UMC 205 Automata Theory and Computability

Midsem Examination 2024

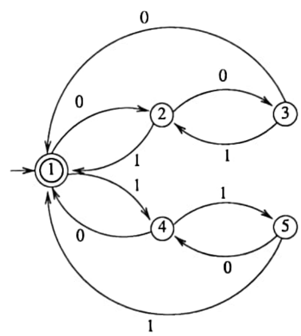
Max Marks: 50, Weightage: 20%, Time: 2 hours. Write your answers briefly and to the point. If required write your answers on a rough sheet first.

1. Consider the languages over the alphabet $\{0, 1\}$ below. One of them is regular while the other is not. Which is which? Justify your answer. (*10*)

   (a) $L_1 = \{xy \mid |x| = |y|, \text{ and } xy \text{ contains a } 1\}$

   (b) $L_2 = \{xy \mid |x| = |y|, \text{ and } y \text{ contains a } 1\}$.

2. Consider the DFA $\mathcal{A}$ below over the alphabet $\{0, 1\}$. (*10*)



   (a) Minimize the DFA.

   (b) Using the McNaughton-Yamada construction or otherwise, construct a regular expression for the language of $\mathcal{A}$. You may use either $\mathcal{A}$ or its minimized version.

3. Consider a language L over the alphabet $\{a, b\}$. What can you say about the canonical Myhill-Nerode relation $\equiv_L$ for L and $\equiv_{\bar{L}}$ where $\bar{L} = \{a, b\}^* - L$? (*5*)

4. Consider strings over the alphabet $\{0, 1\}$. We say that two strings u and v are within a Hamming distance of d from each other, written $h(u, v) \leq d$, if they have the same length and differ from each other in at most d positions. (*15*)

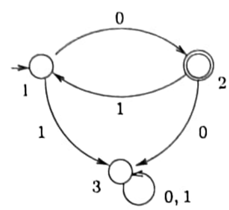
   (a) List out all strings that are within a Hamming distance of 2 from the string 01101: i.e. the set $\{v \in \{0, 1\}^* \mid h(01101, v) \leq 2\}$.

   (b) For a language $L \subseteq \{0, 1\}^*$ and $d \in \mathbb{N}$, we can define

$$H_d(L) = \{v \in \{0, 1\}^* \mid \exists u \in L : \ h(u, v) \leq d\}.$$

   Prove that if L is regular, then so is $H_2(L)$.

(c) Using this construction or otherwise, give a DFA/NFA for $H_2(L)$ where L is the language accepted by the DFA below:



5. Describe the language generated the following context-free grammar G over the alphabet $\{a, b, c\}$: *(5)*

$$S \rightarrow aS \mid aSbS \mid c$$

6. Give a context-free grammar for the language *(5)*

$$\{a^l b^m c^n \mid 0 \leq l < m,\ 0 \leq n\}.$$