Focs Crib Sheet - Saaif Ahmed - ahmeds7

$p \to q \equiv \neg q \to \neg p \equiv \neg p \vee q$

$\sum_{i=k}^{n} 1 = n + 1 - k$	$\sum_{i=1}^{n} i = \frac{1}{2}n(n+1)$	$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$
$\sum_{i=1}^{n} f(x) = n * f(x)$	$\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$	$\sum_{i=0}^{n} \frac{1}{2^i} = 2 - \frac{1}{2^n}$
$\sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r} (r \neq 1)$	$\sum_{i=1}^{n} i^3 = \frac{1}{4}n^2(n+1)^2$	$\sum_{i=1}^{n} \log i = \log n!$

For polynomials, growth rate is the highest order. For nested sums, growth rate is number of nesting plus order of summand. Analyze the largest order term to find behavior

$\int_{m-1}^{n} f(x) \leq \sum_{i=m}^{n} f(i) \leq \int_{m}^{n+1} f(x)$ Linear combination of m, n: Z=mx + ny >0

Any number Q = (quotient)(divisor) +(remainder)

$\gcd(m, n) = \gcd(m, rem(n, m))$

$$\begin{aligned}
\gcd(42, 108) &= \gcd(24, 42) && 24 = \mathbf{108} - 2 \cdot \mathbf{42} \\
&= \gcd(18, 24) && 18 = 42 - 24 = 42 - \underbrace{(108 - 2 \cdot 42)}_{24} = 3 \cdot \mathbf{42} - \mathbf{108} \\
&= \gcd(6, 18) && 6 = 24 - 18 = \underbrace{(108 - 2 \cdot 42)}_{24} - \underbrace{(3 \cdot 42 - 108)}_{18} = 2 \cdot \mathbf{108} - 5 \cdot \mathbf{42} \\
&= \gcd(0, 6) && 0 = 18 - 3 \cdot 6 \\
&= 6 && \gcd(0, n) = n
\end{aligned}$$

Remainders in Euclid's algorithm are integer linear combinations of 42 and 108.

In particular, $\gcd(42, 108) = 6 = 2 \times 108 - 5 \times 42$.

(A+B) mod C = (A mod C + B mod C) mod C If ac = bc (mod d), you can cancel c if gcd(c, d) = 1

Graphs are Isomorphic if the Vertices and Edges can be relabeled and be shown to be equal

Handshake Theorem: $\sum_{i=1}^{n} \delta_i = 2|E|$ Sum of degrees from all vertices is Twice # of edges

Sum Rule: N objects of two types: N1 of type1 and N2 of type2. Then, N = N1 + N2 .

Product Rule

Let N be the number of choices for a sequence X1, X2, X3...Xr

Let Nr be the number of choices for Xr after you choose X1 X2 X3 · · · Xr−1. N = N1 × N2 × N3 × N4 × · · · × Nr.

$\boxed{\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}}$ (sum rule) base cases: $\binom{n}{0} = 1; \binom{n}{n} = 1.$

Binomial Theorem:

$(x + y)^n = \sum_{i=1}^{n} \binom{n}{i} x^i y^{n-i}$

	No rep	rep
k-sequence	$\frac{n!}{(n-k)!}$	n^k
k-subset	$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$	$\binom{k+n-1}{n-1}$
(k1,k2,...kr)		$\frac{k!}{k1! + k2! \ldots kr!}$

$|A \cup B| = |A| + |B| - |A \cap B|$
$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$\binom{nodes}{2} = x\ ; 2^x\ number\ of\ graphs; x = edges$ **Pigeon hole principle

#of subsets of a set : $2^n\ ; n = |set|$

Probability:

Binomial Distribution: 1) Count success in binary trial (1/0 or Win\Fail)
2) Each trail has fixed probability P
3) each trial is independent
PDF: $P_x(k) = B(k, n, p) = \binom{n}{k} p^k (1-p)^n - k$

Linearity of expectation : $Z = aX + bX^2 \rightarrow E[Z] = aE[X] + bE[X^2]$

$E[edge] = \sum_{i=0}^{n} degree_i node_i$ --> Chance to pick any node *sum of degree

$P[A|B] = \frac{P[A \cap B]}{P[B]}\ ; P[B|A] = \frac{P[A \cap B]}{P[A]}\ \ thus\ P[A|B] = P[B|A] * \frac{P[A]}{P[B]}$

$E[X] = E[X|A] * P[A] + E[X|\neg A] * P[\neg A]$

Waiting time: for geometric series is $\frac{1}{p}\ where\ p\ is\ probability\ of\ sucess$
Waiting time for x children with p success : $E[children] = \frac{x}{p}$
Independence: $P[A \cap B] = P[A]\ * P[B]$ if independent; $P[A|B] = P[A]$

Asymptotic Behavior

$T \in o(f)$	$T \in O(f)$	$T \in \theta(f)$	$T \in \omega(f)$	$T \in \Omega(f)$
$T < f$	$T \leq f$	$T = f$	$T > f$	$T \geq f$

Comparison:

$$\lim_{n \to \infty} \frac{T(n)}{f(n)} = \begin{cases} \infty\ ; T \in \omega(f) \\ Constant > 0\ ; T \in \theta(f) \\ 0\ ; T \in o(f) \end{cases}$$

DFA <CFG< TM

Not a function: think vertical line test. One input matched to two outputs
Injection: Everything in A can go to B with no repeats. 1-1; $A \leq B$
Bijection: Everything in A can go to B and B can go to A ; $A \leq B \leq A\ ; A = B$
Surjection: Everything in A can go to B and at least 1 repeat while still being a function. Think pigeon hole principle. If B is output, one output can 2 inputs is surjective

Decidable: always halts
Recognizable: can infinite loop

If A is harder than B
A is decidable --> B is decidable
B is undecidable --> A is undecidable

Transducer: edits the tape, Regular Turing machine halts

A computing problem is a set containing finite binary strings