Quiz 6

UM 205: Introduction to Algebraic Structures (Winter 2023-24)
Indian Institute of Science

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1. Suppose G is a set that is closed under a binary operation $*$, that is associative, and that satisfies the following properties:

   - there exists an element $e \in G$ such that $e * a = a * e = a$ for all $a \in G$,
   - for every $a \in G$, there exists an element a^{-1} such that $a^{-1} * a = e$.

   Prove that $(G, *)$ is a group.

2. Prove that the group of symmetries of the regular three-dimensional cube is not abelian.

1) For a group, we need a binary operation, associativity, identity and inverse.

The operation $*$ is already binary & associative.

Claim: e is the identity

From 1, $e*e = e*e = e$ (1 holds for all $a \in G$)

Clearly, e is an identity

Now $a^{-1} * a = e$ for all $a \in G$ need To show $a * a^{-1} = e$

Thus inverses also exist

$\therefore$ $(G, *)$ is a group

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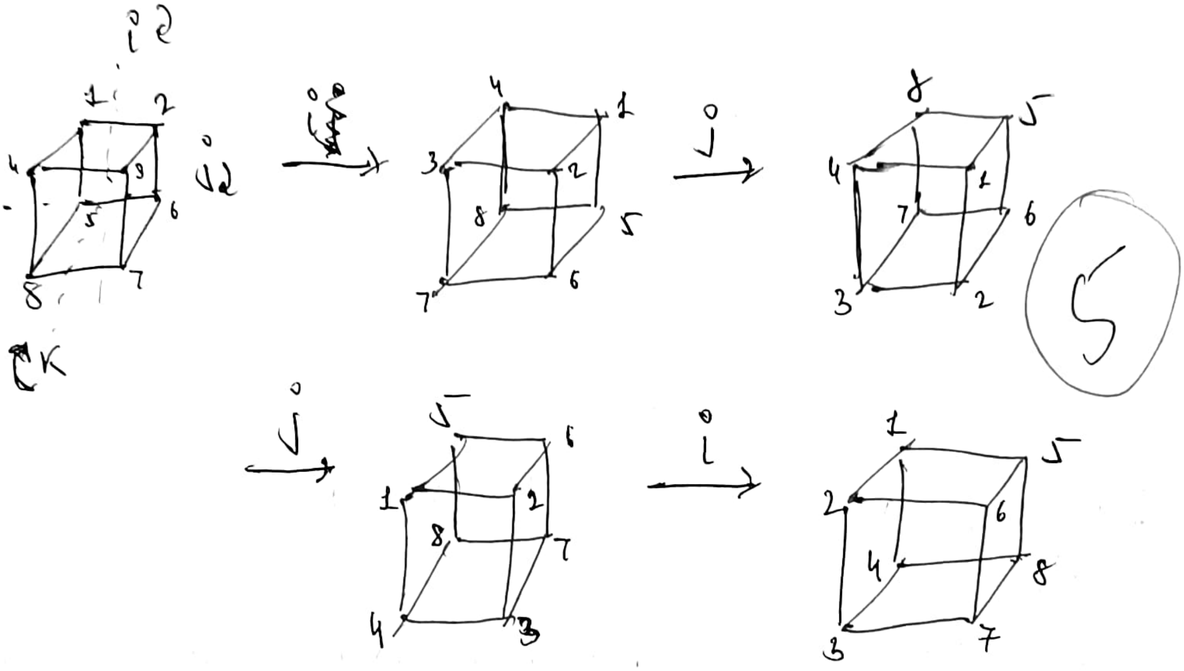
2) Let the symmetries for a 3D cube be of the form $\langle i, j, k \mid i^4 = j^4 = k^4 = 1 \rangle$

We have to show this is not abelian

We know that for the regular 2D square with D_8 symmetries, it is not abelian. Extend this to the cube.



Clearly, $ij \neq ji \Rightarrow$ not abelian

If 3D cube was abelian, we could use flask ~~and~~ it to make D_8 - abelian.