Saaif Ahmed - Assignment #5

Wednesday, October 16, 2019 7:52 PM

Problem 10.10

Use Euclid's GCD algorithm to compute gcd(356250895, 802137245) and express the GCD as an integer linear combination of the two numbers.

Euclid Theorem: $\gcd(m, n) = \gcd(rem(m, n), m)$

$\gcd(356250895, 802137245): 89635455 = 802137245 - (2 * 356250895)$
$\gcd(89635455, 356250895): 87344530 = 356250895 - (3 * 89635455)$
$\gcd(87344530, 89635455): 2290925 = 89635455 - 87344530$
$\gcd(2290925, 87344530): 289380 = 87344530 - (38 * 2290925)$
$\gcd(28930, 2290925): 265265 = 2290925 - (7 * 289380)$
$\gcd(266265, 289380): 24115 = 289380 - 265265$
$\gcd(24115, 265265): 0 = 265265 - (11 * 24115)$

$24115 = 289380 - \left(2290925 - (7 * 289380)\right)$
$24115 = (8 * 289380) - (89635455 - 87344530)$
$24115 = \left(8 * \left(87344530 - \left(38 * (89635455 - 87344530)\right)\right)\right) - (89635455 - 8744530)$
$24115 = \left(8 * \left(87344530 - 38(89635455) + 38(87344530)\right)\right) - (89635455 - 8744530)$
$24115 = (8 * 87344530) - (304 * 89635455) + (304 * 87344530) - 89635455 + 8744530$
$24115 = -(305 * 89635455) + (313 * 87344530)$
$24115 = -(305 * 89635455) + \left(313 * \left(356250895 - (3 * 89635455)\right)\right)$
$24115 = -(305 * 89635455) + (313 * 356250895) - (939 * 896354555)$
$24115 = (313 * 356250895) - 1244(802137245 - 2 * 356250895)$
$24115 = 2801 * 356250895 - 1244 * 802137245$

Answer: GCD is 24115.

$24115 = 2801 * 356250895 - 1244 * 802137245$

Assignment #5

Wednesday, October 16, 2019 8:41 PM

Problem 10.27

(a) Using 6 and 15 gallon jugs

(i) 3 gallons

(0,0)
(6,0)
(0,6)
(6,6)
(0,12)
(6, 12)
(3,15)
(3,0) <-- Done

(ii) 4 gallons

Answer: Cannot be done. The gcd(6,15) is 3. With the gallon problem you can only fill up amounts that are a multiple of the gcd. 4 is not a multiple of 3

(iii) 5 gallons

Answer: Cannot be done. The gcd(6,15) is 3. With the gallon problem you can only fill up amounts that are a multiple of the gcd. 5 is not a multiple of 3

Assignment #5

Wednesday, October 16, 2019 8:50 PM

Problem 10.40

(c) Prove $2^{70} + 3^{70}$ is divisible by 13

$3^3 \equiv 1 \quad mod(13)$

$3^{69} \equiv 1 \quad mod(13)$

$3(3^{69}) \equiv 3(1)\ mod(13)$

$3^{70} \equiv 3 \quad mod(13)$

$2^{12} \equiv 1 \quad mod(13)$

$2^{60} \equiv 1 \quad mod(13)$

$2^{10} * 2^{60} \equiv 2^{10} * 1\ mod(13)$

$2^{10} \equiv 10\ mod(13)$

$2^{70} \equiv 10\ mod\ (13)$

$2^{70} + 3^{70} \equiv 10 + 3\ mod(13)$

$2^{70} + 3^{70} \equiv 0 \quad mod(13)$

Answer: 13 divides $2^{70} + 3^{70}$

Assignment #5

Wednesday, October 16, 2019 9:23 PM

Problem 11.6

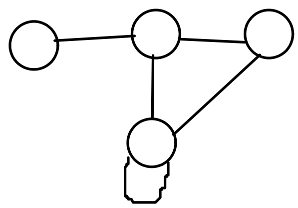
(a) The graph has 5 vertices each of degree 3.

Answer: Not possible due to the Hand-Shaking theorem. The number of edges is always 2 times the sum of all the degrees and thus must be even. In this case that sum is 15 which is odd and therefore this graph cannot exist.

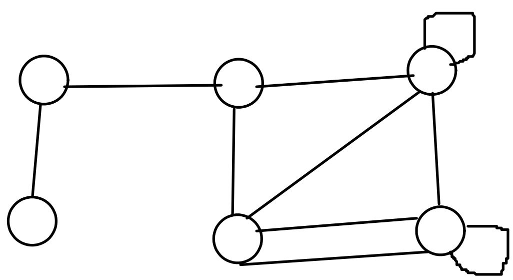
(b) The graph has 4 edges and vertices of degrees 1,2,3,4.

Answer: Not possible due to the Hand-Shaking theorem. The number of edges is always 2 times the sum of all the degrees and thus must be even. In this case that sum is 10 which is not 2 * 4.

(c)



(d)



Assignment #5

Wednesday, October 16, 2019 9:38 PM

Problem 11.40

(a) Can you place all the dominos in a ring?

Answer: No because the appearance of a number must be even in order for it to fit into the circle. Because the number 0 appears an odd number of times there will always be a piece that fails to make it in. This is directly related to the handshaking theorem that states that the sum of node-degrees has to be even. The numbers themselves are node-degrees so they individually cannot be odd.

(b) How many dominos are there for [0, . . . , n]?

Tinker : {0,1,2,3}

{n+1, n , n-1, n-2}

Total for 3 was 10
Set with number of dominoes per number {4,3,2,1}
Total = 4+3+2+1 = 10

$$n + 1 + n + n - 1 + n - 2 \ldots + 2 + 1 = \sum_{i=0}^{n+1} i$$

Answer: $\frac{1}{2}(n + 1)(n + 2)$

(c) For which n can you place all the dominos in a ring?

Answer: We know that if each number is a vertex then the dominoes are edges. We know the total number of dominoes will be equal to the sum of the node-degree values which must be even.

Therefore n must be even.

$$n \text{ is even} \rightarrow \frac{1}{2}(n + 1)(n + 2) \text{ is even}$$

Direct Proof:

Let $n = 2k$; $k \in \boldsymbol{N}$

$$\frac{1}{2}(2k + 1)(2k + 2)$$

$\left(\frac{1}{2}(2k + 1)(k + 1)\right)2$ <--- is even

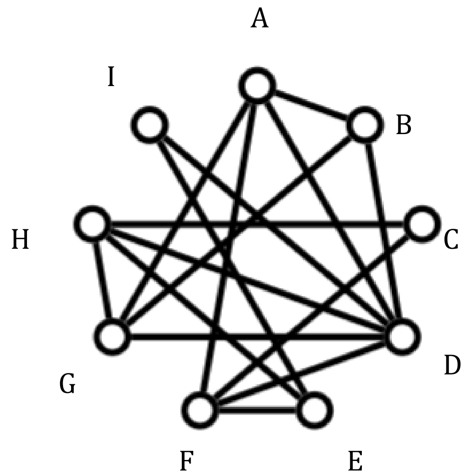
Assignment #5

Wednesday, October 16, 2019 9:59 PM

Problem 12.73

(l) We show a friendship network. The vertices are people and the edges are the friendship links. Can the people be seated at a round table:

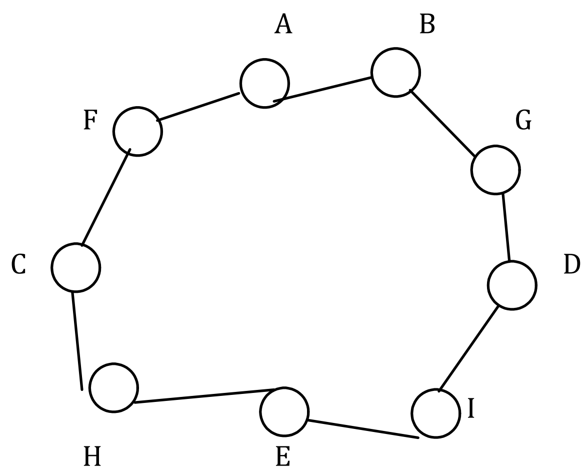
(i) Harmoniously, so that every person has a friend to their left and their right?



Can represent graph as list of connections:

A: B, D, F, G
B: A, G, D
C: H, F
D: A, B, I, H, G, F
E: F, H, I
F: A, C, D, E
G: I, A, B, D
H: C, D, E, G
I: E, D

Valid path: $A \to B \to G \to D \to I \to E \to H \to C \to F \to A$



Assignment #5

Wednesday, October 16, 2019 10:10 PM

Problem 12.73 (continued)

(ii) Sadistically, so that every person has an enemy to their left and their right?

Can represent enemies as a list of connections

A: C, E, H, I
B: C, E, F, H, I
C: A, B, D, E, I, G
D: C, E
E: A, B, C, D, G
F: B, G, H, I
G: C, E, F, H
H: A, B, F, I
I: A, B, C, F, G, H

Valid path : $A \to H \to B \to I \to F \to G \to E \to D \to C \to A$