

• Pseudocode

- a) Expressions: Standard mathematical symbol to express numeric & ~~boolean~~ boolean expression.
- we use ' $\leftarrow$ ' sign as the '=' operator in c/c++/java.
  - we use ' $=$ ' sign as the ' $\equiv$ ' operation in c/c++/java
- b) Method Declarations: name (parameter<sub>1</sub>, parameter<sub>2</sub>, ...) declares a new method "name" ~~with~~ with parameters.  
 → we can call & define the method in the ~~pseudocode~~ pseudocode as we do in c/c++/java
- c) Decision structure:
- ```

if (<Condition>) then
  { action }
else
  { action }.
  
```
- just like c/c++/java
- d) while loops:
- ```

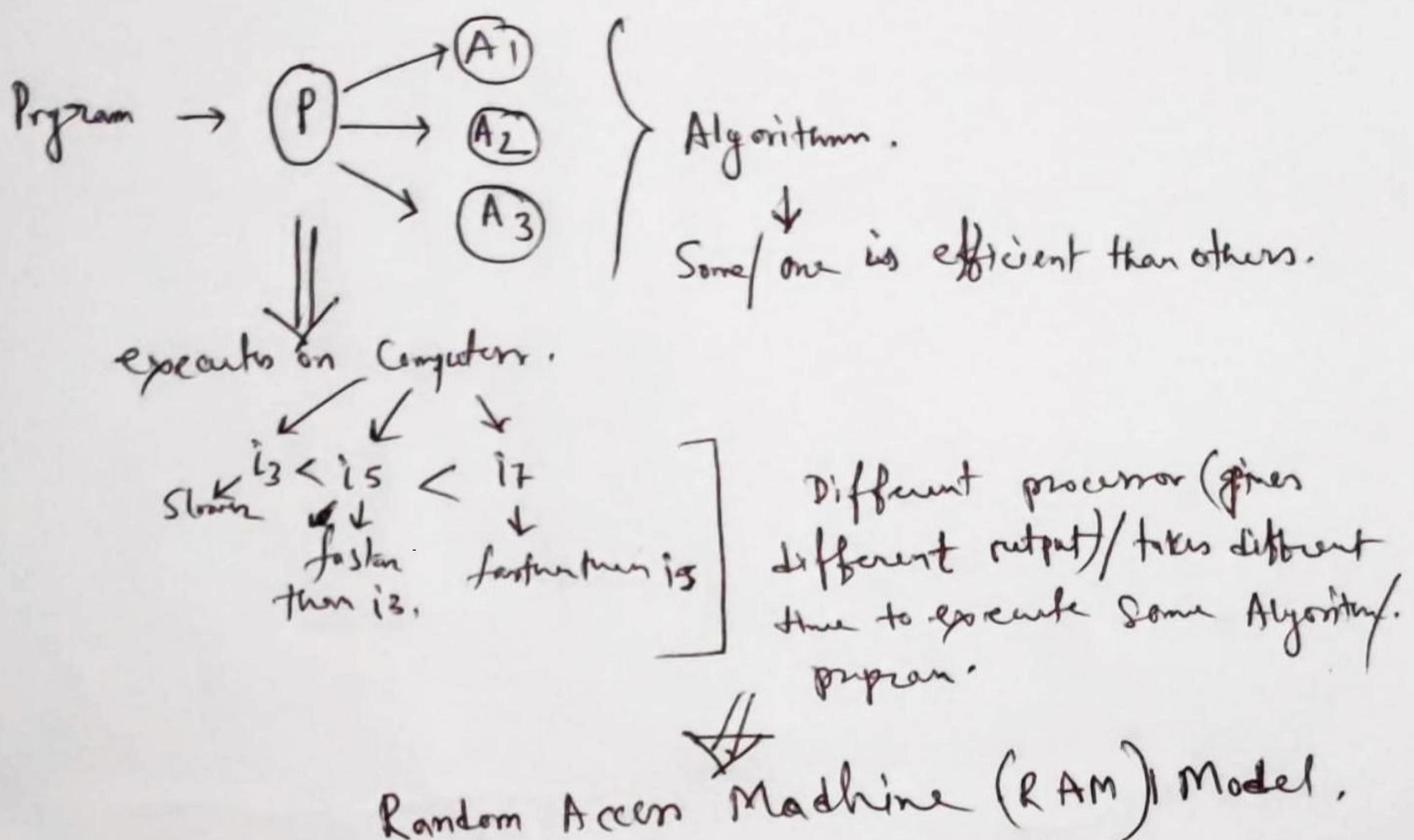
while (<condition>) then
  { action }.
  
```
- e) Repeat loops:
- ```

repeat
  { action }
until (<condition>)
  
```
- f) for loops:
- ```

for ( variable initialization, condition, increment ),
  { action }.
  
```
- g) Array Indexing: A[i] represents the 'i'th cell in the array 'A'.
- h) Method calls: object. name ("xyz", "pr"). [object is optional].
- i) Method Returns: return value: just like c/c++/java.

## RAM MODEL

( Random Access Machine Model ).



~~RAM~~ Random Access Machine (RAM) Model.

Hypothetical Machine

- $\rightarrow$  not concerned about the processor.
- $\Rightarrow$  Only considered how many number of steps an algorithm will take.

$$\left. \begin{array}{l} A_1 \rightarrow 100 \text{ Steps} \\ A_2 \rightarrow 20 \quad " \\ A_3 \rightarrow 5 \quad " \end{array} \right\} A_3 \text{ is faster than } A_1 \text{ & } A_2.$$

Proutine Operations: It corresponds to a low level instruction with an execution time that depends on the H/W & S/W environment but is nevertheless constant. The following are considered as primitive, -.

- $\rightarrow$  assigning a value to a variable
- $\rightarrow$  calling a method
- $\rightarrow$  performing an arithmetic operation
- $\rightarrow$  comparing two numbers.
- $\rightarrow$  Indexing into an array.
- $\rightarrow$  Following an object reference
- $\rightarrow$  Returning from a method.

### Algorithm array Max (A, n):

Amit (3)

Input: An array A storing  $n \geq 1$  integers.

Output: The maximum element in A.

- ① current Max  $\leftarrow A[0]$  ----- 2
- ② for  $i \leftarrow 1$  to  $n-1$  do -----  $2+n$
- ③ if current Max  $< A[i]$  then -----  $2(n-1)$
- ④ current Max  $\leftarrow A[i]$  -----  $2(n-1)$
- ⑤ return current Max. ----- 1

Line ①

corresponds to two primitive operations.

$\rightarrow$  indexing into an array.  $\Rightarrow A[0]$

$\rightarrow$  assigning to a variable.  $\Rightarrow \text{current Max} \leftarrow A[0]$

Thus it contributes 2 units of time.

Line ②

at the beginning the counter initialized to '1'  
 $\rightarrow$  giving value to a variable.

Thus it contribute an unit amount of time

③ before entering the body of the loop a condition  $i < n$  is verified.  
this action corresponds to executing one primitive instruction  
 $\rightarrow$  (comparing two numbers).  
it will iterate for  $n+1$  time.

(3, 4, 4.1)

The body of the for loop is executed ' $n-1$ ' time.  
at each iteration

3  $\Rightarrow A[i]$  is compared to current Max.  
 $\rightarrow$  Indexing into the array  
 $\rightarrow$  comparing with current Max.

Thus it involves two operations, & takes  $2(n-1)$  time.

Similarly 4  $\Rightarrow A[i]$  is possibly assigned to current Max.

$\rightarrow$  indexing into the array,  
 ~~$\rightarrow$  assigning  $A[i]$  to current Max.~~

$\rightarrow$  Assigning  $A[i]$  to current Max.

thus it takes two operations &  $2(n-1)$  time.

Ans. ④

<4.1> increment :  $i = i + 1$

- addition operation
- assignment operation.

But it also involves two primitive operations -  
 & takes  $2(n-1)$  times.

- ⑤ . Returning the value of variable currentMayo  
 corresponds to one primitive operation & executes  
 only one .  
 → One unit of time .

Thus summing up all the steps .

$$t(n) = 2 + 2+n + 2(n-1) + 2(n-1) + 2(n-1) + 1$$

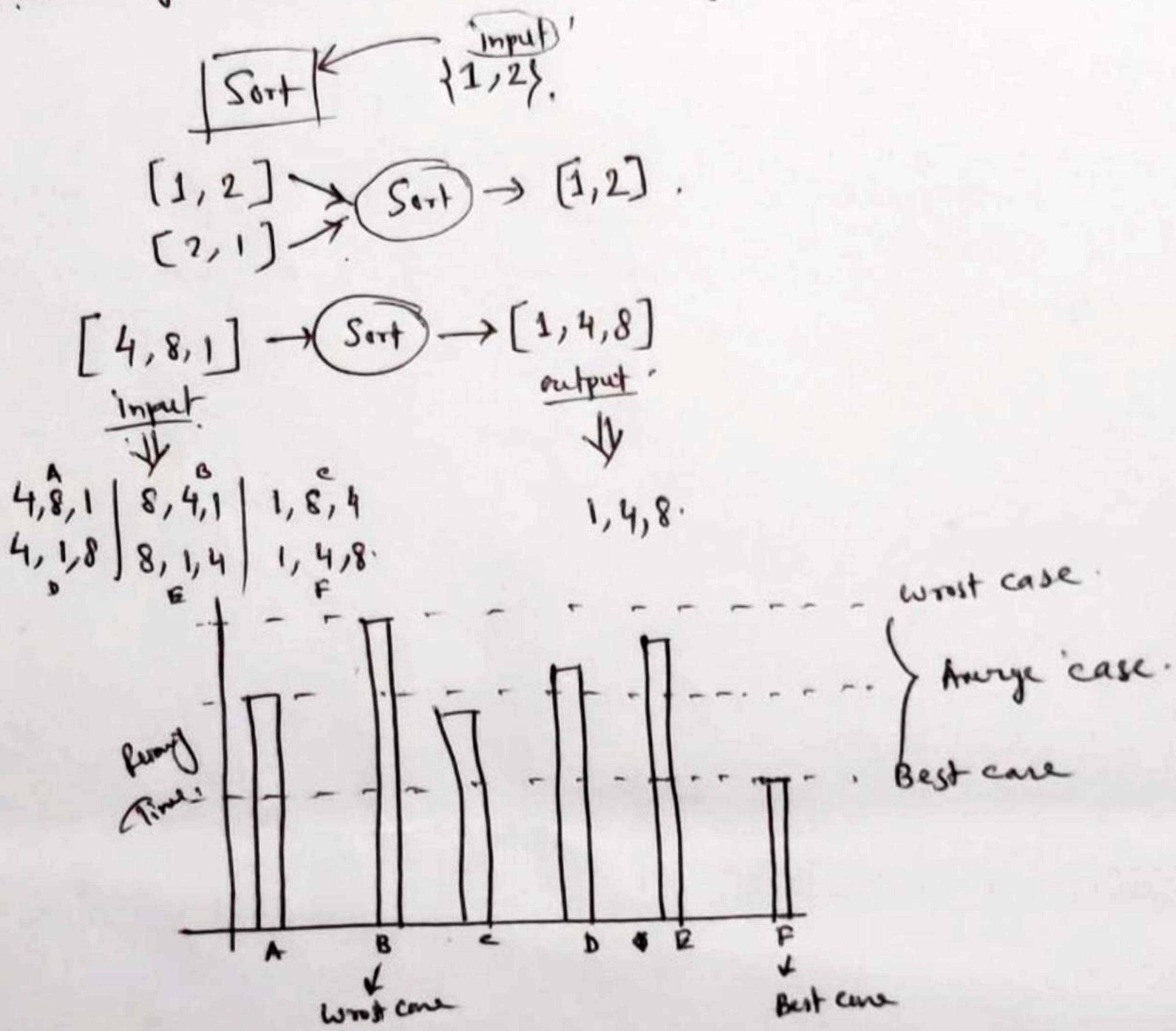
$$t(n) = 7n-2 .$$

## Best - Average - Worst

Amit.

(5)

⇒ Read Again the RAM model in page-2.



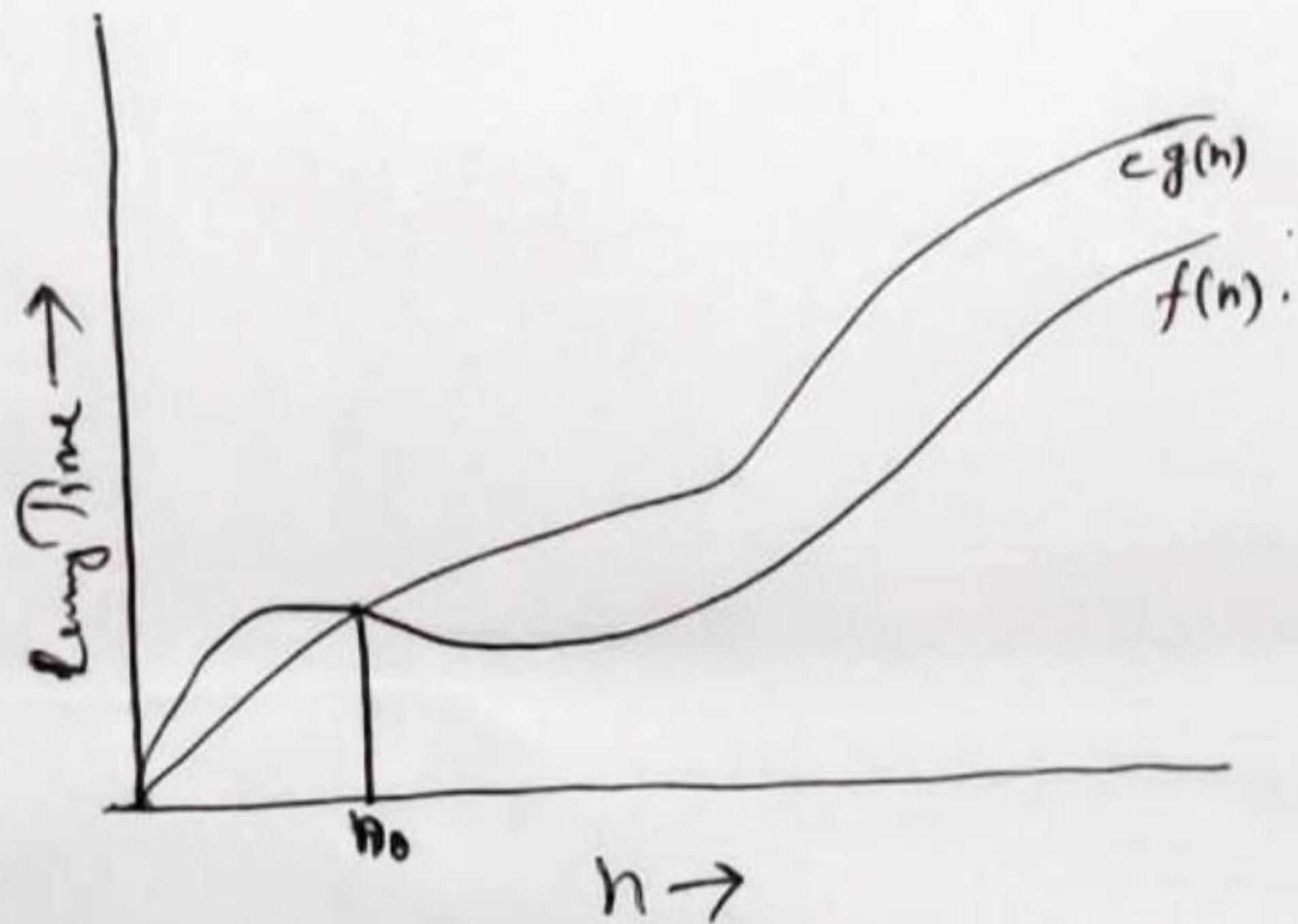
## The Big-oh notation

Let  $f(n)$  &  $g(n)$  are functions mapping non-negative integers to real numbers.

We say that  $f(n)$  is  $O(g(n))$  if there is a real constant  $c > 0$  & an integer constant  $n_0 \geq 1$

Such that  $f(n) \leq c g(n)$  for every integer  $n \geq n_0$ .

This condition is referred as big-oh or we can say  $f(n)$  is of the order of  $g(n)$ .



Example:-  $7n^2$  is  $O(n)$ .

Proof:- By the definition of big-oh we have to find a constant ~~c > 0~~  $c > 0$  &  $n_0 (\geq 1)$  such that  $7n^2 \leq cn$  for every  $n \geq n_0$ .

It's easy to see that  $c = 7$  &  $n_0 = 1$ . However, This is one of the many possible solution.

Ans 7

Let  $d(n)$ ,  $e(n)$ ,  $f(n)$  &  $g(n)$  are functions mapping non-negative integers to non-negative reals then, -

- 1) If  $d(n)$  is  $O(f(n))$ , then  $ad(n)$  is  $O(f(n))$  for any constant  $a > 0$ .
- 2) If  $d(n)$  is  $O(f(n))$  and  $e(n)$  is  $O(g(n))$ , then  $d(n)+e(n)$  is  $O(f(n)+g(n))$ .
- 3) If  $d(n)$  is  $O(f(n))$  &  $e(n)$  is  $O(g(n))$  then  $d(n)e(n)$  is  $O(f(n)g(n))$ .
- 4) If  $d(n)$  is  $O(f(n))$  &  $f(n)$  is  $O(g(n))$  then  $d(n)$  is  $O(g(n))$ .
- 5) If  $f(n)$  is a polynomial of degree  $d$  (i.e.  $f(n) = a_0 + a_1n + \dots + a_dn^d$ ), then  $f(n)$  is  $O(n^d)$ .
- 6)  $n^x$  is  $O(a^n)$  for any fixed  $x > 0$  &  $a > 1$ .
- 7)  $\log n^x$  is  $O(\log n)$  for any fixed  $x > 0$
- 8)  $\log_n^x$  is  $O(n)$  for any fixed constants  $x > 0$  &  $y > 0$ .

$\Rightarrow$  It is considered poor test to include constant factors & lower order terms in the big-O notation.

Example :  $2n^3 + 4n^2 \log n$ .

by applying theorem ~~of O~~ as follows:-

$\rightarrow \log n$  is  $O(n)$  (rule 8)

$\rightarrow 4n^2 \log n$  is  $O(n^3)$  (Rule 3)

$\rightarrow 2n^3 + 4n^2 \log n$  is  $O(2n^3 + 4n^3)$  (Rule 2).

$\rightarrow 2n^3 + 4n^3$  is  $O(n^3)$  (rule 5 or rule 1)

$\rightarrow 2n^3 + 4n^2 \log n$  is  $O(n^3)$ .

$\Rightarrow$  Commonly used functions which appear often in the algorithm, -

logarithmic	linear	quadratic	Polynomial	exponential
$O(\log n)$	$O(n)$	$O(n^2)$	$O(n^k)$ $k > 1$	$O(a^n)$ $a > 1$