ASSIGNMENT 6

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PROBLEM 1

a) $n=7$

$t_0=0.94$

$df=6$

P-value ≥ 0.1 from table

$t_0=0.94 < t_{0.1}=1.44$.

$\Rightarrow$ p-value ≥ 0.1

b) $n=12$

$t_0=2.41$

$df=12-1=11$

$2.201 < t_0=2.41 < 2.218$

$t_{0.25} < t_0=2.41 < t_{0.01}$

$\Rightarrow 0.01 <$ P-value < 0.25

c) $n=41$

$t_0=3.49$

by using excel function

tdist (3.49, 40, 1)

p-value $=0.000596$

d) $n=13$, $t_0=0.52$

tdist(0.52, 12, 1)

$\Rightarrow$ P.value $=0.306261$

Problem 2

given $\alpha = 0.05$

a) $H_0 : \mu \geq 5$: mean tensile strength is greater than (or) equal to 5 pounds per millimeter.

$H_1 : \mu < 5$: mean tensile strength is less than 5 pounds per mm.

as σ is unknown we use t-distribution.

From the data given we calculate the following.

$$\bar{y} = \frac{\sum_{i=1}^{n} x_i}{n} = 4.6939$$

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{y})^2}{n-1}} = 0.2564$$

$$t_0 = \frac{\bar{y} - \mu}{s/\sqrt{n}} = \frac{4.6939 - 5}{0.2564\sqrt{20}}$$

$$= -5.339$$

By using the critical value approach.

dif = 19

$\alpha = 0.05$

C.V = 1.729 from table

as C.V < t_0 for left tail test we reject H_0

i.e. we reject mean tensile strength greater than (or) equal to 5 pounds per mm.

i.e. $\mu \geq 5$ is rejected.

b) given $\alpha = 0.05$ ②

Here out of 20 plates we consider plates whos thickness is > 2.37

Therefore the total of 9 plates are considered out of 20.

$\Rightarrow$ mean of these plates is $\bar{y} = 0.45$

and S.D of this sample is $s = 0.51$

given in the question

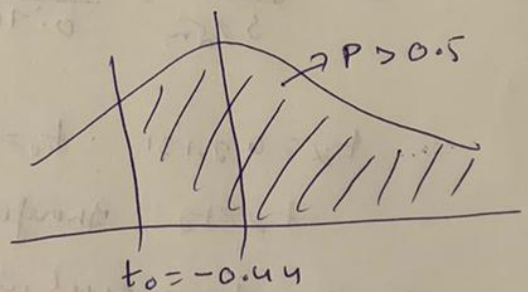
$H_0 = \mu \leq 0.5$ $\therefore$ less than (or) equal to half of plates thicker than 2.37.

$H_1 = \mu > 0.5$ $\therefore$ more than half of plates thickness than 2.37

as σ is unknown we follow t-distribution

$$t_0 = \frac{\bar{y} - \mu}{s/\sqrt{n}} = \frac{0.45 - 0.5}{0.51/\sqrt{20}} = -0.438$$
$$= -0.44$$

df = 19



p-value > 0.5

$\alpha = 0.05$

$\therefore$ p-value $> \alpha$

Do not reject Null Hypothesis (H_0)

Therefore we donot have enough evidence to reject plates less than (or) equal to half of plates thicker than 2.37

PROBLEM 3:

From the given data we find mean and S.D

mean of sample $= \bar{y} = \frac{\sum_{i=1}^{n} x_i}{n} = 13.52$

$$S.D. = s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{y})^2}{n-1}}$$

$= 0.7056$

$H_0: \mu = 13.5$ · power supplies that are supposed to produce 13.5 volts

$H_1 = \mu \neq 13.5$ power supplies that are not supposed to produce 13.5 volts

as σ is not given we follow t-distribution

$$t_0 = \frac{\bar{y} - \mu}{s/\sqrt{n}} = \frac{13.52 - 13.5}{0.7056/\sqrt{16}} = 0.113$$

$t_\alpha = 2.131$ · $t_0 = 0.113$

$t_\alpha > t_0$ therefore do not reject H_0

p-value from excel is 0.91

p-value $> \alpha$

therefore Do not reject Null Hypothesis

PROBLEM 4:

given $\alpha = 0.01$

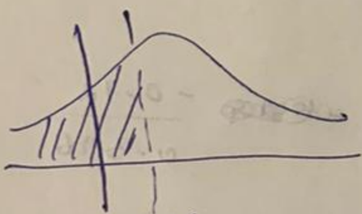
$H_0: P_0 \geq 0.08$ qualities in tank if it shown to produce greater than (or) equal to 8% defective parts.

$H_1: P_0 \leq 0.08$ qualities in tank if it shown to produce less than 8% defective parts

$n = 300 \quad y = 12$

$$\hat{P} = \frac{12}{300} = 0.04$$

$$Z_0 = \frac{\hat{P} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{0.04 - 0.08}{\sqrt{\frac{0.08 * (1-0.08)}{300}}}$$



$= -2.55$ using critical value approach

$c.v = 2.325$

$c.v > Z_0$

$Z_0 = -2.55$

Therefore we reject Null Hypothesis

qualities in tank if it shown to produce greater equal to 80% defective parts is rejected.

PROBLEMS :

given $n=25$ $\alpha = 0.05$

a) $H_0 : \mu \geq 8.73$

$H_1 : \mu < 8.73$

σ is unknown as we follow t-distribution

from data given mean $\bar{y} = \sum_{i=1}^{n} \frac{x_i}{n} = 8.01$

SD for sample $s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{y})^2}{n-1}}$

$= 1.98$

$t_0 = \frac{\bar{y} - \mu}{s/\sqrt{n}} = \frac{8.01 - 8.73}{1.98/\sqrt{25}} = \frac{-0.72}{0.396}$

$= -1.818$

$n = 25$

$df = 24.$ $\alpha = 0.05$

$C.V = 1.711$

as $C.V > t_0$

we reject Null hypothesis

∴ avg pick times of the ware house greater than (or) equal to 8.75 min after layout changes is rejected

b) using p-value approach

$\alpha = 0.05 \cdot n = 25$

$t_{0.025} = -2.064 < t_0 = -1.818 < t_{0.050} = -1.711$

$0.025 < \text{p-value} < 0.05$

$\text{p-value} < \alpha$

$\therefore$ we reject H_0

$\therefore$ average pick items off the warehouse greater than or equal to 8.75 min after layout changes is rejected.

PROBLEM 6:

a) P-value approach (as data is given in hrs we are changing days to hrs)

$\alpha = 0.1$ $\quad \bar{y} = \dfrac{\sum_{i=1}^{n} x_i}{n}$

$\bar{y} = 97.10 \text{ hrs}$

S.D. of sample $= s = \sqrt{\dfrac{\sum_{i=1}^{n} (x_i - \bar{y})^2}{n-1}}$

$= 6.24$

σ^2 is unknown, we use t-distribution

$t_0 = \dfrac{\bar{y} - \mu}{s/\sqrt{n}} = \dfrac{97.10 - 96}{6.24/\sqrt{10}}$

$= 0.557$

$df = 9 \qquad \alpha = 0.1$

p-value > 0.1

p-value $> \alpha$

Therefore we do not reject H_0.

we do not reject delivery on time with lead time greater than (or) equal to 4 days

b) given $\alpha = 0.05$ $\qquad n = 10,\ df = 9$

$t_0 = \dfrac{\bar{y} - \mu}{s/\sqrt{n}}$

$1.833 = \dfrac{\bar{y} - 96}{6.24/\sqrt{10}}$

$\bar{y} = 99.616$

PROBLEM 7:

a) ~~[illegible]~~

Given $n=10$

data mean $\bar{y} = \sum_{i=1}^{n} \frac{x_i}{n} = 178.64$

$S.D = 19.18$

$H_0 = \mu \le 175$: final height of insulation to be less than or equal to 175mm

$H_1 = \mu > 175$: final height of insulation to be more than 175mm

σ is unknown we follow t-distribution

$$t_0 = \frac{\bar{y}-\mu}{s/\sqrt{n}} = \frac{178.64-175}{19.18/\sqrt{10}} = 0.605$$

$df = 9 \quad \alpha = 0.1$

$C.V = 1.383$

$C.V > t_0 \therefore$ we do not reject H_0

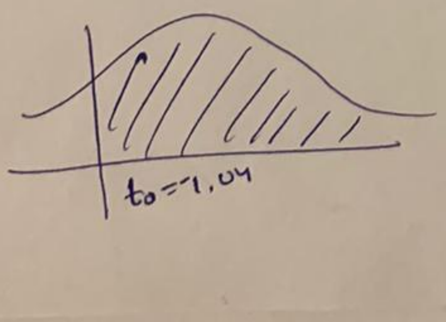
We do not have enough evidence to reject H_0.

b) $\beta = P(H_0 \text{ is not rejected} / H_0 \text{ is false})$

$H_0: \mu \le 175$

$H_1: \mu > 175$

$$t_0 = \frac{\bar{y}-\mu}{s/\sqrt{n}} = \frac{178.67-185}{19.18/\sqrt{10}} = -1.04$$



c) $H_0: \mu \le 175$

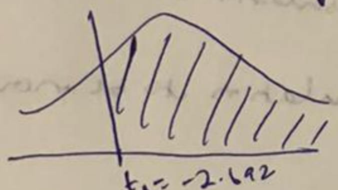
$H_1: \mu > 175$

$\mu = 195$ true value

$$t_0 = \frac{\bar{y}-\mu}{s/\sqrt{n}} = \frac{178.67-195}{19.18/\sqrt{10}}$$

$$= -2.692.$$

Probability of $t_0 > -2.692$ given $\mu = 195$ is 0.987

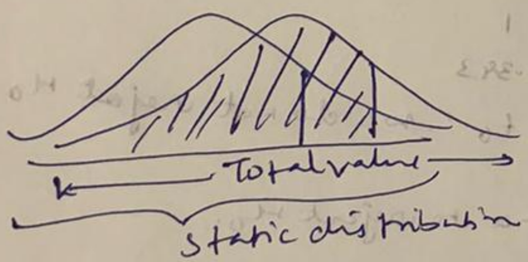


$\beta = 1 - \text{Probability}$

$= 1 - 0.987 = 0.012$.

d) type-II error 'β' changes because of change in area, as 'μ' value increases ~~β value decreases~~ probability decreases as true mean increases β value decreases.

$\beta = 1 - \text{Probability}$



Problem 8:

$H_0 : \mu \leq 0.1$

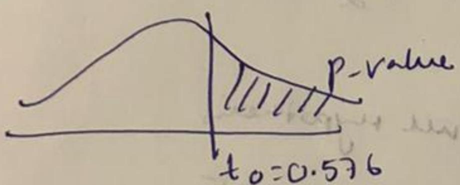
$H_1 : \mu > 0.1$

From the total data of 85 we consider based on given condition based on given condition i.e do not want surface roughness to exceed 6.0 microns.

mean of data is $\bar{y} = \sum_{i=1}^{n} \frac{x_i}{n} = 0.12$

SD of sample $s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{y})^2}{n-1}} = 0.32$

$t_0 = \frac{\bar{y} - \mu}{s/\sqrt{n}} = \frac{0.12 - 0.1}{0.32/\sqrt{85}} = 0.578$



from excel p-value $= 0.283$

$\alpha = 0.05$

p-value > 0.05

$\therefore$ we do not Null Hypothesis

i.e. we do not reject Null Hypothesis.

i.e we do not have enough evidence to reject H_0.

Problem 9: given $n=10$

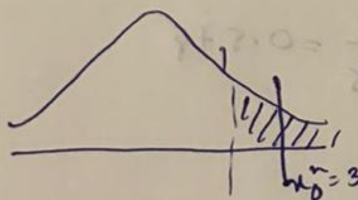
$$\bar{y} = \sum_{i=1}^{n} \frac{x_i}{n} = 0.19$$

$$S.D \text{ is } s = \sqrt{\frac{\sum_{i=1}^{n} (x_i-\bar{y})^2}{n-1}} = 0.19$$

$$s^2 = 0.0361$$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = (10-1)\left(\frac{0.0361}{0.01}\right)$$

$$= 32.49$$



for $\alpha = 0.05$ $df = 9$ $\chi_\alpha^2 = 16.9190$

$$\chi_\alpha^2 < \chi_0^2$$

$\therefore$ We reject Null hypothesis

p-value < 0.005 $\alpha = 0.05$

p-value $< \alpha$

$\therefore$ we reject variance in each can is 0.01

Ho is rejected.

PROBLEM 10:

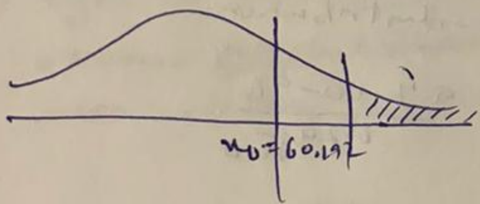
$$\bar{y} = \frac{\sum_{i=1}^{n} x_i}{n} = 53.93 \text{ mins}$$

Sample SD is $s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{y})^2}{n}} = 32.69$

$n = 70$

$$x_0 = (n-1)\left(s/\sigma\right)^2$$

$$= (70-1)\left(\frac{32.69}{35}\right)^2 = 60.192$$



$\alpha = 0.01 \quad df = 69$

$x^2_\alpha = 100.425$ from table

$x^2_\alpha > x_0^2$ ∴ we do not reject Null hypothesis

$x_{0.9} = 55.3290 < x_0^2 = 60.192 < x^2_{0.1} = 85.5271$

$0.1 < \text{P-value} < 0.9$ from spreadsheet. P-value = 0.766

p-value > α

∴ we do not reject Ho

We do not have enough evidence to reject Ho.