Project 3 - Multiple Alignment

<u>**Group - 7**</u>

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Introduction:

In this project we have implemented two programs for global multiple sequence alignment (MSA):

- **sp_exact_3:** An exact algorithm for computing the optimal sum-of-pairs (SP) alignment score and alignment for exact three sequences. This method uses a three-dimensional dynamic programming approach and guarantees an optimal solution.
- **sp_approx:** A 2-approximation algorithm for MSA that can handle an arbitrary (>=3) number of sequences. This method selects a "center" sequence based on pairwise alignment costs and then aligns all sequences to the center. Thus, this method does not guarantee an optimal solution.

Overall, both programs work as expected on the test data (*testseqs* folder). The exact algorithm produces the optimal alignment for three sequences (as verified using provided test data in *testdata_short.txt* and *testdata_long.txt*), while the approximate method scales to more sequences. one unsolved challenge is that for relatively short sequences (e.g., 200 nucleotides), our highly optimized *sp_exact_3* (compiled with *Numba*) sometimes runs faster than *sp_approx* - this is due to the parallelization and merging overhead inherent in the approximate method. However, as sequence lengths increase or the number of sequences grows, *sp_approx* is expected to show its scalability advantages.

Method:

sp_exact_3

Algorithm Overview:

The *sp_exact_3* program aligns three DNA sequences exactly by minimizing the sum-of-pairs (SP) score. It uses a 3D dynamic programming (DP) table to account for every position in each sequence simultaneously. The user can provide sequences either manually or through a FASTA file, customize the scoring scheme with a substitution matrix and a gap penalty, and then receive the final alignment along with its total SP score.

Implementation and Design Choices:

1. Input:

- **Substitution Matrix**: Supplied by default or loaded from a file or entered manually (4×4 format for A, C, G, T).
- **Gap Penalty**: A single integer for scoring gaps, applied uniformly to each gap position.
- **Sequence Input (Manual)**: Prompts for three sequences, replacing ambiguous nucleotides with 'A'.
- **Sequence Input (FASTA)**: Reads sequences from a file. If there are exactly three, they are used directly. If there are more, the user chooses which three to align. If fewer than three, missing sequences are obtained either from more FASTA files or from manual input.

2. Output:

- **Alignment**: Shows three aligned sequences and the computed alignment score. The DP ensures the global optimal SP alignment of three sequences.
- **Optional Save**: Exports the alignment as FASTA.
- **Optional Verification**: Compares the computed score with a test function in $msa_sp_score_3k.py$.

3. Optimization:

 We implemented Numba's JIT compilation to optimize the innermost loops of our DP computation, which reduces constant overhead and enables rapid execution even with cubic complexity.

4. Visualization:

• The program visualizes the substitution matrix and the optimal alignments in the program. Also, for the manual entry of substitution matrix if a user input different phylip-like format than the program also suggests the user with a visualization of supported matrix format.

5. Verification:

 We verified the correctness of sp_exact_3 by testing on the provided testdata_short.txt and testdata_long.txt files, ensuring that our computed SP scores match those computed by the instructor's implementation.

Usage:

- Run the *sp_exact_3* module by typing *python sp_exact_3.py* (It will show the CLI-interface).
- We recommend to install numba and rich (for the beautiful interface) pip install numba, rich.
- Select, enter or load a substitution matrix.
- Enter a gap penalty integer.
- Choose how to input sequences (manual or FASTA).
- Let the program perform the 3D DP alignment and display results.
- Save the alignment if desired.
- Optionally verify the SP score using the msa_sp_score_3k.py test function.

sp_approx

Algorithm Overview:

The sp_approx implements a 2-approximation method for multiple sequence alignment, focusing on DNA sequences that may contain ambiguous letters. It automatically replaces any character outside of A, C, G, T with 'A,' preserving alignment feasibility. The key objective is to produce a reasonably good alignment in a more efficient manner compared to an exact 3D approach, especially when dealing with three or more sequences.

Design and Implementation Choices:

1. Input:

- **Manual**: Users can type in an arbitrary number of sequences, each of which is substituted to remove non-standard nucleotides.
- **FASTA File**: One file can contain multiple sequences, from which the user selects how many to include in the alignment.

2. Output:

- **Computed Alignment**: The program displays the center-star alignment of all chosen sequences and calculates a sum-of-pairs (SP) score.
- **Optional Save**: Results can be saved to a FASTA file.
- **Optional Verification**: The script can verify the output against a separate scoring function.

3. Center Selection:

• Identifies a center sequence by computing pairwise alignment (Needleman) scores between every pair of sequences and picking the one with the smallest total distance to others. Then the algorithm aligns each remaining sequences to the chosen center sequence.

4. Pairwise Scoring:

Uses a gap cost and a substitution matrix mapping each nucleotide pair to an integer cost.
 (Default scoring matrix). – we did not extend it but we will implement it and update it in my GitHub.

5. Parallelism:

• Pairwise scoring for all sequence pairs runs in parallel with a "*ProcessPoolExecutor*" speeding up the center determination step for larger numbers of sequences.

6. Verification:

• Verification was performed by comparing the SP scores computed by sp_exact_3 and sp_approx on test datasets. We also computed the approximation ratio, which for three sequences should be at most 4/3.

Usage:

- Run the script (*python sp_approx.py*).
- **Select Input Mode** (1 for manual, 2 for FASTA).
- **Enter or Select** the DNA sequences. If fewer than two exist, alignment cannot proceed.
- **View** the final alignment and SP score.
- **Optionally Save** the alignment to FASTA.
- **Optionally Verify** the alignment score with the *msa_sp_score_3k.py* function.

Experiments:

By running the "python QA.py" program we can get the answers of these experimental questions -

• Question 1: What is the score of an optimal alignment of the first 3 sequences in brca1
testseqs.fasta (i.e. brca1_bos_taurus, brca1_canis_lupus and brca1_gallus_gallus) as computed by your program sp_exact_3? How does an optimal alignment look like?

Answer:

```
Welcome to the MSA QA Program!
Aligning sequences using sp_exact_3 and sp_approx methods

Question 1: Optimal Alignment (sp_exact_3)

Sp_exact_3 took 3.523 seconds
Peak Memory: 79.338 MB

Exact Alignment Score: 790

>brcal_bos_taurus:
ATGGATTTATCTGCGGATCATGTTGAAGAAGTGCCTCAATGCTATGCA-GAAAATCTTAG--AGTGTCCAAT-ATGTCTGGAGTTGATCAAAGAG-CCT-GTCTCTAC
AAAGTGTGA--CCA-CA-TATTTTGCAAATTTTG-TATGCTGAA-AC-TTCTCAACCA-GAAGAAAGGGCCTTCACAATGTCC--TTTGTGTAAGAATGA-

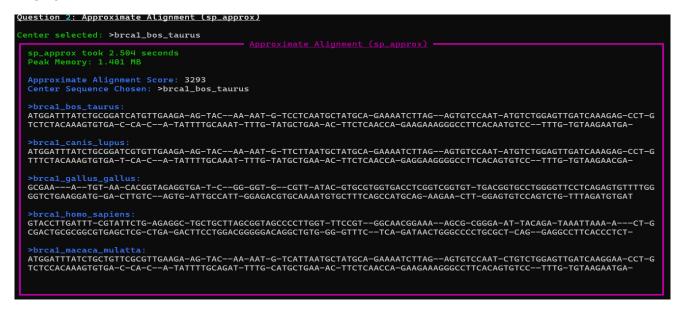
>brcal_canis_lupus:
ATGGATTTATCTGCGGATCGTGTTGAAGAAGTACAAAATGTTCTTAATGCTATGCA-GAAAATCTTAG--AGTGTCCAAT-ATGTCTGGAGTTGATCAAAGAG-CCT-GTTTCTAC
AAAGTGTGA--CCA-CA-TATTTTGCAAATTTTG-TATGCTGAA-AC-TTCTCAACCA-GAAGAAACCTTAG--AGTGTCCAAT-ATGTCTGGAGTTGATCAAAGAG-CCT-GTTTCTAC
AAAGTGTGA-TCA-CA-TATTTTGCAAATTTTG-TATGCTGAA-AC-TTCTCAACCA-GAGGAAGGGGCCTTCACAGTGTCC--TTTGTGTAAGAACGA-

>brcal_gallus_gallus:
GCGAA---ATGTA-ACA-CG-GTAGAGGTGAT-CGGGGTG-CGTT-ATAC-GTGCCTGGTGACCTCGGTGGTGTTGACGGTGCCTGGGGTTCCTCAGAGTGTTTTGGGGTCTGA
AGGATG-GACTTGTCAGTG-ATTGCCATTGGGAGACGTGCAAAAATGTGCTTTCAGCCATGCAGAA-GAA-CTT-GGAGTGTCCAGTCTGTTTAGATGTGAT
```

• **Question 2:** What is the score of the alignment of the first 5 sequences in brca1testseqs.fasta (i.e. brca1_bos_taurus, brca1_canis_lupus, brca1_gallus_gallus, brca1_homo_sapiens,

and brca1_macaca_mulatta) as computed by your program sp_approx? Which of the 5 sequences is choosen as the 'center string'?

Answer:

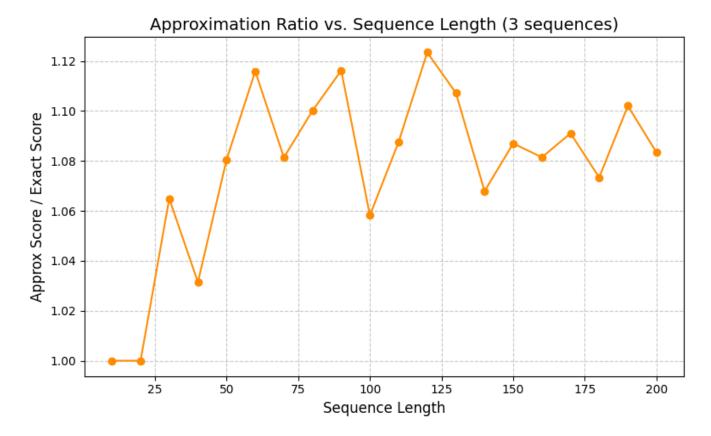


• **Question 3:** Make an experiment comparing the scores of the alignments computed by sp_exact_3 and sp_approx that validates that the approximation ratio of sp_approx is 2(*k*-1)/*k* for *k* sequences. i.e 4/3 for three sequences. For each triplet of sequences (i.e. each fasta file), you should compute the optimal score of an MSA using sp_exact_3 and the score of the alignment produced by sp_approx. Make a graph in which you plot the ratio of the computed scores for each sequence length. Comment on what you observe.

Answer:

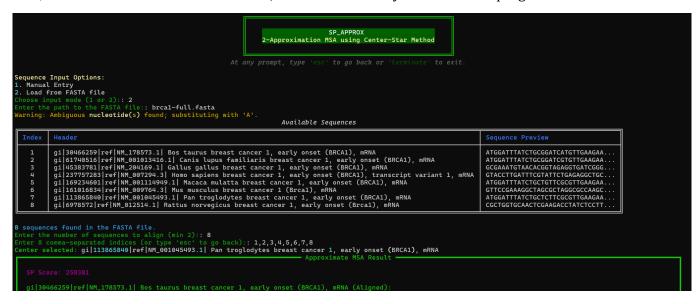
Comparison: Exact vs. Approximate Alignment							
Seq Length	Exact Score	Approx Score	Ratio	Time (Exact)	Mem (Exact) MB	Time (Approx)	Mem (Approx) MB
10	70	70	1.000	0.001	0.012	0.787	0.051
20	135	135	1.000	0.001	0.073	0.817	0.050
30	231	246	1.065	0.004	0.230	0.809	0.048
40	318	328	1.031	0.007	0.529	0.818	0.048
50	385	416	1.081	0.011	1.016	0.792	0.048
60	440	491	1.116	0.022	1.736	0.807	0.048
70	516	558	1.081	0.029	2.735	0.823	0.056
80	589	648	1.100	0.042	4.060	0.817	0.084
90	628	701	1.116	0.065	5.755	0.879	0.121
100	687	727	1.058	0.099	7.867	0.879	0.170
110	754	820	1.088	0.113	10.441	0.886	0.223
120	810	910	1.123	0.167	13.523	0.894	0.298
130	895	991	1.107	0.195	17.159	0.981	0.395
140	957	1022	1.068	0.239	21.395	1.032	0.502
150	1023	1112	1.087	0.288	26.276	1.031	0.629
160	1080	1168	1.081	0.352	31.849	1.094	0.728
170	1186	1294	1.091	0.410	38.158	1.178	0.868
180	1158	1243	1.073	0.489	45.251	1.256	0.996
190	1323	1458	1.102	0.617	53.171	1.274	1.146
200	1379	1494	1.083	0.708	61.966	1.362	1.290

N.B: While the approximate method's theoretical complexity is $O(n^2 \cdot L^2)$ (with k being the number of sequences), its overhead (especially for small sequences) can be higher than sp_exact_3 due to parallelization and the merging process.



The sp_approx method, which leverages parallelized pairwise alignment and a center heuristic, offers a scalable approximation for larger datasets, with a theoretical guarantee on the approximation ratio (2(k-1)/k) or for 3 sequence its 4/3. As sequence length increases, the approximation ratio remains within the theoretical bound (approximately 1.333 for three sequences), validating the theoretical guarantee of the center–star algorithm.

Also, we consider to test **brca1-full.fasta**, and we successfully did it with our program.



Download:

For download the alignment

https://drive.google.com/drive/folders/1NduCVqLIBLg Kmk3o98lABS5mWwc9Lv6?usp=drive link

Appendix:

1. Run sp_exact_3.py D:\Algorithms in Bioinformatics\Project 3>python sp_exact_3.py SP_EXACT_3 Exact Multiple Sequence Alignment for 3 Sequences At any prompt, type 'esc' to go back or 'terminate' (or 'quit') to exit. Substitution Matrix Input Options: Use default substitution matrix Load substitution matrix from a file Input substitution matrix manually Choose option (1, 2, or 3):: 1
Using default substitution matrix. Substitution Matrix Α Θ 5 2 5 C 5 5 0 2 G 2 5 0 5 Enter the gap penalty (an integer):: 5 Sequence Input Options: Manual Entry Load from FASTA file

Choose input mode (1 or 2):: 2
Enter the path to the FASTA file:: testdata_long.txt

Exactly 3 sequences found. They will be used for alignment.

Computing the optimal alignment using exact dynamic programming...

Alignment Score: 1482

Sequence 1:
GTTCCGAAAGGCTAGCGCTAGGCGCCAAGCGGCCGGTTTCCTTGGCGACGGAG-AGCGCGG-GAATTTTAG-ATAGA
TTGTA-AT-TGCGGCT-G-CGCGGCCGCTGCCCGTGCAGCCAGAGGATCCAGC-ACCT-CTCTTG-GGGCTTCTC
CG-TCCTCGGCGCTT-GGAAGTAC-GGATCT-TTTT-T-CT-CGGAGAAAAGTTC-A-C-TGGAA-CTG--Sequence 2:
A--TGGATTTATCTGCTCTTCGCGTTGAA-GAAGTA-CAAAATGTCATTAACGCTATGCAGAAAATCTTAGAGTGTC
CCATCTGTCTGGAGTTGATCAAGG-AACCTGTC-T-CCA-CAAAGTGT-GACC-ACAT-ATTTTGCAAATTT-TG
CA-TGCTGAA-ACTTCTCAACCAGAAGAAAG-GGCC-T--T-CACAGTGTCCTTT-A-TGTAAGA-ATG--A
Sequence 3:
---C-G----CTGGTGC-A-AC-TCGAA-GACCTATCTCCTTCCCGGGGGGGGCTTCTCCG-GCAT-TTAG-GC--C
TCGGC-GTTTGGAAGT-A-CG-GA-GGTTTTTC-T-CGG-AAGAAAGTTCACTGGAAGTGGAAGAAATGGATTTATC
TGCTGTTCGA-ATTCAAGAAGTACAAAATGTCCTTCATGCTATGCAGAAAATCTTGGAGTGTCCAATCTGTTT

```
Would you like to save the alignment to a FASTA file? (yes/no):: y
Enter the file name to save the alignment:: testdata_long_aln
Alignment successfully saved to testdata_long_aln.fasta.
Would you like to verify the alignment using the test program? (yes/no):: y
Alignment is verified and passed!
```

SP APPROX 2-Approximation MSA using Center-Star Method

Sequence Input Options:

1. Manual Entry

2. Load from FASTA file

Enter the path to the FASTA file:: brcal-testseqs.fasta

Available Sequences

Index	Header	Sequence Preview
1 2 3 4 5 6 7 8	brca1_bos_taurus brca1_canis_lupus brca1_gallus_gallus brca1_homo_sapiens brca1_macaca_mulatta brca1_mus_musculus brca1_pan_troglodytes brca1_rattus_norvegicus	ATGGATTTATCTGCGGATCATGTTGAAGAA ATGGATTTATCTGCGGATCGTGTTGAAGAA GCGAAATGTAACACGGTAGAGGTGATCGGG GTACCTTGATTTCGTATTCTGAGAGGCTGC ATGGATTTATCTGCTGTTCGCGTTGAAGAA GTTCCGAAAGGCTAGCCCTAGGCGCCAAGC ATGGATTTATCTGCTCTTTCGCGTTGAAGAA CGCTGGTGCAACTCGAAGACCTATCTCCTT

8 sequences found in the FASTA file.

Enter the number of sequences to align (min 2):: 8
Enter 8 comma-separated indices (or type 'esc' to go back):: 1,2,3,4,5,6,7,8
Center selected: brcal_macaca_mulatta

- Approximate MSA Result -

brcal_bos_taurus (Aligned):

-A-T-GGAT-TTA-TC--TG--CGGA-T-CATGTTGAAGAAG-TAC--AA-AAT-G-TCCTCAATG-C-TA-TGCA-GAAAATCTTAG--A-GTGT C-CA-AT-A-TGTCT-GGAGTTGATCAAA-G-AGCC-T-G-TCTCTACAAAGTGTGA-C-CA-C--A-TATTTTGCAAAT-T-TTG-TATGC-T-G AA-AC-TTC-TCA-ACCA-GAAGAAAGGGCCT-TCACAATGTC-C--TTTG-TGTAAGAATGA-

brca1_canis_lupus (Aligned)

-A-T-GGAT-TTA-TC--TG--CGGA-T-CGTGTTGAAGAAG-TAC--AA-AAT-G-TTCTTAATG-C-TA-TGCA-GAAAATCTTAG--A-GTGT C-CA-AT-A-TGTCT-GGAGTTGATCAAA-G-AGCC-T-G-TTTCTACAAAGTGTGA-T-CA-C--A-TATTTTGCAAAT-T-TTG-TATGC-T-G AA-AC-TTC-TCA-ACCA-GAGGAAGGGGCCT-TCACAGTGTC-C--TTTG-TGTAAGAACGA-

brca1_gallus_gallus (Aligned)

-G-C-GĀA----A---TG--TAĀC-A-CG-GTAGAGGTGA-T-C--GG-GGT-G--CGTT-ATA-C--G-TGCGTGGTGACCTCGGTCG-GTGT -TG-ACGG-TGCCT-GGGGTTCCTCAGA-GTGTTT-TGG-GGTCTGAAGGATG-GA-CTTGTC--AGTG-ATTGC-CAT-TGGAGACGTGC-A-A AATGTGCTT-TCA-GCCATGCAG-AAGAA-CT-T-GGAGTGTC-CAGTCTG-TTTAGATGTGAT

sapiens (Aligned

-G-T-ACCT-TGA-TT--TC--GTAT-T-C-TG-AGAGGCTGCTTAGCGGTAGCCCCTTGGT--T-TC-CGT--GGCAACGGAAA--A-GCGC G-GG-AA-T-TA-C--AGA-TAAATTAA----AACTGC-G-ACTGCGCGGCGTGAGC-T-CG-CTGA-GACTTCCTGGACGG-GGGACAGGC-T-G TG-GG-GTT-TC---TCA-GATAACTGGGCCC-CTGC-GC-TCAG--GAGGCCTTCACCCTCT-

-A-T-GGAT-TTA-TC--TG--CTGT-T-CGCGTTGAAGAAG-TAC--AA-AAT-G-TCATTAATG-C-TA-TGCA-GAAAATCTTAG--A-GTGT C-CA-AT-C-TGTCT-GGAGTTGATCAAG-G-AACC-T-G-TCTCCACAAAGTGTGA-C-CA-C--A-TATTTTGCAGAT-T-TTG-CATGC-T-G AA-AC-TTC-TCA-ACCA-GAAGAAAGGGCCT-TCACAGTGTC-C--TTTG-TGTAAGAATGA-

brca1_mus_musculus (Aligned)

-GTTCCGA--AAG-GC--TA--GCGC-TAGGCGCC-AAGCGG-C-C-----GGT-T-TCCTTGGCGACGGAGAGCGCGGGAATTTTAG--ATAGAT TGTA-AT-TGCGGCT--GCG-CGGCCGCT-G-CCCG-T-G-CAGCCAGAGGATCCAG---CA-C--C-TCTCTTGGGGCT-T-CTC-CGTCC-TCG GC-GC-TT--GGA-AGTA--CGGATCTTTTTTCTCGGAGAAAA-G--TTCA-C-T-GGAACTG-

brca1_pan_troglodytes (Aligned

-A-T-GGAT-TTA-TC--TG--CTCT-T-CGCGTTGAAGAAG-TAC--AA-AAT-G-TCATTAACG-C-TA-TGCA-GAAAATCTTAG--A-GTGT C-CC-AT-C-TGTCT-GGAGTTGATCAAG-G-AACC-T-G-TCTCCACAAAGTGTGA-C-CA-C--A-TATTTTGCAAAT-T-TTG-CATGC-T-G AA-AC-TTC-TCA-ACCA-GAAGAAAGGGCCT-TCACAGTGTC-C--TTTA-TGTAAGAATGA-

brcal_rattus_norvegicus (Aligned)

CG-C-TGGTGCAACTCGAAGACCTATCTCCTTCCCGGGGGGGG-C-T--TC-TCC-G-GCATTTAGG-C-C--T-C--G-GCGTTTGGA--A-GTAC G-GAGGT-T-TTTCTCGGAA--GA--AAGTT-CACT-G-GAAGTGGAAGAAATG-GATT-TA-T--C-TGCTGTTCGAAT-T-CAA-GAAGTAC-A AA-ATGTCCTTCATGCTATGCAGAAA--ATCT-T-GGAGTGTC-C--AATC-TGT--

Enter the file name to save the alignment:: approx_brc1-testseqs_algn Alignment successfully saved to approx_brc1-testseqs_algn.fasta. Would you like to verify the alignment using the test program? (yes/no):: **y** Alignment is verified and passed!

3. Run python QA.py

```
Algorithms in Bioinformatics\Project 3>python QA.py

MSA Project QA

Melcone to the MSA QA Program!

Alicotop cognonces using sp_exact_3 and sp_apprex meth
  estion 1: Optimal Alignment (sp exact 3)
                                                                                                                                                          Exact Alignment (sp exact 3) -
   e<mark>rcal dos, taurus:</mark>
Negattancteregatcatettembangtacamatetecatectateca-gamatettag—netetecat-atetetegagitgateambag-ect-eteteac
Wigitga-eca-ca-tattitgematitte-tatectga-ac-itetemeca-gamagesectteacatetec—ittegtembatea-
         real_canis_lupus:
BBATTIATCTBCBBATCBTGTGMBAWGTACAMATGTTCTTAATGCTATGCA-GAMATCTTAG—AGTGTCCAAT-ATGTCTGBAGTTGATCAMBAG-CCT-GTTTCTAC
AGTGTGA-TCA-CA-TATTTTGCAAATTTTG-TATGCTGAA-AC-TTCTCAACCA-GAGAAAGGGGCCTTCACAATGTCC—TTTGTGTAAGAACGA-
                al gallus
A—Atgia-Aca-co-giagagggat-cogggg-cott-atac-gigcgigggacctcggtgacctcggggt-tgacggigcctggggticctcaagggtttiggggictga
16-aactigicagg-attoccattgaagacgigcaaagggggtticagcatgagaa-gaa-cit-gaagggiccagictgittiagatggat
     stion 2: Approximate Alignment (sp approx)
     ter selected: >brcal_bes_taurus
     pproximate Alignment Score: 3293
enter Sequence Chosen: >brcal_bos_taurus
   encal_bos_taurus:
1766atttatctgcggatcatgttgagg-ag-tac—aa-aat-g-tcctcaatgctatgca-gamatcttag—agtgtccaat-atgtctggagttgatcaagg-cct-g
ICTCTACAMGTGTGA-C-CA-C—A-TATTTTGCAAAT-TTTG-TATGCTGAA-AC-TTCTCAACCA-GAAGAMGGGCCTTCACAATGTCC—TTTG-TGTAAGAATGA-
        rcal_cands_lupus:
GGATTTATC1GCGGATCGTGTGAAGA-AG-TAC—AA-AAT-G-TTCTTAATGCTATGCA-GAAAATCTTAG—AGTGTCCAAT-ATGTCTGGAGTTGATCAAAGAG-CCT-G
TCTACAAAGTGTGA-T-CA-C—A-TATTTTGCAAAT-TTTG-TATGCTGAA-AC-TTCTCAACCA-GAGGAAGGGGCCTTCACAGTGTCC—TTTG-TGTAAGAACGA-
             cal_gallus_gallus:
N——A—TGT—M—CACGGTAGAGGTGA—T—C—GG—GGT—G—CGTT—ATAC—GTGCGTGGTGACCTCGGTGGTGT—TGACGGTGCCTGGGGTTCCTCAGAGTGTTTTGG
TGAAGGATG—GA—CTTGTC—AGTG—ATTGCCATT—GGAGACGTGCAAAATGTGCTTTCAGCCATGCAG—AAGA—CTT—GGAGTGTCCAGTCTG—TTTAGATGTGAT
    breal_hong_sapions:
Taccttgatti-cgtattctg-agaggc-tgctgcttagggfagccccttggt-ticcgt-ggcaacggaaa—aggg-ggga-at-tacaga-taaattaaa-a---
Gactggggggggggggctgagctgg-ggg-gagag-gagtggggggacaggctgtg-gg-gtttc-tca-gataactggggcccttggggct-cag-gaggccttcacctct
          real_magas_milatta:
Bantianciactritoggitiquak-ag-112—al-ant-g-12Atiangctang22-gawatchiag—agtriccat-ctrictgbathgatcaagax-cct-g
CCC/CAMARIGA-C-C-X-C—A-TANTHG/BAT-THG-CANGCTGAX-AC-HCTCAACCX-GAAGAWGGGCCTHCA/ARIGICC—THG-16TMAGANGA-
tion 3: Experiment to Validate Approximation Ratio
                                                                                                            70
135
246
328
416
491
558
648
701
727
820
91
1022
1112
1168
1294
1243
1490
                                                                                                                                                  1.990
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0.592
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1.146
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