Automata Theory and Computability

Assignment 3

(Total marks 65. Due on Thu 15th Feb 2024)

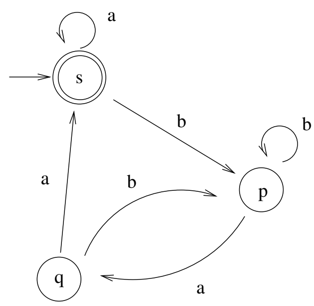
1. Give a language $L \subseteq \{a,b\}^*$ such that neither L nor $\{a,b\}^* - L$ contains an infinite regular set. *(10)*

2. For a language L over an alphabet A define

$$\textit{first-halves}(L) = \{x \in A^* \mid \exists y : |x| = |y| \text{ and } xy \in L\}.$$

Prove or disprove: if L is regular, then so is *first-halves*(L). *(10)*

3. Use the McNaughton-Yamada construction done in class to construct a regular expression corresponding to the language accepted by the DFA below (i.e. the expression corresponding to $L_{ss}^{\{s,p,a\}}$). *(10)*



4. In the McNaughton-Yamada construction of an RE from an NFA, we inductively define $L(p,X,q)$ to be the words accepted by paths from state p to state q possibly using intermediate states in the set of states X. Inductively define $LA(p,Y,q)$, the words accepted by paths from state p to state q, but avoiding using intermediate states in Y. What would be the base case? *(5)*

5. Consider the languages L and M below over the alphabet $\{a,b\}$.

   - L is the language of all strings in which the difference between the number of a's and b's is at most 2. That is:

   $$L = \{w \in \{a,b\}^* \mid |\#_a(w) - \#_b(w)| \leq 2\}.$$

   - M is the language of all strings which satisfy the property that in *every* prefix the difference between the number of a's and b's is at most 2. That is:

   $$M = \{w \in \{a,b\}^* \mid \text{for all prefixes } u \text{ of } w,\ |\#_a(u) - \#_b(u)| \leq 2\}.$$

Describe the classes of the canonical MN relation $\equiv_L$ for L, and similarly for M. Finally, conclude whether L and M are regular or not. *(20)*

6. Minimize the DFA below using the algorithm done in class: (*10*)