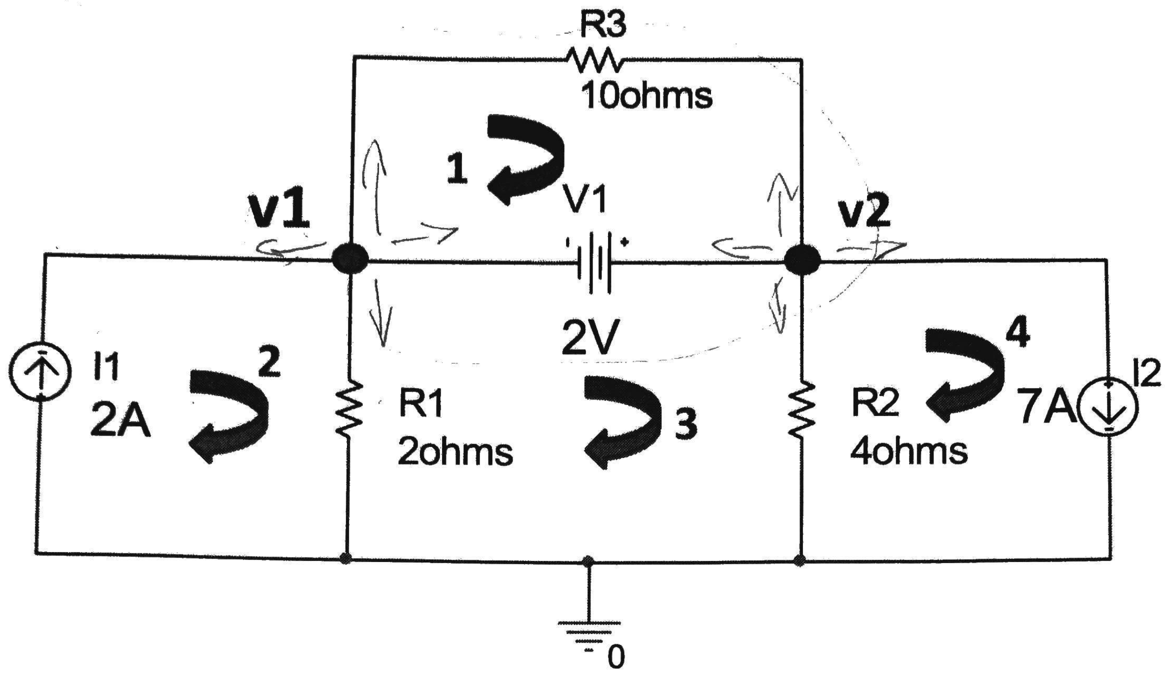
$$1.2: \; V = RI \qquad \frac{V}{R} = I \qquad \frac{9V}{8000\,\Omega} = I \qquad \boxed{I = 1.125\,mA}$$

$$1.3: \; V_{R45678} = \frac{V_1(4000)}{8000} = \frac{1}{2}V_1 = 4.5V \; ; \; V_{678} = \frac{V_{45678}(3k\Omega)}{12k\Omega}$$

$$V_{678} = \frac{1}{4}V_{R45678} = \frac{4.5}{4} = 1.125V \qquad V_{678} = V_{R8} \text{ (parallel)}$$

$$\boxed{V_8 = 1.125V}$$

2) Node/Mesh Analysis



2.1: Find the node voltages V1 and V2 using nodal analysis.

2.2: Using Mesh Analysis, find the voltage across R2.

2.1: $V_2 - V_1 = 2V$

$\frac{V_1}{R_1} - 2A + \frac{V_2}{R_2} + 7A = 0$

$$\underbrace{\begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ -1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} -5 \\ 2 \end{bmatrix}}_{B}$$

$A^{-1} \cdot B = X = \begin{bmatrix} -7.33 \\ -5.33 \end{bmatrix}$

$$\boxed{\begin{aligned} V_1 &= -7.33\ V \\ V_2 &= -5.33\ V \end{aligned}}$$

2.2 $i_1 R_3 + 2V = 0$

$i_3 R_1 - 2A R_1 - 2V + i_3 R_2 - 7A R_2 = 0$

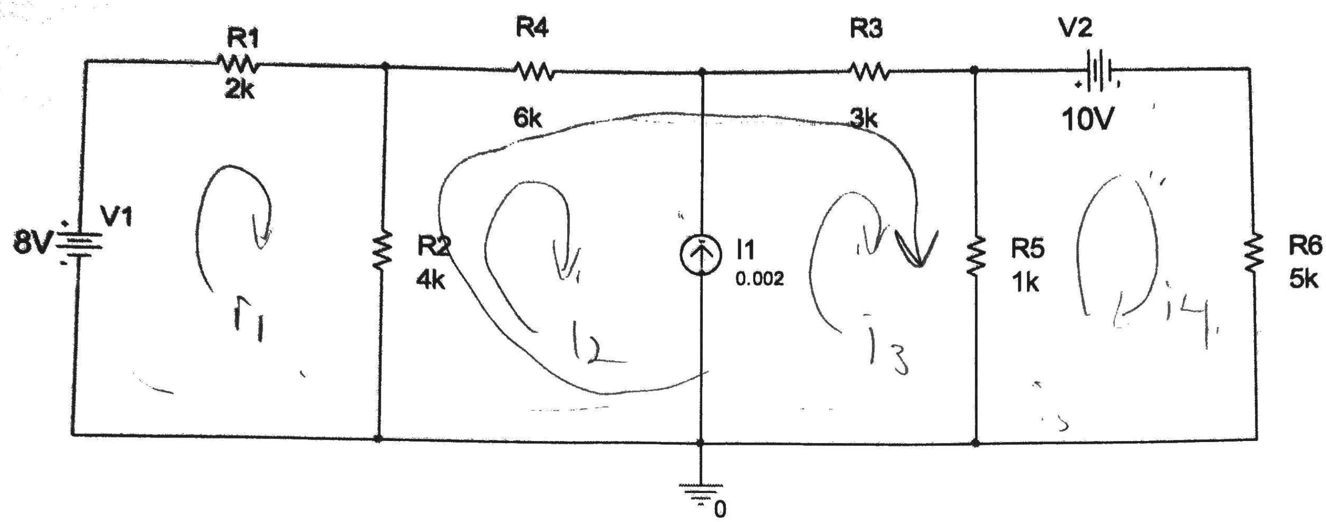
$$\underbrace{\begin{bmatrix} 10 & 0 \\ 0 & 6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_1 \\ i_3 \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} -2 \\ 11 \end{bmatrix}}_{B}$$

$A^{-1} \cdot B = X = \begin{bmatrix} -.2 \\ 5.66 \end{bmatrix}$

$V R_2 = (i_3 - 7A) R_2$

$$\boxed{V R_2 = -5.33\ V}$$

3) Superposition



3.1: Use any method in the parentheses to determine the voltage across R2 (node, mesh, circuit reduction, source transformation)

3.2: Find VR2 using superposition. *(For each source, draw the schematic).*

3.1 Mesh Analysis: $i_3 - i_2 = 0.002A$

$i_1 R_2 + -8V + i_1 R_1 - i_2 R_2 = 0$

$i_2 R_2 - i_1 R_2 + i_2 R_4 + i_3 R_3 + i_3 R_5 - i_4 R_5$

$i_4 R_5 - i_3 R_5 + 10V + i_4 R_6 = 0$

$i_1 6k - i_2 4k = 8$

$i_2 10k - i_1 4k + i_3 4k - i_4 1k = 0$

$i_4 6k - i_3 1k = -10$

$i_3 - i_2 = 0.002$

$$\begin{bmatrix} 6000 & -4000 & 0 & 0 \\ -4000 & 10000 & 4000 & -1000 \\ 0 & 0 & -1000 & 6000 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

A

$$\begin{bmatrix} 8 \\ 0 \\ -10 \\ .002 \end{bmatrix}$$

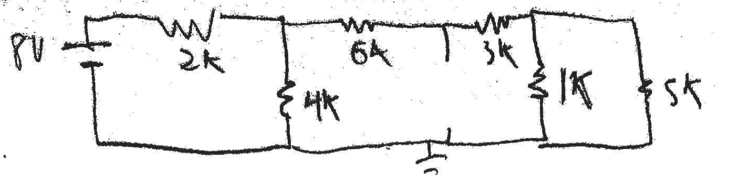
$$A^{-1} \cdot B = \begin{bmatrix} 0.001 \\ -3.58 \\ .0016 \\ -.0014 \end{bmatrix}$$

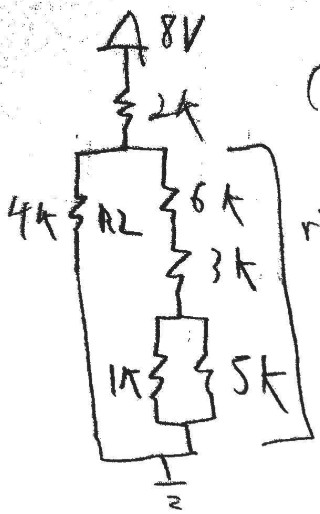
$VR_2 = (i_1 - i_2) R_2$

$0.0011 - (-3.58 \times 10^{-4}) R_2$

$VR_2 = \boxed{5.81 V}$

2: 8V source:



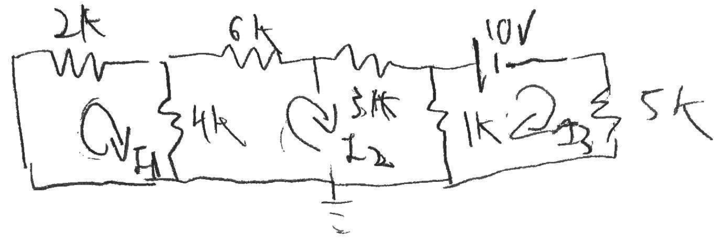


Circuit reduction: $\left(\frac{1}{1k}+\frac{1}{5k}\right)^{-1}+3k+6k = R_{right\ hand} = 9833.33\ \Omega$

$\left(\frac{1}{R_{right\ hand}}+\frac{1}{4k}\right)^{-1} = R_{Bottom} = 2843.37\ \Omega$

$\frac{8V\,(R_{bottom})}{R_{bottom}+2k} = \boxed{VR2 = 4.70\ V}$

10V source:



Mesh Analysis:

$I_1 2k + I_1 4k - I_2 4k = 0$

$I_2 4k - I_1 4k + I_2 6k + I_2 3k + I_2 1k - I_3 1k = 0$

$I_3 1k - I_2 1k + 10V + I_3 5k = 0$

$I_1 6k - I_2 4k = 0$

$I_2 14k - I_1 4k - I_3 1k = 0$

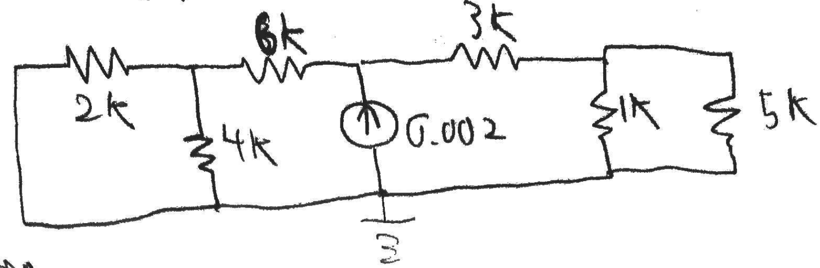
$I_3 6k - I_2 1k = -10V$

$$\begin{bmatrix} 6000 & -4000 & 0 \\ -4000 & 14000 & -1000 \\ 0 & -1000 & 6000 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix} \qquad A^{-1}\cdot B = \begin{bmatrix} 9.95\times10^{-5} \\ -1.49\times10^{-4} \\ -0.00169 \end{bmatrix}$$

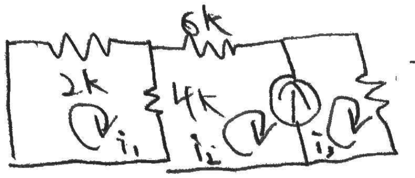
$VR2 = (I_1 - I_2)\,R2$

$\boxed{VR2 = 0.199\ V}$

3.2 I_1 current source:



Circuit Reduction



3833.$\overline{33}$ Mesh Analysis:

definition: $i_3 - i_2 = 0.002\,A$

$i_1 6k - i_2 4k = 0$

$i_2 10k - i_1 4k + i_3(3833.\overline{33}) = 0$

$$\begin{bmatrix} 6000 & -4000 & 0 \\ -4000 & 10000 & 3833.\overline{33} \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^{-1} \cdot B = X = \begin{bmatrix} -4.577\times10^{-4} \\ -6.87\times10^{-4} \\ 0.0013 \end{bmatrix}$$

$V_{R2} = (i_1 - i_2) R_2$

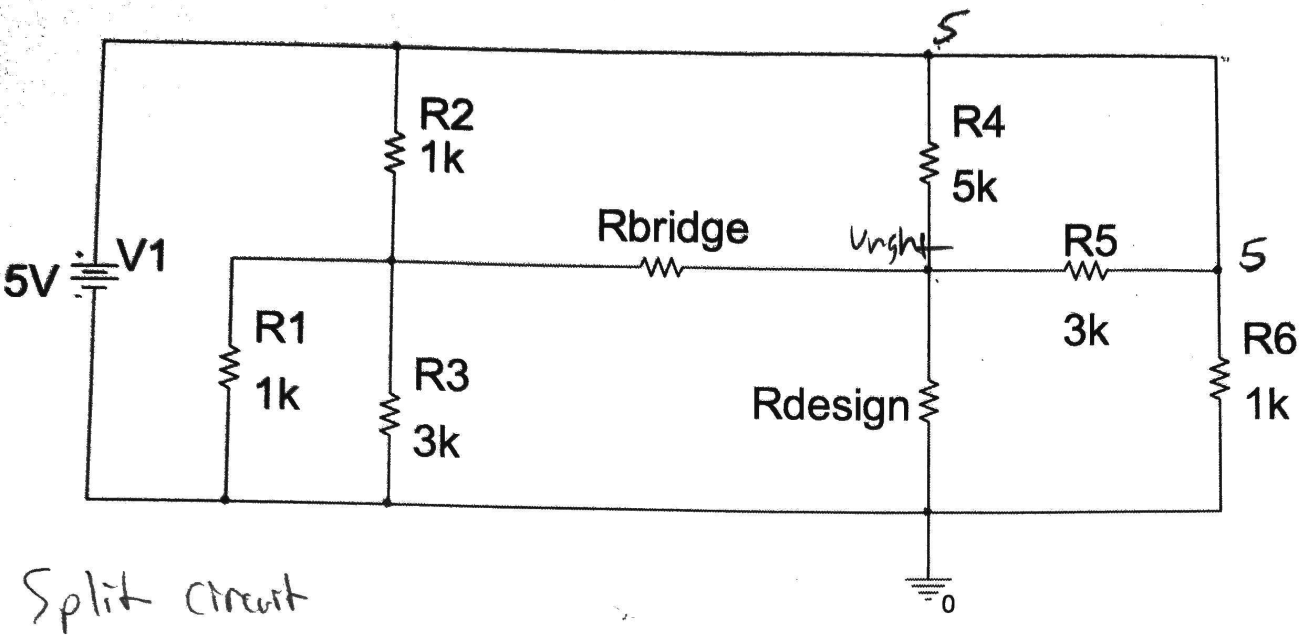
$V_{R2} = .91544\ V$

Final: $10V_{source} + 8V_{source} + 0.002\,A_{source} = V_{R2}$

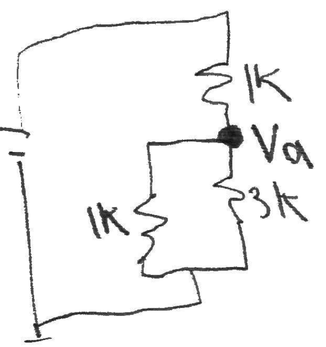
$0.199 + 4.700 + 0.91544 = \boxed{5.81\,V = V_{R_2}}$

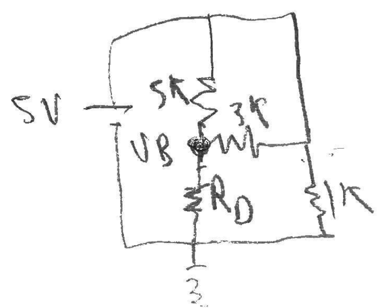
Bridge Circuit

Find the value of Rdesign so the bridge is balanced (no current through Rbridge).

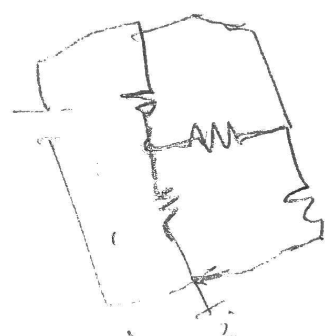


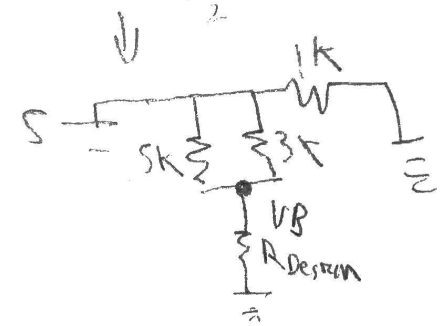
Split circuit





Find Rdesign for $V_a = V_B$





$$V_a = \frac{5V\left(\frac{1}{1000}+\frac{1}{3000}\right)^{-1}}{\left(\frac{1}{1000}+\frac{1}{3000}\right)^{-1}+1000} = 2.14V$$

$$V_B = \frac{5(R_D)}{\left(\frac{1}{5000}+\frac{1}{3000}\right)^{-1}+R_D}$$

$$2.14 = \frac{5(R_D)}{1875+R_D}$$

$$2.14R_D + 4012.5 = 5R_D$$

$$4012.5 = 2.857R_D$$

$$\boxed{R_{Design} = 1406.25\,\Omega}$$