Automata Theory and Computability

Assignment 5 (Turing Machines and Decidability)

Total 65 marks. Due on Thu 11th April 2024.

1. Show that the function $square : \mathbb{N} \to \mathbb{N}$, given by $square(n) = n^2$ is computable by a Turing machine in the sense discussed in class. Give a complete description of the moves of the TM in a modular way. *(10)*

2. Is the following question decidable: Given a Turing machine M and a state q of M, does M ever enter state q on *some* input? Justify your answer. *(5)*

3. Let $L, K \subseteq \Sigma^*$. Define *(10)*

$$L/K = \{x \mid \exists y \in K,\ xy \in L\}$$

   (a) Show that if L is regular and K is *any* language, then L/K is regular.

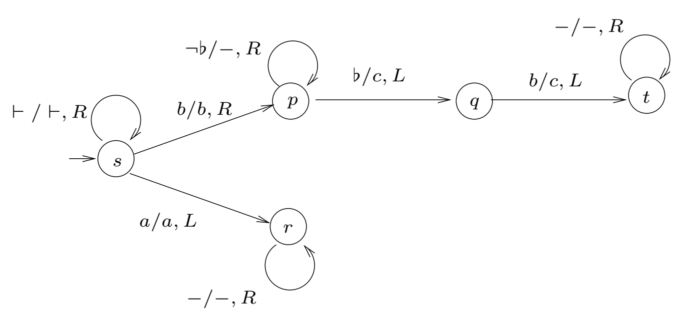
   (b) Show that even if we are given a DFA for L and a Turing machine for K, we cannot always construct an automaton for L/K.

4. Show that neither the language

$$\text{TOTAL} = \{M \mid M \text{ halts on all inputs}\}$$

   nor its complement is r.e. *(10)*

5. Consider the TM M below, with input alphabet $\{a, b\}$.



   (a) Give any string in $Valcomp_{M,baabb}$. *(5)*

   (b) Recall the notion of matching *triples* of symbols used in class. Give the entire set of matching triples for M. *(5)*

   (c) Justify the claim that for two valid configurations c_1 and c_2 of M, which are of the same length, we have: $c_1 \overset{1}{\Rightarrow} c_2$ iff for each position in c_1, the triple of symbols in c_1 and the corresponding triple in c_2 match. *(5)*

(d) Define an analagous notion of matching *pairs* of symbols. What if we weaken the criterion above to say that at each position the *pairs* of symbols in c_1 and c_2 "match"? (*5*)

6. Show that it is undecidable whether the intersection of two given CFLs is a CFL. (*10*)