Mini Project 1 (Solution)

Mini Project Duo Group # 12
Contribution of each group member
Chetan Siddappareddy – 50%
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Both of us have contributed equally to the project. We learnt R through collaboration and then write the R scripts for the corresponding and report all the findings.

Section 1

a) Given $f_T(t)$ in the question, the probability that the lifetime of the satellite exceeds 15 years can be computed by integrating $f_T(t)$ over 15 to ∞ as follows:

$$P(T > 15) = \int_{15}^{\infty} f_T(t)dt = \int_{15}^{\infty} (0.2\exp(-0.1t) - 0.2\exp(-0.2t))dt$$

$$=0.2\int_{15}^{\infty} \exp(-0.1t)dt - 0.2\int_{15}^{\infty} \exp(-0.2t)dt$$

$$=(0.2*\exp(-0.1*15)/0.1) - \exp(-0.2*15)$$

$$=0.44626 - 0.04978$$

$$=0.39647$$

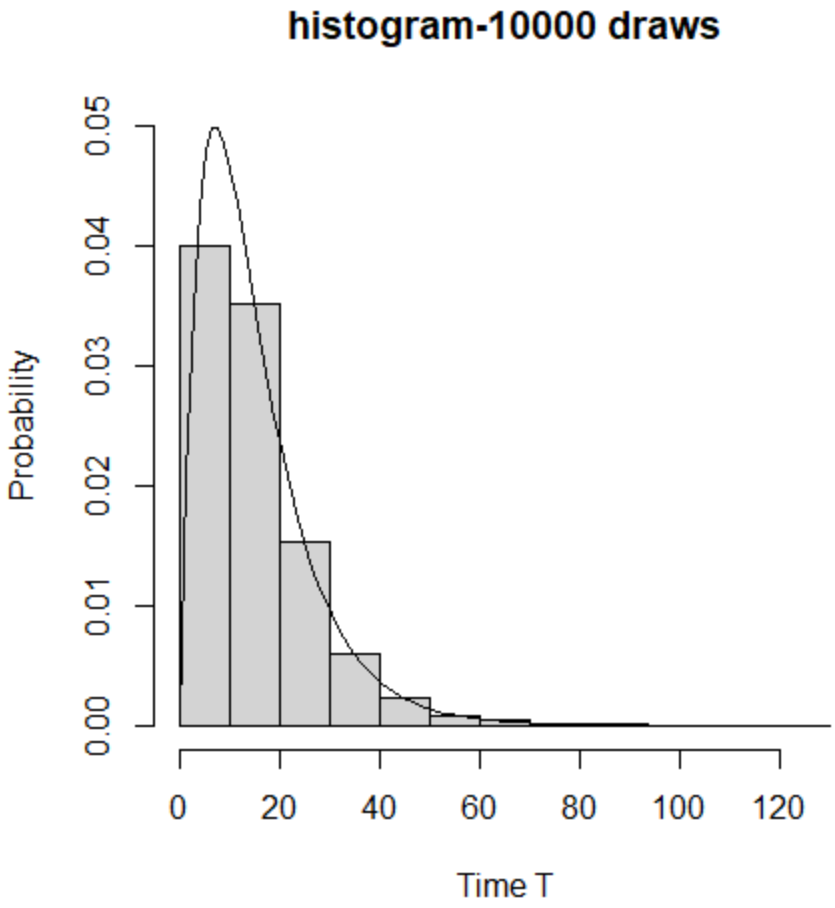
$$\approx 0.3965$$

b)

```
> ############################
> # R code for Section 1b #
> ############################
>
> # i. below is probability density function
> density.fun <- function(x){   return(0.2*exp(-0.1*x) - 0.2*exp(-0.2*x))}
> xOne <- replicate(1, max(rexp(1, 0.1), rexp(1, 0.1)))
> set.seed(100)
> # ii. simulating 10000 draws from T
> xtenK <- replicate(10000, max(rexp(1, 0.1), rexp(1, 0.1)))
```

xtenK	num [1:10000] 31.19 7.98 16.14 11.26 22.17 ...

```
> # iii. creating a histogram
> hist(xtenK, prob = T, ylim = c(0, 0.05), xlab = "Time T", ylab = "Probability", main = "histogram-10000 draws")
> # superimposing the density function of T
>
> curve(density.fun, from = 0, to = max(xtenK), add = T)
>
```



Both the curve and histogram sees the peak at time 15, which happens to be the expected value of T.

> # iv. Estimating E[T] using saved draws
> mean(xtenK)
[1] 15.04825
>

The simulated value 15.04825 is close to the actual value of E[T], which is 15.
> # v. Estimating prob using saved draws
> 1-pexp(15, rate = 1 / mean(xtenK))
[1] 0.3690609
>

The simulated value 0.3690609 is close to the calculated value of probability that the satellite lasts more than 15 years, which is 0.3965.

> # vi. Repeating the process four more times
> # Test 1 – 10000 draws
> xtenK <- replicate(10000, max(rexp(1, 0.1), rexp(1, 0.1)))
> mean(xtenK)
[1] 14.95094
> 1-pexp(15, rate = 1 / mean(xtenK))
[1] 0.3666743
>
> # Test 2 - 10000 draws
> xtenK <- replicate(10000, max(rexp(1, 0.1), rexp(1, 0.1)))
> mean(xtenK)
[1] 14.90976
> 1-pexp(15, rate = 1 / mean(xtenK))
[1] 0.3656596
>
> # Test 3 - 10000 draws
> xtenK <- replicate(10000, max(rexp(1, 0.1), rexp(1, 0.1)))
> mean(xtenK)
[1] 15.1616
> 1-pexp(15, rate = 1 / mean(xtenK))
[1] 0.3718214
>
> # Test 4 – 10000 draws
> xtenK <- replicate(10000, max(rexp(1, 0.1), rexp(1, 0.1)))
> mean(xtenK)
[1] 14.88354
> 1-pexp(15, rate = 1 / mean(xtenK))
[1] 0.3650121
>

Results Table for 1b:

Number of draws – 10000	**E(T)**	**P(T>15)**
First Run	15.04825	0.3690609
Test 1	14.95094	0.3666743
Test 2	14.90976	0.3656596
Test 3	15.1616	0.3718214
Test 4	14.88354	0.3650121

Upon repetition, the expected value and probability values are varying in the same manner, either increasing or decreasing, but remains in the proximity of the actual or calculated values in 1a. This justifies the central limit theorem.

c)

```
> ############################
> # R code for Section 1c #
> ############################
>
> # Test 1 - 1000 draws - oneK => 1000
> xOneK <- replicate(1000, max(rexp(1, 0.1), rexp(1, 0.1)))
> hist(xOneK, prob = T, ylim = c(0, 0.05), xlab = "Time T", ylab = "Probability", main = "histogram-
1000 draws")
> mean(xOneK)
[1] 14.99978
> 1-pexp(15, rate = 1 / mean(xOneK))
[1] 0.3678741
>
> # Test 2 - 1000 draws - oneK => 1000
> xOneK <- replicate(1000, max(rexp(1, 0.1), rexp(1, 0.1)))
> hist(xOneK, prob = T, ylim = c(0, 0.05), xlab = "Time T", ylab = "Probability", main = "histogram-
1000 draws")
> mean(xOneK)
[1] 14.46897
> 1-pexp(15, rate = 1 / mean(xOneK))
[1] 0.3546224
>
> # Test 3 - 1000 draws - oneK => 1000
> xOneK <- replicate(1000, max(rexp(1, 0.1), rexp(1, 0.1)))
> hist(xOneK, prob = T, ylim = c(0, 0.05), xlab = "Time T", ylab = "Probability", main = "histogram-
1000 draws")
> mean(xOneK)
[1] 14.59851
> 1-pexp(15, rate = 1 / mean(xOneK))
[1] 0.3578997
>
> # Test 4 - 1000 draws - oneK => 1000
> xOneK <- replicate(1000, max(rexp(1, 0.1), rexp(1, 0.1)))
> hist(xOneK, prob = T, ylim = c(0, 0.05), xlab = "Time T", ylab = "Probability", main = "histogram-
1000 draws")
> mean(xOneK)
[1] 15.2096
> 1-pexp(15, rate = 1 / mean(xOneK))
[1] 0.3729841
>
> # Test 5 - 1000 draws - oneK => 1000
> xOneK <- replicate(1000, max(rexp(1, 0.1), rexp(1, 0.1)))
> hist(xOneK, prob = T, ylim = c(0, 0.05), xlab = "Time T", ylab = "Probability", main = "histogram-
1000 draws")
```

```
> mean(xOneK)
[1] 15.15149
> 1-pexp(15, rate = 1 / mean(xOneK))
[1] 0.371576
>
> # Test 1 - 100000 draws - hunK => 100000
> xhunK <- replicate(100000, max(rexp(1, 0.1), rexp(1, 0.1)))
> hist(xhunK, prob = T, ylim = c(0, 0.05), xlab = "Time T", ylab = "Probability", main = "histogram-
hunK draws")
> mean(xhunK)
[1] 14.95058
> 1-pexp(15, rate = 1 / mean(xhunK))
[1] 0.3666655
>
> # Test 2 - 100000 draws - hunK => 100000
> xhunK <- replicate(100000, max(rexp(1, 0.1), rexp(1, 0.1)))
> hist(xhunK, prob = T, ylim = c(0, 0.05), xlab = "Time T", ylab = "Probability", main = "histogram-
hunK draws")
> mean(xhunK)
[1] 15.02762
> 1-pexp(15, rate = 1 / mean(xhunK))
[1] 0.3685562
>
> # Test 3 - 100000 draws - hunK => 100000
> xhunK <- replicate(100000, max(rexp(1, 0.1), rexp(1, 0.1)))
> hist(xhunK, prob = T, ylim = c(0, 0.05), xlab = "Time T", ylab = "Probability", main = "histogram-
hunK draws")
> mean(xhunK)
[1] 15.0187
> 1-pexp(15, rate = 1 / mean(xhunK))
[1] 0.3683378
>
> # Test 4 - 100000 draws - hunK => 100000
> xhunK <- replicate(100000, max(rexp(1, 0.1), rexp(1, 0.1)))
> hist(xhunK, prob = T, ylim = c(0, 0.05), xlab = "Time T", ylab = "Probability", main = "histogram-
hunK draws")
> mean(xhunK)
[1] 15.01801
> 1-pexp(15, rate = 1 / mean(xhunK))
[1] 0.3683208
>
> # Test 5 - 100000 draws - hunK => 100000
> xhunK <- replicate(100000, max(rexp(1, 0.1), rexp(1, 0.1)))
> hist(xhunK, prob = T, ylim = c(0, 0.05), xlab = "Time T", ylab = "Probability", main = "histogram-
hunK draws")
```

```
> mean(xhunK)
[1] 14.9987
> 1-pexp(15, rate = 1 / mean(xhunK))
[1] 0.3678476
```

Results Table for 1c:

Number of draws – 1000	**E(T)**	**P(T>15)**
Test 1	14.99978	0.3678741
Test 2	14.46897	0.3546224
Test 3	14.59851	0.3578997
Test 4	15.2096	0.3729841
Test 5	15.15149	0.371576
Number of draws – 100000	**E(T)**	**P(T>15)**
Test 1	14.95058	0.3666655
Test 2	15.02762	0.3685562
Test 3	15.0187	0.3683378
Test 4	15.01801	0.3683208
Test 5	14.9987	0.3678476

It can be inferred from the above results of mean and probability estimation using 100000 draws that there is not much deviation from the corresponding manually calculated values in section 1a. However, as the number of draws reduces, say 1000, the deviation of mean E[T] and P(T > 15) from the manually computed values increases.

Section 2

From the given hint, consider that a circle is inscribed in a square of unit area. Square of unit area can be simulated by taking independent draws from a Uniform (0, 1) distribution for each of x and y- coordinates. Intersection point of diagonals of the unit square and center of circle coincides at (1/2, 1/2).

The probability of a point 'p' falling in the circle inscribed in a unit square is as follows:

$$P(p) = \frac{\text{Area of circle}}{\text{Area of sqaure}} = \frac{\pi}{4}$$

This implies that π is four times P(p).
For a point to be within the circle, it must be within the radius of the circle. Radius of the circle is 0.5. Hence to get the probability of points within the circle

$$P(p) = P\left(\sqrt{(x-0.5)^2 + (y-0.5)^2} \leq 0.5\right)$$

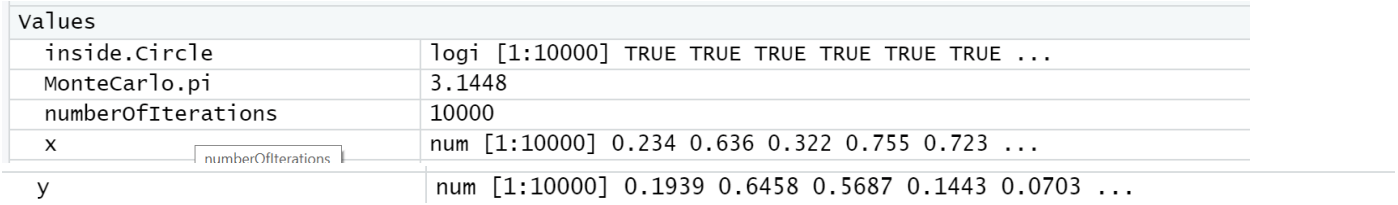
$$\pi = P(p) * 4$$

Below is the RCode implemented based on above Monte Carlo approach.

> ############################
> # R code for Section 2 #
> ############################

```
> numberOfIterations <- 10000
> x <- runif(numberOfIterations, min = 0, max = 1)
> y <- runif(numberOfIterations, min = 0, max = 1)
> inside.Circle <- (sqrt((x-0.5)^2 + (y-0.5)^2) <= 0.5)
> MonteCarlo.pi <- (sum(inside.Circle)/numberOfIterations)*4
> MonteCarlo.pi
```

[1] **3.1448**



Values	
inside.Circle	logi [1:10000] TRUE TRUE TRUE TRUE TRUE TRUE ...
MonteCarlo.pi	3.1448
numberOfIterations	10000
x	num [1:10000] 0.234 0.636 0.322 0.755 0.723 ...
y	num [1:10000] 0.1939 0.6458 0.5687 0.1443 0.0703 ...

The value of π simulated using R is 3.1448 which is very much closer to the actual value of π 3.14159.