

Multivar Exam #1 Saaif Ahmed PG 1

Honor Pledge:

"I have neither given nor received any illegal aid on this exam"

-Saaif Ahmed 9/30/20

Problem 2:

Use double integrals to compute the volume contained in the first octant, $x \geq 0$, $y \geq 0$, $z \geq 0$ under the plane $x + y + z = 1$.

Bounded by points $(1,0,0)$, $(0,1,0)$, $(0,0,1)$

$$x + y + z = 1$$

$$z = 1 - x - y$$

$$\text{Thus } 0 \leq x \leq 1, 0 \leq y \leq 1 - x$$

$$\int_0^1 \int_0^{1-x} (1 - x - y) \, dy \, dx$$

$$\int_0^1 (1-x)y - \frac{1}{2}y^2 \Big|_0^{1-x} \, dx$$

$$\int_0^1 \left(x^2 - 2x - \frac{(1-x)^2}{2} + 1 \right) \, dx$$

$$\frac{1}{3}x^3 - x^2 + x - \frac{1}{6}(x+1)^3 \Big|_0^1 = \frac{1}{6}$$

Answer: $\frac{1}{6}$