Project Report

Electrodynamics SPHY0602

Simulating EM Waves in a lossy dielectric medium

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Abstract

The finite difference time domain method is by far the simplest and most effective method to study the evolution of electric and magnetic fields with time. The method relies on replacing partial derivatives with their central differences. The FDTD method is a powerful tool that finds applications in the biomedical imaging, wireless communication devices, antennas, radar signature technology, waveguides, etc. In this project, we have explored the FDTD method in 1-D to simulate electromagnetic waves in different materials with absorbing boundary conditions.

Keywords - FDTD, electromagnetics, gaussian pulse, dielectrics, conductivity, lossy dielectrics, Maxwell's equations

Contents

1 Introduction

An electromagnetic wave propagates through a medium with a speed equal to or less than the speed of light. James Clerk Maxwell mathematically modelled the electric and magnetic fields arising from distributions of electric charges and currents, and how those fields change in time. He gave forth 4 such equations which are called Maxwell's equations. In this project, we aim at using Maxwell's equations and simulating electric field and magnetic field, as they pass through different mediums with respect to time. We see cases of the EM waves travelling through a vacuum, then entering a dielectric medium. The next case we look at is EM waves travelling through a medium with a loss term associated with it, specified by the medium's conductivity. To perfectly simulate all these cases, we use Finite-Difference Time-Domain (FDTD) method, also called Yee's method, which is a numerical analysis technique used for modelling computational electrodynamics. For propagation in a dielectric medium, two sources were taken into consideration as the pulse, a Gaussian wave and a Sinusoidal wave. The wave first travels in a vacuum and then after a certain point of time enters the dielectric medium with a dielectric constant of 4. The movement is observed at different time stamps. For the case of lossy dielectric medium, a sinusoidal wave is taken as the pulse and its movement is observed as it travels from a vacuum to a dielectric medium with conductivity. The trends are observed for two different dielectric mediums with the same conductivity. The last analysis is of the pulse travelling into a metal wall.

2 Theory

2.1 Maxwell's Equations

Maxwell's equations for time-varying electric and magnetic fields are as follows,

$$\epsilon \frac{\partial E}{\partial t} = \Delta \times H - J \tag{1}$$

$$J = \sigma . E$$

where ϵ is the permittivity and σ is the conductivity of the material.

$$\frac{\partial H}{\partial t} = -\frac{1}{\mu_o} \Delta \times E \tag{2}$$

where μ_o is the permeability of free space. Maxwell's equations essentially tells us that a time-varying electric field induces a magnetic field and a time-varying magnetic field induces an electric field.

2.2 Finite Difference Time Domain Method

The simplest way of solving Maxwell's equation in the time-domain is using the method of Finite Difference Time Domain Method. As the name suggests, the method relies on the usage of finite differences to estimate field values at any point in space with the passage of time.

In this method, the continuous space-time is discretized and hence the partial derivative are approximated by finite differences. This discretization of space-time allows us to implement the FDTD method on a computer.

In this report, we are interested in working with the 1D FDTD method. So, let's say that we have an electromagnetic wave with mutually perpendicular electric and magnetic fields. Suppose, the electric field is oriented in the x-direction (E_x) and the magnetic field is oriented in the y-direction (H_y). In that case, we can compute the curls in Equation 1 and Equation 2 to get,

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\epsilon_r \epsilon_o}\frac{\partial H_y}{\partial z} - \frac{\sigma}{\epsilon_r \epsilon_o} E_x$$

Before we process, we must implement a change of variables with the help of Gaussian units. This is done so that both electric and magnetic fields have the same order of magnitude.

$$\tilde{E} = \sqrt{\frac{\epsilon_o}{\mu_o}} E$$

$$\frac{\partial \tilde{E}_x}{\partial t} = -\frac{1}{\epsilon_r \sqrt{\epsilon_o \mu_o}}\frac{\partial H_y}{\partial z} - \frac{\sigma}{\epsilon_r \epsilon_o} \tilde{E}_x \tag{3}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\sqrt{\epsilon_o \mu_o}}\frac{\partial \tilde{E}_x}{\partial z} \tag{4}$$

2.2.1 Central Difference

Forward Difference

$$\frac{\partial f}{\partial x} = \frac{f(x+h) - f(x)}{\Delta h}$$

Backward Difference

$$\frac{\partial f}{\partial x} = \frac{f(x) - f(x-h)}{\Delta h}$$

Central Difference

$$\frac{\partial f}{\partial x} = \frac{f(x+h/2) - f(x-h/2)}{\Delta h}$$

2.2.2 Python Implementation of FDTD

In order to implement the FDTD on a computer, we must replaces the derivatives in Equation 3 and 4 by their central differences.

$$\frac{\tilde{E}_x^{\,n+1/2}(k) - \tilde{E}_x^{\,n-1/2}(k)}{\Delta t} = -\frac{1}{\epsilon_r\sqrt{\epsilon_o\mu_o}}\left[\frac{H_y^n(k+1/2) - H_y^n(k-1/2)}{\Delta x}\right] - \frac{\sigma}{\epsilon_r\epsilon_o}\frac{\tilde{E}_x^{\,n+1/2}(k) + \tilde{E}_x^{\,n-1/2}(k)}{2}$$

Rearranging the terms in the above equation,

$$\tilde{E}_x^{\,n+1/2}(k) = \left[\frac{1-\frac{\sigma\Delta t}{2\epsilon_r\epsilon_o}}{1+\frac{\sigma\Delta t}{2\epsilon_r\epsilon_o}}\right]\tilde{E}_x^{\,n-1/2}(k) - \frac{0.5}{\epsilon_r\left[1+\frac{\sigma\Delta t}{2\epsilon_r\epsilon_o}\right]}[H_y^n(k+1/2) - H_y^n(k-1/2)] \tag{5}$$

Similarly, replacing the derivatives in Equation 4 with central differences, we get

$$H_y^{n+1/2}(k) = H_y^{n-1/2}(k) - 0.5[\tilde{E}_x^{\,n}(k+1/2) - \tilde{E}_x^{\,n}(k-1/2)] \tag{6}$$

Note that,

$$\frac{1}{\sqrt{\epsilon_o\epsilon_r}}.\frac{\Delta t}{\Delta x} = \frac{1}{2}$$

This relationship is carefully chosen such that the electromagnetic wave doesn't travel faster than light.

Now, what remains to be done is implementing Equation 5 and 6 in the programming language Python which is what we have used for this project. These transformed equations which will be used for the writing the program are known as the 'update' equations because they update the value of electric and magnetic fields in space with the passage of time.

$$e_x[k] = \left[\frac{1-eaf}{1+eaf}\right]e_x[k] - \frac{0.5}{\epsilon_r(1+eaf)}(h_y[k+1] - h_y[k-1]) \tag{7}$$

$$h_y[k] = h_y[k] - 0.5\,(e_x[k+1] - e_x[k-1]) \tag{8}$$

where,

$$eaf = \frac{\sigma \Delta t}{2\epsilon_r \epsilon_o}$$

3 Computational Experimental Details

3.1 Algorithm

1. Derive the update equations for a lossy dielectric medium (using Yee's method) and from Maxwell's equation for changing electric and magnetic fields
2. Create arrays for electric and magnetic fields
3. Put all the elements Ex and Hz array as 0
4. Take tmax = 1000 and t = 0
5. Update Hz and Ex array using the update equations
6. Take a sinusoidal source function
7. Apply the absorbing boundary conditions
8. Repeat for tmax iterations
9. Plot the electric and magnetic fields as a function of space for appropriate time stamps
10. Analyse results

3.2 Code

```
import numpy as np
import matplotlib.pyplot as plt

#array for electric and magnetic fields
ez = np.zeros(200)
hy = np.zeros(200)
```

```
#total iterations of time
tmax = 1000
#arrays
ez_1 = np.zeros(tmax)
ez_197 = np.zeros(tmax)
#step size
dx = 0.01
dt = dx/(2*3*10**8)
f = 700*(10**6)
#initialising
t = 0
#iterating
while t<tmax:
    pulse= np.sin(2*np.pi*f*dt*t)  #gaussian source
    ez[5] = pulse #putting gaussian source in the middle of sample space

    #update equations
    for p in range(100,len(ez)-1):
        sigma = 0.04
        e_r = 4
        e_0 = 8.5*(10**-12)
        eaf = (sigma*dt)/(2*e_r*e_0)
        ca = (1-eaf)/(1+eaf)
        cb = (0.5)/(e_r*(1+eaf))
        ez[p] = (ca*ez[p] ) - (cb*(hy[p] - hy[p-1]))

    for p in range(1,100):
        e_r = 1
        ez[p] = ez[p] - ((0.5/e_r)*(hy[p] - hy[p-1]))

    #absorbing boundary conditions
    ez_1[t] = ez[1]
    ez_197[t] = ez[197]
```

```
if t>2 or t==2:
    ez[0] = ez_1[t-2]
    ez[198] = ez_197[t-2]

for m in range(len(hy)-2):
    hy[m] = hy[m] - (0.5*(ez[m+1] - ez[m]))

t += 1          #iterating time

#plotting
def constant_function(x):
    return np.full(x.shape, 100)
if t==100 or t==250 or t==300 or t==500:
    p = np.linspace(0,199,200)

    fig, ax = plt.subplots(2)

    ax[0].plot(p,ez,label='T = {}'.format(t))
    ax[0].set_title('EM wave propagation in space')
    ax[0].plot(constant_function(p),p,'--',color='black',label='Dielectric
        Medium')
    ax[0].plot(-constant_function(p),p,'--',color='black')
    ax[0].set_ylabel('$E_{z}$')
    ax[0].set_xlabel('FDTD Cells')
    ax[0].set_xlim(0,199)
    ax[0].set_ylim(-1.5,1.5)
    ax[0].legend()

    ax[1].plot(p,hy,label='T = {}'.format(t))
    ax[1].plot(constant_function(p),p,'--',color='black',label='Dielectric
        Medium')
    ax[1].plot(-constant_function(p),p,'--',color='black')
    ax[1].set_ylabel('$H_{y}$')
```

```
ax[1].set_xlabel('FDTD Cells')
ax[1].set_ylim(-1.5,1.5)
ax[1].set_xlim(0,199)
ax[1].legend()
```

4 Results & Discussion

4.1 Propagation in Dielectric Medium

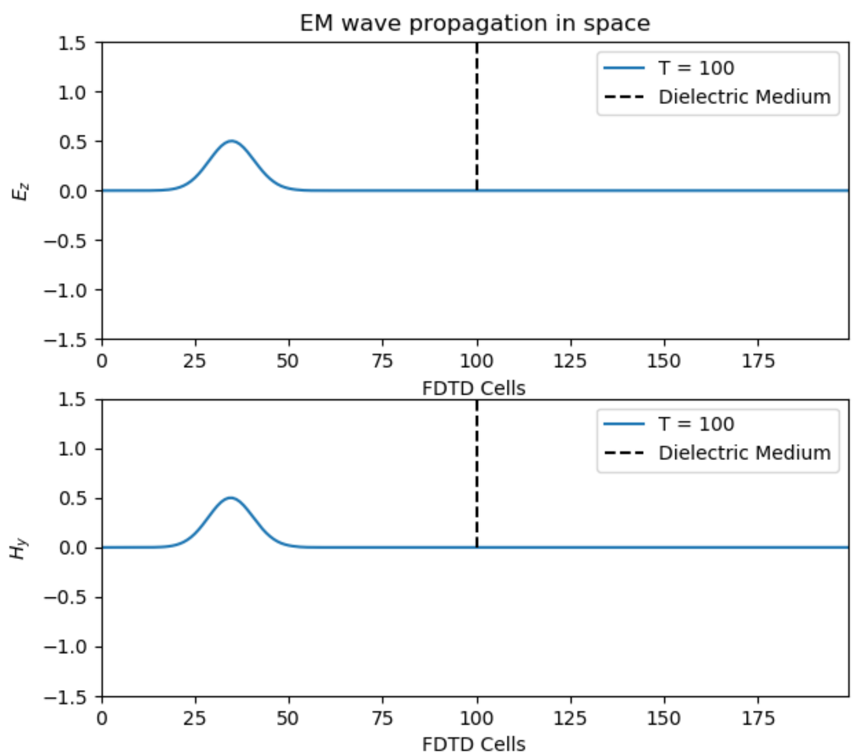

Figure 1: Propagation of Gaussian pulse into dielectric medium of $\epsilon_r = 4$ at t=100

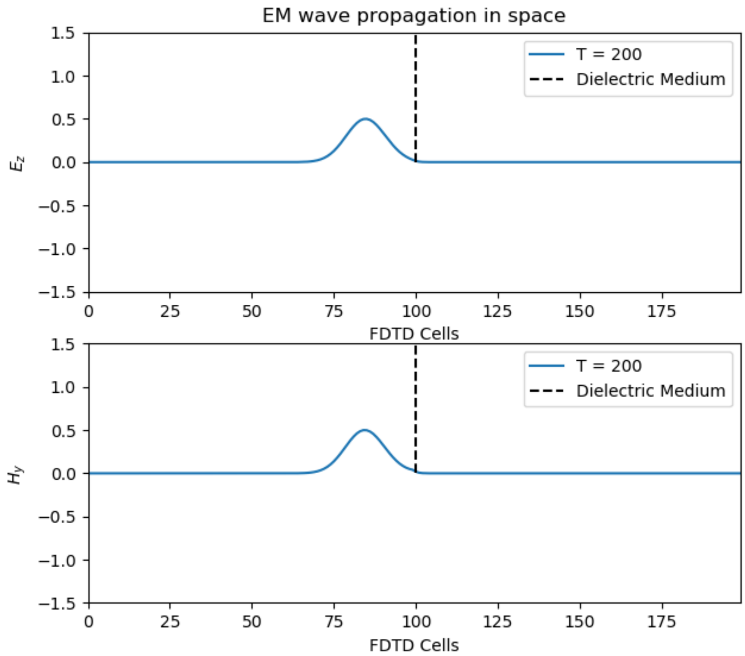

Figure 2: Propagation of Gaussian pulse into dielectric medium of $\epsilon_r = 4$ at t=200

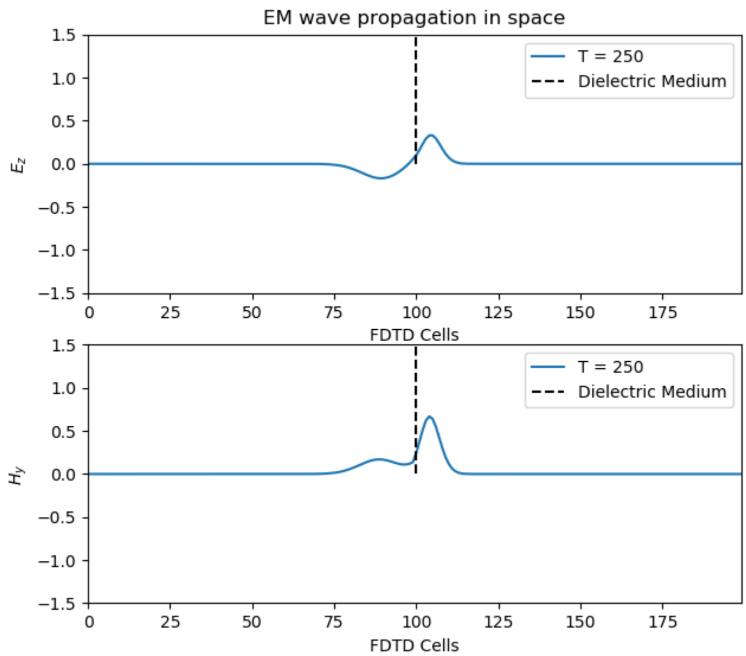

Figure 3: Propagation of Gaussian pulse into dielectric medium of $\epsilon_r = 4$ at t=250

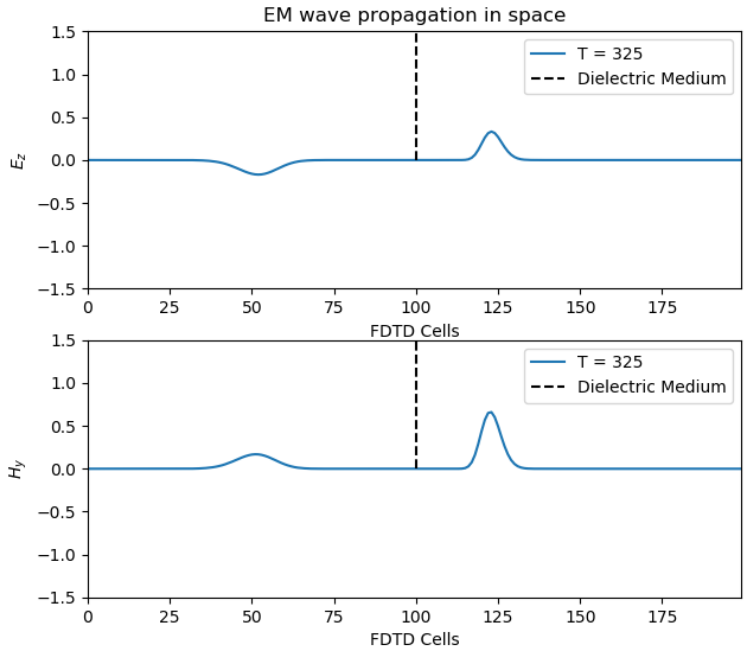

Figure 4: Propagation of Gaussian pulse into dielectric medium of $\epsilon_r = 4$ at t=325

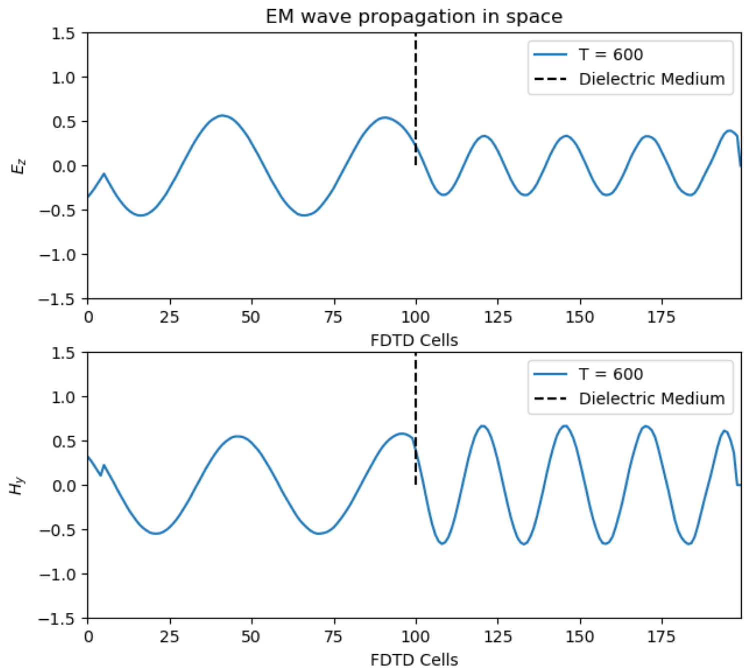

Figure 5: Propagation of 400 GHz sinusoidal pulse into dielectric medium of $\epsilon_r = 4$ at t=500

4.2 Propagation in Lossy Dielectric Medium

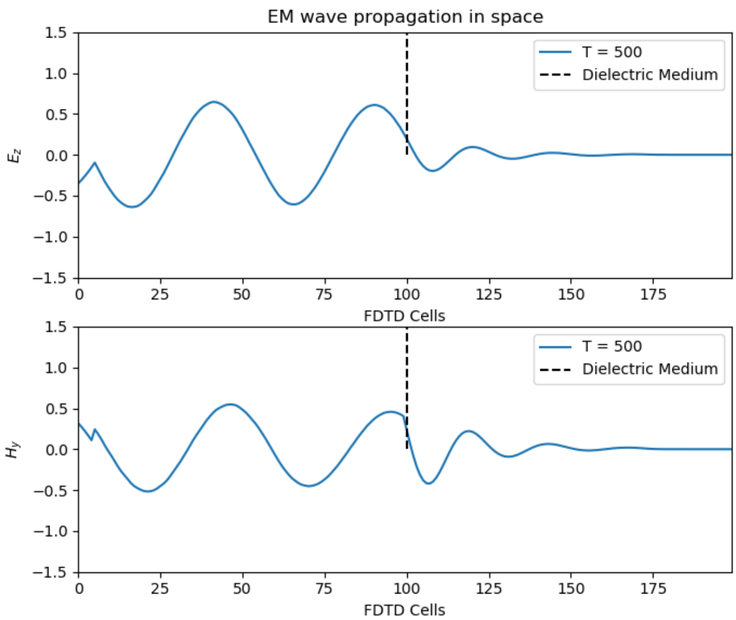

Figure 6: Propagation of 400 GHz sinusoidal pulse into dielectric medium of $\epsilon_r = 4$ and $\sigma = 0.04$

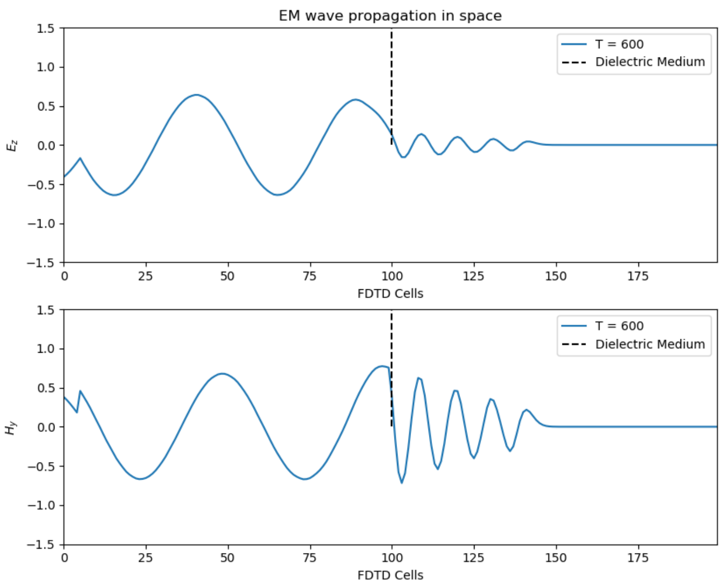

Figure 7: Propagation of 400 GHz sinusoidal pulse into dielectric medium of $\epsilon_r = 20$ and $\sigma = 0.04$

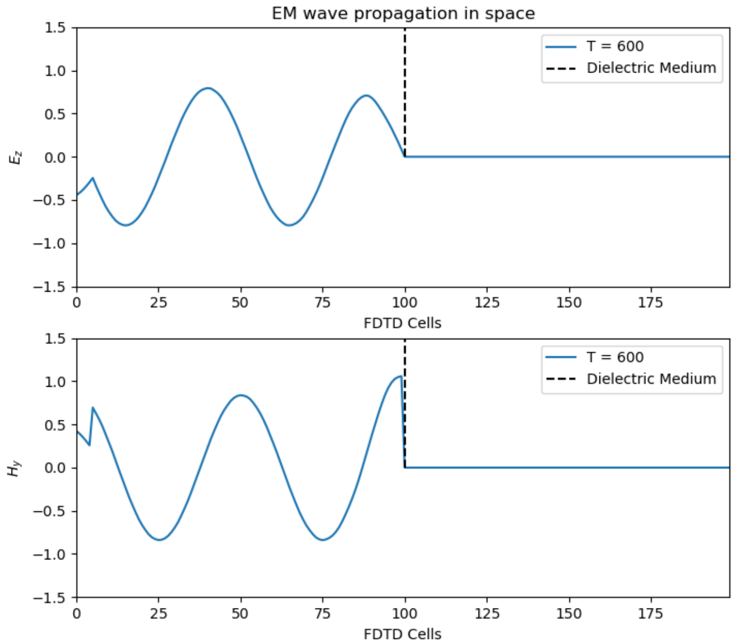

Figure 8: Propagation of 400 GHz sinusoidal pulse into dielectric medium of $\epsilon_r = 4$ and $\sigma = 10^6$, i.e., a metal wall

When a gaussian pulse hits a dielectric medium (non-conductive), a part of the pulse is transmitted and a part of the wave is reflected back as it can be seen in Figure 4.

When a sinusoidal pulse hits a dielectric medium (Figure 5), there is a change in the frequency of the sinusoidal wave.

When a sinusoidal pulse hits a lossy dielectric medium (Figure 6), not only does the frequency of the wave change but it is also attenuated until it becomes 0.

When the sinusoidal pulse hits a metal wall (conductivity tending to ∞, the EM wave is obstructed and cannot enter it.

5 Applications

Current FDTD modelling applications range from ultralow frequency geophysics involving the entire Earth-ionosphere waveguide, through microwaves, including radar signature technology, antennas, wireless communication devices, biomedical imaging/treatment to visible light (photonic crystals, nanoplasmonics, solitons and biophotonics).
1D FDTDs are also used to solve some household problems like calculating reflection at an interface, determination of propagation constant, designing of absorber materials and simulation of lossy, dispersive materials.

6 Conclusion

In this project, we have successfully implemented the FDTD method to simulate electromagnetic waves in a lossy and non-lossy dielectric medium using a computer program.

References

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