EE 338 : Digital Signal Processing

Additonal Filter Design Report

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Contents

Assigned Filter Number: 18

1 Filter-1 (Bandpass) Details

1.1 Un-normalised discrete time filter specifications

Since Filter number is < 75, m = 18 and nature of filter is elliptic
q(m) = $\lfloor 0.1m \rfloor$ = 1
r(m) = m - 10.q(m) = 18 - 10 = 8
B_L(m) = 150 + 17.q(m) + 13.r(m) = 271
B_H(m) = B_L(m) + 45 = 316

The first filter is given to be a **Band-Pass** filter with passband from B_L(m) kHz to B_H(m) kHz. Therefore the specifications are : $-$

- **Passband :** 271 kHz to 316 kHz
- **Transition Band :** 20 kHz on either side of passband
- **Stopband :** 0-251 kHz and 336-600 kHz ($\because$ Sampling rate is 1.2 MHz)
- **Tolerance :** 0.15 in magnitude for both Passband and Stopband
- **Passband Nature :** Equiripple
- **Stopband Nature :** Equiripple

1.2 Normalized Digital Filter Specifications

Sampling Rate = 1.2 MHz
In the normalized frequency axis, sampling rate corresponds to 2π
Thus, any frequency(Ω) upto 600 kHz ($= \frac{SamplingRate}{2}$) can be represented on the normalized axis(ω) as : $-$

$$\omega = \frac{\Omega * 2\pi}{SamplingRate}$$

Therefore, the corresponding normalized digital filter specifications are : $-$

- **Passband :** 0.45167π to 0.5267π
- **Transition Band :** 0.033π on either side of passband
- **Stopband :** 0-0.41833π and 0.56π-π
- **Tolerance :** 0.15 in magnitude for both Passband and Stopband
- **Passband Nature :** Equiripple
- **Stopband Nature :** Equiripple

1.3 Analog filter specifications using Bilinear Transformation

The bilinear transformation is given as : −

$$\Omega = tan(\frac{\omega}{2})$$

Applying the Bilinear transform to the frequencies at the band-edges, we get : −

ω	Ω
0	0
0.41833π	0.771
0.45167π	0.859
0.5267π	1.088
0.56π	1.209
π	∞

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are : −

- **Passband :** $0.859(\Omega_{P_1})$ to $1.088(\Omega_{P_2})$
- **Transition Band :** 0.771 to 0.859 and 1.088 to 1.209
- **Stopband :** 0 to $0.771(\Omega_{S_1})$ and $1.208(\Omega_{S_2})$ to ∞
- **Tolerance :** 0.15 in magnitude for both Passband and Stopband
- **Passband Nature :** Equiripple
- **Stopband Nature :** Equiripple

1.4 Frequency Transformation and Relevant Parameters

We need to transform a Band-Pass analog filter to a Lowpass analog filter. We require two parameters in such a case. We can make use of the Bandpass transformation which is given as : −

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

The two parameters in the above equation are B and Ω_0. They can be determined using the specifications of the bandpass analog filter using the following relations : −

$$\Omega_0 = \sqrt{\Omega_{P_1}\Omega_{P_2}} = \sqrt{0.859 * 1.088} = 0.967$$
$$B = \Omega_{P_2} - \Omega_{P_1} = 1.088 - 0.859 = 0.229$$

Ω	Ω_L
0^+	$-\infty$
$0.771(\Omega_{S_1})$	$-1.478(\Omega_{L_{S_1}})$
$0.859(\Omega_{P_1})$	$-1(\Omega_{L_{P_1}})$
$0.967(\Omega_0)$	0
$1.088(\Omega_{P_2})$	$1(\Omega_{L_{P_2}})$
$1.209(\Omega_{S_2})$	$1.902(\Omega_{L_{S_2}})$
∞	∞

1.5 Frequency Transformed Lowpass Analog Filter Specifications

- **Passband Edge :** $1(\Omega_{L_P})$
- **Stopband Edge :** $\min(-(\Omega_{L_{S_1}}), (\Omega_{L_{S_2}})) = 1.478(\Omega_{L_S})$
- **Tolerance :** 0.15 in magnitude for both Passband and Stopband
- **Passband Nature :** Equiripple
- **Stopband Nature :** Equiripple

1.6 Analog Lowpass Transfer Function

We need an Analog Filter which has a equiripple passband and a equiripple stopband. Therefore we need to design using the **Elliptic** approximation. Since the tolerance(δ) in both passband and stopband is 0.15, the transfer function is of the form : $-$

$$H_{analog,LPF}(s_L) = \frac{H_0}{D_0(s_L)} \prod_{i=1}^{r} \frac{s_L^2 + a_{0i}}{s_L^2 + b_{1i}s_L + b_{0i}}$$

where,

$$r = \begin{cases} \frac{n-1}{2} & \text{for odd n} \\ \frac{n}{2} & \text{for even n} \end{cases}$$

$$D_0(s) = \begin{cases} s + \sigma_0 & \text{for odd n} \\ 1 & \text{for even n} \end{cases}$$

The selectivity factor k $= \frac{\Omega_{L_P}}{\Omega_{L_S}} = 0.677$
The maximum passband loss, $A_p = -20log_{10}(0.85) = 1.41$dB and minimum stopband loss $A_a = -20log_{10}(0.15) = 16.45$dB.

The transfer function coefficients and multiplier constant H_0 can be computed using the following formulae in the sequence:

$$k' = \sqrt{1-k^2}$$

$$q_0 = \frac{1}{2}\left(\frac{1-\sqrt{k'}}{1+\sqrt{k'}}\right)$$

$$q = q_0 + 2q_0^5 + 15q_0^9 + 150q_0^{13}$$

$$D = \frac{10^{0.1A_a}-1}{10^{0.1A_p}-1}$$

$$n \geq \frac{\log 16D}{\log(1/q)}$$

$$\Lambda = \frac{1}{2n}\ln\frac{10^{0.05A_p}+1}{10^{0.05A_p}-1}$$

$$\sigma_0 = \left|\frac{2q^{1/4}\sum_{m=0}^{\infty}(-1)^m q^{m(m+1)}\sinh[(2m+1)\Lambda]}{1+2\sum_{m=1}^{\infty}(-1)^m q^{m^2}\cosh 2m\Lambda}\right|$$

$$W = \sqrt{\left(1+k\sigma_0^2\right)\left(1+\frac{\sigma_0^2}{k}\right)}$$

$$\Omega_i = \frac{2q^{1/4}\sum_{m=0}^{\infty}(-1)^m q^{m(m+1)}\sin\frac{(2m+1)\pi\mu}{n}}{1+2\sum_{m=1}^{\infty}(-1)^m q^{m^2}\cos\frac{2m\pi\mu}{n}}$$

where

$$\mu = \begin{cases} i & \text{for odd } n \\ i-\frac{1}{2} & \text{for even } n \end{cases} \qquad i = 1, 2, \ldots, r$$

$$V_i = \sqrt{\left(1-k\Omega_i^2\right)\left(1-\frac{\Omega_i^2}{k}\right)}$$

$$a_{0i} = \frac{1}{\Omega_i^2}$$

$$b_{0i} = \frac{(\sigma_0 V_i)^2 + (\Omega_i W)^2}{\left(1+\sigma_0^2\Omega_i^2\right)^2}$$

$$b_{1i} = \frac{2\sigma_0 V_i}{1+\sigma_0^2\Omega_i^2}$$

$$H_0 = \begin{cases} \sigma_0 \prod_{i=1}^{r} \dfrac{b_{0i}}{a_{0i}} & \text{for odd } n \\ 10^{-0.05A_p} \prod_{i=1}^{r} \dfrac{b_{0i}}{a_{0i}} & \text{for even } n \end{cases}$$

Solving the above equations, we get:

$$k' = 0.736,\ q_0 = 0.0382,\ q = 0.0382,\ D = 112.515$$
$$\text{Hence, } n \geq 2.29\ \ or\ n = 3$$
$$\text{Similarly, } \Lambda = 0.4189\ \sigma_0 = 0.4283,\ W = 1.1953,\ \Omega_1 = -0.7376$$
$$a_{01} = 1.8383,\ b_{01} = 0.6614,\ b_{11} = 0.2744,\ and\ H_0 = 0.1541$$

Hence, it is a **third** order filter with Elliptic Approximation, while an order **seven** filter was needed with Butterworth Approximation to satisfy the given constraints for both!
The Analog Lowpass Transfer function is:-

$$H_{analog,LPF}(s_L) = \frac{0.1541.(s_L^2 + 1.8383)}{(s_L + 0.4283).(s_L^2 + 0.2744s_L + 0.6614)}$$

Also note that the poles of the above transfer function are -

$$p_1 = -0.4283$$
$$p_2 = -0.1372 - j0.8016$$
$$p_3 = -0.1372 + j0.8016$$

located in the Left Half Plane. Hence, the filter is stable.

1.7 Analog Bandpass Transfer Function

The transformation equation is given by : $-$

$$s_L = \frac{s^2 + \Omega_0^2}{Bs}$$

Substituting the values of the parameters B(0.229) and Ω_0(0.967), we get : $-$

$$s_L = \frac{s^2 + 0.935}{0.229s}$$

Substituting this value into $H_{analog,LPF}(s_L)$ we get $H_{analog,BPF}(s)$ as : $-$

$$\frac{0.0353s^5 + 0.0693s^3 + 0.0308s}{s^6 + 0.1608s^5 + 2.842s^4 + 0.3037s^3 + 2.6537s^2 + 0.1402s + 0.8141}$$

1.8 Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use of the Bilinear Transformation which is given as : −

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using above equation we get $H_{discrete,BPF}(z)$ from $H_{analog,BPF}(s)$(s) as : −

$$\frac{(1.71 \times 10^{-2}).(1 - 0.1336z^{-1} + 0.9042z^{-2} - 0.9042z^{-4} + 0.1336z^{-5} - z^{-6})}{1 - 0.1989z^{-1} + 2.8188z^{-2} - 0.3746z^{-3} + 2.6687z^{-4} - 0.1781z^{-5} + 0.8472z^{-6}}$$

1.9 Realization Using Direct Form II

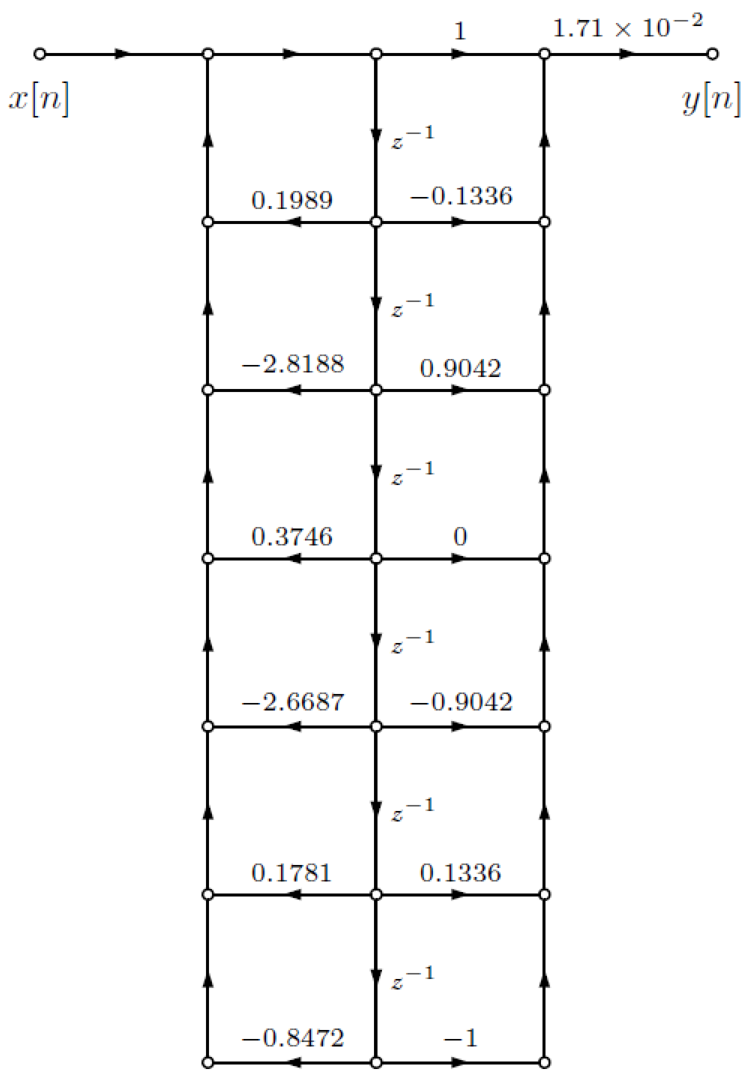

Figure 1: Direct Form II Block Diagram for $H_{discrete,BPF}(z)$

Direct Form II is obtained by treating the transfer function H(z) = B(z)/(1 - A(z)) as a cascade of 1/(1 - A(z)) followed by B(z). The intermediate signal formed is the one whose samples get stored in the buffer. Thus, it is advantageous in comparison to Direct Form I since it saves memory space.
The negative of the denominator coefficients appear as gains on the side of the input sequence x[n] while the numerator coefficients appear on the side of the output y[n] as gains in the signal-flow graph representation of the Direct Form II.

2 Filter-2 (Bandstop) Details

2.1 Un-normalised discrete time filter specifications

Since Filter number is < 75, m = 18 and nature of filter is elliptic
q(m) = $\lfloor 0.1m \rfloor$ = 1
r(m) = m - 10.q(m) = 18 - 10 = 8
B_L(m) = 157 + 17.q(m) + 13.r(m) = 278
B_H(m) = B_L(m) + 55 = 333

The second filter is given to be a **Band-stop** filter with stopband from B_L(m) kHz to B_H(m) kHz. Therefore the specifications are : $-$

- **Stopband :** 278 kHz to 333 kHz
- **Transition Band :** 20 kHz on either side of stopband
- **Passband :** 0-258 kHz and 353-600 kHz ($\because$ Sampling rate is 1.2 MHz)
- **Tolerance :** 0.15 in magnitude for both Passband and Stopband
- **Passband Nature :** Equiripple
- **Stopband Nature :** Equiripple

2.2 Normalized Digital Filter Specifications

Sampling Rate = 1.2 MHz
In the normalized frequency axis, sampling rate corresponds to 2π
Thus, any frequency(Ω) upto 600 kHz ($= \frac{SamplingRate}{2}$) can be represented on the normalized axis(ω) as : $-$

$$\omega = \frac{\Omega * 2\pi}{SamplingRate}$$

Therefore, the corresponding normalized digital filter specifications are : $-$

- **Stopband :** 0.4633π to 0.555π
- **Transition Band :** 0.033π on either side of passband
- **Passband :** 0-0.43π and 0.5883π-π
- **Tolerance :** 0.15 in magnitude for both Passband and Stopband
- **Passband Nature :** Equiripple
- **Stopband Nature :** Equiripple

2.3 Analog filter specifications using Bilinear Transformation

The bilinear transformation is given as : −

$$\Omega = tan(\frac{\omega}{2})$$

Applying the Bilinear transform to the frequencies at the band-edges, we get : −

ω	Ω
0	0
0.43π	0.801
0.4633π	0.891
0.555π	1.19
0.5883π	1.324
π	∞

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are : −

- **Stopband :** $0.891(\Omega_{S_1})$ to $1.19(\Omega_{S_2})$
- **Transition Band :** 0.801 to 0.891 and 1.19 to 1.324
- **Passband :** 0 to $0.801(\Omega_{P_1})$ and $1.324(\Omega_{P_2})$ to ∞
- **Tolerance :** 0.15 in magnitude for both Passband and Stopband
- **Passband Nature :** Equiripple
- **Stopband Nature :** Equiripple

2.4 Frequency Transformation and Relevant Parameters

We need to transform a Band-Stop analog filter to a Lowpass analog filter. We require two parameters in such a case. We can make use of the Bandpass transformation which is given as : −

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

The two parameters in the above equation are B and Ω_0. They can be determined using the specifications of the bandpass analog filter using the following relations : −

$$\Omega_0 = \sqrt{\Omega_{P_1}\Omega_{P_2}} = \sqrt{0.801 * 1.324} = 1.03$$
$$B = \Omega_{P_2} - \Omega_{P_1} = 1.324 - 0.801 = 0.526$$

Ω	Ω_L
0^+	0^+
$0.801(\Omega_{P_1})$	$1(\Omega_{L_{P_1}})$
$0.891(\Omega_{S_1})$	$1.755(\Omega_{L_{S_1}})$
$1.03(\Omega_0^-)$	$+\infty$
$1.03(\Omega_0^+)$	$-\infty$
$1.19(\Omega_{S_2})$	$-1.762(\Omega_{L_{S_2}})$
$1.324(\Omega_{P_2})$	$-1(\Omega_{L_{P_2}})$
∞	0^-

2.5 Frequency Transformed Lowpass Analog Filter Specifications

- **Passband Edge :** $1(\Omega_{L_P})$
- **Stopband Edge :** $\min((\Omega_{L_{S_1}}), -(\Omega_{L_{S_2}})) = 1.755(\Omega_{L_S})$
- **Tolerance :** 0.15 in magnitude for both Passband and Stopband
- **Passband Nature :** Equiripple
- **Stopband Nature :** Equiripple

2.6 Analog Lowpass Transfer Function

We need an Analog Filter which has a equiripple passband and a equiripple stopband. Therefore we need to design using the **Elliptic** approximation. Since the tolerance(δ) in both passband and stopband is 0.15, the transfer function is of the form : −

$$H_{analog,LPF}(s_L) = \frac{H_0}{D_0(s_L)} \prod_{i=1}^{r} \frac{s_L^2 + a_{0i}}{s_L^2 + b_{1i}s_L + b_{0i}}$$

where,

$$r = \begin{cases} \frac{n-1}{2} & \text{for odd n} \\ \frac{n}{2} & \text{for even n} \end{cases}$$

$$D_0(s) = \begin{cases} s + \sigma_0 & \text{for odd n} \\ 1 & \text{for even n} \end{cases}$$

The selectivity factor k = $\frac{\Omega_{L_P}}{\Omega_{L_S}} = 0.57$
The maximum passband loss, $A_p = -20log_{10}(0.85) = 1.41$dB and minimum stopband loss $A_a = -20log_{10}(0.15) = 16.45$dB.
Solving the same set of equations mentioned during the analysis of the previous filter, we get:

$$k' = 0.822,\ q_0 = 0.0245,\ q = 0.0245,\ D = 112.515$$
$$\text{Hence, } n \geq 2.02\ or\ n = 3$$
$$\text{Similarly, } \Lambda = 0.4189,\ \sigma_0 = 0.3667,\ W = 1.1535,\ \Omega_1 = -0.6689$$
$$a_{01} = 2.2352,\ b_{01} = 0.5488,\ b_{11} = 0.2769\ and\ H_0 = 0.09$$

Hence, it is a **third** order filter with Elliptic Approximation, **same** order as the filter was needed with Chebyshev Approximation to satisfy the given constraints for both!
The Analog Lowpass Transfer function is:-

$$H_{analog,LPF}(s_L) = \frac{0.09.(s_L^2 + 2.2352)}{(s_L + 0.3667).(s_L^2 + 0.2769 s_L + 0.5488)}$$

Also note that the poles of the above transfer function are -

$$p_1 = -0.3667$$
$$p_2 = -0.1384 - j0.7277$$
$$p_3 = -0.1384 + j0.7277$$

located in the Left Half Plane. Hence, the filter is stable.

2.7 Analog Bandstop Transfer Function

The transformation equation is given by : $-$

$$s_L = \frac{Bs}{s^2 + \Omega_0^2}$$

Substituting the values of the parameters B(0.526) and Ω_0(1.03), we get : $-$

$$s_L = \frac{0.526s}{s^2 + 1.061}$$

Substituting this value into $H_{analog,LPF}(s_L)$ we get $H_{analog,BSF}(s)$ as : $-$

$$\frac{0.9998s^6 + 3.3058s^4 + 3.5082s^2 + 1.1950}{s^6 + 1.6920s^5 + 4.0603s^4 + 4.3041s^3 + 4.3089s^2 + 1.9055s + 1.1952}$$

2.8 Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use of the Bilinear Transformation which is given as : $-$

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using above equation we get $H_{discrete,BSF}(z)$ from $H_{analog,BSF}(s)$(s) as : $-$

$$\frac{(0.4879).(1 + 0.1749z^{-1} + 2.8981z^{-2} + 0.3434z^{-3} + 2.8981z^{-4} + 0.1749z^{-5} + z^{-6})}{1 + 0.1366z^{-1} + 1.6048z^{-2} + 0.1575z^{-3} + 1.0551z^{-4} + 0.0441z^{-5} + 0.1442z^{-6}}$$

2.9 Realization Using Direct Form II

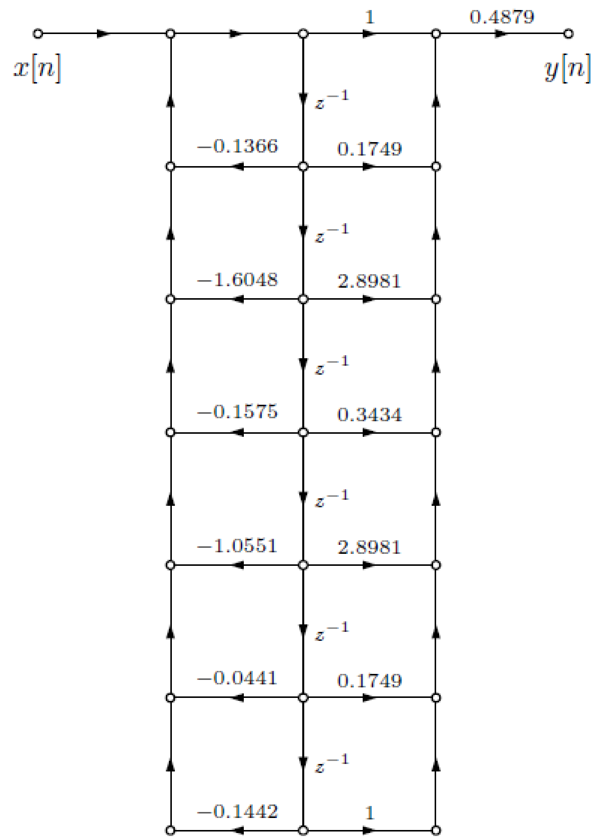

Figure 2: Direct Form II Block Diagram for $H_{discrete,BSF}(z)$

Direct Form II is obtained by treating the transfer function H(z) = B(z)/(1 - A(z)) as a cascade of 1/(1 - A(z)) followed by B(z). The intermediate signal formed is the one whose samples get stored in the buffer. Thus, it is advantageous in comparison to Direct Form I since it saves memory space.

The negative of the denominator coefficients appear as gains on the side of the input sequence x[n] while the numerator coefficients appear on the side of the output y[n] as gains in the signal-flow graph representation of the Direct Form II.

3 MATLAB Plots and Comparisons

3.1 Filter 1 - Bandpass

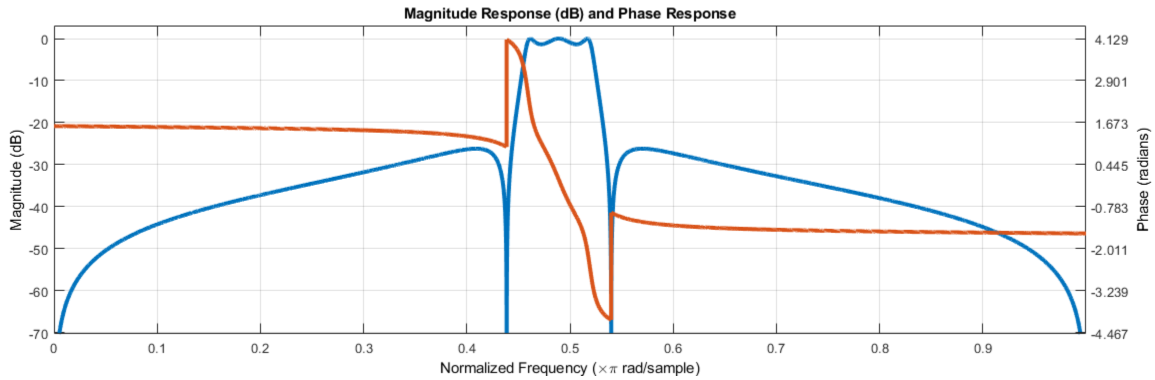

Figure 3: Frequency Response

From the above plot, it can be seen that the passband and stopband attenuation have been satisfied. It can be seen that the **phase response** is **not linear**

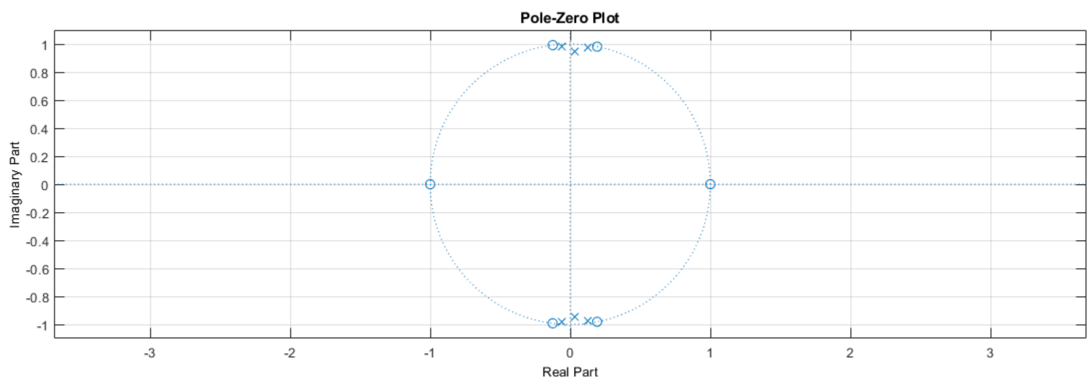

Figure 4: Pole-Zero map (all poles within unit circle, hence stable)

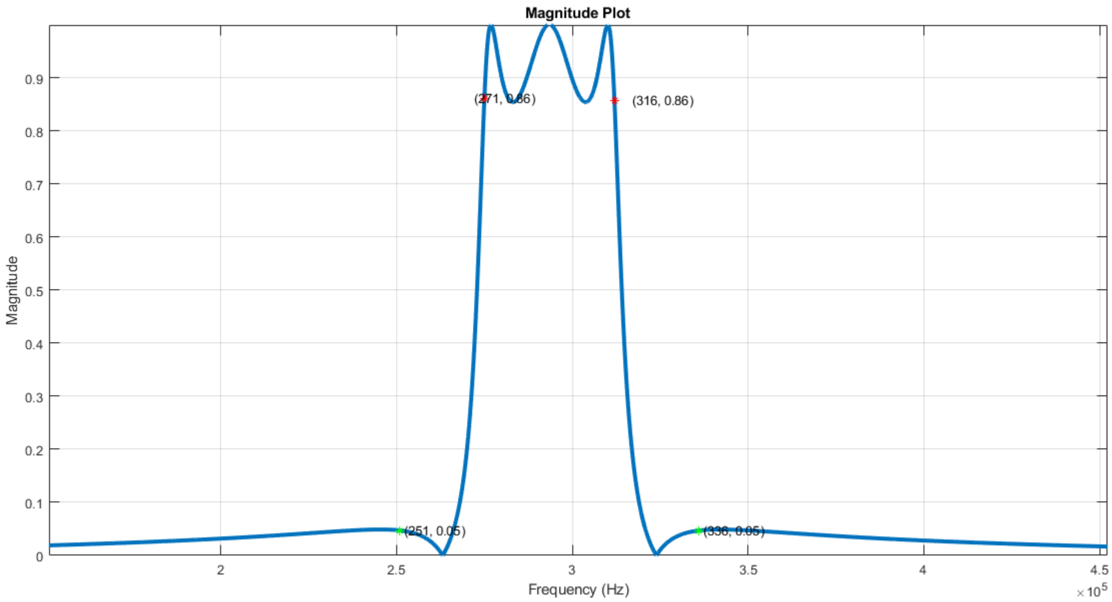

Figure 5: Magnitude Response

In the above plot, the band edge frequencies (in kHz) have been marked. From the magnitude at these frequencies it can be seen that the specications required in the passband and the stopband have been met.

3.2 Filter 2 - Bandstop

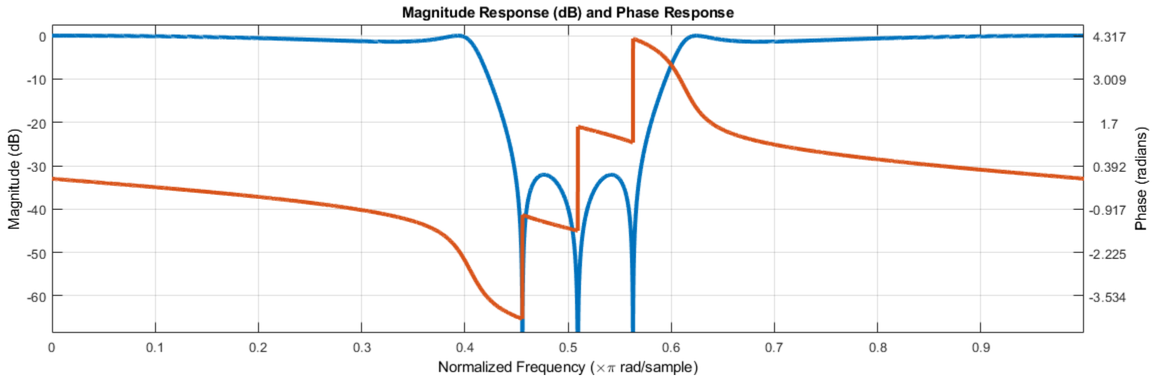

Figure 6: Frequency Response

From the above plot, it can be seen that the passband and stopband attenuation have been satisfied. It can be seen that the **phase response** is **not linear**

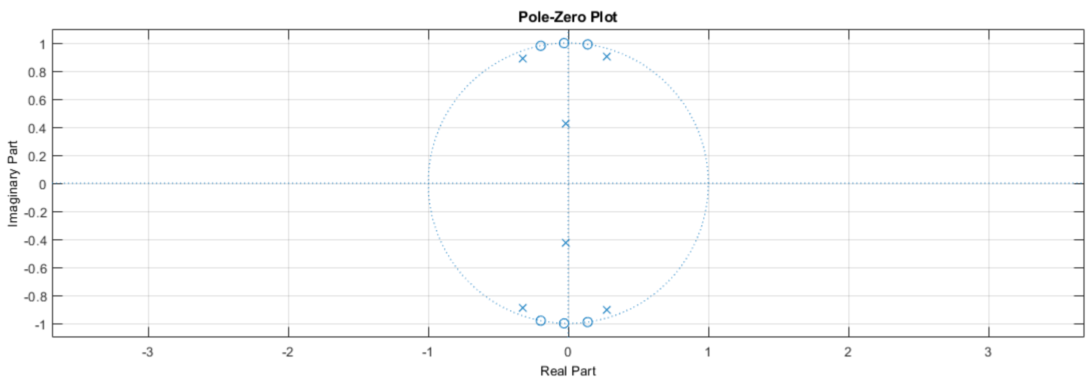

Figure 7: Pole-Zero map (all poles within unit circle, hence stable)

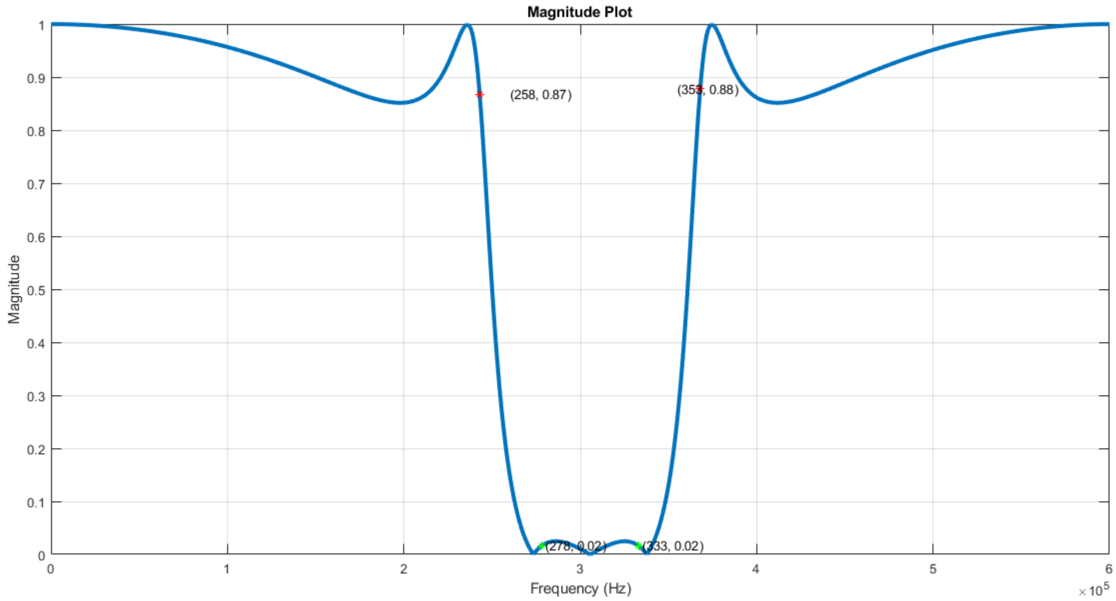

Figure 8: Magnitude Response

In the above plot, the band edge frequencies (in kHz) have been marked. From the magnitude at these frequencies it can be seen that the specications required in the passband and the stopband have been met.

4 Peer Review Acknowledgement

To whosoever it may concern,
I have successfully reviewed this filter design report done by **Anugole Sai Gaurav** as a part of the additional course assignment for EE338, Digital Signal Processing . This includes the design process, context, calculations, simulation programs and results.

Thank you

Regards,
Jayesh Choudhary
170070038