Assignment 4 Question 5 Report

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1 Framing the problem

The problem given is an optimization problem. We want to maximize $f^T Cf$ where C is the covariance matrix.
However, we have the constraints:-

- $f^T f = 1$
- $f^T e = 0$, e is the eigen vector of C with the largest eigen value.

Here e is the eigen vector of C with the largest eigen value.

2 Using Lagrange Multipliers

Now we can apply the formulation of Largrange multipliers to solve this problem. We have to maximize the cost function:-

$$J(f) = f^T Cf - \lambda_1(f^T f - 1) - \lambda_2 f^T e$$

subject to:-

- $f^T f = 1$
- $f^T e = 0$, e is the eigen vector of C with the largest eigen value.

3 Optimization

Differentiating $J(f)$ with respect to f and equating to zero, we get,

$$2f^T C - 2\lambda_1 f^T - \lambda_T e^T = 0$$

Now, we can take transpose on both sides. Seeing that C^T is symmetric, we have,

$$Cf = \lambda_1 f + \frac{\lambda_2}{2} e$$

If we pre-multiply by e^T, we get an expression for λ_2 as $e^T f = 0$.

$$\lambda_2 = 2e^T Cf$$

Substituting this back,

$$Cf = \lambda_1 f + 2e^T Cfe - - - (1)$$

But we have that $Ce = \mu e$. Taking transpose on both sides, we get,

$$e^T C = \mu e^T$$

Substituting this into (1),

$$Cf = \lambda_1 f + 2\mu e^T f e$$

Again, using $e^T f = 0$, we get,

$$Cf = \lambda_1 f$$

Hence f is an eigen vector of C. We have to maximize $f^T C f$. But since $Cf = \lambda_1 f$, we have,

$$f^T C f = \lambda_1$$

Above we used $f^T f = 1$
We want λ_1 to be as large as possible. But e has the largest eigen value and it is perpendicular to f. Since the eigen values are unique(given in the question), f will have the second largest eigen value.

Hence proved.

4 Conclusions

Hence we have proved the given statement.