Bead on a Rotating Hoop

By

Roshni Sahoo
Email: roshni.sahoo@xaviers.edu.in
UID: 192131

Apoorva Bhatia
Email: apoorva.bhatia@xaviers.edu.in
UID: 192216

Nomya Mahana
Email: nomya.mahana@xaviers.edu.in
UID: 192138

September 2021

Department of Physics

St. Xavier's College (Autonomous), Mumbai, India – 400001

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Abstract

The bead on a rotating hoop is one of the typical problems solved in Classical Mechanics. It is a simple yet dynamic system. In this project we take into consideration a bead with fixed mass that is free to move on a vertical hoop rotating with some angular velocity. The Lagrangian for this system is obtained using the kinetic and potential energy terms. These values are put in the Lagrange's equation and the ODE is solved numerically using Python. We even analyze the trajectory of the bead for different values of angular velocity and the initial angle that it forms with the vertical. Points of stable and unstable equilibrium have also been examined. The study is further extended for the case where the hoop is tilted at some angle with respect to the vertical. For all the cases friction has been neglected.

Keywords: Lagrange, mechanics, classical mechanics, rotating hoop, non-inertial frame of reference, Lagrangian

1 Introduction

The system containing a bead on a rotating hoop can be easily solved using Lagrangian mechanics. This is a traditional problem that has been reported in many standard references in Classical mechanics in the context of Lagrangian mechanics [1]. The Lagrangian formulation has two important advantages over the earlier Newtonian formulation. First, Lagrange's equations, unlike Newton's, take the same form in any coordinate system. Second, in treating constrained systems, much like this one, the Lagrangian approach eliminates the forces of constraint (such as the normal force of the wire, which constrains the bead to remain on the wire). This greatly simplifies most problems. Lagrangian equation for any constrained system is given by the difference of kinetic and potential energy. By plugging in these values in the Lagrange equation in this case a second order ODE is obtained which can be numerically solved using Python.

A more complicated system can be imagined. If the hoop is tilted, i.e., its rotational axis does not coincide with the vertical axis, the Potential Energy of the system changes. Hence, Lagrangian of such a system change but since it's Lagrangian, we can solve it similarly. For both the cases a number of parameters like the angular velocity of the hoop, angle subtended by the bead with respect to the vertical and tilt of the hoop can be varied to find equilibrium points and to effectively analyze this mechanical motion.

The report is organized as follows. In Section 2 , the theoretical framework of the system in which a bead placed on a rotating hoop (both vertical and tilted) is laid out. The Lagrangian for both the systems is written and the ordinary differential equation in θ (the generalized coordinate) is obtained. This ODE is then solved using numerical methods for which the code is provided in the Appendix. The program has been written in Python and the module Scipy has been used for solving the ODE. In Section 3, the θ $vs\ time$ graphs are obtained. These graphs essentially tell us how the angle that the bead subtends with the axis of rotation of the hoop varies with time. Furthermore, the trajectories of the bead in three-dimensional space are also presented for both vertical and tilted rotating hoop. While analyzing the motion of the bead, we specifically look at the initial angles for which the bead would remain at rest w.r.t to the hoop (i.e., it will not slide along the hoop). Additionally, we also vary the angular velocity of the hoop while keeping all other initial conditions constant to see how the motion of the bead changes. Alternatively, the angular velocity of the hoop is kept constant and the initial angle is varied. Completing the analysis of the motion of the bead by various certain parameters of the hoop allows one to fully understand this system.

Significant amount of research has been done in classical mechanics for the bead on a rotating hoop. In this study, the ordinary differential equation obtained for the system of a bead on a rotating hoop on substituting its Lagrangian in the Lagrange equations is solved using numerical methods. However, this differential equation is solved analytically using Lie symmetries [2]. Another similar approach using symmetries and elementary calculus to solve this system theoretically has been proposed by Dutta & Ray [3]. Another interesting paper by Johnson & Rabchuk [4] describes the bead on a rotating hoop as a ponderomotive particle trap. Such a trap is a time-varying and spatially inhomogeneous field that produces a restoring force on a particle and pushes it towards a minimum in the field amplitude.

So far, the system was such that the rotating hoop was vertical. Rotating hoops which are tilted have also been studied by several authors. Both Lee, S [5] and Dutta, S., & Ray, S. [6] have investigated the bead-hoop system where the hoop is tilted with respect to the horizontal axis. The observed bifurcations at the equilibrium points of the bead as well as non-linear oscillations in the bead’s motion. Bifurcation is an interesting phenomenon in dynamical systems where the system has stable and unstable equilibrium points. The bead-on-a-hoop is such a system which has several points of equilibrium while not all of them stable.

2 Theoretical Framework

2.1 Bead on a vertical rotating hoop

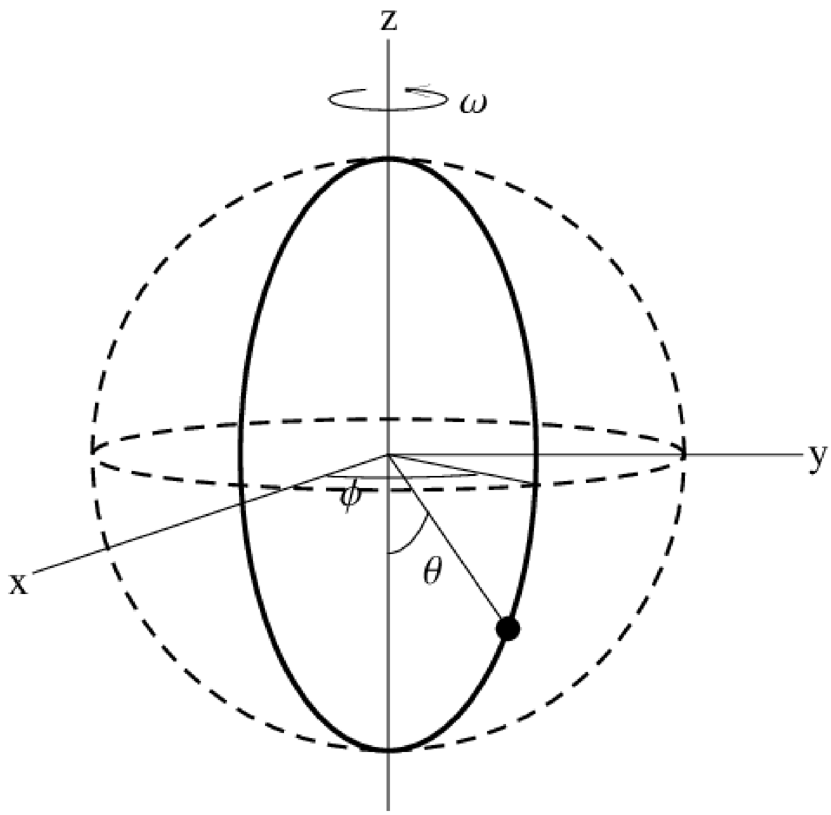


Figure 1. Bead on a vertical rotating hoop

Consider the above system where the bead is rotating about the vertical z-axis at an angular velocity ω.

$$\omega = \dot{\phi}$$

The degree of freedom of this system is 1 as there is only one generalized co-ordinate θ which is the angle subtended by the bead with the vertical axis of the rotating hoop. To obtain the equation of motion of the bead (i.e., $\theta(t)$), we make use of Lagrangian mechanics. Lagrangian mechanics makes solving such an equation extremely easy.

The Lagrangian of the system is given by L = T - V, where T is its kinetic energy and V is its potential energy.

The only source of potential energy for this system comes from the external gravitation force, which acts vertically downwards, and whose magnitude (after taking the bottom of the hoop to be V = 0) is given by:

$$V = mgh = mg(1 - cos\theta)$$

The kinetic energy equals:

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m(v_t^2 + v_n^2) = \frac{1}{2}mR^2(\dot{\theta}^2 + \omega^2 \sin^2\theta),$$

where R is the radius of the hoop, v_t, v_n are the tangential and normal components of the velocity respectively (relative to the hoop).

Thus, we have

$$L = T - V = \frac{1}{2}mR^2(\dot{\theta}^2 + \omega^2 sin^2\theta) - mg(1 - cos\theta)$$

The Euler-Lagrange equation is as follows:

$$\frac{\partial L}{\partial \theta} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right)$$

We have,

$\frac{\partial L}{\partial \theta} = mR^2\omega^2 sin\theta cos\theta - mgRsin\theta$ and $\frac{\partial L}{\partial \dot{\theta}} = mR^2\dot{\theta}$.

Thus, the Euler-Lagrange equation yields:

$$mR^2\omega^2 sin\theta cos\theta - mgRsin\theta = mR^2\ddot{\theta} \Rightarrow \ddot{\theta} = \left(\omega^2 cos\theta - \frac{g}{R}\right) sin\theta$$

Having found the equation of motion for this system in terms of its generalized coordinates, let us discuss its equilibrium positions.

An equilibrium position is a point such that, if the bead is at rest at that point, it remains at rest. Clearly, a necessary condition for this is,

$\ddot{\theta} = 0$.

Thus, we set $\left(\omega^2 cos\theta - \frac{g}{R}\right) sin\theta = 0$.

The second factor is zero when $\theta = 0, \pi$. Thus, the top and bottom of the hoop are equilibrium positions.

The first factor is zero when $cos\,\theta = \frac{g}{\omega^2 R} \Rightarrow \omega^2 \geq \frac{g}{R}$, since $|\cos\theta| \leq 1$. The equilibrium positions are then given by $\theta = \pm \arccos\frac{g}{\omega^2 R}$.

Thus, when the hoop is rotating fast enough, we have two more equilibrium positions.

It is of interest to check which equilibrium positions happen to be *stable* ones, i.e., which ones are such that, if the bead is given a slight nudge, it moves back to the equilibrium position. How shall we check for this?

First, consider the **equilibrium at $\boldsymbol{\theta = 0}$.**

$\theta \sim 0 \Rightarrow cos\theta \sim 1, sin\theta \sim 0; \ddot{\theta} = \left(\omega^2 - \frac{g}{R}\right)\theta$. Now, if $\omega^2 \geq \frac{g}{R}$ and we give the bead a slight nudge to the right, we have $\theta > 0, \ddot{\theta} < 0$; the bead will thus fall back into place. This happens similarly if we nudge the bead to the left. Equilibrium is stable.

However, if $\omega^2 \leq \frac{g}{R}$, the signs of a small angular displacement and angular acceleration will be the same; therefore, it will accelerate away from the bottom upon being nudged, and equilibrium is unstable.

Next, consider **the equilibrium at θ=π.**

A qualitative argument suffices to show that it is unstable. Both gravitational force as well as centrifugal force will be acting downwards. There is no "incentive" for a nudged bead to move back to the top, and it would continue to accelerate away. Thus, equilibrium is unstable.

In case $\omega^2 \geq \frac{g}{R}$, we had two more equilibrium positions. Recall that the equation of motion is given by $\ddot{\theta} = \left(\omega^2 cos\theta - \frac{g}{R}\right) sin\theta$. These equilibrium positions were such that one of them lay between 0 and $\frac{\pi}{2}$, and the other was its additive inverse. Consider the first case. Upon giving the bead a slight nudge to the right, we have either the parenthetical term becoming negative (since cosine is a decreasing function), while the second factor remains positive. The displacement and acceleration being at odds with each other with regards to their signs, we conclude that the bead will fall back into place. A similar argument will establish the same result for a slight nudge to the left, as well as for the other equilibrium position.

Here is another figure that might give a better view of a bead rotating on a vertical hoop.

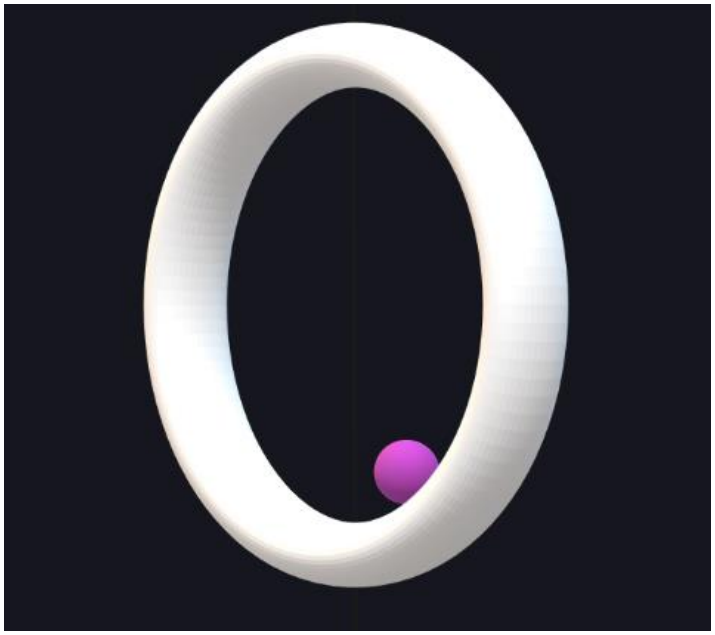


Figure 2. Bead on a rotating vertical hoop 3-D visualization

2.2 Bead on a tilted rotating hoop

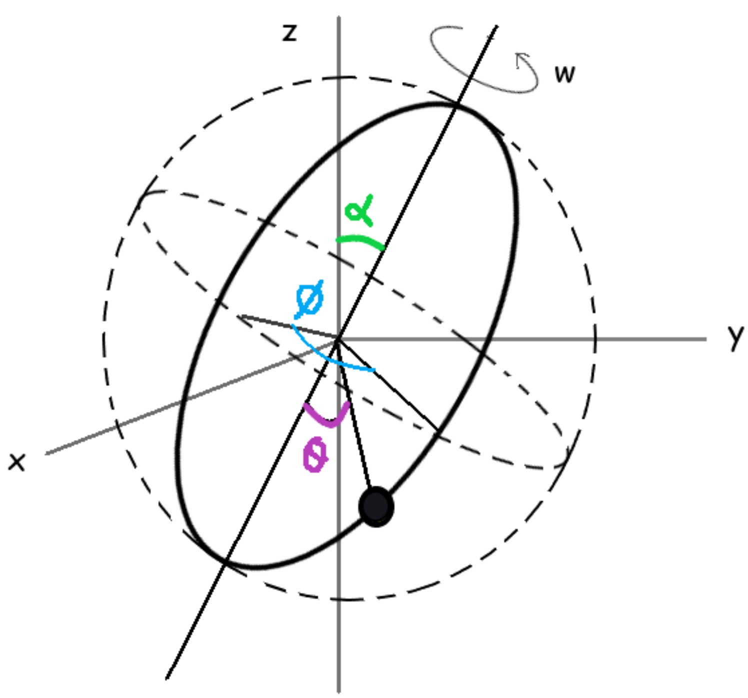


Figure 3. Bead on a tilted rotating hoop.

In this system, the rotating hoop's vertical axis is tilted w.r.t to the vertical axis of the non-inertial frame of reference. Note that, θ here is angle subtended by the bead with the axis of rotation of the hoop. In the previous system also the angle θ was w.r.t to the axis of the rotation of the hoop, however, that axis happened to coincide with the vertical axis of the non-inertial frame of reference. In this case, however, it doesn't.

The analytic for this situation is broadly similar to the previous one. There is no change in the expression for the total kinetic energy of the system. Hence,

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m(v_t^2 + v_n^2) = \frac{1}{2}mR^2\left(\dot{\theta}^2 + \omega^2 \sin^2\theta\right)$$

However, the potential energy is now given by**,**

$$V = -mgRcos(\theta - \alpha).$$

Therefore, the Lagrangian is,

$$L = \frac{1}{2}mR^2\left((\dot{\theta^2}) \ + \omega^2 si\,n^2\,\theta\right) + mgRcos(\theta - \alpha).$$

Plugging this into the Euler-Lagrange equation yields

$$\ddot{\theta} = \omega^2 sin\theta cos\theta - \frac{gsin(\theta-\alpha)}{R}.$$

This second order differential equation is now solved numerically as its analytical solution is not straightforward. Further, the trajectories of the bead for different values of α and initial angle θ_o are obtained.

The bead will remain at rest w.r.t the hoop when

$$\dot{\theta} \ = \ 0$$

$$\Rightarrow \ \ddot{\theta} = \ 0$$

$$0 \ = \ \omega^2 sin\theta cos\theta - \frac{gsin(\theta - \alpha)}{R}$$

$$\Rightarrow \omega^2 sin\theta cos\theta \ = \ \frac{gsin(\theta - \alpha)}{R}$$

$$\frac{sin(\theta - \alpha)}{\sin 2\theta} \ = \ \frac{2\mathrm{g}}{\mathrm{R}\,\omega^2}$$

The above equation needs to be solved to find the angles for which the bead would remain at rest w.r.t the hoop. However, solving for θ in the above equation is not straightforward and is hence, omitted in this study.

A 3-D visualization of the bead on a tilted rotating hoop. See how the hoop is slightly tilted with respect to the z-axis.

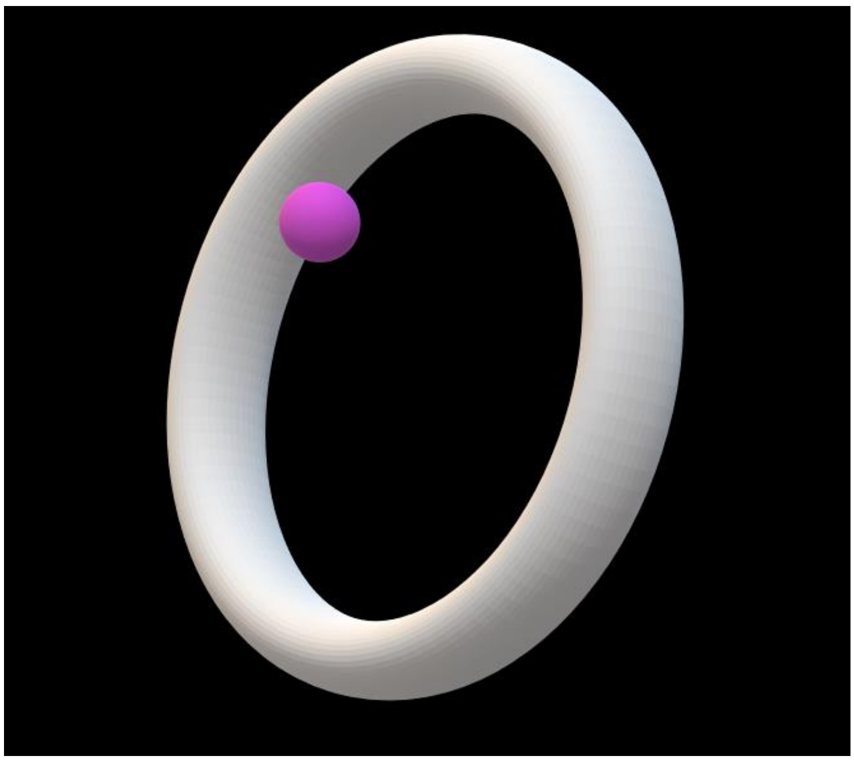


Figure 4. Bead on a tilted rotating hoop. 3D visualization

3 Results & Discussion

3.1 Bead on a vertical rotating hoop

The bead on a vertical rotating hoop is a classic example solved in many textbooks on classical mechanics. We analyze this system at its stable and unstable equilibrium points to and plot the motion of the bead at various other initial angles as well.
Note that, the radius of the rotating hoop is taken as = 5 cm

Analysis of equilibrium points
From Section 2 we have learnt that the bead is at equilibrium when $\boldsymbol{\theta_o}$ **= 0 or** $\boldsymbol{\pi}$ irrespective of the angular velocity.

Angular velocity of the hoop = $20\ rad/s$

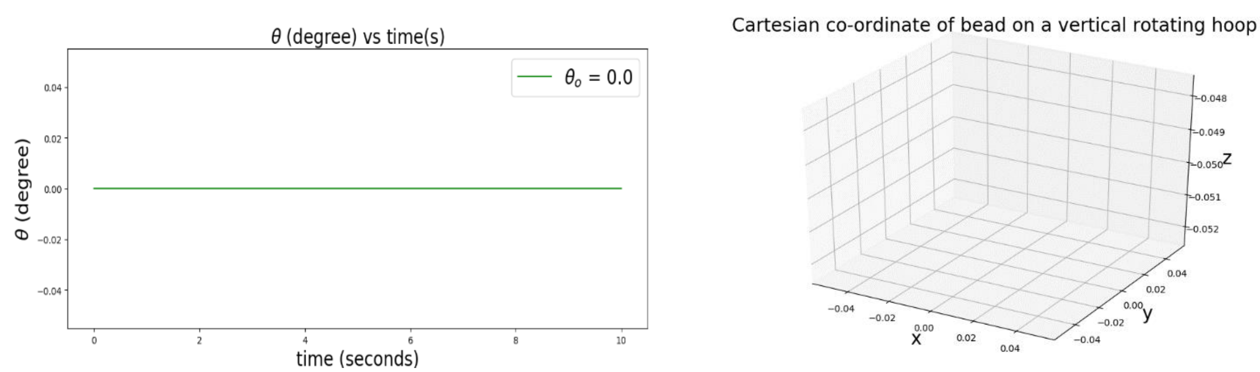


Figure 5. ***Initial angle*** $\boldsymbol{\theta_0 = 0°}$**.**
$\theta\ vs\ time\ graph\ (left)\ and\ trajectory\ of\ the\ bead\ for\ 10\ seconds\ (right)$

Clearly the bead remains at rest when it is placed on the rotating hoop (with 0 initial velocity) at an angle of 0 degrees with the vertical. The important question is whether the bead is in a stable equilibrium or not. To be in a stable equilibrium means that, if the angle of the bead is increased or decreased by a small amount, it will immediately try to come back to the rest position. In this scenario, the bead performs small oscillations about the rest position. However, there are specific conditions for the angular velocity of the hoop for which $\boldsymbol{\theta_o}$ **= 0** will be a stable equilibrium point.

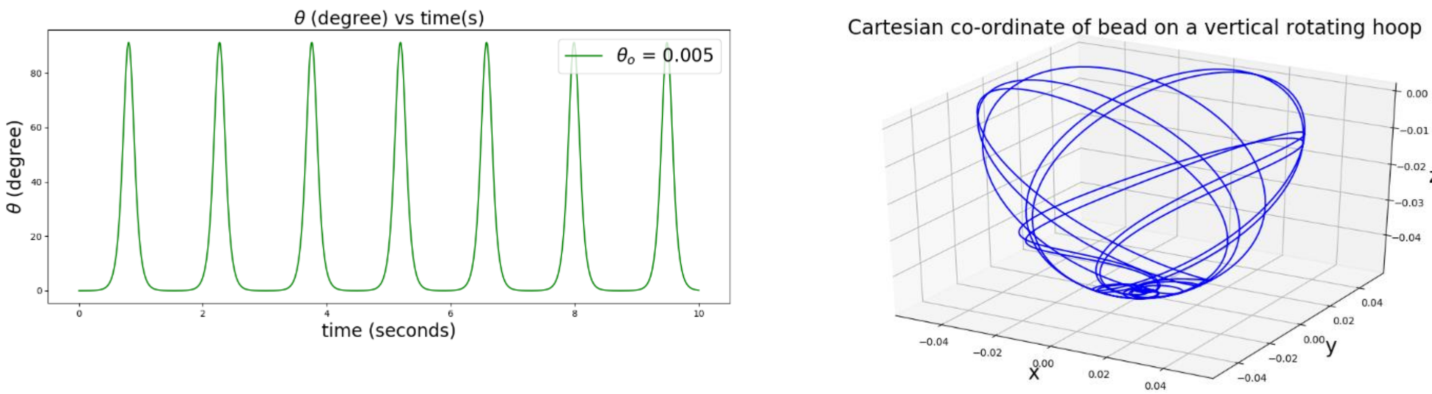


Figure 6. ***Initial angle*** $\boldsymbol{\theta_0 = 0.005°}$***.***
$\theta\ vs\ time\ graph\ (left)\ and\ trajectory\ of\ the\ bead\ for\ 10\ seconds\ (right).$

In the previous figure, we can see that the bead is clearly not in a stable equilibrium as it is making large oscillations about $\boldsymbol{\theta_o = 0}$**.** The angular velocity in this case was 20 rad/sec. If we reduce the angular velocity to 10 rad/s, we observe that the bead is in a stable equilibrium.

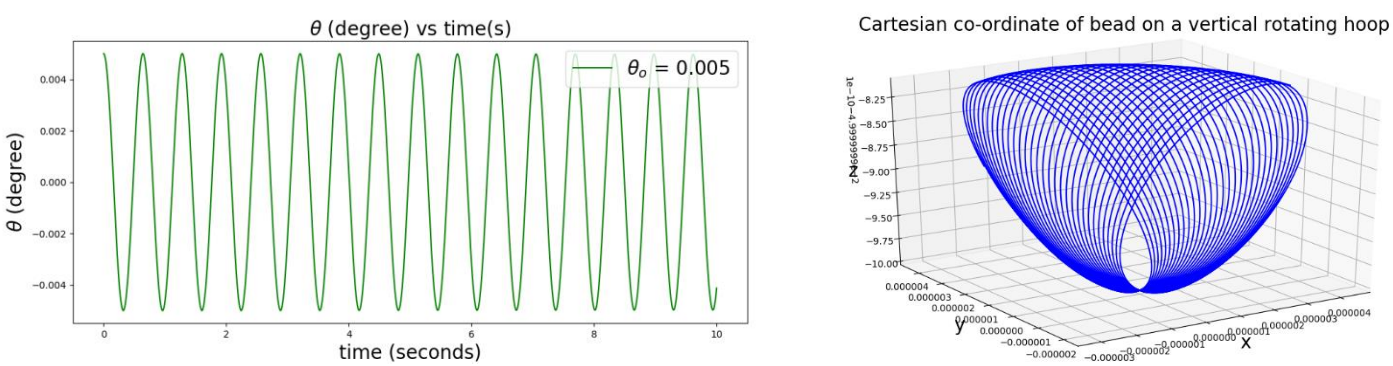


Figure 7. ***Initial angle*** $\boldsymbol{\theta_0 = 0.005°}$***.***
$\theta\ vs\ time\ graph\ (left)\ and\ trajectory\ of\ the\ bead\ for\ 10\ seconds\ (right).$

Here, we can clearly see that the bead is in a stable equilibrium as θ is varying sinusoidally with time. This means that the bead is oscillating sinusoidally about $\theta = 0$ while the hoop is rotating.

Now, let's consider a random angle of the θ between 0 and 90 and observe the motion of the bead.

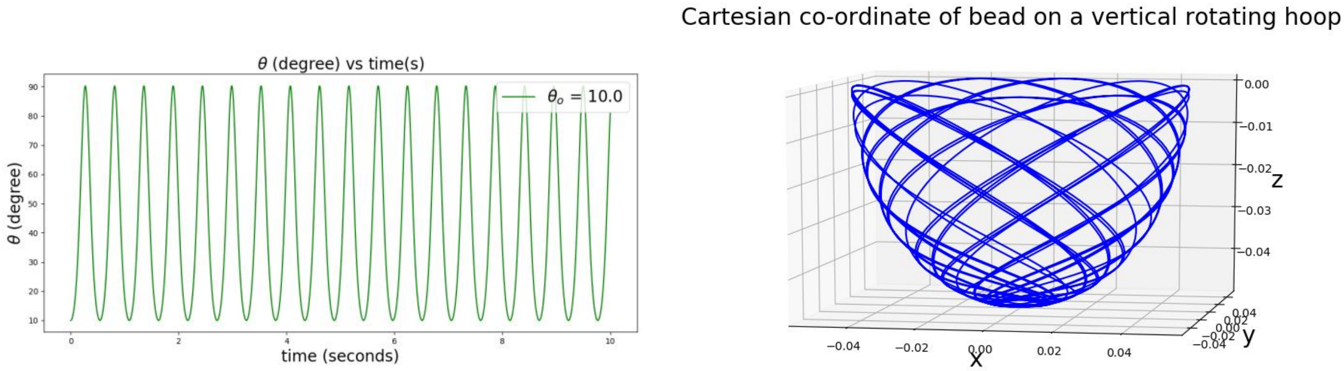


Figure 8. ***Initial angle*** $\boldsymbol{\theta_0 = 10°}$.
θ *vs time graph* (*left*) *and trajectory of the bead for* 10 *seconds* (*right*).

Here, we can clearly see that the bead is not at rest and is moving freely along the circumference of the rotating hoop. The angular velocity of the hoop here is 20 rad/s.

Let's now consider the next equilibrium point, $\boldsymbol{\theta = \pi}$.

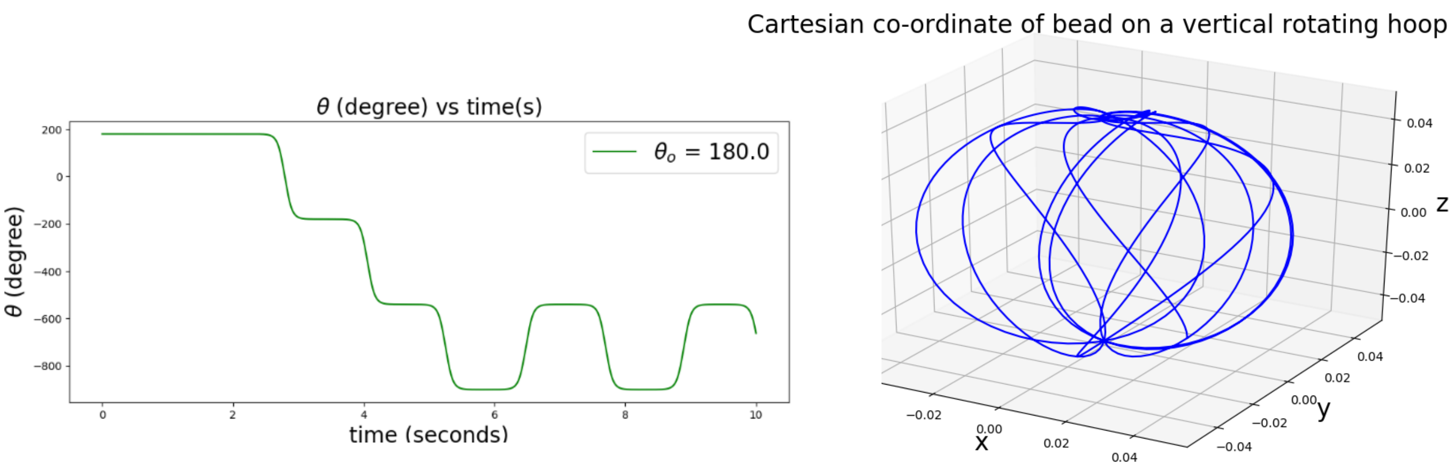


Figure 9. ***Initial angle*** $\boldsymbol{\theta_0 = 10°}$.
θ *vs time graph* (*left*) *and trajectory of the bead for* 10 *seconds* (*right*).

The bead at this angle is highly unstable for any given value of the angular velocity and this we see that the bead is unable to remain at rest in this position.

From Section 2, we know that when $\omega^2 R > g$, we get 2 more equilibrium points.

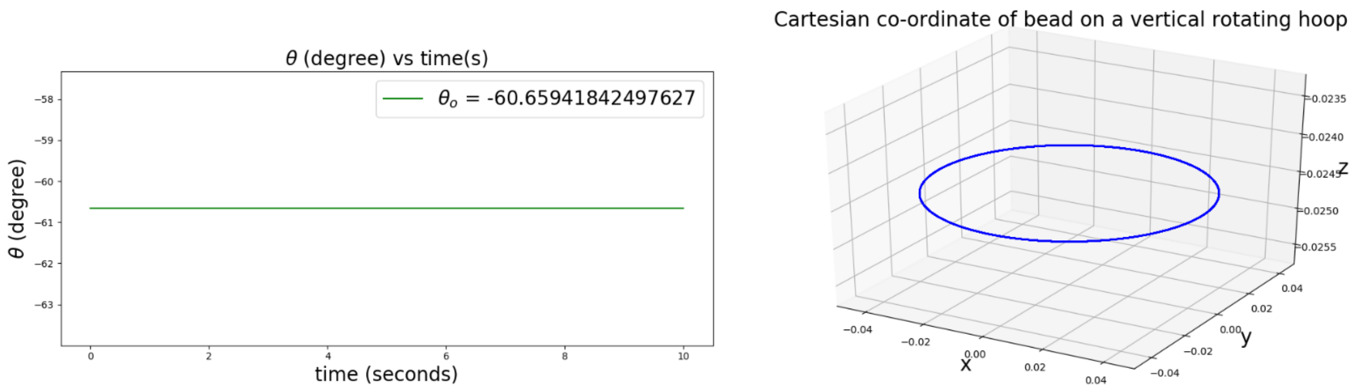


Figure 10. ***Initial angle $\boldsymbol{\theta_0 = -60.66°}$.***
θ *vs time graph (left) and trajectory of the bead for* 10 *seconds (right).*

This is indeed true; we see that the bead remains at rest and follows a circular path as the hoop rotates. This point also happens to be a stable equilibrium point. If we disturb the initial angle by 0.005 degrees, we will observe small oscillations about $\theta_o = -60.66°$. What this implies is that the bead tries to come back to its initial position and hence it oscillates about it.

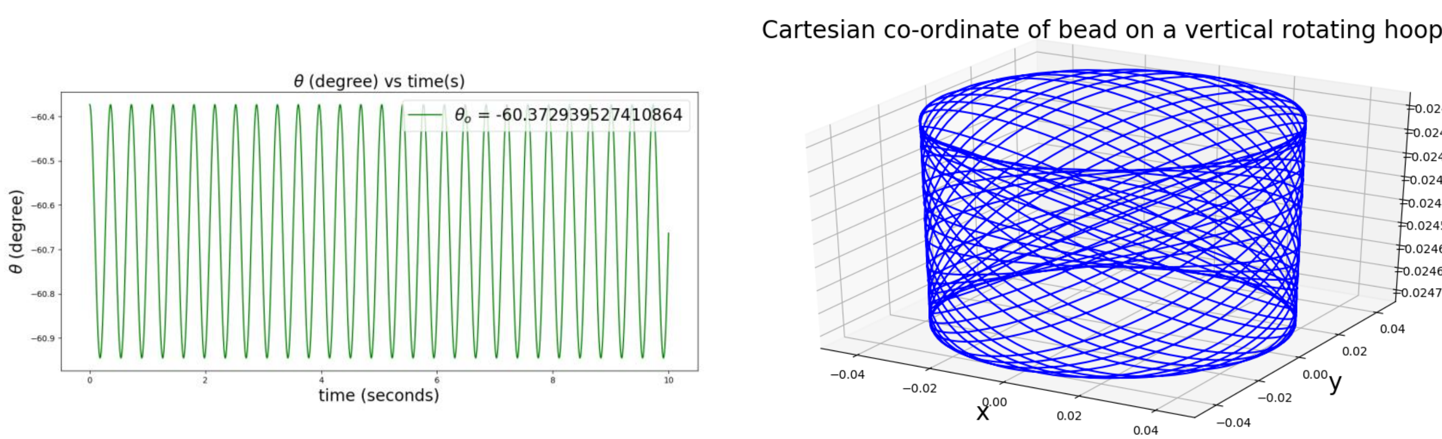


Figure 11. ***Initial angle $\boldsymbol{\theta_0 = -60.37°}$.***
θ *vs time graph (left) and trajectory of the bead for* 10 *seconds (right).*

Now that we have analyzed the points of equilibrium of the bead, let's look at how the motion of the bead changes with the angular velocity of the rotating hoop.

Trajectory of bead at varying angular velocities of hoop

The radius of the hoop = 5 cm

The angular velocity of the hoop is varied from 20 rad/s to 40 rad/s. The initial angle and velocity of the bead are kept same for both cases.

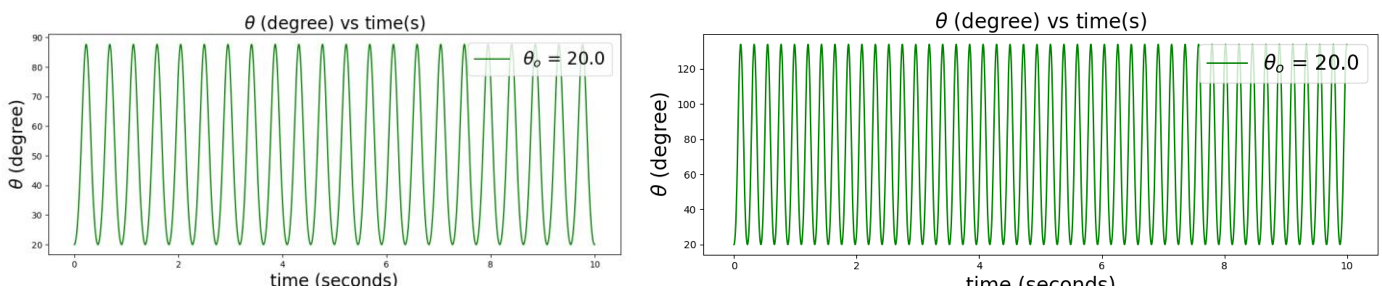


Figure 12. θ vs time graphs of bead with initial angle $\theta_o = 20°$.
Angular velocity $\omega = 20\ rad/s$ (left) and $\omega = 40\ rad/s$ (right)

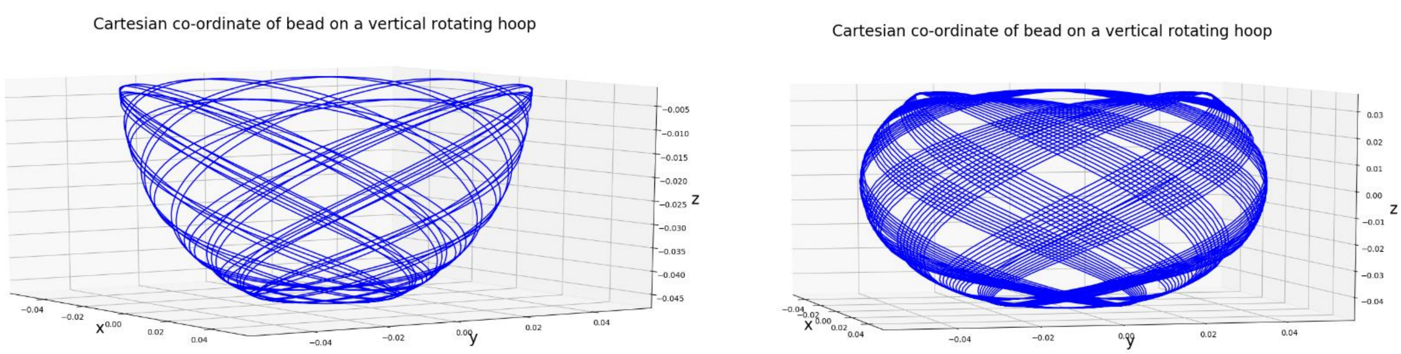


Figure 13. $Trajectory$ of bead with initial angle $\theta_o = 20°$.
Angular velocity $\omega = 20\ rad/s$ (left) and $\omega = 40\ rad/s$ (right)

When the angular velocity of the hoop is increased from 20 rad/s to 40 rad/s, it is observed that the oscillation of θ becomes very rapid (not that both systems run for 10 seconds only) . It is also observed in Figure 12 that for the hoop that is rotating more rapidly, the maximum amplitude of the oscillations of θ is greater than that of the hoop that is rotating slower. In addition to that, the frequency of oscillations of θ for the faster hoop is also larger. These reasons account for the change in the appearance of the trajectories of bead in both cases. It is observed that for the hoop rotating faster, the trails of the bead are denser and form a globular shape where as for the hoop rotating at 20 rad/s, the trails left by the bead are less dense and the shape resembles a paraboloid.

Trajectory of bead at varying initial angle of bead

Now, we would like to observe the motion of the bead at varying initial angles of the bead while the angular velocity of the hoop remains the same.

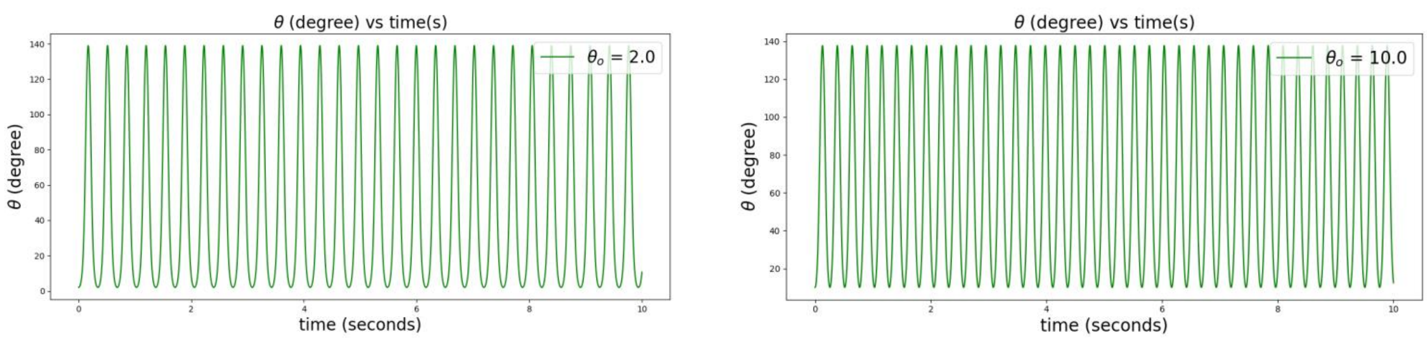


Figure 14. θ *vs time graphs of bead with angular velocity* $\omega = 40$ *rad/s.*
Initial angle $\theta_o = 2°$ ***(left) and*** $\theta_o = 10°$ ***(right)***

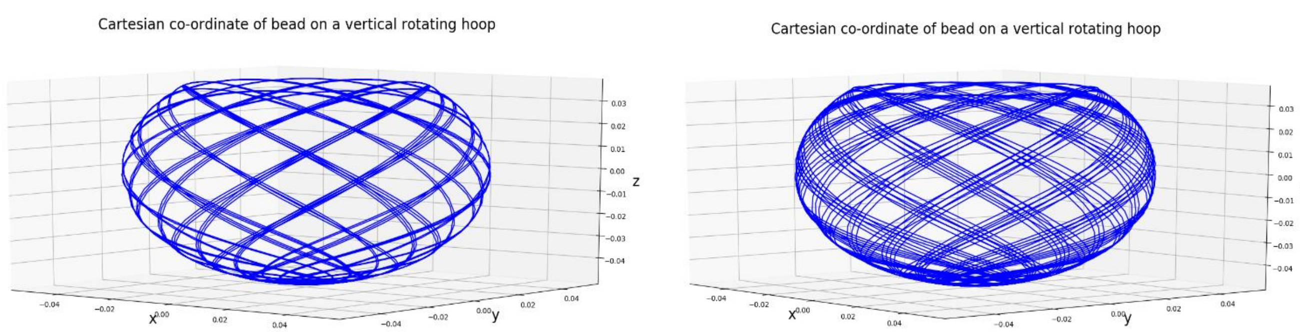


Figure 15. . Trajectories of bead with angular velocity $\omega = 40$ *rad/s.*
Initial angle $\theta_o = 2°$ ***(left) and*** $\theta_o = 10°$ ***(right)***

On increasing the initial angle of at which the bead is placed on the rotating hoop, the frequency of oscillations of theta increases which results in results in a larger number of densely packed trails left by the bead (Figure 15). So, although the geometric pattern formed by the trajectory of the beads looks more or less similar in both the figures in Figure 15, the one on the right has larger number of lines.

Hence, all our theoretical observations are indeed true and treatment of the bead on a vertical rotating hoop is now complete.

3.2 Bead on tilted rotating hoop

Bead rotating on a tilted hoop is not a problem that is commonly solved in many textbooks; thus, the results of this simulation can be rather unexpected.

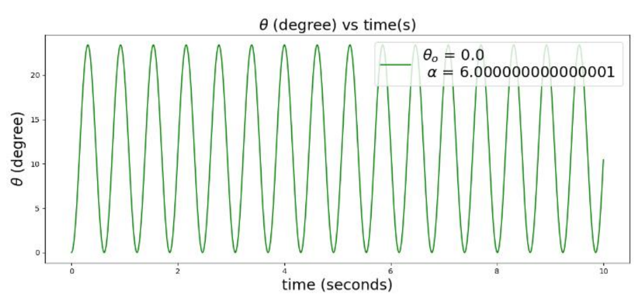
Varying initial conditions of the system (θ_o & α)

Let us keep in mind the follow initial conditions for the system:

Radius of rotating hoop = <u>0.05 m</u>

Angular velocity = <u>10 rad/s</u>

Tilt of the rotating hoop w.r.t the vertical axis <u>α = 6°</u>



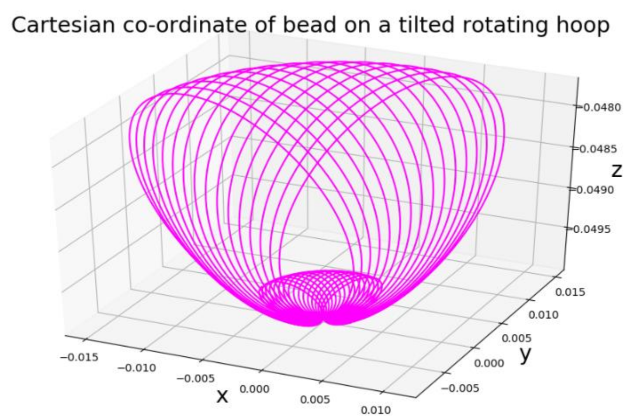


Figure 16. ***Initial angle*** $\boldsymbol{\theta_0 = 0°,\ \alpha = 6°}$

θ *vs time graph (left) and trajectory of the bead for 10 seconds (right)*

Clearly, when the rotating hoop is tilted by an angle of 6°, the bead doesn't remain at rest when it is placed on the hoop at initial angle of 0° and initial velocity 0 rad/s. This is because 0° is not a stable equilibrium point for the bead to remain at rest on the hoop.

It is however, very important that we check if the Lagrangian for this new system is correct. We can do so by substituting $\alpha = 0°$. When we do that, we see that the Lagrangian resembles that of the system where the rotating hoop was vertical instead of tilted at an angle.

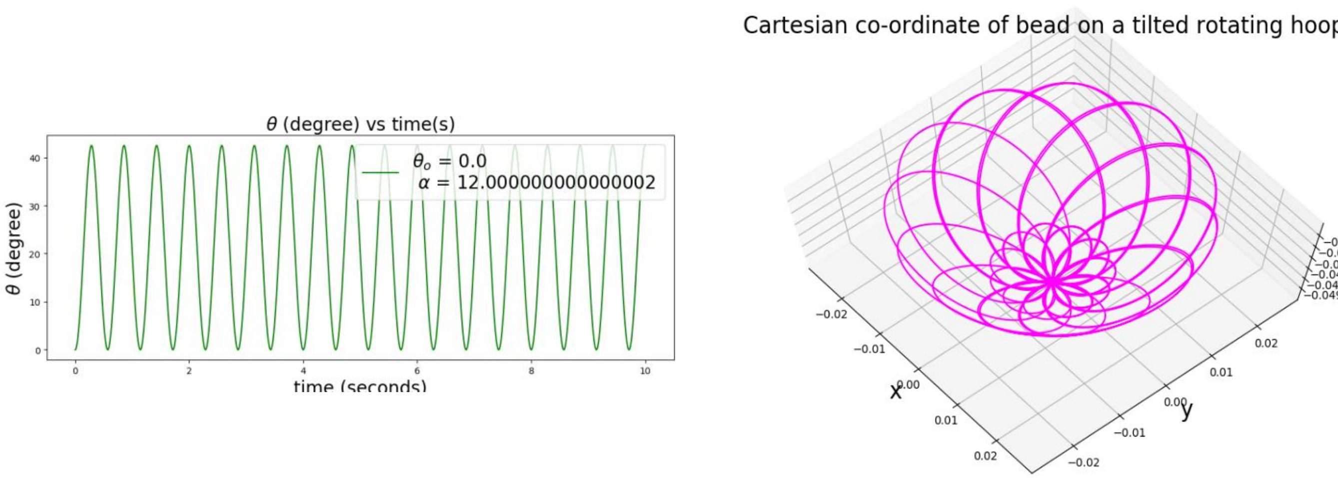


Figure 17. . ***Initial angle*** $\boldsymbol{\theta_0 = 0°,\ \alpha = 12°}$
$\theta\ vs\ time\ graph\ (left)\ and\ trajectory\ of\ the\ bead\ for\ 10\ seconds\ (right)$

On increasing $\alpha = 12°$, we see a rather interesting geometric pattern forming due to the trajectory of the bead.

On further increasing θ_o to 12°, we see that the trajectory of the beat becomes more intricate. Finding the angles at which the bead would remain at rest is not as straightforward and hence has been omitted in our study for the time being.

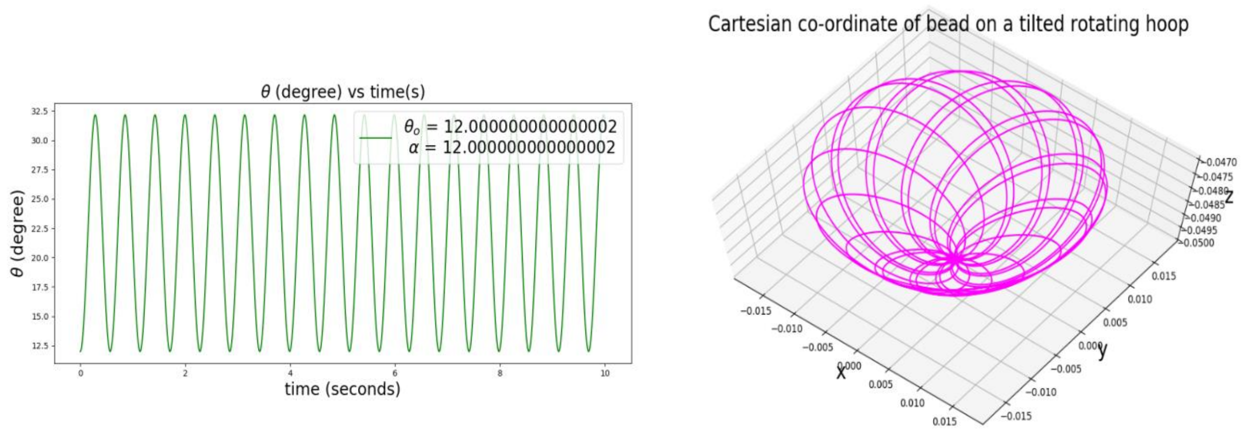


Figure 18. ***Initial angle*** $\boldsymbol{\theta_0 = 12°}$, $\boldsymbol{\alpha = 12°}$
θ *vs time graph (left) and trajectory of the bead for* 10 *seconds (right)*

Trajectory of the bead at varying angular velocity of hoop

The angular velocity of the hoop is increased from 20 rad/s to 40 rad/s while the tilt of the axis of rotation (w.r.t to the z-axis) and the initial angle θ_o is kept constant.

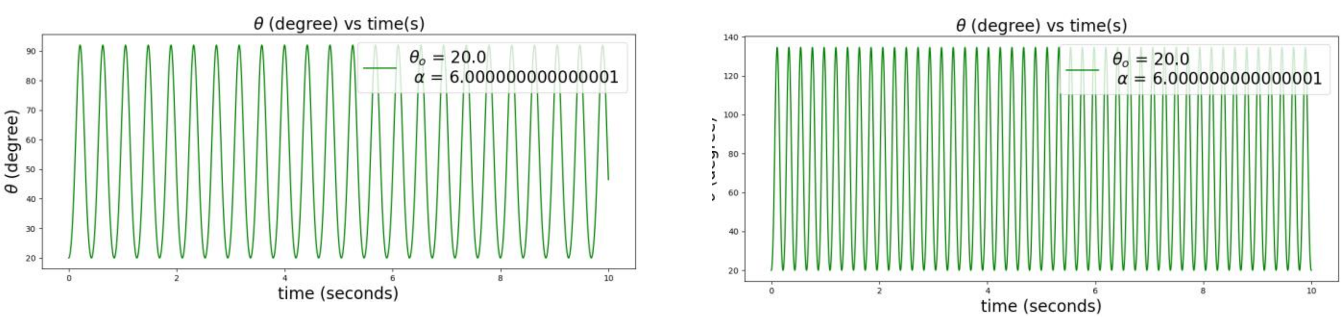
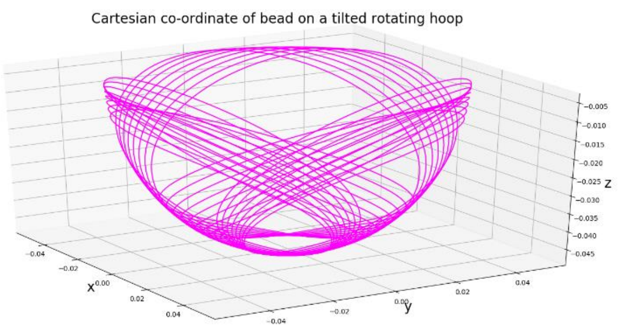


Figure 19. θ *vs time*
Angular velocity of the hoop $\boldsymbol{\omega = 20}$ ***rad/s (left) and 40 rad/s (right)***

On increasing the angular velocity of the hoop, the frequency of oscillations of the bead along the hoop increases this results in a trajectory of the bead to form denser lines. Further, the increased angular velocity of the hoop also results in the bead to reach a higher point on the hoop as it can be seen that the maximum angle of the bead when $\omega = 40$ rad/s is 140 °.



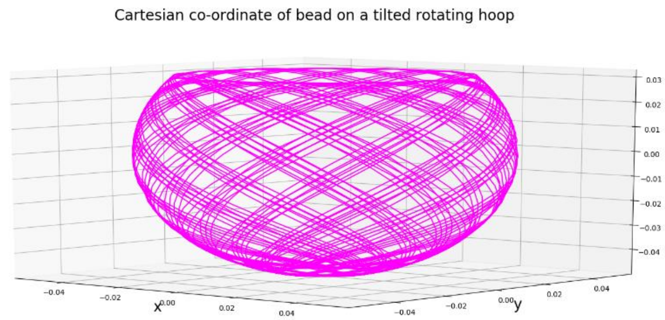


Figure 20. Trajectory of the bead.
angular velocity of the hoop $\boldsymbol{\omega} = 20$ ***rad/s (left) and 40 rad/s (right)***

In this study, the bead is assumed to be a point particle, however, a much more realistic analysis would be if the bead was considered to be a spherical body. This has been done by Raviola *et al* [7] where they observed resonant behavior that has not been reported before.

4 Applications

Methods used in Lagrangian mechanics can be applied to various areas of physics to easily find solutions.

1. Lagrangian mechanics can be used in geometrical optics to successfully trace the path followed by the light rays. This is done by solving the Euler Lagrange equations.
2. Lagrangian mechanics can be formulated in special relativity and general relativity as well. Although the Lagrangian is altered and is not simply the difference between the kinetic and potential energies, the Euler Lagrange equation holds for every case.
3. Lagrangian is used in path integral calculations in quantum field theory.
4. A rather physical example where Lagrangian mechanics comes in handy is of automobile and rocket engineering wherein it is used to achieve optimal control status of the different dissipative systems employed.

5 Conclusion

In this report, the traditional problem of a bead on a vertical rotating hoop has been investigated. The equation of motion of the bead has been solved numerically using Python and the trajectory of the bead for various initial angles has been plotted. Furthermore, the equilibrium points of the bead have been analyzed theoretically and then verified numerically. Furthermore, whether the equilibrium points are stable or unstable has also been examined. In addition to this, a small modification to the vertical rotating hoop has been made, where the axis of the hoop has been slightly tilted with respect to the vertical axis of the non-inertial reference frame. An attempt has been made to analyze the equilibrium points of the bead in such a system. Furthermore, the trajectory of the bead for various initial angles of the bead, tilt of the hoop axis and angular velocity of the hoop has been examined.

6 Acknowledgement

We thank the Department of Physics, St. Xavier's College for their continued support and guidance in the completion of this project. We would like to extend our thanks to our Classical Mechanics course professor, Dr. Radhekrishna Dubey for this guidance and important observations about our project.

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Appendix: Source Code

Bead on a vertical rotating hoop

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint,ode

#mass of bead
m = 0.01 #kg

#angular velocity of rotating hoop
w = 20 #rads-1

#radius of the rotating hoop
R = 0.05 #m

#gravitational constant
g = 9.8 #ms-2

def bead_on_hoop(x,t,w,R):
    theta,z = x
    thetadot = z
    zdot = np.sin(theta)*((w**2)*np.cos(theta) - (g/R))
    return (thetadot, zdot)

#initial condition: theta(radian), thetadot(radian/s)
x0 = [45*(np.pi/180),0]
#x0 = [np.arccos(g/(R*w**2)),0]

#time coordinate to solve the ODE for: from 0 to 60 seconds
t = np.linspace(0,10,3000)

#solving the ODE
sol = odeint(bead_on_hoop,x0,t,args=(w,R))

# plot the angle as a function of time
fig= plt.figure(figsize=(12,4))
ax = plt.axes()
ax.plot(t, sol[:, 0]*(180/np.pi), 'g',label='$\\theta_o$ = {}'.format(x0[0
]*(180/np.pi)))   #unpacking theta
ax.set_title('$\\theta$ (degree) vs time(s)',fontsize='20')
ax.set_xlabel("time (seconds)",fontsize='20')
ax.set_ylabel("$\\theta$ (degree)", fontsize = '20')
```

```
ax.legend(loc='upper right',fontsize = '20')

#3-D plotting
from mpl_toolkits.mplot3d import Axes3D

theta = sol[:,0]        #in radian
phi = w*t                #wt in radian

X = R*np.cos(phi)*np.sin(theta)
Y = R*np.sin(phi)*np.sin(theta)
Z = -R*np.cos(theta)
fig= plt.figure(figsize=(12,4))
ax = plt.axes(projection ='3d')
p = ax.plot3D(X,Y,Z,c='b')
ax.set_title('Cartesian co-
ordinate of bead on a vertical rotating hoop',fontsize='20')
ax.set_xlabel("x",fontsize='20')
ax.set_ylabel("y", fontsize = '20')
ax.set_zlabel("z", fontsize = '20')
```

Bead on a rotating tilted hoop

```
# -*- coding: utf-8 -*-
"""
Created on Tue Sep 28 12:52:11 2021

@author: roshni
"""

import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint,ode


#mass of bead
m = 0.01 #kg

#angular velocity of rotating hoop
w = 20 #rads-1

#radius of the rotating hoop
R = 0.05 #m
```

```
#gravitational constant
g = 9.8 #ms-2

#angle that hoop makes with vertical
alpha = 2*(np.pi/180)  #radian

def bead_on_tilted_hoop(x,t,w,R,a):
    theta,z = x
    thetadot = z
    zdot = np.sin(theta)*(((w**2)*np.cos(theta)) - (g*np.cos(alpha)/R)) +
((g*np.cos(theta)*np.sin(alpha))/R)
    return (thetadot, zdot)

#initial condition: theta(radian), thetadot(radian/s)
x0 = [0*(np.pi/180),0]
#x0 = [np.arccos(g/(R*w**2)),0]

#time coordinate to solve the ODE for: from 0 to 60 seconds
t = np.linspace(0,10,3000)

#solving the ODE
sol = odeint(bead_on_tilted_hoop,x0,t,args=(w,R,alpha))

# plot the angle as a function of time
fig= plt.figure(figsize=(12,4))
ax = plt.axes()
ax.plot(t, sol[:, 0]*(180/np.pi), 'g', label="theta")   #unpacking theta
ax.set_title('theta (degree) vs time(s)',fontsize='20')
ax.set_xlabel("time (seconds)",fontsize='20')
ax.set_ylabel("theta vs time", fontsize = '20')
#ax.set_ylim(77.0,77.9)

#3-D plotting
from mpl_toolkits.mplot3d import Axes3D

theta = sol[:,0]        #in radian
phi = w*t                #wt in radian

X = R*np.cos(phi)*np.sin(theta-alpha)
Y = R*np.sin(phi)*np.sin(theta-alpha)
Z = -R*np.cos(theta-alpha)
fig= plt.figure(figsize=(12,4))
ax = plt.axes(projection ='3d')
p = ax.plot3D(X,Y,Z,c='b')
```

```
ax.set_title('Cartesian co-
ordinate of bead on a tilted rotating hoop',fontsize='20')
ax.set_xlabel("x",fontsize='20')
ax.set_ylabel("y", fontsize = '20')
ax.set_zlabel("z", fontsize = '20')
```