ASSIGNMENT 7

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PROBLEM 1.

Given $n_1 = 191$, $n_2 = 202$, $y_1 = 38$, $y_2 = 21$, $\hat{P}_1 = 38/191 = 0.19$

$\hat{P}_2 = 21/202 = 0.1$

$\alpha = 0.05$

$H_0 : P_1 \le P_2 : P_1 - P_2 \le 0$

$H_1 : P_1 > P_2 : P_1 - P_2 > 0$

$$Z_0 = \frac{(\hat{P}_1 - \hat{P}_2) - (P_1 - P_2)}{\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_2(1-\hat{P}_2)}{n_2}}}$$

$$= \frac{(0.19 - 0.1) - 0}{\sqrt{\frac{(0.19)(1-0.19)}{191} + \frac{(0.1)(1-0.1)}{202}}}$$

$$\cancel{= \frac{0.09}{\sqrt{\frac{(0.19)(1-0.19)}{10}}}}$$

$$= \frac{0.09}{\sqrt{8.05 * 10^{-4} + 4.45 * 10^{-4}}} = 2.54$$

P-value $= 5.5 \times 10^{-3}$

P-value $< \alpha$

so we reject Null hypothesis H_0, we reject

$P_1 \le P_2 : P_1 - P_2 \le 0$ is rejected.

PROBLEM 2 :

a) given $\alpha = 0.1$

$H_0 : \mu_d = 2500$ $\qquad \mu_1 = 1\,min$

$H_1 : \mu_d \neq 2500$ $\qquad \mu_2 = 4\,weeks$

$\mu_1 - \mu_2$

$$\Rightarrow t_0 = \frac{\bar{d} - D_0}{s_d/\sqrt{n}}$$

checking data from excel $\bar{d} \xrightarrow{} $ (mean of difference) $= 2635.6$

S_d = Standard deviation of difference

$S_d = 508.6$

$$t_0 = \frac{\bar{d} - D_0}{s_d/\sqrt{n}} = \frac{2635.6 - 2500}{\frac{508.6}{\sqrt{16}}} = 1.066$$

p-value from excel by using tdist function

P-value $= 0.303 > \alpha = 0.1$

Since p-value $> \alpha$ there is no evidence

$\Rightarrow$ Reject Null Hypothesis H_0 and DO not reject $\mu_d = 2500$

b) given $\alpha = 0.01$, $d_f = n - 1 \Rightarrow 16 - 1 = 15$, confidence interval

$$\bar{d} \pm t_{df, \alpha/2} \left(s_d/\sqrt{n} \right)$$

$$\Rightarrow 2635.6 \pm t_{15, 0.005} \left(\frac{508.6}{\sqrt{16}} \right)$$

$$\Rightarrow 2635.6 \pm (2.947) * \left(\frac{508.6}{4} \right)$$

$$= [2260.8, 3010.3]$$

PROBLEM3:

given $n_1 = 8$

$n_2 = 11$

$\bar{y_1} = 14.7$

$\bar{y_2} = 16.4$

$s_1 = 0.8$

$s_2 = 2.1$

$\alpha = 0.05$

$H_0 = \sigma_1^2 = \sigma_2^2$

$H_1 = \sigma_1^2 \neq \sigma_2^2$

$$F(n, d, \alpha) = \frac{1}{F(d, n, (1-\alpha))}$$

$$F_{7,10,0.025} = 3.14$$

$$F_{7,10,(1-0.025)} = \frac{1}{F_{10,7,0.025}} = \frac{1}{4.76} = 0.21$$

$$F_0 = s_1^2 / s_2^2 = \left(\frac{0.8}{2.1}\right)^2 = 0.145$$

F_0 is in rejection region i.e H_0 is rejected that mean $\sigma_1^2 = \sigma_2^2$ is rejected.

Therefore we consider σ_1^2 and σ_2^2 are not same

$\Rightarrow H_0: \mu_1 = \mu_2 ; \mu_1 - \mu_2 = 0$

$H_1: \mu_1 \neq \mu_2 : \mu_1 - \mu_2 \neq 0$

as σ_1^2 and σ_2^2 are not same

so

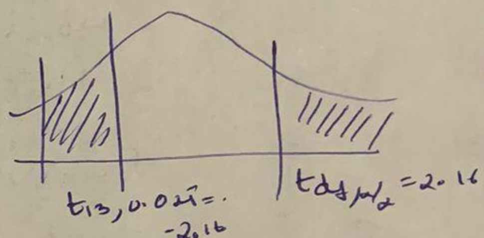
$$df = \frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_1^2/n_2)^2}{n_2 - 1}}$$

$$= \frac{\left(\frac{(0.8)^2}{8} + \frac{(2.1)^2}{11}\right)^2}{\frac{\left(\frac{(0.8)^2}{8}\right)^2}{7} + \frac{\left(\frac{(2.1)^2}{11}\right)^2}{10}}$$

$$\Rightarrow \frac{(0.08 + 0.4)^2}{9.14 * 10^{-4} + 0.016}$$

$$= 13.6 \approx 13$$

$$t_0 = \frac{(\bar{y}_1 - \bar{y}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$



$$= -2.45$$

as t_0 is in rejection region $t_0 < c.v$

we reject null hypothesis H_0

we reject $\mu_1 = \mu_2$.

PROBLEM 4:

Given $\alpha = 0.05$

Assume varriance are same.

$H_0 : \mu_0 \le \mu_2 : \mu_1 - \mu_2 \le 0$

$H_1 : \mu_1 > \mu_1 : \mu_1 - \mu_2 > 0$

$n_1 = 12, n_2 = 10, \bar{y}_1 = 8.78, \bar{y}_2 = 6.82$

$S_1 = 0.89 \quad s_2 = 0.92$

$$s.d = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{(n-1)}}$$

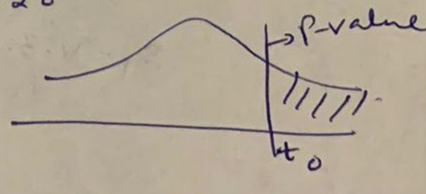
$$t_0 = \frac{(\bar{y}_1 - \bar{y}_2) - (\mu_1 - \mu_2)}{\left(\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + (n_2 - 2}}\right)\left(\sqrt{\frac{1}{n_1} + 1/n_2}\right)}$$

$$t_0 = \frac{(8.78 - 6.82) - (0)}{\sqrt{\frac{(11)(0.89)^2 + (9)(0.92)^2}{12 + 10 - 2}} \cdot \sqrt{\frac{1}{12} + \frac{1}{10}}}$$

$$= \frac{1.96}{0.9 * 0.428} = 5.08$$

$df = n_1 + n_2 - 2 = 12 + 10 - 2 = 20$

P-value < 0.0005



P-value < α so we reject H_0

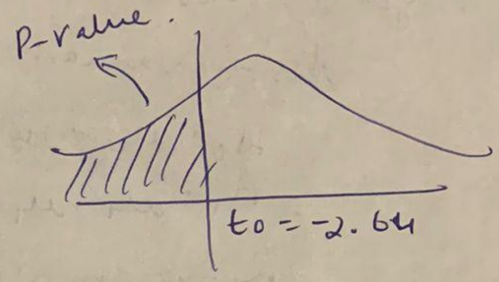
i.e we reject $\mu_1 \le \mu_2$.

PROBLEM 5:

$H_0: \mu_d \geq 0$

$H_1: \mu_d < 0$

$(\mu_1 - \mu_2)$



$\alpha = 0.05$

$\bar{d} = -0.51$.

$S_d = 0.61$

$$t_0 = \frac{\bar{d} - D_0}{s_d/\sqrt{n}} = \frac{-0.51 - 0}{0.61/\sqrt{10}} = -2.64$$

$d_f = n - 1 = 9$

p-value $0.01 < \text{p-value} < 0.025$

$\therefore$ p-value $< \alpha$

$\therefore$ we reject Null Hypothesis H_0.

PROBLEM 6:

a)

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

$\alpha = 0.1$

$n_1 = 13, n_2 = 16, \bar{y}_1 = 53.7 \quad \bar{y}_2 = 57.5$

$S_1 = 4.38, S_2 = 6.08$

$$F_0 = S_1^2 / S_1^2 = \left(\frac{4.38}{6.08}\right)^2 = 0.52$$

$$F_{(n, d, \alpha)} = \frac{1}{F_{(d, n; (n-\alpha))}}$$

$$= \frac{1}{F_{(16, 13, 0.05)}} = \frac{1}{2.51} = 0.39$$

F_0 lies in non rejection region

We don't have evidence to reject H_0 therefore we assume variance to be same.

b)

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$.

$\alpha = 0.01$

$n_1 = 13$

$y_1 = 53.7$

$s_1 = 4.38$

$n_2 = 16$

$\bar{y}_2 = 57.5$

$s_2 = 6.08$

$df = n_1 + n_2 - 2$

$= 13 + 16 - 2 = 27$

$$t_0 = \frac{(y_1 - y_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$= \frac{(53.7 - 57.5) - 0}{\sqrt{\frac{(12)(4.38)^2 + (15)(6.08)^2}{27}}\sqrt{1/13 + 1/16}}$$

$$= \frac{-3.8}{5.39 \times 0.373} = -1.88$$

P-value = $(2) \times (0.025) <$ P-value $< (0.05)/2$

$\Rightarrow 0.05 <$ P-value < 0.8

P-value $\not> \alpha$

Do not reject Null Hypothesis H_0 and so we donot have evidence to reject H_0.

c) This falls in population proportion
we check for both brands of tires that last atleast 55000 miles

$n_1 = 13 \quad n_2 = 16$

$y_1 = 5 \quad y_2 = 12$

$\hat{P}_1 = 5/13 = 0.38 \quad \hat{P}_2 = 12/16 = 0.75$

$H_0: P_1 \geq P_2 : P_1 - P_2 \geq 0$

$H_1: P_1 < P_2 : P_1 - P_2 < 0$

given $\alpha = 0.01$

$$Z_0 = \frac{(\hat{P}_1 - \hat{P}_2) - (P_1 - P_2)}{\sqrt{\frac{\hat{P}_1(1-\hat{P}_1)}{n_1} + \frac{\hat{P}_1(1-\hat{P}_1)}{n_2}}}$$

$$\Rightarrow \frac{0.36 - 0.75}{\sqrt{\frac{(0.38)(1-0.38)}{13} + \frac{(0.75)(1-0.75)}{16}}}$$

$$= \frac{-0.35}{\sqrt{0.018 + 0.017}} = -2.17$$

$C.V < Z_0$

$-2.575 < -2.77$

We donot reject Null hypothesis H_0 i.e. we donot have evidence to reject $P_1 - P_2 \geq 0$ H_0.