UM 204: QUIZ 1

Jan. 12, 2024

Duration. 15 minutes

Maximum score. 10 points

You are allowed to compute limits of real sequences without proof.

Problem. Let $A : \mathbb{Q} \to [0, \infty)$ be an absolute value function on $\mathbb{Q}$, i.e.,

(1) $A(x) = 0$ if and only if $x = 0$,

(2) $A(xy) = A(x)A(y)$ for all $x, y \in \mathbb{Q}$,

(3) $A(x + y) \leq A(x) + A(y)$ for all $x, y \in \mathbb{Q}$.

Suppose there is a $C > 0$ such that $A(n) \leq C$ for all $n \in \mathbb{N}$. Show that

$$A(x + y) \leq \max\{A(x), A(y)\}, \quad \forall x, y \in \mathbb{Q}.$$

Hint. Estimate $A\left((x + y)^m\right)$ *from above, take the* m^{th} *root, and take limits as* $m \to \infty$.