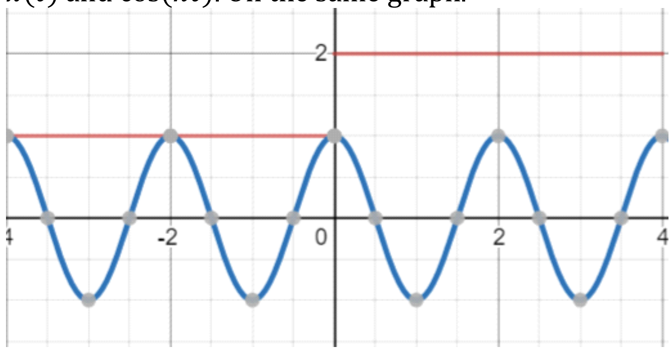
Signals & Systems HW#6

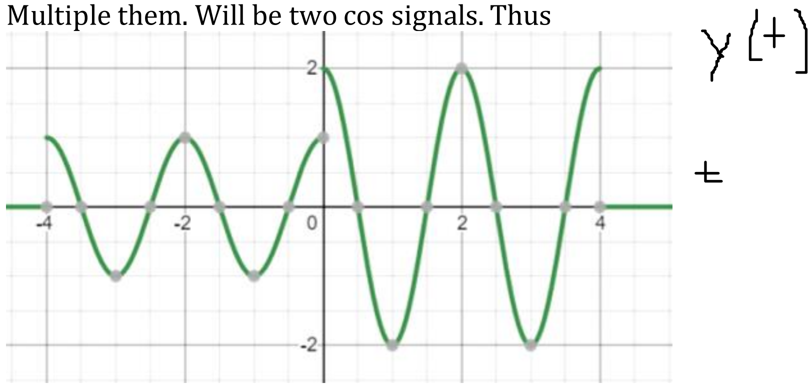
1.

a)

$x(t)$ and $\cos(\pi t)$. On the same graph.

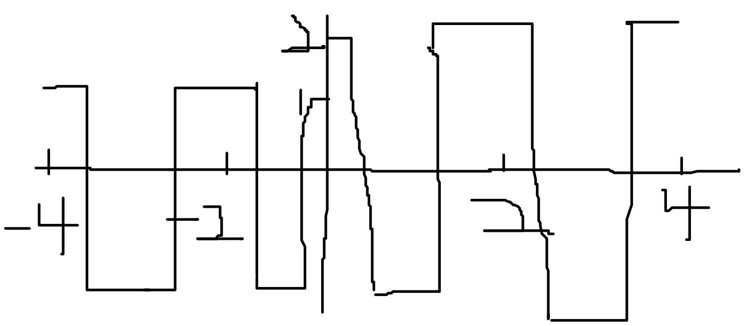


Multiple them. Will be two cos signals. Thus



b)

$y(t)$ is just amplitude shift thus.



2.

a)

find the FT of $\cos(\pi t)$

$FT\{\cos(\pi t)\} = \pi[\delta(\omega - \pi) + \delta(\omega_\pi)]$

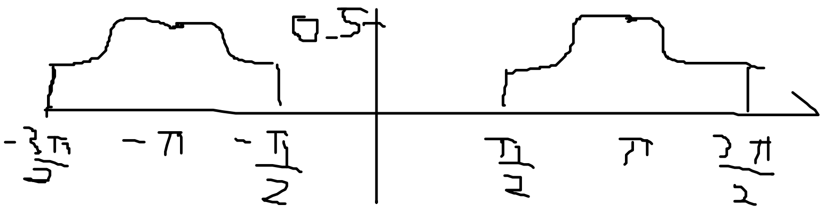
if $y(t) = x(t)(\cos(\pi t))$ then

$FT\{y(t)\} = Y(\omega) = \frac{\pi}{2\pi}\left[x(\omega) * \left(\delta(\omega - \pi) + \delta(\omega + \pi)\right)\right]$

$= \frac{1}{2}[x(\omega) * \delta(\omega - \pi) + x(\omega) * \delta(\omega + \pi)]$

$= \frac{1}{2}[x(\omega - \pi) + x(\omega + \pi)]$

Thus it's 1/2 the amplitude shifted to the left and right by π



b)

$p(t)$ is periodic need to find Fourier series coefficients

$\frac{1}{T}\int_T x(t)e^{-j\omega_0 t}dt$

$= \frac{1}{2}\left(\int_{-\frac{1}{2}}^{\frac{1}{2}} 1 * e^{-jk\pi t} + \int_{\frac{1}{2}}^{\frac{3}{2}} (-1)e^{-jk\pi t}\right)$

$= \frac{1}{2}\left(-\frac{1}{jk\pi}\left(e^{-\frac{jk\pi}{2}} - e^{\frac{jk\pi}{2}}\right) + \frac{1}{jk\pi}\left(e^{-\frac{jk3\pi}{2}} - e^{\frac{jk\pi}{2}}\right)\right)$

$= \frac{1}{2jk\pi}\left(2j\sin\left(\frac{k\pi}{2}\right) + e^{\frac{-jk\pi}{2}}\left(e^{-jk\pi} - 1\right)\right)$

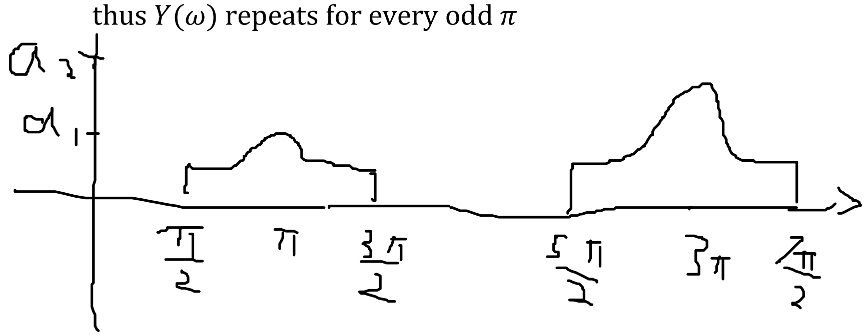
$= \frac{1}{2jk\pi}\left(2j\sin\left(\frac{k\pi}{2}\right) + e^{\frac{-jk\pi}{2}}\left((-1)^k - 1\right)\right)$

$FT\{p(t)\} = 2\pi \sum_{k=-\infty} a_k \delta(\omega - k\pi)$

$Y(\omega) = \frac{1}{2\pi}[X(\omega) * p(\omega)]$

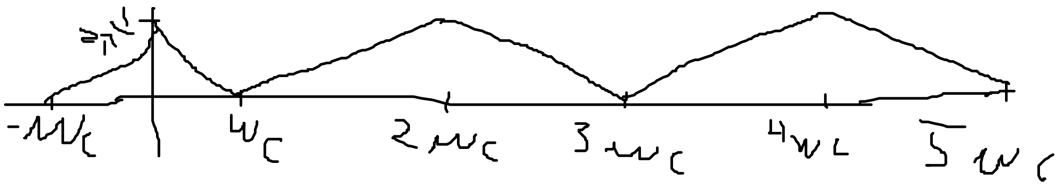
$= \sum_{k=-\infty}^{\infty} a_k\, X(\omega) * \delta(\omega - k\pi) = \sum_{k=-\infty}^{\infty} a_k\, X(\omega - k\pi)$

thus $Y(\omega)$ repeats for every odd π

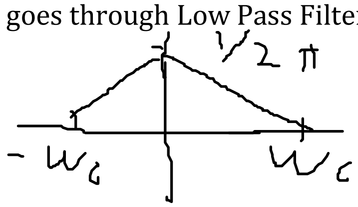


3.

a)



it goes through Low Pass Filter

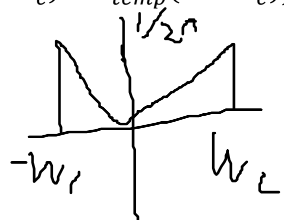


b)

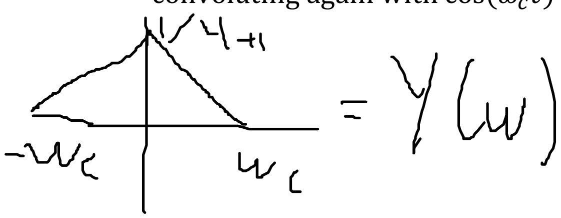
we know $\frac{1}{2\pi}X(\omega) * S(\omega) = x_{temp}(\omega)$ from above.

We multiply (convolute) that with $FT(\cos(\omega_c t))$

$= \frac{1}{2}(x_{temp}(\omega - \omega_c) + x_{temp}(\omega + \omega_c))$

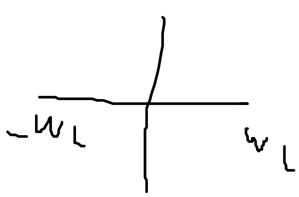


convoluting again with $\cos(\omega_c t)$ will phase shift it. Then you put it through LPF



c)

only difference from $\cos(\omega_c t)$ is coefficient is $\frac{\pi}{j}$. Thus bringing it to complex plane for the LPF period. Meaning not physically drawn.



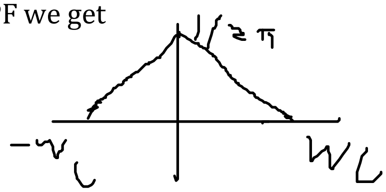
3 (continued):

d)

$$x_{temp}(\omega) = \frac{1}{2\pi}\Big(\pi\big(\delta(\omega - 2\omega) + \delta(\omega - 2\omega)\big) * \big(X(\omega) * S(\omega)\big)\Big)$$

$$= \frac{1}{2}\big(x(\omega - 2\omega) + x(\omega + 2\omega)\big)$$

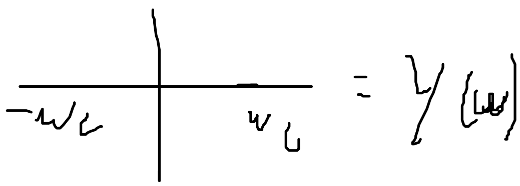
Through LPF we get



The next $\cos(2\omega_c t) \to x_{temp2}(\omega) = \frac{\pi}{2\pi}\big(x_{temp}(\omega) * \cos(2\omega_c t)\big)$

$$= \frac{1}{2}\Big(x_{temp}(\omega - 2\omega) + x_{temp}(\omega + 2\omega)\Big)$$

Expansion and through LPF



4.

a)

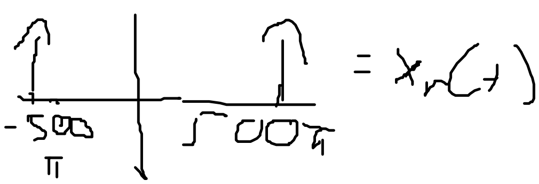
First do FT of system $X_p(\omega) = \frac{1}{2\pi}\big(x(\omega) * p(\omega)\big)$

$$= \frac{2\pi}{2\pi(10^{-3})}\sum_{n=-\infty}^{\infty}\delta(\omega - n\omega)$$

$x_r(\omega) = x_p(\omega) * H(\omega)$

$x_p(\omega)$ is delta functions centered at 0 with period of 1000π

$H(\omega)$ is a LPF so we analyze just between $\pm 1000\pi$



$\theta = \frac{\pi}{4}$ so coefficient is $\frac{\sqrt{2}}{2}$

$$x_r(t) = \cos\left(500\pi t + \frac{\pi}{4}\right)$$

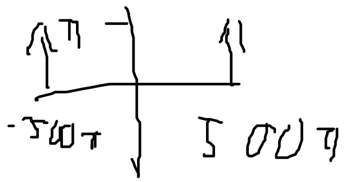
4 (continued)

b)

Same as before. Centered at 0 but period of $x_p(\omega)$ is 3000π and starting at -500π and 500π

$H(\omega)$ is a LPF so we analyze only for $\pm 1000\ \pi$

$\theta = \frac{1}{2}$ so scale is 1



$x_r(t) = \sin(500\pi t)$

5.

a)

$$\frac{1}{T}\int_T x(t)e^{-j\omega_0 t}dt$$

$$= \frac{1}{2\Delta}\int_0^{2\pi} \delta(t)e^{-\frac{jn2\pi}{\Delta}t} - \delta(t-\Delta)e^{-\frac{jn2\pi}{\Delta}t}dt$$

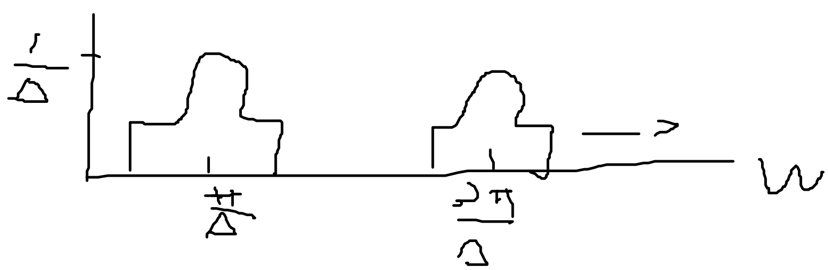
$$= \frac{1}{2\Delta}(1\ - e^{-jn\pi})$$

odd vs even

only non zero when odd. thus

$$p(t) = \sum_{n=-\infty}^{\infty} \frac{1}{\Delta}e^{\frac{j(2n+1)2\pi}{2\Delta}t}$$

$x_p(\omega) = \sum_{n=-\infty}^{\infty} \frac{x(t)}{\Delta}e^{\frac{j(2n+1)2\pi}{2\Delta}t}$. This means $x_p(\omega)$ is $x(\omega)$ repeating at every odd multiple of π/Δ



b)

$$x_p(t)d(t) = x_p(t)\cos\left(\frac{\pi}{\Delta}t\right)$$

$$x_p(\omega) * d(\omega) = \frac{1}{2\pi}X_p(\omega) * \pi\left[\delta\left(\omega - \frac{\pi}{\Delta}\right) + \delta\left(\omega + \frac{\pi}{\Delta}\right)\right]$$

$$= \frac{1}{2}X_p\left(\omega - \frac{\pi}{\Delta}\right) + \frac{1}{2}X_p\left(\omega + \frac{\pi}{\Delta}\right)$$

Pass this through $\mathrm{H}(\omega)$ filter.