Assignment 2

1. **(Computer Center Staffing)** You are the Director of the Computer Center for Gaillard College and responsible for scheduling the staffing of the center. It is open from 8 am until midnight. You have monitored the usage of the center at various times of the day and determined that the following numbers of computer consultants are required.

Time of day	Minimum number of consultants required to be on duty
8 am–noon	4
Noon–4 pm	8
4 am–8 pm	10
8 am–midnight	6

Two types of computer consultants can be hired: full-time and part-time. The full-time consultants work for eight consecutive hours in any of the following shifts: morning (8 am – 4 pm), afternoon (noon – 8 pm), and evening (4 pm – midnight). Full-time consult- ants are paid $14 per hour.
Part-time consultants can be hired to work any of the four shifts listed in the table. Part-time consultants are paid $12 per hour. An additional requirement is that during every time period, at least one full-time consultant must be on duty for every part-time consultant on duty.

a) Determine a minimum-cost staffing plan for the center. In your solution, how many consultants will be paid to work full time and how many will be paid to work part time? What is the minimum cost?

b) After thinking about this problem for a while, you have decided to recognize meal breaks explicitly in the scheduling of full-time consultants. In particular, full-time consultants are entitled to a one-hour lunch break during their eight-hour shift. In addition, employment rules specify that the lunch break can start after three hours of work or after four hours of work, but those are the only alternatives. Part-time consultants do not receive a meal break. Under these conditions, find a minimum-cost staffing plan. What is the minimum cost?

Hint: for this problem, you only need to formulate the LP problem without solving it.

Solution:

a: Minimum-cost staffing plan for the center.

Fulltime (FT) Consultants - **$14/Hr**. Part-time (PT) Consultants - **$12/Hr.**

Req-> Min 1 FT should be there for every PT on Duty. ------ Condition

			Working time for PT and FT Consultants			
Time of Day	**Shift**	**Min Consultants Required on Duty**	**PT Working Hrs.**	**PT Variable**	**FT Working Hrs.**	**FT Variable**
8 am–Noon	1	4	4	X_1	8	Y_1
Noon–4 pm	2	8	4	X_2	8	Y_2
4 am–8 pm	3	10	4	X_3		
8 pm–Midnight	4	6	4	X_4	8	Y_3

Table:1

Total Count of Consultants working all Day - 28

Total Shift Hrs. 4 Hrs. each shift for 4 shifts = 4*4 = 16 Hrs.in a Day

Equations based on constrains and with reference to table: (Consider min Consultants required on shift)

$$X_1 + Y_1 \geq 4$$

$$X_2 + Y_1 + Y_2 \geq 8$$

$$X_3 + Y_2 + Y_3 \geq 10$$

$$X_4 + Y_3 + Y_4 \geq 6$$

***From the condition (1FT is required for every 1PT)**

Max PT consultants are half of min consultants On Duty

i.e. (2,4,5,3) consultants respectively for 4 Shifts = $2X_1 + 4X_2 + 5X_3 + 3X_4$

Total PT Consultants = **14**

Each PT works for 4 Hrs. i.e. $12*4 = $48 ------ Salary for Each PT Consultant

Entire Payment for PT Consultants = 14*$48 = **$672 ------ Total FT Payment**

FT can work for 8 Conti Hrs. i.e. 1 FT is available for 2 Shifts so ½ of MAX PT consultants = 7 FT

Total FT Consultants = **7**

Reference to above equations and table.

i.e. (2,2,3) consultants respectively for 4 Shifts = $2\ Y_1 + 2\ Y_2 + 3\ Y_3$

Each FT works for 8 Hrs. i.e. $14*8 = $112 ----- Salary of Each FT Consultant

Entire Payment for FT Consultants = 7*$112 = **$784 ------ Total FT Payment**

$2\ Y_1 + 2\ Y_2 + 3\ Y_3 = \$784$

Sum of Total FT an PT Payment per day = $784 + $672 = **$1456 --** Minimum-Cost Staffing Plan

b: 1 Hour break for FT Consultants

FT Works 8 Hrs. in that 1 Hr. is for Lunch = 8-1 = 7Hrs.

$= 112(2\ Y_1 + 2\ Y_2 + 3\ Y_3) – 14(2\ Y_1 + 2\ Y_2 + 3\ Y_3)$ --- FT Payment post lunch

$= 98(2\ Y_1 + 2\ Y_2 + 3\ Y_3)$

Entire FT Payment of consultants = $98*7 = **$686 -------Total FT Payment Post Lunch**

No change in PT Payment = $672 from above.

Sum of Total FT and PT Post lunch break = $686 + $ 672 = **$1358 --** Minimum-Cost Staffing Plan

2. Consider the problem from the previous assignment.

   Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week. Solve this problem graphically.

Solution: (Solved in the Previous Assignment - 1)

C_p= Collegiate to be Produced
M_p = Mini to be Produced
Profit Max $\$_M = [32C_p + 26M_p]$ ------ ($32 & $24 values are from the given unit profits above)
35 Labors work for 40 Hrs./Week = 35*40 = 1400 Hours
Since Labor time is in Hrs. Converting the above minutes to Hours.
Labor Time – (Mins to Hours - Dividing by 60)
Collegiate 45 Min = 0.75 Hrs.
Mini 40 Min = 0.66 Hrs.
Equations:
Nylon Used Per bag - **$3\ C_p + 2\ M_p \leq 5000$ Sft**
Labor Hours - **$0.75\ C_p + 0.66\ M_p \leq 1400$ Hrs.**

Sales - $C_p \leq 1000$, $M_p \leq 1200$
and $C_p \geq 0$, $M_p \geq 0$

Graphical Representations:

Solve the 3 Equations to get the X, Y Coordinates so that we can plot the graph.

Replacing C_p and M_P with X and Y Respectively (i.e. $\mathbf{C_p = X}$ and $\mathbf{M_P = Y}$)

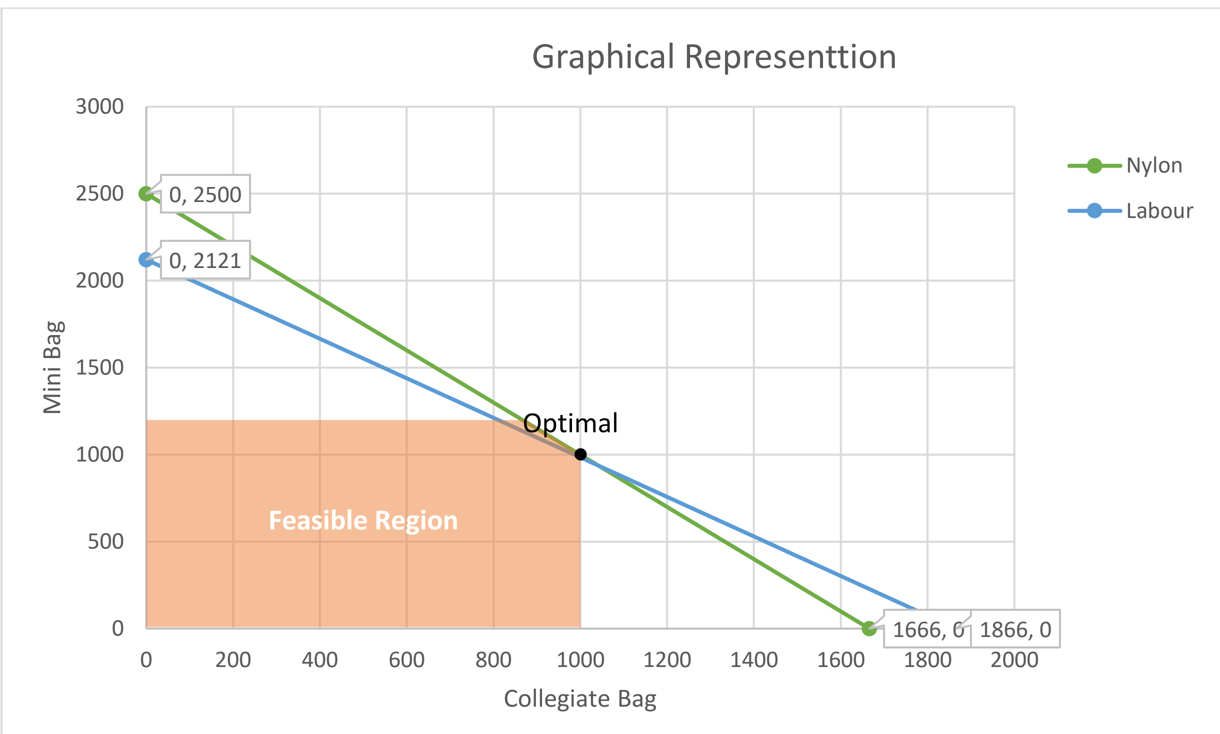
3X + 2Y = 5000 --------- (Nylon)

X = 0	Y = 0
2Y = 5000 Y = 5000/3 Points = (0,2500)	3X = 5000 X = 5000 / 3 Points = (1666,0)

0.75X+ 0.66Y = 1400 --------- (Labor Hours)

X = 0	Y = 0
0.66 Y = 1400 Points = (0,2121)	0.75 X = 1400 Points = (1866 ,0)

X ≤ 1000; Y ≤ 1200 --- (Sales)



3. **(Weigelt Production)** The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

   The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

   Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

   At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

   Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

   a. Define the decision variables
   b. Formulate a linear programming model for this problem.
   c. Solve the problem using lpsolve, or any other equivalent library in R.

Solution:

The problem above says the following information about

Existing Plant details:

Weigelt Production Plants	Plant 1 **(P_1)**	Plant 2 **(P_2)**	Plant 3 **(P_3)**
Excess Capacity to Produce	750	900	450
In- Process Storage Space (Sq. Ft)	13000	12000	5000
In – Process Storage Per Day (Sq. Ft)	20	15	12

Table 1

New Product **(NP)** details:

Product Sizes	Large **(L)**	Medium **(M)**	Small **(S)**
Net Unit Profit	$420	$360	$300
Sales Forecast Per day	900	1200	750

Table 2

a. Decision Variables

As we have 3 Plants and each plant produces 3 of the new products sizes, we have a total of 9 decision variables.

NP.L. P_1 = No. of Large units generated in Plant 1 per day
NP.L. P_2 = No. of Large units generated in Plant 2 per day
NP.L. P_3 = No. of Large units generated in Plant 3 per day

NP.M. P_1 = No. of Medium units generated in Plant 1 per day
NP.M. P_2 = No. of Medium units generated in Plant 2 per day
NP.M. P_3 = No. of Medium units generated in Plant 3 per day

$NP.S.\ P_1$ = No. of Small units generated in Plant 1 per day
$NP.S.\ P_2$ = No. of Small units generated in Plant 2 per day
$NP.S.\ P_3$ = No. of Small units generated in Plant 3 per day

b. Formulate a linear programming model for this problem.

Capacity to produce in each plant.

$$\left.\begin{aligned} &NP.L.\ P_1 + NP.M.\ P_1 + NP.S.\ P_1 \le 750 \\ &NP.L.\ P_2 + NP.M.\ P_2 + NP.S.\ P_2 \le 900 \\ &NP.L.\ P_3 + NP.M.\ P_3 + NP.S.\ P_3 \le 450 \end{aligned}\right\} \text{Ref: Table 1}$$

Storage Space (Inc. Of Per day Storage)

$$\left.\begin{aligned} &[20\,(NP.L.\ P_1) + 15\,(NP.M.\ P_1) + 12\,(NP.S.\ P_1)] \le 13000 \\ &[20\,(NP.L.\ P_2) + 15\,(NP.M.\ P_2) + 12\,(NP.S.\ P_2)] \le 12000 \\ &[20\,(NP.L.\ P_3) + 15\,(NP.M.\ P_3) + 12\,(NP.S.\ P_3)] \le 5000 \end{aligned}\right\} \text{Ref: Table 1}$$

New Product Sales Forecast

$$\left.\begin{aligned} &NP.L.\ P_1 + NP.L.\ P_2 + NP.L.\ P_3 \le 900 \\ &NP.M.\ P_1 + NP.M.\ P_2 + NP.M.\ P_3 \le 1200 \\ &NP.S.\ P_1 + NP.S.\ P_2 + NP.S.\ P_3 \le 750 \end{aligned}\right\} \text{Ref: Table 2}$$

Total Net Profit Per Day **(TP)**

$$\left.\begin{aligned} TP = {} &420\,(NP.L.\ P_1 + NP.L.\ P_2 + NP.L.\ P_3) + 360\,(NP.M.\ P_1 + NP.M.\ P_2 + NP.M.\ P_3) \\ &+ 300(NP.S.\ P_1 + NP.S.\ P_2 + NP.S.\ P_3) \end{aligned}\right\} \text{Ref: Table 2}$$

And

$NP.L.\ P_1 \ge 0$, $NP.L.\ P_2 \ge 0$, $NP.L.\ P \ge 0$, $NP.M.\ P_1 \ge 0$, $NP.M.\ P_2 \ge 0$, $NP.M.\ P_3 \ge 0$, $NP.S.\ P_1 \ge 0$, $NP.S.\ P_2 \ge 0$, $NP.S.\ P_3 \ge 0$

c. Solve the problem using lpsolve, or any other equivalent library in R.

Solution: Code Uploaded in GIT

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