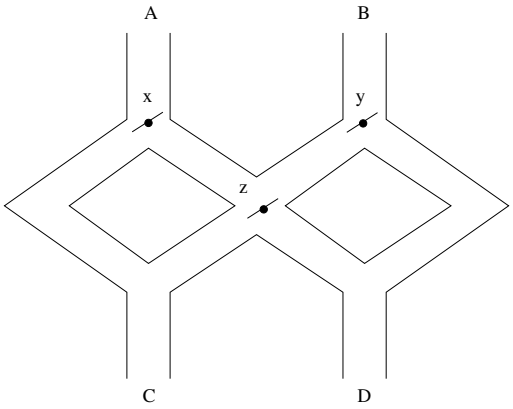
Automata Theory and Computability

Assignment 1

(Total marks 60. Due on Tue 23rd Jan 2024)

1. Give a DFA for the language of all strings over the alphabet $\{0, 1\}$ which contain an occurrence of 010 or 100 as a contiguous substring. (*5*)

2. Consider the toy shown below. A marble is dropped in at either A or B and the position of the levers x, y, and z cause it to come out at either C or D. Each time a marble hits a lever, it causes the lever to change direction, so that the next marble coming its way will take the opposite branch. Thus if the levers of the toy are initially as shown in the diagram, a marble dropped in at B will come out at C but if a marble is dropped in at B a second time it will come out at D.



   Model the toy as an automaton which runs on inputs 'A' and 'B' (representing a marble dropped in at A and B respectively), and which accepts all sequences of inputs in which the last marble comes out at D. (*10*)

3. Show that the set of strings in $\{0, 1, 2\}^*$ which are base 3 representations of even numbers, is regular. (*10*)

4. Let $\mathcal{A} = (Q, \delta, s, F)$ be a DFA over an alphabet A. Give a formal proof that for any strings $x, y \in A^*$: (*5*)

$$\widehat{\delta}(q, xy) = \widehat{\delta}(\widehat{\delta}(q, x), y).$$

5. Consider the language of nested C-style comments. The aphabet comprises characters "`/`", "`*`", and "`c`" (the latter symbol representing any ASCII character apart from "`/`" and "`*`"). The language allows all well-nested and complete comments. Thus strings like "`cc/*ccc*/c`" and "`cc/*cc/*ccc*/c*/ccc`" are in the language, but not "`cc/*c/*cc*/cc`". Is this language regular? Justify your answer. (*10*)

6. For a set of natural numbers X, define $binary(X)$ to be the set of binary representations of numbers in X. Similarly define $unary(X)$ to be the set of "unary" representations of numbers in X: $unary(X) = \{1^n \mid n \in X\}$. Thus for $X = \{2, 3, 6\}$, $binary(X) = \{10, 11, 110\}$ and $unary(X) = \{11, 111, 111111\}$.

   Consider the two propositions below:

   (a) For all X, if $binary(X)$ is regular then so is $unary(X)$.

   (b) For all X, if $unary(X)$ is regular then so is $binary(X)$.

   One of the statements above is true and the other is false. Which is which? Justify your answer. (*20*)