ASSIYNMENT-5

SUJATA SAHU

PROBLEM 1:

The conclusions drawn from sample/statistics computed on the sample is empeerical. Empeerical distributions are distributions of observed data such as data in random sample.

PROBLEM 2:

a. NO

b. Yes

c. Yes

d. NO

e. yes

f. NO

g. Yes

h. Yes

i. NO

PROBLEM 3:

a) given $n=50$ $y=18$ $\hat{p} = 18/50 = 0.36$

95% confidence $\alpha = 0.05$

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\Rightarrow 0.36 \pm z_{0.025} \sqrt{\frac{0.36(1-0.36)}{50}}$$

$$\Rightarrow 0.36 \pm 1.96 \sqrt{\frac{0.36 * 0.64}{50}}$$

$$\Rightarrow 0.36 \pm 0.133$$

$$= [0.22, 0.49]$$

95% confidence interval lies in [0.22, 0.49] interval.

b) 95% confidence . $\alpha = 0.05$

given true proportion is 0.02

$$0.02 = Z\alpha_{/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.02 = Z_{0.025} \sqrt{\frac{(0.36)(0.64)}{n}}$$

$$0.02 = 1.96 \sqrt{\frac{0.36 * 0.64}{n}}$$

$$n = 2212.7$$

$$n = 2213$$

c) Given true proportion is 0.02

$$0.02 = Z\alpha_{/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

here . $Z_{\alpha/2} = Z_{0.025}$ as $\alpha = 0.05$ 95% confidence int follows proportion of population

$\therefore Z_{\alpha/2} = 1.96$

$n = 50$

$$0.02 = 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{50}}$$

$$\hat{p}(1-\hat{p}) = 5.2 \times 10^{-3}$$

$$\hat{p}^2 - \hat{p} + 0.0052 = 0$$

$$\hat{p} = \frac{1 \pm \sqrt{1 - 4 * 0.0052}}{2}$$

$$\left[\begin{array}{l} ax^2 + bx + c = 0 \\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{array} \right]$$

$$\hat{p} = 0.9947, 0.0053$$

$\therefore$ if $\hat{p} = 0.9947$ then $\hat{q} = 1 - \hat{p} = 0.0053$

if $\hat{p} = 0.00530$ then $\hat{q} = 1 - \hat{p} = 0.9947$

PROBLEM 4

given data

3.481	3.448	3.485	3.475	3.472
3.477	3.472	3.464	3.472	3.470
3.470	3.470	3.477	3.473	3.474

$$\text{mean } \mu = \frac{\sum_{i=1}^{15} x_i}{n} \qquad n = 15$$

$$\bar{y} = 3.472$$

$$\text{sample variance} = s^n = \sum_{i=1}^{15} \frac{(\bar{y} - y_i)^2}{n-1}$$

$$= \frac{(8.1*10^{-5} + 5.76*10^{-4} + 1.69*10^{-4} + 9*10^{-6} + 0 + 2.5*10^{-5} + 0 + 6.4*10^{-5} + 0 + 4*10^{-6} + 4\times10^{-6} + 4*10^{-6} + 2.5*10^{-5} + 10^{-6} + 4*10^{-6}}{15-1}$$

$$s^n = \frac{9.66*10^{-4}}{14}$$

$$s^n = 6.9*10^{-5}$$

$$s = 0.008307$$

99% confidence interval for standard deviation it follows chai square distribution.

$\alpha = 0.01$

$n = 15$

$df = n-1 = 14$

$$\Rightarrow \sqrt{\frac{(n-1)s^2}{X^n_{df,\alpha/2}}} \leq \sigma \leq \sqrt{\frac{(n-1)s^2}{X^2_{df,1.\alpha/2}}}$$

$$\sqrt{\frac{(14)*(0.008307)^2}{X^2_{14,0.005}}} \leq \sigma \leq \sqrt{\frac{14*(0.008307)^2}{X^2_{14,0.995}}}$$

$$\sqrt{\frac{(14)*(0.008307)^2}{31.3193}} \leq \sigma \leq \sqrt{\frac{14*(0.0083075)}{4.07468}}$$

$$5.5*10^{-3} \leq \sigma \leq 0.0153$$

99% confidence interval lies in b/w $[5.5*10^{-3}, 0.0153]$ for standard deviation.

PROBLEM 5 :

FROM Data we know that $n = 70$

$df = 70 - 1 = 69$

here σ is unknown so it follows, t - distribution

a) 95% of confidence interval $\alpha = 0.05$

from Data mean $= 53.927$

std $= 32.693$

confidence interval $\bar{y} \pm t_{(df, \alpha/2)} s/\sqrt{n}$

$= 53.927 \pm t_{69;0.025} * \frac{32.693}{\sqrt{70}}$

$= 53.927 \pm 7.791$

$= [46.135, 61.718]$

95% of confidence interval is $[46.135, 61.718]$

b) 99% confidence interval for standard deviation this follows chai square distribution.

$\alpha = 0.01$ $\quad df = n-1 \Rightarrow 70-1 = 69$

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{df, \alpha/2}}} \leq \sigma \leq \sqrt{\frac{(n-1)s}{\chi^2_{df, 1-\alpha/2}}}$$

$$\sqrt{\frac{(70-1)(32.693)^2}{\chi^2_{69, 0.005}}} \leq \sigma \leq \sqrt{\frac{(69)(32.693)^2}{\chi^2_{69; 0.995}}}$$

$$\Rightarrow \sqrt{\frac{73749.42}{102.996}} \leq \sigma \leq \sqrt{\frac{73749.42}{42.493}}$$

$$26.75 \leq \sigma \leq 41.66$$

99% confidence interval for standard deviation is

$[26.75, 41.66]$

c)

90% confidence interval to estimate proportion, this follows proportion of population.

$\alpha = 0.1$

non critical patients waiting more than an hour is 26

this

$$\hat{p} = \frac{26}{70}$$

$$= \hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.371 \pm Z_{0.05} \sqrt{\frac{0.371(1-0.371)}{70}}$$

$$= 0.371 \pm 1.645 \sqrt{\frac{0.371 * 0.628}{70}}$$

$$= 0.371 \pm 0.0949$$

$$= [0.276, 0.4659]$$

90% confidence interval to estimate proportion is [0.276, 0.4659]

[0.276, 0.4659]

PBRO

PROBLEM 6 ;

a) given for thickness and strength of 20 plates

$n = 20$

Thickness mean $= \frac{\sum_{i=1}^{n} x_i}{n}$

$= 2.3611$ mm

Specification for plate thickness is 2.37mm. The mean of all plates is less than the specified limit.

strength mean $= \frac{\sum_{i=1}^{n} x_i}{n} = 4.693$

specification for plate strength is 5 pound per mm. Here mean of all plates is less than the specified limit. Here point estimate of thickness and strength for plates are less than the specified limit of thickness and strength respectively.

b) thickness

95% confidence interval for mean thickness from data :

$$s = \sqrt{\frac{\sum_{i=1}^{n} (\bar{y} - y_i)^2}{n-1}}$$

sample standard deviation $s = \sqrt{\frac{\sum_{i=1}^{n} (\bar{y} - y_i)^2}{n-1}}$

$= 0.197$

as here σ is unknown we use t-distribution

$$\Rightarrow \bar{y} \pm t_{df, \alpha/2} (s/\sqrt{n})$$

$$\Rightarrow 2.3611 \pm t_{19, 0.025} \left(\frac{0.197}{\sqrt{20}}\right)$$

$$\Rightarrow 2.3611 \pm (2.093) \left(\frac{0.197}{\sqrt{20}}\right)$$

$= 2.3611 \pm 0.0921$

$= [2.26, 2.453]$

specified value 2.37mm lies within 95% confidence interval.

c) strength

95% of confidence interval for mean strength from data sample standard deviation

$$s = \sqrt{\frac{\sum_{i=1}^{n} (\bar{y} - y_i)^2}{n-1}} = 0.256$$

and here σ is unknown we use t-distribution

$$= \bar{y} \pm t_{df, \alpha/2} (s/\sqrt{n})$$

$$= 4.693 \pm t_{19, 0.025} \left(\frac{0.256}{\sqrt{20}}\right)$$

$$= 4.693 \pm (2.093) \left(\frac{0.256}{\sqrt{20}}\right)$$

$$= 4.693 \pm 0.119$$

$$= [4.57, 4.812]$$

specified value 5 is outside the 95% confidence interval.

d) 95% confidence interval for proportion of plates where thickness is greater than specifications.

Here by using countif function in excell to known whose thickness is greater than specification.

countif (. , ">=2.37") = 9

$\alpha = 0.005$, $\hat{p} = 9/20 = 0.45$

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.45 \pm 1.96 \sqrt{\frac{0.45 * 0.55}{20}}$$

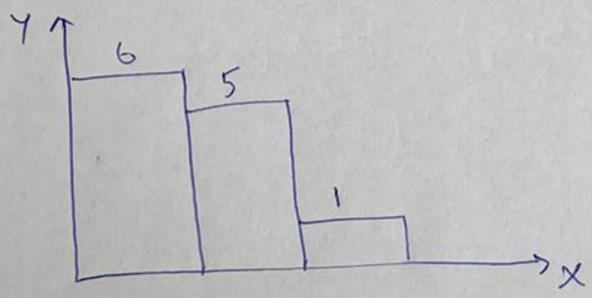
$= 0.45 \pm 0.218$

$= [0.231, 0.668]$

95% confidence interval for proportion of plates thickness is greater than specification is [0.231, 0.668]

PROBLEM7

a) given data 2216 2237 2249 2204 2225 2301
2281 2263 2318 2255 2275 2295



The distribution is not symmetric and there is skewness unlike normal distribution which is symmetric with zero skewness.

b) 95% confidence interval for mean compressive strength we follow t-distribution as σ is unknown.

from Data mean $\bar{y} = \frac{\sum_{i=1}^{n} x_i}{n}$

$= 2259.91$

sample standard deviation $s = \sqrt{\frac{\sum_{i=1}^{n} (\bar{y} - x_i)^2}{n-1}}$

$s = 35.56$

confidence interval

$$= \bar{y} \pm t_{df, \alpha/2} (s/\sqrt{n})$$

$$= 2259.91 \pm t_{11, 0.025} \left(\frac{35.56}{\sqrt{12}}\right)$$

$$= 2259.91 \pm 2.201 \left(\frac{35.56}{\sqrt{12}}\right)$$

$$= 2259.91 \pm 22.59$$

$$= [2237.31, 2282.50]$$

95% confidence interval for mean strength is [2237.31, 2282.50]

c) 95% confidence interval for standard deviation of compressive strength will follow chai square, distribution.

sample standard deviation, $s = \sqrt{\frac{\sum_{i=1}^{n} (\bar{y} - y_i)^2}{n-1}}$

$= 35.56$

confidence interval is

$$\sqrt{\frac{(n-1)s^2}{\chi^n_{df, \alpha/2}}} \le \sigma \le \sqrt{\frac{(n-1)s^2}{\chi^2_{df, 1-\alpha/2}}}$$

$$= \sqrt{\frac{11 * (35.56)^2}{\chi^n_{11, 0.05}}} \le \sigma \le \sqrt{\frac{11 * (35.56)^2}{\chi^n_{11, 0.95}}}$$

$$= \sqrt{\frac{11 * (35.56)^2}{19.6751}} \le \sigma \le \sqrt{\frac{11 * (35.56)^2}{4.57481}}$$

$$26.59 \le \sigma \le 55.154$$

95% confidence interval for std of strength is [26.59, 55.154]

PROBLEM 8 :

given $n = 1600$ $y = 8$

$$\hat{p} = 8/_{1600} = 0.005$$

99% confidence interval

$$\alpha = 0.01$$

proportion of population confidence interval

$$\rightarrow \hat{p} = \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= 0.005 \pm Z_{0.005} \sqrt{\frac{(0.005)(1-0.005)}{1600}}$$

$$= 0.005 \pm 2.575 \sqrt{\frac{0.005 * 0.995}{1600}}$$

$$= 0.005 \pm 4.34 * 10^{-3}$$

$$= [4.59 * 10^{-4}, 9.54 * 10^{-3}]$$

99% confidence interval for proportion of aircraft that have wiring errors is $[4.59 \times 10^{-4}, 9.54 \times 10^{-3}]$