Signals & Systems HW#3

Monday, October 5, 2020 8:46 PM

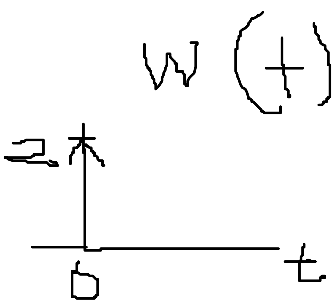
1.

$$x(t) * y(t) = \int_{-\infty}^{\infty} g\left(\frac{t-\tau+1}{2}\right)\left(\sum_{k=-\infty}^{\infty} \delta(t-4k)\right) d\tau$$

$$\sum_{k=-\infty}^{\infty} \delta(t-4k)\ (\int_{-\infty}^{\infty} g\left(\frac{t-\tau+1}{2}\right) d\tau)$$

The gate function is non zero from $\tau - 1\ to\ \tau + 1$
Thus the integral is $\big((\tau+1)-(\tau-1)\big) * 1 = 2$

$$w(t) = 2\left(\sum_{k=-\infty}^{\infty} \delta(t-4k)\right)$$



2.

$$x(t) = (1-t^2)g\left(\frac{t+2}{4}\right)$$

Meaning the bounds are determined by $g(t+1)$ because the function resides in $x(t)$

$$\int_{-\infty}^{\infty} x(\tau)g(t-\tau)d\tau = \int_{0}^{t+1} (1-\tau^2)d\tau = \tau - \frac{1}{3}\tau^3\Bigg|_0^{t+1} = (t+1) - \frac{1}{3}(t+1)^3$$

3:

a)

$$h(t) = s(t) - s(t-1) = 2e^{-t}u(t) - \left(2e^{-(t-1)}u(t-1)\right)$$
$$= 2e^{-t}u(t) - 2e^{-t+1}u(t-1)$$

b)

$$\int_{-\infty}^{\infty} (t-\tau)2e^{-\tau}u(\tau)u(t-\tau)\, d\tau\ - \int_{-\infty}^{\infty} (t-\tau)2e^{-\tau+1}u(\tau-1)u(t-\tau)d\tau$$
$$\int_0^t (t-\tau)2e^{-\tau}\, d\tau\ - \int_0^{t+1} (t-\tau)2e^{-\tau+1}d\tau$$

$$w(t) = 2(e^{-t}+t-1) - 2(-e+et+2e^{-t})$$

4:

$y(t) = x(t) * x(t)$ is non zero for $[-6,6]$
Means $x(t)$ range is $[-3,3]$
$g\left(\frac{t+3}{6}\right) = g\left(at + \frac{1}{2}\right)$
Thus $a = \frac{1}{6}$
Maximum Value is given when graphs overlap meaning max of $x(t)$ times max of $x(t)$
Thus $A = \sqrt{48}$

Answer: $A = \sqrt{48}$; $a = \frac{1}{6}$; $x(t) = \sqrt{48}\, g\left(\frac{1}{6}t + \frac{1}{2}\right)$

5:

$y(t) = h(t) * x(t)$
$y(t - t_1 - t_2) = h(t - t_1) * x(t - t_2)$
$h(t-2); t_1 = 2$
$e^{-2\left(t+\frac{1}{2}\right)}$; $t_2 = -\frac{1}{2}$
$y\left(t + \frac{1}{2} - 2\right) = \left(t + \frac{1}{2} - 2\right)^2$

6:

a)
$h(t) = t\big(u(t)\big)$

b)
System is memoryless

c)
System is BIBO stable since $\int x(t) < \infty$

d)
$h(t) = (t)g(t)$

e)
System is causal

f)
System is BIBO stable since $\int x(t) < \infty$