

## CPS 843 (CP 8307) Problem Set 5

Due Monday, December 1st, @ 11:59pm

(5 assignments in total, final grade is evaluated based on the best 4 out of 5)

### Purpose

- Familiar with the algorithm in computer vision
- Understand the basic concepts of camera calibration, single-view and two-view geometry

### Part 1:

Problem 1. (1) Give the canonical form of the plane at infinity. (2) Verify that the plane at infinity is a fixed plane under a 3D affine transformation. (3) Given the 3D points transformation  $\mathbf{X}' = \mathbf{H}\mathbf{X}$ , verify that a 3D plane is transformed as  $\boldsymbol{\pi}' = \mathbf{H}^{-T}\boldsymbol{\pi}$ . (6 points)

Problem 2. (1) Verify that coplanar 3D points and their images are related by a 2D homography given by  $\mathbf{H} = \mathbf{K}[\mathbf{r}_1, \mathbf{r}_2, \mathbf{t}]$ . (2) Verify that the back-projection of an image line is a 3D plane given by  $\Pi = \mathbf{P}^T \mathbf{l}$ . (3) Verify that the images captured by a zooming camera are related by a 2D homography given by  $\mathbf{H} = \mathbf{K}'\mathbf{K}^{-1}$ . (6 points)

Problem 3. (1) Verify that the image of the absolute conic is given by  $\boldsymbol{\omega} = (\mathbf{K}\mathbf{K}^T)^{-1}$ . (2) Verify that two image points correspond to orthogonal directions satisfy  $\mathbf{x}_1^T \boldsymbol{\omega} \mathbf{x}_2 = 0$ . (3) Verify that two constraints on the image of the absolute conic can be obtained from a homography between a 3D plane and its image  $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3]$ . (4) Show the constraints on the image of the absolute conic obtained under the square pixel assumption. (6 points)

### Part 2: (7 points)

#### Software:

The Structure from Motion Package of the Computer Vision Toolbox for Matlab as outlined in the <https://www.mathworks.com/help/vision/ug/structure-from-motion-from-two-views.html>

#### Work to do:

1. Read the camera calibration paper “Flexible Camera Calibration by Viewing a Plane from Unknown Orientations - Zhang, ICCV’99” (the Calibration App is based on this paper). You may download the paper here <https://www.microsoft.com/en-us/research/wp-content/uploads/2016/11/zhan99.pdf>
2. Read the lecture notes about two-view geometry and 3D reconstruction (Lectures 10, 11)
3. Follow the instructions and calibrate your camera using the Single Camera Calibration App given in the following link. Follow the workflow of the app to calibrate your camera and export the parameters to an object. <https://www.mathworks.com/help/vision/ug/using-the-single-camera-calibrator-app.html>

4. Using the calibrated camera, take a pair (or more pairs) of stereo images of a rigid object or static scene from two different viewpoints (to facilitate feature detection, it's better to choose the object or scene with rich texture).
5. Carefully read and follow the steps in the Structure from Motion Package, and perform the following steps: 1) read images; 2) load camera parameters; 3) remove lens distortion (optional); 4) find correspondences; 5) estimate the essential matrix; 6) compute the camera pose; 7) 3D reconstruction; and 8) display the reconstructed 3D points cloud.  
<https://www.mathworks.com/help/vision/ug/structure-from-motion-from-two-views.html>
6. You can directly use the given packages for calibration and reconstruction. You may make some necessary modifications based on your needs and understanding.

### Report requirements:

- A brief technical overview of the camera calibration paper “Flexible Camera Calibration by Viewing a Plane from Unknown Orientations” (in about 0.5-1 page using your own works).
- A brief technical overview of the theory for 3D reconstruction we taught during class (in about 0.5-1 page using your own works).
- Following each step (except for the sphere fitting step), give a brief description of what that step is used for, and the results of that step, such as the camera parameters, essential matrix, matching results, etc.
- Make a reasonable analysis and discussion of your results based on your understanding.

### Available resources

- Reconstruction package:  
<https://www.mathworks.com/help/vision/ug/structure-from-motion-from-two-views.html>
- Calibration App:  
<https://www.mathworks.com/help/vision/ug/using-the-single-camera-calibrator-app.html>
- Structure from motion:  
<https://www.mathworks.com/help/vision/structure-from-motion.html>

### Requirements

- The assignment is due on **Monday, December 1st @ 11:59pm. You have 24 hours of grace time without penalty. Submissions after the grace period will not be accepted.**
- Submit all your work in **a single PDF file** through D2L (multiple submissions are allowed, but only the last submission will be considered for evaluation).
- Please **resize all images properly** in line with the text of your report.
- Submit the **source code** in line with the report of each part.
- You can directly use any available functions of Matlab for your work. Other programming languages and packages are also allowed.
- Complete the report by yourself. We will use Turnitin® for similarity check.

### TA for HW#5:

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You may contact the TA directly if you have any questions about this assignment.