## AI & ML Course Quiz 2(Mar 11, 2024)

Time: 30 minutes

## Instructions

- Answer all questions
- $\bullet\,$  See upload instructions in the form

Question:	1	2	Total
Points:	10	10	20
Score:			

1. Consider the problem of Binary Classification using the following Soft margin SVM on a Dataset of total 100 observations, equally divided between the positive and negative classes.

$$min_{\mathbf{w},b,\xi} \frac{1}{2} \|\mathbf{w}\|^2 + 2 \sum_{i=1}^{N} \xi_i$$

$$y_i \left( \mathbf{w}^{\top} \mathbf{x}^{(i)} + b \right) \ge 1 - \xi_i, \quad \xi_i \ge 0, i \in \{1, \dots, 100\}$$

Following information is available. At optimality,

$$\mathbf{w}^* = -2\sum_{i=1}^{3} \mathbf{x}^{(i)} + r\sum_{i=11}^{16} \mathbf{x}^{(i)}$$

The value of r is not known.

- The training set error(expressed as percentage) of the optimal classifier is 2%.
- It is given that  $\xi = \beta \mathbf{e}$ ,  $\beta \sum_{i=1}^{N} e_i = 10$  where  $\mathbf{e} \in \{0,1\}^N$  and  $\beta$  is an unknown scalar

Based on the information provided, answer the following questions

- (a) (2 points) What is the value of r
- (b) (2 points) What percentage of positive class examples are mis-classified \_\_\_\_\_
- (c) (2 points) What percentage of negative class examples are mis-classified \_\_\_\_\_
- (d) (2 points) What is the cardinality of the set  $M = \{i | \xi_i > 1, i \in [100]\}$
- (e) (2 points) What is the value of  $\beta$
- 2. A dataset  $\mathcal{D} = \{(y_i, x_i) | i \in [n]\}$  was generated as follows

$$y_i = 2x_i + \epsilon_i, \quad \epsilon_i \sim N(0, 1)$$

where  $x_i \in \mathbb{R}$ . Consider implementing Ridge regression

$$w_{RR} = \operatorname{argmin}_{w} w^2 + \frac{1}{2} \sum_{i=1}^{n} (y_i - wx_i)^2$$

on  $\mathcal{D}$ . Following information is available  $\frac{1}{n}\sum_{i=1}^n x_iy_i = 0.1$ ,  $\frac{1}{n}\sum_{i=1}^n x_i^2 = 2$ , n = 100 Answer the following.

- (a) (2 points) What is the value of  $w_{RR}$ ?  $\bigcirc$   $\frac{50}{100}$   $\bigcirc$   $\frac{0.1}{4}$   $\bigcirc$   $\frac{5}{101}$   $\bigcirc$  2 (b) (4 points) What is  $E(w_{RR})$   $\bigcirc$   $\frac{200}{101}$   $\bigcirc$   $\frac{100}{101}$   $\bigcirc$   $\frac{1}{101}$   $\bigcirc$  2
- (c) (4 points) What is the squared bias of the prediction of the ridge regression model on a test point x.  $\bigcirc \frac{4}{202^2}x^2 \bigcirc \frac{4}{101^2}x^2 \bigcirc \frac{8}{202^2}x^2 \bigcirc x^2$