Assignment #9 - Saaif Ahmed

Sunday, November 24, 2019 9:23 PM

Problem 22.2

The answer to Problem 22.2 is neither YES nor NO. Both responses to that statement results in an infinite loop of indecision. Take for example answering YES. We would be stating that the answer to Problem 22.2 is NO while simultaneously answering YES to Problem 22.2. This is invalid. Take answering NO. We would be denying the answer NO to Problem 22.2 which means we are implying YES. But, while we are implying YES we are simultaneously answer NO to Problem 22.2

The problem here lies within the fact that we are attempting to reconcile a state within itself. This cannot be done without outside information or a clear definition. So the answer is:

Answer: Neither

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Problem 22.9

(a) Injective

Disprove:

Proof by contradiction:

Assume f is injective to **N**. Therefore every combination of A and B must have a unique output. Take A = 4 and B = 15. Therefore $\frac{AB}{2} = 30$.

Now take A = 12 and B = 5. $\frac{AB}{2} = 30$.

Thus two entirely distinct inputs match to the same value in **N**.

Original statement is false. f is not injective to **N**.

Answer: Disprove. f is not injective

(b) Surjective

Proof by Pigeon Hole Principle:

f will be surjective to **N** so long as one output in **N** matches to two inputs in f. This was shown above with A = 4 ; B = 15 and A = 12 ; B = 5. All other inputs of f can match to every other value in **N** but the one example shown proves that there is overlap.

Answer: Prove. f is surjective

(c) Bijective

Disprove:

Proof by definition:

Two sets X and Y are bijective if $|X| \leq |Y|$ and $|Y| \leq |X|$ or in other terms, they are both injective to each other. Because f was shown to be not injective, it cannot be bijective with **N**

Answer: Disprove. f is not bijective

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Problem 22.25

(a)

Proof:

Let A be the set of all functions that maps possible combinations of {x, y} into some value of **N**, where
$x = f(0)$ and $y = f(1)$.
Formally written that is $A = \{f: \{0,1\} \to \boldsymbol{N}\}$.

Let's look at the definition of *f*. It takes in either 0 or 1 and maps it to some number in **N**.

Which means that for all possible functions $f(0)$ the cardinality is the same as $|\boldsymbol{N}|$ the same is true for $f(1)$.

The total cardinality of $A = |\boldsymbol{N}| \times |\boldsymbol{N}|$. This is countable because $|\boldsymbol{N}|$ is countable.

(b)

Proof:

Let's define a set $A = \{f: \boldsymbol{N} \to \{0,1\}\}$ that contains all the functions that turn **N** into 0 or 1.
There is a bijection between this set and the set of infinite binary strings. This is because there are infinite ways to create a function that does this mapping with each number in **N**.
Thus the cardinality of the set of infinite binary strings is present in this set A.

A is uncountable.

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Problem 23.33

(b)

(i)

Looking for strings that do not have number of 0 divisible by 4.
Find the strings that do have number of 0 divisible by 4 and take complement.

String with multiple of 4 zeros:
Must have all combinations of 1's before it
Must have all combinations of 1's after it
Must have total number of zeros divisible by 4.

$* = Kleene\ Star$
All combinations of 1 including none : 1^*
Total 0's divisible by 4 : $\{0\}^{\cdot n} \mid n = 4k\ ; k \in \boldsymbol{Z}$

Resultant expression: $\left\{\{1\}^* \cdot \{0\} \cdot \{1\}^*\right\}^{\cdot\, n}$ where $n = 4k; n \in \boldsymbol{N}$

Answer: $\neg(\left\{\{1\}^* \cdot \{0\} \cdot \{1\}^*\right\}^{\cdot\, n}$ where $n = 4k\ ; n \in \boldsymbol{N}$)

(ii)

Looking for strings that do not have number of 0 divisible by 4.
Means that string must have either $4n + 1$; $4n + 2$; $4n + 3$ values of 0.

Union each expression:

Resultant expression:
$* = Kleene\ Star$

Answer:
$(\left\{\{1\}^* \cdot \{0\} \cdot \{1\}^*\right\}^{\cdot 1} \cdot \left\{\{1\}^* \cdot \{0\} \cdot \{1\}^*\right\}^{\cdot\, n}) \cup (\left\{\{1\}^* \cdot \{0\} \cdot \{1\}^*\right\}^{\cdot 2} \cdot \left\{\{1\}^* \cdot \{0\} \cdot \{1\}^*\right\}^{\cdot\, n}) \cup$
$(\left\{\{1\}^* \cdot \{0\} \cdot \{1\}^*\right\}^{\cdot 3} \cdot \left\{\{1\}^* \cdot \{0\} \cdot \{1\}^*\right\}^{\cdot\, n}) \mid n = 4k\ ; k \in \boldsymbol{Z}\ ; n \in \boldsymbol{N}$

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Problem 24.3

(d)

The language produced by this automaton is that of binary strings whose total number of 1's is not a multiple of 3.
It can have as many 0's before and after the amount of 1's decided so long as the number of 1's is not a multiple of 3.

Thus the resultant expression is:

$* = Kleene\ Star$
$\{\{0\}^* \cdot \{1\} \cdot \{0\}^*\}^n \mid n = 3k\ ; k \in \boldsymbol{Z}$

This is the expression for strings that have a multiple of 3 1's. So the complement is what we want.

Answer: $\neg \left(\{\{0\}^* \cdot \{1\} \cdot \{0\}^*\}^n \,\middle|\, n = 3k\ ; k \in \boldsymbol{Z} \right)$

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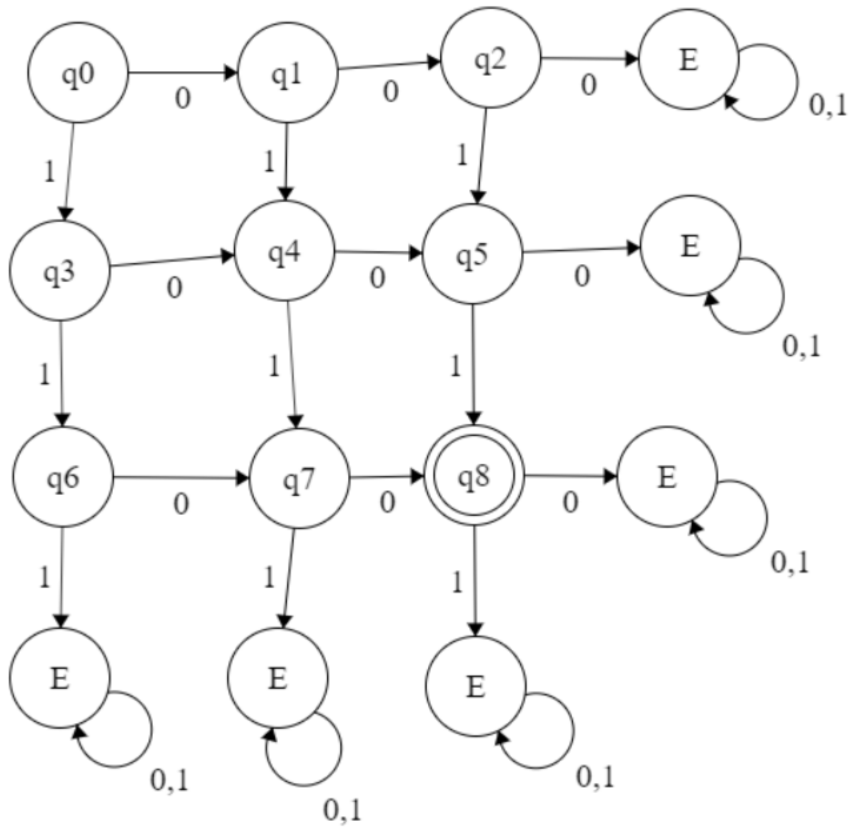
Problem 24.11

(z)

Answer:



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Problem 24.51

(b)

It is impossible for either design to exist. A DFA is not able to design something that keeps tracks of an undefined variable. Furthermore it will have no indication of when to stop and start building w. In addition, it will have no indication of when to stop w because there was no indication of when to stop n

Answer: Impossible.