Duration. 15 minutes

Maximum score. 10 points

You are free to use basic set-theoretic facts.

Problem. Let (X, d) be a metric space, and $E \subseteq X$. Recall that the *boundary* of E is the set

$$bE = \overline{E} \setminus E^{\circ}.$$

Let $x \in X$. Show that the following are equivalent.

(1) $x \in bE$,

(2) for every $\varepsilon > 0$, there is a $z \in E$ and $w \in E^c$ such that $z, w \in B(x; \varepsilon)$,

(3) $x \in \overline{E} \cap \overline{(X \setminus E)}$.