Signals & Systems HW#1

Tuesday, September 1, 2020 10:57 AM

1:

a)

$x^2 - x + 2 = 0$

$x = \frac{1}{2} + \frac{j\sqrt{7}}{2}, \frac{1}{2} - \frac{j\sqrt{7}}{2}$

Polar Form:

$x = \frac{1}{2} - \frac{j\sqrt{7}}{2} \quad a = \sqrt{\frac{1}{2}^2 + \frac{\sqrt{7}}{2}^2} = \sqrt{2}$

$x = \sqrt{2}e^{-j69.9°}$
$x = \sqrt{2}e^{j69.9°}$

b)

$x^2 + 2 = 0\, ; x = \pm\sqrt{-2}$

Polar Form:

$x = \sqrt{2}e^{-j90°}$
$x = \sqrt{2}e^{j90°}$

2:

a)

$1 + j \rightarrow r_1 e^{\theta_1} = \sqrt{2}e^{j45°}$
$(1 + 2j)^* \ \rightarrow (1 - 2j) \rightarrow r_2 e^{\theta_2} = \sqrt{5}e^{-j63.4°}$

Thus $\frac{1+j}{(1+2j)^*} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)} = \frac{\sqrt{2}}{\sqrt{5}} e^{j(108.4°)}$

b)

$j^{2+j} = j^2 * j^j = -j^j = -(j^j)$
$= e^{-90°}\{\cos(1 * \ln 1) + j \sin(1 * \ln 1\,)\} \ * -1$
$= -e^{90°}$

c)

$(3 - 2j)^4 = (3 - 2j)(3 - 2j)(3 - 2j)(3 - 2j) = (9 - 12j + 4j^2)(9 - 12j + 4j^2)$
$(5 - 12j)(5 - 12j) = (25\ - 120j + 144j^2) = -119 - 120j$

$= 169e^{-45°}$

3:

a)

$$(1+2j)(2-j)^* = (1+2j)(2+j) = (2+5j+2j^2) = 5j$$

b)

$$e^{2j} + (1-j)\,e^{\left(\frac{j\pi}{2}\right)} = e^{2j} + e^{\left(\frac{j\pi}{2}\right)} \; - je^{\left(\frac{j\pi}{2}\right)}$$

$$= (1 * \cos(2) + j\sin 2) + \left(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right) - j\,\left(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right)$$
$$= -0.41 + j\,0.91 + 1 \; - j$$
$$= 0.58 \; - 0.091j$$

4:

a)

$\cos\left(3t + \frac{\pi}{3}\right)$ is a periodic with a period of $\frac{2\pi}{3}$

b)

$2\tan\left(\frac{\pi t}{4}\right)$ is not periodic

c)

$3\cos(\sqrt{2}t)$ is a periodic with a period of $\frac{2\pi}{\sqrt{2}}$

d)

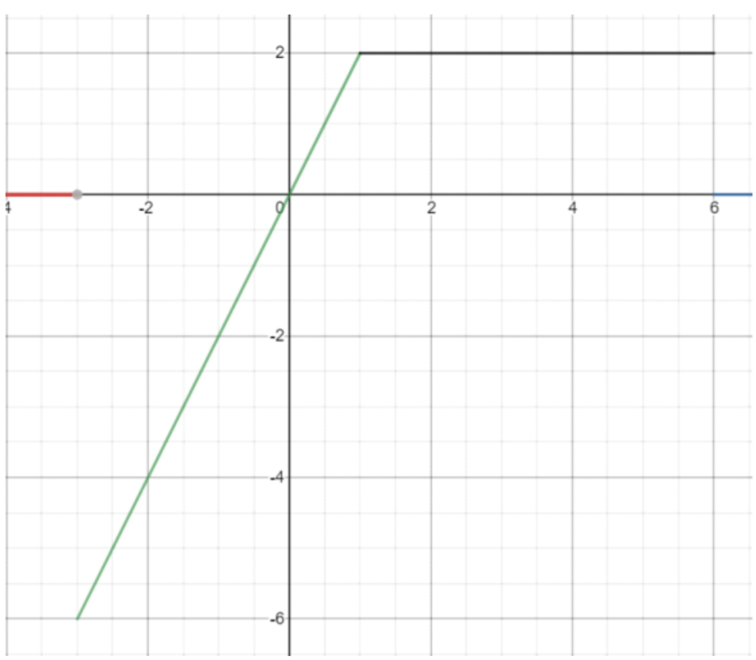
$e^{j\left(\frac{\pi t}{4}\right)} = \cos\left(\frac{\pi}{4}t\right) + j\sin\left(\frac{\pi}{4}t\right)$ is periodic with a period of 8

e)

$e^{t+j\pi t} = e^t * e^{j\pi t} = e^t(\cos(\pi t) + j\sin(\pi t))$ is not periodic

5:

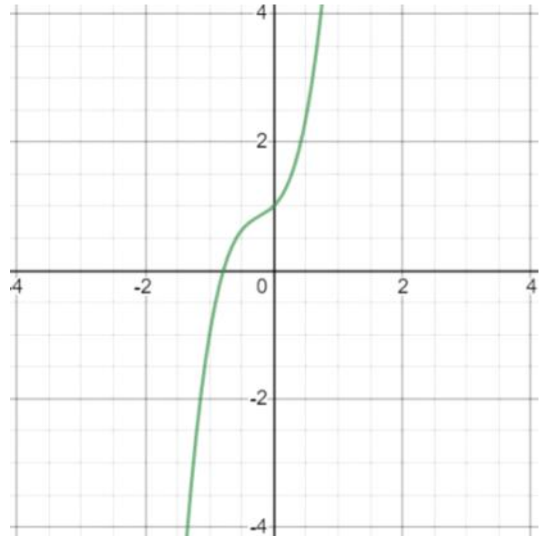
a)



Even for $t < -3 \;\cup t > 6$
Odd for $-1 \leq t \leq 1$

5 (continued):

b)

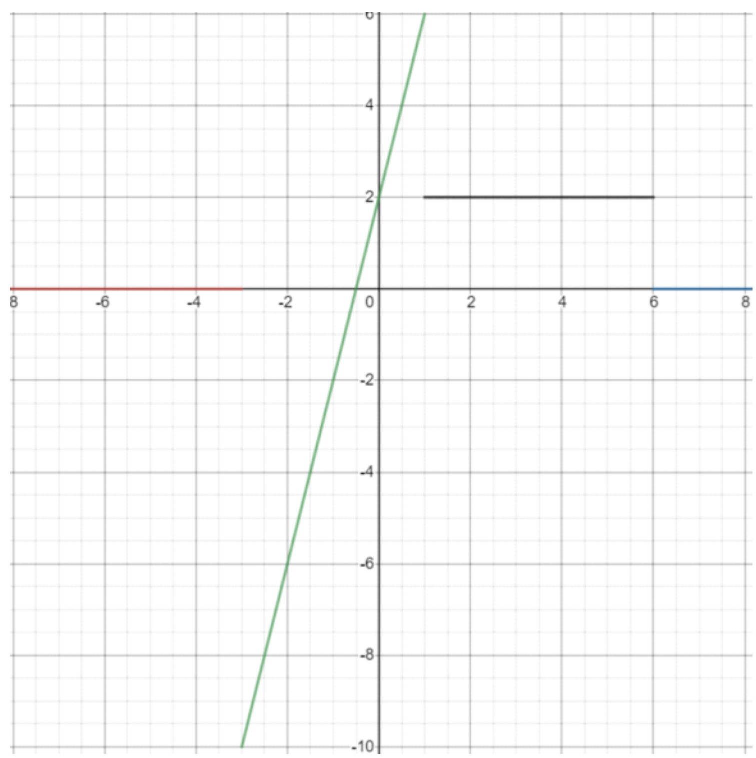


Never even
Never odd

6:

a)

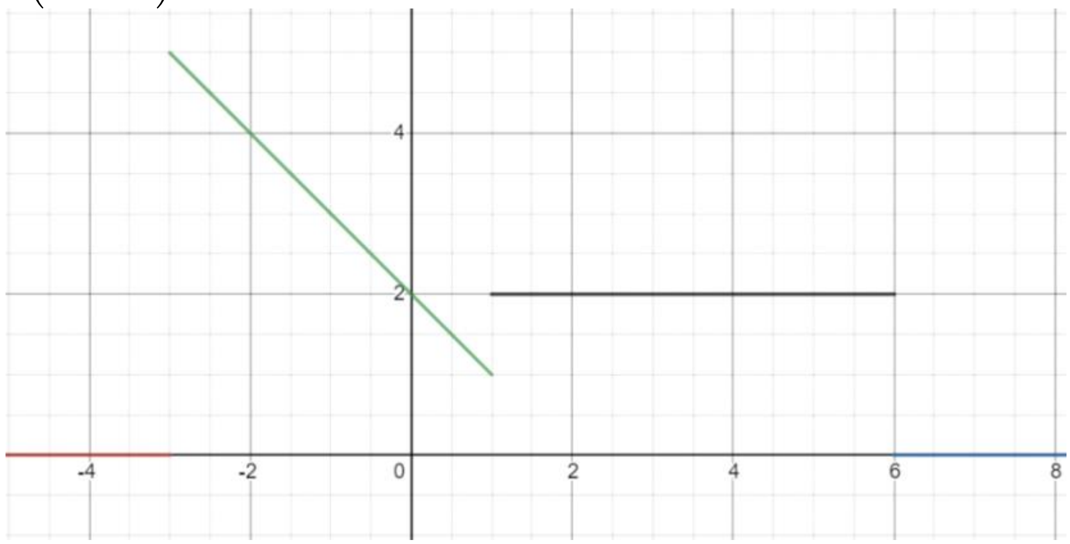
$f(2t+1) = \{\, 4t + 2 \quad -3 \le t \le 1\,, 2 \quad 1 < t \le 6\,, 0 \; otherwise$



6 (continued):

b)

$$f\left(-\frac{1}{2}t+1\right)=\{-t+2\quad -3\le t\le 1\,,2\quad 1<t\le 6\,,0\ otherwise$$



7:

$$f(t) = 3\sin\left(\omega_0 t+\frac{\pi}{3}\right)+\cos(\omega_0 t)+2\cos\left(\omega_0 t+\frac{\pi}{4}\right)$$

$$= 3\left(\sin(\omega_0 t)\cos\left(\frac{\pi}{3}\right)+\cos(\omega_0 t)\sin\left(\frac{\pi}{3}\right)\right)+\cos(\omega_0 t)+2\left(\cos(\omega_0 t)\cos\left(\frac{\pi}{4}\right)-\sin(\omega_0 t)\sin\left(\frac{\pi}{4}\right)\right)$$

$$= \frac{3}{2}\sin(\omega_0 t)+\frac{3\sqrt{3}}{2}\cos(\omega_0 t)+\cos(\omega_0 t)+\sqrt{2}\cos(\omega_0 t)-2\sin(\omega_0 t)$$

$$= \frac{3\sqrt{3}+2+2\sqrt{2}}{2}\cos(\omega_0 t)+\frac{1}{2}\sin(\omega_0 t)$$

$$= \sqrt{\left(\frac{3\sqrt{3}+2+2\sqrt{2}}{2}\right)^2+\left(\frac{1}{4}\right)^2}\cos\left(\omega_0 t\ -\arctan\left(\frac{\frac{1}{2}}{\frac{3\sqrt{3}+2+2\sqrt{2}}{2}}\right)\right)$$

$$= C\cos(\omega_0 t+\theta)$$

$$= 5.04\cos(\omega_0 t\ -0.1)$$