

HW 3

Monday, September 21, 2020

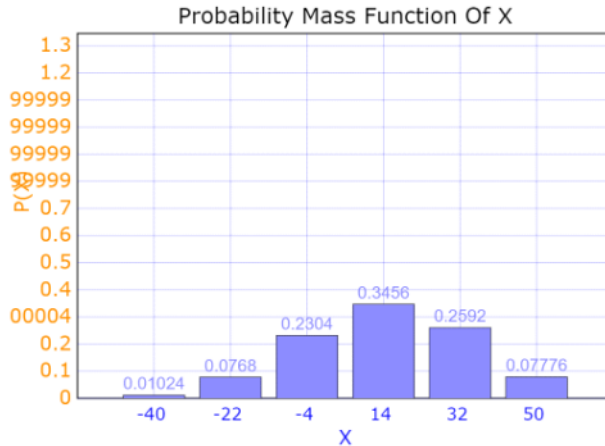
10:01 AM

1:

a)

Any value of X will be given by $PMF_x = ((10 * i) - (8(5 - i))) * (.4)^{5-i} * (.6)^i * \binom{5}{i}$

Where i is the number of crashes



b)

$$E(X) = \sum_{i=0}^5 ((10 * i) - (8 * (5 - i))) * (.4)^{5-i} * (.6)^i * \binom{5}{i}$$

$$= 14$$

c)

$$Var(X) = E(X^2) - (E(X))^2$$

$$= \sum_{i=0}^5 ((10 * i) - (8 * (5 - i)))^2 * (.4)^{5-i} * (.6)^i * \binom{5}{i} - (14)^2$$

$$= 584.8 - (14)^2$$

$$= 388.8$$

d)

$$Y = \text{\# of crashes} \rightarrow \text{binomial} = \binom{5}{i} (0.4)^{5-i} (0.6)^i$$

$$X = aY + b$$

$$10i - 40 + 8i \text{ where } i \text{ is the number of crashes}$$

$$X = 18Y - 40$$

e)

$$E(X) = \sum_{Y=0}^5 (18Y - 40) \binom{5}{Y} (0.4)^{5-Y} (0.6)^Y$$

$$= 14$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= \sum_{Y=0}^5 (18Y - 40)^2 \binom{5}{Y} (0.4)^{5-Y} (0.6)^Y - 196$$

$$= 388.8$$

The values match

2:

a)

$$\begin{aligned} E(Y) &= E(Y|good)P(good) + E(Y|bad)P(bad) \\ &= (1.5)(0.3) + (12)(0.7) \\ &= 8.85 \end{aligned}$$

b)

$$\begin{aligned} E(Y^2) &= E(Y^2|good)P(good) + E(Y^2|bad)P(bad) \\ &> E(X^2): X \text{ is Poisson} \rightarrow \lambda^2 + \lambda \\ &= (1.5^2 + 1.5)(0.3) + (12^2 + 12)(0.7) \\ &= 110.325 \end{aligned}$$

3:

a)

$$\begin{aligned} PMFp_x(x|X \leq 4) &= \frac{P(X = x \cap X \leq 4)}{P(X \leq 4)} \\ > P(X \leq 4) &= \sum_{i=0}^4 (0.6)^i (0.4) = 0.92224 \\ > \text{if } X > 4 &\rightarrow PMF(x) = 0 \end{aligned}$$

$$= \frac{1}{P(X \leq 4)} * P(X = x)$$

b)

$$E(x) = \frac{1}{P(X \leq 4)} \sum_{i=0}^4 (0.6)^{i-1} (0.4) = 1$$

c)

$$PMF_x p(X = x) = P(x|X \leq 4)P(X \leq 4) + P(x|X > 4)P(X > 4)$$

Use the law of total probability

$$PMFp_x(x|X > 4) = \frac{PMFp_x(X = x) - PMFp_x(x|X \leq 4)P(X \leq 4)}{P(X > 4)}$$