ASSIGNMENT 4
SUJATA SAHU

ASSIGNMENT 4

SUJATA SAHU

①

PROBLEM 1

$$Var(Y) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \int_0^\infty x^2 f(x)\,dx$$

$$= \int_0^\infty x^2 \lambda (\exp)^{(-\lambda x)}\,dx$$

$$= \left[-x^2 \exp^{(-\lambda x)}\right]_0^\infty + \int_0^\infty 2x \exp^{(-\lambda x)}\,dx$$

$$= 0 + \frac{2}{\lambda}\int_0^\infty x\lambda \exp^{(-\lambda x)}\,dx$$

$$= (2/\lambda)\,E(x) = (2/\lambda)(1/\lambda)$$

$$= 2/\lambda^2$$

$$Var(Y) = E(x^2) - (E(x))^2$$

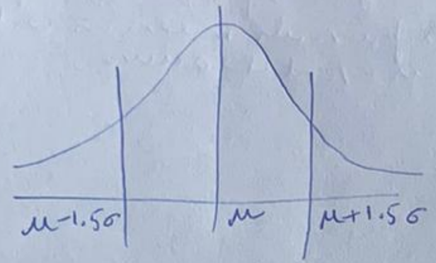
$$= 2/\lambda^2 - 1/\lambda^2$$

$$= 1/\lambda^2$$

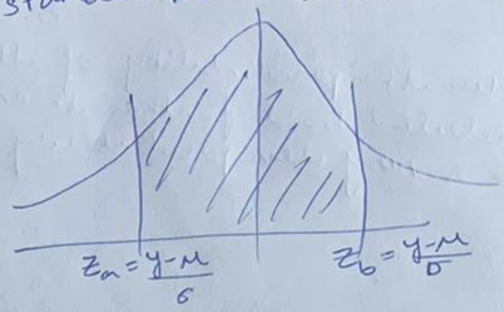
Therefore proved that variance of y is $1/\lambda^2$

PROBLEM2

a) Normal distribution



standard Normal distribution



here $Z_a = \frac{y_1-\mu}{\sigma} = \frac{\mu-1.5\sigma-\mu}{\sigma}$

$$Z_a = -1.5$$

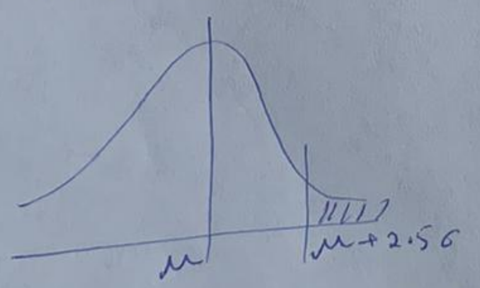
$$Z_b = \frac{y_2-\mu}{\sigma} = \frac{\mu+1.5\sigma-\mu}{\sigma}$$

$$Z_b = 1.5$$

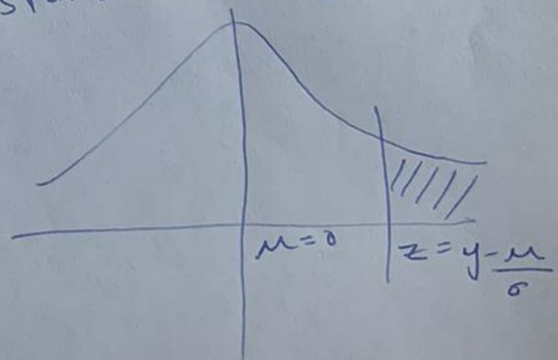
$$P(\mu-1.5\sigma < x < \mu+1.5\sigma) = P(Z_a < Z < Z_b)$$
$$= P(-1.5 < Z < 1.5)$$
$$= 0.4332 + 0.4332$$
$$= 0.8664$$

Here the probability of thread length of randomly selected bolt within 1.5 S.D of its mean is 86.64%

b) Normal distribution



Standard Normal distribution



here $Z = \frac{y-\mu}{\sigma}$

$$= \frac{\mu+2.5\sigma-\mu}{\sigma} = 2.5$$

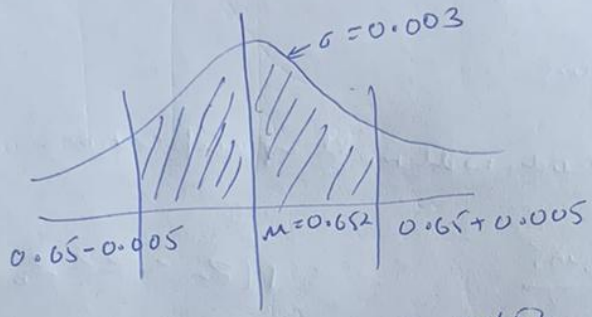
$$P(x > \mu + 2.5\sigma) = P(Z > 2.5)$$
$$= (0.5 - 0.4936)$$
$$= 0.0062$$

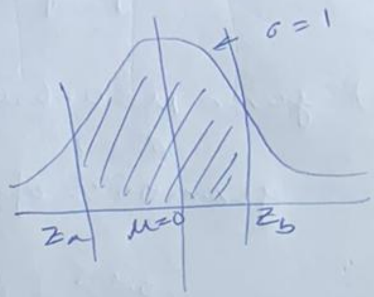
Here the probability of thread length of randomly selected bolt farther than 2.5 SD from its mean is 0.62% which is a very less probability for this event to occur

PROBLEM 3

given $\mu = 0.652$ cm, $\sigma = 0.003$ cm

a) $P(0.65 - 0.005 < x < 0.65 + 0.005) = ?$





$$z_a = \frac{0.65 - 0.005 - 0.652}{0.003} = -2.33$$

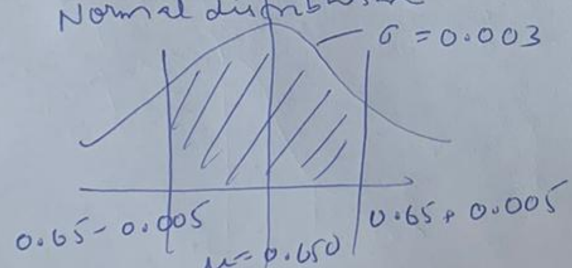
$$z_b = \frac{0.650 + 0.005 - 0.652}{0.003} = 1$$

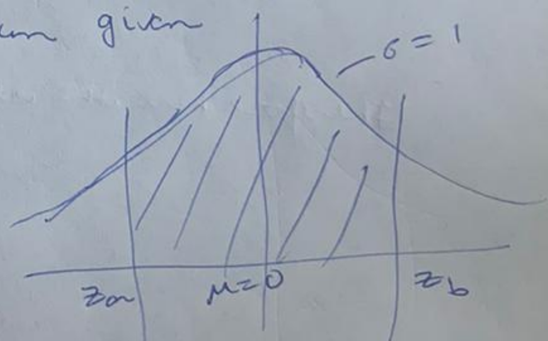
$$P(0.65 - 0.005 < x < 0.65 + 0.005) = P(-2.33 < Z < 1)$$
$$= 0.4901 + 0.3413$$
$$= 0.8314$$

Probability of meeting this specification is 83.14%

b) $\mu = 0.650$ cm, $\sigma = 0.003$ cm given

Normal distribution





$$z_a = \frac{y - \mu}{\sigma} = \frac{0.65 - 0.005 - 0.650}{0.003} = -1.66$$

$$z_b = \frac{0.65 + 0.005 - 0.650}{0.003} = +1.66$$

(5) $P(0.65-0.005 < x < 0.65+0.005) = P(-1.66 < z < 1.66)$

$= 0.4515 + 0.4515$

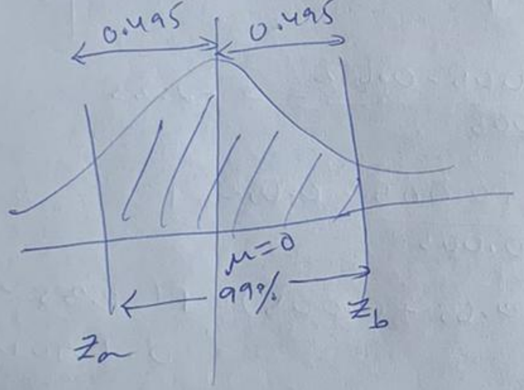
$= 0.903$

probability of meeting the specification is 90.3%

c) given $\mu = 0.650$ cm

$\sigma = ?$

given 99% of rods will meet specification



$P(Z_a < Z < Z_b) = 0.99$

$Z_a = -2.575$

$Z_b = 2.575$

$Z_a = \frac{y_1 - \mu}{\sigma}$, $Z_b = \frac{y_2 - \mu}{\sigma}$

$Z_b = \frac{0.650 + 0.005 - 0.650}{\sigma}$

$= \frac{0.005}{\sigma} = 2.575$

$\sigma = 0.00194$

Therefore here the standard deviation is 0.00194

PROBLEM 4;

a) given $n=20$

$$\mu = \frac{\Sigma(x)}{n} = \frac{1.8+8.2+1.8+2.7+4.2+0.4+6.3+1.1+2.8+2.1+0.3+4.9+9.5+4.3+4.2+2.4+0.4+15.6+1.3+3.8}{20}$$

$$= \frac{78.1}{20} = 3.905$$

b) $\lambda = \frac{1}{\mu} = \frac{1}{3.905} = 0.256$

c) $P(x < 4.5\,\text{kpa}) = 1 - \exp^{(-\lambda y)}$

$= 1 - \exp^{(-0.256 * 4.5)}$

$= 1 - \exp^{-1.152}$

$= 0.6839$

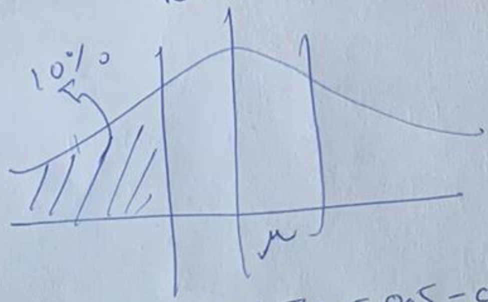
Normal distribution (1)

PROBLEM 5:

given $P(x<18.3)=0.1$

$P(x>19.76)=0.05$

$P(x>18)=?$



$P(x<18.3)=0.1$

$P\left(Z<\frac{\mu-18.3}{\sigma}\right)$

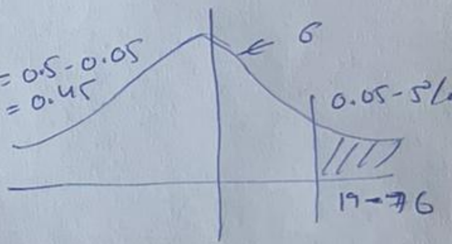
here $Z_a=0.5-0.1$
$=0.4$

$\frac{\mu-18.3}{\sigma}=1.285$

$\mu-18.3=1.285\sigma \longrightarrow (1)$

$P(x>19.76)=0.05$

$Z_b=0.5-0.05$
$=0.45$



$P\left(Z>\frac{19.76-\mu}{\sigma}\right)$

$=\frac{19.76-\mu}{\sigma}=1.645$

$\Rightarrow 19.76-\mu=1.645\sigma \longrightarrow (2)$

Adding eq (1) and (2) $\Rightarrow \mu-18.3+19.76-\mu=1.285\sigma+1.645\sigma$

$1.46=2.93\sigma$

$=\sigma=0.49$

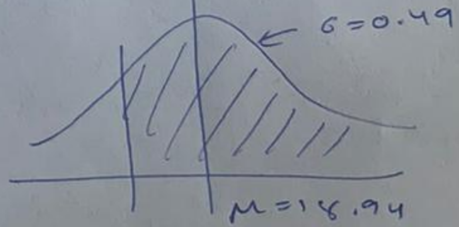
$\mu=18.94$

$P(x>18)=P\left(Z>\frac{18-18.94}{0.49}\right)$

$P(Z>-1.92)$

$=0.5+0.4726$

$=0.9726$

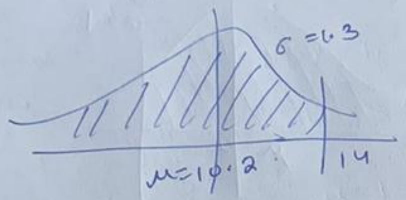


Probability of $x>18=90.26\%$

PROBLEM 6

(i) $P(x<14)$ for critical patients in both ER designs

$P(A<14) =$.



$$Z_a = \frac{y-\mu}{\sigma} = \frac{14-10.2}{1.3}$$

$$= 2.92$$

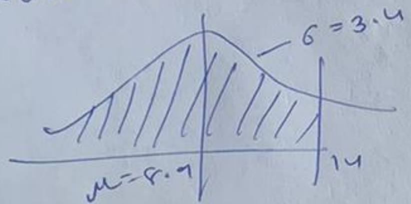
$$P(A<14) = P(Z_A < 2.92)$$

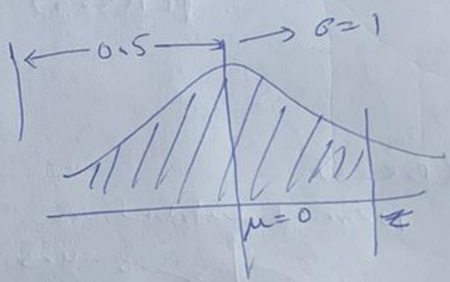
$$= 0.5 + 0.4982$$

$$= 0.9982$$

$P(B<14)$

Normal distribution





$$Z_b = \frac{y-\mu}{\sigma} = \frac{14-8.9}{3.4}$$

$$= 1.5$$

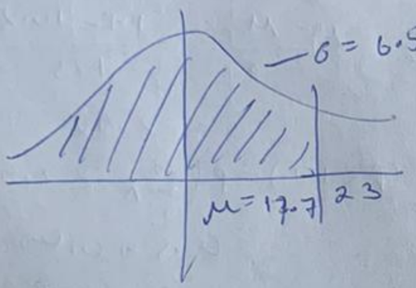
$$P(B<14) = P(Z_b < 1.5) = 0.5 + 0.4332 = 0.9332$$

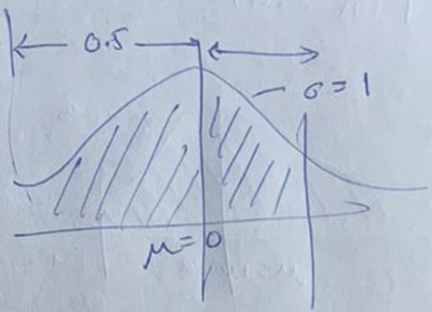
Probability of A design is more, so waiting time is more for 'A'

b) $P(Y<23) = ?$ for Non critical patients in both ER designs

$P(A<23) = p$

Normal distribution



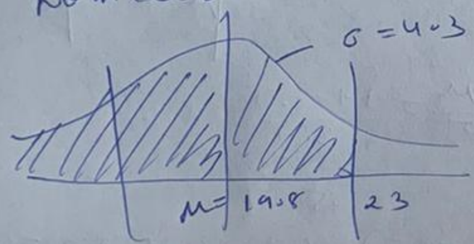


$$z_a = \frac{y-\mu}{\sigma} = \frac{23-17.7}{6.5} = 0.82$$

$$P(A<23) = P(Z_a < 0.82)$$
$$= 0.5 + 0.2939$$
$$= 0.7939$$

$P(B<23)$

Normal distribution



$$Z_b = \frac{23-19.8}{4.3}$$

$$P(B<23) = P(Z_b < 0.74) = 0.5 + 0.2704$$
$$= 0.7704$$

Probability of 'A' Design is more, so waiting time for 'A' is more

c) maximum probability is maximum for A Design in both the cases i.e

$P(A < 14$ for critical patients $) = 99.82\%$

$P(B < 14$ for critical patients $) = 93.32\%$

$P(A < 23$ for Non critical patients $) = 79.37\%$

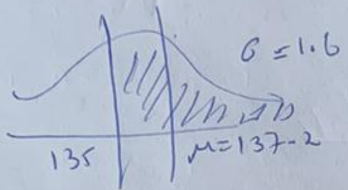
$P(B < 23$ for Non critical patients $) = 77.04\%$

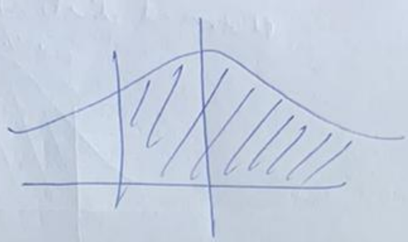
Therefore probability is more for 'A' Design in both the cases therefore Design B is better than Design A as waiting time for Design B is less than that of 'A'. So Design B is better than 'A' in both the cases.

PROBLEM 7

given $\mu = 137.2$ $\quad \sigma = 1.6$

a) $P(x > 135) = ?$





$$Z = \frac{y-\mu}{\sigma} = \frac{135-137.2}{1.6}$$

$$= -1.375$$

$$P(x>135) = P(Z > -1.37) = 0.4147 + 0.5$$

$$= 0.9147$$

$$P(x > 135) = 91.47\%$$

b) given $n = 10$ this follows binomial Distribution

$$P(x=8) = 10C_8\,(0.9147)^8\,(1-0.9147)^{10-8}$$

$$= 10C_8\,(0.9147)^8\,(0.0853)^2$$

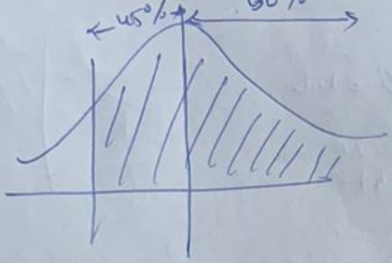
$$= \frac{10 \times 9 \times 8!}{8! \times 2}\,(0.9147)^8\,(0.0853)^2$$

$$= 5 \times 9\,(0.9147)^8\,(0.0853)^2$$

$$= 0.16$$

c) given $\mu = 137.2$. $\sigma = ?$

Normal distribution



$P(x > 135) = 95\%$

$$Z = \frac{135 - 137.2}{\sigma} = \frac{-2.2}{\sigma}$$

$$1.645 = \frac{-2.2}{\sigma}$$

$$\sigma = 1.337$$

S.D should be 1.337 so that 95% of Jars contains more than the stated contents.

PROBLEM 8

given $a = 1.43$ $b = 1.6$

a) expectation of voltage for battery $= \frac{a+b}{2}$

$$= \frac{1.43 + 1.6}{2}$$

$$= \frac{3.03}{2} = 1.515$$

b) $p(x < 1.5) = \frac{1.5 - a}{b - a} = \frac{1.5 - 1.43}{1.6 - 1.43} = \frac{0.07}{0.17} = 0.41$

c) $p(x < 1.5) * 50 = 0.411 * 50 = 20.55$

~~PROBLEM~~

PROBLEM 9

a) given $\mu = 1.25$ mins

$\lambda = 1/1.25 = 0.8$

$P(x < 1\,min) = 1 - e^{-\lambda y}$ → exponential Distribution

$= 1 - e^{-(0.8)(1)}$

$= 0.55$

b) given $\mu = 540$ mins

$\lambda = 1/540$

$P(x > 720) = 1 - e^{-\lambda y}$

$= 1 - e^{-(1/540)(720)}$

$= 1 - e^{-1.333}$

$= 0.736$

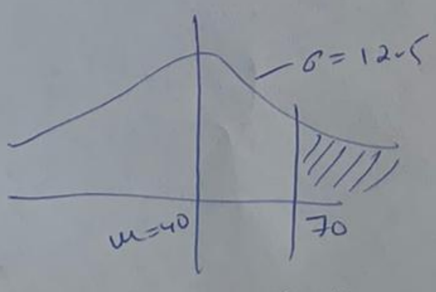
c) $P(x > 70\,mins) = ?$

given $\mu = 40$ mins $\quad \sigma = 12.5$ mins

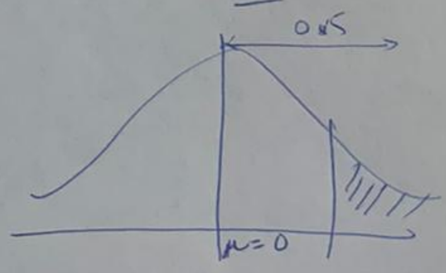
$Z = \frac{y - \mu}{\sigma} = \frac{70 - 40}{12.5} = 2.4$

$P(x > 70) = P(Z > 2.4) = 0.5 - 0.4918$

$= 0.0082$



Normal Distribution



Standard Normal distribution

PROBLEM 10 :

$$h(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1-F(t)}$$

$$= \frac{\lambda e^{-\lambda t}}{1-\int_0^{\infty} \lambda e^{-\lambda t} dt}$$

$$\int_0^{\infty} \lambda e^{-\lambda t} dt = 1 - \exp(-\lambda t)$$
$$= 1 - e^{-\lambda t}$$

$$h(t) = \frac{\lambda e^{-\lambda t}}{1-(1-e^{-\lambda t})} = \frac{\lambda e^{-\lambda t}}{1-1+e^{-\lambda t}} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}}$$

$$h(t) = \lambda$$

mean of exp distribution is $1/\lambda$

as mean time increases that means there will be less no. of failures.

hazard rate and mean time between failure are inversely proportional.