Graph Theory Reference

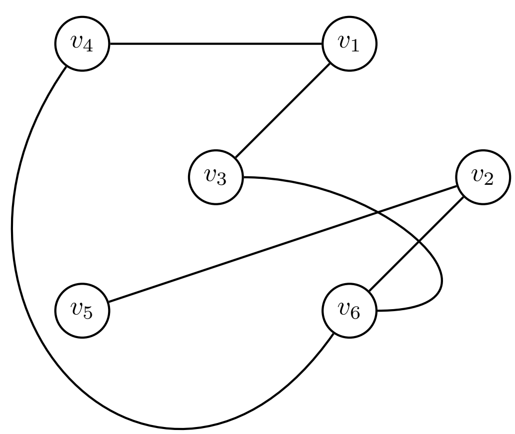
December 26, 2022

1 Introduction

$\mathbb{N}$ is the set of natural numbers. The set $\mathbb{Z}/n\mathbb{Z}$ of integers module N is denoted by $\mathbb{Z}_n$. For example, $\mathbb{Z}_2$ is $\{\bar{0}, \bar{1}\}$. Base 2 logarithm is written as 'log'. The expressions $x := y$ and $y =: x$ mean that x is being defined as y. A partition is a set $B = \{A_1, A_2, ..., A_k\}$ such that all sets of B are disjoint with each other. $[A]^k$ is the set of subsets of size k in A.

2 Graphs

A graph is a pair $G = (V, E)$ of sets such that $E \subset [V]^2$. Always assume $V \cap E = \emptyset$. The elements of V are vertices (or nodes, or points) of the graph G, while the elements of V are the edges (or lines) of graph G.



The graph on $V = \{v_1, \ldots, v_6\}$ *with edge set*
$E = \{\{v_1, v_4\}, \{v_1, v_3\}, \ldots, \{v_4, v_6\}\}$

A graph with vertex set V is said to be a graph on V. The vertex of a graph G is referred to as $V(G)$, its edge set as $E(G)$. Sometimes, a graph might not be distinguished from its edge and vertex set. For example, a vertex $v \in G$ instead of $v \in V(G)$, and an edge $e \in G$ instead of $e \in E(G)$.

The number of vertices of a graph G is its order, written as $|G| = |V(G)|$; its number of edges is denoted by $||G|| = |E(G)|$. According to the graph's order, they are finite, infinite, countable and so on.

The empty graph $(\emptyset, \emptyset)$ is denoted simply as $\emptyset$. A graph with order 0 or 1 is called trivial. Trivial graphs are useful e.g. to start an induction; they are also silly counterexamples. The text generally disregards the trivial graphs.

A vertex v is incident with an edge e if $v \in e$; then e is an edge at v. An edge $\{x, y\}$ is usually written as xy or yx. If $x \in X$ and $y \in Y$, then xy is an $X - Y$ edge. The set of all $X - Y$ edges in a set E is denoted by $E(X, Y)$; instead of $E(x, Y)$ and $E(X, y)$ we simply write $E(x, Y)$ and $E(X, y)$. The set of all edges in E at a vertex v is denoted by $E(v)$.

Two vertices x, y of G are adjacent, or neighbors, if $x, y \in E(G)$. Two edges $e \neq f$ are adjacent if they have an end in common ($e \cap f \neq \emptyset$). If all the vertices of G are pairwise adjacent, then G is complete. A complete graph on n vertices is a K^n; a K^3 is called a triangle.

Pairwise non-adjacent vertices are independent. A set of vertices or edges is independent if no two elements of its elements are adjacent. Independent sets of vertices are called stable.

Let $G = (V, E)$ and $G' = (V', E')$ be two graphs.