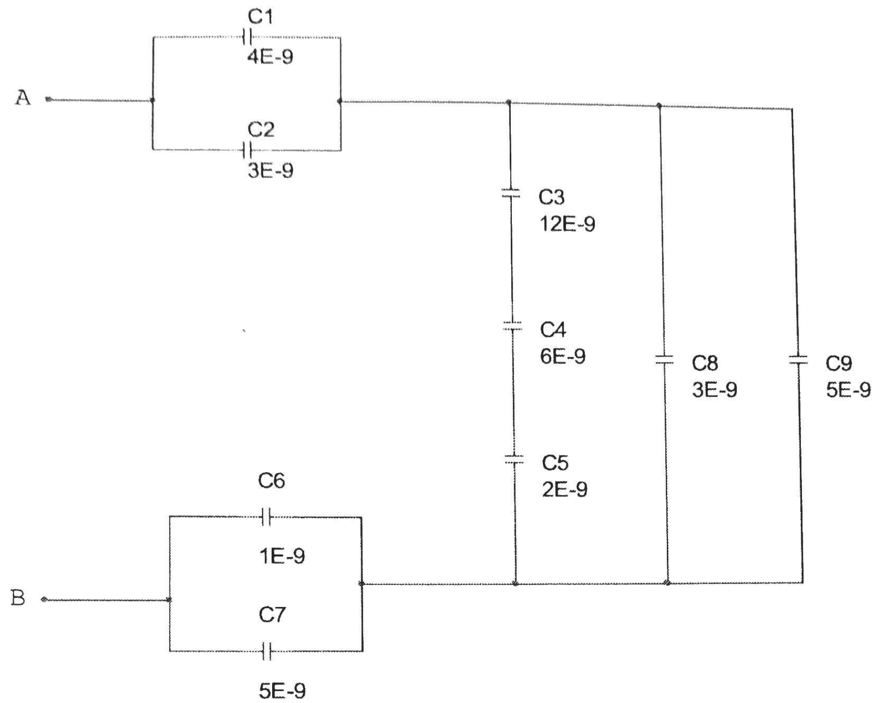
1) Capacitors and Inductors



1.1: For the above circuit, determine the equivalent capacitance between A and B.

$C_{12} = C_1 + C_2 = 4 \times 10^{-9} + 3 \times 10^{-9} = 7 \times 10^{-9}$ F

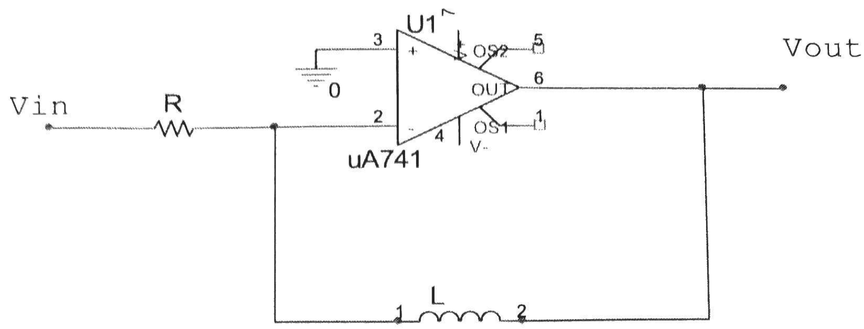
$C_{345} = \left(\frac{1}{C_3} + \frac{1}{C_4} + \frac{1}{C_5}\right)^{-1} = 1.33 \times 10^{-9}$ F

$C_{34589} = C_{345} + C_8 + C_9 = 9.33 \times 10^{-9}$ F

$C_{67} = C_6 + C_7 = 6 \times 10^{-9}$ F

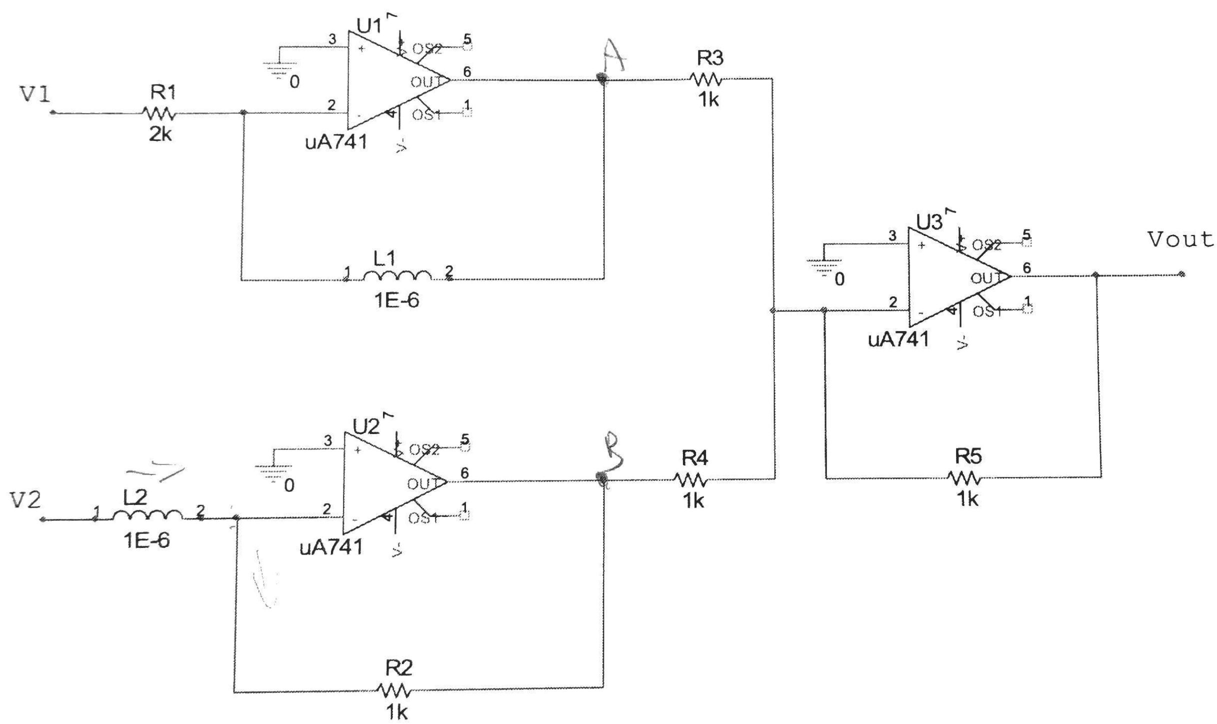
$C_{123456789} = \left(\frac{1}{C_{12}} + \frac{1}{C_{34589}} + \frac{1}{C_{67}}\right)^{-1} = \boxed{2.4 \times 10^{-9} \text{ F} = C_{AB}}$

2) Amplifier circuits

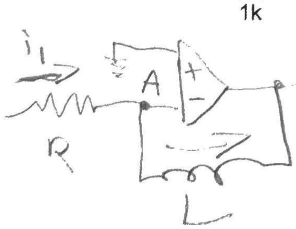


2.1: For the RL amplifier circuit, determine the relationship between Vout and Vin. As with RC amplifier circuits, KCL is a good starting point. (The power is taken out for simplicity but the op amp is powered).

2.2: In the circuit below V1 = V2 = 4 cos (300t). Determine Vout



2.1:



ideal op amp KCl $V_A = 0$

$\frac{V_{in}}{R} = i_1$ $\quad V_{out} = 0 - V_L = -L\frac{di}{dt}$ the $\frac{di}{dt}$ refers to i_1 $\quad V_{out} = -L\frac{di}{dt}$

$$V_{out} = -L\frac{d}{dt}\left(\frac{V_{in}}{R}\right) = \boxed{V_{out} = \frac{-L}{R}\frac{dV_{in}}{dt}}$$

2.2: $V_{out} = 0 - \frac{V R_2}{\cancel{\ }} = -I_1 R_2$ what is I_1? $V_S = \frac{di}{dt} L$

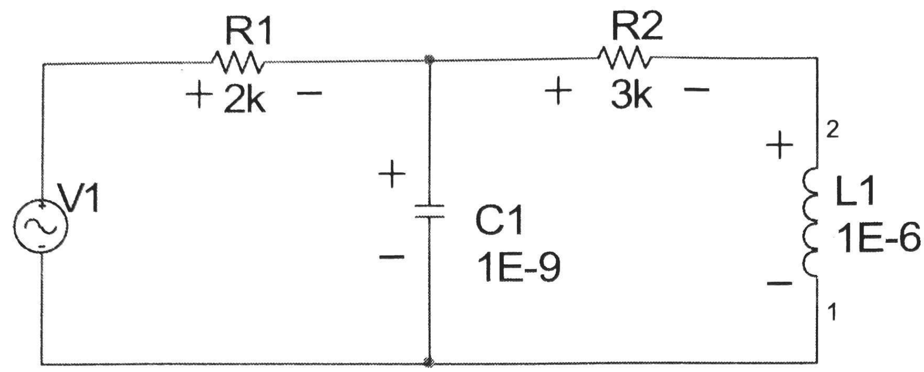
$\frac{1}{L}\int V_s = I_1$ $\quad V_{out} = -\frac{R_2}{L}\int V_s\,dt$

At A: $\frac{-1\times10^{-6}}{2000}\left(-\sin(300t)1200\right) = 6\times10^{-7}\sin(300t)$

At B: $\frac{-1000}{1\times10^{-6}}\left(\frac{4}{300}\sin(300t)\right) = -1.33\times10^{7}\sin(300t)$

$V_{out} = -\left(6\times10^{-7}\sin(300t)\right) - \left(-1.33\times10^{7}\sin(300t)\right) = \boxed{1.33\times10^{7}\sin(300t)}$

3) Voltage/Current continuity



In the above circuit, the voltage is defined as follows:

$$V1 = \begin{cases} 5V & t < 0 \\ 10V & 0 < t \end{cases}$$ (the voltage source turns on at t = 0)

3.1: Determine a mathematical expression for the source. **Meaning use the unit step function u(t) in your expression.**

3.2: At t =0- (just before the voltage changes), for the polarities indicated, determine the voltage across each component and the current through each component.

3.3: At $t=0^+$ (just after the voltage changes), determine the voltage across each component and the current through each component for the polarities indicated in the circuit.

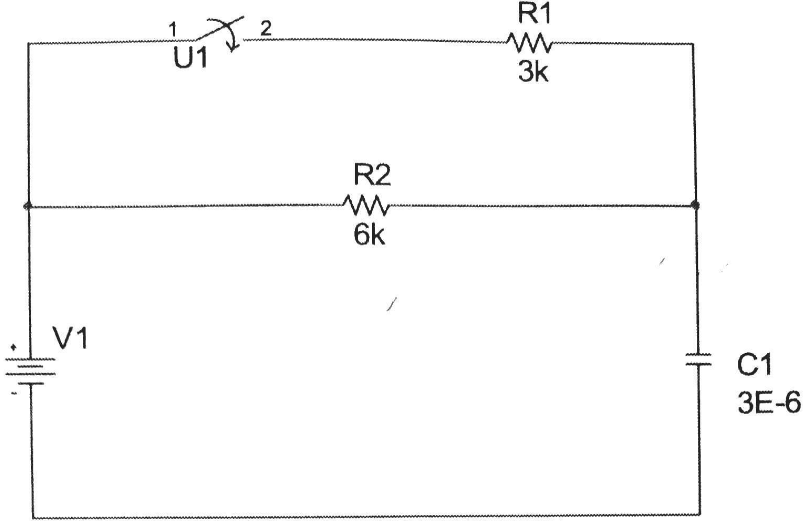
3.1: $V(t) = V_a u(t)$ $V_a u(t) = \begin{cases} V_a & 0 < t \\ \frac{V_a}{2} & t < 0 \end{cases}$ $V_a = 10V$

3.2:

Component	Voltage	Current
R1	2V	1mA
R2	3V	1mA
C1	3V	0mA
L1	0	1mA

Inductor is a short
cap is open.
Simple voltage divider

4) RC circuits



In the above circuit, the voltage source turns on at t=0, V1 = 15u(t). At t = 1ms, switch U1 closes.

4.1: Determine the voltage across the capacitor for a function of time for 0<t<1ms

4.2: Determine the voltage across the capacitor for t > 1ms.

4.1:

$$\frac{dv_1}{dt} + \frac{V_{C1}}{R_1C_1} = \frac{15}{R_1C_1} \qquad \tau = RC = 0.001s$$

$$V_c(t) = A_1 e^{-t/\tau} + B_1$$

$$A_1 e^{-55.6t} + B_1$$

∞

$$0 + B_1 = 15 \Rightarrow B_1 = 15$$

0

$$A_1 + B_1 = 0 \Rightarrow A_1 = -15$$

$$\boxed{V_c(t) = -15e^{-55.6t} + 15}$$

4.2 : circuit reduction

$R_{eq} = 2k$

$\frac{dV_c}{dt} R_{eq} C + V_c = 15$ $\quad \tau = RC$

$= 0.006$

$V_c(t) = A_1 e^{-t/\tau} + B_1$ $\quad t < 1ms$

$= A_1 e^{-167(t-0.001)} + B_1$

∞ $\quad$ 0

$0 + B_1 = 15 \quad B_1 = 15$ $\quad A_1 + B_1 = 0 \rightarrow A_1 = -15$

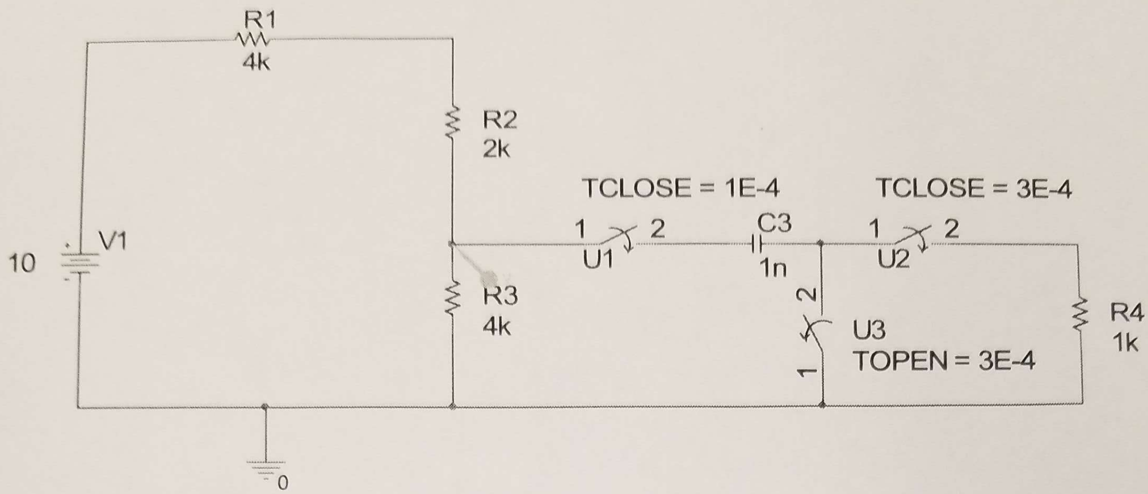
$-15 e^{-55.6(0.001)} + 15 = 0.811$

$A_1 + B_1 = 0.811$

$A_1 = 0.811 - 15 = -14.189$

$$\boxed{V_c(t) = -14.2 e^{-167(t-0.001)} + 15}$$

5) Thevenin Equivalent and Switching Circuits



In the above circuit, the voltage source turns on at t = 0. Switch U1 closes at t = 0.1 ms. Switch U2 closes and switch U3 opens at t = 0.3 ms (effectively putting resistor R3 in series with C3 at t = 0.3 ms).

5.1: Determine the voltage across R3 as a function of time for t > 0.

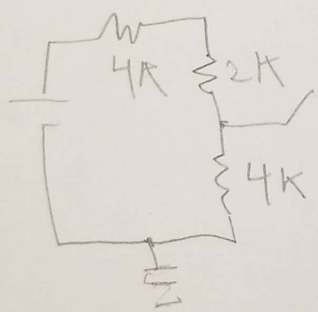
Three regions of interest

$t < 0.1\text{ms}$

$0.1\text{ms} < t < 0.3\text{ms}$

$0.3\text{ms} < t$

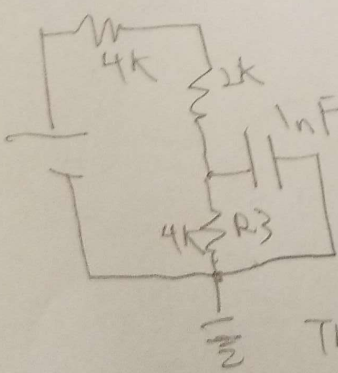
$t < 0.1\text{ms}$



$V_{R_3} = 10\,\frac{R_3}{R_1+R_2+R_3} = \boxed{4V \text{ at } t<0.1\text{ms}}$

$0.1\text{ms} < t < 0.3\text{ms}$

Thevenin:



$V_{TH} = 4V$

$R_{TH} = \frac{4V}{0.00166667A} = 2400\,\Omega$

$I_{norton} = \frac{10V}{6k} = 0.001666\,A$

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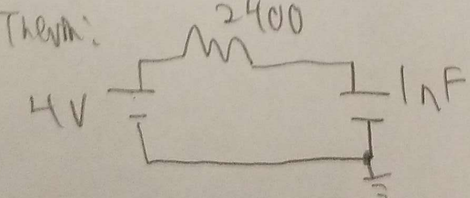
$\tau = R \cdot C$

$= 2.4\times10^{-6}$

Thevin:



@ $0.1ms < t < 0.3ms$: $V_C(t) = V_{CN} + V_{CF}$

$$= A_1 e^{-\frac{(t-0.0001)}{\tau}} + A_2$$

$V_{R3} = V_C$ $\quad V_C(t=0.1^-_{ms}) = V_C(t=0.1^+_{ms}) = 0$

$0 = A_1 e^0 + A_2 \quad A_2 = -A_1$

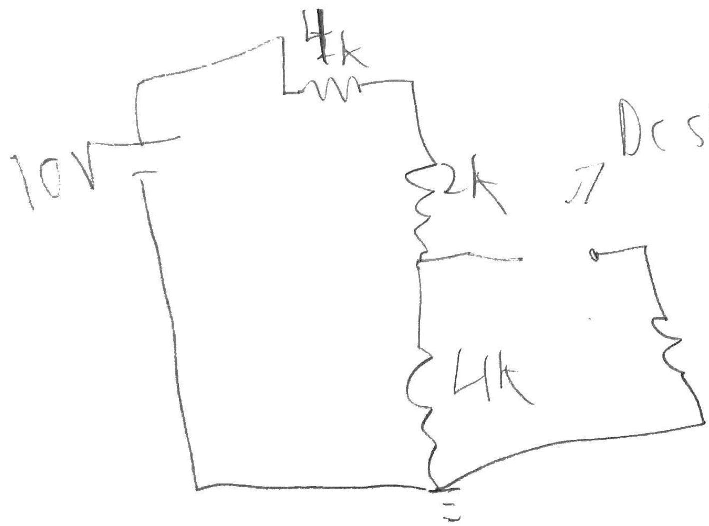
final: $V_C(t) = 4V$

$$4 = \underbrace{A_1 e^{\frac{(-0.0003-0.0001)}{2.4\times10^{-6}}}}_{\text{ignore}} - A_1$$

$4 = -A_1 \quad A_1 = -4 \quad A_2 = 4$

$$\boxed{V_C(t) = V_{R_3}(t) = -4e^{-\frac{(t-0.0001)}{2.4\times10^{-6}}} + 4}$$

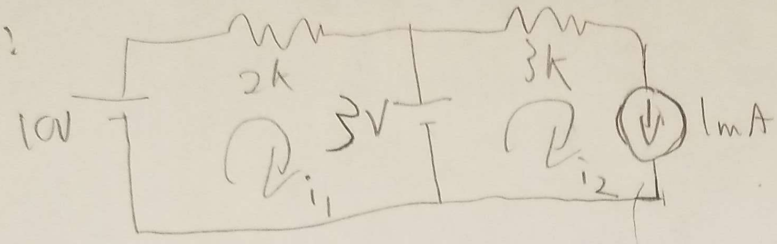
t ~~ANA~~ $> 0.3ms$



DC steady state

$$\frac{10v(4k)}{4k+2k+4k} = \boxed{V_{R3} = 4V}$$

3.3:



$i_1 2k = 7V$
$i_1 = 0.0065 A$

$i_1 - i_2 = 2.5mA$

$-5V + i_2 3k + V_4 = 0$
$V_4 = 5V + i_2 3k$
$=$

Component	Current	Voltage
R_1	3.5mA	7V
R_2	1mA	3V
C1	2.5mA	5V
4	-1mA	7V