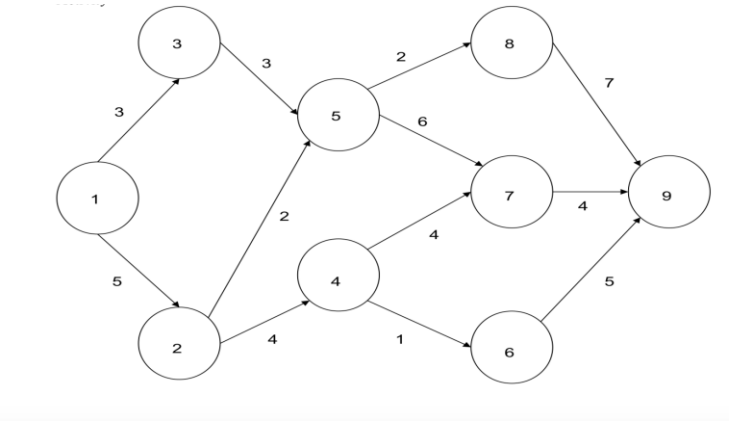
Assignment – 5

1. Consider the following activity-on-arc project network, where the 12 arcs (arrows) represent the 12 activities (tasks) that must be performed to complete the project and the network displays the order in which the activities need to be performed. The number next to each arc (arrow) is the time required for the corresponding activity. Consider the problem of finding the longest path (the largest total time) through this network from start (node 1) to finish (node 9), since the longest path is the critical path.
   Formulate and solve the binary integer programming (BIP) model for this problem using library *lpsolve* or equivalent in R.

Solution:



From the Diagram:

Finding the Longest Path:

Max Longest Path: (Time- Arc-In- Out)

Max L = $3A_{13} + 5A_{12} + 3A_{35} + 2A_{25} + 2A_{58} + 6A_{57} + 4A_{24} + 4A_{47} + 1A_{46} + 7A_{89} + 4A_{79} + 5A_{69}$

Origin and Destination Node = 1

A_{IO}(Arc In-Out)

Origin Node:

Node 1: $3A_{13} + 5A_{12} = 1$

Destination Node:

Node 9: $7A_{89} + 4A_{79} + 5A_{69} = 1$

Intermediate Nodes:

Node 2: $5A_{12} = 2A_{25} + 4A_{24}$

Node 3: $3A_{13} = 3A_{35}$

Node 4: $4A_{24} = 1A_{46} + 4A_{47}$

Node 5: $3A_{35} + 2A_{25} = 2A_{58} + 6A_{57}$

Node 6: $1A_{46} = 5A_{69}$

Node 7: $6A_{57} + 4A_{47} = 4A_{79}$

Node 8: $2A_{58} = 7A_{89}$

Where A_{IO} are Binary

R File is attached in the git hub

Running the LP solver, the critical path is:

From node **1** to **2**

From node **2** to **5**

From node **5** to **7**

From node **7** to **9**

The duration of the project is, therefore **17-time units**.

2. **Selecting an Investment Portfolio** An investment manager wants to determine an Opti- mal portfolio for a wealthy client. The fund has $2.5 million to invest, and its objective is to maximize total dollar return from both growth and dividends over the course of the coming year. The client has researched eight high-tech companies and wants the portfolio to consist of shares in these firms only. Three of the firms (S1 – S3) are primarily software companies, three (H1–H3) are primarily hardware companies, and two (C1–C2) are internet consulting companies. The client has stipulated that no more than 40 percent of the investment be allocated to any one of these three sectors. To assure diversification, at least $100,000 must be invested in each of the eight stocks. Moreover, the number of shares invested in any stock must be a multiple of 1000.

   The table below gives estimates from the investment company's database relating to these stocks. These estimates include the price per share, the projected annual growth rate in the share price, and the anticipated annual dividend payment per share.

Solution:

Particulars	S1	S2	S3	H1	H2	H3	C1	C2
Price Per Share $	40	50	80	60	45	60	30	25
Growth Rate	0.05	0.1	0.03	0.04	0.07	0.15	0.22	0.25
Dividend $	2.00	1.50	3.50	3.00	2.00	1.00	1.80	0.00
Total Profit #	4	6.5	5.9	5.4	5.15	10	8.4	6.25

= (Price Per Share + Growth Rate) / Dividend

Max Z: = $4 X_{S1} + 6.5 X_{S2} + 5.9 X_{S3} + 5.4 X_{H1} + 5.15 X_{H2} + 10 X_{H3} + 8.4 X_{C1} + 6.25 X_{C2}$

Investment:

$1000 X_{S1} >= 0$

$1000 X_{S2} >= 0$

$1000 X_{S3} >= 0$

$1000 X_{H1} >= 0$

$1000 X_{H2} >= 0$

$1000 X_{H3} >= 0$

$1000\ X_{C1} >= 0$

$1000\ X_{C2} >= 0$

Investment constraint:

$40\ X_{S1} + 50\ X_{S2} + 80\ X_{S3} + 60\ X_{H1} + 45\ X_{H2} + 60\ X_{H3} + 30\ X_{C1} + 25\ X_{C2} <= 2500000$

Min $100,000 must be invested in each stock

$40\ X_{S1} >= 100000$; $50\ X_{S2} >= 100000$; $80\ X_{S3} >= 100000$;

$60\ X_{H1} >= 100000$; $45\ X_{H2} >= 100000$; $60\ X_{H3} >= 100000$;

$30\ X_{C1} >= 100000$; $25\ X_{C2} >= 100000$

No more than 40% investment.

$40\ X_{S1} + 50\ X_{S2} + 80\ X_{S3} <= 1000000$

$60\ X_{H1} + 45\ X_{H2} + 60\ X_{H3} <= 1000000$

$30\ X_{C1} + 25\ X_{C2} <= 1000000$

Where $X_{S1}, X_{S2}, X_{S3}, X_{H1}, X_{H2}, X_{H3}, X_{C1}, X_{C2} >= 0$ (Int)

2a. Using lpsolve, maximum returns as ***487145.2*** and number of stocks are

S1= *2500, S2= 6000, S3= 1250, H1= 1667, H2= 2223, H3= 3332, C1= 30000, C2= 4000.*

Amount invested

S1= 100000, S2= 300000, S3= 100000, H1= 100020, H2= 100035, H3= 799920, C1= 900000, C2= 100000.

2b. Without integer restriction we get the objective function, maximum returns as ***487152.8*** and number of stocks are S1= *2500.0, S2= 6000.0, S3= 1250.0, H1= 1667.667, H2= 2222.222, H3= 13333.333, C1= 30000.0, C2= 4000.0*

The amount invested in each stock S1= 100000, S2= 300000, S3= 100000, H1= 100000, H2= 100000, H3= 800000, C1= 900000, C2= 100000.

Percentage difference in objective functions with and without integer restriction is 0.00156

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