Dynamics of Language Death

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1 Introduction

Language death is the phenomenon in which the speech communities witness a decrease in the linguistic competence of its speakers, eventually leading to lesser language variety or no fluent speakers of that language. The influence of dominating languages on endangered ones slowly leads to the extinction of the lesser used languages. It is estimated that 90% of the world's languages will die with the current generation and no native speakers of that language will be left.

2 Modelling and stability analysis of language death

2.1 Assumptions made

1. We assume that the language in itself is constant, ignoring the grammar, syntax and other language rules that need to be learnt for knowing the language. Effectively we assume that learning any language is as difficult as learning any other language. There is no increase or decrease in the vocabulary.
2. We assume that languages compete with each other only for speakers. Speakers may change their alignment from one language to another. There is no competition in terms of vocabulary.
3. We assume a highly connected population. Thus changes,if any, reflect instantaneously throughout the population.
4. We assume that the population has no spatial or social structure and does not differ in these parameters. Thus spatially, the dynamics of the language vary uniformly.
5. We assume that every speaker is monolingual at any given time.

2.2 Modelling two competing languages

1. We consider two competing languages, X and Y.
2. Every language has an attractiveness value that depends upon

- Number of speakers
- Perceived status-which is determined by how influential the language is socially and economically.

3. A speaker converts from Y to X with a probability, per unit of time, of $P_{yx}(x, s)$ where x is a fraction of the population speaking X and $0 < s \leq 1$ is a measure of X's relative status.

2.3 Equations used to model the language death

$\frac{dx}{dt} = yP_{yx}(x, s) - xP_{xy}(x, s)$

$\frac{dx}{dt} = (1 - x)P_{yx}(x, s) - xP_{yx}(1 - x, 1 - s)$

$P_{yx}(x, s) = cx^a s$

$\frac{dx}{dt} = c(1 - x)x^a s - cx(1 - x)^a(1 - s)$

2.4 Aspects to be noted about the equations

- $y = 1 - x$, since x and y are fractions
- $P_{xy}(x, s) = P_{yx}(1 - x, 1 - s)$
- No speaker will embrace a language which has no speakers, i.e $P_{yx}(0, s) = 0$
- Similarly no speaker will embrace a language which has no status i.e, $P_{yx}(x, 0) = 0$
- Statistically, the exponential constant a is found out to be between 1.06 to 1.56 with the mean being $a = 1.31$

2.5 Graphs for stability analysis

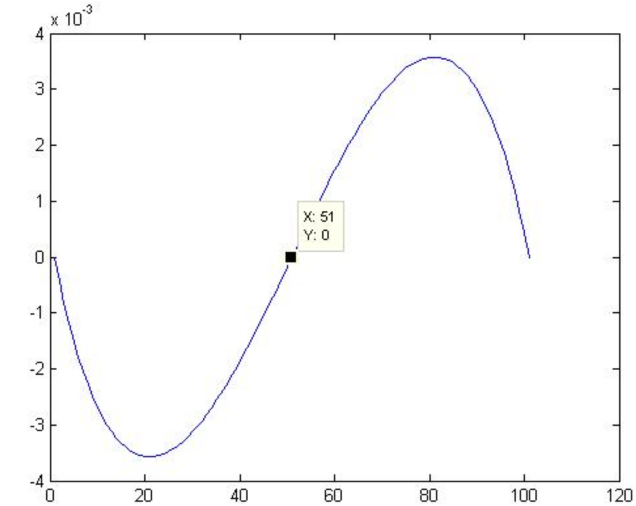

Figure 1: Phase plane with equal status

The phase plot of the language X, shows three fixed points with $x = 0$ and $x = 1$ as stable and $x = 0.5$ as unstable fixed point. This holds true whenever the status of the two languages is neither 0 nor 1, there exists an unstable fixed point somewhere midway ($0 < x < 1$).

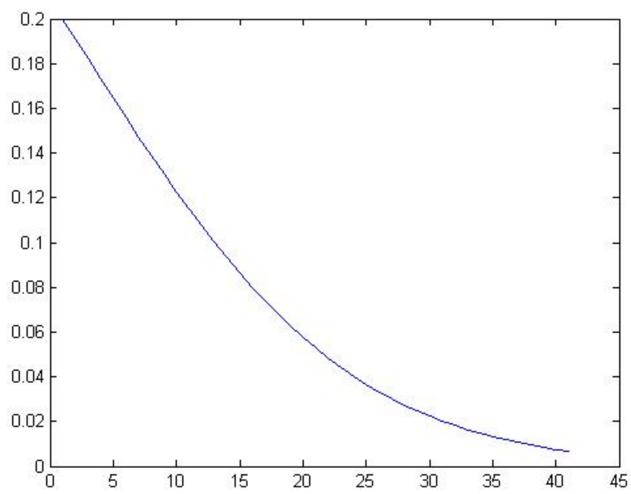

Figure 2: Population Fraction vs time

The plot above shows the fraction of population speaking language X and the plot shows the death of the language x eventually when the status of language Y is more.

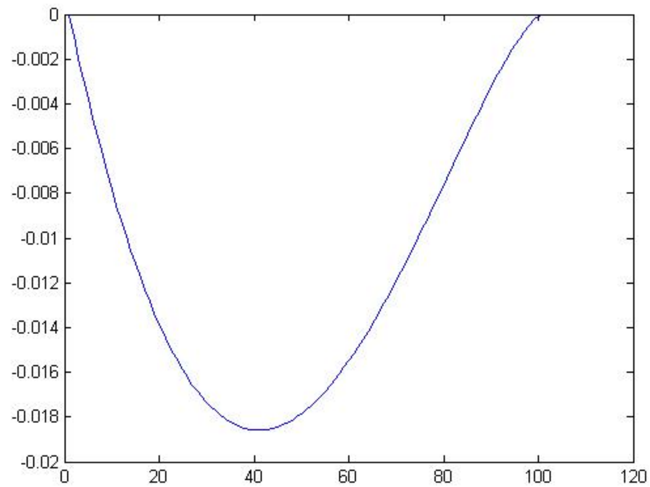

Figure 3: Phase plane for s=0

The plot above shows the phase plot of the population fraction speaking language X. There are two fixed points. Here, $s = 0$ shows that no one will adopt a language with no status. Thus $x = 0$ is a stable fixed point and $x = 1$ is the unstable fixed point.

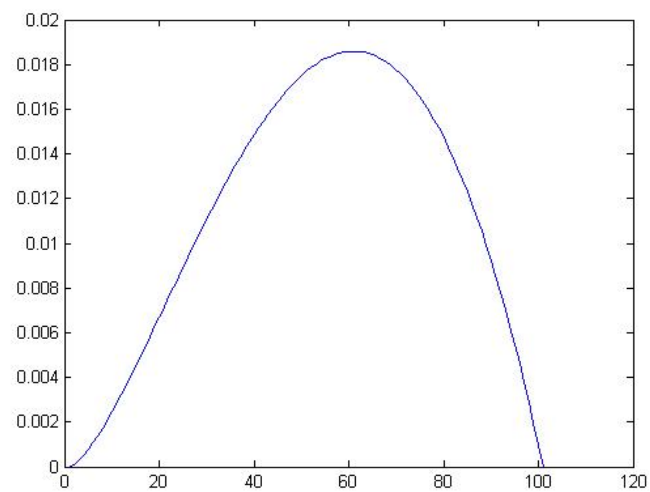

Figure 4: Phase plane for s=1

The plot above shows the phase plot of the population fraction speaking language X. Again, there are two fixed points. Here, $s = 1$ shows that everyone will adopt a language with very high status. Thus $x = 0$ is an unstable fixed point and $x = 1$ is the stable fixed point.

2.6 Understanding the dynamics of language death- stability analysis

This model works with a population of speakers who are limited to knowing one language, and only have 2 languages to chose from, one of which is say vernacular and hence endangered, and the other is a prestigious and hence dominating language. As the number of speakers of a particular language grow, the status of the language improves, and it becomes more and more prestigious.

According to the current model, there are two stable fixed points at $x = 0$ and $x = 1$ which means that two languages cannot co-exist stably. One is bound to drive the other to extinction. There is a third fixed point such that $0 < x < 1$ which is unstable which is generated as a result of bifurcation when the parameter s is lies between $0 < s < 1$. When $s = 0$, there will be only one fixed point at $x = 0$, and when $s = 1$, there will be one fixed point at $x = 1$.

The bifurcation is a *transcritical bifurcation*.

3 Extension to the 2 languages model - Bilingual community

3.1 Introduction

The model described above assumes that a speaker embraces only one language at any given point of time. In the bilingual community model, a speaker can choose to speak in both languages at the same instant of time. The equations and their stability is discussed below.

3.2 Equations used to include bilingual speakers

$\frac{dx}{dt} = yP_{yx}(x, s_x) + bP_{bx}(x, s_x) - x[P_{xy}(y, s_y) + P_{xb}(b, s_b)]$

$P_{xb}(b, s_b) = cs_b b^a = cks_y(1 - x)^a$

$P_{xy}(y, s_y) = cs_y y^a = c(1 - k)s_y(1 - x)^a$

$P_{yb}(b, s_b) = cs_b b^a = cks_y(1 - y)^a$

$P_{yx}(x, s_x) = cs_x x^a = c(1 - k)s_x(1 - y)^a$

$\frac{dx}{dt} = c[(1 - x)(1 - k)s_x(1 - y)^a - x(1 - s_x)(1 - x)^a]$

$\frac{dy}{dt} = c[(1-y)(1-k)(1-s_x)(1-x)^a - ys_x(1-y)^a]$

3.3 Aspects about the equations

- $x+y+b=1$, where x, y, and b are fractions of the population and b represents the bilingual fraction of the population.

- The parameter k represents the ease of bilingualism, or the similarity between two languages. $k = 0$ would suggest no similarity, ergo there would be no communication between X and Y, therefore $P_{xb} = 0$, whereas $k = 1$ implies $X = Y$ where $P_{xy} = 0$ and P_{xb} becomes the same as P_{xy} in the non-bilingual model.

- For transfer from B to X, we may assume $P_{bx} = P_{yx}$, considering both B to X and X to Y involve loss of language Y, which is caused mainly by the death of the speaker. Similarly, $P_{by} = P_{xy}$.

3.4 Graphs

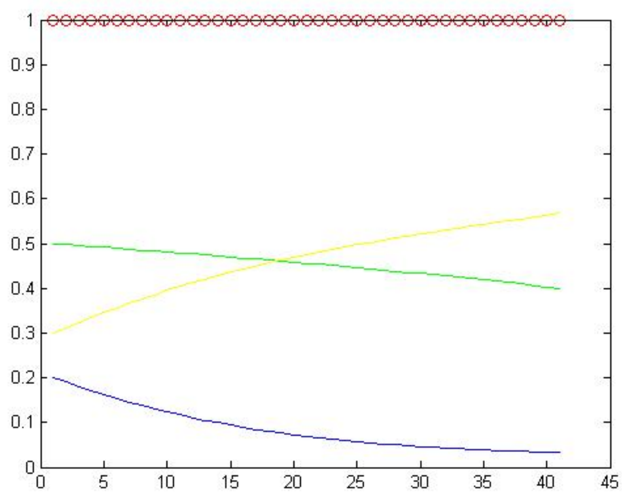

Figure 5: Population Fractions for similar languages

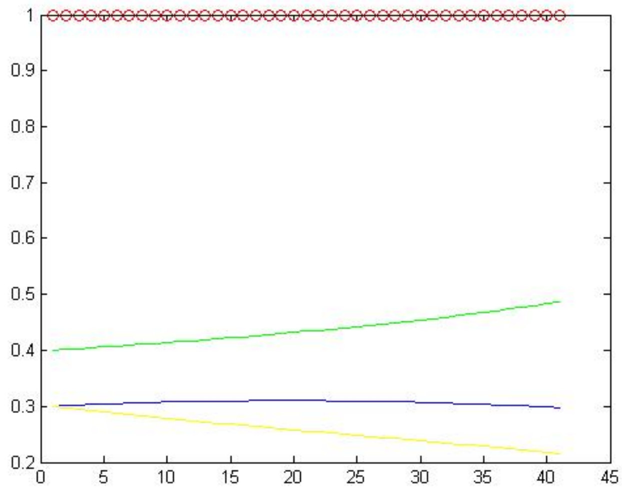

Figure 6: Population Fractions for dissimilar languages

Population speaking language X in blue, language Y in green, and B in yellow. The total population is shown in red circles.

The total fraction of the population remains constant, the population speaking languages X and Y reduce in the first graph, producing more number of people speaking both. This is due to the similarity between languages ($K = 0.8$).The similarity between languages, if reduced to 0.1, increases the population speaking language X and Y, leaving behind both the languages. The figure explains the nature of the two languages spoken individually.

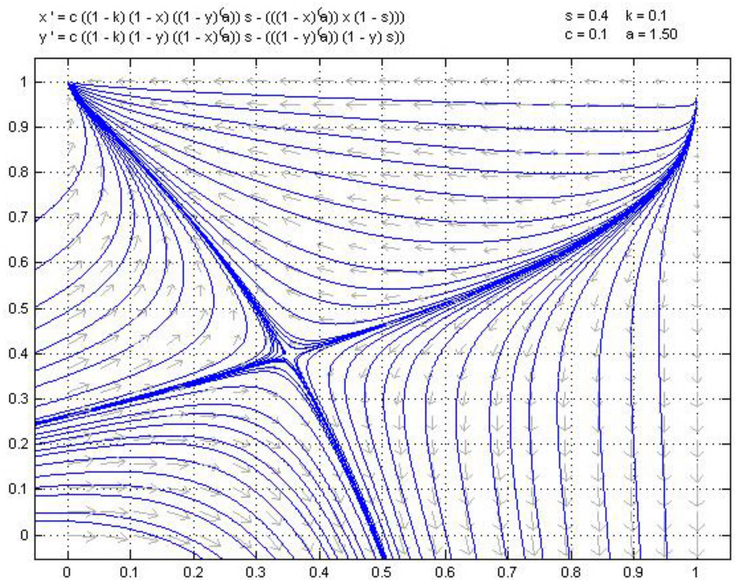

Figure 7: Phase plane of bilingual population

The phase plane plot of the two languages when the similarity is very low leads to a *saddle node bifurcation* where both the languages co-exist.

3.5 Analysis of the stability of the bilingual community equations

When two languages spoken in a community are similar (say, $k \approx 0.8$), they can co-exist together by having bilingual speakers. Analysed further, the model reflects that for each s_x, there is a threshold value $k_{min}(s_x, a)$, such that if $k < k_{min}$, the language with lesser status becomes extinct over time. However if $k > k_{min}$, then both B and X exist. The similarity of the competing languages play a major role in determining the stability of bilingualism. The model shows that bilingualism is possible.

A

Code

The Code for the langugae death function language
xo=0.8;
iv=[xo];
tspan=[0 50];
$[t, u] = ode45(@deriv, tspan, iv);$
plot(u(:,1));
end
$functionydot$=$deriv(t, y)$ a=1.50;
s=0.33;
x=y(1);
$xdot = 0.1 * x * (1 - x) * (x^{(a-1)} * s - ((1 - x)^{(a-1)}) * (1 - s));$
dot=[xdot];
end

The code for the phase plane, to find the fixed points
function lang
a=1.50;
s=0.8;
xdot=zeros();
u=1;
for x=0:0.01:1;
$xdot(u) = 0.1 * x * (1 - x) * (x^{(a-1)} * s - ((1 - x)^{(a-1)}) * (1 - s));$
disp(xdot(u));
u=u+1;
end
plot(xdot);
end

References

[1] $http : //guava.physics.uiuc.edu/\ nigel/courses/569/Essays\_Fall2009/files/liu.pdf$

[2] $https : //www.math.uh.edu/\ zpkilpat/teaching/math4309/project/nature03\_abrams.pdf$