FUNDAMENTALS OF ENGINEERING STATISTICAL ANALYSIS

ISE/DSA 5013

Assignment 4

Show your work for calculation problems. You will receive no credit if you only provide the answer. Define any random variables which are necessary in solving probability problems. *As with all homework this semester, spend time to be neat and organized. Any disorganized submissions are subject to a zero grade.*

Problem 1

Assume random variable Y follows an exponential distribution with rate λ. Show that the variance of Y is $\frac{1}{\lambda^2}$.

Problem 2

If bolt thread length is normally distributed, what is the probability that the thread length of a randomly selected bolt is

a. Within 1.5 standard deviations of its mean value?
b. Farther than 2.5 standard deviations from its mean value?

Problem 3

Johnson rods for use in a particular vehicle have diameters that are normally distributed with mean 0.652 cm and standard deviation 0.003 cm. The specification for the Johnson rod is 0.650 ± 0.005 cm.

a. What proportion of the Johnson rods manufactured by this process meet specifications?
b. The process mean can be adjusted through calibration. If the mean is set to 0.650 cm, what proportion of the rods will meet specifications?
c. If the mean is set to 0.650 cm, what must the standard deviation be so that 99% of rods will meet specifications?

Problem 4

A mechanical engineer collected the bending strength of a sample of 20 support beams that are used in a transmission tower. The data, in GPa, are as follows.

1.8	8.2	1.8	2.7	4.2	0.4	6.3	1.1	2.8	2.1
0.3	4.9	9.5	4.3	4.2	2.4	0.4	15.6	1.3	3.8

a. These data, in a sense, represent the "load per bend." What is the average "load per bend"?
b. Assume that the load data follow an exponential distribution. As such, the rate, λ, would have to represent "average bend per load." What is λ for these data?
c. What is the probability that the beam can support a load of 4.5 GPa?

Problem 5

Back at Tinker Air Force Base, the specifications for a bolt used in an aircraft MRO application require that the ultimate tensile strength be at least 18 kN. It is known that 10% of the bolts have strengths less than 18.3 kN and that 5% of the bolts have strengths greater than 19.76 kN. Assume that the uncertain tensile strengths of bolts follow a normal distribution. What proportion of the bolts meet the strength specification?

Problem 6

As a process engineering consultant, you've been asked to work on a project with a hospital in a large city. In particular, you've been asked to make some changes to the emergency room to alter patient waiting times. Assume that patients fall into two categories: critical and non-critical. You developed two designs (A, B), and you used computer simulations to determine the behavior of the two designs (so you wouldn't have to accrue the time and expense of implementing each design at the hospital). After the simulations, you found that patient waiting times were normally distributed for each patient category and each design, the parameters of which are as follows (in minutes).

ER	Critical patients		Non-critical patients	
design	μ	σ	μ	σ
A	10.2	1.3	17.7	6.5
B	8.9	3.4	19.8	4.3

a. Assume that an industry standard for maximum waiting time for critical patients is 14 minutes. Compare designs A and B.
b. Assume that an industry standard for maximum waiting time for non-critical patients is 23 minutes. Compare designs A and B.
c. What are your thoughts on which design to choose?

Problem 7

You're a quality control engineer for Select Recipe, and you're in charge of the production line for big nasty jars of mustard. A data collection effort suggests that jars of mustard that come from the production line have weights that are normally distributed with a mean of 137.2 ounces and a standard deviation of 1.6 ounces. The label on the jar suggests that the jar is marketed at 135 ounces.

a. What is the probability that a randomly chosen jar contains more than the stated contents?
b. Among ten randomly suggested jars, what is the probability that at least eight contain more than the stated contents?
c. Assuming that the mean weight remains at 137.2 ounces, to what value would the standard deviation have to be changed so that 95% of all jars contain more than the stated contents?

Problem 8

A new battery labeled as 1.5 volts actually has a voltage described by a uniform distribution between 1.43 and 1.60 volts.

a. What is the expectation of the voltage for a battery?
b. What is the probability that a battery has a voltage less than 1.5 volts?
c. If a box contains 50 batteries, what is the expected number of batteries in the box with a voltage less than 1.5 volts?

Problem 9

You're a manufacturing engineer working at Kruger Industrial Smoothing, and you're examining the operation of one of the key machines on the shop floor. You collected some data on the following three characteristics of the machine, and you estimated the distributions and parameters as follows. (adapted from [Mendenhall and Sincich 2016], exercise 5.103).

(i) The interarrival times of jobs (that is, the time between arrivals of raw material to the machine) are exponential distributed with a mean of 1.25 minutes.
(ii) The amount of time the machine operates before breaking down is exponential distributed with a mean of 540 minutes.
(iii) The repair time for the machine is normally distributed with a mean of 40 minutes and standard deviation of 12.5 minutes.

a. Find the probability that two jobs arrive for processing at most one minute apart.
b. Find the probability that the machine operates for at least 720 minutes (12 hours) before breaking down.
c. Find the probability that the repair time for the machine exceeds 70 minutes.

Problem 10

Random variable T describes the time between failures of a particular machine on the shop floor. As T is always positive, the hazard function, $h(t)$, describes the instantaneous failure rate of the machine. Naturally, some systems may fail more frequently early in their lifecycles and more frequently later in their lifecycles, while the middle of their useful lives display a lower likelihood of failure. This concept is referred to as the bathtub curve. The equation for the hazard function is as follows. Here, $R(t)$ represents the reliability of the machine at time t, or the probability that failure hasn't occurred prior to time t. As T represents the time at which failure occurs, $R(t)$ is calculated as $1 - F(t)$, as $F(t)$ represents the cumulative probability of failure time, or the probability that failure occurred prior to time t.

$$h(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - F(t)}$$

a. Spend two minutes reading about the bathtub curve concept here: http://en.wikipedia.org/wiki/Bathtub_curve. This phenomenon exists in many manufactured systems.
b. Find the hazard function when T follows an exponential distribution. What do you conclude?