The Duckworth-Lewis Method

E0 259 Data Analytics

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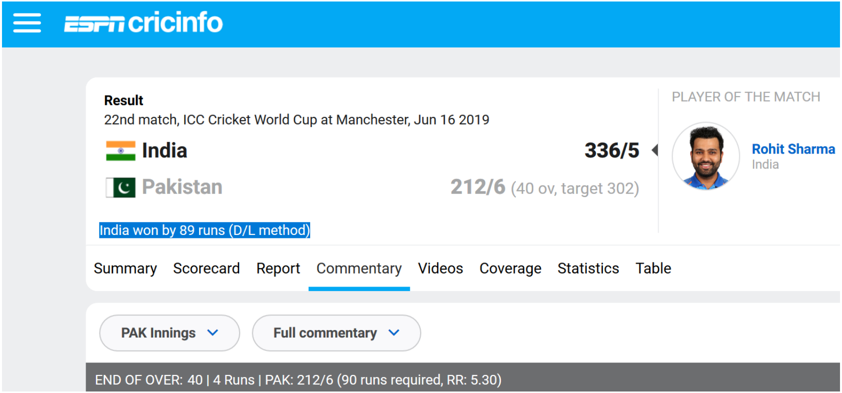
The Indo-Pak match, ICC World Cup 2019



- 3.24pm The drizzle has gotten heavier ...
- 3.30pm Pakistan's 20-over target: 97/0, 109/1, 125/2 or 146/3 ... Do you preserve wickets, do you go for the dash and risk losing a wicket or two?

Our goal is to understand how this comes about

One-day Cricket

- Each team plays an innings.
  Both get 50 overs and have 10 wickets.
- The team batting first (Team 1) tries to maximise its score. The bowling team (Team 2) tries to restrict this score.
- The bowling team (Team 2) then gets to bat, and tries to reach this score.
- Shorter version than 'test cricket' with a win/lose outcome.

Matches affected by bad weather

- Bad weather often leads to interesting twists and turns in test cricket, but is not tolerated in result-oriented one-day games. (Ind-Eng 2021 series test 1: Eng 183 & 303, Ind 278 & 52/1)
- There simply isn't time, unlike in test cricket, for the match to continue another day, though reserve days have been used on occasions. (India-NZ WC 2019 semifinal.)
- "Draw" is not a good outcome in knock-out competitions.
- Continue with a shortened match and revised targets.
  Decide a winner based on **the state of the match** if play can't continue.
- First, some prior approaches through examples.

The average run-rate method

- Third-final of the 1988/89 Benson and Hedges World Series Cup between Australia and The West Indies.
- AUS scored 226/4 off 38 overs. Two hours delay during Australia's innings. (Dean Jones 93 n.o.)
- WI still needed 180 off 31.2 overs when rain again stopped play for 1 hour 25 minutes.
- Target revised to 61 off the remaining 11.2 overs. (108 in 18 overs) Criterion used: Average run-rate.
- Most sides would achieve this target. WI (with Haynes and Richards) won easily with 4.4 overs remaining.
- WI had it too easy. Why?
- Post-match, Border called for a revision of the regulations; Richards was happy the existing regulations.

Another example - hypothetical

- Suppose Team 1 plays 50 overs and scores 250.
  Run-rate is 5 per over.

- Team 2 replies, and is 120/0 off 25 overs when rain stops play.

- Who is the winner?

- Par score under ARR method is $25 \times 5 = 125$, or 126 to win.
  ARR method: Team 2 loses. Is it fair?
  What if 120/2?
  120/9?

The 'most productive overs' or MPO method

- 1992 Cricket World Cup Semifinal: ENG vs. RSA.
- ENG made 252/6 off 45 overs.
- RSA were 231/6 and needed 22 runs off 13 balls when rain stopped play for 12 minutes.
- RSA target revised to 22 runs off 7 balls, then 21 runs off 1 ball. Criterion used: Most productive overs.
- The two good overs that RSA bowled were struck off. RSA was being penalised for bowling those overs well. (Actually, Wessels went slow and denied ENG the five final overs of acceleration because innings was scheduled to end latest by 6:10 pm).
- Christopher Martin-Jenkins on radio immediately after the game: "Surely someone, somewhere could come up with something better."

Enter F.C.Duckworth and A.J.Lewis

- The WI/AUS game during the 1988/89 BH WSC:
  - WI initial target would have been 232 off 38 overs due to the 2 hour interruption during AUS innings.
  - WI revised target would have been 139 off 11.2 overs after the second interruption.
  - AUS could have been more aggressive if they had known that their innings were shorter.
  - WI had many wickets in hand and could afford a risk of a much faster scoring rate.

- The RSA/ENG game during the 1992 WC:
  - RSA would need 2 to tie and 3 to win in 1 ball.
    (Updated version: 3 to tie, 4 to win in 1 ball.)
  - Both are reasonable targets.

The kinds of interruptions

- Before first team's innings ... (shorten the game)
- During first team's innings ...
  Repeated interruptions possible
- In between the two innings ...
- During second team's innings ...
  Repeated interruptions possible ... (shorten + revise target)
- Stoppage with no resumption ... (determine winner)

D/L method since 1997

- D/L method was tried out first on 01 Jan 1997, ZIM vs. ENG.
  ZIM scored 200 in 50 overs.
  Rain during ENG innings reduced the game to 42 overs.
  ARR target 169.
  D/L target 186.
  ENG scored 179 off 42 and lost (D/L method).

- Has been the preferred method, with some modifications, for resetting targets in shortened games.

- Other methods: ARR; MPO;
  Discounted MPO 0.5% for every over lost;
  PARAB provides diminishing returns for overs in terms of runs;
  Adaptation of PARAB in WC1996 (ignores wickets in hand);
  CLARK method applies different rules for different kind of stoppages;
  VJD method, nearest contender to D/L.

What is desired of a good method (from the D/L paper)

- Revision must be fair to both sides.
  “Relative positions of the two teams should be the same after the interruption as they were before it.”
- Must provide sensible results in all situations.
  Recall RSA-ENG semifinal of 1992.
- Should be independent of the first team’s scoring pattern ... because it is so before the interruption.
- Easy to apply, requiring no more than a table of numbers and a pocket calculator.
- Understandable by all those involved - players, officials, spectators, reporters.

The latest version deviates from the last two principles.

The basis: two resources and their valuation

- The batting side has two resources at their disposal to set a target: *overs to go and wickets in hand*.
- Both matter, and in a combined way.
  Twenty overs when ten wickets are in hand is much more valuable than when only 1 wicket is in hand.
- Team 2's target must be reset based on its *resources* before the interruption and after the interruption, such that *relative positions of the two teams should be the same before and after interruption*.
- View total runs that can be scored as a function of overs to go (u) and wickets in hand (w) as the net value of the resources.
  Call it $Z(u, w)$.
  Come up with a model, and then fit to data.

The run production function

- Suppose a team has all 10 wickets and starts playing.
  They play to set a score target, but don't have any over restrictions.
  The total runs scored before they lose all ten wickets is a random quantity.

- Let Z_0 be its average.

- Model for the average score in u overs:

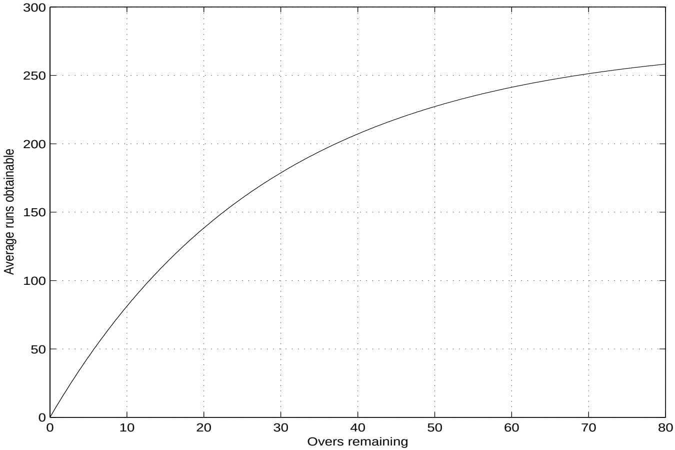
$$Z(u) = Z_0[1 - \exp\{-bu\}].$$

- A plot with $Z_0 = 275, b = 0.035$ ...

- Average first innings score in 50 overs:
  217 (since 1971) and 235 (2005 - 2015)

Run production function example

- Model for the average score in u overs: $Z(u) = Z_0[1 - \exp\{-bu\}]$. A plot with $Z_0 = 275, b = 0.035$



Getting the curve

- How would you get this curve?
- Find all data points with ...
  50 overs remaining, all 10 wickets in hand: get data points for runs scored in 50 overs;
  49 overs remaining, all 10 wickets in hand: get data points for runs scored in 49 overs;
  48 overs remaining, all 10 wickets in hand: get data points for runs scored in 48 overs;
  ...
  Fit a curve, and extrapolate

Nonlinear regression

- For a candidate (Z_0, b), given the data: $D = (u_1, y_1), \ldots, (u_K, y_K)$
- Error: $\ell(Z(u_i; Z_0, b), y_i)$ for the ith data point
  - Usually squared difference: $\ell(\hat{y}, y) = (\hat{y} - y)^2$
  - Across all data points:

$$L(Z_0, b; D) = \sum_{i=1}^{K} \ell(Z(u_i; Z_0, b), y_i)$$

- Curve fitting:

$$(Z_0^*, b^*) = \arg\min_{Z_0, b} L(Z_0, b; D)$$

Resource fraction

- Resources available at the start of an N overs game: $Z(N)$.
  Call this 1 unit.

- Fraction of resources available when only u overs remain is then:

$$\frac{Z(u)}{Z(N)}.$$

What if wickets have fallen?

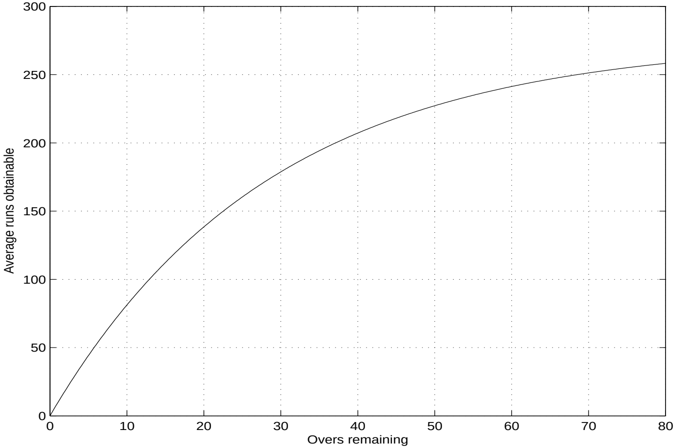
- A revised relationship:
  If u overs to go and w wickets in hand, then

$$Z(u, w) = Z_0(w)[1 - \exp\{-b(w)u\}].$$

- $Z_0(w)$ depends on the number of wickets in hand.
- Similarly, growth rate too depends number of wickets in hand.
- Anticipate $Z_0(10) > Z_0(9) > \cdots > Z_0(1)$.
- What about the growth rate parameter?

More on the run production function

- $Z(u, 10) = Z_0(10)[1 - \exp\{-b(10)u\}]$.
  A plot with $Z_0(10) = 275, b(10) = 0.035$



- Suffices to know $Z(\infty, 10)$ and $L := \partial_u Z(u, 10)|_{u=0} = b(10)Z_0(10)$.

Rate of increase, a connection across curves

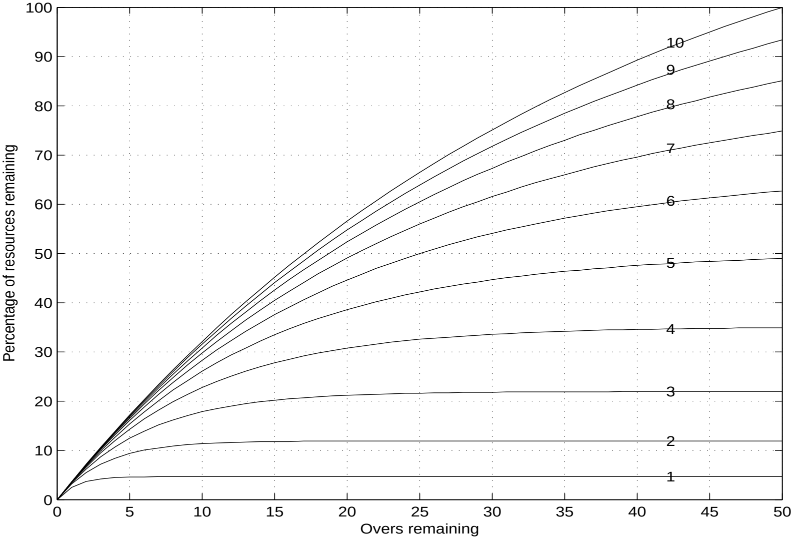
- If only one ball remains, regardless of the number of wickets in hand, anticipate that the (average) increment to the score is the same.
- OK assumption if a good batsman is on strike.
- In mathematical terms, with slope denoted L:

$$Z(u, w) = Z_0(w)[1 - \exp\{-Lu/Z_0(w)\}].$$

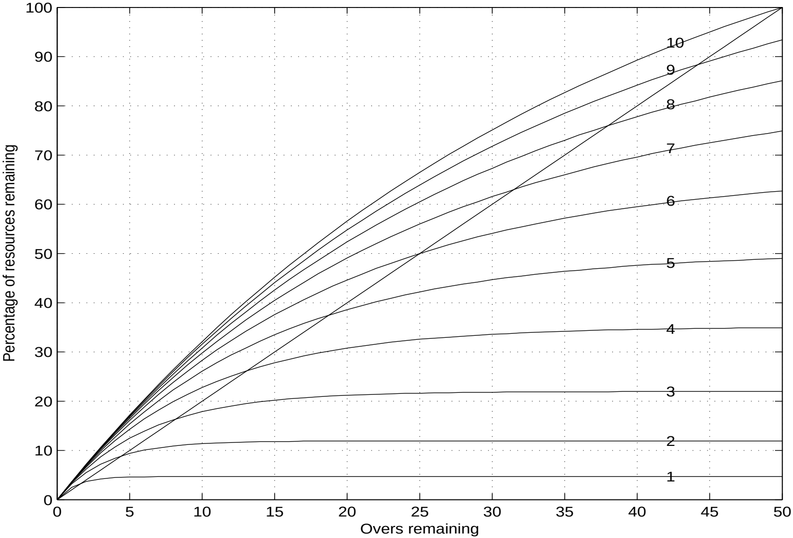
- Fraction of resources remaining

$$P(u, w) := \frac{Z(u, w)}{Z(N, 10)}.$$

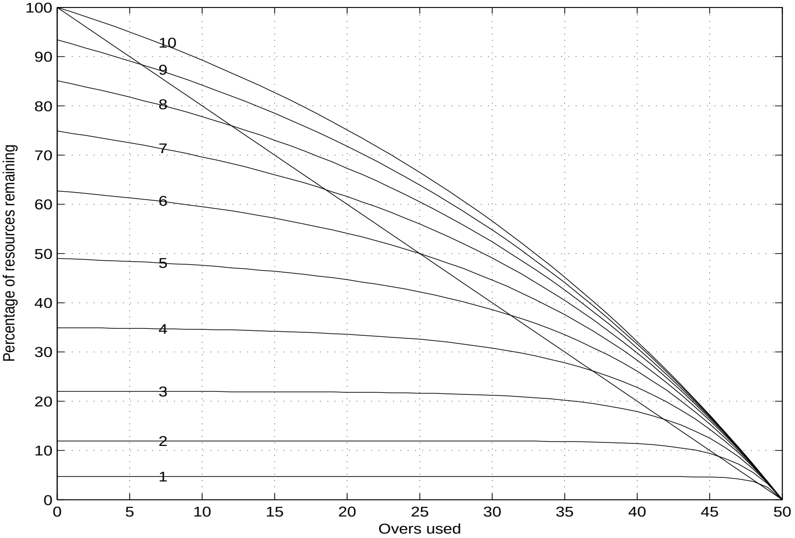
The resources remaining



The resources remaining, and a valuation ignoring wickets



The overs used picture



D/L method: Interruptions in the second team's innings

- Team 1 played 50 overs and scored S runs. They used up 100% of their resources to do this.
- Team 2 replies, and has w wickets in hand and u overs remaining, when rain interrupts play.
  When play resumes, team 2 has only v overs where $v < u$.
- Team 2 has been deprived of $u - v$ overs. They still have w wickets.
- Proportion of the resources R_2 available for use by team 2:

$$R_2 = \underbrace{1 - P(u, w)}_{\text{fraction used up before interruption}} + \underbrace{P(v, w)}_{\text{fraction remaining}}$$

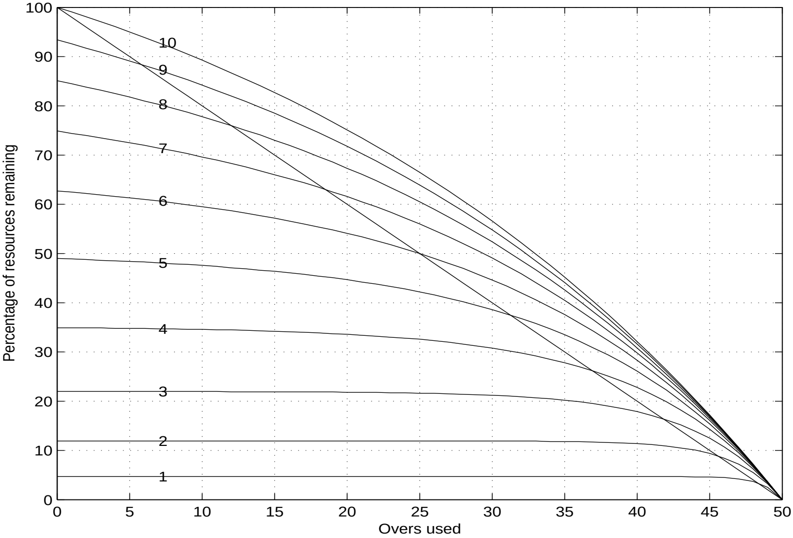
- Par score $T = SR_2$. Target is the next integer.

Example applications from indicated curves

Team 1 scores 250 off 50 overs.
Team 2's innings stopped at 25 overs.

	I	II	III
Team 2 score at stoppage	120/0	120/5	120/9
Resources lost	0.66	0.42	0.05
R_2	0.34	0.58	0.95
$T = SR_2$	85	145	247.5

The overs used picture



Interruption and resumption

Team 1 scores 250 off 50 overs.
Team 2's innings interrupted at $u = 25$ overs.
When play resumes, team 2 has $v = 10$ overs.

	I	II	III
Team 2 score	120/0	120/5	120/9
Resources rem. at stoppage	0.66	0.42	0.05
Resources rem. at resumption	0.34	0.26	0.05
R_2	1-0.66+0.34	1-0.42+0.26	1-0.05+0.05
	= 0.68	= 0.84	= 1.00
$T = SR_2$	170	210	250

Prima facie, it seems unfair: The weaker have to cross a higher target.

What would be your strategy?

- Australia scored 250 runs off 50 overs.
- India goes in to bat. Forecast says rain in about 1.5 hours.
- What would be your strategy?
- What did you try to optimise?

Stoppages during the first innings

- If the interruption occurs before start of play, there is no issue.
  If 20 overs in total are lost, the game is shortened by 10 overs, and both teams know this before they begin.
- But often, interruptions occur *during* the first innings.
- Match officials still try to arrange that both sides play the same number of overs.
- But Team 1 started out thinking 50 overs, and suddenly, find that their innings is shortened.
  Team 2 knows, from the start of their innings, that it is shortened.
- Whose loss is greater?
- Mostly Team 1's loss is greater, except ... when they have already lost a lot of wickets.

The D/L method in such interruptions

- Let R_1 be the proportion of the resources available to Team 1:

  $$R_1 = 1 - P(u, w) + P(v, w)$$

  if a stoppage occurs when Team 1 has u overs left and w wickets in hand, and at resumption, had lost $u - v$ overs.

- Similarly compute R_2.

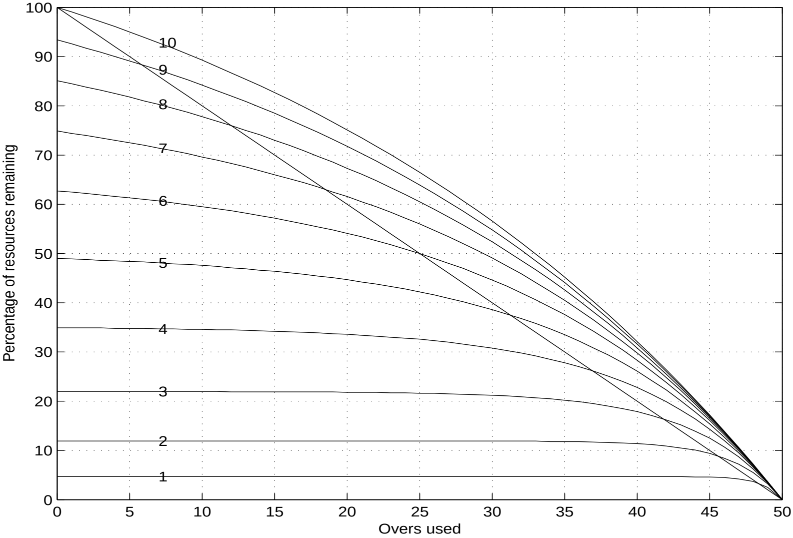
- If $R_2 \leq R_1$, then $T = SR_2/R_1$.

- We will deal with the other case soon enough.

Premature termination of first innings (D/L paper)

- IND vs. PAK, Singer Cup, Singapore, April 1996.
- IND scored 226/8 off 47.1 overs out of 50 overs when rain terminated IND innings.
- PARAB method gave PAK a target of 186/33 overs.
- D/L method:
  - IND used up $R_1 = 0.919$ fraction of their resources and lost 0.081.
  - PAK had $R_2 = 0.815$ fraction of resources available.
  - Since $R_2 < R_1$, D/L par-score is $SR_2/R_1 = 200.42$, or 201 off 33 to win.
- PAK won easily (PARAB target) with 30 balls to spare. (SRT 100, Aamer Sohail and Saeed Anwar 70s).

The overs used picture: for quick reference



Of course, there are some issues ...

$R_2 > R_1$ anomaly, example from D/L paper

- Team 1 scores 80/0 off 10 overs. Rain reduces match to 10 overs.
- $R_1 \approx 1 - 0.9 = 0.1$.
- $R_2 \approx 0.34$. $R_2 > R_1$.
- Clearly, team 2 must have a higher target. But what target?
- $SR_2/R_1 \approx 80 \times 0.34/0.1 = 272$ off 10 overs!
- Can the *well above average* scoring rate really be sustained for 50 overs?

An inelegant fix

- If $R_2 > R_1$, then

$$T = S + G(N) \times (R_2 - R_1).$$

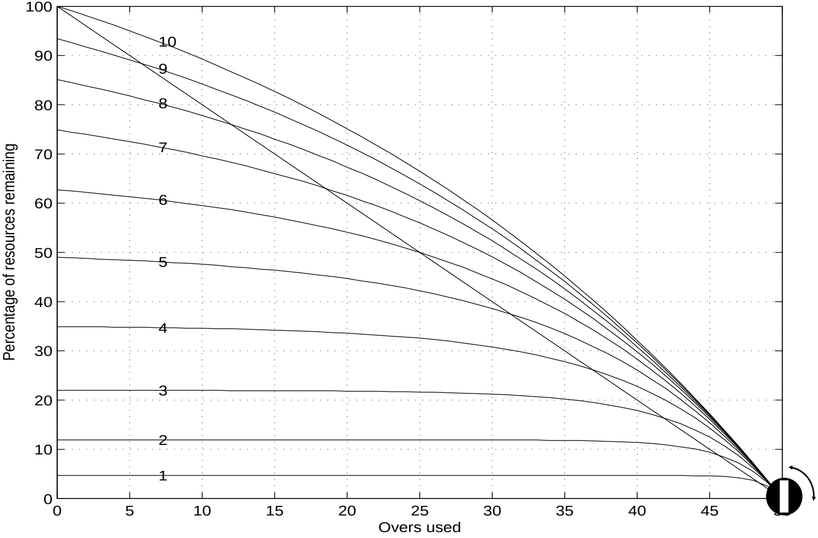
- $G(N)$ is the average first innings score in an N over match. (Compare with $G(N)$ replaced by S/R_1.)
- $G(50)$ was
  225 during 1999-2002,
  235 during 2002-2009, and
  245 in the last version.
  Also, it was different for ICC full member nations, associates, under-19.
- The Professional Edition (DLS) fixes this in a more elegant way.

A recent example

- WC2019, semifinal IND vs. NZ.
- NZ 211/5 in 46.1 overs when rain caused stoppage.
- Had NZ's innings been terminated then and if IND had 46 overs,
  - NZ would have used $R_1 = 0.86$.
  - IND's $R_2 = 0.96$ if 46 overs.
  - Target for 46 overs: $T = 211 + 245 \times (0.96 - 0.86) = 235.5$. (The latest D/L puts it at 237).

Fix high first innings scores: D/L Professional Edition

Introduce dependence of curves on the first innings score.



The fix for high first innings scores

- Higher the first innings score, closer to a straight line.
- Resources remaining after 25 overs is lesser,
  if Team 2 gets only 25 overs, R_2 is lower,
  so $T = SR_2$ is lower.
- Again, the choice of parameters based on data and is in the Professional Edition.

A discussion on the "relative positions" criterion

- The relative positions of the two teams before and after interruption should be the same.
- Think about the hypothetical IND-AUS game. What was your strategy? What did you try to optimise?
- One appealing criterion is *isoprobability*:
  The (estimated) probability of winning before and after the interruption must be the same.
- Does D/L satisfy this?

An example to bring home the point

- 20 July 2003, Cambridge vs. Oxford, at the Lord's.
- Cambridge scored 190 off their 50 overs.
- Oxford were 162/1 off 31 overs when rain interrupted play.
  (29 to win off 19 overs)
- When rain stopped, 12 overs remained.
  But Oxford had already exceeded any target that D/L would set.
  Oxford was declared winner by D/L method.
- Before the rain, Oxford had a huge advantage, but cricket is a game of 'glorious uncertainties'.
  The probability of Oxford winning was not 1. Yet, after the interruption, Oxford was declared winner at resumption.
- Isoprobability criterion would have given Cambridge a positive chance to bowl Oxford out. Low probability, but still positive.
  Would spectators have preferred that? Players?

An insightful pair of comparable games

- Two adjacent grounds A and B hosting two matches.
  Both Teams 1 scored 250 off 50 overs.
  Both Teams 2 played 20 overs, lost 3 wickets, when it rained.
  10 overs lost due to rain, and now 20 overs remain.
- Team 2A: 120/3
  Team 2B: 50/3.
- Before the break, Teams 2A and 2B needed 131 and 201 (resp.) off 30 overs with 7 wickets in hand.
- But since both teams used up the same amount of resources, and get the same (reduced) resources at resumption, their D/L targets are identical: 221.
  - Team 2A must score 101 off 20
  - Team 2B must score 171 off 20.
- More difficult for Team 2B. D/L improved the advantage for the team that was ahead before the interruption.

Isoprobability criterion

- Isoprobability targets: Team 2A - 228, Team 2B - 216.
  - Team 2A must score 108 off 20. (7 runs more than D/L).
  - Team 2B must score 166 off 20. (5 runs less than D/L).

- A case of "from each according to their ability"?

Adding a third match (Carter and Guthrie)

Three adjacent grounds A, B, C hosting three matches.
Teams 1A and 1B scored 250 off 50 overs.
Team 1C scored 180 off 50 overs.
All Teams 2 played 20 overs, lost 3 wickets, when it rained.
10 overs lost due to rain. 20 overs remain.

	Team 1 score	Team 2 at int	Target at int	D/L target	D/L to-go	IsoP target	IsoP to-go
A	250	120/3	131 (in 30)	221	101 (20)	228	108 (20)
B	250	50/3	201 (in 30)	221	171 (20)	216	166 (20)
C	180	50/3	131 (in 30)	159	109 (20)	158	108 (20)

- Isoprobability: Teams 2A and 2C must go the same distance.
  D/L: Team 2A has it easier.
- Team 2A was the poorer bowling team. D/L gives it a discount.
- Is that "cricket"?

Incentive to alter strategy under D/L rule

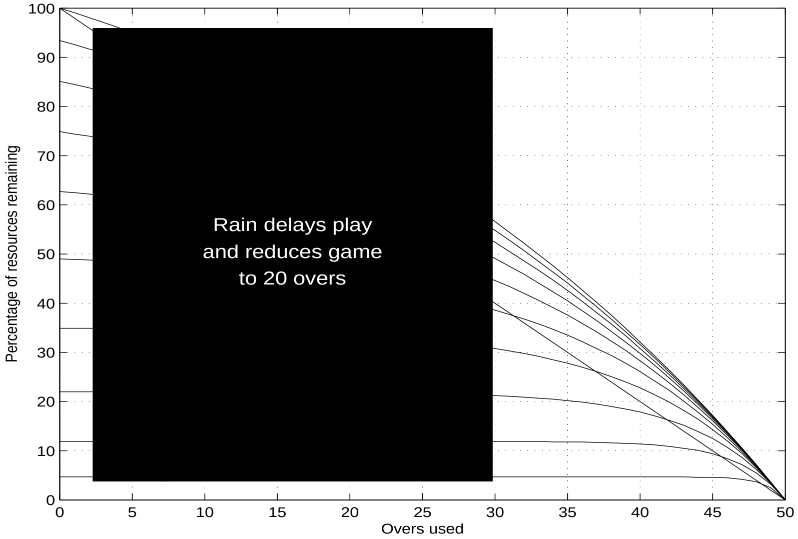
- Team 1 is at 160/4 in 39 overs.
  Rain is expected in the next over. Predicted duration of rain is such that their innings will be terminated, and Team 2 will have about 40 overs.

- Consider two options for Team 1:
  (i) bat carefully and lose no further wickets (Team 2 target 206)
  (ii) bat to maximise score but lose two wickets (Team 2 target 194).

- We'd like Team 1 will bat normally as in an uninterrupted match, and maximise their expected score.

- Will Team 1 do this?
  Team 1's sole objective is also maximising its chances of winning.
  D/L method is likely to distort their strategy.
  Is it "cricket" that Team 1 plays contrary to maximising runs?
  What about ARR method?

Applying D/L to T20



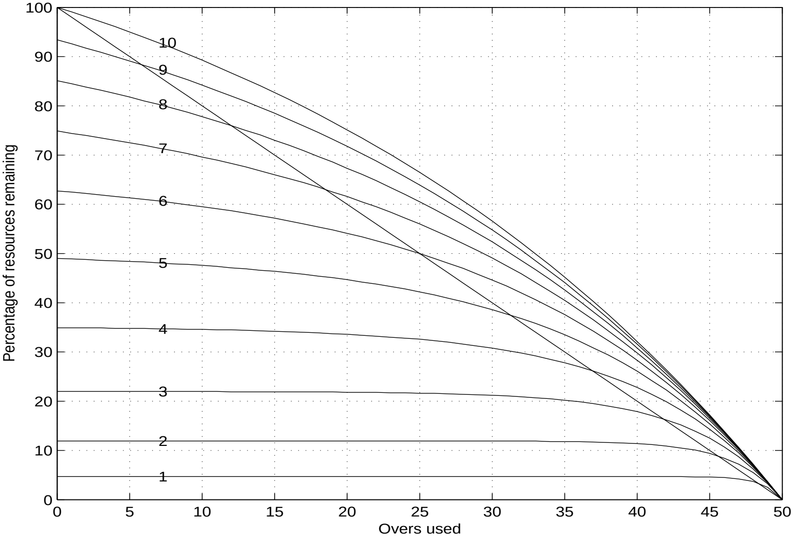
Applying D/L to T20

- Curves are a lot flatter - D/L is now much closer to ARR method.
- ICC World T20: ENG-WI (May 2010).
- ENG 191/5 off 20. WI 30/0 in 2.2 overs. Rain stops play.
  At resumption, WI target reduced to 60 (in 6 overs).
- If rain had come before start of WI's play, RR method target = 58 (in 6 overs).
  But D/L target is only 66 (in 6 overs).
  Too much of an advantage for WI.
- But WI did much better. They consumed very little, and lost quite a bit of resources to rain.
  Revised target was much smaller.
- Is there a fix? ... Shrink the curves. Revised targets for shrink-the-curves method:
  Rain at start: 87 off 6 overs.
  Rain as in game: 69 (in 6 overs).
  The VJD method apparently fares better than D/L here.

Applying D/L to T20: A recent example

- IND vs. AUS on 07 October 2017
- AUS 118/8 in 18.4 overs. Rain stops play.
  At resumption only 6 overs available for IND.
- ARR method PAR score $= 118/(18.4\text{overs}) \times 6 = 37.9$.
- Truncate-the-curves: 20 overs 10 wickets $= 0.57$ remain.
  $R_1 = 0.57 - 0.04 = 0.53, \qquad R_2 = 0.20.$
  PAR score $= 0.20/0.53 \times 118 = 44.5$.
- Shrink-the-curves: Equivalent overs lost in 50 overs:
  $8\ b/120\ b \times 50 = 3.2$ overs.
  Equivalent overs for IND is $6/20 \times 50 = 15$ overs.
  $R_1 = 1 - 0.07 = 0.93, \qquad R_2 = 0.45.$
  PAR score $= 0.45/0.93 \times 118 = 57.1$.
- D/L target for the match somewhere in-between: 48 from 6 overs.

The overs used picture for calculations



Another issue with the D/L method

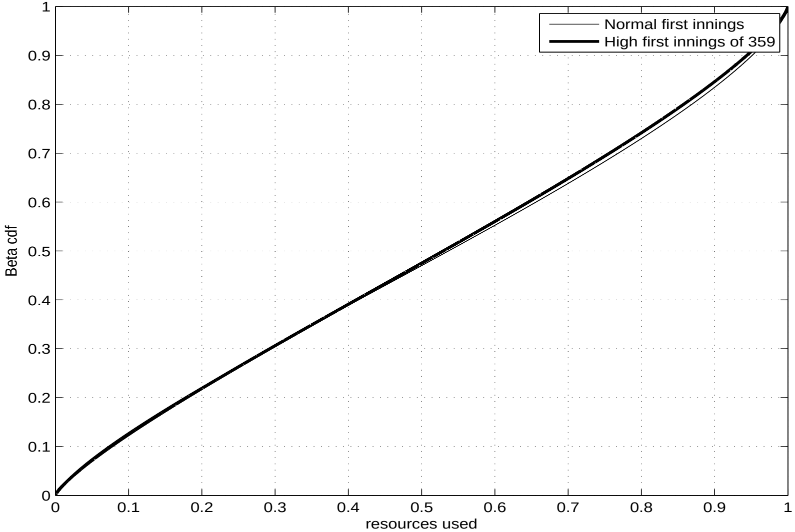
- Given u overs to go and w wickets in hand, D/L method assumes that both Team 1 and Team 2 will have the same scoring pattern.
- Run scoring potential at any stage mapped to remaining resources. For this Team 1 data alone suffices.
- But Team 2 is maximising probability of winning. It's pattern of scoring is perhaps different.
- Stern (2009) analysed the pattern of play in the second innings.

The typical pattern in successful chases ...

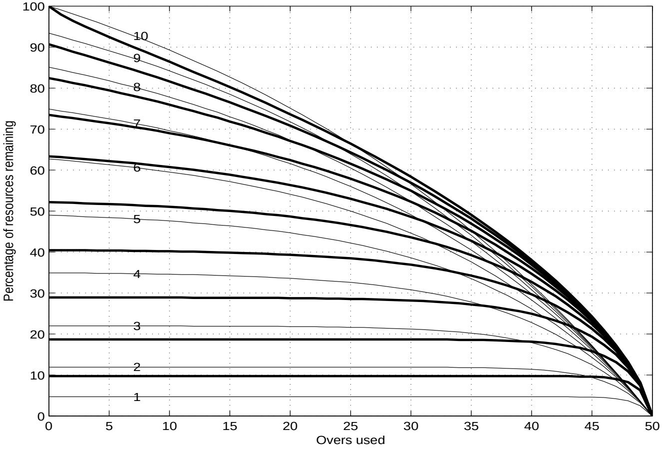
- If the first innings score is low, use lesser amount of resources.
- If the first innings score is average or high, use the full quota of overs and resources.
- Also, a fast start, then slow middle overs, followed by acceleration near the end.

Stern's correction to resources used

$R_2'(u, w) = F(R_2(u, w))$: D/L Standard edition resources used R_2 transformed to Stern's resources used R_2'



Stern's correction to resources remaining



Lose overs where curve is steep $\rightarrow$ lose more resources $\rightarrow$ lower target.
Lose overs either initially (all 10 in hand) or at the end $\rightarrow$ lower target.
Lose more than 3 wickets initially $\rightarrow$ steeper target.

Cricket as an Operations Research (OR) problem

- What is the ‘optimal’ batting strategy for Team 1 at any stage of the game?
  What is the ‘optimal’ batting strategy for Team 2?
- Suppose that there are n balls to go and w wickets in hand.
- The batsman can play a risky shot that fetches more runs or a safe shot with fewer runs.
  p_d = probability of losing wicket.
  p_x = probability of getting x runs, $x = 0, 1, 2, 3, 4, 5, 6$.
  Through his choice, the batsman can shape this probability vector.
- What is his best strategy at any stage of Team 1’s innings?

Clarke's simplified model - Team 1

- Ignore bowler quality or varying quality of later batsmen.
- Also, let us assume that if batsman's run rate (per ball) is r then the probability of losing wicket is $p_d(r)$. Assume known (from historical data).
- $r = \sum_{x=0}^{6} x p_x$.
- Let $Z_b(n, w)$ = expected number of runs in n balls, with w wickets.
- Goal: Maximise $Z_b(300\ balls, 10\ wickets)$.
- A recursive formula:

$$Z_b(n, w) = \max_{p_0, \dots, p_1} \left[p_d Z_b(n-1, w-1) + (1 - p_d) Z_b(n-1, w) + \sum_x x p_x \right]$$

  This is called the dynamic programming equation or Bellman equation.

- Boundary conditions: $Z_b(0, w) = 0$ for all w, $Z_b(n, 0) = 0$ for all n. Enough information to determine $Z_b(n, w)$ for all relevant n, w.

Clarke's simplified model - Team 2

- Maximise probability of reaching the target.
- $q(s, n, w) =$ probability of scoring s with n balls to go and w wickets in hand.
- Again a dynamic programming equation:

$$q(s, n, w) = \max_{p_d, p_0, \ldots, p_6} \left[p_d q(s, n-1, w-1) + \sum_x p_x q(s-x, n-1, w) \right]$$

- Simplify even further.
  Assume batsman can score 0 or a runs only. He can choose p_a.
  This gives a run rate of ap_a.
  Suppose this fixes the p_d via $p_d(r)$. Then $p_0 = 1 - p_a - p_d$.
- Boundary conditions: $q(l, 0, w) = 0$ for all $l < s$ and all w.
  $q(l, n, 0) = 0$ for all $l < s$ and all n.
  $q(0, n, w) = 1$ for all $n \geq 0$ and all $w \geq 0$.
  Enough to determine $q(n, w)$ for all relevant n, w.

In summary

- D/L Method to revise targets in an interrupted ODI game.
- Handles many kinds of stoppages.
- Reasonable targets in most situations.
- Some anomalies fixed by the Professional Edition, Stern's corrections.
- Not isoprobability, incentivises alteration of strategy, ...

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