Automata Theory and Computability

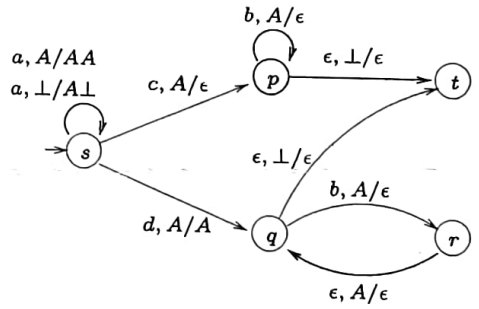
Final Examination 2024

Max Marks: 70, Weightage: 40%, Time: 3 hours. Write your answers neatly, briefly, and to the point. If required write your answers on a rough sheet first. There is no need to write proofs, except when explicitly asked to justify your answer.

1. A *run* in a string is a maximal contiguous subsequence of length 1 or more of the same letter. For example the string "*aabaaaabb*" has runs (of a's) of length 2 and 4, and runs (of b's) of length 1 and 2; but no run of length 3. Give a DFA for the language of all strings over the alphabet $\{a, b\}$ in which every run is of length 3 or more. *(5)*

2. Give a context-free grammar for the language of strings over $\{0, 1\}$ which have strictly more 1's than 0's. *(5)*

3. Consider the PDA M below, which runs on the input alphabet $A = \{a, b, c, d\}$, has stack alphabet $\Gamma = \{\perp, A\}$, and accepts by empty stack. Describe the language accepted by the PDA. *(5)*



4. Consider the language L of all strings of the form

$$w_1 \# w_2 \# \cdots \# w_n \quad (n \geq 1)$$

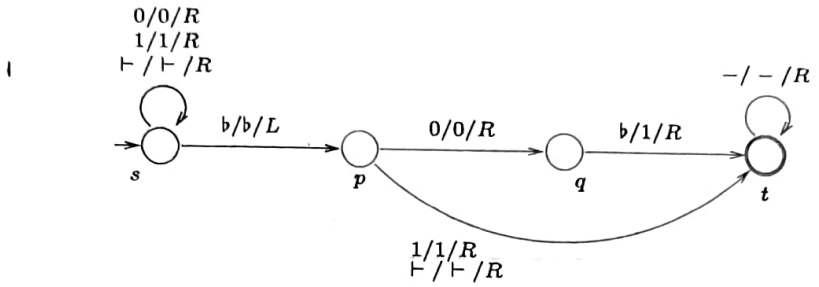
where each $w_i \in \{a, b\}^+$ and all w_i's are of the same length. Is L a context-free language? Justify your answer. *(10)*

5. Recall that two languages L and L' over an ordered alphabet A are called "letter equivalent" iff $\psi(L) = \psi(L')$, where ψ is the Parikh map corresponding to A. In the questions below assume that A is the alphabet $\{a, b\}$. *(10)*

   (a) Prove every CFL L over A is letter-equivalent to a regular language R over A.

   (b) Is it true that *every* language over A is letter-equivalent to a regular language over A? Justify your answer.

6. Describe the function on binary strings computed by the Turing machine M below. (5)



7. Let L and M be arbitrary languages over the alphabet $\{a, b\}$ such that L is recursive and M is recursively enumerable. Then which of the following statements *must* be true?

   (a) $L \cap M$ is recursive.

   (b) $L \cap M$ is recursively enumerable but not necessarily recursive.

   (c) $L \cap M$ is not recursively enumerable.

   Justify your answer. (5)

8. Consider the languages containing encodings of Turing machines below. One of them is recursively enumerable and the other is not. Which is which? Justify your answer. (10)

   - TOTAL $= \{enc(M) \mid M$ halts on all inputs$\}$
   - PARTIAL $= \{enc(M) \mid M$ halts on some input$\}$

9. Is it decidable whether a given CFG accepts a *regular* language? Justify your answer. (10)

10. In the proof of Gödel's Incompleteness Theorem done in class, an important step is to be able to pick out the digit at a particular position in a number. Give a formula, $3digit(v, y, a, b, c)$, which says that in the base-7 representation of the number v, the 7-ary digits at position y, $y + 1$, and $y+2$ are a, b and c respectively. Assume that the position y is represented as a power of 7, and position 0 corresponds to the least significant 7-ary digit. (5)