Chapter 1: Groups

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A **group** is a pair $G = (G, *)$ consisting of a set of elements G, and a binary operation * on G, such that:

1. G has an **identity element**, usually denoted 1_G or just 1, with the property that

$$1_G * g = g * 1_G = g \text{ for all } g \in G.$$

2. The operation is associative, meaning $(a * b) * c = a * (b * c)$ for any $a, b, c \in G$. Consequently we generally don't write the parantheses.

3. Each element $g \in G$ has an **inverse**, that is, an element $h \in G$ such that

$$g * h = h * g = 1_G$$

For the symmetric group, the group elements are not the numbers, but the functions/permutations themselves.

Fact 1.2.3 Let G be a group.

1. The identity of a group is unique. Let 1 and $1'$ are identities, then $1 = 1 * 1' = 1'$.

2. If h and h' are inverses to g, then $1_G = g*h \Rightarrow h' = (h'*g)*h = 1_g*h = h$.

3. For any $g \in G$, $(g^{-1})^{-1} = g$ Let $h = g^{-1}$. $gh = 1_G$, $hh^{-1} = 1_G$, therefore $g = h^{-1} = (g^{-1})^{-1}$.