Quiz 4

UM 205: Introduction to Algebraic Structures (Winter 2023-24)
Indian Institute of Science

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March 5, 2024

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1. Let R be a principal ideal domain (PID). Show that if two elements $a, b \in R$ are associates, then $(a) = (b)$, i.e. the ideals generated by them are the same.

2. Let k be a field and let $R = k[x, y]$ be the set of polynomials over k in variables x and y. Assuming that R is an integral domain, either prove that it is a PID or construct an ideal I and prove that I is not principal.

1) If $a, b \in R$ are associates, then either $a = bu$ (where u is a unit) or $a = bc$ (where $c \in R$) ? ✓

If $a = bu$, then clearly the ideals are same since ✓
$a = bu \Rightarrow ra = rbu \ \forall r \in R$ this implies $(a) \subseteq (b)$
what about other side ?

If $a = b \cdot c$, then we use the fact that we have a PID $\Rightarrow$ every finitely generated ideal is of the form (a). Clearly $(a) \subseteq (b)$. Assume $(b) \not\subseteq (a)$. Then $\exists$ b' in (b) which doesn't lie in (a).
This is a violation of PID property. Hence $(b) \subseteq (a)$
how ?

$\therefore$ $(a) = (b)$.

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2) ~~Let~~ Not PID

Consider (x, y) ideal

This generates all linear polynomials with constant 0. need to show why!

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However, we can't reduce it further.