Assignment 2

General Instructions

- The Python standard library is not enough to do solve these questions. You will need to import appropriate libraries for each task. Generally, you might import and use any library you wish unless otherwise stated.
- Where detail instructions like variable or function names, required libraries, and etc are not given by the question, feel free to do it the way you would like to.
- After each question, add the needed number of new cells and place your answers inside the cells.
- When you are required to explain or answer in text format open a Markdown cell and enter your answer in it.
- Do not remove or modify the original cells provided by the instructor.
- Comment your code whenever needed using # sign at the beginning of the row.
- Do not hesitate to communicate your questions to the TAs or instructors. Good luck!

In [1]:
```
# The following piece of code gives the opportunity to show multiple outputs
# in one cell:
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"


# Colorful outputs
class bcolors:
    RED       = '\033[91m'
    OKBLUE    = '\033[94m'
    BOLD      = '\033[1m'
    UNDERLINE = '\033[4m'
    ENDC      = '\033[0m'
```

Question 1 (40 points)

1. Download `Income2.csv` from http://www-bcf.usc.edu/~gareth/ISL/Income2.csv (http://www-bcf.usc.edu/~gareth/ISL/Income2.csv)
2. Load the data into this Jupyter notebook.
3. Describe the data using descriptive statistics such as measures of central tendancy, dispersion or association.
4. Explore the data by vizualing it through various figures. Clearly explain or interpret each figure, justify the appropriateness of the tool used for visualization, highlight the information it provides, and, finally, explain how this information affect your further analysis.
5. Model `Income` as a linear function of `Years of Education`. What are your independent and dependent variables? What type of model did you use? Why?
6. Scatterplot the dependent and independent variables used in the model versus each other. Based on the scatterplot, do you think a linear model is an adequate model for the data in hand? Discuss your answer.

7. Print the slope of the fitted line out and provide a 95% confidence interval for the estimate slope.
8. Add the fitted line over the scatterplot.
9. Using the **discrete uniform** distribution, randomly generate 10 numbers between the 25 and 75 percentiles of the variable `Years of Education` in the original data. If these numbers represent years of education for 10 employees, then predict each person's `Income` due to your model.
10. Now, model `Income` as a linear function of both `Years of Education` and `Seniority`. What type of model did you use? How many parameters (coefficients) does this model have?
11. Print out the estimated coefficients after the model has been fitted.
12. How much would be the `Income` of a new individual with 18 years of education and 60 years of seniority?
13. Argue which of `Years of Education` or `Seniority` is a stronger predictor of `Income`? Justify your comparison. (**Hint**: take into consideration that these variables are in different units)

In [2]:
```
import sklearn
import statsmodels
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats  as stats
import statsmodels.formula.api as smf
import math
```

In [3]:
```
#1.Download Income2.csv from http://www-bcf.usc.edu/~gareth/ISL/Income2.csv
#2.Load the data into this Jupyter notebook
path='data/'
filename = path+'Income2.csv'
income_data = pd.read_csv(filename)
income_data.head()
```

Out[3]:

	Unnamed: 0	Education	Seniority	Income
0	1	21.586207	113.103448	99.917173
1	2	18.275862	119.310345	92.579135
2	3	12.068966	100.689655	34.678727
3	4	17.034483	187.586207	78.702806
4	5	19.931034	20.000000	68.009922

In [4]:
```
#3.Describe the data using descriptive statistics such as measures of central te
income_data.describe()
```

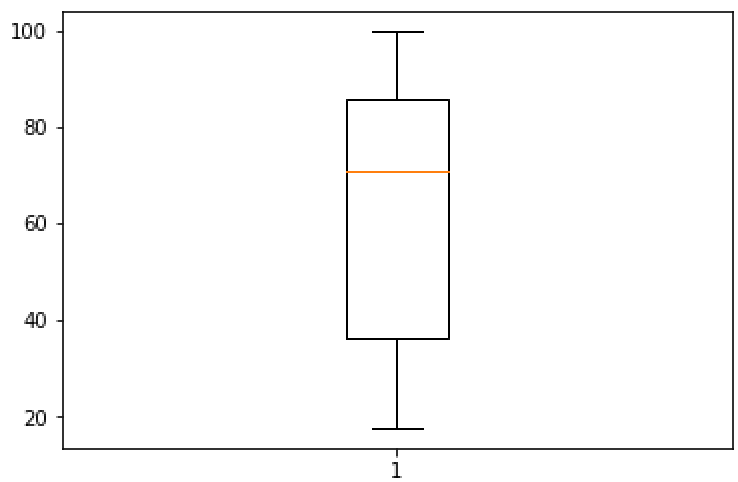
Out[4]:

	Unnamed: 0	Education	Seniority	Income
count	30.000000	30.000000	30.000000	30.000000
mean	15.500000	16.386207	93.862069	62.744733
std	8.803408	3.810622	55.715623	27.013285
min	1.000000	10.000000	20.000000	17.613593
25%	8.250000	12.482759	44.827586	36.392043
50%	15.500000	17.034483	94.482759	70.804791
75%	22.750000	19.931034	133.275862	85.930608
max	30.000000	21.586207	187.586207	99.917173

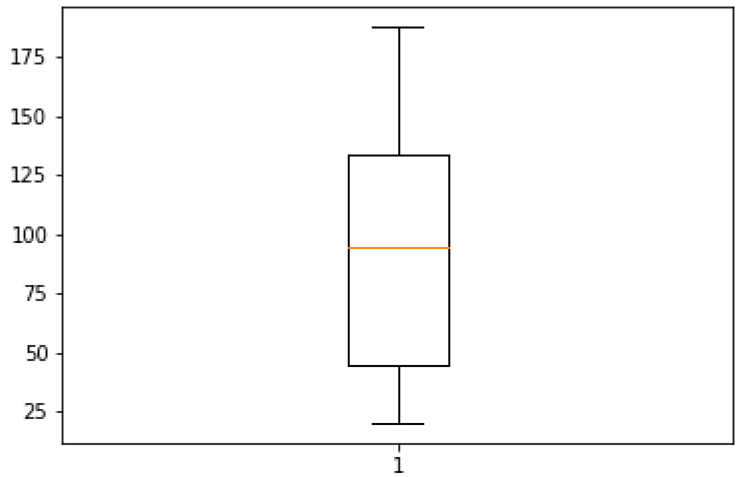
#3. Describe the data using descriptive statistics such as measures of central tendancy, dispersion or association

Ans:There are total 30 observations with 3 major variables to be consider. They are Education,Seniority and Income and all the 3 variable are continuous. Education:It is describing the years of education and the range is in between 10 to 21.586207.The mean value is 16.38 and median in 17.034. Seniority:It is describing the serniority in the unit of months and its range is in between 20 to 187.58. The mean and median value are 93.86 and 94.48 respectively. Income:It is describing the income in thousand units. The range of this variable is 17.613 to 99.917 and the mean and median value are 62.744 and 70.804 respectively

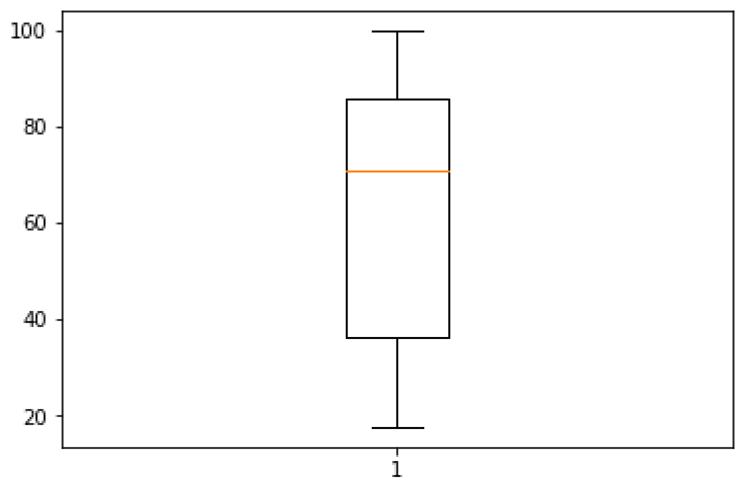
In [5]:
```
plt.boxplot(income_data.Income);
```



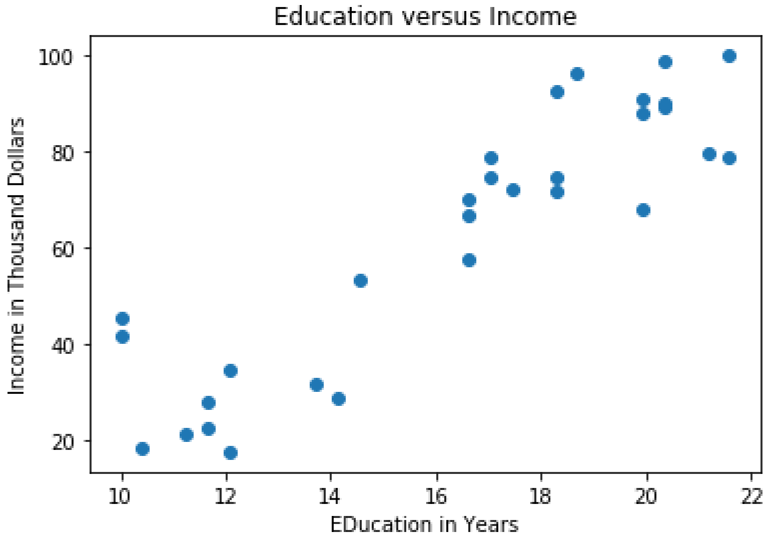
In [6]:
```
plt.boxplot(income_data.Seniority);
```



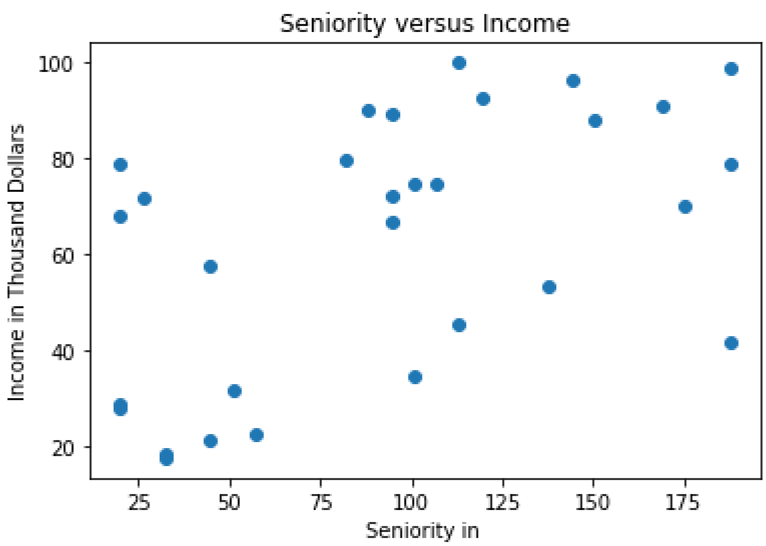
In [7]:
```
plt.boxplot(income_data.Income);
```



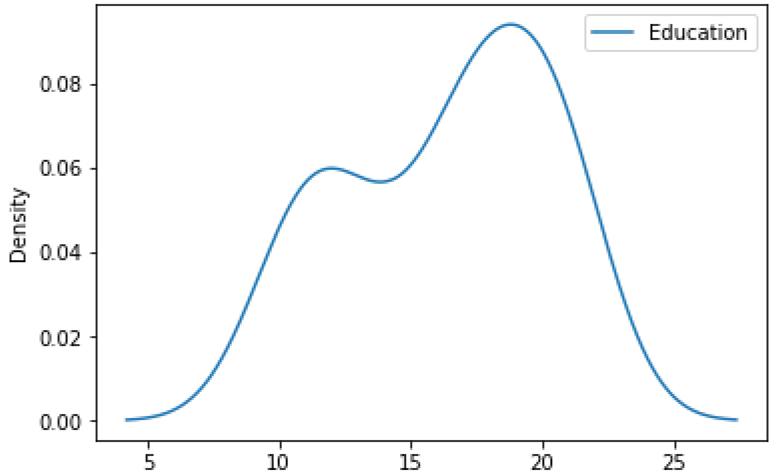
In [8]:
```
plt.scatter(income_data.Education, income_data.Income)
plt.title('Education versus Income')
plt.xlabel('EDucation in Years')
plt.ylabel('Income in Thousand Dollars');
```



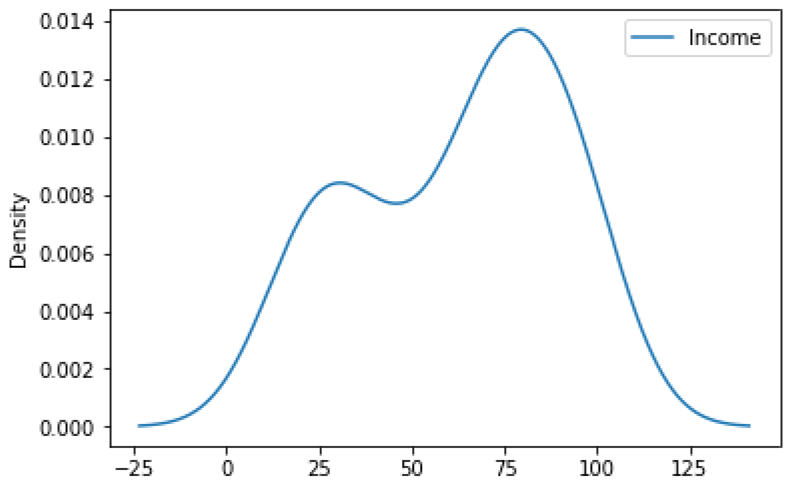
In [9]:
```
plt.scatter(income_data.Seniority, income_data.Income)
plt.title('Seniority versus Income')
plt.xlabel('Seniority in ')
plt.ylabel('Income in Thousand Dollars');
```



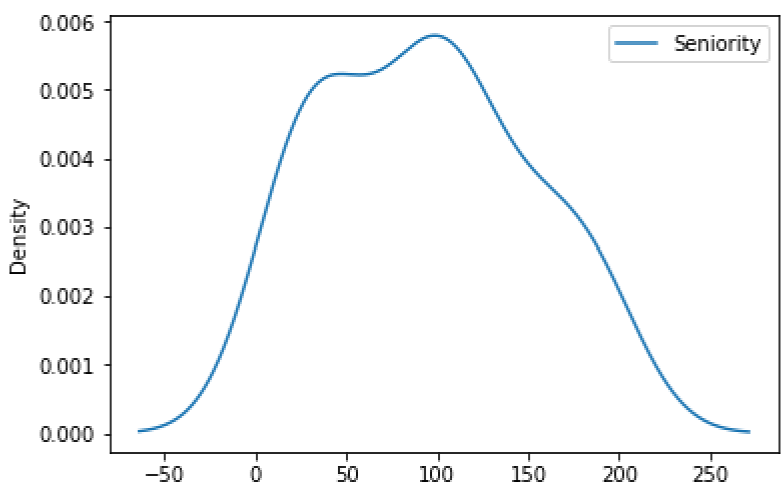
In [10]:
```
pd.DataFrame(income_data['Education']).plot(kind="density", figsize=(6,4));
```



In [11]:
```
pd.DataFrame(income_data['Income']).plot(kind="density", figsize=(6,4));
```



In [12]:
```
pd.DataFrame(income_data['Seniority']).plot(kind="density", figsize=(6,4));
```



4.Explore the data by vizualing it through various figures. Clearly explain or interpret each figure, justify the appropriateness of the tool used for visualization, highlight the information it provides, and,finally, explain how this information affect your further analysis.

Ans:Here income is treated as the dependent variable while Education and seniority are considered to be independent variables.

Scatter plot is drawn to visualize the linear relationship between dependent and Independent variables. When Years of Education increases the Income also increases. Similarly when seniority increases the income also increases however here variable are more dispersed.

Box plot is drawn to spot any outlier observations in the variable. Density plot is drawn to see the distribution of the independent variable. From the above density plot it is observed that all the 3 data have normally distributed (a bell shaped curve), without being skewed to the left or right.

In [13]:
```
#5.Model Income as a linear function of Years of Education. What are your indepen
#What type of model did you use? Why?

from sklearn.linear_model import LinearRegression

# initialize the model first
lr = LinearRegression()
# fit the model and feed the data
lr.fit(X = income_data[ ['Education'] ], y = income_data['Income'])
```

Out[13]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None,
         normalize=False)
```

In [14]:
```
print(lr.intercept_, lr.coef_)
```

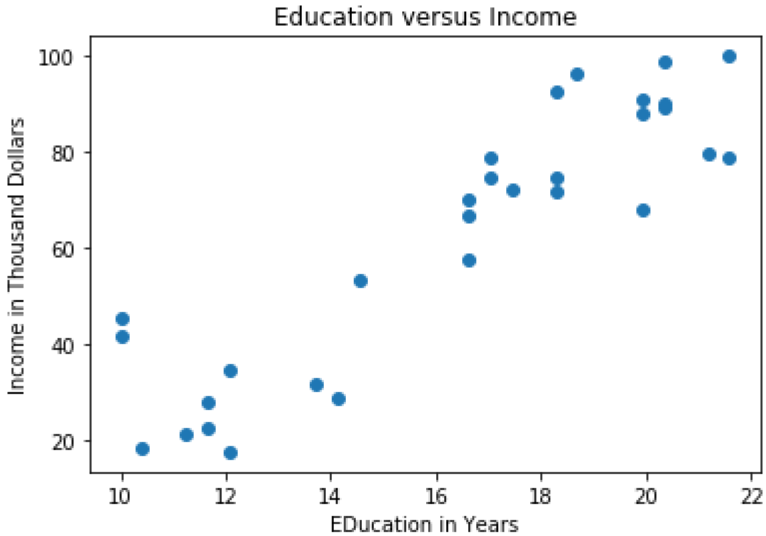
```
-41.91661220978736 [6.38716122]
```

5.Ans:Years of Education is the independent and Income is the dependent variables

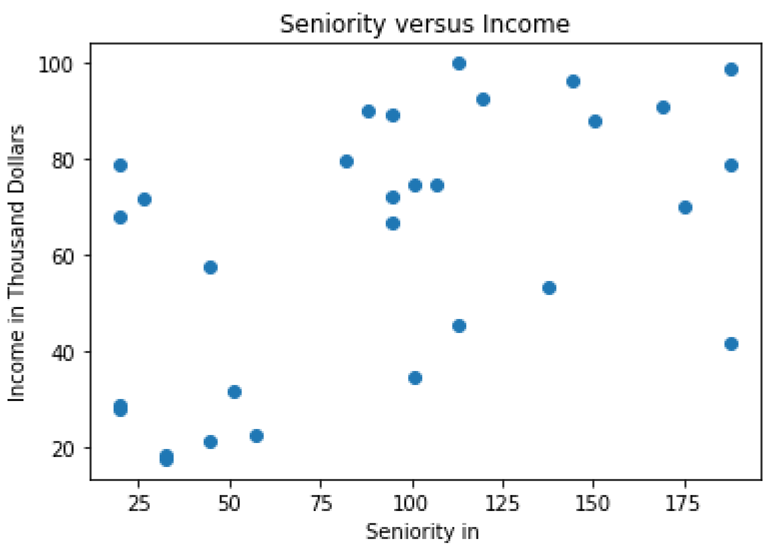
Ans:Here Income is treated as the dependent variable while Education and Seniority are considered to be independent variables.

From the Scatter plot we can see when Years of Education increases the Income also increases. Similarly when seniority increases the income also increases however here variable are more dispersed. Therfore we can use Linear regression model for this .

In [15]:
```
#6.Scatterplot the dependent and independent variables used in the model versus e
plt.scatter(income_data.Education, income_data.Income)
plt.title('Education versus Income')
plt.xlabel('EDucation in Years')
plt.ylabel('Income in Thousand Dollars');
```



In [16]:
```
plt.scatter(income_data.Seniority, income_data.Income)
plt.title('Seniority versus Income')
plt.xlabel('Seniority in ')
plt.ylabel('Income in Thousand Dollars');
```



6.Based on the scatterplot, do you think a linear model is an adequate model for the data in hand? Discuss your answer.

Ans:From the Scatter plot we can see when Years of Education increases the Income also increases. Similarly when seniority increases the income also increases however here variable are more dispersed. Therefore we can use Linear regression model for this .

In [17]:
```
#7.Print the slope of the fitted line out provide a  95%  confidence interval fo
model = smf.ols('Income ~ Education', data=income_data)
lr_model = model.fit()

#Print out the statistics
lr_model.summary()
```

Out[17]:

OLS Regression Results

Dep. Variable:	Income	**R-squared:**	0.812
Model:	OLS	**Adj. R-squared:**	0.805
Method:	Least Squares	**F-statistic:**	120.8
Date:	Fri, 05 Apr 2019	**Prob (F-statistic):**	1.15e-11
Time:	21:53:48	**Log-Likelihood:**	-115.90
No. Observations:	30	**AIC:**	235.8
Df Residuals:	28	**BIC:**	238.6
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	-41.9166	9.769	-4.291	0.000	-61.927	-21.906
Education	6.3872	0.581	10.990	0.000	5.197	7.578

Omnibus:	0.561	**Durbin-Watson:**	2.194
Prob(Omnibus):	0.756	**Jarque-Bera (JB):**	0.652
Skew:	0.140	**Prob(JB):**	0.722
Kurtosis:	2.335	**Cond. No.**	75.7

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [18]:
```
#8.Add the fitted line over the scatterplot

plt.plot(income_data.Education, income_data.Income, 'or', mfc='none')
# add a regression line
plt.plot(income_data.Education, lr_model.params.Intercept+lr_model.params.Educati
plt.xlabel('Education')
plt.ylabel('Income')
plt.title('Education vs Income')
```

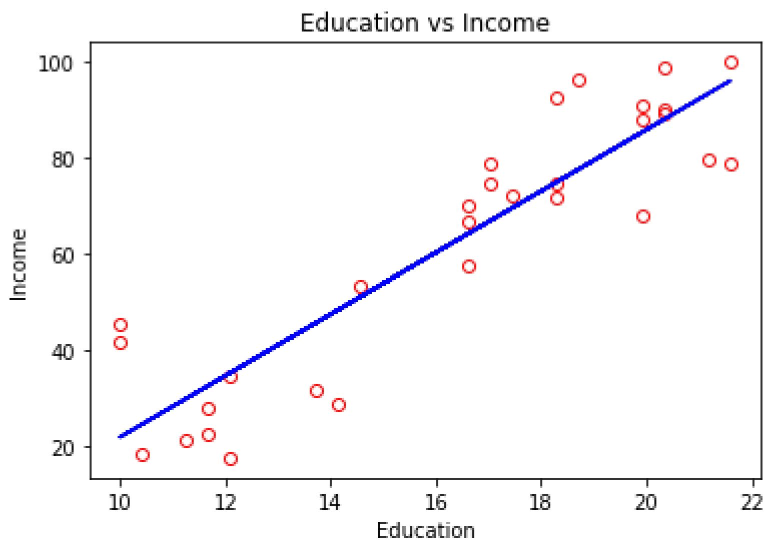
Out[18]: [<matplotlib.lines.Line2D at 0x3522229fd0>]

Out[18]: [<matplotlib.lines.Line2D at 0x352220c5f8>]

Out[18]: Text(0.5, 0, 'Education')

Out[18]: Text(0, 0.5, 'Income')

Out[18]: Text(0.5, 1.0, 'Education vs Income')



In [19]:

```
#9.Using the discrete uniform distribution, randomly generate  10  numbers betwee
# Years of Education in the original data. If these numbers represent years of ed
# then predict each person's Income due to your model.

#25  and  75  percentiles of the variable
q1 = np.percentile(income_data.Education, 25) # 25%
q2 = np.percentile(income_data.Education, 75) # 75%

#Using the discrete uniform distribution, randomly generate  10  numbers between
#'Years of Education' in the original data.
uniform_data = stats.uniform.rvs(size=10,  # Generate 100000 numbers
                                 loc = q1 ,      # From 25%
                                 scale= q2 - q1)     # To 75%
print(uniform_data)

#If these numbers represent years of education for  10  employees,then predict ea
# initialize the model first
lr = LinearRegression()
# fit the model and feed the data
lr.fit(X = income_data[ ['Education'] ], y = income_data['Income'])
print('The predicted Income are:',lr.predict(uniform_data.reshape(10,1)).reshape
```

```
[13.93484388 18.88646113 14.47151966 17.10986556 15.86496152 16.19373642
 18.90681902 14.97606386 13.78939511 13.82538145]
```

Out[19]:

```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None,
         normalize=False)

The predicted Income are: [[47.08748221]
 [78.71425987]
 [50.51531689]
 [67.36685753]
 [59.4154547 ]
 [61.51539299]
 [78.84428896]
 [53.73792204]
 [46.15847743]
 [46.38832798]]
```

10.Now, model Income as a linear function of both Years of Education and Seniority. What type of model did you use? How many parameters (coefficients) does this model have?

In [20]:
```
model_mult = smf.ols('Income ~ Education + Seniority', data=income_data)
lr_model_mult = model_mult.fit()

#Print out the statistics
lr_model_mult.summary()
```

Out[20]:

OLS Regression Results

Dep. Variable:	Income	**R-squared:**	0.934
Model:	OLS	**Adj. R-squared:**	0.929
Method:	Least Squares	**F-statistic:**	191.4
Date:	Fri, 05 Apr 2019	**Prob (F-statistic):**	1.13e-16
Time:	21:53:49	**Log-Likelihood:**	-100.15
No. Observations:	30	**AIC:**	206.3
Df Residuals:	27	**BIC:**	210.5
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-50.0856	5.999	-8.349	0.000	-62.394	-37.777
Education	5.8956	0.357	16.513	0.000	5.163	6.628
Seniority	0.1729	0.024	7.079	0.000	0.123	0.223

Omnibus:	3.352	**Durbin-Watson:**	2.102
Prob(Omnibus):	0.187	**Jarque-Bera (JB):**	2.672
Skew:	0.729	**Prob(JB):**	0.263
Kurtosis:	2.892	**Cond. No.**	502.

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Ans: From the model we can see it is a Linear Regression model as the method used is least squares. There are 2 parameters (coefficients) are here one for Education and one is for Seniority. For coeffiecients for 'Education' and 'Seniority' is:5.8956 and 0.1729 respectively.

In [21]:
```
#11.Print out the estimated coefficients after the model has been fitted
lr_mult = LinearRegression()
lr_mult.fit(X = income_data[ ['Education', 'Seniority'] ], y = income_data['Incon
print('The coefficients are', lr_mult.coef_)
```

Out[21]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None,
         normalize=False)

The coefficients are [5.89555596 0.17285547]
```

In [22]:
```
#12.How much would be the Income of a new individual with  18  years of educatio
X_new = np.array([18, 60])
print(X_new.reshape(1,2))

print("The income will be:", lr_mult.predict(X_new.reshape(1,2)).reshape(1,1))
```

```
[[18 60]]
The income will be: [[66.4056967]]
```

In [23]:
```
#13.Argue which of Years of Education or Seniority is a stronger predictor of In
#(Hint: take into consideration that these variables are in different units)
model_Edu = smf.ols('Income ~ Education', data=income_data)
lr_model_Edu = model_Edu.fit()
lr_model_Edu.summary()
```

Out[23]:

OLS Regression Results

Dep. Variable:	Income	**R-squared:**	0.812
Model:	OLS	**Adj. R-squared:**	0.805
Method:	Least Squares	**F-statistic:**	120.8
Date:	Fri, 05 Apr 2019	**Prob (F-statistic):**	1.15e-11
Time:	21:53:49	**Log-Likelihood:**	-115.90
No. Observations:	30	**AIC:**	235.8
Df Residuals:	28	**BIC:**	238.6
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	-41.9166	9.769	-4.291	0.000	-61.927	-21.906
Education	6.3872	0.581	10.990	0.000	5.197	7.578

Omnibus:	0.561	**Durbin-Watson:**	2.194
Prob(Omnibus):	0.756	**Jarque-Bera (JB):**	0.652
Skew:	0.140	**Prob(JB):**	0.722
Kurtosis:	2.335	**Cond. No.**	75.7

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [24]:
```
model_Sen = smf.ols('Income ~ Seniority', data=income_data)
lr_model_Sen = model_Sen.fit()
lr_model_Sen.summary()
```

Out[24]:

OLS Regression Results

Dep. Variable:	Income	**R-squared:**	0.269
Model:	OLS	**Adj. R-squared:**	0.243
Method:	Least Squares	**F-statistic:**	10.28
Date:	Fri, 05 Apr 2019	**Prob (F-statistic):**	0.00335
Time:	21:53:49	**Log-Likelihood:**	-136.26
No. Observations:	30	**AIC:**	276.5
Df Residuals:	28	**BIC:**	279.3
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	39.1583	8.516	4.598	0.000	21.714	56.602
Seniority	0.2513	0.078	3.207	0.003	0.091	0.412

Omnibus:	7.403	**Durbin-Watson:**	2.410
Prob(Omnibus):	0.025	**Jarque-Bera (JB):**	2.253
Skew:	-0.208	**Prob(JB):**	0.324
Kurtosis:	1.724	**Cond. No.**	216.

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Ans:Adjusted. R-squared reflects the fit of the model. R-squared values range from 0 to 1, where a higher value generally indicates a better fit. Comparing the R-Squared value for both the predictor Education and Seniority (which is 0.805 and 0.243 respectively) we can see that Education~Income model has a better fitting. Therefore it can act as a stronger predictor for income. Also from the Prob (F-statistic)(p value) value shows that Education is a stronger predictor than Seniority.(Prob (F-statistic) for Education and Seniority are: :1.15e-11 & 0.00335 respectively)

Question 2 (30 points)

1. Download `Credit.csv` from [http://www-bcf.usc.edu/~gareth/ISL/Credit.csv (http://www-bcf.usc.edu/~gareth/ISL/Credit.csv)](http://www-bcf.usc.edu/~gareth/ISL/Credit.csv)
2. Load the data into this Jupyter notebook.
3. Describe the data using descriptive statistics such as measures of central tendancy, dispersion or association.

4. Explore the data by vizualing it through various figures. Clearly explain or interpret each figure, justify the appropriateness of the tool used for visualization, highlight the information it provides, and, finally, explain how this information affect your further analysis.
5. Which variables of this dataset are **qualitative** and which ones are **quantitative**? Create an attribute (also called design) matrix $\mathbf{X}$ that includes only the following attributes: `Income` , `Limit` , `Rating` , `Cards` , `Age` , and `Education` .
6. Create a binary variable `Balance_1500` which equals 1 for each observation if `Balance` > 1500 for that observation and equals 0 otherwise.
7. Model `Balance_1500` by the explanatory variables mentionned in Step 5 using the following models:

   - logistic regression,
   - linear discriminant, and
   - quadratic discriminant.
8. Interpret the coefficients of `Income` , `Age` , and `Education` for the logistic regression model.
9. Find the probability of (`Balance` > 1500), for the following values, using all three aforementionned methods:

Income	Limit	Rating	Cards	Age	Education
63	8100	600	4	30	13
186	13414	950	2	41	13

   `Compare the probabilities and comment.`

10. For each method, print the confusion matrix, the accuracy score and the AUC using all observations. Compare these metrics and comment.
11. Plot the ROC Curve of the three methods on the same figure. Comment.

In [25]:
```
#1.Download Credit.csv from http://www-bcf.usc.edu/~gareth/ISL/Credit.csv
#2.Load the data into this Jupyter notebook.
path='data/'
filename = path+'Credit.csv'
Credit_data = pd.read_csv(filename)
Credit_data.head()
```

Out[25]:

	Unnamed: 0	Income	Limit	Rating	Cards	Age	Education	Gender	Student	Married	Ethnicity
0	1	14.891	3606	283	2	34	11	Male	No	Yes	Caucasian
1	2	106.025	6645	483	3	82	15	Female	Yes	Yes	Asian
2	3	104.593	7075	514	4	71	11	Male	No	No	Asian
3	4	148.924	9504	681	3	36	11	Female	No	No	Asian
4	5	55.882	4897	357	2	68	16	Male	No	Yes	Caucasian

In [26]:
```
#3.Describe the data using descriptive statistics such as measures of central te
Credit_data.describe()
```

Out[26]:

	Unnamed: 0	Income	Limit	Rating	Cards	Age	Education	
count	400.000000	400.000000	400.000000	400.000000	400.000000	400.000000	400.000000	40(
mean	200.500000	45.218885	4735.600000	354.940000	2.957500	55.667500	13.450000	52(
std	115.614301	35.244273	2308.198848	154.724143	1.371275	17.249807	3.125207	45(
min	1.000000	10.354000	855.000000	93.000000	1.000000	23.000000	5.000000	(
25%	100.750000	21.007250	3088.000000	247.250000	2.000000	41.750000	11.000000	68
50%	200.500000	33.115500	4622.500000	344.000000	3.000000	56.000000	14.000000	45(
75%	300.250000	57.470750	5872.750000	437.250000	4.000000	70.000000	16.000000	86:
max	400.000000	186.634000	13913.000000	982.000000	9.000000	98.000000	20.000000	199(

3.Describe the data using descriptive statistics such as measures of central tendancy, dispersion or association.

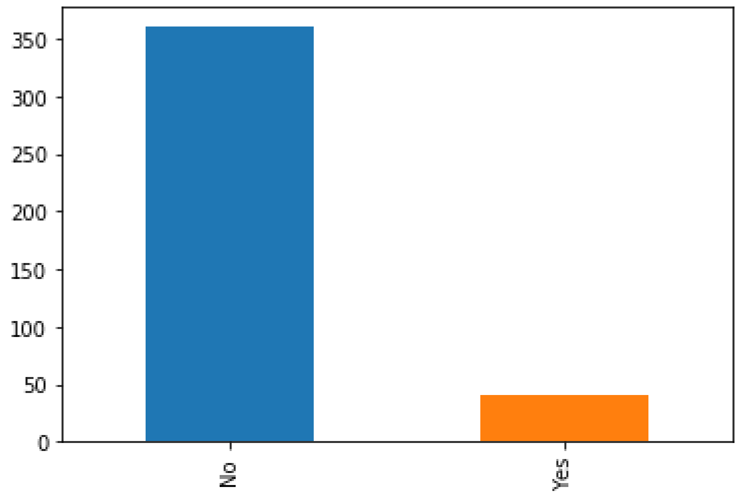
Ans:There are total 400 observations with data for Income,Limit,Rating,Cards,Age,Education,Gender,Student,Married,Ethinicity,Balance.Out of which ncome, Limit, Rating, Cards, Age, Education and Balance are continues, while Gender, Student, Married and Ethnicity are categorical.

Income:It is describing the Income in thousand og units and the range is in between 10.35 to 186.63.The mean value is 45.21 and median is 33.11. Limit:It is describing the Credit limit and its range is in between 855 to 13913. The mean and median value are 4735.60 and 4622.50 respectively. Rating:It is describing the rating of the credit card. The range of this variable is 93 to 982 and the mean and median value are 354.94 and 344 respectively. Cards:It is describing the no of cards the card holder is having. The range of this variable is 1 to 9 and the mean and median value are 2.95 & 3 respectively. Age:It is describing the Age of the card holder in unit years.The range is in between 23 to 98 and the mean and median are 55.66 & 56 Respectively. Education:It is describing the years education the card holder is having. The range for this data is in between 5 to 20 yrs and the mean and median values are 13.45 and 14 respectively. Balance:It describes the Balance in the Credit card. The range for this data is in between 0 to 1999 and the mean and median values are 520 and 459.5 respectively.

Gender: describes the gender of the card holder having 2 classes 'Male','Female'.
Student:Describes whether the card holder is astudent or not and has 2 classes 'Yes' and 'No'
Married:Describes whether the card holder is Married or not and 2 classes 'Yes' and 'No'
Ethinicity:Describes the Ethnicity of the card holder and is having 3 classes 'Caucasian','Asian','African American'

4.Explore the data by vizualing it through various figures. Clearly explain or interpret each figure, justify the appropriateness of the tool used for visualization, highlight the information it provides, and, finally, explain how this information affect your further analysis.

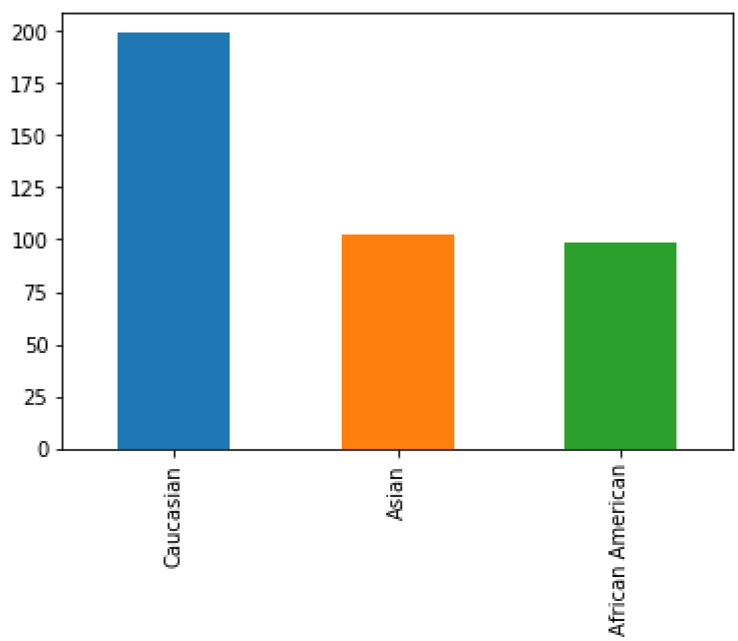
In [27]:
```
Credit_data['Student'].value_counts().plot(kind='bar');
```



A bar chart is generally used to present relative quantities for multiple categories.In the above figure we can interprete how many card holders are student or not. Simlarly we cal plot for other categorical data to show different classes of those cariables.

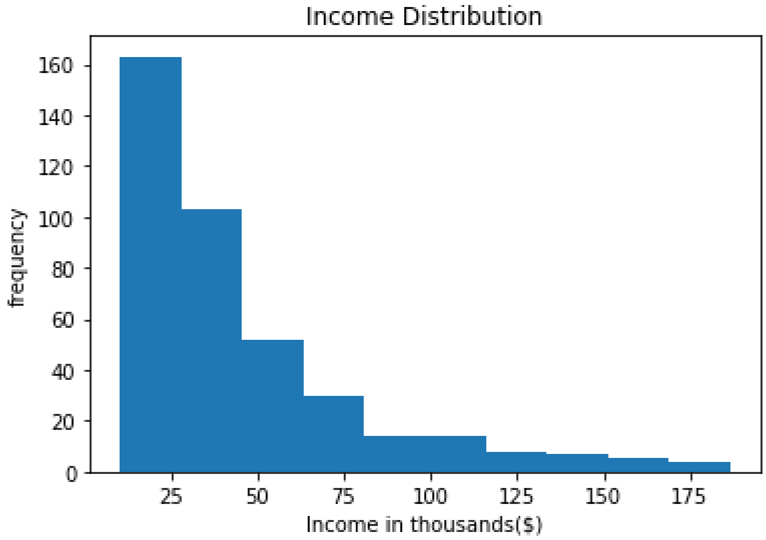
In [28]:
```
Credit_data['Ethnicity'].value_counts().plot(kind='bar')
```

Out[28]: <matplotlib.axes._subplots.AxesSubplot at 0x3522b327f0>

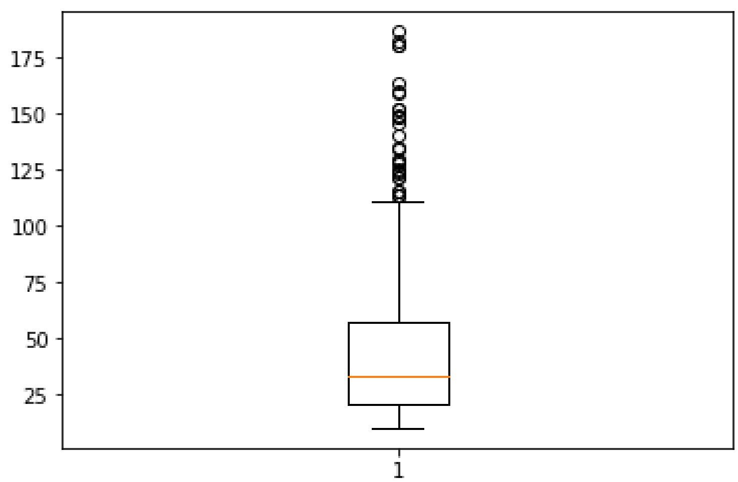


A histogram plot is generally used to summarize the distribution of a data sample. A boxplot is generally used to summarize the distribution of a data sample

In [29]:
```
plt.hist(Credit_data['Income'])
plt.title('Income Distribution')
plt.xlabel('Income in thousands($)')
plt.ylabel('frequency');
```

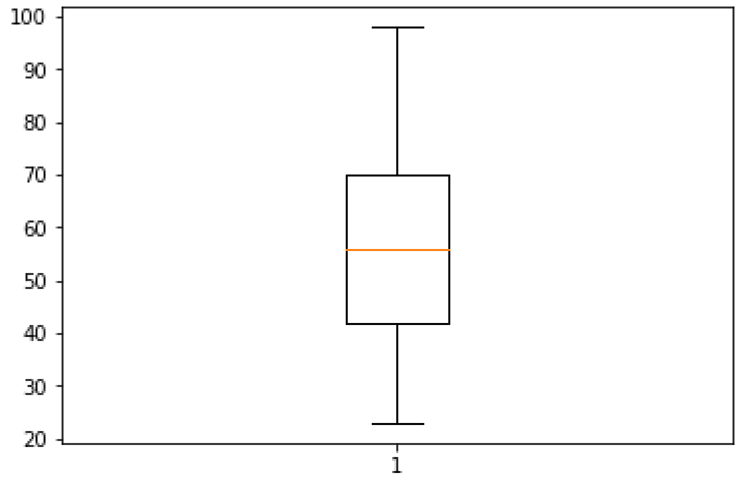


In [30]:
```
plt.boxplot(Credit_data.Income);
```

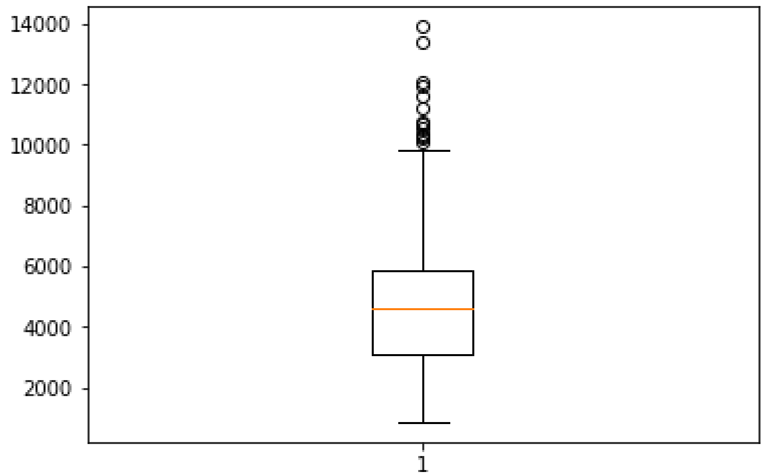


For Income data many of the datas are outlier

In [31]:
```
plt.boxplot(Credit_data.Age);
```

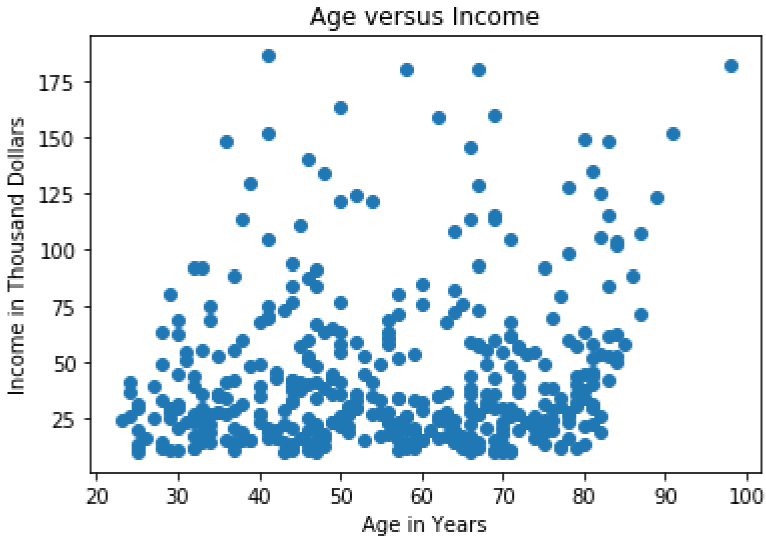


In [32]:
```
plt.boxplot(Credit_data.Limit);
```



Scatterplot matrix shows the relationship between two variables as dots in two dimensions

In [33]:
```
plt.scatter(Credit_data.Age, Credit_data.Income)
plt.title('Age versus Income')
plt.xlabel('Age in Years')
plt.ylabel('Income in Thousand Dollars');
```



5.Which variables of this dataset are qualitative and which ones are quantitative?

Ans:In this dataset 'Income','Limit','Rating','Cards','Age','Education' & 'Balance' are quantitative data.The other data as 'Gender','Student','Married','Ethinicity' are qualitative data.

In [34]:
```
#5.Create an attribute (also called design) matrix  X  that includes only the
# following attributes:Income, Limit, Rating, Cards, Age, and Education
X = Credit_data[['Income', 'Limit', 'Rating', 'Cards', 'Age', 'Education', 'Bala
X.head()
```

Out[34]:

	Income	Limit	Rating	Cards	Age	Education	Balance
0	14.891	3606	283	2	34	11	333
1	106.025	6645	483	3	82	15	903
2	104.593	7075	514	4	71	11	580
3	148.924	9504	681	3	36	11	964
4	55.882	4897	357	2	68	16	331

In [35]:
```
#6.Create a binary variable Balance_1500 which equals  1  for each observation i
# and equals  0  otherwise
X['Balance_1500'] = np.where(X['Balance']>1500, 1, 0)
X.head()
```

```
C:\Users\mana\Anaconda3\lib\site-packages\ipykernel_launcher.py:3: SettingWithC
opyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas-docs/stab
le/indexing.html#indexing-view-versus-copy (http://pandas.pydata.org/pandas-doc
s/stable/indexing.html#indexing-view-versus-copy)
  This is separate from the ipykernel package so we can avoid doing imports unt
il
```

Out[35]:

	Income	Limit	Rating	Cards	Age	Education	Balance	Balance_1500
0	14.891	3606	283	2	34	11	333	0
1	106.025	6645	483	3	82	15	903	0
2	104.593	7075	514	4	71	11	580	0
3	148.924	9504	681	3	36	11	964	0
4	55.882	4897	357	2	68	16	331	0

In [36]:
```
#7.Model Balance_1500 by the explanatory variables mentionned in Step 5 using the
X5 = X[['Income', 'Limit', 'Rating', 'Cards', 'Age', 'Education']]
y = X['Balance_1500']
#logistic regression
from sklearn.linear_model import LogisticRegression
lr = LogisticRegression()
lr.fit(X5, y)
```

```
C:\Users\mana\Anaconda3\lib\site-packages\sklearn\linear_model\logistic.py:433:
FutureWarning: Default solver will be changed to 'lbfgs' in 0.22. Specify a sol
ver to silence this warning.
  FutureWarning)
```

Out[36]:
```
LogisticRegression(C=1.0, class_weight=None, dual=False, fit_intercept=True,
          intercept_scaling=1, max_iter=100, multi_class='warn',
          n_jobs=None, penalty='l2', random_state=None, solver='warn',
          tol=0.0001, verbose=0, warm_start=False)
```

In [37]:
```
#LinearDiscriminantAnalysis
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
lda = LinearDiscriminantAnalysis()
lda.fit(X5,y)
```

Out[37]:
```
LinearDiscriminantAnalysis(n_components=None, priors=None, shrinkage=None,
              solver='svd', store_covariance=False, tol=0.0001)
```

In [38]:
```
from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis
qda = QuadraticDiscriminantAnalysis()
qda.fit(X5,y)
```

Out[38]:
```
QuadraticDiscriminantAnalysis(priors=None, reg_param=0.0,
               store_covariance=False, store_covariances=None, tol=0.0001)
```

In [39]:
```
#8.Interpret the coefficients of Income, Age, and Education for the Logistic regr
import statsmodels.formula.api as smf
lr_smf = smf.Logit.from_formula(formula = "Balance_1500~Income+Limit+Rating+Cards
                                   data= X).fit()
```

```
Optimization terminated successfully.
         Current function value: 0.040712
         Iterations 11
```

In [40]:
```
lr_smf.summary()
```

Out[40]:

Logit Regression Results

Dep. Variable:	Balance_1500	**No. Observations:**	400
Model:	Logit	**Df Residuals:**	393
Method:	MLE	**Df Model:**	6
Date:	Fri, 05 Apr 2019	**Pseudo R-squ.:**	0.6217
Time:	21:53:56	**Log-Likelihood:**	-16.285
converged:	True	**LL-Null:**	-43.046
		LLR p-value:	9.206e-10

	coef	std err	z	P>|z|	[0.025	0.975]
Intercept	-16.8495	6.048	-2.786	0.005	-28.703	-4.996
Income	-0.0840	0.041	-2.043	0.041	-0.165	-0.003
Limit	0.0019	0.004	0.520	0.603	-0.005	0.009
Rating	0.0125	0.053	0.238	0.812	-0.091	0.116
Cards	0.1227	0.416	0.295	0.768	-0.693	0.938
Age	-0.0204	0.031	-0.654	0.513	-0.082	0.041
Education	-0.0747	0.142	-0.526	0.599	-0.353	0.204

Possibly complete quasi-separation: A fraction 0.49 of observations can be perfectly predicted. This might indicate that there is complete quasi-separation. In this case some parameters will not be identified.

8.Interpretation

Ans:After analysing the coefficients of all the variable, it is observed that 'Income','Age','Education' has inverse relationship with Balance_1500. The other datas such as 'Limit', 'Rating' and 'Cards' has +ve coefficient which indicates these variables are positively corelated with Balance_1500.

9.Find the probability of (Balance >1500), for the following values, using all three aforementionned methods: Income Limit Rating Cards Age Education 63 8100 600 4 30 13 186 13414 950 2 41 13 Compare the probabilities and comment.

In [41]:
```
X_pred = np.array([63,8100,600,4,30,13,186,13414,950,2,41,13]).reshape(2,6)
print(X_pred)
```

```
[[   63  8100   600     4    30    13]
 [  186 13414   950     2    41    13]]
```

In [42]:
```
lr.predict_proba(X_pred)
y_pred_lr = lr.predict(X_pred)
```

Out[42]:
```
array([[0.90629018, 0.09370982],
       [0.17306781, 0.82693219]])
```

In [43]:
```
print(lda.predict_proba(X_pred))
```

```
[[0.94050988 0.05949012]
 [0.00721199 0.99278801]]
```

In [44]:
```
print(qda.predict_proba(X_pred))
```

```
[[9.99999995e-01 4.93626009e-09]
 [7.83057752e-04 9.99216942e-01]]
```

In [45]:
```
#10.For each method, print the confusion matrix, the accuracy score and the AUC u
#For Logistic regression
from sklearn.metrics import confusion_matrix
y_pred_lr = lr.predict(X5)
print(confusion_matrix(y, y_pred_lr))

from sklearn.metrics import roc_auc_score
log_AUC_LR = roc_auc_score(y, y_pred_lr)
print('AUC_logistic:%.3f'% log_AUC_LR)
```

```
[[390   1]
 [  6   3]]
AUC_logistic:0.665
```

In [46]:
```
#For Linear discriminant
y_pred_lda = lda.predict(X5)
print(confusion_matrix(y, y_pred_lda))
log_AUC_LDA = roc_auc_score(y, y_pred_lda)
print('AUC_LDA:%.3f'% log_AUC_LDA)
```

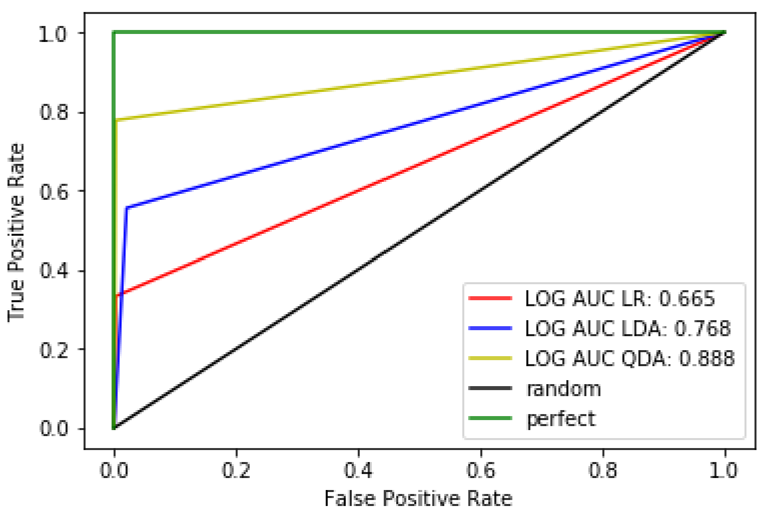
```
[[383   8]
 [  4   5]]
AUC_LDA:0.768
```

In [47]:
```
#For Quadratic discriminant
y_pred_qda = qda.predict(X5)
print(confusion_matrix(y, y_pred_qda))
log_AUC_QDA = roc_auc_score(y, y_pred_qda)
print('AUC_QDA:%.3f'% log_AUC_QDA)
```

```
[[390   1]
 [  2   7]]
AUC_QDA:0.888
```

Compring the confusion matrix and AUC value of Logistic regression and LDA and QDA, we can say that the accuracy of the QDA is better than the rest of the 2.

In [48]:
```
#11.Plot the ROC Curve of the three methods on the same figure. Comment.
#logistic Regression
from sklearn.metrics import roc_curve
log_fpr1, log_tpr1, log_thresholds1 = roc_curve(y, y_pred_lr)
log_fpr2, log_tpr2, log_thresholds2 = roc_curve(y, y_pred_lda)
log_fpr3, log_tpr3, log_thresholds3 = roc_curve(y, y_pred_qda)
plt.plot(log_fpr1, log_tpr1,'r-',label = 'LOG AUC LR: %.3f'%log_AUC_LR)
plt.plot(log_fpr2, log_tpr2,'b-',label = 'LOG AUC LDA: %.3f'%log_AUC_LDA)
plt.plot(log_fpr3, log_tpr3,'y-',label = 'LOG AUC QDA: %.3f'%log_AUC_QDA)
plt.plot([0,1],[0,1],'k-',label='random')
plt.plot([0,0,1,1],[0,1,1,1],'g-',label='perfect')
plt.legend()
plt.xlabel('False Positive Rate')
plt.ylabel('True Positive Rate');
```



Question 3 (30 points)

1. From the dataset `Credit.csv` , extract the variable `Student` and save it as `Student` .
2. Save the number of observations in `Student` as `population_size` .
3. Factorize `Student` and compute the proportion of "students" and save it as `true_p` .

Let us consider the following simple logistic regression model

$$\Pr(\text{Student} = \text{Yes}) = \frac{e^{\beta}}{1 + e^{\beta}}.$$

Here, we do not consider any predictor. The objective is to estimate β by manipulating the likelihood of the model.

4. Define a variable `sample_size` $= 100$. Now, sample `sample_size` number of observations from `Student` and call it `sample`.
5. Define a function called `likelihood` which takes one argument `beta` and computes the likelihood of `beta` based on the `sample`.
6. Randomly generate 50 numbers from the **continuous uniform** distribution $U[-5; 5]$. Save these numbers as `beta_candidate`.
7. Using the `likelihood` function defined in Step 5, compute the likelihood of `beta_candidate` and save it as `likelihood_candidate`. Plot the `likelihood_candidate` versus `beta_candidate`.
8. Based on the plot, which value of `beta_candidate` would you choose as the estimate of β? Explain why.
9. Based on the chosen `beta_candidate`, estimate the `true_p` (or $\Pr(\text{Student} = \text{Yes})$).

In [49]:
```
#1.Read dataset Credit.csv
path='data/'
filename = path+'Credit.csv'
Credit_data = pd.read_csv(filename)
Credit_data.head()
```

Out[49]:

	Unnamed: 0	Income	Limit	Rating	Cards	Age	Education	Gender	Student	Married	Ethnicity
0	1	14.891	3606	283	2	34	11	Male	No	Yes	Caucasian
1	2	106.025	6645	483	3	82	15	Female	Yes	Yes	Asian
2	3	104.593	7075	514	4	71	11	Male	No	No	Asian
3	4	148.924	9504	681	3	36	11	Female	No	No	Asian
4	5	55.882	4897	357	2	68	16	Male	No	Yes	Caucasian

In [50]:
```
#1.From the dataset Credit.csv, extract the variable Student and save it as Stude
Student = Credit_data[['Student']]
Student.head()
```

Out[50]:

	Student
0	No
1	Yes
2	No
3	No
4	No

In [51]:
```
#2.Save the number of observations in Student as population_size.
population_size = Student.size
print(population_size)
```

```
400
```

In [52]:
```
#3.Factorize Student and compute the proportion of "students" and save it as true
Student['Default'] = Student.Student.factorize()[0]
print(Student['Default'].value_counts()/population_size)
Student['true_p'] = np.where(Student['Default']== 0, 0.9, 0.1)
Student.head()
```

```
0    0.9
1    0.1
Name: Default, dtype: float64

C:\Users\mana\Anaconda3\lib\site-packages\ipykernel_launcher.py:2: SettingWithC
opyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas-docs/stab
le/indexing.html#indexing-view-versus-copy (http://pandas.pydata.org/pandas-doc
s/stable/indexing.html#indexing-view-versus-copy)

C:\Users\mana\Anaconda3\lib\site-packages\ipykernel_launcher.py:4: SettingWithC
opyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas-docs/stab
le/indexing.html#indexing-view-versus-copy (http://pandas.pydata.org/pandas-doc
s/stable/indexing.html#indexing-view-versus-copy)
  after removing the cwd from sys.path.
```

Out[52]:

	Student	Default	true_p
0	No	0	0.9
1	Yes	1	0.1
2	No	0	0.9
3	No	0	0.9
4	No	0	0.9

In [53]:
```
#4.Define a variable sample_size  =100 . Now, sample sample_size number of obser
np.random.seed(123)
sample_size  =100
sample = np.random.choice(a= Student['true_p'], size = sample_size)
```

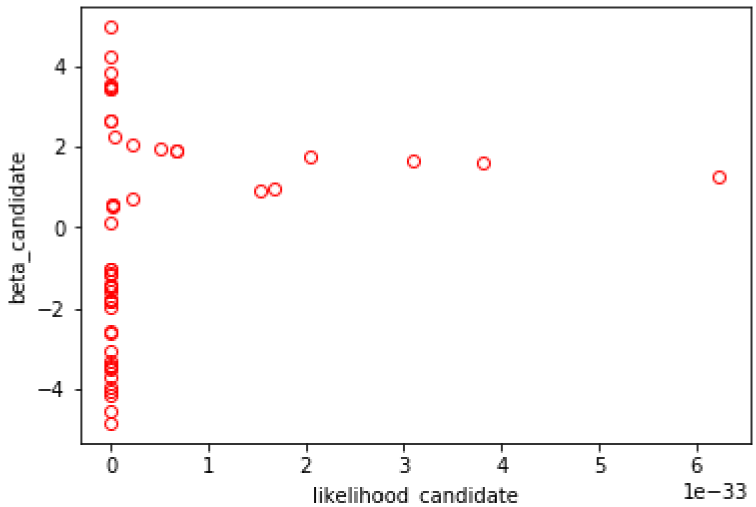
In [54]:
```
#5.Define a function called likelihood which takes one argument beta and computes

def likelihood(beta):
    result = np.array([])
    for i in sample:
        value =  math.exp(beta)/(1+math.exp(beta))
        value1 = value ** i
        value2 = (1-value1) ** (1-i)
        result = np.append(result,value1 * value2)
    #print(result)
    final_result = np.prod(result)
    return final_result
```

In [55]:
```
#6.Randomly generate  50  numbers from the continuous uniform distribution  U[-5
import scipy.stats  as stats
beta_candidate = stats.uniform.rvs(size=50,  # Generate 50 numbers
                                 loc = -5,  # From -5
                                 scale= 10) # To 5
print(beta_candidate)
```

```
[ 1.74689051  3.42342438 -4.16805012  2.63682841 -2.56333625 -3.05777039
  0.72456957 -4.04287483  3.85326826  1.27248972  2.23416358 -4.83870793
  0.94431879  0.56785192 -3.41040356 -3.46929485  1.95529529 -1.81233574
  1.91970296  0.5438325  -1.11049426  4.2513249   3.41669997 -1.42602433
 -4.56408536 -1.95231927 -1.01814318  2.0495883   4.95358482 -1.44085134
  2.62547814  0.93176917  1.91701799 -3.48872548 -1.01123707 -2.59144102
 -1.56543986  0.13128154  1.6662455  -3.94091515 -3.69105049 -1.78019394
  1.61564337  3.46506225  0.53257345  3.54452488 -1.15162189 -1.83212103
 -1.45735324 -3.28918171]
```

In [56]:
```
#7.Using the likelihood function defined in Step 5, compute the likelihood of bet
likelihood_candidate = np.array([])
for i in beta_candidate:
    likelihood_candidate = np.append(likelihood_candidate,likelihood(i))
```

In [57]:
```
#8.Plot the likelihood_candidate versus beta_candidate.
import matplotlib.pyplot as plt
plt.plot(likelihood_candidate, beta_candidate, 'or', mfc='none')
plt.xlabel('likelihood_candidate')
plt.ylabel('beta_candidate');
```



8.Based on the plot, which value of beta_candidate would you choose as the estimate of β ? Explain why.

Ans:beta_candidate =

In [58]:
```
#9.Based on the chosen beta_candidate, estimate the true_p (or  Pr(Student=Yes)
```