Lecture - 1

UM - 205, Office $\rightarrow$ X - 15, Mail $\rightarrow$ arvind@iisc.ac.in

math.iisc.ac.in/~arvind/um205 ; Prereq :- UMA 101, 102

Grading & Announcements on Teams.

6 or 7 Quizzes.
The lowest will be dropped.

Midterm :- Week of Feb 17^{th} (Tentative)

Grading :-

Quizzes :- 20%
Mid :- 30%
End :- 50%

$\rightarrow$ Closed Books, Closed Notes, No electronic devices.

Relative Grading

Office Hours :- Tuesday 4 to 5 pm

Algebraic Structures :-

- Set Theory
- Combinatorics
- Graph Theory
- Number Theory
- Group Theory

Aims :-
- To deal with structures formally.
- To learn the axiomatic methods.
- To build complex structures from simpler ones.
- To learn how to prove intuitively obvious statements.

Natural Numbers :-

$$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$$

$\rightarrow$ We will develop '$\mathbb{N}$' using these axioms.

* Peano Axioms [Reference :- 1, Analysis - I by T. Tao
2, Naive Set Theory]

$\rightarrow$ Two fundamental constructs :-

1, 0, the number zero

2. A successor function (++)

Axioms :-

1. $0 \in \mathbb{N}$
2. If $n \in \mathbb{N}$, then so does $n{+}{+}$

So, $0{+}{+} \in \mathbb{N}$, $(0{+}{+}){+}{+} \in \mathbb{N}$, ...

As a matter of notation, let $0{+}{+} = 1$, $1{+}{+} = 2$, etc.

As of now, we could have $2{+}{+} = 0$ [We don't want this to happen]

3. 0 is not the successor of any natural number, i.e., $n{+}{+} \neq 0 \ \forall \ n \in \mathbb{N}$

Exercise :- Prove $4 \neq 0$

$4 = 3{+}{+}$ and $3 \in \mathbb{N}$

$\Rightarrow \ 3{+}{+} \neq 0 \Rightarrow \boxed{4 \neq 0}$

One can have $5{+}{+} = 3$ or $5{+}{+} = 5$ [We don't want this to happen]

4. Different natural numbers have different successors, i.e., if $n{+}{+} = m{+}{+}$, then $n = m$

Amazingly, this is still not enough !!

Eg:- $\{0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \text{......}\}$

5. (Principle of Mathematical Induction)

Let $P(n)$ be any 'property' for $n \in \mathbb{N}$. Suppose $P(0)$ is True & suppose $P(n{+}{+})$ is True whenever $P(n)$ is true, then $P(n)$ is True $\forall n$.

Property :- It is a statement or an assertion. Eg :- n is even, n is odd, etc.

Finally, there exists a number system $\mathbb{N}$, whose elements we will call natural numbers, for which axioms 1–5 hold

—— X ——