Assignment - 1 (Computational Neuroscience)

S SAI SRINIVAS(22CS30053)

Van der Pol Equation

$$\frac{d^2y}{dt^2} - \mu(1-y^2)\frac{dy}{dt} + y = 0, \quad \mu > 0$$

(a) Conversion to First-Order Equations

Given state variables:

$$x_1 = y, \quad x_2 = \mu^{-1}\frac{dy}{dt}$$

From the definition:

$$\frac{dx_1}{dt} = \mu x_2$$

Also,

$$\frac{d^2y}{dt^2} = \mu\frac{dx_2}{dt}$$

Substitute into the Van der Pol equation:

$$\mu\frac{dx_2}{dt} - \mu(1-x_1^2)(\mu x_2) + x_1 = 0$$

$$\Rightarrow \frac{dx_2}{dt} = (1-x_1^2)x_2 - \frac{x_1}{\mu} \tag{1}$$

$$\Rightarrow \frac{dx_1}{dt} = \mu x_2. \tag{2}$$

(b) Observation

Oscillations were observed in MATLAB simulations for various μ values.

(c) Speed Comparison

Table 1: Solver speed for different μ values (time in seconds).

μ	0.1	1	100	1000
`ode45`	0.0018	0.0031	0.0507	0.7121
`ode15s`	0.0140	0.0262	0.0072	0.0030

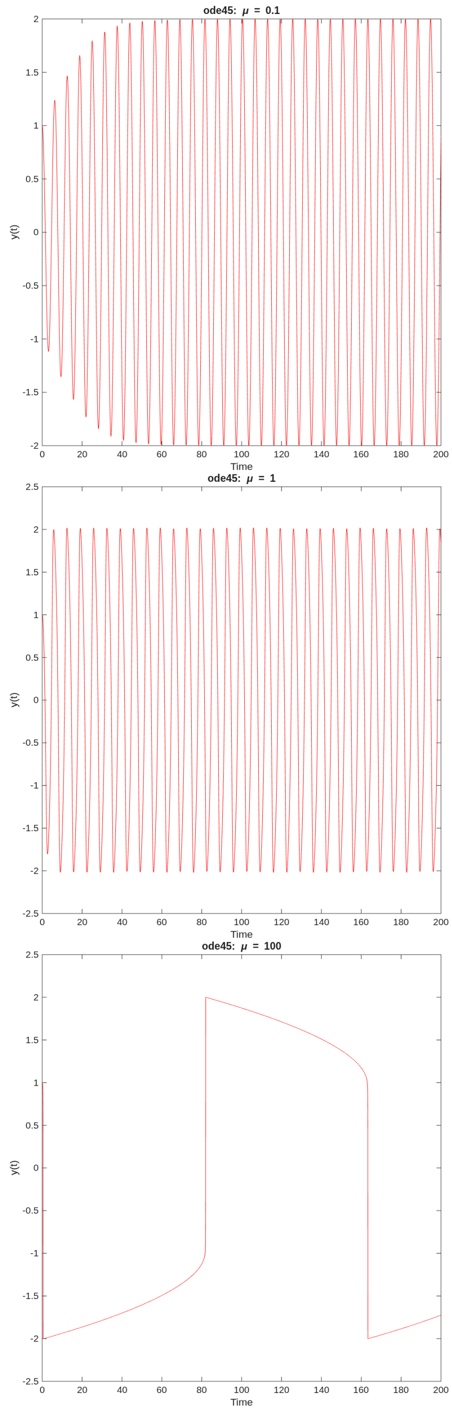
(d) Initial Conditions and Convergence

Table 2: Convergence behavior for different μ values and initial conditions $[1,0]$, $[1,2]$, $[4,-2]$.

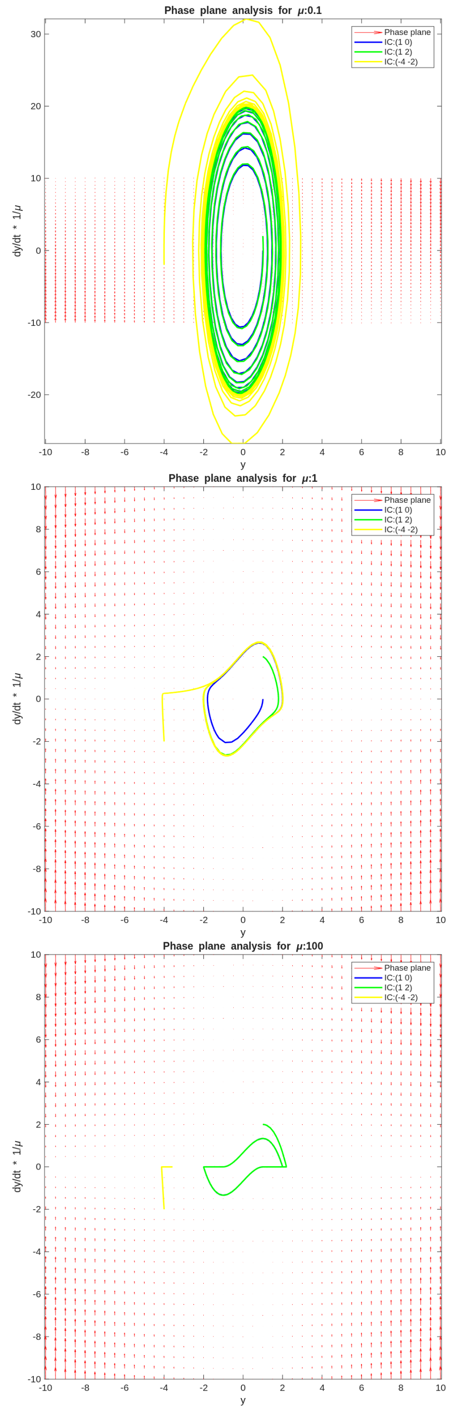
μ	Observation
1000	All graphs converged instantaneously.
100	$[1,2]$ took one cycle; others converged instantly.
1	All took about one cycle to converge.
0.03	All took multiple cycles to converge.

(e) Figures

Figure 1: Comparison of oscillation patterns (left) and phase-plane analysis (right).



(a) Oscillations for different μ values and initial conditions.



(b) Phase-plane trajectories for corresponding cases.