ASSIGNMENT 2

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PROBLEM 1:

Given total number of rods = 500

Number of defective rods = 5

Therefore number of non defective rods = 500 - 5 = 495

a) To determine whether the probabily of selecting defective or non defective Johnson rods is independent can be found out by the concept of conditional probability

Lets say A = Event that the first rod selected is defective

B = Event that the second rod selected is defective

To check the independence, we can compare P(B) (the probability of B occuring) to P(B|A) (the probability of B occuring given that A has occured)

If P(B) = P(B|A) then the events are independent

$$P(B) = \frac{\text{Number of defective rods in the batch}}{\text{Total number of rods in the batch.}}$$

$$= \frac{5}{500} = 1/100$$

$$P(B|A) = \frac{(\text{Number of defective rods left in the batch})}{\text{Total number of rods left in the batch}}$$

$$= 4/499$$

Since P(B) is not equal to P(B|A), the probability of selecting defective rods is dependent to each other.

b) The probability that the second one selected is defective given that the first one was defective can be found out by conditional probability

$$P(B|A) = \frac{(\text{Number of ways to choose a defective second rod given the first is defective})}{\text{Total number of ways to choose the second rod})}$$

Since the first rod is already defective, there are 4 defective rods left and 499 rods left in total.

$$\Rightarrow \frac{4}{499}$$

c) Probability of both are defective $= \dfrac{5C_2}{500C_2}$

$$= \left(\frac{5\times4\times3!}{2!\times3!}\right) \Big/ \left(\frac{500\times499\times498!}{2!\times498!}\right)$$

$$= \left(\frac{5\times4}{2}\right) \Big/ \left(\frac{500\times499}{2}\right)$$

$$= \frac{5\times4}{500\times499}$$

$$= \frac{4}{499\times100}$$

$$= 1.6\times10^{-5}$$

d) Probability that both are nondefective $= \dfrac{495C_2}{500C_2}$

$$= \frac{495\times494}{2} \times \frac{2}{500\times499}$$

$$= \frac{495}{500} \times \frac{494}{499}$$

$$= 0.98$$

PROBLEM 2 :

Given $P(\text{width}) = 0.86$

$P(\text{width} \cap \text{Height} \cap \text{length}) = 0.8$

$P(\text{Width} \cap \text{Length} \cap \text{Height}^c) = 0.02$

$P(\text{width} \cap \text{Height} \cap \text{Length}^c) = 0.03$

$P(\text{width} \cup \text{Height}) = 0.92$

Let us represent the above as

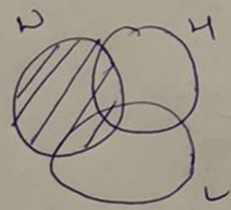
$P(\text{width}) = P(W) = 0.86$

$P(\text{width} \cap \text{Height} \cap \text{Length}) = P(W \cap H \cap L) = 0.8$

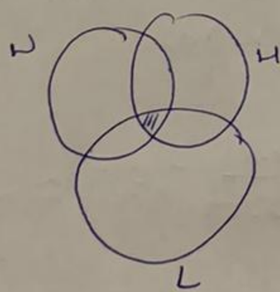
$P(\text{width} \cap \text{Length} \cap \text{Height}^c) = P(W \cap H^c \cap L) = 0.02$

$P(\text{width} \cap \text{Height} \cap \text{Length}^c) = P(W \cap H \cap L^c) = 0.03$

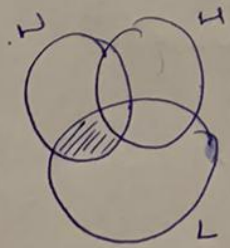
$P(\text{width} \cup \text{Height}) = P(W \cup H) = 0.92$



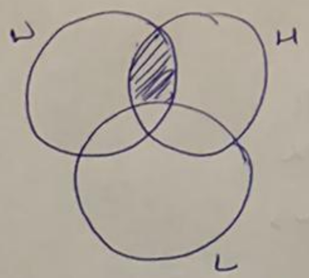
$= P(W) = 0.86 \longrightarrow (1)$



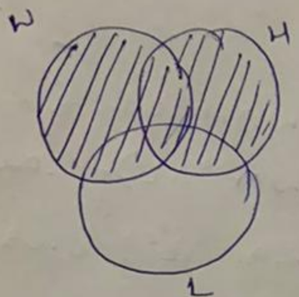
$P(W \cap H \cap L) = 0.8 \longrightarrow (2)$



$P(W \cap H^c \cap L) = 0.02 \longrightarrow (3)$



$$P(W \cap H \cap L^c) = 0.03 \longrightarrow 4$$



$$P(W \cup H) = 0.92 \longrightarrow 5$$

$$P(W|H) = \frac{P(W \cap H)}{P(H)}$$

$$= \frac{P(W \cap H \cap L) + P(W \cap H \cap L^c)}{P(H)}$$

$$= \frac{0.8 + 0.03}{P(W \cup H) - P(W) + P(W \cap H)}$$

$$= \frac{0.83}{0.92 - 0.86 + 0.83} = \frac{0.83}{0.89} = 0.932$$

PROBLEM 3 :

Let $P(E)$ = the probability that a particular beam will not be directly usable in commercial construction.

given in the question

$$P(A_1) = 0.20$$
$$P(A_2) = 0.20$$
$$P(A_3) = 0.60$$
$$P(E/A_1) = 0.1$$
$$P(E/A_2) = 0.12$$
$$P(E/A_3) = 0.04$$

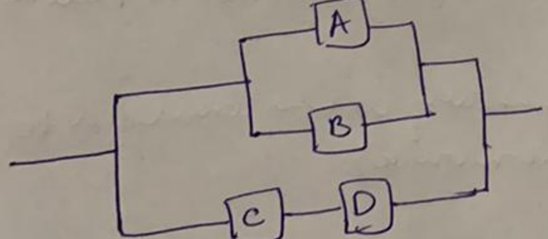
$$P(E) = P(E/A_1)P(A_1) + P(E/A_2)P(A_2) + P(E/A_3)P(A_3)$$
$$= 0.1 * 0.2 + 0.12 * 0.2 + 0.04 * 0.6$$
$$= 0.068$$

PROBLEM 4 :

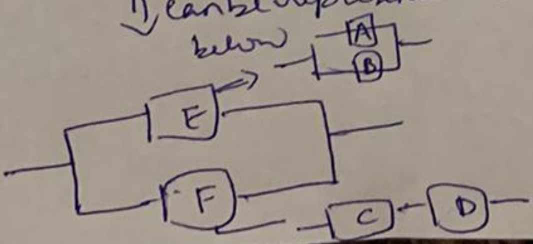
given $P(A) = 0.9$. $P(B) = 0.88$ $P(C) = 0.94$ $P(D) = 0.98$

a) $P(\text{system work}) = 1 - P(\text{system doesn't work})$

$$= 1 - P(S^c)$$
$$= 1 - P(E^c) * P(F^c)$$
$$= 1 - (P(A^c)P(B^c))(1 - P(C) * P(D))$$
$$= 1 - ((1-0.9)(1-0.88))(1-0.94*0.98)$$
$$= 1 - (0.1*0.12)(1-0.9212)$$
$$= 1 - 0.012 * 0.0788$$
$$= 0.999$$



It can be represented as below



b) The quantity for P (system work) if A alone works, B alone work, 'C and D' alone work, A, C and D works, B, C and D works, A and B works, all working. So there are 7 possibilities for system 'P' work.

PROBLEM 5:

$P(\text{Low risk}) = P(L) = 0.70$

$P(\text{medium risk}) = P(m) = 0.2$

$P(\text{High risk}) = P(H) = 0.1$

$P(\text{Not on Time}/\text{Low risk}) = P(T^c/L) = 0.03$

$P(\text{Not on Time}/\text{medium}) = P(T^c/M) = 0.11$

$P(\text{Not on Time}/\text{High}) = P(T^c/H) = 0.23$

a) $P(\text{Low risk and on Time})$

$$= P(L \cap T) = P(L)\,P(T/L)$$

$$= P(L)(1 - P(T^c/L))$$

$$= 0.7(1 - 0.03)$$

$$= 0.679$$

b) $P(T^c) = P(L)P(T^c/L) + P(m)P(T^c/m) + P(H)P(T^c/H)$

$$= 0.7 * 0.03 + 0.2 * 0.11 + 0.1 * 0.23$$

$= 0.066$ (Probability that a project is not completed on time)

c) $P(L/T^c) = \dfrac{P(T^c/L)(P(L))}{P(T^c)}$ (Probability it was considered to be low risk)

$$= \frac{0.03 * 0.7}{0.066}$$

$$= 0.318$$

PROBLEM 6:

Given in the question

$P(S_1) = 0.15$ $\quad$ $P(D/S_1) = 0.001$

$P(S_2) = 0.05$ $\quad$ $P(D/S_2) = 0.0003$

$P(S_3) = 0.10$ $\quad$ $P(D/S_3) = 0.0007$

$P(S_4) = 0.20$ $\quad$ $P(D/S_4) = 0.0060$

$P(S_5) = 0.12$ $\quad$ $P(D/S_5) = 0.0002$

$P(S_6) = 0.20$ $\quad$ $P(D/S_6) = 0.0002$

$P(S_7) = 0.18$ $\quad$ $P(D/S_7) = 0.001$

$\Rightarrow P(D) = P(D/S_1)P(S_1) + P(D/S_2)P(S_2) + P(D/S_3)P(S_3) + P(D/S_4)P(S_4) + P(D/S_5)P(S_5) + P(D/S_6)P(S_6) + P(D/S_7) \cdot P(S_7)$

$\Rightarrow 0.001 * 0.15 + 0.0003 * 0.05 + 0.0007 * 0.1 + 0.006 * 0.2 + 0.0002 * 0.12 + 0.0002 * 0.2 + 0.001 * 0.18$

$\Rightarrow 1.679 * 10^{-3}$

$$P(S_1/D) = \frac{P(D/S_1)\,P(S_1)}{P(D)}$$

$$= \frac{0.001 * 0.15}{1.679 * 10^{-3}} = 0.0893$$

$$P(S_2/D) = \frac{P(D/S_2)\,P(S_2)}{P(D)}$$

$$= \frac{0.0003 * 0.05}{1.679 * 10^{-3}} = 8.93 * 10^{-3}$$

$$P(S_3/D) = \frac{P(D/S_3)\,P(S_3)}{P(D)}$$

$$= \frac{0.0007 * 0.1}{1.679 * 10^{-3}} = 0.0416$$

$$P(S_4/D) = \frac{P(D/S_4)\,P(S_4)}{P(D)}$$

$$= \frac{0.006 * 0.2}{1.679 * 10^{-3}} = 0.714$$

$$P(S_5/D) = \frac{P(D/S_5)\,P(S_5)}{P(D)}$$

$$= \frac{0.0002 * 0.12}{1.679 \times 10^{-3}} = 0.0142$$

$$P(S_6/D) = \frac{P(D/S_6)\,P(S_6)}{P(D)}$$

$$= \frac{0.0002 * 0.2}{1.679 * 10^{-3}} = 0.0238$$

$$P(S_7/D) = \frac{P(D/S_7)\,P(S_7)}{P(D)}$$

$$= \frac{0.001 * 0.18}{1.679 * 10^{-3}} = 0.107$$

b) $P(D/S_1) = P(D/S_2) = P(D/S_3) = P(D/S_4) = P(D/S_5) = P(D/S_6) = P(D/S_7)$
$= 0.0005$

Let this be $P(D/S) = 0.0005$

$$P(D) = [P(S_1) + P(S_2) + P(S_3) + P(S_4) + P(S_5) + P(S_6) + P(S_7)] * P(D/S)$$

$$= [0.15 + 0.05 + 0.1 + 0.2 + 0.12 + 0.2 + 0.18] * 0.0005$$

$$= 0.0005$$

$$P(S_1/D) = \frac{P(D/S_1)\,P(S_1)}{P(D)} = \frac{0.0005 * 0.15}{0.0005} = 0.15$$

$$P(S_2/D) = \frac{P(D/S_2)\,P(S_2)}{P(D)} = \frac{0.0005 * 0.05}{0.0005} = 0.05$$

$$P(S_3/D) = \frac{P(D/S_3)\,P(S_3)}{P(D)} = \frac{0.0005 * 0.10}{0.0005} = 0.1$$

$$P(S_4/D) = \frac{P(D/S_4)\,P(S_4)}{P(D)} = \frac{0.0005 * 0.2}{0.0005} = 0.2$$

$$P(S_5/D) = \frac{P(D/S_5)\,P(S_5)}{P(D)} = \frac{0.0005 * 0.12}{0.0005} = 0.12$$

$$P(S_6/D) = \frac{P(D/S_6)\,P(S_6)}{P(D)} = \frac{0.0005 * 0.20}{0.0005} = 0.20$$

$$P(S_7/D) = \frac{P(D/S_7)\,P(S_7)}{P(D)} = \frac{0.0005 * 0.18}{0.0005} = 0.18$$

PROBLEM 7

Given in the question

Line 1 produced 500 non conforming cars
Line 2 produced 400 non conforming cars
Line 3 produced 600 non conforming cars.

a) The probability that the car was produce by line 1 $= \frac{500}{1500} = 1/3$

b) $P(\text{surface defect}) = \frac{0.10 + 0.08 + 0.15}{\text{Total}} \Rightarrow \frac{0.33}{0.15 + 0.12 + 0.20 + 0.50 + 0.44 + 0.4 + 0.21 + 0.28 + 0.24 + 0.01 + 0.08 + 0.15 + 0.04 + 0.08 + 0.01}$

the reason for the non conformance is a surface defect

$$\Rightarrow \frac{0.33}{3} = 0.11$$

c) p(Line3/surface defect) = $\frac{\text{P(Line3} \cap \text{surface defect)}}{\text{p(surface defect)}}$

$$= \frac{(0.15)/3}{0.11}$$

$$= \frac{0.05}{0.11} = 0.45$$

PROBLEM 8 :

P(workers attended the facility's training program meet the desired production quota) = 0.86

$$P(Q/T) = 0.86$$

P(workers do not attended the facility's training program meet the desired production quota) = 0.35

P(training program) = 0.80.

$$P(T) = 0.8$$

To find the probability that a new worker will meet the production quota.

$$P(Q) = P(Q/T)\,P(T) + P(Q/T^c)\,P(T^c)$$

$$= 0.86 * 0.8 + 0.35(1-0.8)$$

$$= 0.86 * 0.8 + 0.35 * 0.2$$

$$= 0.758$$

PROBLEM 9:

a) Given No. of rivets during an HRO operation = 25

$P(\text{Rivets Defective}) = P(R_D) = ?$

$P(\text{Seams Defective}) = P(S_D) = 0.2$

$P(\text{Seams working}) = P(S) = (P(R))^{25}$

$P(\text{Rivets working}) = P(R) = (P(S))^{1/25}$

$= (1-0.2)^{1/25}$

$= (0.8)^{1/25}$

$P(R_D) = 1 - P(R)$

$= 1 - (0.8)^{1/25}$

$= 8.88 * 10^{-3}$

b) $P(SD) = 0.10$

$P(R_D) = 1 - P(R)$

$= 1 - (1 - P(S_D))^{1/25}$

$= 1 - (1 - 0.1)^{1/25}$

$= 1 - (0.9)^{1/25}$

$= 4.205 * 10^{-3}$

PROBLEM 10:

$P(A) = 0.8$

$P(B) = (0.2$

$P(A/D) = \frac{P(D/A)\,P(A)}{P(D)}$

$P(B) = 0.2$

$P(D/B) = 0.03$

$P(B/D) = \frac{P(D/B)\,P(B)}{P(D)}$

$$P(D) = P(D/A)\,P(A) + P(D/B)\,P(B)$$

$$= 0.05 * 0.8 + 0.03 * 0.2$$

$$= 0.046$$

$$P(A/D) = \frac{P(D/A)\,P(A)}{P(D)}$$

$$= \frac{0.05 * 0.8}{0.046}$$

$$= 0.869$$

$$P(B/D) = \frac{P(D/B)\,P(B)}{P(D)}$$

$$= \frac{0.03 * 0.2}{0.046}$$

$$= 0.13$$

PROBLEM 11:

a) $P(\text{Rainy condition}) = \dfrac{226+6}{21+226+228+7+185+0+6+6+3+10}$

$$= \frac{232}{692} = 0.335$$

b) $P(\text{missed intruder}) = \dfrac{6+6+3+10}{692}$

$$= \frac{25}{692} = 0.0361$$

c) $P(\text{clear and windy during test}) = P(\text{clear}) + P(\text{windy}) - P(\text{clear and windy})$

$$= \frac{21}{692} + \frac{185+10}{692} - 0 = 0.312$$

d) $P(\text{detected intruder} / \text{snowy}) = \dfrac{P(\text{Snowy} \cap \text{detected intruder})}{P(\text{Detected intruder})}$

$$= \frac{7}{21+228+226+7+185}$$

$$= \frac{7}{667} = 0.010$$

e) $P(\text{missed intruder} / \text{cloudy condition}) = \dfrac{P(\text{missed intruder} \cap \text{cloudy})}{P(\text{cloudy})}$

$$= \frac{6}{228+6}$$

$$= \frac{6}{234} = 0.025$$