26/30

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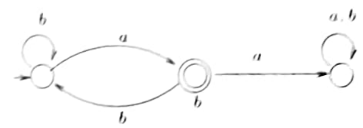
Automata Theory and Computability

Quiz 1 (2024)

Time: 30 minutes. Total marks: 30

Instructions: Write your answers neatly and to the point in the space provided below each question. If necessary write your answers in rough first. Do not attach rough sheets.

1. Consider the DFA over the alphabet $\{a, b\}$ given below.



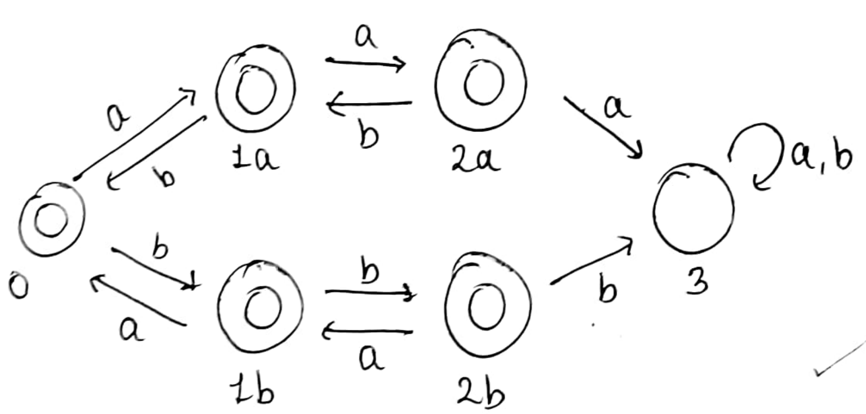
Describe the language accepted by the automaton. (5)

5

The language is ~~a~~ the set of strings of a's & b's where there are no consecutive a's and the last letter of the string is 'a'. ✓

2. Consider the language of all strings over the alphabet $\{a, b\}$ which satisfy the property that in *every prefix* the difference between the number of a's and b's is at most 2. Thus, $aabab$ is in the language, while $abaaab$ is not. Give the state diagram of a DFA for this language. Label your states meaningfully. (10)

10



✓ good.

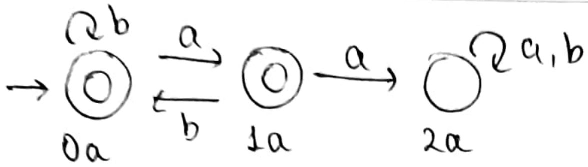
0 → no difference
1a → # a = # b + 1
2a → # a = # b + 2
1b → # b = # a + 1
2b → # b = # a + 2

3. For a language $L \subseteq A^*$, the language of prefixes of L, denoted $pref(L)$, is defined to be the set

$$\{u \in A^* \mid \exists v \in A^* \text{ such that } u \cdot v \in L\}.$$

(a) Give a DFA that accepts the language $pref(L(\mathcal{A}))$ for the DFA $\mathcal{A}$ of the Q 1. (5)

(5)



0a → last letter b
1a → last letters ab
2a → last letters aa
We have to prevent consecutive a's

(b) Prove that the class of regular languages is closed under the prefix operation. (10)

6

Consider DFA

We need to show that there is a ~~unique~~ DFA which accepts pref(L(A))

Let DFA for ~~L~~ A be (Q, s, δ, F)

Consider the DFA$_1$ $\tilde{A} = (Q, s, \delta, F \cup s \cup t)$

where t is the set of ~~stat~~ intermediate states from which there is a finite path to a final state.

Don't add s.

Claim: $L(\tilde{A}) = pref(L(A))$. $uv \in L(A)$

$w \in pref(L(A)) \Rightarrow_1 \exists v \in A^* \text{ st } \hat{\delta}(\hat{\delta}(s, w), v) \in F$

$\Rightarrow \delta(s, w) \in F \cup \{s\} \cup t$

$\Rightarrow w \in L(\tilde{A})$

$\therefore pref(L(A)) \subseteq L(\tilde{A})$

$w \in \mathcal{L}(\tilde{A}) \Rightarrow \delta(s, w) \in F \cup S \cup t$

$\Rightarrow \exists v_1 \in A^* \text{ st } \delta(\delta(s,w), v) \in F$] why?

$\Rightarrow w \in \text{pref}(\mathcal{L}(A))$

$\therefore \mathcal{L}(\tilde{A}) \subseteq \text{pref}(\mathcal{L}(A))$

$\therefore \mathcal{L}(\tilde{A}) = \text{pref}(\mathcal{L}(A))$

Thus $\text{pref}(\mathcal{L}(A))$ ~~is~~ ~~closed~~ has unique DFA $(Q, s, \delta, F \cup S \cup t)$

4/10 good attempt