**PT-Symmetric local resonance phononic crystal**

**Abstract**

In this study, we study a type of PT-Symmetric local resonance phononic crystal through numberical computation and simulation respectively, and find two novel phenomenons. The first one is extending the effective mass from real number domain to complex domain, the second one is asymmetric propagation in the subwavelength frequency domain under PT-Symmetric condition. Our findings greatly enrich the physical connotation of local resonance phononic crystals.

**I. INTRODUCTION**

Phononic crystal is an artificial composite structure composed of periodic arrangements of elastic media with different material parameters. Analogue to photonic crystal, its main property is elastic wave band gap means that if the frequency of elastic or acoustic wave in band gap can not propogate. There are two main mechanisms for generating band gaps: one is Bragg scattering, the band gap of this phononic crystal structure is produced by interaction between elastic waves with scattering body. And this would make vibration mode at certain frequencies not exist, so these frequencies consis of the band gap. Another one is local resonance, ie the wavelength of the elastic wave in the band gap frequency is much larger than the lattice constant of the structure. The band gap is produced by mutual coupling when the frequency of incidence elastic wave closes to the scatter body’s resonance frequency. The concept of local resonance phononic crystal is proposed by Liu et al in 2000[2], after that, local resonance phononic crystals cause widespread concern[5-7], such structures both have negative modules at certain frequencies. For electromagnetic waves, negative values of the electric permittivity and magnetic permeability yield many new phenomena such as negative phase velocity, evanescent wave, and superlensing [8-13]. So for acoustic waves, the appearance of negative modulus will also have many places worth studying.

Parity-time (PT)-symmetry was initially proposed by Bender et al in quantum mechanics[1], which is neither parity (P) symmetric nor time (T) reversal symmetric individually, but are invariant under the combined operations of P inversion and T reversal. According to that, it typically requires a symmetric relation [14–18] and so, the most important characteristic of non-Hermitian structures is the complex conjugate modulus composed of paired positive and negative imaginary

parts. PT-transition results in the broken phase, where exception points and complex eigen-values arise. As a result, many intriguing properties and the potential of non-Hermitian systems have also been explored as a topical issue in various fields of physics within the last decade, such as unidirectional behaviors, nontrivial anisotropic-transmission resonances[19-21], defect states and topological edgemodes[3].

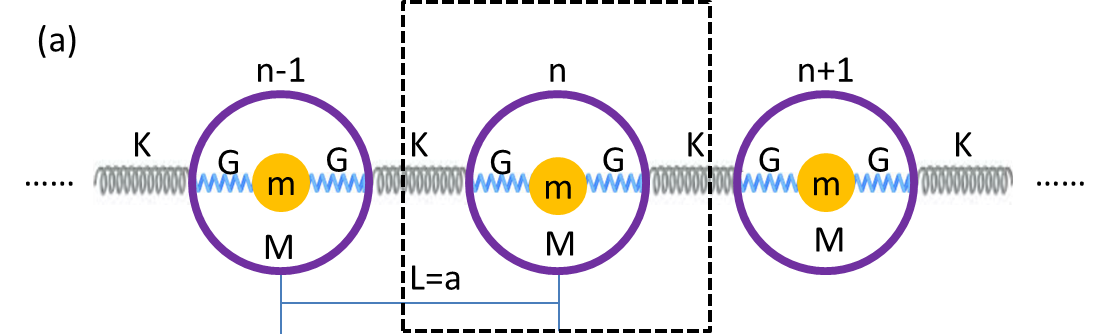
In this study, we try to apply PT-Symmetric to local resonance phononic crystals and obtain two novel conclusions. One is the complex effective modules, this greatly expands the physical meaning of the local resonance phononic crystal; another one is the unidirectional transmission in the subwavelength frequency domain, the conventional extraordinary wave behaviors base on PT-Symmetric are arise on wavelength frequency domain mostly, and that greatly limits application senario.

**II. SPRING-MASS MODEL ANALYZE**

The basic structure shows in FIG. 1. (a) is the simple of a local resonance phononic crystal proposed by Liu et al. in 2000[2]. It is consisted of hard core material and coating with elastically soft material.

According to the appendix A in Ref [3], we could deduce the dispersion relation of this structure as below

(1)   
where denotes the effective mass of unit cell, is the local resoance frequency, a is the lattice constant, q is the Bloch wave number. The dispersion relation curve of Eq. (1) in the Irreducible Brillouin Zone (IBZ) has shown in FIG.1.(b) with . Effective mass is another way to identify local resonance phenomen, because the value of effective mass becomes infinity sharply when the frequency enter local resonance band gap as shown in FIG.1.(C).



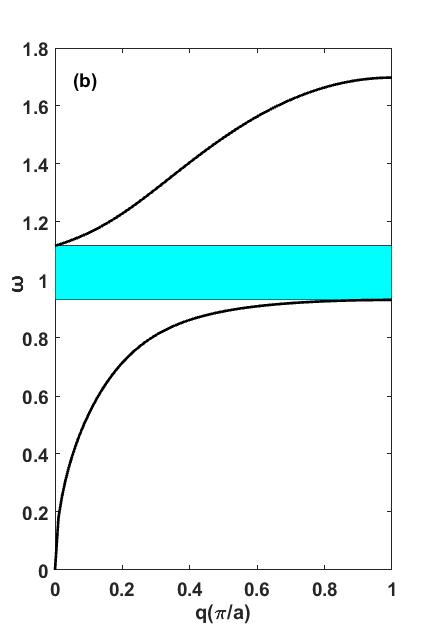
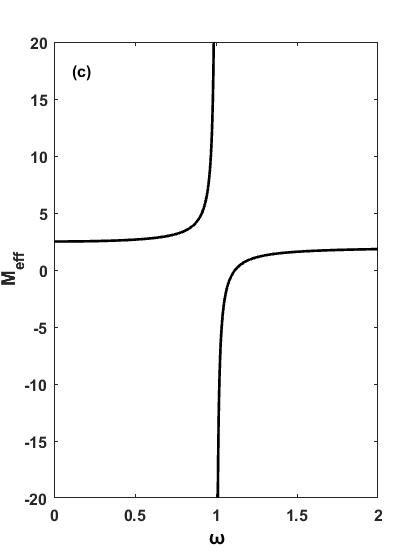
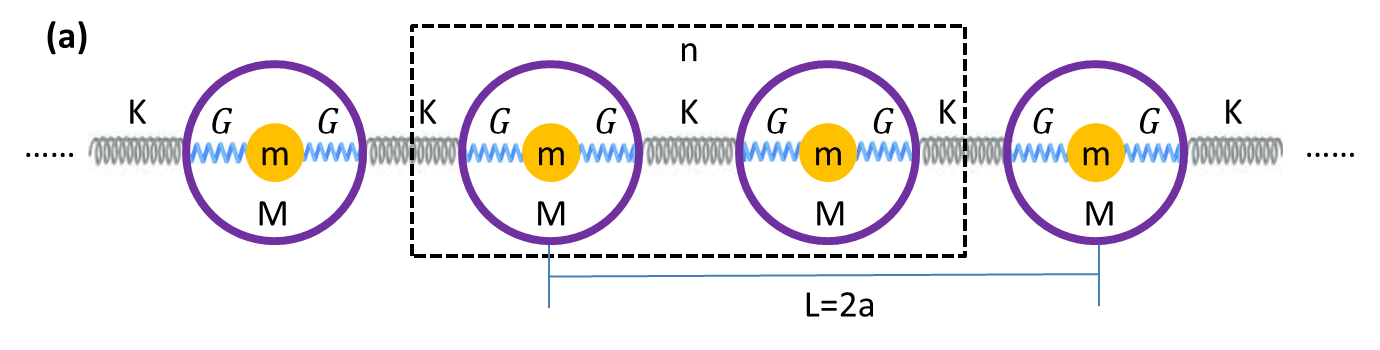
 

FIG. 1. (a) mass-in-mass monatomic chain model, K is the spring constant between each unit cell, G is the spring constant between core and coating, m is the mass of core, M is the mass of coating and lattice constant ; (b) dispersion relation curve with with , , cyan area corresponds to local resonance band gap; (c) effective mass curve of unite cell in (a), its value would becomes positive or negative infinity when the frequency approximates , and this corresponds to the local resonance band gap.

Then concern about supercell that consists of two unit cells but other parameters remain invariant, and the band structure is deduced by folding FIG.1.(b) simply as shown in FIG.2.(b) because the IBZ of FIG.1.(a) is the half of FIG.1.(a).



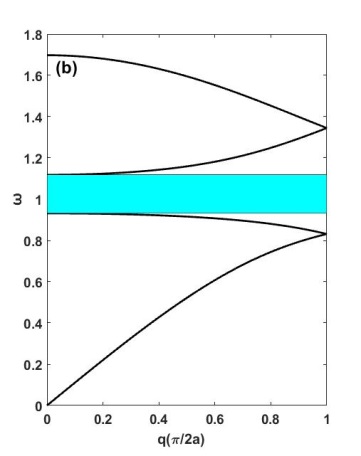
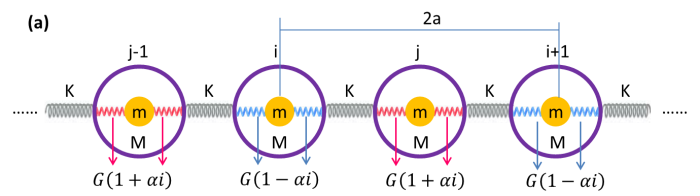


FIG. 2. (a) two unit cells in FIG.1.(a) consist of one supercell structure; (b) dispersion relation of supercell structure, it could be deduced by folding FIG.1.(b) and the cycan area local resonance band gap is the same.

In this fundamental , we exert loss and gain factor to the spring constant G alternatively to form a PT-Symmetric structure. And define the loss unit cell for G contains positive imaginary part, i.e. , gain unit cell for G contains negative imaginary part, i.e. . FIG.3.(a) and FIG.3.(b) are corresponding PT-Symmetric structure and dispersion relation diagram respectively, where FIG.3.(b) is the case of .

Compare with FIG.2.(b), degeneracy appears in the dispersion relation curve clearly when we exert the loss and gain factor, and the local resonance band gap (cycan area) becomes more narrow. The dispersion relation curve evolute from non-degeneracy to degeneracy means phase transition from PT-exact phase to PT -broken phase and the point between these two phases is called exception point [22]. The magenta points A and B signed in the figure are two exception points. In addition, the effective mass has two different value --- loss effective mass for the loss unit and gain effective mass for the gain unit, and they are a pair of conjugate complex numbers. FIG.3.(c) and FIG.3.(d) shows the real part and imaginary part of them respectively. We could see that, in the frequency domain of local resonance band gap, the real part of effective mass oscillates sharply rather than goes to positive or negative infinite after exert PT-Symmetric, while the imaginary part would have peak. So this is a well way to identify if a structure exists local resonance band gap or not.



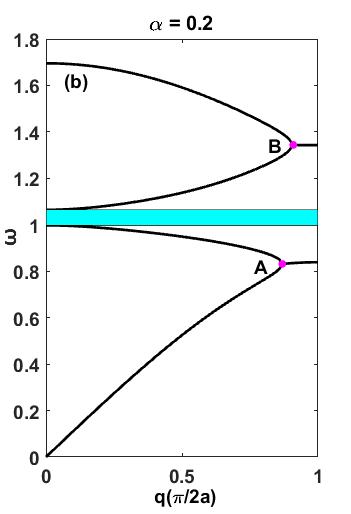
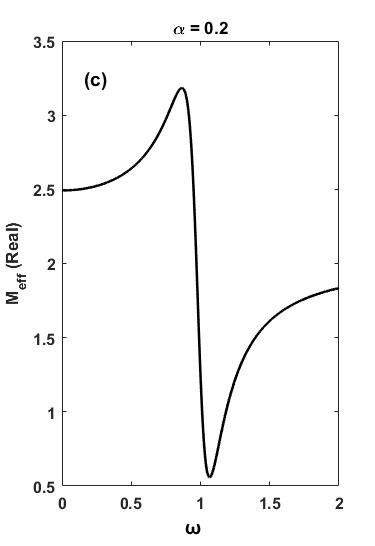
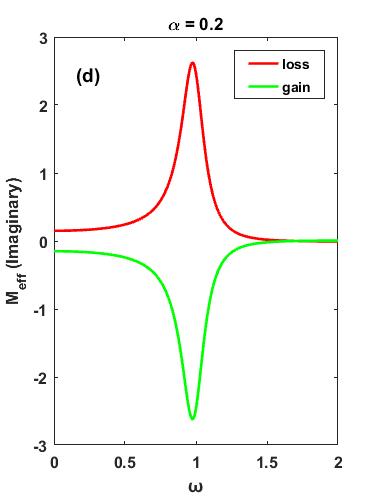
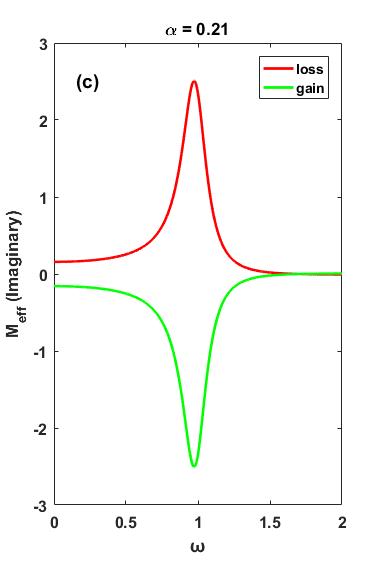
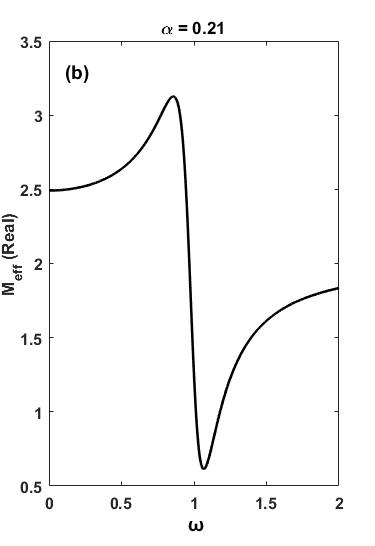
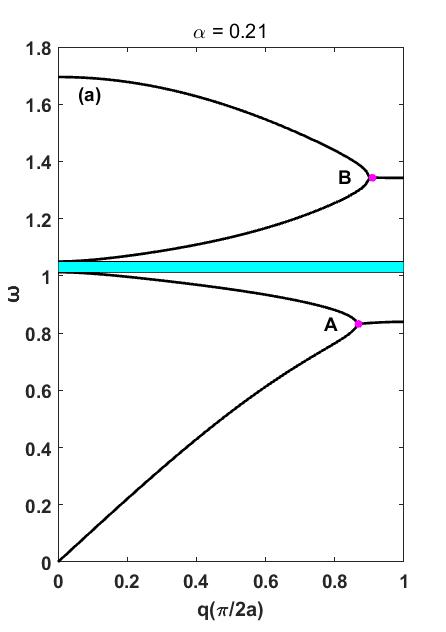
  

FIG.3. (a) The structure introduces loss and gain to spring constant G alternatively that satisfies PT-Symmetric; (b) Dispersion relation curve of the structure in (a) when , points A and B are two exception points which are the demarcation of degeneracy area and non-degeneracy area; (c) Real part of effective mass for both loss and gain unit cell, sharply oscillates in the local resonance band gap frequency domain; (d) Imaginary part of effective mass for both loss and gain unit cell, peaks appear in the local resonance band gap frequency domain.

Next, we increase value of furthermore and to observe the evoluation of dispersion relation curve. The relevant results all has been plotted in FIG.4, local resonance band gap would become narrower and disappears completely finally then produces a new exception point. The degeneracy woluld also enhance as becomes bigger and bigger.

And the size of extreme value of effective mass (both real part and imaginary part) would becomes smaller and smaller at the same time. We could see this by comparing the effective mass in FIG.4 and FIG.3.



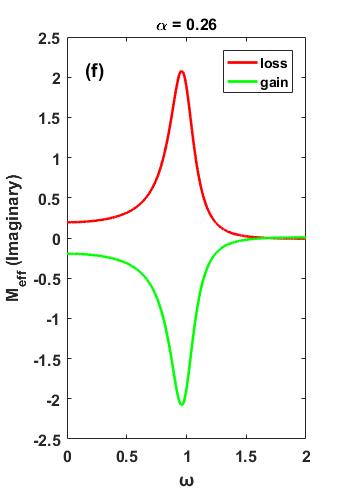
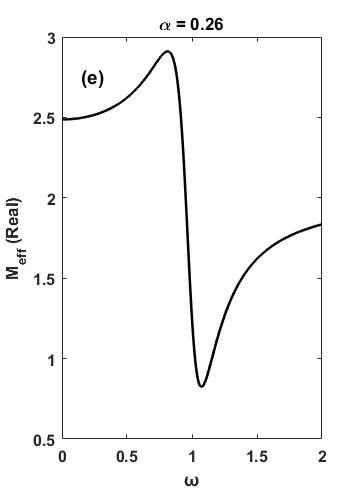
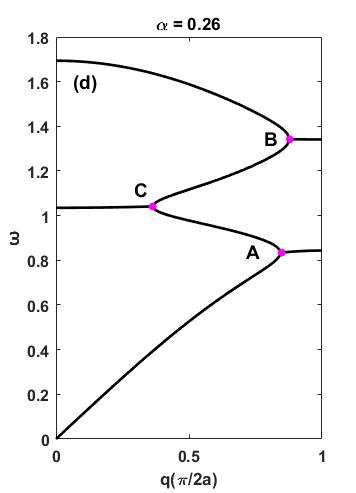


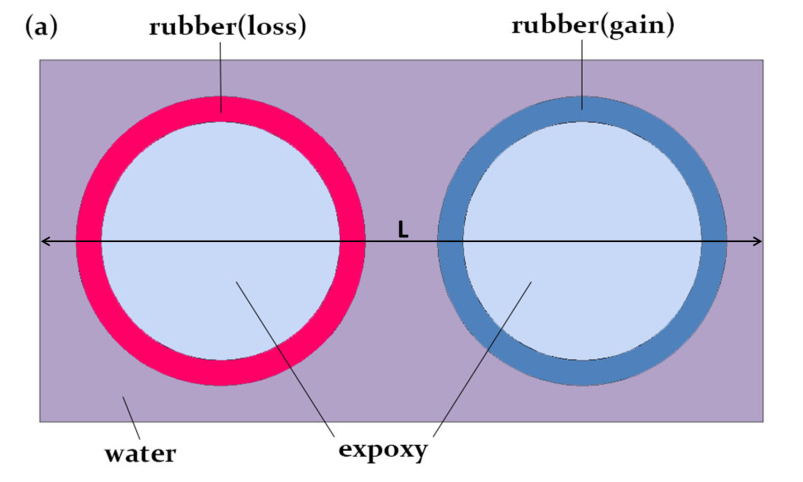
FIG.4. (a) The dispersion relation curve when , we can see that the local resonance band gap hardly ever closes; (b) Real part of effective mass for both loss and gain unit cell when ; (c) Imaginary part of effective mass for both loss and gain unit cell when ; (d) ~ (f) show the dispersion relation curve, real part of effective mass for both loss and gain unit cell and imaginary part of effective mass for both loss and gain unit cell respectively when .

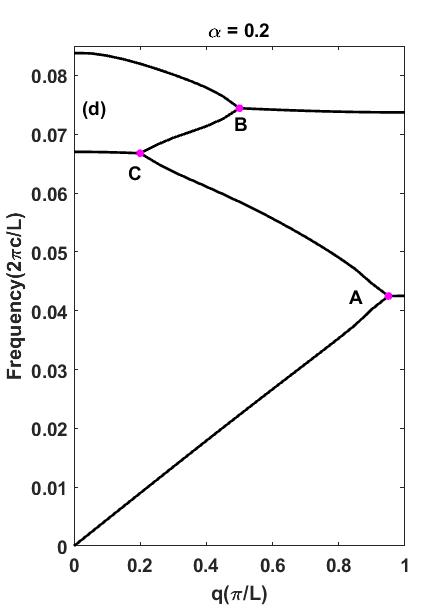
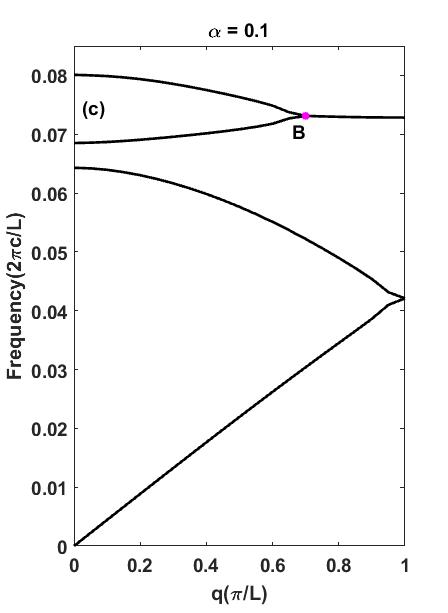
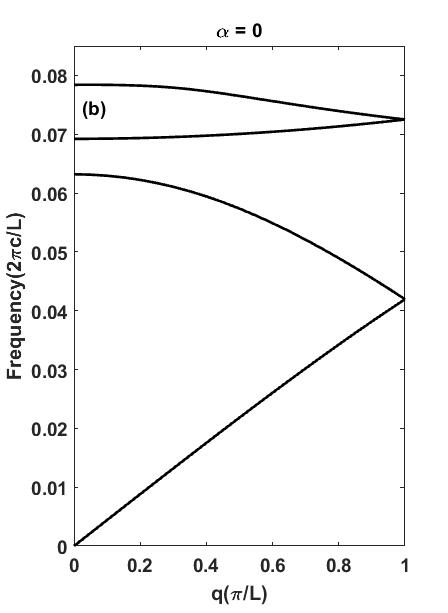
In the next section, we will construct a pratical structure through commercial FEM software comsol multiphysics to investigate relevant physics property.

**III. SIMULATION ANALYZE**

Analog to the unit cell of model in last section, we use a pratice structure consists of an expoxy core (mass density , first lame constant , and the second lame constant ) coated by soft rubber mass density ( , first lame constant , and the second lame constant ) and they are immersed in water background (mass density , longitudinal wave velocity c = 1490 m/s). We exert imaginary part on the lame constant and to mimic exert imaginary part to spring constant G in FIG.3.(a). Assume the width of this structure is , then the height will be 0.5L, diameter of each cylinder is where the thickness of soft rubber is and the diameter of expoxy core is .

We compute the dispersion relation curve where the condition of periodic boundary exerts on left and right boundary and the condition of continuity boundary exerts on up and down boundary, relevant results have been plotted in FIG.5.(b) ~ (e) correspond to the changes from 0~0.4 with steps of 0.1 which have similar trendency with the numberical computation results in previous figure . Though lack the effective mass curves of this structure , we still could claim that use this structure as the equivalence of previous model is rational through the dispersion relation curve.





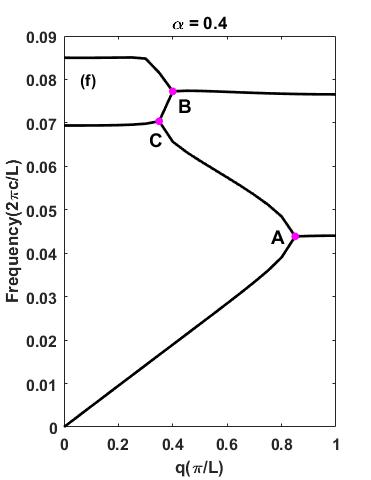
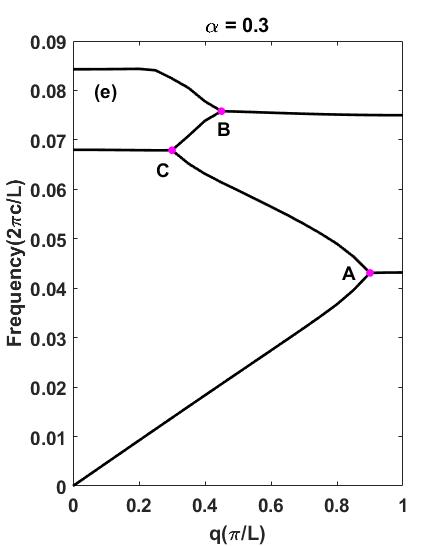


FIG.5. (a) The unit cell of structure we concern in text, expoxy core in the left is coated with loss soft rubber whose two lame constants have positive imaginary part and counterpart in the right has negative imaginary part; (b) Dispersion relation curve without any loss and gain; (c) Dispersion relation curve with , one exception point in the diagram means the degeneracy exists in this case; (d) Dispersion relation with , in this situation ,we could see three exception points; (e) and (f) Show dispersion relation when and respectively, the total tredence is degeneracy would enhance as increases, and this matches the spring-mass model in section II.

In order to observe the trendence in this pratical structure clearly, we also plot the ── degenercy curves in FIG.6. There are three curves in the chart because we have three exception points (A, B and C) in the dispersion relation curve.

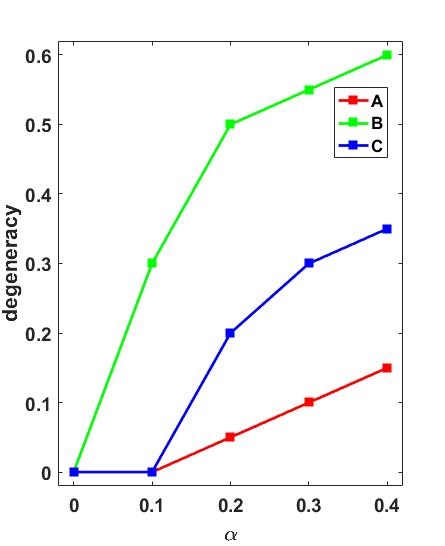
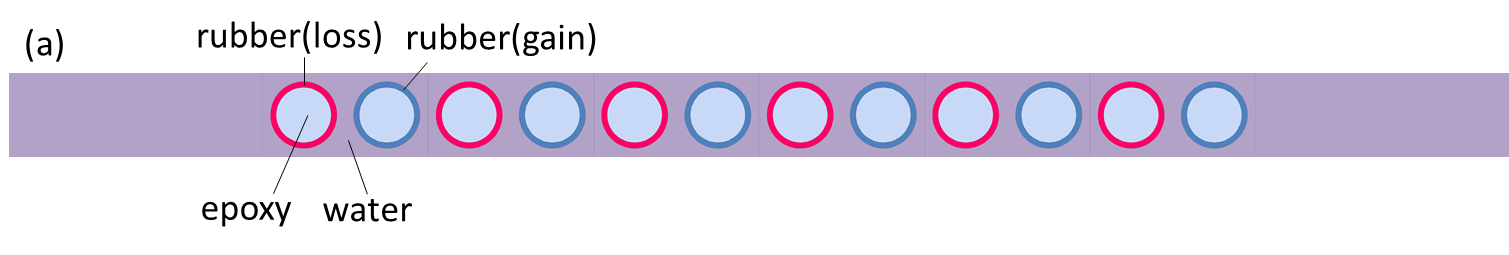
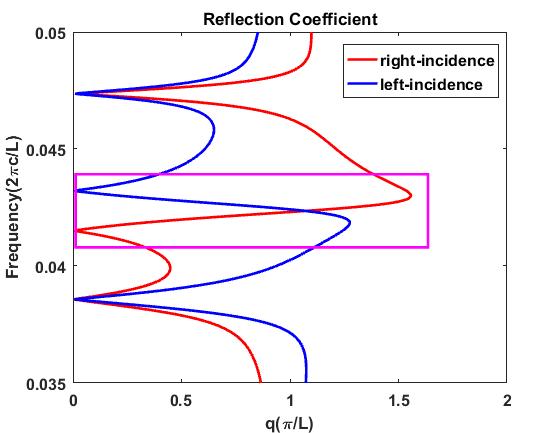


FIG.6. ── degenercy curves for the structure in FIG.5. (a), in this chart we can see that degeneracy enhances as the imaginary part increases, three curves correspond exception point A, B and C respectively.

Then we concern about an array of six unite cells with and compute its transmission spectrum. Near the normalize frequency 0.04, two high reflection coefficients appear corresponds to left- and right-incidence situation respectively. These two situtions both hold for extremed low reflection coefficient from other side, so asymmetric propagation phnomenons exist at these two frequency point [21].





**(b)**

FIG.6. (a) The structure of super-cell which is consisted of six unite cells; (b) reflection coefficient curve for the situation of left-incidence and right-incidence (blue and red), and the dispersion relation curve (black), magenta square marks out the asymmetric propagation part of two reflection coefficient curves.

In order to observe the phenomenon clearer, the spatial distribution of scatter acoustic pressure in two corresponded frequency points (about 3092Hz and 3220Hz) also have been plotted in FIG.7. As we can see, at frequency 3092 Hz, spatial distribution of scatter acoustic pressure is almost 0 in the incidence area when the wave propagates from right to left, and but it is not 0 when wave propagates from left to right. While at frequency 3220 Hz, the situation is oppsite.

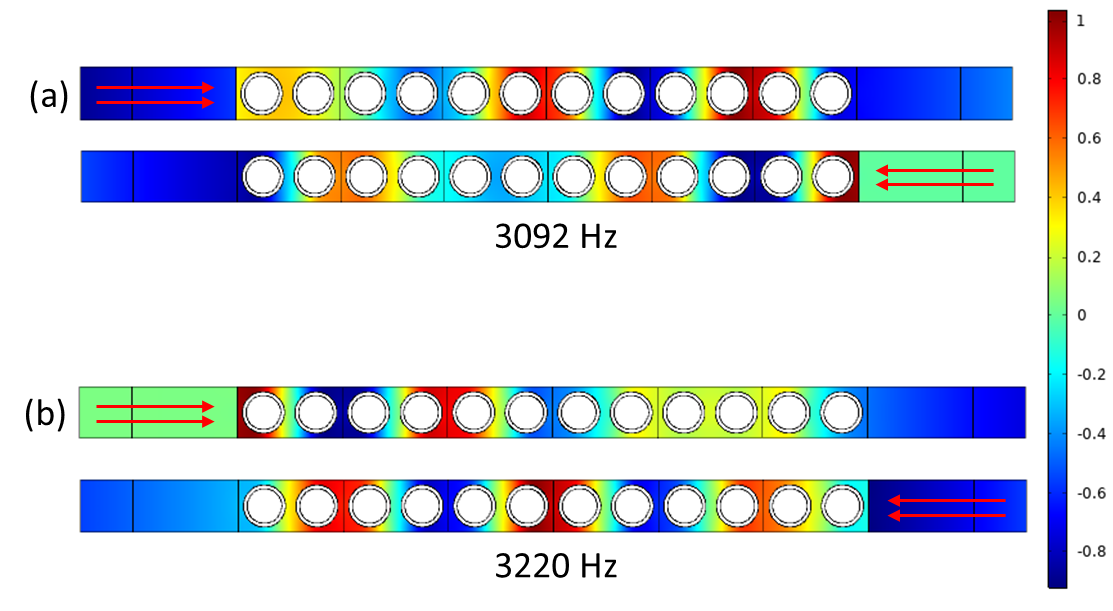


FIG.7. (a) The scatter acoustic pressure field contribution in the frequency 3092 Hz, above is left-incidence, below is right-incidence; (b) same situation except the frequency of wave is 3220 Hz.

In addition, these two frequencies are near the exception point A in FIG.5.(d) belong to subwavelength domain because they are even lower than local resonance frequency domain is shown in FIG.5.(a) and FIG.5.(b). So we have achieved asymmetric propagation in low frequency domain by using PT-symmetric on the local resonance phononic crystal.

**IV. CONCLUSION**

In summary, we first studied the spring-mass unit cell model with balance gain and loss to form PT-Symmetric, and extend effective mass to complex number domain, then use a pratical structure to research its transmisson spectrum, achieve asymmetric propagation through PT-Symmetric method in the subwavelength frequency area successively. This founding means potential for the fabrication of novel functional devices in low frequency domain, such as acoustic diode and perfect sensor.

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