Assignment 1 Question 3 Report

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Consider a (non-discrete) image $I(x)$ with a continuous domain and real-valued intensities within $[0,1]$. Let the image histogram be $h(I)$, with mass 1. Consider the histogram $h(I)$ is split into two histograms (i) $h_1(I)$ over the domain $[0,a]$ and (ii) $h_2(I)$ over the domain $(a,1]$, for some arbitrary $a \in (0,1)$. Assume that the histogram mass within $[0,a]$ is $\alpha \in (0,1)$.

Question Suppose you perform histogram equalization over the two intensity intervals $[0,a]$ and $(a,1]$ separately, in a way that preserved the masses of the two histograms $h_1(I)$ and $h_2(I)$ after the transformation. Derive the mean intensity for the resulting histogram (or, equivalently, image).

It is given that: $\int_0^1 h(I)dI = 1$

From mass conservation over the domain $(0,a]$: $h_1(I)dI = p_1(b)db$

From mass conservation over the domain $(a,1]$: $h_2(I)dI = p_2(b)db$

On integrating over $(0,a]$: $\int_0^a h_1(I)dI = \int_0^1 p_1(b)db = \alpha$

On integrating over $(a,1]$: $\int_a^1 h_2(I)dI = \int_0^1 p_2(b)db = 1$ - α

The output image has a uniform distribution thus on scaling it down : $p_1(b) = \frac{\alpha}{a}$ and $p_2(b) = \frac{1-\alpha}{1-a}$

The mean intensity for the resulting histogram is : $\int_0^a bp_1(b)db + \int_a^1 bp_2(b)db$

Using above equations : $\int_0^a b\frac{\alpha}{a}db + \int_a^1 b\frac{1-\alpha}{1-a}db$

On simplification it yields : $\frac{a\alpha}{2} + \frac{(1-\alpha)(1+a)}{2} = \frac{1+a-\alpha}{2}$

The mean intensity for the resulting histogram is :$\frac{1+a-\alpha}{2}$

Question Let the chosen intensity a be the median intensity for the original histogram $h(I)$. Assume that the mean intensity for the original histogram $h(I)$ is also a. Then, what is the mean intensity for the resulting histogram (or, equivalently, image).

Since it is given that a is the median intensity, $\int_0^a h_1(I)dI = \frac{1}{2} = \alpha$.

Also since a is the mean mean intensity of the original histogram, $\int_0^1 Ih_1(I)dI =$ a

Putting the value of α in the equation obtained in the above question for mean output intensity, we get it to be : $\frac{1+2a}{4}$

Question : Describe a scenerio where the above described histogram-based intensity transform with $a = median\ intensity$ will do a better job in intensity transformation than a simple histogram equalization. Explain the reasons clearly.

Whenever histogram equalization is performed on the entire histogram of the image, the new function is always intended to be of uniform distribution and hence the mean intensity would be close to 0.5 most of the times even if input images are quite different. However with the above described histogram-based intensity transform and median intensity a, the output mean is at least similar to input mean, thereby preserving the nature of image's contents. Its applications might include digital photography, medical image analysis, etc.

Question : Do an online search to find an image along the lines of your reasoning. Write a code for this intensity transformation and demonstrate the better performance on the image you obtained. Note that the better performance should be distinctly evident.

The statue image was transformed according to the given algorithm and compared with the histogram equalised output. The threshold was set to 120 and it was output of bi-histogram equalised image seemed more realistic and similar to the input with image content well maintained!!