ASSIGNMENT 3
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PROBLEM 1

a. given $P(A_1)=0.5$ of detecting the object
$P(A_2)=0.3$ of detecting the object
P(object is detected)

$$P(D) = P(A_1 \cup A_2)$$
$$= P(A_1) + P(A_2) - P(A_1 \cap A_2)$$
$$= 0.5 + 0.3 - 0.5 * 0.3$$
$$= 0.65$$

We can solve Above in the following way

$$P(D) = P(A_1)P(A_2^c) + P(A_2)P(A_1^c) + P(A_1)P(A_1)$$
$$= (0.5)(1-0.3) + (0.3)(1-0.5) + 0.5 * 0.3$$
$$= 0.65$$

b. P(object detected exactly one of radar prototype)

$$= \underbrace{P(A_1)P(A_2^c)}_{\text{Detected by } A_1 \text{ and Not detected by } A_2} + \underbrace{P(A_2)P(A_1^c)}_{\text{Detected by } A_2 \text{ Not detected by } A_1}$$

$$= (0.5)(1-0.3) + (0.3)(1-0.5)$$
$$= 0.5$$

c. P(A1/Detected by only one among them)

$$\frac{P(A_1 \cap A_2^c)}{P(\text{Detection by only one})}$$

$$= \frac{(0.5)(1-0.3)}{0.5}$$

$$= 0.7$$

PROBLEM2

Given in the question 8 thick bolts, 3 thin bolts, 5 medium bolts and 6 thick nuts, 4 medium nuts, 2 thin nuts

Let event A = nuts fits into bolt

$$P(A) = \frac{\text{NO. of ways for A to occur}}{\text{NO. of ways to draw 1 Nut, 1 bolt}}$$

The ways in which 1 Nut and 1 bolt can be drawn = 16 * 12

16 = Total bolts

12 = Total nuts

NOW, number of ways for A to occur is

no. of matching thick pair = 8 * 6

NO. of matching medium pair = 5 × 4

NO. of matching thin pair = 3 × 2

Number of ways for A to occur = 8×6 + 5×4 + 3×2

$$P(A) = \frac{(8\times6) + (5\times4) + (3\times2)}{16\times12}$$

$$= \frac{48 + 20 + 6}{192} = \frac{74}{192} = 0.385$$

PROBLEM 3

Given 12 components which are to be stacked into a cylindrical casing

a) all components are different

$$= 12! = 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 479001600$$

~~[illegible]~~. There are total 12! ways to design it all components are different

b) if 7 components are identical to one another, but the others are different then the number of different design configurations possible are $\frac{12!}{7!}$ ways

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$= 95040$ ways.

c) If three components are of one type and identical to each others and four components are of another type and identical to each other, but the others are different then number of different designs configurations are possible is equal $= \frac{12!}{3! \times 4!}$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 4 \times 3 \times 2 \times 1}$$

$= 3326400$ ways.

PROBLEM 4:

a. If eight people are randomly selected, then the probability that all have different birthdays is

$$P(A) = \frac{\text{diffrent birthdays}}{\text{Total possibilities}}$$

$$= \frac{365 \times 364 \times 363 \times 362 \times 361 \times 360 \times 359 \times 358}{(365)^8}$$

$$= 0.925$$

b. Atleast 2 having same birthday among '8' = P(B)

$$P(B) = 1 - P(A)$$
$$= 1 - 0.925$$
$$= 0.075$$

Problem 4 c 365

k	Days	Prob	1-prob
1	365	1	0
2	364	0.997260274	0.00274
3	363	0.991795834	0.008204
4	362	0.983644088	0.016356
5	361	0.972864426	0.027136
6	360	0.959537516	0.040462
7	359	0.943764297	0.056236
8	358	0.925664708	0.074335
9	357	0.905376166	0.094624
10	356	0.883051822	0.116948
11	355	0.858858622	0.141141
12	354	0.832975211	0.167025
13	353	0.805589725	0.19441
14	352	0.776897488	0.223103
15	351	0.74709868	0.252901
16	350	0.716395995	0.283604
17	349	0.684992335	0.315008
18	348	0.653088582	0.346911
19	347	0.620881474	0.379119
20	346	0.588561616	0.411438
21	345	0.556311665	0.443688
22	344	0.524304692	0.475695
23	343	0.492702766	0.507297

at k = 23 50-50 chance that at least two people will have the same birthday.

PROBLEM 5

given $N = 20$

$n = 6$

$r = 20 - 8 = 12$

$y = 6$

$N - r = 20 - 12 = 8$

$$P(y=6) = \frac{({}^{r}C_{y})({}^{N-r}C_{n-y})}{({}^{N}C_{n})}$$

$$= \frac{({}^{12}C_{6})({}^{20-12}C_{6-6})}{{}^{20}C_{6}}$$

$$= \frac{({}^{12}C_{6})({}^{8}C_{0})}{{}^{20}C_{6}}$$

$$= \left(\frac{12!}{6! \times 6!}\right) \Big/ \frac{20!}{6! \times 14!}$$

$$\Rightarrow \frac{12! \times 14!}{6! \times 20!} = \frac{12! \times 14!}{6! \times 20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14!}$$

$$= 0.0238$$

PROBLEM 6:

given in the question $\lambda = 7.2$ times/per month

a) $P(x \geq 5) = 1 - P(x=0) - P(x=1) - P(x=2) - P(x=3) - P(x=4)$

$$\Rightarrow 1 - \frac{(\lambda)^0 e^{-\lambda}}{0!} - \frac{(\lambda)^1 e^{-\lambda}}{1!} - \frac{(\lambda)^2 e^{-\lambda}}{2!} - \frac{(\lambda)^3 e^{-\lambda}}{3!} - \frac{\lambda^4 e^{-\lambda}}{4!}$$

Substituting the value of λ

$$\Rightarrow 1 - (7.2)^0 e^{-7.2} - (7.2 e^{-7.2} - \frac{(7.2)^2 e^{-7.2}}{2} = \frac{(7.2)^3 e^{-7.2}}{6} - \frac{(7.2)^4 e^{-7.2}}{4 \times 6}$$

$$\Rightarrow 1 - e^{-7.2}\left(1 + 7.2 + \frac{(7.2)^2}{2} + \frac{(7.2)^3}{6} + \frac{(7.2)^4}{4 \times 6}\right)$$

$$\Rightarrow 1 - e^{-7.2}(1 + 7.2 + 25.92 + 62.208 + 111.9744)$$

$$\Rightarrow 1 - e^{-7.2}(208.3024)$$

$$\Rightarrow 1 - (2.718)^{-7.2}(208.3024)$$

$$\Rightarrow 1 - 0.155 \Rightarrow 0.844$$

b) given $\lambda = 4.1$ times/month

$P(x \geq 5) = 1 - P(x=0) - P(x=1) - P(x=2) - P(x=3) - P(x=4)$

$$\Rightarrow 1 - \frac{(\lambda)^0 (e)^{-\lambda}}{0!} - \frac{(\lambda)^1 e^{-\lambda}}{1!} - \frac{(\lambda)^2 e^{-\lambda}}{2!} - \frac{(\lambda)^3 e^{-\lambda}}{3!} - \frac{(\lambda)^4 e^{-\lambda}}{4!}$$

$$\Rightarrow 1 - (e^{-\lambda})\left(1 + 4.1 + \frac{(4.1)^2}{2} + \frac{(4.1)^3}{6} + \frac{(4.1)^4}{24}\right)$$

$\hookrightarrow$ we got this by substituting $\lambda = 4.1$

$$\Rightarrow 1 - e^{-4.1}(1 + 4.1 + 8.405 + 11.486 + 11.774)$$

$$\Rightarrow 1 - e^{-4.1} * 36.765 \Rightarrow 1 - 0.609 = 0.39$$

c) Given $\lambda_1 = 7.2$ times / fail hr

$\lambda_2 = \cancel{\$}4.1$ times / fail hr

for 1 unit of failure cost incurred is 165 \$. The amount of savings recieved from change in machine from 1st to 2nd $= (7.2 - 4.1) \times 165 = 511.5\$$

$\Rightarrow \dfrac{12000}{511.5} = 23.4$ months to pay off.

~~PROBLEM~~.

Problem 7	Detected objects				
Individual object detec	0	1	2	3	4
0.75	0.0039	0.0469	0.2109	0.4219	0.3164
0.8	0.0016	0.0256	0.1536	0.4096	0.4096
0.85	0.0005	0.0115	0.0975	0.3685	0.5220
0.9	0.0001	0.0036	0.0486	0.2916	0.6561

PROBLEM 8:

given $\lambda = 0.05$ / per square foot

a) $p(x=0) = \frac{(\lambda)^0 (e)^{-\lambda}}{0!}$

$= \frac{(0.5)^0 (e)^{-0.5}}{1} \Rightarrow e^{-0.5}$

$\Rightarrow 0.606$

b) $p(x \geq 9) \Rightarrow p(x=9) + p(x=10)$

$\Rightarrow ({}^{10}C_9)(y)^9(1-y)^1 + {}^{10}C_{10}(y)^{10}(1-y)^0$

$\Rightarrow ({}^{10}C_9)(0.5)^9(0.5)^1 + {}^{10}C_{10}(0.5)^{10}(0.5)^0$

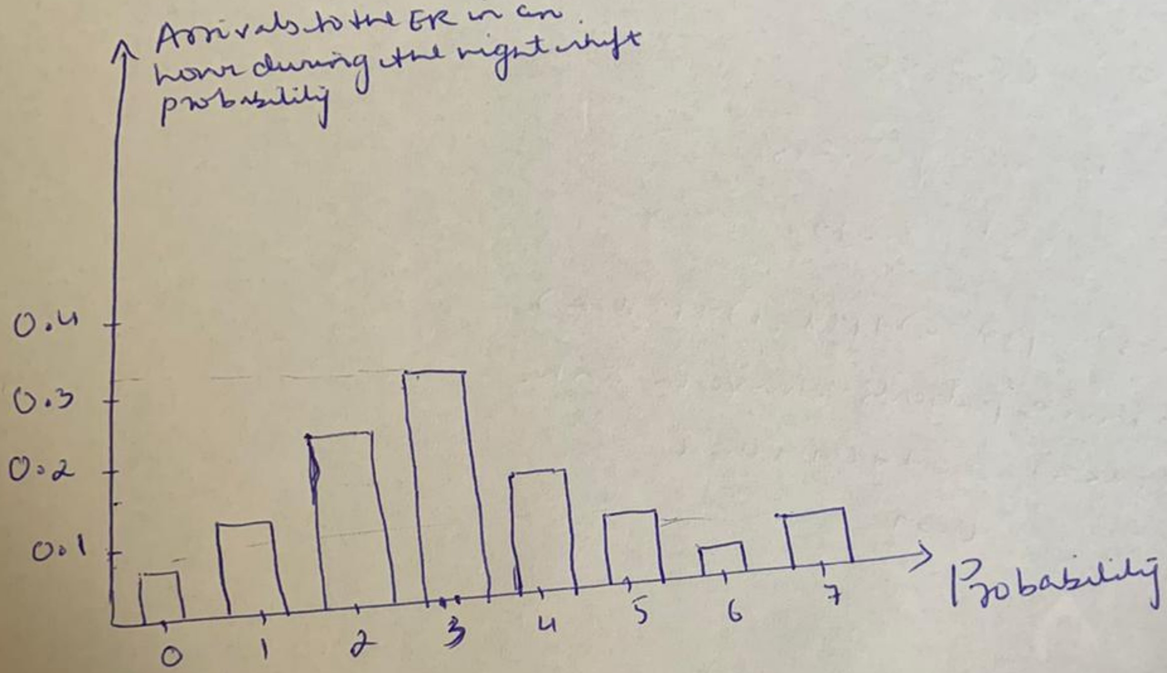
$\Rightarrow (0.5)^{10}({}^{10}C_9 + {}^{10}C_{10})$

$\Rightarrow (0.5)^{10}(10+1) \Rightarrow 11 * (0.5)^{10} \Rightarrow 0.0107$

PROBLEM 9

a)

Arrivals to the ER in an hour during the night shift	0	1	2	3	4	5	6	7
Probability	0.04	0.11	0.24	0.32	0.14	0.08	0.02	0.05



b) mean $\mu = \sum (i) P(i)$

$\Rightarrow (0)P(0) + 1 * P(1) + 2 * P(2) + 3 * P(3) + 4 * P(4) + 5 * P(5) + 6 * P(6) + 7 * P(7)$

$\Rightarrow 0 * 0.04 + 1 * 0.11 + 2 * 0.24 + 3 * 0.32 + 4 * 0.14 + 5 * 0.08 + 6 * 0.02 + 7 * 0.05$

$\Rightarrow 2.98$

$$\text{Variance} = \sigma^2 = \sum_{i=0}^{n} (y_i - \mu)^2$$

$$= (0.04-2.98)^2 + (0.11-2.98)^2 + (0.24-2.98)^2 + (0.32-2.98)^2$$

$$+ (0.14-2.98)^2 + (0.08-2.98)^2 + (0.02-2.98)^2 + (0.05-2.98)^2$$

$$\sigma^2 = 8.6436 + 8.2369 + 7.5076 + 8.0656 + 8.41 + 8.7616 + 8.5849$$

$$\sigma^2 = 65.2858$$

$$S.D = \sigma = \sqrt{65.2858}$$

$$S.D = 8.0799$$

c) $P(x<3) = P(x=0) + P(x=1) + P(x=2)$

Less than 3 patients arrive in an hr

$P(x<3) = 0.04 + 0.11 + 0.24$

$= 0.39$

PROBLEM 10

a) given $N = 387$

$r = 41$

$N - r = 387 - 41 = 346$

$n = 5$

$P(y<2) = P(y=0) + P(y=1)$

$$P(y=y) = \frac{({}^{r}C_{y})({}^{N-r}C_{n-y})}{{}^{N}C_{n}}$$

$$P(y<2) = \frac{({}^{41}C_{0})({}^{346}C_{5})}{{}^{387}C_{5}} + \frac{({}^{41}C_{1})({}^{346}C_{4})}{{}^{387}C_{5}}$$

$= 0.569 + 0.3413$

$= 0.9108$

b) given $N = 283$

$r = 245$

$N - r = 38$

$n = 3$

$$P(y=3) = \frac{({}^{245}C_{3})({}^{38}C_{0})}{({}^{283}C_{3})} = 0.647769$$

c) $\Rightarrow \dfrac{({}^{38}C_{1})({}^{41}C_{5})}{({}^{79}C_{6})}$

$\Rightarrow 0.1024$

here P(B) fail to meet standards = 38

P(A) fail to meet standards = 41

Total fail to meet standards = 79