1351. (d1, 2022, Giochi di Archimede) On the shore of a circular lake are three docks. On a map, they look like points A, B, C on a circle and their angles can be measured to be $\angle CAB = 57°$, $\angle ABC = 48°$, $\angle BCA = 75°$. In case a boat on the lake needs emergency, the nearest dock sends a rescue boat. What portion of the lake is assisted by the dock that covers the biggest area?

1289. (d1, 2019 AIME I, P3 of 15) In $\triangle PQR$, $PR = 15$, $QR = 20$, and $PQ = 25$. Points A and B lie on $\overline{PQ}$, points C and D lie on $\overline{QR}$, and points E and F lie on $\overline{PR}$, with $PA = QB = QC = RD = RE = PF = 5$. Find the area of hexagon $ABCDEF$.

1281. (d1, 2022 NZMO1, P1 of 8) $ABCD$ is a rectangle with side lengths $AB = CD = 1$ and $BC = DA = 2$. Let M be the midpoint of AD. Point P lies on the opposite side of line MB to A, such that triangle MBP is equilateral. Find the value of $\angle PCB$.

1239. (d1, Alternate Segment Theorem) Let ABC be a triangle with circumcircle Γ. Let ℓ be a line tangent to Γ at B, and let D be a point on ℓ such that D and A are on opposite sides of BC. Prove that $\angle BAC = \angle DBC$.

1198. (d1, 2021 Irish MO, P2 of 10) An isosceles triangle ABC is inscribed in a circle with $\angle ACB = 90^o$ and EF is a chord of the circle such that neither E nor F coincide with C. Lines CE and CF meet AB at D and G respectively. Prove that $|CE| \cdot |DG| = |EF| \cdot |CG|$.

1177. (d1, 2018 UK JMO, B4) A rectangular sheet of paper is labelled $ABCD$, with AB one of the longer sides. The sheet is folded so that vertex A is placed exactly on top of the opposite vertex C. The fold line is XY, where X lies on AB and Y lies on CD. Prove that triangle CXY is isosceles.

1127. (d1, 2016 Senior Maths Challenge, P24 of 25) Let $PQRS$ be a square and let U be the midpoint of QR. The point T lies on SR such that the line TU is tangent to the circle centred at P with radius PQ. What is the ratio of the length of TR to the length of UR?

1043. (d1, 2018 AMC10, P15 of 25) Two circles of radius 5 are externally tangent to eachother, and internally tangent to a circle of radius 13 at points A and B, respectively. The distance AB can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

1022. (d1, 1997 AMC12, P9 of 30) Let $ABCD$ be a square of side length 2. If E is the midpoint of AD, and F is the foot of the perpendicular from C to BE, what is the area of the quadrilateral $CDEF$?

1008. (d1, 2018 AMC10B, P7 of 25) N congruent semicircles lie on the diameter of a large semicircle, with their diameters covering the diameter of the large semicircle with no overlap. Let A be the combined area of the small semicircles, and B the area inside the large semicircle but outside the small semicircles. If $A : B = 1 : 18$, what is N?

959. (d1, 2004 SMC, P22 of 25, adapted) Let $ABCD$ be a quadrilateral, with AB parallel to CD and lengths $\overline{AB} = x$ and $\overline{CD} = y$. Suppose AC and BD meet at Z. Suppose the line through Z parallel to AB meets BC and DA at P and Q respectively. Show that the length $\overline{PQ}$ can be determined just from the values of x and y. Then find a formula for $\overline{PQ}$ in terms of x and y.

952. (d1, 2013 Bosnia & Herzegovina Regional Grade 9, P2 of 4)

In a triangle ABC, $\angle ACB = 50^\circ$ and $\angle CBA = 70^\circ$. Let D be the foot of the perpendicular from point A to side BC and E the antipode of A in the circumcircle of ABC. Find $\angle DAE$.

854. (d1, 2019 ASC, P1 of 5) For $n \geq 3$, the sequence of points $A_1, A_2, \ldots, A_n$ in the Cartesian plane has increasing x -coordinates. The line A_1A_2 has positive gradient, the line A_2A_3 has negative gradient, and the gradients continue to alternate in sign, up to the line $A_{n-1}A_n$. So the zigzag path $A_1A_2 \cdots A_n$ forms a sequence of alternating peaks and valleys at $A_2, A_3, \ldots, A_{n-1}$

The angle less than 180° defined by the two line segments that meet at a peak is called a peak angle. Similarly, the angle less than 180° defined by the two line segments that meet at a valley is called a valley angle. Let P be the sum of all the peak angles and let V be the sum of all the valley angles.

Prove that if $P \leq V$, then n must be even.

784. (d1, Folklore) Let M be the midpoint of side BC of triangle ABC. Prove that $AB + AC > 2AM$.

764. (d1, 2018 UK IMOK, M6) Let ABC be a triangle. Let points T and U lie on segment AB, P and Q lie on segment BC and R and S lie on segment CA. Suppose SP and AB are parallel, UR and BC are parallel, and QT and CA are parallel. Suppose also that SP, UR and QT also meet at a point. Suppose also that the lengths PQ, RS and TU are equal. Prove that

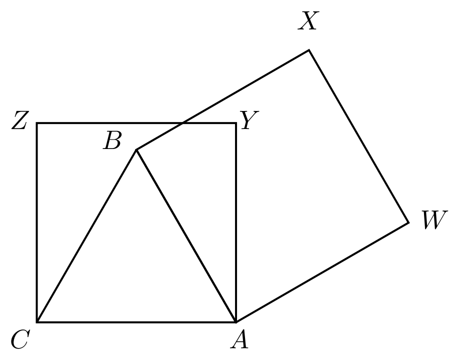
$$\frac{1}{PQ} = \frac{1}{AB} + \frac{1}{BC} + \frac{1}{CA}.$$

763. (d1, 2015 BMO1, P2 of 6) Let $ABCD$ be a cyclic quadrilateral and let the lines CD and BA meet at E. The line through D which is tangent to the circle ADE meets the line CB at F. Prove that the triangle CDF is isosceles.

735. (d1, 2015 UK JMO, B4) The point F lies inside the regular pentagon $ABCDE$ so that $ABFE$ is a rhombus. Prove that EFC is a straight line.

700. (d1, Thales' theorem and converse) Points A, B, and C lie on a circle Γ. Show that $\angle ABC = 90^\circ$ if and only if AC is the diameter of Γ.

581. (d1, 2016 UK IMOK, C4) The diagram shows an equilateral triangle ABC and two squares $AWXB$ and $AYZC$. Prove that triangle AXZ is equilateral.



497. (d1, 2020 NZMO1, P2 of 8) Let $ABCD$ be a square and let X be any point on side BC between B and C . Let Y be the point on line CD such that $BX = YD$ and D is between C and Y . Prove that the midpoint of XY lies on diagonal BD .

1366. (d2, 1989 Cono Sur Olympiad, P1 of 6) Two isosceles triangles with sidelengths x, x, a and x, x, b ($a \neq b$) have equal areas. Find x.

1352. (d2, Power of a Point) Let Γ be a circle and P be a point not on the circumference of Γ. Suppose that a line ℓ through P either intersects Γ at two distinct points A and B, or it is tangent to Γ, making A and B the same point. Prove that $PA \times PB$ is independent of ℓ.

1344. (d2, 1997 CMO, P4 of 5) Let $ABCD$ be a parallelogram, and P a point in its interior. Show that, if $\angle APB + \angle CPD = 180°$, then $\angle PBC = \angle PDC$.

1310. (d2, 2021 IOQM) A bug travels in the coordinate plane moving only along the lines that are parallel to the x-axis or y-axis. Let $A = (-3, 2)$ and $B = (3, -2)$. Consider all possible paths of the bug from A to B of length at most 14. How many points with integer coordinates lie on at least one of these paths?

1303. (d2, 2013 HMMT) Let triangle ABC satisfy $2BC = AB + AC$ and have incenter I and circumcircle ω. Let D be the intersection of AI and ω (with A, D distinct). Prove that I is the midpoint of AD.

1267. (d2, Folklore) Let ABC be a triangle, and L, M and N the midpoints of BC, CA and AB, respectively. Prove that $\angle LAC = \angle ABM$ if, and only if, $\angle ANC = \angle ALB$.

1205. (d2, PST 4.6) In rectangle $ABCD$, let M and N be the midpoints of BC and CD, respectively. Let DM and BN intersect at P.

Prove that $\angle MAN = \angle BPM$.

1170. (d2, 2018 Austria Beginners' Competitions, P2 of 4) Let ABC be an acute-angled triangle, M the midpoint of the side AC and F the foot on AB of the altitude through the vertex C. Prove that $AM = AF$ holds if and only if $\angle BAC = 60^o$.

1155. (d2, Pitot's theorem) Prove that a quadrilateral with side lengths a, b, c, d, in that order, has an inscribed circle if, and only if, $a + c = b + d$.

1106. (d2, 1997 BMO1, P4 of 5) Let $ABCD$ be a convex quadrilateral. The midpoints of AB, BC, CD and DA are P, Q, R, and S, respectively. Given that the quadrilateral $PQRS$ has area 1, prove that the area of the quadrilateral $ABCD$ is 2.

1085. (d2, Pi) Let C be a circle of diameter 1. Show that the length of the circumference of C is bigger than 3, but smaller than 4.

1078. (d2, 2003 BMO1, P2 of 5) $ABCD$ is a rectangle, P is the midpoint of AB, and Q is the point on PD such that CQ is perpendicular to PD. Prove that the triangle BQC is isosceles.

1057. (d2, 2018 Polish Junior MO Round 2, P2 of 5) Let ABC be an acute angled triangle with $AC \neq BC$. Let K be the foot of the altitude from C, and O be the circumcentre of ABC. Show that the quadrilaterals $AKOC$ and $BKOC$ have the same area.

988. (d2, 2021 ICMC Round 1, P1 of 6) Let T_n be the number of non-congruent triangles with positive area and integer side lengths summing to n. Prove that $T_{2022} = T_{2019}$.

889. (d2, 2019 IGO Elementary, P4) Quadrilateral $ABCD$ is given such that

$$\angle DAC = \angle CAB = 60^\circ,$$

and

$$AB = BD - AC.$$

Lines AB and CD intersect each other at point E. Prove that

$$\angle ADB = 2\angle BEC.$$

868. (d2, 2021 NZMO1, P2) Let $ABCD$ be a trapezium such that $AB \parallel CD$. Let E be the intersection of diagonals AC and BD. Suppose that $AB = BE$ and $AC = DE$. Prove that the internal angle bisector of $\angle BAC$ is perpendicular to AD.

841. (d2, 2000 BMO1, P3 of 5) Triangle ABC has a right angle at A. Among all points P on the perimeter of the triangle, find the position of P such that

$$AP + BP + CP$$

is minimised.

826. (d2, 2000 BMO1, P1 of 5) Two intersecting circles C_1 and C_2 have a common tangent which touches C_1 at P and C_2 at Q. The two circles intersect at M and N, where N is nearer to PQ than M is. The line PN meets the circle C_2 again at R. Prove that MQ bisects angle PMR.

806. (d2, Centroid) Let ABC be a triangle, and let D, E and F be the midpoints of BC, CA and AB respectively. Prove that AD, BE and CF meet at a point. If this point is G, prove that AG is twice the length of GD.

785. (d2, Catriona Shearer's Twitter) A point P is chosen on the diagonal BD of square $ABCD$. If $DP = CP + BP$ then find all possibilities for the value of $\angle CPD$.

729. (d2, Folklore) Let $ABCD$ be a convex quadrilateral with perpendicular diagonals which intersect at P. Prove that the reflections of P across AB, BC, CD and DA form the vertices of a cyclic quadrilateral.

715. (d2, Folklore) A Platonic solid is a three-dimensional convex shape where all faces are congruent regular polygons, and each vertex has the same number of faces adjacent to it. Prove that, if we count two similar Platonic solids as not distinct, there are at most five distinct Platonic solids.

687. (d2, Folklore) Let ABC be a triangle. Let O be the centre of the circle S that goes through A, B and C. Let A' be diametrically opposite A on S. Let B' be diametrically opposite B on S. Let C' be diametrically opposite C on S. Prove that there exists a point H such that $BA'CH$, $CB'AH$ and $AC'BH$ are all parallelograms.

658. (d2, Praslov Problem 2.88) Let ABC be a triangle with circumcenter O. The foot of the altitude from A is H. Prove that $\angle OAH = |\angle B - \angle C|$.

652. (d2, Folklore) Let ABC be a triangle with incircle ω, tangent to AB at X, BC at Y and CA at Z. The line XY intersects the circle at A through Z at point P and the circle at C through Z at point Q. Prove that $\angle PZX = \angle YZQ$.

630. (d2, 2020 BMO1, P5 of 7) Let points A, B and C lie on a circle Γ. Circle Δ is tangent to AC at A. It meets Γ again at D and the line AB again at P. The point A lies between points B and P. Prove that if $AD = DP$, then $BP = AC$.

617. (d2, 2013/4 BMO1, P2 of 6) In the acute-angled triangle ABC, the foot of the perpendicular from B to CA is E. Let ℓ be the tangent to the circle ABC at B. The foot of the perpendicular from C to ℓ is F. Prove that EF is parallel to AB.

616. (d2, Varignon's Theorem) Prove that the midpoints of the sides of a quadrilateral are the vertices of a parallelogram.

504. (d2, Simson Line) Let ABC be a triangle and P be any point on the circumcircle of ABC. Let X, Y, Z be the feet of the perpendiculars from P onto lines BC, CA, and AB. Prove that points X, Y, Z are collinear.

371. (d2, 2020 Maynooth Olympiad, P8 of 10) Two circles touch at a point T and have a common tangent PS If PQ is the diameter of circle 1 and QR is tangent to circle 2, show that $PQ = QR$.

351. (d2, 2020 AMO, Q6 of 8) Let $ABCD$ be a square. For a point P inside $ABCD$, a *windmill* centered at P consists of two perpendicular lines ℓ_1 and ℓ_2 passing through P, such that

1. ℓ_1 intersects the sides AB and CD at W and Y respectively, and
2. ℓ_2 intersects the sides BC and DA at X and Z respectively.

A windmill is called *round* if the quadrilateral $WXYZ$ is cyclic.
Determine all points P inside $ABCD$ such that every windmill centered at P is round.

273. (d2, LuMaT 2018 Pre-eliminary Round) Given a convex cyclic hexagon with side length of $1, 1, 1, 1, 2, 2$ (in that particular order). Find all possible values of the radius of circumcircle from that hexagon.

231. (d2, 2019 Oral Moscow Geometry Olympiad, Grade 8–9) In the triangle ABC, I is the center of the inscribed circle, point M lies on the side of BC, with $\angle BIM = 90^o$. Prove that the distance from point M to line AB is equal to the diameter of the circle inscribed in triangle ABC

218. (d2, The Incenter-Excenter Lemma) In a triangle ABC, the internal angle bisectors concur at the incenter I, and the external angle bisectors at B and C meet at the A-excenter X. Let M be the midpoint of arc BC. Prove that $MB = MI = MC = MX$.

168. (d2, 2019 ToT Senior O-Level Paper, P1 of 5) The distances from some point inside a regular hexagon to three of its vertices that are consecutive, are equal to 1, 1 and 2, respectively. Determine the side length of the hexagon.

149. (d2, 2019 NZ Senior Maths Competition, Q13) Prove (without a calculator) that $\cos 36° - \sin 18° = \frac{1}{2}$.

77. (d2, 2014 New Zealand Camp Selection Problems, Q2) Let ABC be a triangle in which the length of side AB is 4 units, and that of BC is 2 units. Let D be the point on AB at distance 3 units from A. Prove that the line perpendicular to AB through D, the angle bisector of $\triangle ABC$, and the perpendicular bisector of BC all meet at a single point.

43. (d2, 2019 NZ Squad Selection Test Q1) The rectangle $ABCD$ has longest side AB. The point E lies on the line AD such that BE is perpendicular to AC, and the point F lies on the segment CD such that $AF = AB$. Prove that the lines AF and EF are perpendicular.

24. (d2, 2015/16 BMO1, Q2) Let $ABCD$ be a cyclic quadrilateral and let the lines CD and BA meet at E. The line through D which is tangent to the circle ADE meets the line CB at F. Prove that the triangle CDF is isosceles.

8. (d2, 2017 BMO1, Q3) The triangle ABC has $AB = CA$ and BC is its longest side. The point N is on the side BC and $BN = AB$. The line perpendicular to AB which passes through N meets AB at M. Prove that the line MN divides both the area and the perimeter of triangle ABC into equal parts.

1373. (d3, 1997 AIME, P15 of 15) The sides of rectangle $ABCD$ have lengths 10 and 11. An equilateral triangle is drawn so that no point of the triangle lies outside $ABCD$. The maximum possible area of such a triangle can be written in the form $p\sqrt{q} - r$, where p, q, and r are positive integers, and q is not divisible by the square of any prime number. Find $p + q + r$.

1255. (d3, 2012 EGMO, P1 of 8) Let ABC be a triangle with circumcentre O. The points D, E, F lie in the interiors of the sides BC, CA, AB respectively, such that DE is perpendicular to CO and DF is perpendicular to BO. (By interior we mean, for example, that the point D lies on the line BC and D is between B and C on that line.) Let K be the circumcentre of triangle AFE. Prove that the lines DK and BC are perpendicular.

1184. (d3, 2017 German MO, P2 of 6) Let ABC be a triangle such that $|AB| \neq |AC|$. Prove that there exists a point $D \neq A$ on its circumcircle satisfying the following property:

For any points M, N outside the circumcircle on the rays AB and AC, respectively, satisfying $|BM| = |CN|$, the circumcircle of AMN passes through D.

1135. (d3, 2008 CMO, P1 of 5) $ABCD$ is a convex quadrilateral for which AB is the longest side. Points M and N are located on sides AB and BC respectively, so that each of the segments AN and CM divides the quadrilateral into two parts of equal area. Prove that the segment MN bisects the diagonal BD.

1058. (d3, ACPS 2.4.10) A bug is crawling on the coordinate plane from $(7, 11)$ to $(-17, -3)$. The bug travels at constant speed one unit per second everywhere but quadrant II (negative x- and positive y-coordinates), where it travels at half a unit per second. What path should the bug take to complete its journey in minimal time?

953. (d3, Coffee Time in Memphis) Show that every convex polygon of area 1 is contained in a rectangle of area 2.

800. (d3, 2019 Tournament of Towns Senior O-Level, P3 of 5) Prove that any triangle can be cut into 2019 quadrilaterals such that each quadrilateral is both inscribed and circumscribed.

793. (d3, 2021 Irish MO, P8 of 10) A point C lies on a line segment AB between A and B and circles are drawn having AC and CB as diameters. A common tangent to both circles touches the circle with AC as diameter at $P \neq C$ and the circle with CB as diameter at $Q \neq C$.

Prove that AP, BQ and the common tangent to both circles at C all meet at a single point which lies on the circumference of the circle with AB as diameter.

786. (d3, 2014 STEP 3, P5 of 13) Let $PQRS$ be a quadrilateral in the plane with all internal angles less than $180°$. Squares with centres X, Y, Z and T are constructed externally to the quadrilateral on the sides PQ, QR, RS and ST respectively. Show that $XYZT$ is a square if and only if $PQRS$ is a parallelogram.

673. (d3, 2013/14 BMO1, P5 of 6) Let ABC be an equilateral triangle, and P be a point inside this triangle. Let D, E and F be the feet of the perpendiculars from P to the sides BC, CA and AB respectively. Prove that

a) $AF + BD + CE = AE + BF + CD$ and

b) $[APF] + [BPD] + [CPE] = [APE] + [BPF] + [CPD]$.

The area of triangle XYZ is denoted $[XYZ]$.

637. (d3, Folklore) A finite set of circles, all of the same radius and no two intersecting, are drawn on a plane. Consider the sets of points on the circumference of each circle not visible from any other circle. Prove that the total length of these sets is equal to the circumference of one of the circles.

589. (d3, 2005/6 BMO1, P5 of 6) Let G be a convex quadrilateral. Show that there is a point X in the plane of G with the property that every straight line through X divides G into two regions of equal area if and only if G is a parallelogram.

561. (d3, 2010/11 BMO1, P5 of 6) Circles S_1 and S_2 meet at L and M. Let P be a point on S_2. Let PL and PM meet S_1 again at Q and R

respectively. The lines QM and RL meet at K. Show that, as P varies on S_2, K lies on a fixed circle.

478. (d3, 2010 BMO2, P2 of 4) In triangle ABC the centroid is G and D is the midpoint of CA. The line through G parallel to BC meets AB at E. Prove that $\angle AEC = \angle DGC$ if, and only if, $\angle ACB = 90°$.

470. (d3, 2020 USAJMO, P4 of 6) Let $ABCD$ be a convex quadrilateral inscribed in a circle and satisfying $DA < AB = BC < CD$. Points E and F are chosen on sides CD and AB such that $BE \perp AC$ and $EF \parallel BC$. Prove that $FB = FD$.

448. (d3, 2002 France TST, P5 of 6) Let ABC be a non-equilateral triangle. Denote by I the incenter and by O the circumcentre of the triangle ABC. Prove that $\angle AIO \leq \frac{\pi}{2}$ holds if and only if $2BC \leq AB + AC$.

435. (d3, Classic) Let there be an acute-angled triangle ABC. The altitudes of ABC meet at a point H. Let M be the midpoint of BC. Let H' be the reflection of H about M. Prove that H' lies on the circumcircle of triangle ABC.

427. (d3, 2020 Irish MO Training Handout) Prove the six line segments joining the incentre/excentres of any triangle are bisected by the circumference of the circumcircle.

295. (d3, van Aubel's Theorem) Let $ABCD$ be a convex quadrilateral. We construct squares externally on sides AB, BC, CD, and DA. Let O_1, O_2, O_3, and O_4 be the centers of these squares, respectively. Show that line segments O_1O_3 and O_2O_4 are equal in length and perpendicular.

233. (d3, 2019 UKSMC P23 of 25) The edge-length of a solid cube is 2. Two adjacent edges of the cube are selected, and a plane passes through the midpoints of the chosen edges, as well as the midpoints of the edges opposite to the chosen edges. What is the area of the cross-section of the plane contained within the cube?

196. (d3, 2018 Polish Junior First Round, P2 of 5) Inside parallelogram $ABCD$ is point P, such that $PC = BC$. Show that line BP is perpendicular to line which connects middles of sides of line segments AP and CD.

177. (d3, 2017 BMO1, P5 of 6) Let ABC be a triangle with $\angle A < \angle B < 90°$ and let Γ be the circle through A, B and C. The tangents to Γ at A and C meet at P. The line segments AB and PC produced meet at Q. It is given that

$$[ACP] = [ABC] = [BQC].$$

Prove that $\angle BCA = 90°$.
Here $[XYZ]$ denotes the area of triangle XYZ.

141. (d3, 2007 Italian MO, P3 of 6) Let G be the centroid of triangle ABC, D the reflection of A in G, E the reflection of B in G and M the midpoint of AB. Show that quadrilateral $BMCD$ is cyclic if, and only if, $BA = BE$.

111. (d3, 2000 Swiss TST, P1) A convex quadrilateral $ABCD$ is inscribed in a circle. Show that the line connecting the midpoints of the arcs

AB and CD and the line collecting the midpoints of the arcs BC and DA are perpendicular.

97. (d3, 2001 Croatian TST, Q2) Circles k_1 and k_2 intersect at P and Q, and A and B are the tangency points of their common tangent that is closer to P (where A is on k_1 and B is on k_2). The tangent to k_1 at P intersects k_2 again at C. The lines AP and BC meet at R. Show that the lines BP and BC are tangent to the circumcircle of triangle PQR.

85. (d3, 2002 Japanese MO, Q1) Distinct points A, M, B with $AM = MB$ are given on a circle C_0. Let P be a point on the arc AB not containing M. Circle C_1 is internally tangent to C_0 at P and tangent to AB at Q. Prove that the product $MP \times MQ$ is independent of the position of P.

69. (d3, 2019 EGMO, Q4 of 6) Let ABC be a triangle with incenter I. The circle through B tangent to AI at I meets side AB again at P. The circle through C tangent to AI at I meets side AC again at Q. Prove that PQ is tangent to the incircle of ABC.

68. (d3, 2018 BMO2, Q1 of 4) Consider triangle ABC. The midpoint of AC is M. The circle tangent to BC and B and passing through M meets the line AB again at P. Prove that $AB \times BP = 2BM^2$.

52. (d3, 2000 Mexico MO, Q6 of 6) Let ABC be a triangle with $\angle B > 90^\circ$ such that there is a point H on side AC with $AH = BH$ and BH perpendicular to BC. Let D and E be the midpoints of AB and BC respectively. A line through H parallel to AB cuts DE at F. Prove that $\angle BCF = \angle ACD$.

37. (d3, 2018 NZ Camp Selection Test, Q2 of 9) Let ABC be an equilateral triangle and let P be a point on the minor arc BC of the circumcircle of ABC. Prove that $PB + PC = PA$.

1226. (d4, 2022 Malaysian IMOTST, P1 of 6) Given an acute triangle ABC, mark 3 points X, Y, Z in the interior of the triangle. Let X_1, X_2, X_3 be the projections of X to BC, CA, AB respectively, and define the points Y_i, Z_i similarly for $i = 1, 2, 3$.

1. Suppose that $X_iY_i < X_iZ_i$ for all $i = 1, 2, 3$, prove that $XY < XZ$.
2. Prove that this is not neccesarily true, if triangle ABC is allowed to be obtuse.

1123. (d4, 2022 EGMO, P1 of 6) Let ABC be an acute-angled triangle in which $BC < AB$ and $BC < CA$. Let point P lie on segment AB and point Q lie on segment AC such that $P \neq B$, $Q \neq C$ and $BQ = BC = CP$. Let T be the circumcenter of triangle APQ, H the orthocenter of triangle ABC, and S the point of intersection of the lines BQ and CP. Prove that T, H, and S are collinear.

1116. (d4, 2017 PAMO, P6 of 6) Let ABC be a triangle with H its orthocenter. The circle with diameter $[AC]$ cuts the circumcircle of triangle ABH at K. Prove that the point of intersection of the lines CK and BH is the midpoint of the segment $[BH]$.

1053. (d4, 1999 USAMO, P2 of 6) Let $ABCD$ be a cyclic quadrilateral.

Prove that

$$|AB - CD| + |AD - BC| \geq 2|AC - BD|$$

997. (d4, 2019 PAMO, P4 of 6) The tangents to the circumcircle of $\triangle ABC$ at B and C meet at D. The circumcircle of $\triangle BCD$ meets sides AC and AB again at E and F respectively. Let O be the circumcentre of $\triangle ABC$. Show that AO is perpendicular to EF.

934. (d4, 2012/3 BMO1, P6 of 6) Let ABC be a triangle. Let S be the circle through B tangent to CA at A and let T be the circle through C tangent to AB at A. The circles S and T intersect at A and D. Let E be the point where the line AD meets the circle ABC. Prove that D is the midpoint of AE.

913. (d4, 2019 IMOSL, G1) Let ABC be a triangle. Circle Γ passes through A, meets segments AB and AC again at points D and E respectively, and intersects segment BC at F and G such that F lies between B and G. The tangent to circle BDF at F and the tangent to circle CEG at G meet at point T. Suppose that points A and T are distinct. Prove that line AT is parallel to BC.

885. (d4, 2021 BMO2, P3 of 4) Let ABC be a triangle with $AB > AC$. Its circumcircle is Γ and its incentre is I. Let D be the contact point of the incircle for ABC with BC.

Let K be the point on Γ such that $\angle AKI$ is a right angle.

Prove that AI and KD meet on Γ.

862. (d4, Romantics of Geometry, Post 8700, adapted) Let Ω be a circle and let X and Y be points on Ω.

Suppose circles S_1 and S_2 are tangent to both the line XY at P_1 and P_2 respectively. Suppose they are also tangent to the minor arc XY at Q_1 and Q_2 respectively.

Prove that P_1, P_2, Q_1 and Q_2 are concyclic.

822. (d4, 2010 IMOSL, G1) Let ABC be an acute triangle with D, E, F the feet of the altitudes lying on BC, CA, AB respectively. One of the intersection points of the line EF and the circumcircle is P. The lines BP and DF meet at point Q. Prove that $AP = AQ$.

794. (d4, 2009 BMO2, P2 of 4) Let ABC be an acute-angled triangle with $\angle B = \angle C$. Let the circumcentre be O and the orthocentre be H. Prove that the centre of the circle BOH lies on the line AB.

787. (d4, 2010 BMO2, P2 of 4) In triangle ABC the centroid is G and D is the midpoint of CA. The line through G parallel to BC meets AB at E. Prove that $\angle AEC = \angle DGC$ if, and only if, $\angle ACB = 90°$.

771. (d4, Folklore) A point P is given inside convex polyhedron Γ. Does there necessarily exist a face F of Γ such that the foot from P to the plane containing F lies within F?

765. (d4, 2006 BMO2, P3 of 4) Let ABC be a triangle with $AC > AB$. The point X lies on the side BA extended through A, and the point Y lies on the side CA in such a way that $BX = CA$ and $CY = BA$. The line XY meets the perpendicular bisector of side BC at P. Show that

$$\angle BPC + \angle CAB = 180°.$$

752. (**d4, Japanese theorem for cyclic quadrilaterals**) Let $ABCD$ be a cyclic quadrilateral. Prove that the incentres of triangles $\triangle ABC$, $\triangle BCD$, $\triangle CDA$ and $\triangle DAB$ form a rectangle.

737. (**d4, 1981 IMO 5**) Three circles of equal radius have a common point O and lie inside a given triangle. Each circle touches a pair of sides of the triangle. Prove that the incenter and the circumcenter of the triangle are collinear with the point O.

694. (**d4, 2004 BMO2, P1 of 4**) Let ABC be an equilateral triangle, and D an internal point of the side BC. A circle, tangent to BC at D, cuts AB internally at M and N, and AC internally at P and Q.

Show that $BD + AM + AN = CD + AP + AQ$.

689. (**d4, 2020 Irish MO, P3 of 10**) Circles Ω_1, centre Q, and Ω_2, centre R, touch externally at B. A third circle, Ω_3, which contains Ω_1 and Ω_2, touches Ω_1 and Ω_2 at A and C, respectively. Point C is joined to B and the line BC is extended to meet Ω_3 at D.

Prove that QR and AD intersect on the circumference of Ω_1.

647. (**d4, Monge's Theorem**) Prove that, for any three circles in a plane, none of which is completely inside one of the others, the intersection points of each of the three pairs of external tangent lines are collinear.

640. (**d4, 2013 BMO2, P2 of 4**) The point P lies inside triangle ABC so that $\angle ABP = \angle PCA$. The point Q is such that $PBQC$ is a parallelogram. Prove that $\angle QAB = \angle CAP$.

555. (**d4, 2018/9 BMO1, P4 of 6**) Let Γ be a semicircle with diameter AB. The point C lies on the diameter AB and points E and D lie on the arc BA, with E between B and$D$. Let the tangents to Γ at D and E meet at F. Suppose that $\angle ACD = \angle ECB$.

Prove that $\angle EFD = \angle ACD + \angle ECB$.

548. (**d4, 2020 BMO2, P2 of 4**) Describe all collections S of at least four points in the plane such that no three points are collinear and such that every triangle formed by three points in S has the same circumradius.

(The circumradius of a triangle is the radius of the circle passing through all three of its vertices.)

519. (**d4, 2020 ASC, P4 of 5**) Let ABC be an acute triangle with $AB > AC$. Let O be the circumcentre of triangle ABC and P be the foot of the altitude from A to BC. Denote the midpoints of the sides BC, CA and AB by D, E and F, respectively. The line AO intersects the lines DE and DF at Q and R, respectively. Prove that D is the incentre of triangle PQR.

490. (**d4, 2020 Irish MO, P10 of 10**) Show that there exists a hexagon $ABCDEF$ in the plane such that the distance between every pair of vertices is an integer.

437. (**d4, 2019 BMO2, P1 of 4**) Let ABC be a triangle. Let L be the line through B perpendicular to AB. The perpendicular from A to BC meets L at the point D. The perpendicular bisector of BC meets L at the point P. Let E be the foot of the perpendicular from D to AC.

Prove that triangle BPE is isosceles.

415. (d4, 2020 HMMT Geometry Round, Q5) Let $ABCDEF$ be a regular hexagon with side length 2. A circle with radius 3 and center at A is drawn. Find the area inside quadrilateral $BCDE$ but outside the circle.

387. (d4, 2001 Italian MO, P5 of 6) The incircle γ of a triangle ABC touches AB at T. Let D be the point on γ diametrically opposite to T, and let S be the intersection of lines AB and CD. Show that $AT = SB$.

359. (d4, 2012 BMO2, P1 of 4) The diagonals AC and BD of a cyclic quadrilateral meet at E. The midpoints of the sides AB, BC, CD and DA are P, Q, R and S respectively. Prove that the circumcircles EPS and EQR have the same radius.

324. (d4, 2018 Canada MO Q2) Let five points on a circle be labelled A, B, C, D, and E in clockwise order. Assume $AE = DE$ and let P be the intersection of AC and BD. Let Q be the point on the line through A and B such that A is between B and Q and $AQ = DP$ Similarly, let R be the point on the line through C and D such that D is between C and R and $DR = AP$. Prove that PE is perpendicular to QR.

297. (d4, A special case of Napoleon's theorem) Let A, B, C be three points on a horizontal lie, lying in that order. Construct an equilateral triangle upwards with base AC, and construct two equilateral triangles downwards with bases AB and BC respectively. Show that the centroids of these equilateral triangles form another equilateral triangle, and prove that the centroid of this new equilateral triangle lies on line segment AC.

267. (d4, Moscow MO 2012 Grade 10, Q5 of 6) An acute-angled triangle ABC is given. For an arbitrary line ℓ, we denote by ℓ_a, ℓ_b and ℓ_c the reflections of ℓ with respect to the sides of the triangle, and by I_ℓ the center of the inscribed circle of the triangle formed by these straight lines. Find the geometric locus of I_ℓ.

219. (d4, 2007/8 BMO2 P2) Let triangle ABC have incentre I and circumcentre O. Suppose that $\angle AIO = 90°$ and $\angle CIO = 45°$. Find the ratio $AB : BC : CA$.

143. (d4, 2012 APMO, P1 of 5) Let P be a point in the interior of a triangle ABC, and let D, E, F be the point of intersection of the line AP and the side BC of the triangle, of the line BP and the side CA, and of the line CP and the side AB, respectively. Prove that the area of the triangle ABC must be 6 if the area of each of the triangles PFA, PDB and PEC is 1.

124. (d4, 2016 BMO2, P3) Let $ABCD$ be a cyclic quadrilateral. The diagonals AC and BD meet at P, and DA and CB produced meet at Q. The midpoint of AB is E.

Prove that if PQ is perpendicular to AC, then PE is perpendicular to BC.

106. (d4, 2008 Japan MO, Q3) Suppose there exists an acute-angled triangle ABC with circumcentre O. A circle passing through A and O intersects lines AB and AC at P and Q respectively, distinct from A. Suppose PQ and BC are equal in length. Find the possible angles $\leq 90°$ created between PQ and BC.

91. (d4, 2000 Belarus Team Selection Test 8, Q1) The diagonals of a convex quadrilateral $ABCD$ with $AB = AC = BD$ intersect at P, and O and I are the circumcentre and incentre of triangle ABP, respectively. Prove that if $O \neq I$ then OI and CD are perpendicular.

75. (d4, 2018 APMO, Q1 of 5) Let H be the orthocenter of the triangle ABC. Let M and N be the midpoints of the sides AB and AC, respectively. Assume that H lies inside the quadrilateral $BMNC$ and that the circumcircles of triangles BMH and CNH are tangent to each other. The line through H parallel to BC intersects the circumcircles of the triangles BMH and CNH in the points K and L, respectively. Let F be the intersection point of MK and NL and let J be the incenter of triangle MHN. Prove that $FJ = FA$.

58. (d4, 2017 BMO2, Q3 of 4) Consider the cyclic quadrilateral $ABCD$. The diagonals AC and BDmeet at P, and the rays AD and BC intersect at Q. The internal angle bisector of $\angle BQA$ meets AC at R and the internal angle bisector of $\angle APD$ meets AD at S. Prove that RS is parallel to CD.

56. (d4, 2013 APMO, Q5 of 5) Let $ABCD$ be a quadrilateral inscribed in a circle ω, and let P be a point on the extension of AC such that PB and PD are tangent to ω. The tangent at C intersects PD at Q and the line AD at R. Let E be the second point of intersection between AQ and ω. Prove that B, E, R are collinear.

55. (d4, 2016 APMO, Q1 of 5) We say that a triangle ABC is great if the following holds: for any point D on the side BC, if P and Q are the feet of the perpendiculars from D to the lines AB and AC, respectively, then the reflection of D in the line PQ lies on the circumcircle of the triangle ABC. Prove that triangle ABC is great if and only if $\angle A = 90°$ and $AB = AC$.

47. (d4, 2009 Croatian TST, Q3 of 4) Let ABC be a triangle such that $AB > AC$. Let l be a tangent at A to the circumcircle of ABC. A circle with centre A and radius AC intersects AB at D and the line l at E and F (in such a way that C and E are on the same side of AB). Prove that the line DE passes through the incentre of ABC.

17. (d4, 1999 Balkan MO (16th), Q1)

Let D be the midpoint of the shorter arc BC of the circumcircle of an acute-angled triangle ABC. The points symmetric to D with respect to BC and the circumcenter are denoted by E and F, respectively. Let K be the midpoint of EA.

1. Prove that the circle passing through the midpoints of the sides of $\triangle ABC$ also passes through K.

2. The line through K and the midpoint of BC is perpendicular to AF.

1312. (d5, 2003 USAMO, P4 of 6) Let ABC be a triangle. A circle passing through A and B intersects segments AC and BC at D and E, respectively. Lines AB and DE intersect at F, while lines BD and CF intersect at M. Prove that $MF = MC$ if and only if $MB \cdot MD = MC^2$.

1306. (d5, 2014 IMO, P4 of 6) Let P and Q be on segment BC of an acute triangle ABC such that $\angle PAB = \angle BCA$ and $\angle CAQ = \angle ABC$. Let M

and N be the points on AP and AQ, respectively, such that P is the midpoint of AM and Q is the midpoint of AN. Prove that the intersection of BM and CN is on the circumference of triangle ABC.

1089. (d5, 2007 IMO, P4 of 6) In triangle ABC the bisector of angle BCA intersects the circumcircle again at R, the perpendicular bisector of BC at P, and the perpendicular bisector of AC at Q. The midpoint of BC is K and the midpoint of AC is L. Prove that the triangles RPK and RQL have the same area.

1081. (d5, 2022 INMO, P1 of 3) Let D be an interior point on the side BC of an acute-angled triangle ABC. Let the circumcircle of triangle ADB intersect AC again at $E(\neq A)$ and the circumcircle of triangle ADC intersect AB again at $F(\neq A)$. Let AD, BE, and CF intersect the circumcircle of triangle ABC again at $D_1(\neq A)$, $E_1(\neq B)$ and $F_1(\neq C)$, respectively. Let I and I_1 be the incentres of triangles DEF and $D_1E_1F_1$, respectively. Prove that E, F, I, I_1 are concyclic.

1032. (d5, 2020 USA EGMO TST, P4 of 6) Let ABC be a triangle. Distinct points D, E, F lie on sides BC, AC, and AB, respectively, such that quadrilaterals $ABDE$ and $ACDF$ are cyclic. Line AD meets the circumcircle of $\triangle ABC$ again at P. Let Q denote the reflection of P across BC. Show that Q lies on the circumcircle of $\triangle AEF$.

1025. (d5, 2020 USAMO, P1 of 6) Let ABC be a fixed acute triangle inscribed in a circle ω with center O. A variable point X is chosen on minor arc AB of ω, and segments CX and AB meet at D. Denote by O_1 and O_2 the circumcenters of triangles ADX and BDX, respectively. Determine all points X for which the area of triangle OO_1O_2 is minimized.

1004. (d5, 2019 Swiss TST, P1 of 12) Let ABC be a triangle and D, E, F be the feet of altitudes drawn from A, B, C respectively. Let H be the orthocenter of ABC. Lines EF and AD intersect at G. Let K the point on circumcircle of ABC such that AK is a diameter of this circle. AK cuts BC in M. Prove that GM and HK are parallel.

990. (d5, 2020 IMO, P1 of 6) Consider the convex quadrilateral $ABCD$. The point P is in the interior of $ABCD$. The following ratio equalities hold:

$$\angle PAD : \angle PBA : \angle DPA = 1 : 2 : 3 = \angle CBP : \angle BAP : \angle BPC$$

Prove that the following three lines meet in a point: the internal bisectors of angles $\angle ADP$ and $\angle PCB$ and the perpendicular bisector of segment AB.

948. (d5, 2021 USAMO, P1 of 6) Rectangles BCC_1B_2, CAA_1C_2, and ABB_1A_2 are erected outside an acute triangle ABC. Suppose that

$$\angle BC_1C + \angle CA_1A + \angle AB_1B = 180^\circ.$$

Prove that lines B_1C_2, C_1A_2, and A_1B_2 are concurrent.

906. (d5, 2007 IMOSL, G3) The diagonals of a trapezium $ABCD$ intersect at point P. Point Q lies between the parallel lines BC and AD such that $\angle AQD = \angle CQB$, and line CD separates points P and Q. Prove that $\angle BQP = \angle DAQ$.

899. (d5, Original) Let ABC be a triangle with $AB < AC$. Let ω be a circle passing through B, C and assume that A is inside ω. Suppose X, Y lie on ω such that $< BXA =< AYC$. Suppose also that X and C lie on opposite sides of the line AB and that Y and B lie on opposite sides of the line AC. Show that, as X, Y vary on ω, the line XY passes through a fixed point.

864. (d5, 2000 IMO, P1 of 6) Two circles G_1 and G_2 intersect at two points M and N. Let AB be the line tangent to these circles at A and B, respectively, so that M lies closer to AB than N. Let CD be the line parallel to AB and passing through the point M, with C on G_1 and D on G_2. Lines AC and BD meet at E; lines AN and CD meet at P; lines BN and CD meet at Q. Show that $EP = EQ$.

856. (d5, 1999 IMOSL, G1) Let ABC be a triangle and M be an interior point. Prove that

$$\min\{MA, MB, MC\} + MA + MB + MC < AB + AC + BC$$

851. (d5, 2021 ELMO, P1 of 6) In $\triangle ABC$, points P and Q lie on sides AB and AC, respectively, such that the circumcircle of $\triangle APQ$ is tangent to side BC at D. Let E lie on side BC such that $BD = EC$. Line DP intersects the circumcircle of $\triangle CDQ$ again at X, and line DQ intersects the circumcircle of $\triangle BDP$ again at Y.
Prove that D, E, X, and Y are concyclic.

835. (d5, 2021 MODSMO, P3 of 7) Let D be the foot of the altitude from A to BC in acute triangle ABC. The circle with diameter AD intersects the circumcircle of ABC for a second time at $E \neq A$. Let F be the point such that E is the midpoint of segment FD. Prove that FD bisects $\angle BFC$.

801. (d5, 2013 Irish MO, P5 of 10) A, B and C are points on the circumference of a circle with centre O. Tangents are drawn to the circumcircles of triangles OAB and OAC at P and Q respectively, where P and Q are diametrically opposite O. The two tangents intersect at K. The line CA meets the circumcircle of $\triangle OAB$ at A and X. Prove that X lies on the line KO.

780. (d5, 2008 IMO, P1 of 6) Let H be the orthocenter of an acute-angled triangle ABC. The circle Γ_A centered at the midpoint of BC and passing through H intersects the sideline BC at points A_1 and A_2. Similarly, define the points B_1, B_2, C_1 and C_2.

Prove that the six points A_1, A_2, B_1, B_2, C_1 and C_2 are concyclic.

766. (d5, 2015 IMOSL, G1) Let ABC be an acute triangle with orthocenter H. Let G be the point such that the quadrilateral $ABGH$ is a parallelogram. Let I be the point on the line GH such that AC bisects HI. Suppose that the line AC intersects the circumcircle of the triangle GCI at C and J. Prove that $IJ = AH$.

745. (d5, 1992 IMO, P4 of 6) In the plane let C be a circle, L a line tangent to the circle C, and M a point on L. Find the locus of all points P with the following property: there exists two points Q, R on L such that M is the midpoint of QR and C is the inscribed circle of triangle PQR.

682. (d5, 2012 IMO, P1 of 6) Given triangle ABC the point J is the centre of the excircle opposite the vertex A. This excircle is tangent to the side

BC at M, and to the lines AB and AC at K and L, respectively. The lines LM and BJ meet at F, and the lines KM and CJ meet at G. Let S be the point of intersection of the lines AF and BC, and let T be the point of intersection of the lines AG and BC. Prove that M is the midpoint of ST.

(The *excircle* of ABC opposite the vertex A is the circle that is tangent to the line segment BC, to the ray AB beyond B, and to the ray AC beyond C.)

675. (d5, 1995 IMOSL, G3) The incircle of triangle $\triangle ABC$ touches the sides BC, CA, AB at D, E, F respectively. X is a point inside triangle of $\triangle ABC$ such that the incircle of triangle $\triangle XBC$ touches BC at D, and touches CX and XB at Y and Z respectively. Show that E, F, Z, Y are concyclic.

604. (d5, 2007 Balkan MO, P1 of 4) Let $ABCD$ a convex quadrilateral with $AB = BC = CD$, with AC not equal to BD and E be the intersection point of its diagonals. Prove that $AE = DE$ if and only if $\angle BAD + \angle ADC = 120°$.

590. (d5, 2020 IGO, P1 of 5) Let M, N, P be midpoints of BC, AC and AB of triangle $\triangle ABC$ respectively. E and F are two points on the segment $\overline{BC}$ so that $\angle NEC = \frac{1}{2}\angle AMB$ and $\angle PFB = \frac{1}{2}\angle AMC$. Prove that $AE = AF$.

549. (d5, 2013 IMOSL, G4) Let ABC be a triangle with $\angle B > \angle C$. Let P and Q be two different points on line AC such that $\angle PBA = \angle QBA = \angle ACB$ and A is located between P and C. Suppose that there exists an interior point D of segment BQ for which $PD = PB$. Let the ray AD intersect the circle ABC at $R \neq A$. Prove that $QB = QR$.

542. (d5, 2012 IMOSL, G2) Let $ABCD$ be a cyclic quadrilateral whose diagonals AC and BD meet at E. The extensions of the sides AD and BC beyond A and B meet at F. Let G be the point such that $ECGD$ is a parallelogram, and let H be the image of E under reflection in AD. Prove that D, H, F, G are concyclic.

535. (d5, 2010 Romania TST 4, P1 of 3) Let γ be a fixed circle in the plane and let P be a fixed point not on γ. Two variable lines ℓ and ℓ' through P intersect γ at points X and Y, and X' and Y', respectively. Let M and N be the points directly opposite P in circles PXX' and PYY', respectively. Prove that as the lines ℓ and ℓ' vary, there is a fixed point through which line MN always passes.

528. (d5, 1999 BMO2, P2 of 4) Let $ABCDEF$ be a hexagon (which may not be regular), which circumscribes a circle S. (That is, S is tangent to each of the six sides of the hexagon.) The circle S touches AB, CD, EFat their midpoints P, Q, R respectively. Let X, Y, Z be the points of contact of S with BC, DE, FA respectively. Prove that PY, QZ, RX are concurrent.

507. (d5, 2017 IMOSL G1) Let $ABCDE$ be a convex pentagon such that $AB = BC = CD$, $\angle EAB = \angle BCD$, and $\angle EDC = \angle CBA$. Prove that the perpendicular line from E to BC and the line segments AC and BD are concurrent.

500. (d5, Symmedians) Let ABC be an acute triangle with circumcircle Γ. Let M be the midpoint of BC. Let the tangents to Γ at B and C meet at P. Prove that $\angle CAM = \angle PAB$.

428. (d5, 2020 Irish MO Training Handout) Two chords, AC and

BD, in a circle with centre O, intersect at E. The circumcircles of triangles EAB and ECD intersect again at F. Prove OF is perpendicular to EF.

408. (d5, 2016 EGMO, P4 of 6) Two circle ω_1 and ω_2 of equal radius intersect at different points X_1 and X_2. Consider a circle ω externally tangent to ω_1 at a point T_1, and internally tangent to ω_2 at a point T_2. Prove that lines X_1T_1 and X_2T_2 intersect at a point lying on ω.

379. (d5, 2015 IMOSL, G1) Let ABC be an acute triangle with orthocenter H. Let G be the point such that the quadrilateral $ABGH$ is a parallelogram. Let I be the point on the line GH such that AC bisects HI. Suppose that the line AC intersects the circumcircle of the triangle GCI at C and J. Prove that $IJ = AH$.

311. (d5, Centroid of a tetrahedron) A *median* of a tetrahedron is a line that joins a vertex with the centroid of the opposite face. A *bimedian* of a tetrahedron is a line joining the midpoints of two opposite edges. Prove that the four medians and three bimedians of a tetrahedron are concurrent.

302. (d5, 1986 IMO, P2 of 6) A triangle $A_1A_2A_3$ and a point P_0 are given in the plane. We define $A_s = A_{s-3}$ for all $s \geq 4$. We construct a set of points $P_1, P_2, P-3, \cdots$, such that P_{k+1} is the image of P_k under a rotation with center A_{k+1} through angle 120° clockwise (for $k = 0, 1, 2, \cdots$). Prove that if $P_{1986} = P_0$, then the triangle $A_1A_2A_3$ is equilateral.

254. (d5, 2015 BMO2 P3 of 4) Two circles touch one another internally at A. A variable chord PQ of the outer circle touches the inner circle. Prove that the locus of the incentre of triangle AQP is another circle touching the given circles at A.

249. (d5, 2011 All-Russian Olympiad Day 1, P4 of 4) Triangle ABC has perimeter 4. Points X and Y are on rays AB and AC respectively such that $AX = AY = 1$. Segments BC and XY intersect at M. Prove that the perimeter of either triangle ABM or ACM is 2.

199. (d5, 2014 IMOSL G1) The points P and Q are chosen on the side BC of an acute-angled triangle ABC so that $\angle PAB = \angle ACB$ and $=\angle QAC = \angle CBA$. The points M and N are taken on the rays AP and AQ, respectively, so that $AP = PM$ and $AQ = QN$. Prove that the lines BM and CN intersect on the circumcircle of the triangle ABC.

163. (d5, 1990 USAMO, P2 of 6) Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be concentric circles, with $\mathcal{C}_2$ in the interior of $\mathcal{C}_1$. From a point A on $\mathcal{C}_1$ one draws the tangent AB to $\mathcal{C}_2$ ($B \in \mathcal{C}_2$). Let C be the second point of intersection of AB and $\mathcal{C}_1$, and let D be the midpoint of AB. A line passing through A intersects $\mathcal{C}_2$ at E and F in such a way that the perpendicular bisectors of DE and CF intersect at a point M on AB. Find, with proof, the ratio AM/MC.

126. (d5, 2018 APMO, P4 of 5) Let ABC be an equilateral triangle. From the vertex A we draw a ray towards the interior of the triangle such that the ray reaches one of the sides of the triangle. When the ray reaches a side, it then bounces off following the law of reflection, that is, if it arrives with a directed angle α, it leaves with a directed angle $180^\circ - \alpha$. After n bounces, the ray returns to A without ever landing on any of the other two vertices. Find all possible values of n.

117. (d5, 2003-4 Polish MO (55th), Round 3, P1 of 6) A point D is taken on the side AB of a triangle ABC. Two circles passing through D and touching AC and BC at A and B respectively intersect again at point E. Let F be the point symmetric to C with respect to the perpendicular bisector of AB. Prove that the points D, E, and F lie on a line.

108. (d5, 2018 USA TSTST, Q5) Let ABC be an acute triangle with circumcircle ω, and let H be the foot of the altitude from A to $\overline{BC}$. Let P and Q be the points on ω with $PA = PH$ and $QA = QH$. The tangent to ω at P intersects lines AC and AB at E_1 and F_1 respectively; the tangent to ω at Q intersects lines AC and AB at E_2 and F_2 respectively. Show that the circumcircles of $\triangle AE_1F_1$ and $\triangle AE_2F_2$ are congruent, and the line through their centres is parallel to the tangent to ω at A.

34. (d5, 2013 Australian TST, Q2) Let ABC be a triangle with orthocentre H. Let D be the point such that $AHCD$ is a parallelogram. Let M be the midpoint of BC, and the perpendicular from M to AB meet it at E. Let the line parallel to BD through A intersect ME at G. Suppose F is the midpoint of ME. Show that A, M, C and F are concyclic if, and only if, BF bisects CG.

1326. (d6, 2022 Dutch TST day 1 P3 of 4) Let H, O be the orthocenter and circumcenter respectively of triangle ABC. Let K be the circumcenter of triangle AHO. Show that the reflection of K in line OH is on BC.

1291. (d6, 2021 NZMO2, P4 of 5) Let AB be a chord of circle Γ. Let O be the centre of a circle which is tangent to AB at C and internally tangent to Γ at P. Point C lies between A and B. Let the circumcircle of triangle POC intersect Γ at distinct points P and Q. Prove that $\angle AQP = \angle CQB$.

1285. (d6, 2022 IMO, P4 of 6) Let $ABCDE$ be a convex pentagon such that $BC = DE$. Assume that there is a point T inside $ABCDE$ with $TB = TD, TC = TE$ and $\angle ABT = \angle TEA$. Let line AB intersect lines CD and CT at points P and Q, respectively. Assume that the points P, B, A, Q occur on their line in that order. Let line AE intersect CD and DT at points R and S, respectively. Assume that the points R, E, A, S occur on their line in that order. Prove that the points P, S, Q, R lie on a circle.

1277. (d6, 2001 USAMO, P4 of 6) Let P be a point in the plane of triangle ABC such that the segments PA, PB, and PC are the sides of an obtuse triangle. Assume that in this triangle the obtuse angle opposes the side congruent to PA. Prove that $\angle BAC$ is acute.

1243. (d6, 2017 Canadian MO, P4 of 5) Let $ABCD$ be a parallelogram. Points P and Q lie inside $ABCD$ such that $\triangle ABP$ and $\triangle BCQ$ are equilateral. Prove that the intersection of the line through P perpendicular to PD and the line through Q perpendicular to DQ lies on the altitude from B in $\triangle ABC$.

1236. (d6, 2020 Francophone Mathematical Olympiad, Seniors P1 of 4) Let ABC be an acute triangle with $AC > AB$, Let DEF be the intouch triangle with $D \in (BC)$, $E \in (AC)$, $F \in (AB)$, let G be the intersection of the perpendicular from D to EF with AB and $X = (ABC) \cap (AEF)$.

Prove that B, D, G and X are concyclic.

1215. (d6, 2015 RMM, P4 of 6) Let ABC be a triangle, and let D be the point where the incircle meets side BC. Let J_b and J_c be the incentres of the triangles ABD and ACD, respectively. Prove that the circumcentre of the triangle AJ_bJ_c lies on the angle bisector of $\angle BAC$.

1180. (d6, 1987 IMO, P2 of 6) In an acute-angled triangle ABC the interior bisector of angle A meets BC at L and meets the circumcircle of ABC again at N. From L perpendiculars are drawn to AB and AC, with feet K and M respectively. Prove that the quadrilateral $AKNM$ and the triangle ABC have equal areas.

1173. (d6, 2010 IMO, P4 of 6) Let P be a point interior to triangle ABC (with $CA \neq CB$). The lines AP, BP and CP meet again its circumcircle Γ at K, L, respectively M. The tangent line at C to Γ meets the line AB at S. Show that from $SC = SP$ follows $MK = ML$.

1159. (d6, 2002 IMO, P2 of 6) The circle S has centre O, and BC is a diameter of S. Let A be a point of S such that $\angle AOB < 120°$. Let D be the midpoint of the arc AB which does not contain C. The line through O parallel to DA meets the line AC at I. The perpendicular bisector of OA meets S at E and at F. Prove that I is the incentre of the triangle CEF.

1153. (d6, 8100 (¡@518664855050518548¿)) Let ABC be a triangle and let X and Y be points on rays AB and AC respectively, such that $AX = AY = 2BC$. Let M be the midpoint of XY. Prove that if M is the A-excentre of ABC, then MX equals either MB or MC.

1144. (d6, 2018 RMM, P1 of 6) Let $ABCD$ be a cyclic quadrilateral an let P be a point on the side AB. The diagonals AC meets the segments DP at Q. The line through P parallel to CD mmets the extension of the side CB beyond B at K. The line through Q parallel to BD meets the extension of the side CB beyond B at L. Prove that the circumcircles of the triangles BKP and CLQ are tangent.

1138. (d6, 2020 EGMO, P5 of 6) Consider the triangle ABC with $\angle BCA > 90°$. The circumcircle Γ of ABC has radius R. There is a point P in the interior of the line segment AB such that $PB = PC$ and the length of PA is R. The perpendicular bisector of PB intersects Γ at the points D and E.

Prove P is the incentre of triangle CDE.

1102. (d6, 2015 India TST, Day 1, P1 of 3) Let $ABCD$ be a convex quadrilateral and let the diagonals AC and BD intersect at O. Let I_1, I_2, I_3, I_4 be respectively the incentres of triangles AOB, BOC, COD, DOA. Let J_1, J_2, J_3, J_4 be respectively the excentres of triangles AOB, BOC, COD, DOA opposite O. Show that I_1, I_2, I_3, I_4 lie on a circle if and only if J_1, J_2, J_3, J_4 lie on a circle.

1060. (d6, 2007 UkrMO 11.8) In acute-angled triangle ABC, AA_1 is the angle bisector, and M is its midpoint. Point P on segment BM is such that $\angle APC = 90°$ and point Q on segment CM is such that $\angle AQB = 90°$. Prove that points P, M, Q, A_1 are concyclic.

1018. (d6, 2020 IGO Advanced, P1 of 5) Let M, N, P be midpoints of BC, AC and AB of triangle $\triangle ABC$ respectively. E and F are two points on the segment $\overline{BC}$ so that $\angle NEC = \frac{1}{2}\angle AMB$ and $\angle PFB = \frac{1}{2}\angle AMC$. Prove that $AE = AF$.

991. (d6, 2021 ICMC Round 1, P6 of 6) Is it possible to cover a circle of area 1 with finitely many equilateral triangles whose areas sum to 1.01, all pointing in the same direction?

956. (d6, 2018 USAMTS R2, P5 of 5) Acute scalene triangle $\triangle ABC$ has circumcenter O and orthocenter H. Points X and Y, distinct from B and C, lie on the circumcircle of $\triangle ABC$ such that $\angle BXH = \angle CYH = 90°$. Show that if lines XY, AH, and BC are concurrent, then OH is parallel to BC.

927. (d6, 2003 IMOSL, G4) Let Γ_1, Γ_2, Γ_3, Γ_4 be distinct circles such that Γ_1, Γ_3 are externally tangent at P, and Γ_2, Γ_4 are externally tangent at the same point P. Suppose that Γ_1 and Γ_2; Γ_2 and Γ_3; Γ_3 and Γ_4; Γ_4 and Γ_1 meet at A, B, C, D, respectively, and that all these points are different from P. Prove that

$$\frac{AB \cdot BC}{AD \cdot DC} = \frac{PB^2}{PD^2}.$$

808. (d6, 2021 EGMO, P4 of 6) Let ABC be a triangle with incenter I and let D be an arbitrary point on the side BC. Let the line through D perpendicular to BI intersect CI at E. Let the line through D perpendicular to CI intersect BI at F. Prove that the reflection of A across the line EF lies on the line BC.

773. (d6, 2014 IMOSL, G2) Let ABC be a triangle. The points K, L, and M lie on the segments BC, CA, and AB, respectively, such that the lines AK, BL, and CM intersect in a common point. Prove that it is possible to choose two of the triangles ALM, BMK, and CKL whose inradii sum up to at least the inradius of the triangle ABC.

759. (d6, 2015 IMO, P4 of 6) Triangle ABC has circumcircle Ω and circumcenter O. A circle Γ with center A intersects the segment BC at points D and E, such that B, D, E, and C are all different and lie on line BC in this order. Let F and G be the points of intersection of Γ and Ω, such that A, F, B, C, and G lie on Ω in this order. Let K be the second point of intersection of the circumcircle of triangle BDF and the segment AB. Let L be the second point of intersection of the circumcircle of triangle CGE and the segment CA.

Suppose that the lines FK and GL are different and intersect at the point X. Prove that X lies on the line AO.

754. (d6, 2019 Leaked RMM, P4 of 6) Let there be an equilateral triangle ABC and a point P in its plane such that $AP < BP < CP$. Suppose that the lengths of segments AP, BP and CP uniquely determine the side of ABC. Prove that P lies on the circumcircle of triangle ABC.

717. (d6, 2014 Balkan MO, P3 of 4) Let $ABCD$ be a trapezium inscribed in a circle Γ with diameter AB. Let E be the intersection point of the diagonals AC and BD . The circle with center B and radius BE meets Γ at the points K and L (where K is on the same side of AB as C). The line perpendicular to BD at E intersects CD at M. Prove that KM is perpendicular to DL.

683. (d6, 2019 EGMO, P3 of 6) Let ABC be a triangle such that $\angle CAB > \angle ABC$, and let I be its incentre. Let D be the point on segment BC such that $\angle CAD = \angle ABC$. Let ω be the circle tangent to AC at A and passing through I. Let X be the second point of intersection of ω and the circumcircle of ABC. Prove that the angle bisectors of $\angle DAB$ and $\angle CXB$ intersect at a point on line BC.

633. (d6, 2018 IMO, P1 of 6) Let Γ be the circumcircle of acute triangle ABC. Points D and E are on segments AB and AC respectively such that $AD = AE$. The perpendicular bisectors of BD and CE intersect minor arcs AB and AC of Γ at points F and G respectively. Prove that lines DE and FG are either parallel or they are the same line.

619. (d6, 2016 Balkan MO, P2 of 4) Let $ABCD$ be a cyclic quadrilateral with $AB < CD$. The diagonals intersect at the point F and lines AD and BC intersect at the point E. Let K and L be the orthogonal projections of F onto lines AD and BC respectively, and let M, S and T be the midpoints of EF, CF and DF respectively. Prove that the second intersection point of the circumcircles of triangles MKT and MLS lies on the segment CD.

612. (d6, 2017 Balkan MO, P2 of 4) Consider an acute-angled triangle ABC with $AB < AC$ and let ω be its circumscribed circle. Let t_B and t_C be the tangents to the circle ω at points B and C, respectively, and let L be their intersection. The straight line passing through the point B and parallel to AC intersects t_C in point D. The straight line passing through the point C and parallel to AB intersects t_B in point E. The circumcircle of the triangle BDC intersects AC in T, where T is located between A and C. The circumcircle of the triangle BEC intersects the line AB (or its extension) in S, where B is located between S and A. Prove that ST, AL, and BC are concurrent.

592. (d6, 2013 IMO, P4 of 6) Let ABC be an acute triangle with orthocenter H, and let W be a point on the side BC, lying strictly between B and C. The points M and N are the feet of the altitudes from B and C, respectively. Denote by ω_1 is the circumcircle of BWN, and let X be the point on ω_1 such that WX is a diameter of ω_1. Analogously, denote by ω_2 the circumcircle of triangle CWM, and let Y be the point such that WY is a diameter of ω_2. Prove that X, Y and H are collinear.

577. (d6, 2009 Balkan MO, P2 of 4) Let MN be a line parallel to the side BC of a triangle ABC, with M on the side AB and N on the side AC. The lines BN and CM meet at point P. The circumcircles of triangles BMP and CNP meet at two distinct points P and Q. Prove that $\angle BAQ = \angle CAP$.

570. (d6, 2006 IMOSL, G2) Let $ABCD$ be a trapezoid with parallel sides $AB > CD$. Points K and L lie on the line segments AB and CD, respectively, so that $AK/KB = DL/LC$. Suppose that there are points P and Q on the line segment KL satisfying

$$\angle APB = \angle BCD \qquad \text{and} \qquad \angle CQD = \angle ABC.$$

Prove that the points P, Q, B and C are concyclic.

473. (d6, 2017 European Mathematical Cup, P3 of 4) Let ABC be an acute triangle. Denote by H and M the orthocenter of ABC and the midpoint of side BC, respectively. Let Y be a point on AC such that YH is perpendicular to MH and let Q be a point on BH such that QA is perpendicular to AM. Let J be the second point of intersection of MQ and the circle with diameter MY. Prove that HJ is perpendicular to AM.

466. (d6, 2015 IMOSL G3) Let ABC be a triangle with $\angle C = 90°$, and let H be the foot of the altitude from C. A point D is chosen inside the triangle CBH so that CH bisects AD. Let P be the intersection point of the lines BD and CH. Let ω be the semicircle with diameter BD that meets the segment CB at an interior point. A line through P is tangent to ω at Q. Prove that the lines CQ and AD meet on ω.

451. (d6, 2011 Iran TST, P1 of 12) In acute triangle ABC, $\angle B$ is greater than $\angle C$. Let M be the midpoint of $\overline{BC}$ and let E and F be the feet of the altitudes from B and C, respectively. Let K and L be the midpoints of $\overline{ME}$ and $\overline{MF}$, respectively, and let T be on line KL such that $\overline{TA} \parallel \overline{BC}$. Prove that $TA = TM$.

416. (d6, 2017 IMOSL, G4) In triangle ABC, let ω be the excircle opposite to A. Let D, E and F be the points where ω is tangent to BC, CA, and AB, respectively. The circle AEF intersects line BC at P and Q. Let M be the midpoint of AD. Prove that the circle MPQ is tangent to ω.

339. (d6, 2009 IMOSL G4) Given a cyclic quadrilateral $ABCD$, let the diagonals AC and BD meet at E and the lines AD and BC meet at F. The midpoints of AB and CD are G and H, respectively. Show that EF is tangent at E to the circle through the points E, G and H.

289. (d6, 2014 EGMO, P2 of 6) Let D and E be points in the interiors of sides AB and AC, respectively, of a triangle ABC, such that $DB = BC = CE$. Let the lines CD and BE meet at F. Prove that the incentre I of triangle ABC, the orthocentre H of triangle DEF and the midpoint M of the arc BAC of the circumcircle of triangle ABC are collinear.

277. (d6, 2013 North Korea TST, Q5 of 6) The incircle ω of a quadrilateral $ABCD$ touches AB, BC, CD, DA at E, F, G, H, respectively. Choose an arbitrary point X on the segment AC inside ω. The segments XB, XD meet ω at I, J respectively. Prove that FJ, IG, AC are concurrent.

227. (d6, 2017 IMO, Q4 of 6) Let R and S be different points on a circle Ω such that RS is not a diameter. Let ℓ be the tangent line to Ω at R. Point T is such that S is the midpoint of the line segment RT. Point J is chosen on the shorter arc RS of Ω so that the circumcircle Γ of triangle JST intersects ℓ at two distinct points. Let A be the common point of Γ and ℓ that is closer to R. Line AJ meets Ω again at K. Prove that the line KT is tangent to Γ.

193. (d6, 2015 Spring TOT, A-paper Q7) It is well-known that if a quadrilateral has the circumcircle and the incircle with the same centre then it is a square. Is the similar statement true in 3 dimensions: namely, if a cuboid is inscribed into a sphere and circumscribed around a sphere and the centres of the spheres coincide, does it imply that the cuboid is a cube? (A cuboid is a polyhedron with 6 quadrilateral faces such that each vertex belongs to 3 edges.)

28. (d6, 2016 IMO (57th), Q1) Triangle BCF has a right angle at B. Let A be the point on line CF such that $FA = FB$ and F lies between A and C. Point D is chosen so that $DA = DC$ and AC is the bisector of $\angle DAB$. Point E is chosen so that $EA = ED$ and AD is the bisector of $\angle EAC$. Let M be the midpoint of CF. Let X be the point such that $AMXE$ is a parallelogram (where $AM \parallel EX$ and $AE \parallel MX$). Prove that BD, FX and ME are concurrent.

1355. (d7, 2015 IMOSL, G3) Let ABC be a triangle with $\angle C = 90°$, and let H be the foot of the altitude from C. A point D is chosen inside the triangle CBH so that CH bisects AD. Let P be the intersection point of the lines BD and CH. Let ω be the semicircle with diameter BD that meets the segment CB at an interior point. A line through P is tangent to ω at Q. Prove that the lines CQ and AD meet on ω.

1313. (d7, 2014 IMOSL, G3) Let Ω and O be the circumcircle and the circumcentre of an acute-angled triangle ABC with $AB > BC$. The angle bisector of $\angle ABC$ intersects Ω at $M \neq B$. Let Γ be the circle with diameter BM. The angle bisectors of $\angle AOB$ and $\angle BOC$ intersect Γ at points P and Q, respectively. The point R is chosen on the line PQ so that $BR = MR$. Prove that $BR \parallel AC$. (Here we always assume that an angle bisector is a ray.)

1300. (d7, 2011 IMOSL, G1) Let ABC be an acute triangle. Let ω be a circle whose centre L lies on the side BC. Suppose that ω is tangent to AB at B' and AC at C'. Suppose also that the circumcentre O of triangle ABC lies on the shorter arc $B'C'$ of ω. Prove that the circumcircle of ABC and ω meet at two points.

1265. (d7, 2017 Sharygin Finals, 9.4) Points M and K are chosen on lateral sides AB, AC of an isosceles triangle ABC and point D is chosen on BC such that $AMDK$ is a parallelogram. Let the lines MK and BC meet at point L, and let X, Y be the intersection points of AB, AC with the perpendicular line from D to BC. Prove that the circle with center L and radius LD and the circumcircle of triangle AXY are tangent.

1257. (d7, 2019 IMOSL, G2) Let ABC be an acute-angled triangle and let D, E, and F be the feet of altitudes from A, B, and C to sides BC, CA, and AB, respectively. Denote by ω_B and ω_C the incircles of triangles BDF and CDE, and let these circles be tangent to segments DF and DE at M and N, respectively. Let line MN meet circles ω_B and ω_C again at $P \neq M$ and $Q \neq N$, respectively. Prove that $MP = NQ$.

1222. (d7, 2016 RMM, P1 of 6) Let ABC be a triangle and let D be a point on the segment $BC, D \neq B$ and $D \neq C$. The circle ABD meets the segment AC again at an interior point E. The circle ACD meets the segment AB again at an interior point F. Let A' be the reflection of A in the line BC. The lines $A'C$ and DE meet at P, and the lines $A'B$ and DF meet at Q. Prove that the lines AD, BP and CQ are concurrent (or all parallel).

1187. (d7, 2019 IMOSL, G4) Let P be a point inside triangle ABC. Let AP meet BC at A_1, let BP meet CA at B_1, and let CP meet AB at C_1. Let A_2 be the point such that A_1 is the midpoint of PA_2, let B_2 be the point such that B_1 is the midpoint of PB_2, and let C_2 be the point such that C_1 is the midpoint of PC_2. Prove that points A_2, B_2, and C_2 cannot all lie strictly inside the circumcircle of triangle ABC.

1166. (d7, 2010 IMO, P2 of 6) Let I be the incentre of triangle ABC and let Γ be its circumcircle. Let the line AI intersect Γ again at D. Let E be a point on the arc BDC and F a point on the side BC such that

$$\angle BAF = \angle CAE < \tfrac{1}{2}\angle BAC.$$

Finally, let G be the midpoint of the segment IF. Prove that the lines DG and EI intersect on Γ.

1145. (d7, 2015 EGMO, P6 of 6) Let H be the orthocentre and G be the centroid of acute-angled triangle $\triangle ABC$ with $AB \neq AC$. The line AG intersects the circumcircle of $\triangle ABC$ at A and P. Let P' be the reflection of P in the line BC. Prove that $\angle CAB = 60^\circ$ if and only if $HG = GP'$.

1096. (d7, 2007 IMO, P2 of 6) Consider five points A, B, C, D and E such that $ABCD$ is a parallelogram and $BCED$ is a cyclic quadrilateral. Let ℓ be a line passing through A. Suppose that ℓ intersects the interior of the segment DC at F and intersects line BC at G. Suppose also that $EF = EG = EC$. Prove that ℓ is the bisector of angle DAB.

1075. (d7, 2021 IMO, P4 of 6) Let Γ be a circle with centre I, and $ABCD$ a convex quadrilateral such that each of the segments AB, BC, CD and DA is tangent to Γ. Let Ω be the circumcircle of the triangle AIC. The extension of BA beyond A meets Ω at X, and the extension of BC beyond C meets Ω at Z. The extensions of AD and CD beyond D meet Ω at Y and T, respectively. Prove that

$$AD + DT + TX + XA = CD + DY + YZ + ZC.$$

1061. (d7, 2022 BMO2, P4 of 4) Let ABC be an acute angled triangle with circumcircle Γ. Let ℓ_B and ℓ_C be the lines perpendicular to BC which pass through B and C respectively. A point T lies on the minor arc BC. The tangent to Γ at T meets ℓ_B and ℓ_C at P_B and P_C respectively. The line through P_B perpendicular to AC and the line through P_C perpendicular to AB meet at a point Q. Given that Q lies on BC, prove that the line AT passes through Q.

1042. (d7, 2021 USA TSTST, P8 of 9) Let ABC be a scalene triangle. Points A_1, B_1 and C_1 are chosen on segments BC, CA and AB, respectively, such that $\triangle A_1B_1C_1$ and $\triangle ABC$ are similar. Let A_2 be the unique point on line B_1C_1 such that $AA_2 = A_1A_2$. Points B_2 and C_2 are defined similarly. Prove that $\triangle A_2B_2C_2$ and $\triangle ABC$ are similar.

1040. (d7, 2017 IMOSL, G5) Let $ABCC_1B_1A_1$ be a convex hexagon such that $AB = BC$, and suppose that the line segments AA_1, BB_1, and CC_1 have the same perpendicular bisector. Let the diagonals AC_1 and A_1C meet at D, and denote by ω the circle ABC. Let ω intersect the circle A_1BC_1 again at $E \neq B$. Prove that the lines BB_1 and DE intersect on ω.

1035. (d7, 2010 APMO, P4 of 5) Let ABC be an acute angled triangle satisfying the conditions $AB > BC$ and $AC > BC$. Denote by O and H the circumcentre and orthocentre, respectively, of the triangle ABC. Suppose that the circumcircle of the triangle AHC intersects the line AB at M different from A, and the circumcircle of the triangle AHB intersects the line AC at N different from A. Prove that the circumcentre of the triangle MNH lies on the line OH.

901. (**d7, 2021 Taiwan TST Round 2 Quiz 1, G**) Let $ABCD$ be a convex quadrilateral with pairwise distinct side lengths such that $AC \perp BD$. Let O_1, O_2 be the circumcenters of $\Delta ABD, \Delta CBD$, respectively. Show that AO_2, CO_1, the Euler line of ΔABC and the Euler line of ΔADC are concurrent.

845. (**d7, 2020 RMM, P1 of 6**) Let ABC be a triangle with a right angle at C. Let I be the incentre of triangle ABC, and let D be the foot of the altitude from C to AB. The incircle ω of triangle ABC is tangent to sides BC, CA, and AB at A_1, B_1, and C_1, respectively. Let E and F be the reflections of C in lines C_1A_1 and C_1B_1, respectively. Let K and L be the reflections of D in lines C_1A_1 and C_1B_1, respectively. Prove that the circumcircles of triangles A_1EI, B_1FI, and C_1KL have a common point.

796. (**d7, 2021 EGMO, P5 of 6**) A plane has a special point O called the origin. Let P be a set of 2021 points in the plane such that

- no three points in P lie on a line and
- no two points in P lie on a line through the origin.

A triangle with vertices in P is fat if O is strictly inside the triangle. Find the maximum number of fat triangles.

788. (**d7, 2019 Mathematical Reflections Journal Issue 3**) Consider a triangle ABC with incentre I and an incircle which touches AC and AB at E and F respectively. Also denote by H the orthocentre of ABC and M the midpoint of side BC. Show that if H is on EF then H, I, M are collinear.

760. (**d7, 2020 Cono Sur Olympiad, P3 of 6**) Let ABC be an acute triangle with circumcircle ω such that $AC < BC$, and let M be the midpoint of BC. Points F and E are chosen on AB and BC respectively, such that $AC = CF$ and $EB = EF$. The line AM intersects ω again at D.

Prove that the lines DE and FM intersect on ω.

746. (**d7, 2017 IMOSL G3**) Let O be the circumcenter of an acute triangle ABC. Line OA intersects the altitudes of ABC through B and C at P and Q, respectively. The altitudes meet at H. Prove that the circumcenter of triangle PQH lies on a median of triangle ABC.

725. (**d7, 2019 Balkan MO, P3 of 4**) Let ABC be an acute scalene triangle. Let X and Y be two distinct interior points of the segment BC such that $\angle CAX = \angle YAB$. Suppose that:

1. K and S are the feet of the perpendiculars from from B to the lines AX and AY respectively.
2. T and L are the feet of the perpendiculars from C to the lines AX and AY respectively.

Prove that KL and ST intersect on the line BC.

661. (**d7, 2019 IMO, P2 of 6**) In triangle ABC, point A_1 lies on side BC and point B_1 lies on side AC. Let P and Q be points on segments AA_1 and BB_1, respectively, such that PQ is parallel to AB. Let P_1 be a point on line PB_1, such that B_1 lies strictly between P and P_1, and $\angle PP_1C = \angle BAC$. Similarly, let Q_1 be the point on line QA_1, such that A_1 lies strictly between Q and Q_1, and $\angle CQ_1Q = \angle CBA$.

Prove that points P, Q, P_1, and Q_1 are concyclic.

626. (d7, 2014 BMO2, P4 of 4) Let ABC be a triangle and let P be a point in its interior. Let AP meet the circumcircle of ABC again at A'. The points B' and C' are similarly defined. Let O_A be the circumcentre of BCP. The circumcentres O_B and O_C are similarly defined. Let $O_A{}'$ be the circumcentre of $B'C'P$. The circumcentres $O_B{}'$ and $O_C{}'$ are similarly defined. Prove that the lines $O_AO_A{}'$, $O_BO_B{}'$ and $O_CO_C{}'$ are concurrent.

598. (d7, 2013 IMOSL, G2) Let ω be the circumcircle of a triangle ABC. Denote by M and N the midpoints of the sides AB and AC, respectively, and denote by T the midpoint of the arc BC of ω not containing A. The circumcircles of the triangles AMT and ANT intersect the perpendicular bisectors of AC and AB at points X and Y, respectively; assume that X and Y lie inside the triangle ABC. The lines MN and XY intersect at K. Prove that $KA = KT$.

584. (d7, 2018 IMOSL, G2) Let ABC be a triangle with $AB = AC$, and let M be the midpoint of BC. Let P be a point such that $PB < PC$ and PA is parallel to BC. Let X and Y be points on the lines PB and PC, respectively, so that B lies on the segment PX, C lies on the segment PY, and $\angle PXM = \angle PYM$. Prove that the quadrilateral $APXY$ is cyclic.

514. (d7, 2010 IMOSL, G3) Let $A_1A_2 \ldots A_n$ be a convex polygon. Point P inside this polygon is chosen so that its projections $P_1, \ldots, P_n$ onto lines $A_1A_2, \ldots, A_nA_1$ respectively lie on the sides of the polygon. Prove that for arbitrary points $X_1, \ldots, X_n$ on sides $A_1A_2, \ldots, A_nA_1$ respectively,

$$\max\left\{\frac{X_1X_2}{P_1P_2}, \ldots, \frac{X_nX_1}{P_nP_1}\right\} \geq 1.$$

494. (d7, Morley's Trisector Theorem) In triangle ABC, let X, Y, Z be points such that $\angle ABX = \angle XBY = \angle YBC, \angle BCY = \angle YCZ = \angle ZCA, \angle CAZ = \angle ZAX = \angle XAB$. Show that XYZ is equilateral.

486. (d7, 2012 IMOSL G3) In an acute triangle ABC the points D, E and F are the feet of the altitudes through A, B and C respectively. The incenters of the triangles AEF and BDF are I_1 and I_2 respectively; the circumcenters of the triangles ACI_1 and BCI_2 are O_1 and O_2 respectively. Prove that I_1I_2 and O_1O_2 are parallel.

445. (d7, APMO 2014, P5 of 5) Circles ω and Ω meet at points A and B. Let M be the midpoint of the arc AB of circle ω (M lies inside Ω). A chord MP of circle ω intersects Ω at Q (Q lies inside ω). Let ℓ_P be the tangent line to ω at P, and let ℓ_Q be the tangent line to Ω at Q. Prove that the circumcircle of the triangle formed by the lines ℓ_P, ℓ_Q and AB is tangent to Ω.

438. (d7, 2012 EGMO, Q7) Let ABC be an acute-angled triangle with circumcircle Γ and orthocentre H. Let K be a point of Γ on the other side of BC from A. Let L be the reflection of K in the line AB, and let M be the reflection of K in the line BC. Let E be the second point of intersection of Γ with the circumcircle of triangle BLM.

Show that the lines KH, EM and BC are concurrent. (The orthocentre of a triangle is the point on all three of its altitudes.)

361. (d7, 2020 USA TST, Q2 of 3) Two circles Γ_1 and Γ_2 have common external tangents ℓ_1 and ℓ_2 meeting at T. Suppose ℓ_1 touches Γ_1 at A and ℓ_2 touches Γ_2 at B. A circle Ω through A and B intersects Γ_1 again at C and Γ_2 again at D, such that quadrilateral $ABCD$ is convex.

Suppose lines AC and BD meet at point X, while lines AD and BC meet at point Y. Show that T, X, Y are collinear.

304. (d7, Fermat point) Let ABC be a triangle, and let P be a point inside ABC. Show that if $\angle APB = \angle BPC = \angle CPA$ then $PA + PB + PC$ is minimal.

242. (d7, "Coffin" Problem) The faces of a given tetrahedron all have the same area. Prove that they are mutually congruent.

186. (d7, 2015 Saint Petersburg MO Grade 11, Q7 of 7) Let BL be the angle bisector of acute triangle ABC, with L on AC. Point K is chosen on BL such that $\angle AKC - \angle ABC = 90°$. Point S lies on the extension of BL from L such that $\angle ASC = 90°$. Point T is diametrically opposite the point K on the circumcircle of $\triangle AKC$.

Prove that ST and the perpendicular bisector of AC concur on the circumcircle of ABC.

101. (d7, 2019 Asian Pacific MO, Q3) Let ABC be a scalene triangle with circumcircle Γ. Let M be the midpoint of BC. A variable point P is selected in the line segment AM. The circumcircles of triangles BPM and CPM intersectΓ again at points D and E, respectively. The lines DP and EP intersect (at second time) the circumcircles to triangles CPM and BPM at X and Y, respectively. Prove that as P varies, the circumcircle of $\triangle AXY$ passes through a fixed point T distinct from A.

1378. (d8, 2023 USATST, P2) Let ABC be an acute triangle. Let M be the midpoint of side BC, and let E and F be the feet of the altitudes from B and C, respectively. Suppose that the common external tangents to the circumcircles of triangles BME and CMF intersect at a point K, and that K lies on the circumcircle of ABC. Prove that line AK is perpendicular to line BC.

1371. (d8, 2021 Christmas American Math Olympiad, P3 of 6) Let ABC be an scalene triangle with circumcircle Γ and orthocenter H, and let K and M be the midpoints of $\overline{AH}$ and $\overline{BC}$, respectively. Line AH intersects Γ again at T, and line KM intersects Γ at U and V. Lines TU and TV intersect lines AB and AC at X and Y, respectively, and point W lies on line KM such that $\overline{AW} \perp \overline{HM}$. If Z is the reflection of A over W, prove that X, Y, Z are collinear.

1357. (d8, 2022 Kosovo National MO, Grade 12, P3 of 4) Let $\triangle ABC$ be a triangle and D be a point in line BC such that AD bisects $\angle BAC$. Furthermore, let F and G be points on the circumcircle of $\triangle ABC$ and $E \neq D$ point in line BC such that $AF = AE = AD = AG$. If X and Y are the feet of perpendiculars from D to EF and EG, respectively. Prove that $XY \parallel AD$.

1343. (d8, 10th EMC, Senior league, P2 of 4) Let ABC be a triangle and let D, E and F be the midpoints of sides BC, CA and AB, respectively. Let $X \neq A$ be the intersection of AD with the circumcircle of ABC. Let Ω be the circle through D and X, tangent to the circumcircle of ABC. Let Y and Z be the intersections of the tangent to Ω at D with the perpendicular bisectors of segments DE and DF, respectively. Let P be the intersection of YE and ZF and let G be the centroid of ABC. Show that the tangents at B and C to the circumcircle of ABC and the line PG are concurrent.

1335. (d8, 2015 IMOSL, G4) Let ABC be an acute triangle and let M be the midpoint of AC. A circle ω passing through B and M meets the sides AB and BC at points P and Q respectively. Let T be the point such that $BPTQ$ is a parallelogram. Suppose that T lies on the circumcircle of ABC. Determine all possible values of $\frac{BT}{BM}$.

1308. (d8, 2022 Taiwan IMOC, G5) P is a point inside ABC. BP, CP intersect AC, AB at E, F, respectively. AP intersect $\odot(ABC)$ again at X. $\odot(ABC)$ and $\odot(AEF)$ intersect again at S. T is a point on BC such that $PT \parallel EF$. Prove that $\odot(STX)$ passes through the midpoint of BC.

1287. (d8, 2019 China National Olympiad, P3 of 6) Let O be the circumcenter of $\triangle ABC$ $(AB < AC)$, and D be a point on the internal angle bisector of $\angle BAC$. Point E lies on BC, satisfying $OE \parallel AD$, $DE \perp BC$. Point K lies on EB extended such that $EK = EA$. The circumcircle of $\triangle ADK$ meets BC at $P \neq K$, and meets the circumcircle of $\triangle ABC$ at $Q \neq A$. Prove that PQ is tangent to the circumcircle of $\triangle ABC$.

1266. (d8, 2022 Taiwan IMOC, G6) Let D be a point on the circumcircle of some triangle ABC. Let E, F be points on AC, AB, respectively, such that A, D, E, F are concyclic. Let M be the midpoint of BC. Show that if DM, BE, CF are concurrent, then either $BE \cap CF$ is on the circle $ADEF$, or EF is parallel to BC.

1259. (d8, 2022 MEMO T6) Let $ABCD$ be a convex quadrilateral such that $AC = BD$ and the sides AB and CD are not parallel. Let P be the intersection point of the diagonals AC and BD. Points E and F lie, respectively, on segments BP and AP such that $PC = PE$ and $PD = PF$. Prove that the circumcircle of the triangle determined by the lines AB, CD, EF is tangent to the circumcircle of the triangle ABP.

1174. (d8, 2013 IMOSL, G3) In a triangle ABC, let D and E be the feet of the angle bisectors of angles A and B, respectively. A rhombus is inscribed into the quadrilateral $AEDB$ (all vertices of the rhombus lie on different sides of $AEDB$). Let φ be the non-obtuse angle of the rhombus. Prove that $\varphi \leq \max\{\angle BAC, \angle ABC\}$.

1161. (d8, 2021 Balkan MO SL, G8) Let ABC be a scalene triangle and let I be its incenter. The projections of I on BC, CA, and AB are D, E and F respectively. Let K be the reflection of D over the line AI, and let L be the second point of intersection of the circumcircles of the triangles BFK and CEK. If $\frac{1}{3}BC = AC - AB$, prove that $DE = 2KL$.

1154. (d8, 2022 DAMO, P5 of 6) In triangle ABC, D is an arbitrary point on BC. $(ADC), (ADB)$ cut AB, AC at F and E respectively. Tangents to (ABC) at B and C intersect at X. $Z = EF \cap BX$ and $Y = EF \cap CX$. P is a point on (ABC) such that AP, YZ, BC are concurrent. Prove that P lies on (XYZ).

1097. (d8, 2021-2022 ToT Fall Round, Senior A P5 of 7) Let $ABCD$ be a parallelogram and let P be a point inside it such that $\angle PDA = \angle PBA$. Let ω_1 be the excircle of PAB opposite to the vertex A. Let ω_2 be the incircle of triangle PCD. Prove that one of the common tangents of ω_1, ω_2 is parallel to AD.

1077. (d8, 2021 IMO Malaysian Training Camp 1) Let ABC be an acute triangle with $AB < AC$. Let Γ be its circumcircle, I its incenter and P a point on Γ such that $\angle API = 90°$. Let Q be a point on arc BAC such that

$$QB \cdot \tan \angle B = QC \cdot \tan \angle C.$$

Consider a point R such that PR is tangent to Γ and $BR = CR$. Prove that the points A, Q, R are collinear.

1063. (d8, 2011 IMOSL, G2) Let $A_1A_2A_3A_4$ be a non-cyclic quadrilateral. Let O_1 and r_1 be the circumcentre and the circumradius of the triangle $A_2A_3A_4$. Define O_2, O_3, O_4 and r_2, r_3, r_4 in a similar way. Prove that

$$\frac{1}{O_1A_1^2 - r_1^2} + \frac{1}{O_2A_2^2 - r_2^2} + \frac{1}{O_3A_3^2 - r_3^2} + \frac{1}{O_4A_4^2 - r_4^2} = 0.$$

1007. (d8, Folklore) Let S be a closed (i.e. containing its own boundary) and bounded subset of Euclidean space and let f be a function $S \to S$ that preserves distances between points. Must f be bijective?

977. (d8, 2020 USA TSTST, P2 of 9) Let ABC be a scalene triangle with incenter I. The incircle of ABC touches $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ at points D, E, F, respectively. Let P be the foot of the altitude from D to $\overline{EF}$, and let M be the midpoint of $\overline{BC}$. The rays AP and IP intersect the circumcircle of $\triangle ABC$ again at points G and Q, respectively. Show that the incenter of $\triangle GQM$ coincides with D.

943. (d8, 2016 IMOSL, G5) Let D be the foot of perpendicular from A to the Euler line (the line passing through the circumcentre and the orthocentre) of an acute scalene triangle ABC. A circle ω with centre S passes through A and D, and it intersects sides AB and AC at X and Y respectively. Let P be the foot of altitude from A to BC, and let M be the midpoint of BC. Prove that the circumcentre of triangle XSY is equidistant from P and M.

936. (d8, 2018 IMOSL, G4) A point T is chosen inside a triangle ABC. Let A_1, B_1, and C_1 be the reflections of T in BC, CA, and AB, respectively. Let Ω be the circumcircle of the triangle $A_1B_1C_1$. The lines A_1T, B_1T, and C_1T meet Ω again at A_2, B_2, and C_2, respectively. Prove that the lines AA_2, BB_2, and CC_2 are concurrent on Ω.

915. (d8, 2021 APMO, P3 of 5) Let $ABCD$ be a cyclic convex quadrilateral and Γ be its circumcircle. Let E be the intersection of the diagonals of AC and BD. Let L be the center of the circle tangent to sides AB, BC, and CD, and let M be the midpoint of the arc BC of Γ not containing A and D. Prove that the excenter of triangle BCE opposite E lies on the line LM.

859. (d8, **2020 IMOSL, G6**) Let ABC be a triangle with $AB < AC$, incenter I, and A excenter I_A. The incircle meets BC at D. Define $E = AD \cap BI_A$, $F = AD \cap CI_A$. Show that the circumcircle of $\triangle AID$ and $\triangle I_AEF$ are tangent to each other.

789. (d8, **2021 EGMO, P3 of 6**) Let ABC be a triangle with an obtuse angle at A. Let E and F be the intersections of the external bisector of angle A with the altitudes of ABC through B and C respectively. Let M and N be the points on the segments EC and FB respectively such that $\angle EMA = \angle BCA$ and $\angle ANF = \angle ABC$. Prove that the points E, F, N, M lie on a circle.

783. (d8, **2020 USA TSTST, P6 of 9**) Let A, B, C, D be four points such that no three are collinear and D is not the orthocenter of ABC. Let P, Q, R be the orthocenters of $\triangle BCD$, $\triangle CAD$, $\triangle ABD$, respectively. Suppose that the lines AP, BQ, CR are pairwise distinct and are concurrent. Show that the four points A, B, C, D lie on a circle.

732. (d8, **USA TST 2006, P2 of 6**) Let H be the orthocenter of acute triangle ABC. A circle ω, with center O, passes through A and H and intersects sides AB and AC again at Q and P (other than A), respectively.
The circumcircle of triangle OPQ is tangent to segment BC at R. Finally, let D, E and F be the feet of the altitudes from A, B and C respectively, of $\triangle ABC$.

Prove that $\frac{CR}{BR} = \frac{ED}{FD}$.

704. (d8, **2020 EGMO, P3 of 6**) Let $ABCDEF$ be a convex hexagon such that $\angle A = \angle C = \angle E$ and $\angle B = \angle D = \angle F$ and the (interior) angle bisectors of $\angle A, \angle C$, and $\angle E$ are concurrent. Prove that the (interior) angle bisectors of $\angle B, \angle D$, and $\angle F$ must also be concurrent.
Note that $\angle A = \angle FAB$. The other interior angles of the hexagon are similarly described.

622. (d8, **2011 IMOSL, G6**) Let ABC be a triangle with $AB = AC$ and let D be the midpoint of AC. The angle bisector of $\angle BAC$ intersects the circle through D, B and C at the point E inside the triangle ABC. The line BD intersects the circle through A, E and B in two points B and F. The lines AF and BE meet at a point I, and the lines CI and BD meet at a point K. Show that I is the incentre of triangle KAB.

615. (d8, **2015 IMO, P3 of 6**) Let ABC be an acute triangle with $AB > AC$. Let Γ be its circumcircle, H its orthocenter, and F the foot of the altitude from A. Let M be the midpoint of BC. Let Q be the point on Γ such that $\angle HQA = 90^\circ$ and let K be the point on Γ such that $\angle HKQ = 90^\circ$. Assume that the points A, B, C, K and Q are all different and lie on Γ in this order.

Prove that the circumcircles of triangles KQH and FKM are tangent to each other.

550. (d8, **2019 IMOSL, G6**) Let I be the incentre of acute-angled triangle ABC. Let the incircle meet BC, CA, and AB at D, E, and F, respectively. Let line EF intersect the circumcircle of the triangle at P and Q, such that F lies between E and P. Prove that $\angle DPA + \angle AQD = \angle QIP$.

510. **(d8, 2011 IMOSL, G4)** Let ABC be an acute triangle with circumcircle Ω. Let B_0 be the midpoint of AC and let C_0 be the midpoint of AB. Let D be the foot of the altitude from A and let G be the centroid of the triangle ABC. Let ω be a circle through B_0 and C_0 that is tangent to the circle Ω at a point $X \neq A$. Prove that the points D, G and X are collinear.

459. **(d8, 2012 IMO, P5)** Let ABC be a triangle with $\angle BCA = 90°$, and let D be the foot of the altitude from C. Let X be a point in the interior of the segment CD. Let K be the point on the segment AX such that $BK = BC$. Similarly, let L be the point on the segment BX such that $AL = AC$. Let M be the point of intersection of AL and BK.

Show that $MK = ML$.

340. **(d8, 2004 IMO Q5)** In a convex quadrilateral $ABCD$, the diagonal BD bisects neither the angle ABC nor the angle CDA. The point P lies inside $ABCD$ and satisfies

$$\angle PBC = \angle DBA \quad \text{and} \quad \angle PDC = \angle BDA.$$

Prove that $ABCD$ is a cyclic quadrilateral if and only if $AP = CP$.

305. **(d8, 2019 Korea MO, Senior Q6 of 8)** Acute triangle ABC with $AB > AC$ has incentre I and circumcircle Ω. D is the foot of the altitude from A onto BC. AI meets Ω at $M \neq A$. E and F lie on AD such that $EM \perp AM$ and $IF \perp AD$. Show that $ID \times AM = IE \times AF$.

152. **(d8, 2007 Iranian MO, P4 of 5)** P is a point inside triangle ABC such that $PA = PB + PC$. Let B' and C' be the midpoints of arcs APC and APB respectively. Prove that the circumcircles of BPB' and CPC' are tangent to one another.

62. **(d8, 2018 Sharygin Q15)** The altitudes AH_1, BH_2, CH_3 of an acute-angled triangle ABC meet at point H. Points P and Q are the reflections of H_2 and H_3 with respect to H. The circumcircle of triangle PH_1Q meets for the second time BH_2 and CH_3 at points R and S. Prove that RS is a medial line of triangle ABC.

1356. **(d9, 2021 Taiwan TST Round 3 Independent Study 2-G)** Let ABC be a triangle with $AB < AC$, and let I_a be its A-excenter. Let D be the projection of I_a to BC. Let X be the intersection of AI_a and BC, and let Y, Z be the points on AC, AB, respectively, such that X, Y, Z are on a line perpendicular to AI_a. Let the circumcircle of AYZ intersect AI_a again at U. Suppose that the tangent of the circumcircle of ABC at A intersects BC at T, and the segment TU intersects the circumcircle of ABC at V. Show that $\angle BAV = \angle DAC$.

1329. **(d9, 2022 German National Olympiad, P3 of 6)** Let M and N be the midpoints of segments BC and AC of a triangle ABC, respectively. Let Q be a point on the line through N parallel to BC such that Q and C are on opposite sides of AB and $|QN| \cdot |BC| = |AB| \cdot |AC|$.

Suppose that the circumcircle of triangle AQN intersects the segment MN a second time in a point $T \neq N$. Prove that there is a circle through points T and N touching both the side BC and the incircle of triangle ABC.

1301. (**d9**, **2020 RMM Shortlist, G3**) In the triangle ABC with circumcircle Γ, the incircle ω touches sides BC, CA, and AB at D, E, and F, respectively. The line through D perpendicular to EF meets ω at $K \neq D$. Line AK meets Γ at $L \neq A$. Rays KI and IL meet the circumcircle of triangle BIC at $Q \neq I$ and $P \neq I$, respectively. The circumcircles of triangles KFB and KEC meet EF at $R \neq F$ and $S \neq E$, respectively. Prove that $PQRS$ is cyclic.

1294. (**d9**, **2021 Taiwan TST Round 1 Independent Study 2-G**) Let ABC be a triangle with incenter I and circumcircle Ω. A point X on Ω which is different from A satisfies $AI = XI$. The incircle touches AC and AB at E, F, respectively. Let M_a, M_b, M_c be the midpoints of sides BC, CA, AB, respectively. Let T be the intersection of the lines M_bF and M_cE. Suppose that AT intersects Ω again at a point S.

Prove that X, M_a, S, T are concyclic.

1175. (**d9**, **2022 Israel TST 8, P3**) In triangle ABC, the angle bisectors are BE and CF (where E, F are on the sides of the triangle), and their intersection point is I. Point N lies on the circumcircle of AEF, and the angle $\angle IAN$ is right. The circumcircle of AEF meets the line NI a second time at the point L. Show that the circumcenter of AIL lies on line BC.

1133. (**d9**, **2022 Taiwan TST Round 3 Independent Study 1-G**) Let ABC be an acute triangle with orthocenter H and circumcircle Ω. Let M be the midpoint of side BC. Point D is chosen from the minor arc BC on Γ such that $\angle BAD = \angle MAC$. Let E be a point on Γ such that DE is perpendicular to AM, and F be a point on line BC such that DF is perpendicular to BC. Lines HF and AM intersect at point N, and point R is the reflection point of H with respect to N.

Prove that $\angle AER + \angle DFR = 180°$.

1105. (**d9**, **2020 China National Olympiad, P2 of 6**) In triangle ABC, $AB > AC$. The bisector of $\angle BAC$ meets BC at D. P is on line DA, such that A lies between P and D. PQ is tangent to $\odot(ABD)$ at Q. PR is tangent to $\odot(ACD)$ at R. CQ meets BR at K. The line parallel to BC and passing through K meets QD, AD, RD at E, L, F, respectively. Prove that $EL = KF$.

1056. (**d9**, **2018 Taiwan TST Round 3, P3 of 6**) Let I be the incenter of triangle ABC and ℓ be the perpendicular bisector of AI. Suppose that P is on the circumcircle of triangle ABC, and line AP and ℓ intersect at point Q. Point R is on ℓ such that $\angle IPR = 90°$. Suppose that line IQ and the midsegment of ABC that is parallel to BC intersect at M. Show that $\angle AMR = 90°$.

(Note: In a triangle, a *midsegment* is a line connecting two midpoints.)

1028. (**d9**, **2021 IGO Advanced, P5 of 5**) Given a triangle ABC with incenter I. The incircle of triangle ABC is tangent to BC at D. Let P and Q be points on the side BC such that $\angle PAB = \angle BCA$ and $\angle QAC = \angle ABC$, respectively. Let K and L be the incenter of triangles ABP and ACQ, respectively. Prove that AD is the Euler line of triangle IKL.

1014. (**d9**, **2016 ELMO, P6 of 6**) Elmo is now learning Olympiad geometry. In triangle ABC with $AB \neq AC$, let its incircle be tangent to sides

BC, CA, and AB at D, E, and F, respectively. The internal angle bisector of $\angle BAC$ intersects lines DE and DF at X and Y, respectively. Let S and T be distinct points on side BC such that $\angle XSY = \angle XTY = 90^\circ$. Finally, let γ be the circumcircle of $\triangle AST$.

1. Help Elmo show that γ is tangent to the circumcircle of $\triangle ABC$.
2. Help Elmo show that γ is tangent to the incircle of $\triangle ABC$.

1000. (d9, 2018 China TST Test 1, P3 of 6) Circle ω is tangent to sides AB,AC of triangle ABC at D,E respectively, such that $D \neq B$, $E \neq C$ and $BD + CE < BC$. F,G lies on BC such that $BF = BD$, $CG = CE$. Let DG and EF meet at K. L lies on minor arc DE of ω, such that the tangent of L to ω is parallel to BC. Prove that the incenter of $\triangle ABC$ lies on KL.

993. (d9, Folklore) Is it possible to cut a paper square up into finitely many pieces, all of which are similar to a $1 \times \sqrt{2}$ rectangle?

965. (d9, 2020 Fake USAMO, P3 of 6) Let $\triangle ABC$ be a scalene triangle with circumcenter O, incenter I, and incircle ω. Let ω touch the sides $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$ at points D, E, and F respectively. Let T be the projection of D to $\overline{EF}$. The line AT intersects the circumcircle of $\triangle ABC$ again at point $X \neq A$. The circumcircles of $\triangle AEX$ and $\triangle AFX$ intersect ω again at points $P \neq E$ and $Q \neq F$ respectively. Prove that the lines EQ, FP, and OI are concurrent.

944. (d9, 2021 Taiwan IMOC, G9) Let the incenter and the A-excenter of $\triangle ABC$ be I and I_A, respectively. Let BI intersect AC at E and CI intersect AB at F. Suppose that the reflections of I with respect to EF, FI_A, EI_A are X, Y, Z respectively. Show that $\odot(XYZ)$ and $\odot(ABC)$ are tangent to each other.

930. (d9, 2018 IMOSL, G7) Let O be the circumcentre, and Ω be the circumcircle of an acute-angled triangle ABC. Let P be an arbitrary point on Ω, distinct from A, B, C, and their antipodes in Ω. Denote the circumcentres of the triangles AOP, BOP, and COP by O_A, O_B, and O_C, respectively. The lines ℓ_A, ℓ_B, ℓ_C perpendicular to BC, CA, and AB pass through O_A, O_B, and O_C, respectively. Prove that the circumcircle of triangle formed by ℓ_A, ℓ_B, and ℓ_C is tangent to the line OP.

839. (d9, 2021 MODSMO, P7 of 7) Consider a triangle ABC with incircle ω. Let S be the point on ω such that the circumcircle of BSC is tangent to ω and let the A-excircle be tangent to BC at A_1. Prove that the tangent from S to ω and the tangent from A_1 to ω (distinct from BC) meet on the line parallel to BC and passing through A.

825. (d9, 2018 IMOSL, G5) Let ABC be a triangle with circumcircle Ω and incentre I. A line ℓ intersects the lines AI, BI, and CI at points D, E, and F, respectively, distinct from the points A, B, C, and I. The perpendicular bisectors x, y, and z of the segments AD, BE, and CF, respectively determine a triangle Θ. Show that the circumcircle of the triangle Θ is tangent to Ω.

727. (d9, 2019 IMOSL, G5) Let $ABCDE$ be a convex pentagon with $CD = DE$ and $\angle EDC \neq 2 \cdot \angle ADB$. Suppose that a point P is located in the interior of the pentagon such that $AP = AE$ and $BP = BC$. Prove that P lies on the diagonal CE if and only if area (BCD) + area (ADE) = area (ABD) + area (ABP).

662. (d9, West Australian Geometry Olympiad, P3 of 4) Let ABC be a triangle and ω its circumcircle. Denote by I the incentre and O the circumcentre. Let the A-mixtilinear incircle be internally tangent to ω at T, and let D, E and F be the feet of the internal angle bisectors from A, B and C respectively. Suppose that AT meets EF at P. If I, P and O are collinear, then prove that O, D and T are also collinear.

608. (d9, 2019 USA TSTST, P9 of 9) Let ABC be a triangle with incenter I. Points K and L are chosen on segment BC such that the incircles of $\triangle ABK$ and $\triangle ABL$ are tangent at P, and the incircles of $\triangle ACK$ and $\triangle ACL$ are tangent at Q. Prove that $IP = IQ$.

545. (d9, 2014 IMO, P3 of 6) Convex quadrilateral $ABCD$ has $\angle ABC = \angle CDA = 90^\circ$. Point H is the foot of the perpendicular from A to BD. Points S and T lie on sides AB and AD, respectively, such that H lies inside triangle SCT and

$$\angle CHS - \angle CSB = 90^\circ, \quad \angle THC - \angle DTC = 90^\circ.$$

Prove that line BD is tangent to the circumcircle of triangle TSH.

516. (d9, 2016 China TST 2, P6 of 6) The diagonals of a cyclic quadrilateral $ABCD$ intersect at P, and there exists a circle Γ tangent to the extensions of AB, BC, AD, DC at X, Y, Z, T respectively. Circle Ω passes through points A, B, and is externally tangent to circle Γ at S. Prove that $SP \perp ST$.

474. (d9, 2007 IMOSL G8) Point P lies on side AB of a convex quadrilateral $ABCD$. Let ω be the incircle of triangle CPD, and let I be its incenter. Suppose that ω is tangent to the incircles of triangles APD and BPC at points K and L, respectively. Let lines AC and BD meet at E, and let lines AK and BL meet at F. Prove that points E, I, and F are collinear.

440. (d9, 2019 USEMO, P6 of 6) Let ABC be an acute scalene triangle with circumcenter O and altitudes $\overline{AD}$, $\overline{BE}$, $\overline{CF}$. Let X, Y, Z be the midpoints of $\overline{AD}$, $\overline{BE}$, $\overline{CF}$. Lines AD and YZ intersect at P, lines BE and ZX intersect at Q, and lines CF and XY intersect at R.

Suppose that lines YZ and BC intersect at A', and lines QR and EF intersect at D'. Prove that the perpendiculars from A, B, C, O, to the lines QR, RP, PQ, $A'D'$, respectively, are concurrent.

293. (d9, Japanese theorem for cyclic polygons) Given a cyclic polygon P, let T be some triangulation of P, and let S be the sum of the inradii of the triangles in T. Prove that S is independent of T.

215. (d9, 2017 IGO Q5 of 5) Sphere S touches a plane. Let A, B, C, D be four points on this plane such that no three of them are collinear. Consider the point A' such that S is tangent to the faces of the tetrahedron $A'BCD$. Points B', C', D' are defined similarly. Prove that A', B', C', D' are coplanar and the plane $A'B'C'D'$ touches S.

131. (d9, 2017 IMOSL G7) A convex quadrilateral $ABCD$ has an inscribed circle with center I. Let I_a, I_b, I_c and I_d be the incenters of the triangles DAB, ABC, BCD and CDA, respectively. Suppose that the common external tangents of the circles AI_bI_d and CI_bI_d meet at X, and the common external tangents of the circles BI_aI_c and DI_aI_c meet at Y. Prove that $\angle XIY = 90°$.

1364. (d10, 2022 IGO Advanced, P4 of 5) Let $ABCD$ be a trapezoid with $AB \parallel CD$. Its diagonals intersect at a point P. The line passing through P parallel to AB intersects AD and BC at Q and R, respectively. Exterior angle bisectors of angles DBA, DCA intersect at X. Let S be the foot of X onto BC. Prove that if quadrilaterals $ABPQ$, $CDQP$ are circumcribed, then $PR = PS$.

1336. (d10, 2015 Geolympiad Fall Contest, P3 of 3) Let ABC be a triangle and let P_A be the point where the circle passing through B and C different from the circumcircle of ABC that is tangent to the A-mixtilinear-incircle is tangent to the A-mixtilinear-incircle, and define P_B and P_C similarly. Prove that AP_A, BP_B, and CP_C concur.

1245. (d10, 2019 Taiwan TST Round 3, P6 of 6) Given is a triangle $\triangle ABC$ with circumcircle Ω. Denote its incenter and A-excenter by I, J, respectively. Let T be the reflection of J w.r.t BC and P is the intersection of BC and AT. If the circumcircle of $\triangle AIP$ intersects BC at $X \neq P$ and there is a point $Y \neq A$ on Ω such that $IA = IY$, then show that $\odot(IXY)$ is tangent to the line AI.

1238. (d10, 2020 Iran TST, P9 of 12) Given a triangle ABC with circumcircle Γ. Points E and F are the foot of angle bisectors of B and C, I is incenter and K is the intersection of AI and EF. Suppose that T be the midpoint of arc BAC. Circle Γ intersects the A-median and circumcircle of AEF for the second time at X and S. Let S' be the reflection of S across AI and J be the second intersection of circumcircle of $AS'K$ and AX. Prove that quadrilateral $TJIX$ is cyclic.

1224. (d10, 2022 Israel TST Test 10, P3 of 3) Scalene triangle ABC has incenter I and circumcircle Ω with center O. H is the orthocenter of triangle BIC, and T is a point on Ω for which $\angle ATI = 90°$. Circle (AIO) intersects line IH again at X. Show that the lines AX, HT intersect on Ω.

1217. (d10, 2021 IMOSL, G8) Let ABC be a triangle with circumcircle ω and let Ω_A be the A-excircle. Let X and Y be the intersection points of ω and Ω_A. Let P and Q be the projections of A onto the tangent lines to Ω_A at X and Y respectively. The tangent line at P to the circumcircle of the triangle APX intersects the tangent line at Q to the circumcircle of the triangle AQY at a point R. Prove that $\overline{AR} \perp \overline{BC}$.

1203. (d10, 2022 AUS → UNK F3, P3 of 3) Let H be the orthocenter of acute scalene triangle $\triangle ABC$ with circumcircle Γ, and let D, E, F be the feet of the altitudes from A, B, C. Let M be the midpoint of BC and let the circle γ with diameter AH meet Γ at Q. Suppose ME, MF meet the line through A parallel to BC at E', F' and QE', QF' meet BC at S, T, and further suppose that AM meets Γ again at K, and the circumcircle of $\triangle ADK$ meets γ again at W. Show that SE, TF and the tangents to γ at Q, W concur.

1189. (d10, 2022 DIMO, P6 of 6) In triangle $\triangle ABC$, M is the midpoint of arc BAC, I is the incenter and I_A is the A-excenter. Let AC, AB, MI cut $BI, CI, (ABC)$ in E, F, P respectively and let S be the intersection of the circumcircles of triangles AEF and ABC. If X, Y are the reflections of I across I_AE, I_AF respectively, then prove that $(BYF), (CXE), (PXY)$ and PS are concurrent.

1126. (d10, 2021 USA TSTST, P6 of 9) Triangles ABC and DEF share circumcircle Ω and incircle ω so that points A, F, B, D, C, and E occur in this order along Ω. Let Δ_A be the triangle formed by lines AB, AC, and EF, and define triangles $\Delta_B, \Delta_C, \ldots, \Delta_F$ similarly. Furthermore, let Ω_A and ω_A be the circumcircle and incircle of triangle Δ_A, respectively, and define circles $\Omega_B, \omega_B, \ldots, \Omega_F, \omega_F$ similarly.

1. Prove that the two common external tangents to circles Ω_A and Ω_D and the two common external tangents to ω_A and ω_D are either concurrent or pairwise parallel.

2. Suppose that these four lines meet at point T_A, and define points T_B and T_C similarly. Prove that points T_A, T_B, and T_C are collinear.

1112. (d10, 2022 Taiwan TST Round 2 Independent Study 1-G) Let I, O, H, and Ω be the incenter, circumcenter, orthocenter, and the circumcircle of the triangle ABC, respectively. Assume that line AI intersects with Ω again at point $M \neq A$. line IH and BC meets at point D, and line MD intersects with Ω again at point $E \neq M$. Prove that line OI is tangent to the circumcircle of triangle IHE.

986. (d10, 2021 Brazilian Olympic Revenge, P3 of 6) Let I, C, ω and Ω be the incenter, circumcenter, incircle and circumcircle, respectively, of the scalene triangle XYZ with $XZ > YZ > XY$. The incircle ω is tangent to the sides YZ, XZ and XY at the points D, E and F. Let S be the point on Ω such that XS, CI and YZ are concurrent. Let $(XEF) \cap \Omega = R$, $(RSD) \cap (XEF) = U$, $SU \cap CI = N$, $EF \cap YZ = A$, $EF \cap CI = T$ and $XU \cap YZ = O$.

Prove that $NARUTO$ is cyclic.

902. (d10, 2019 Taiwan TST Round 1 Mock 2, P3 of 3) Given a triangle $\triangle ABC$. Denote its incenter and orthocenter by I, H, respectively. If there is a point K with

$$AH + AK = BH + BK = CH + CK$$

Show that H, I, K are collinear.

867. (d10, 2021 IMO, P3 of 6) Let D be an interior point of the acute triangle ABC with $AB > AC$ so that $\angle DAB = \angle CAD$. The point E on the segment AC satisfies $\angle ADE = \angle BCD$, the point F on the segment AB satisfies $\angle FDA = \angle DBC$, and the point X on the line AC satisfies $CX = BX$. Let O_1 and O_2 be the circumcenters of the triangles ADC and EXD, respectively. Prove that the lines BC, EF, and O_1O_2 are concurrent.

790. (d10, 110 Geometry Problems, 77) Let ABC be a triangle with circumcircle Γ and incircle γ. Let A' be the mixtilinear excircle touch point, and let $A'B'$ and $A'C'$ be tangent to γ with $B', C' \in \Gamma$. Let X be the tangency point of $B'C'$ with γ which exists by Poncelet's Porism. Show that (XBC) is tangent to γ.

776. (d10, 2018 Real IMOSL, G6) Let ABC be an scalene acute-angled triangle. The tangents to its circumcircle at points A and B meet the opposite sidelines at A_1 and B_1, respectively. Let L be the Lemoine point of the triangle (where the symmedians meet), and let P be a point inside the triangle such that its projections onto the sides form an equilateral triangle. Prove that $LP \perp A_1B_1$.

713. (d10, 2016 USA TSTST) Let ABC be a triangle with incenter I, and whose incircle is tangent to $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ at D, E, F, respectively. Let K be the foot of the altitude from D to $\overline{EF}$. Suppose that the circumcircle of $\triangle AIB$ meets the incircle at two distinct points C_1 and C_2, while the circumcircle of $\triangle AIC$ meets the incircle at two distinct points B_1 and B_2. Prove that the radical axis of the circumcircles of $\triangle BB_1B_2$ and $\triangle CC_1C_2$ passes through the midpoint M of $\overline{DK}$.

692. (d10, 2008 IMO, P6 of 6) Let $ABCD$ be a convex quadrilateral with $BA \neq BC$. Denote the incircles of triangles ABC and ADC by ω_1 and ω_2 respectively. Suppose that there exists a circle ω tangent to ray BA beyond A and to the ray BC beyond C, which is also tangent to the lines AD and CD. Prove that the common external tangents to ω_1 and ω_2 intersect on ω.

650. (d10, 2012 RMM, P6 of 6) Let ABC be a triangle and let I and O denote its incentre and circumcentre respectively. Let ω_A be the circle through B and C which is tangent to the incircle of the triangle ABC; the circles ω_B and ω_C are defined similarly. The circles ω_B and ω_C meet at a point A' distinct from A; the points B' and C' are defined similarly. Prove that the lines AA', BB' and CC' are concurrent at a point on the line IO.

643. (d10, 2019 RMMSL, G4) Let Ω be the circumcircle of an acute-angled triangle ABC. A point D is chosen on the internal bisector of $\angle ACB$ so that the points D and C are separated by AB. A circle ω centered at D is tangent to the segment AB at E. The tangents to ω through C meet the segment AB at K and L, where K lies on the segment AL. A circle Ω_1 is tangent to the segments AL, CL, and also to Ω at point M. Similarly, a circle Ω_2 is tangent to the segments BK, CK, and also to Ω at point N. The lines LM and KN meet at P. Prove that $\angle KCE = \angle LCP$.

544. (d10, 2014 IMOSL, G7) Let ABC be a triangle with circumcircle Ω and incentre I. Let the line passing through I and perpendicular to CI intersect the segment BC and the arc BC (not containing A) of Ω at points U and V, respectively. Let the line passing through U and parallel to AI intersect AV at X, and let the line passing through V and parallel to AI intersect AB at Y. Let W and Z be the midpoints of AX and BC, respectively. Prove that if the points I, X, and Y are collinear, then the points I, W, and Z are also collinear.

538. (d10, 2018 IMO, P6 of 6) A convex quadrilateral $ABCD$ satisfies $AB \cdot CD = BC \cdot DA$. Point X lies inside $ABCD$ so that

$$\angle XAB = \angle XCD \quad \text{and} \quad \angle XBC = \angle XDA.$$

Prove that $\angle BXA + \angle DXC = 180^\circ$.

531. (d10, 2011 RMM, P3 of 6) A triangle ABC is inscribed in a circle ω. A variable line ℓ chosen parallel to BC meets segments AB, AC at points

D, E respectively, and meets ω at points K, L (where D lies between K and E). Circle γ_1 is tangent to the segments KD and BD and also tangent to ω, while circle γ_2 is tangent to the segments LE and CE and also tangent to ω. Determine the locus, as ℓ varies, of the meeting point of the common inner tangents to γ_1 and γ_2.

524. (d10, 2020 HMIC, P5 of 5) A triangle has area T, a circle has area C, and their overlap has area A. Prove

$$A \leq \frac{T}{3} + \frac{C}{2}.$$

496. (d10, 2011 IMOSL G7) Let $ABCDEF$ be a convex hexagon all of whose sides are tangent to a circle ω with centre O. Suppose that the circumcircle of triangle ACE is concentric with ω. Let J be the foot of the perpendicular from B to CD. Suppose that the perpendicular from B to DF intersects the line EO at a point K. Let L be the foot of the perpendicular from K to DE. Prove that $DJ = DL$.

488. (d10, 2016 USAMO, P5 of 6) An equilateral pentagon $AMNPQ$ is inscribed in triangle ABC such that $M \in \overline{AB}, Q \in \overline{AC}$, and $N, P \in \overline{BC}$. Let S be the intersection of $\overleftrightarrow{MN}$ and $\overleftrightarrow{PQ}$. Denote by ℓ the angle bisector of $\angle MSQ$. Prove that $\overline{OI}$ is parallel to ℓ, where O is the circumcentre of triangle ABC, and I is the incentre of triangle ABC.

334. (d10, 2019 UNK $\rightarrow$ AUS F3, P3 of 3) $ABCD$ is a convex quadrilateral. AB and CD intersect at E. Let H_A, H_B, H_C, H_D be the orthocentres of triangles DAB, ABC, BCD, CDA. H_AH_B intersects H_CH_D at F. H is the orthocentre of triangle EAD. Prove that $\frac{EH}{HF} = \frac{AD}{BC}$.

328. (d10, 2018 AUS $\rightarrow$ UNK F3, P3 of 3) Let B, C, D be collinear points in that order. Find all points A satisfying:

i) A is not on line BCD,

ii) $AB \leq AC$,

iii) there is a unique point X on segment AD such that $\frac{AX}{BX} = \frac{CX}{DX}$.

300. (d10, 2014 Taiwan TST 3, P3 of 3) Let M be any point on the circumcircle of triangle ABC. Suppose the tangents from M to the incircle meet BC at two points X_1 and X_2. Prove that the circumcircle of triangle MX_1X_2 intersects the circumcircle of ABC again at the tangency point of the A-mixtilinear incircle.

229. (d10, 2010 IMOSL G7) Three circular arcs γ_1, γ_2, and γ_3 connect the points A and C. These arcs lie in the same half-plane defined by line AC in such a way that arc γ_2 lies between the arcs γ_1 and γ_3. Point B lies on the segment AC. Let h_1, h_2, and h_3 be three rays starting at B, lying in the same half-plane, h_2 being between h_1 and h_3. For $i, j = 1, 2, 3$, denote by V_{ij} the point of intersection of h_i and γ_j (see the Figure below). Denote by $\widehat{V_{ij}V_{kj}}\widehat{V_{kl}V_{il}}$ the curved quadrilateral, whose sides are the segments $V_{ij}V_{il}$, $V_{kj}V_{kl}$ and arcs $V_{ij}V_{kj}$ and $V_{il}V_{kl}$. We say that this quadrilateral is *circumscribed* if there exists a circle touching these two segments and two arcs. Prove that if the curved quadrilaterals $\widehat{V_{11}V_{21}}\widehat{V_{22}V_{12}}, \widehat{V_{12}V_{22}}\widehat{V_{23}V_{13}}, \widehat{V_{21}V_{31}}\widehat{V_{32}V_{22}}$ are circumscribed, then the curved quadrilateral $\widehat{V_{22}V_{32}}\widehat{V_{33}V_{23}}$ is circumscribed, too.

139. (d10, 2013 IMO P3 of 6) Let the excircle of triangle ABC opposite the vertex A be tangent to the side BC at the point A_1. Define the points B_1 on CA and C_1 on AB analogously, using the excircles opposite B and C, respectively. Suppose that the circumcentre of triangle $A_1B_1C_1$ lies on the circumcircle of triangle ABC. Prove that triangle ABC is right-angled.

972. (d11, 2020 USEMO, P3 of 6) Let ABC be an acute triangle with circumcenter O and orthocenter H. Let Γ denote the circumcircle of triangle ABC, and N the midpoint of OH. The tangents to Γ at B and C, and the line through H perpendicular to line AN, determine a triangle whose circumcircle we denote by ω_A. Define ω_B and ω_C similarly.
Prove that the common chords of ω_A, ω_B and ω_C are concurrent on line OH.

195. (d11, 2015 USATST P6 of 6) Let ABC be a non-equilateral triangle and let M_a, M_b, M_c be the midpoints of the sides BC, CA, AB, respectively. Let S be a point lying on the Euler line. Denote by X, Y, Z the second intersections of M_aS, M_bS, M_cS with the nine-point circle. Prove that AX, BY, CZ are concurrent.

419. (d12, Cauchy's Rigidity Theorem) P and Q are two convex polyhedra, for which there exists a bijective mapping ϕ of faces from P to Q such that $\phi(F)$ is congruent to F and F_i, F_j are adjacent iff $\phi(F_i)$, $\phi(F_j)$ are. Prove P and Q are congruent.

342. (d12, 2020 USATST, P6 of 6) Let $P_1P_2\cdots P_{100}$ be a cyclic 100-gon and let $P_i = P_{i+100}$ for all i. Define Q_i as the intersection of diagonals $\overline{P_{i-2}P_{i+1}}$ and $\overline{P_{i-1}P_{i+2}}$ for all integers i.
Suppose there exists a point P satisfying $\overline{PP_i} \perp \overline{P_{i-1}P_{i+1}}$ for all integers i. Prove that the points $Q_1, Q_2, \ldots, Q_{100}$ are concyclic.

167. (d12, 2011 IMO, P6 of 6) ABC has circumcircle Γ, and line ℓ is tangent to Γ. Consider the triangle Θ formed by reflecting ℓ over the three sides of ABC. Prove that the circumcircle of Θ is tangent to Γ.

153. (d12, Problem of Apollonius) Show how to construct a circle tangent to three given circles using only straightedge and compass.

412. (d14, Hilbert's third problem) Given two polyhedra of equal volume, is it always possible to dissect one into finitely many polyhedral pieces which can be rearranged to make the other?