Saad Ahmed 661425916

"I have not witnessed any wrongdoing, nor have I personally violated any conditions of the Honor Code, while taking this examination".

Ahmed 4/13/20

1. $a_0 = \frac{1}{T}\int_0^T x_1(t)\,dt \quad T = 2$

$a_0 = \frac{1}{T}\int_0^1 1\,dt + \int_1^2 0\,dt$

$a_0 = \frac{1}{2}\cdot 1 = \boxed{\frac{1}{2}}$

2. A) $a_k = \frac{1}{T}\int_0^T x_1(t) e^{-jk\omega_0 t}$

$T = 2$

$\omega_0 = \pi$

$$a_k = \frac{1}{2}\int_0^2 x(t) e^{-jk\pi t}\, dt$$

$$a_k = \frac{1}{2}\int_0^2 e^{-jk\pi t}\, dt$$

$$= \frac{1}{2} \cdot \frac{j}{\pi k} \int_0^{-jk\pi 2} e^u\, du$$

$$= \frac{j}{\pi k 2} \cdot e^u \Big|_0^{-jk\pi 2}$$

$$\boxed{a_k = \frac{j(e^{-2j\pi k} - 1)}{2\pi k}}$$

B) $T = 2$ $\quad b_k = \frac{1}{2}\int_1^3 e^{-jk\pi t}$

$\omega_0 = \pi$

$$\frac{1}{2} \cdot \frac{j}{\pi k} \int_1^{3-jk\pi} e^u\, du$$

$$= \frac{j}{2\pi k} \cdot e^u \Big|_1^{-jk\pi 3}$$

$$\boxed{= \frac{j e^{-3\pi k j} - j e^{-j\pi k}}{2\pi k}}$$

C) ~~same as~~ $z(t) = x(t) - y(t)$

thus coefficients $=$ $c_k = a_k - b_k$

$$\boxed{c_k = \frac{j(e^{-2j\pi k} - 1) - j e^{-j\pi k}(e^{-2\pi k j} - 1)}{2\pi k}}$$

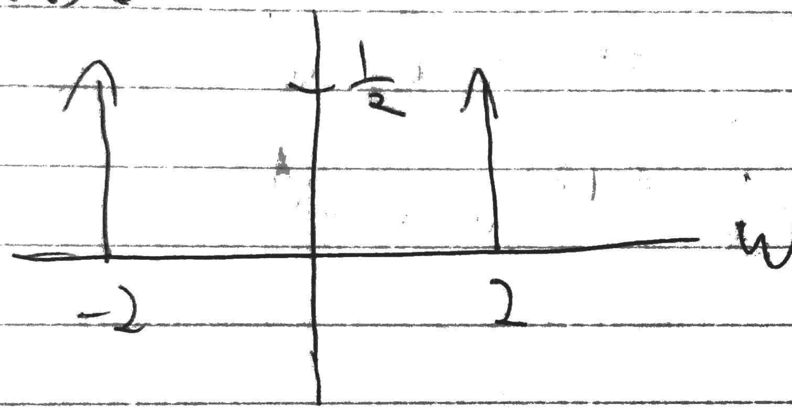
3. $y(t) = x(t)\cos(2t)$

$$Y(\omega) = \frac{1}{2\pi} X(\omega) * F[(\cos(2t))]$$

$$\left[Y(\omega) = \frac{1}{2\pi} X(\omega) * \pi\left(\delta(\omega-2) + \delta(\omega+2)\right)\right]$$

* only applicable at delta function ~~points~~ places.

$Y(\omega)$:



5. $X(w) = j\frac{d}{dw}\left(\frac{e^{j3w}}{1+jw}\right)$

$\cancel{F(X)}\ t\,x(t) \to j\frac{d}{dw}(X(w))$

$X(w) = e^{-jw(-3)} \cdot \frac{1}{1+jw}$

$\downarrow$

$\cancel{F}(x(t-t_0))\quad t_0 = -3$

$e^{-at}u(t) \to FT = \frac{1}{a+jw} \to \frac{1}{1+jw}\quad a=1$

$x(t) = t\left(e^{-a(t+3)}u(t+3)\right)$

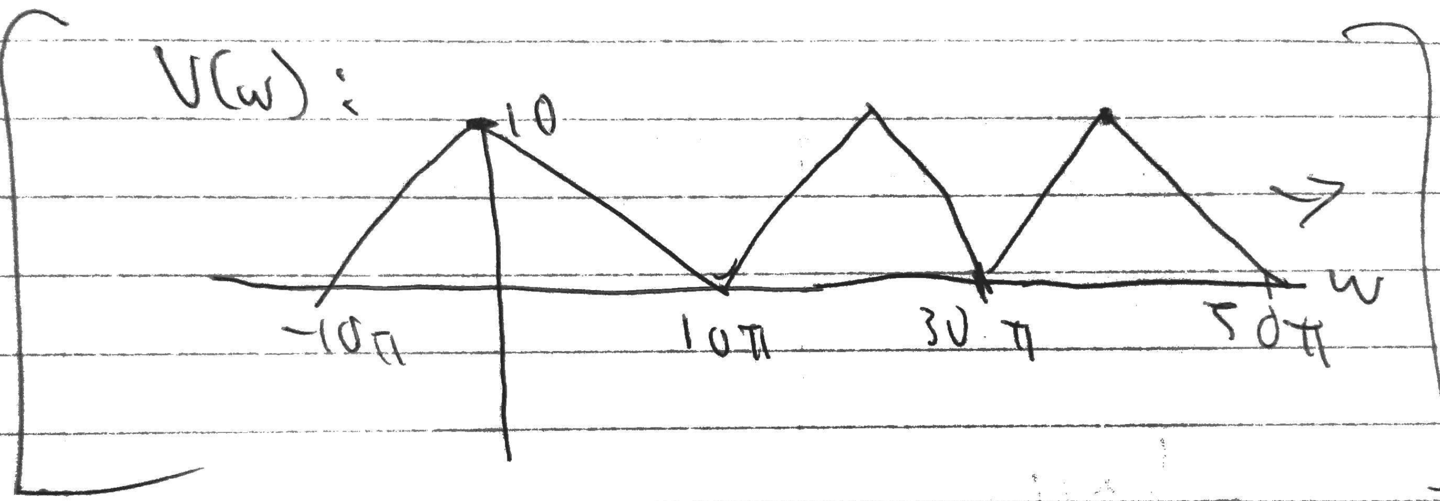
7. $v(t) = x(t) \cdot s(t)$

$V(\omega) = \left(\cancel{X}(\omega) * S(\omega)\right)\frac{1}{2\pi}$

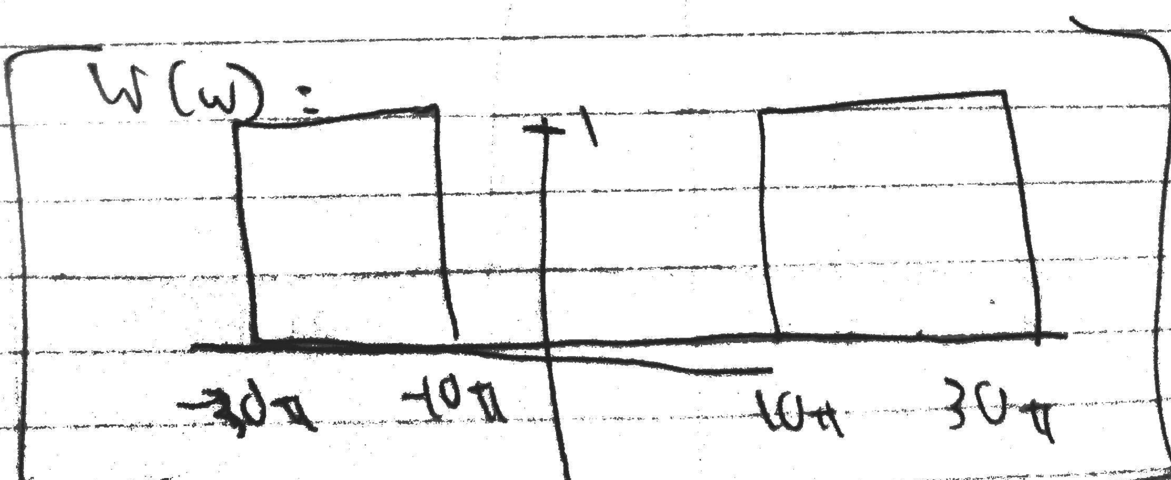
$S(\omega) = \frac{2\pi}{0.1}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi}{0.1}k\right)$

$$\left[V(\omega) = 10\sum_{k=-\infty}^{\infty} X(\omega - 20\pi k) \right]$$

shifts by 20π to the right



W(w): will be a low pas filter



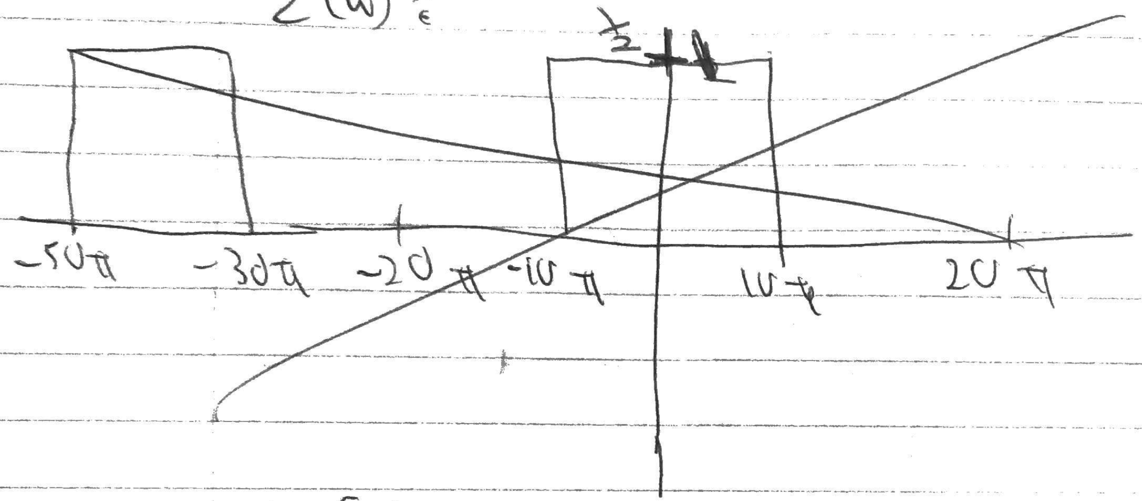
7. $z(t) = \cancel{\frac{1}{2\pi}}\, w(t) \cdot r(t)$

$Z(w) = \frac{1}{2\pi}\left(W(\cancel{w}w) * R(w)\right)$

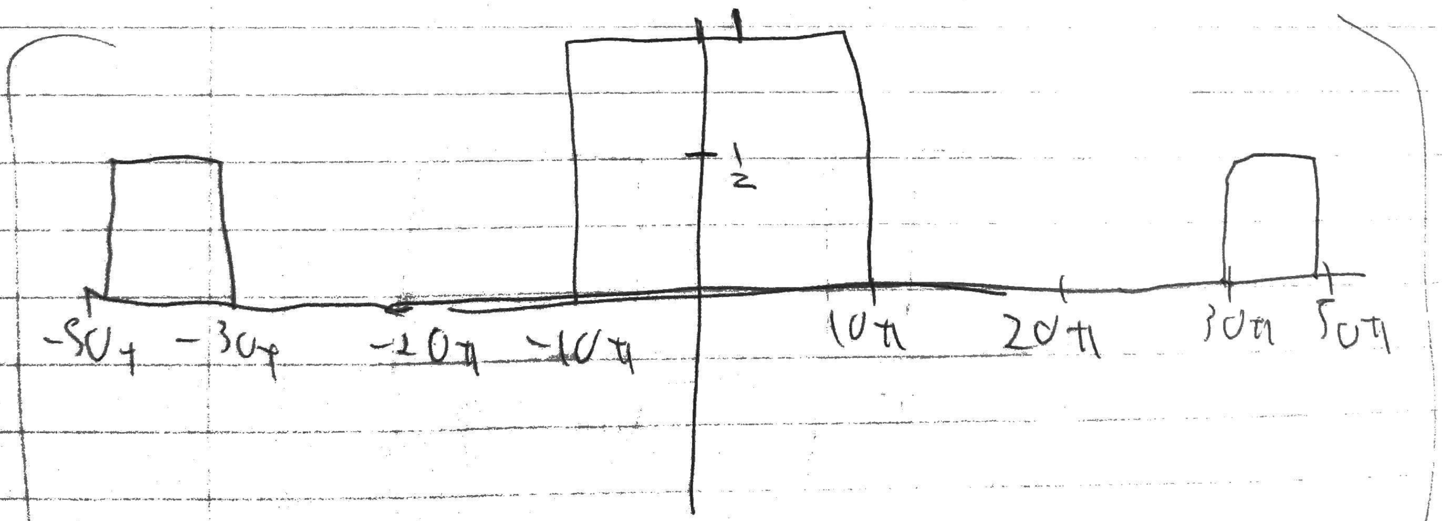
$R(w) = \pi\left[\delta(w - 20\pi) + \delta(w + 20\pi)\right]$

$Z(w) = \frac{1}{2}\left(W(w) * \delta(w - 20\pi)\right) + \frac{1}{2}\left(W(w) * \delta(w + 20\pi)\right)$

$Z(w)$:

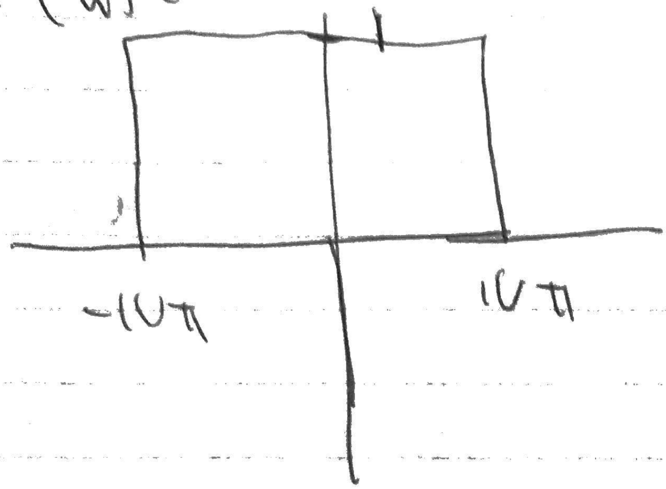


$Z(w)$:



7. $Y(\omega) =$ through low pass filter

$Y(\omega) =$



8 A) $x(t) = 2\cos(2\pi t) + \cos(3\pi t) + \cos(4\pi t)$

$f = 1 \quad f = 1.5 \quad f = 2$

The rate $\boxed{\omega_s = 8\pi}$

B) $y(t) = \cancel{2\cos(2\pi t)\cos(2000\pi}$

$= \left(2\cos(2\pi t) + \cos(3\pi t) + \cos(4\pi t)\right)$

$\cos(2000\pi t)$

$\cancel{FT}\{\cos(2000\pi t)\} = \pi\left(\delta(\omega - 2000\pi) + \delta(\omega + 2000\pi)\right)$

$X(\omega) * \pi[\delta(\omega - 2000\pi) + \delta(\omega + 2000\pi)]$

Things happens every 2000π

the highest frequency is

$\boxed{2000\pi}$