Graph Theory Chapter 1 Exercises

January 5, 2023

1. **What is the number of edges in a K^n?** Since there are n vertices, $|E(K^n)| = |[V]^2| = \binom{n}{2} = \frac{n!}{(n-2)!2!} = \frac{n(n-1)}{2}$.

2. **Let $d \in \mathbb{N}$ and $V := \{0,1\}^d$; thus, V is the set of all 0-1 sequences of length d. The graph on V in which two such sequences form an edge if and only if they differ in exactly one position is called the d-*dimensional cube*. Determine the average degree, number of edges, diameter, girth, and circumference of this graph.** Each vertex $v \in V$ has d neighbors because there are d positions where the sequence can vary exactly once. Since there are 2^d vertices, and each vertex has d neighbors, there are $\frac{d2^d}{2} = d2^{d-1}$ edges. The average degree is $\frac{d2^n}{d} = d$. Notice that the distance between two vertices is the number of times their sequences differ at a position. Two sequences can differ at position i atmost d times. This means that the diameter is d.