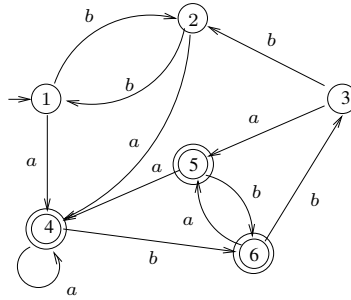


Automata Theory and Computability

Assignment 2 (Myhill-Nerode and CFGs)

(Total marks 80. Due on Thu 25th Feb 2025)

1. Let L be a regular language over the alphabet A . Which of the following languages over A are regular? Justify your answers. (20)
 - (a) $\text{mid-thirds}(L) = \{v \mid \exists u, w : |u| = |v| = |w| \text{ and } uvw \in L\}$.
 - (b) $\text{half}(L) = \{w \mid ww \in L\}$.
2. Describe the equivalence classes of the Myhill-Nerode relation \equiv_L for the language $L = \{x \in \{a, b\}^* \mid \#_a(x) = \#_b(x)\}$. Depict the canonical DA for this language. (10)
3. Find a language L for which the canonical Myhill-Nerode relation for L , \equiv_L , has all singleton equivalence classes. (10)
4. Minimize the DFA below using the algorithm done in class: (10)



5. Give a context-free grammar for the following language. Prove that your grammar is correct: (10)

“Equal a ’s and b ’s” – i.e. $\{x \in \{a, b\}^* \mid \#_a(x) = \#_b(x)\}$. (Hint: give a grammar similar to the one for balanced parenthesis).
6. Give a context grammar for the language (10)

$$L_2 = a^*b^*c^* - \{a^n b^n c^n \mid n \geq 0\}.$$

7. Give an equivalent grammar in Chomsky Normal Form for the following CFG: (10)

$$\begin{aligned} S &\rightarrow aSbb \mid T, \\ T &\rightarrow bTaa \mid S \mid \epsilon. \end{aligned}$$