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Quiz 2

UM 205: Introduction to Algebraic Structures (Winter 2023-24)
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10

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1. Let $n \in \mathbb{N}, n \geq 3$. Suppose you have to place $n$ balls in $n-2$ boxes. Prove that you can find either one box with at least 3 balls or two boxes with at least 2 balls.

2. A restaurant offers five different soups, four main courses, and three desserts. You decide to order at most one soup, at most one main course, and at most one dessert. In how many ways can you do this?

1) Let us label the boxes to be $a_1, \ldots, a_{n-2}$. Now, introduce another box $a_0$ in this. We distribute $n$ balls in $a_0, a_1, \ldots, a_{n-2}$ boxes and then add the contents of box $a_0$ to $a_1$.

5

Since we have $n$ balls and $n-1$ boxes, by the Pigeon hole principle, we have atleast one box with 2 balls. Let that be $a_i$. ✓

If we have more than one box with two balls, or one with 3, we are done. If we have exactly one, and none with 3, then every box has 1 ball but $a_i$ has 2. Now put balls from $a_0$ to $a_1$. If $\#a_1 = 1$ (ie $i \neq 1$), then $\#a_1 + \#a_0 \geq 2$. If $\#a_1 = 2$ (ie $i = 1$), then $\#a_1 + \#a_0 = 3$. Hence we always get two boxes with two balls or one with three.

2) 5 soups, 4 main course, 3 desert.

No. of ways for atmost 1 soups $= \binom{5}{0} + \binom{5}{1} = 6$

" " 1 main course $= \binom{4}{0} + \binom{4}{1} = 5$

" " 1 desert $= \binom{3}{0} + \binom{3}{1} = 4$

Now, all these three acts are independent. So, no of ways to choose a meal is the product of the three.

∴ No of meals $= 6 \times 5 \times 4$

$= \boxed{120}$ ✓

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