Saarif Ahmed HW #7

1 A: $L\{t\} \to \frac{1}{s^2}$, $L(e^{-2t}t) \to \frac{1}{(s+2)^2}$

$L\{te^{-2t}u(t-1)\} \to \frac{e^{-s}}{(s+2)^2}$

$\boxed{L\{x(t)\} = e^{-s}/(s+2)^2}$

B: $L(e^{-at}) = \frac{1}{s+a}$

thus

$L\{e^{-2t}u(t)\} = \frac{1}{s+2}$

$L\{e^{-4t}u(t+2)\} = \frac{e^{2s}}{s+4}$

$\boxed{L\{x(t)\} = \frac{1}{s+2} + \frac{e^{2s}}{s+4}}$

2 A: Transfer to s-domain

$6s^2X(s) + 5s\cancel{X}(s) + X(s) = L\{e^{-3(t-1+1)}u(t-1) + 2u(t)\}$

$X(s)\cdot(6s^2+5s+1) = e^{-(s+3)}/(s+3) + \frac{2}{s}$

$6s^2+5s+1 = (2s+1)(3s+1)$

$\boxed{X(s) = \frac{e^{-(s+3)}}{(s+3)(3s+1)(2s+1)} + \frac{2}{s(2s+1)(3s+1)}}$

B: $1 = A(3s+1)(2s+1) + B(s+3)(2s+1) + C(s+3)(3s+1)$

pick $s=-3$ | pick $s=-\frac{1}{3}$ | pick $s=-\frac{1}{2}$

$A = \frac{1}{40}$ | $B = \frac{9}{8}$ | $C = -\frac{4}{5}$

2 B: $x(s) = \left(\frac{\frac{1}{40}}{s+3} + \left(\frac{9}{8}\right)\frac{1}{3s+1} - \left(\frac{4}{5}\right)\frac{1}{2s+1}\right)\left(e^{-(s+3)}+2\right)$

$L^{-1}(X(s)) = e^{-3} L^{-1}\{e^{-s}(\frac{1}{40}\frac{1}{s+3})\} + e^{-3} L^{-1}\{e^{-s}(\frac{9}{8}\frac{1}{3s+1})\} + e^{-3} L^{-1}\{e^{-s}(-\frac{4}{5}\frac{1}{2s+1})\} + L^{-1}\{\frac{2}{40}\frac{1}{s+3}\} + \frac{9}{4}L^{-1}\{\frac{1}{3s+1}\} - L^{-1}\{\frac{8}{5}\frac{1}{2s+1}\}$

$$\left[x(t) = (e^{-3}/40)\, e^{-3(t-1)}u(t-1) + \frac{3}{8}e^{-\frac{1}{3}(t-1)-3}u(t-1) + \frac{2}{40}e^{-3t}u(t) + \frac{3}{4}e^{-t/3}u(t) - \frac{4}{5}e^{-t/2}u(t)\right]$$

3 A: $\int_0^{\infty} e^{-3t}e^{-st}\,dt + \int_{-\infty}^{0} e^{-2t}e^{-st}\,dt$

$= \frac{e^{-t(s+3)}}{-(s+3)}\Big|_0^{\infty} + \frac{e^{-t(s+2)}}{-(s+2)}\Big|_{-\infty}^{0}$

$= \underline{\frac{1}{s+3} - \frac{1}{s+2} = X(s)}$

B: just normal Laplace transform.

$$\left[X(s) = \frac{s+3}{(s+3)^2+9}\right]$$

C: $\int_0^1 e^{3t}e^{-st}\cos(2t)\,dt$

$= \int_0^1 e^{(3-s)t}\cos(2t)\,dt$

$= \frac{e}{(3-s)^2+4t^2}\left((3-s)\sin 2t + 2t\cos(2t)\right)\Big|_0^1$

$$\left[= \frac{e}{(3-s)^2+4}\left((3-s)\sin(2) + 2\cos(2)\right) - \frac{1}{(3-s)^2}\right]$$

4 A: $\frac{1}{(s+2)(s^2+4)} = \frac{A(s^2+4)+B(s+2)s+C(s+2)}{(s+2)(s^2+4)}$

$1 = A(s^2+4)+B(s+2)s+C(s+2)$

$1 = (A+B)s^2+(2B+C)s+(2C+4A)$

$(A+B)=0$ $\quad 2C+4A$

$A=-B$ $\quad 2C-4B$

$2B+C=0$ $\quad -4B-4B=1$

$C=-2B$ $\quad B=-\frac{1}{8} \to A=\frac{1}{8} \to \frac{1}{4}=C$

$X(s)=\frac{1}{8}\frac{1}{s+2}-\frac{1}{8}\frac{s}{s^2+4}+\frac{1}{4}\frac{1}{s^2+4}$

right-side

$L^{-1}(X(s)) = \frac{1}{8}e^{-2t}u(t)-(\frac{1}{8}\cos(2t)+\frac{1}{8}\sin(2t))u(t)$

$\boxed{= \frac{1}{8}(e^{-2t}u(t)-\cos(2t)u(t)-\sin(2t)u(t))}$

B: left side $\to$ flip sign

$\boxed{= \frac{1}{8}(-e^{-2t}u(-t)+\cos(2t)(u(-t))+\sin(2t)u(-t))}$

C: $X(s)=\frac{1}{8}\cdot\frac{1}{s+2}-\frac{1}{8}\frac{s}{s^2+4}+\frac{1}{8}\frac{2}{s+4}$

$\sigma>-2$ $\quad \sigma<0$ $\quad \sigma<0$

right $\quad$ left

$x(t)=\frac{1}{8}(e^{-2t}u(t)+\cos(2t)u(-t)-\sin(2t)u(-t))$

5 A: $X(s) = \frac{A}{s+2} + \frac{B}{s+3}$

$\frac{s+1}{(s+2)(s+3)} = \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$

$(A+B) = 1$
$3A+2B = 1$
$A=-1,\ B=2$

$s+1 = (B+A)s + 3A+2B$

$X(s) = \frac{-1}{s+2} + \frac{2}{s+3}$

$L^{-1}(X(s)) = (-e^{-2t} + 2e^{-3t})\,u(t)$

B: $X(s) = \frac{A}{s+2} + \frac{B}{s+3}$ $Re\{s\} < -3$

thus $A=-1$ $B=2$ $\Downarrow$

$u(t)$ is out of range need $u(-t)$

so flip across y axis

$x(t) = e^{-2t}u(-t) - 2e^{-3t}u(-t)$

C: $X(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1}$

$s^2 - s + 1 = As(s-1) + B(s-1) + Cs^2$

$= (A+C)s^2 + (-A+B)s - B$

$A+C = 1$

$B-A = -1$

$-B = -1 \rightarrow A=0 \rightarrow C=1$

$X(s) = -\frac{1}{s^2} + \frac{1}{s-1}$

$\boxed{L^{-1}(X(s)) = -t\,u(t) - e^{t}u(-t)}$

6 A: $\mathcal{L}(y(t)) = s\,Y(s) + 2\,Y(s) = X(s)$

$$\boxed{\frac{Y(s)}{X(s)} = \frac{1}{s+2}}$$

B: for $x(t) = \delta(t)$

$X(s) = 1$

$Y(s) = \frac{1}{s+2}$

$$\boxed{y(t) = e^{-2t}u(t)}$$