Automata Theory and Computability

Assignment 4 (Context-Free Grammars and PDAs)

(Total marks 70. Due on Thu 21st Mar 2024)

1. Construct a context-free grammar for the language (*5*)

$$\{xy \in \{0,1\}^* \mid |x| = |y| \text{ and } y \text{ contains a } 1\}.$$

2. Convert the following context-free grammar to Chomsky Normal Form (CNF): (*10*)

$$\begin{array}{lcl} S & \longrightarrow & AAA \mid B \\ A & \longrightarrow & aA \mid B \\ B & \longrightarrow & \epsilon \end{array}$$

3. Consider the language $L = \{a^n b^{n^2} \mid n \geq 0\}$. Use the Pumping Lemma for CFLs to show that L is not a CFL. (*10*)

4. Consider the CFG G below:

$$\begin{array}{lcl} S & \rightarrow & XC \mid AY \\ X & \rightarrow & aXb \mid ab \\ Y & \rightarrow & bYc \mid bc \\ A & \rightarrow & aA \mid a \\ C & \rightarrow & cC \mid c \end{array}$$

   (a) Describe the language accepted by G. (*5*)

   (b) Use the construction in Parikh's theorem to construct a semi-linear expression for $\psi(L(G))$. That is

      i. Identify the basic pumps for G. (*5*)

      ii. Identify the $\leq$-minimal parse trees. (*5*)

      iii. Use these to obtain an expression for $\psi(L(G))$. (*5*)

   (c) Use the semi-linear expression obtained above to give a regular expression that is letter-equivalent to $L(G)$. (*5*)

5. Give the transition diagram of a PDA for the language $L = \{a^l b^m c^n \mid l = m + n\}$. (*10*)

6. Is the class of context-free languages closed under the prefix operation? Justify your answer. (*10*)