Assignment 5 \ Question 2 Report

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1 1D Image

It is given that, $g = h * f$ where g is the gradient image (in 1D), h is the convolution kernel to represent the gradient operation, and f is the original 1D image
To determine f from g and h we can perform Fourier Transform.
Taking the Fourier Transform on both sides, we have, $G(f) = H(f)F(f)$
This gives us : $F(f) = G(f)/H(f)$
On taking the Inverse Fourier Transform :

$$f = F^{-1}(\frac{G(f)}{H(f)})$$

Thus we can perfectly reconstruct the original 1D image till $H(f)$ is non-zero.
Fourier Transform of Gradient Operator is like a high-pass filter thus there would be no difficulty in case of high frequencies but it may go down to zero in case of low frequencies thus we may lose information in that case.

2 2D Image

For 2D image we have:

$$g_x = h_x * f$$
$$g_y = h_y * f$$

Using the same approach as in case of 1D image:

$$F(f) = G_x(f)/H_x(f) \qquad and \qquad F(f) = G_y(f)/H_y(f)$$

which gives:

$$f = F^{-1}(\frac{G_x(f)}{H_x(f)}) \qquad\qquad f = F^{-1}(\frac{G_y(f)}{H_y(f)})$$

Using the similar argument that gradient operator act as a high pass filter in frequency domain, we see that we can perfectly reconstruct the image in case of high frequencies.
Since in this case we have two equations to get f, even if $H_x(f)$ is zero we can use the second equation to get back f and vise versa. Problem occurs when both $H_x(f)$ and $H_y(f)$ are zero in which case, we lose the information.