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"On my honor I have neither given nor received unauthorized aid on this exam.

-Saait Ahmed
12/15/20

2. A: N is geometric

B: $\frac{2}{3} \cdot \frac{2}{3}$ first two are failure

$= \frac{4}{9} \rightarrow \boxed{4/9}$

C: $P(n>4 \mid 2 \text{ unsucess}) = \dfrac{P(2 \text{ unsucss} \mid n>4) \cdot P(n>4)}{P(2 \text{ unsuccess})}$

$= 1 \cdot \left(\frac{2}{3}\right)^4 / \left(\frac{2}{3}\right)^2 = \boxed{\frac{4}{9}}$

D: The independence of trials for a geometric random variable.

2. A: $\binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3}$

$1 + 3 + 3 + 1$

S = { nothing, mushrooms, sausage, onions, mushrooms + sausage, mushroom + onions, ~~mushrooms~~ * sausage + onion, all 3 }

[8 outcomes]

B: $\binom{6}{4}$ ways to get first 4.

$P(\text{mush}) = \frac{1}{2}$

$P(4 \text{ of } 6 \text{ have mushroom}) = \left(\frac{1}{2}\right)^4 \cdot \binom{6}{4} = \boxed{0.9375}$

C: $P(\text{onion}) = \frac{1}{2}$

$P(\text{failure} > 2) \cdot P(\text{success})$

$P(\text{failure}) = \frac{1}{2}$

$\left(\frac{1}{2} \cdot \frac{1}{2}\right) \cdot \left(\frac{1}{2}\right) = \boxed{\frac{1}{8}}$

3 A: let Y = minutes [illegible] on force

$E(\cancel{E}(Y|\cancel{X}))$

$\cancel{= E(X \cdot E(Y))}$

$\cancel{E(Y) = \frac{1}{2}\left(\frac{1}{2}x^2 + \frac{3}{2}x^2\right)}$

$\cancel{= \frac{1}{2}(2x^2)}$

$= x^2$

$E(Y) = E(E(Y|X))$

$E(Y|X) = x^2$

$E(Y) = E(x^2)$

$Var(x) = E(x^2) - (E(x))^2$

$Var(X) = \frac{1}{12}(b-a)^2 = \frac{1}{12}(10-0)^2 = \frac{25}{3}$

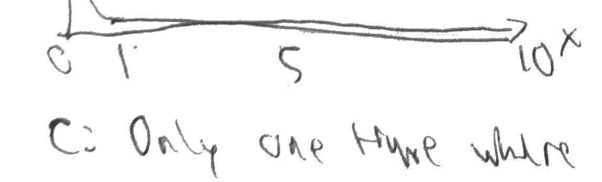
$E(x) = 5$

$E(x2) = \boxed{\frac{100}{3}} = E(Y)$

B: $f(Y|x) \cdot f_x(x) = \frac{1}{10} \cdot \frac{1}{x^2} = \boxed{\frac{1}{10x^2} = f(x,y)}$

for $y \geq 0$

for $x \in [0, 10]$



C: Only one time where $f(x,y) = 48$

$x = 0.045$

$f_Y(y) = \cancel{10(0.0456)^2} = \boxed{0.0456}$

4. A: ~~$f_{S|H_0}(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-}$~~

sum of 5 gaussians, independent

$$f_{S|H_0}(x) = \frac{1}{\sqrt{5}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-15}{\sqrt{5}}\right)^2}$$

$$\left[f_{S|H_1}(x) = \frac{1}{\sqrt{5}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-75}{\sqrt{5}}\right)^2} \right]$$

B: ~~f(x)~~ Choose H_1 since less standard deviations away ~~and~~.

5 ~~B~~ Ai ~~85~~ 85 hp (>15)

$$P(85hp \mid x>15) = \frac{P(x>15 \mid 85hp)\, P(85hp)}{\cancel{P(x>15)}}$$

$$P(85) = \frac{1}{4} e^{-\frac{85}{4}} = 1.47 \times 10^{-10}$$

$P(15 \text{ bites } \cancel{\text{health}} > 85) = \cancel{P(85hp \geq 15 \text{ bites})}$

~~P(85)~~ ~~P(85~~ $= \frac{\lambda}{1-\lambda} = \frac{1}{3}$

$P(x>15) = \frac{1}{3}$

Bi u - ? $P\left(\frac{16(82-u)}{4 \cdot \sqrt{3}} > \frac{16c}{\sqrt{16} \cdot \sqrt{3}}\right) = 0.01$

z $\quad z = 2.3263$

$2.3263 = 2c = 1.196$

$[80.836, 83.163]$

6. A: $P(X<Y) = \int_0^2 \int_0^y \frac{1}{3}(x+y)\,dx\,dy$

$\int_0^2 \int_0^y \frac{1}{3}x + \frac{1}{3}y \,dx\,dy$

$\int_0^2 \frac{1}{3}(\frac{1}{2}x^2) + \frac{1}{3}yx \Big|_0^y$

$\int_0^2 \frac{y^2}{2}\,dy$

$\frac{1}{2}(\frac{1}{3}y^3)\Big|_0^2 = \boxed{\frac{4}{3}}$

B: $Cov(X,Y) = E(XY) - E(X)E(Y)$

$E(XY) = \int_0^2 \int_0^1 \frac{1}{3}x^2y + \frac{1}{3}xy^2 \,dx\,dy$

$\int_0^2 \frac{1}{9}yx^3 + \frac{1}{6}x^2y^2 \Big|_0^1$

$\int_0^2 \frac{1}{9}y + \frac{1}{6}y^2 \,dy$

$\frac{1}{18}y^2 + \frac{1}{18}y^3 \Big|_0^2 = \boxed{\frac{12}{18}}$

C: It will be positive because $Cov(X,Y)$ is positive

D: They are not independent because $Cov \neq 0$

7. A: $\frac{F(x|H_0)}{F(x|H_1)} \gtrless \frac{P(H_0)}{P(H_1)}$

Using graph $f_{Y|X}(y|X=1)$

we want the maximum so

$MAP(Y) = .16$

B: from graph $f_{X|Y}(1|Y=y)$ take max

$ML(Y) = 0.6$

C: $\hat{Y}_{mmse} = E(Y|X=1)$

using graph $E(Y|X=x)$

where $x=1$

$\hat{Y}_{mmse} = 3$

8. A: for independent $f(x,y) = f(x) \cdot f(y)$

$\int\int \frac{1}{4} e^{-\frac{1}{4}r} \cdot \frac{1}{5} e^{-\frac{1}{5}b} \, db \, dr$

$\int_0^\infty \int_0^\infty$

$\int_0^\infty \int_0^\infty \frac{1}{20} e^{-\frac{1}{4}r} e^{-\frac{1}{5}b} \, db \, dr$

$\int_0^\infty -\frac{1}{4} e^{-\frac{1}{4}r} e^{-\frac{1}{5}b} \Big|_0^\infty$

$\int_0^\infty + \frac{1}{4} e^{-\frac{1}{4}r} \Big|_0^\infty$

$-1 e^{-\frac{1}{4}r} \Big|_0^\infty$

$CDF = e^{-\frac{1}{4}R} = P$

B: calculated above.

$\frac{1}{4} e^{-\frac{1}{4}r} \cdot \frac{1}{5} e^{-\frac{1}{5}b} = PDF$

C: ~~multiply means = 20 minutes~~

minimum of means = 4 minutes

6. A: You can have the time between arrivals be exponentially distributed with arrival rate 4.

B: You can have the value ~~exponentially~~ dependent on the ~~ber~~ coin flip. so for 0 you do not do exponential. And for 1 you do.

$\cancel{h^x(1-p}$

$$\left[h^x(1-h)^{1-x} = e^{hx}\right] \quad x \in \{0,1\}$$

for 0 you get 1
and for 1 you get e^h (the exp distribution)