# Signals & Systems HW#5

**1.**

$$\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-4}^{-2} e^{-j\omega t}dt + \int_{1}^{4} e^{-j\omega t}dt$$
$$-j\omega e^{-j\omega t}\Big|_{-4}^{-2} + (-j\omega)e^{-j\omega t}\Big|_{1}^{4}$$
$$= j\omega e^{2j\omega} - j\omega e^{4j\omega} + j\omega e^{-4j\omega} - je^{-j\omega}$$
$$= j\omega(\cos(2\omega) + j\sin(2\omega)) - j\omega(\cos(4\omega) + j\sin(4\omega)) + j\omega(\cos(-4\omega) + j\sin(-4\omega))$$
$$- j\omega(\cos(-\omega) + j\sin(-\omega))$$
$$= j\omega(\cos(2\omega) + j\sin(2\omega) - \cos(4\omega) - j\sin(4\omega) + \cos(-4\omega) + j\sin(-4\omega) - \cos(-\omega) - \sin(-\omega))$$
$$= j\omega(\cos(2\omega) + j\sin(2\omega) - \cos(-\omega) - \sin(-\omega))$$

**2.**

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t}\left(e^{3j}\delta(\omega-2) + e^{-3j}\delta(\omega+2)\right)d\omega$$
$$= \frac{1}{2\pi}\left(\int_{-\infty}^{\infty} e^{j\omega t}\left(e^{3j}\delta(\omega-2)\right)d\omega + \int_{-\infty}^{\infty} e^{j\omega t}\left(e^{-3j}\delta(\omega+2)\right)d\omega\right)$$
$$= \frac{1}{2\pi}\left(\int_{2} e^{j\omega t}e^{3j} + \int_{-2} e^{j\omega t}e^{-3j}\right)$$
$$= \frac{1}{2\pi}\left(\frac{e^{3j+tj(2)}}{j(2)} + \frac{e^{-3j+tj(-2)}}{j(-2)}\right)$$
$$= \frac{1}{2\pi}\left(\frac{e^{3j+2jt} - e^{-(3j+2jt)}}{2j}\right)$$
$$= \frac{1}{2\,\pi}(\sin(3j + 2jt))$$

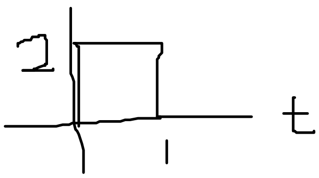
**3.**

$$\int_{-\infty}^{\infty} |\sin(2t)|e^{-j\omega t}dt = \int_{0}^{\pi} 2\sin(2t)\,e^{-j\omega t}dt = \frac{2e^{j\omega t}(j\omega\sin(2t) - 2\cos(2t))}{-\omega^2+4}\Bigg|_{0}^{2\pi}$$
$$= \frac{4e^{j\omega 2\pi} + 2}{-\omega^2 + 4}$$

**4.**

**a)**

$$\frac{dx(t)}{dt}$$




**b)**

$$\int_0^1 2e^{-j\omega t}\,dt = -2j\omega e^{-j\omega t}\Big|_0^1 = -2j\omega(e^{-j\omega}-1)$$

**c)**

Knowing $FT\{x(t)\} = X(\omega)$ and $FT\left\{\frac{dx(t)}{dt}\right\} = j\omega X(\omega)$

$FT\{x(t)\} = -2(e^{-j\omega}-1)$

**d)**

$$FT\left\{\frac{dz(t)}{dt}\right\} = -2j\omega\left(e^{-j\omega}-1\right) + (2j\omega)(e^{-4j\omega}-e^{-3j\omega})$$

thus using the rule from before

$FT\{z(t)\} = -2\left(e^{-j\omega}-1\right) + 2(e^{-4j\omega}-e^{-3j\omega})$

**5.**

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{j\omega}{4+2j\omega}e^{j\omega t}dt = \sqrt{\frac{\pi}{2}}\delta(t) - \sqrt{2\pi}e^{2t}u(-t)$$

**6.**

**a)**

using FT property $x(-2t) \rightarrow \frac{j\omega}{2}X\left(\frac{\omega}{-2}\right)$

**b)**

$\cos(4t) \rightarrow \mathrm{FT} = \sqrt{\frac{\pi}{2}}\delta(\omega-4) + \sqrt{\frac{\pi}{2}}\delta(\omega+4)$

thus

$$FT\{\cos(4t)\left(x(t)\right)\} = X(\omega) * \left(\sqrt{\frac{\pi}{2}}\delta(\omega-4) + \sqrt{\frac{\pi}{2}}\delta(\omega+4)\right)$$

**c)**

**7.**

**a)**

Multiply in time domain to convolute in freq domain

$e^{-5t} \rightarrow \dfrac{1}{-5+j\omega}$

$\cos(4t) \rightarrow \dfrac{1}{2}[\delta(\omega-A)+\delta(\omega+A)] \rightarrow \dfrac{1}{2}[\delta(\omega-4)+\delta(\omega+4)]$

thus $FT\left(x(t)\right) = \left(\frac{1}{-5+j\omega}\right) * (\frac{1}{2}[\delta(\omega-4)+\delta(\omega+4)])$

**7(continued).**

**b)**

$$x(t) = \frac{\sin t}{t}\ (\sin 2t)$$

Multiply in time domain to convolute in freq domain

$FT\{\mathrm{sinc}(t)\} = Rect(\omega)$

$$FT\{\sin(2t)\} = \frac{1}{2}[\delta(\omega - A) + \delta(\omega + A)] \to \frac{1}{2}[\delta(\omega - 2) + \delta(\omega + 2)]$$

thus

$$FT\{x(t)\} = Rect(\omega) * \left(\frac{1}{2}[\delta(\omega - 2) + \delta(\omega + 2)]\right)$$

**8.**

energy is conserved through FT

$$\int_{-\infty}^{\infty} \left|e^{-2|\omega|}\right|^2 d\omega = 2\int_0^{\infty} |e^{-2\omega}|^2 d\omega$$

$$2\int_0^{\infty} e^{-4\omega}\ d\omega = 2(-4e^{-4\omega}|_0^{\infty}) = 2(0 + 4) = 8$$

Energy is 8