Photometric Stereo

Computer Vision I

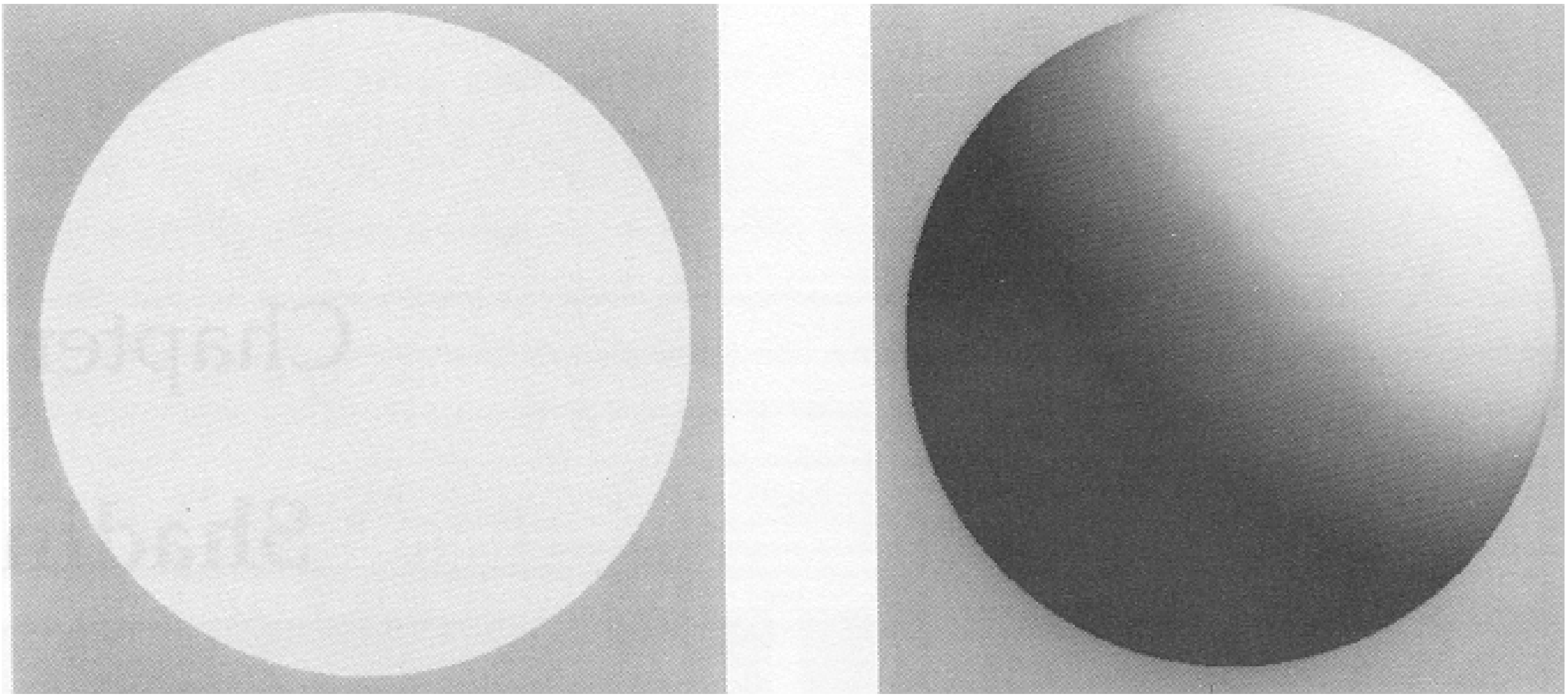
CSE 252A

Lecture 4

Announcements

- Homework 1 is due today by 11:59 PM
- Homework 2 will be assigned today
  - Due Tue, Oct 22, 11:59 PM
- Reading:
  - Section 2.2.4: Photometric Stereo
    - Shape from Multiple Shaded Images

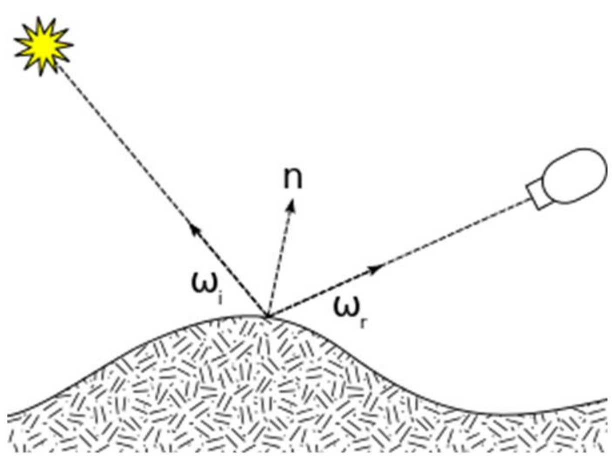
Shading reveals 3-D surface geometry



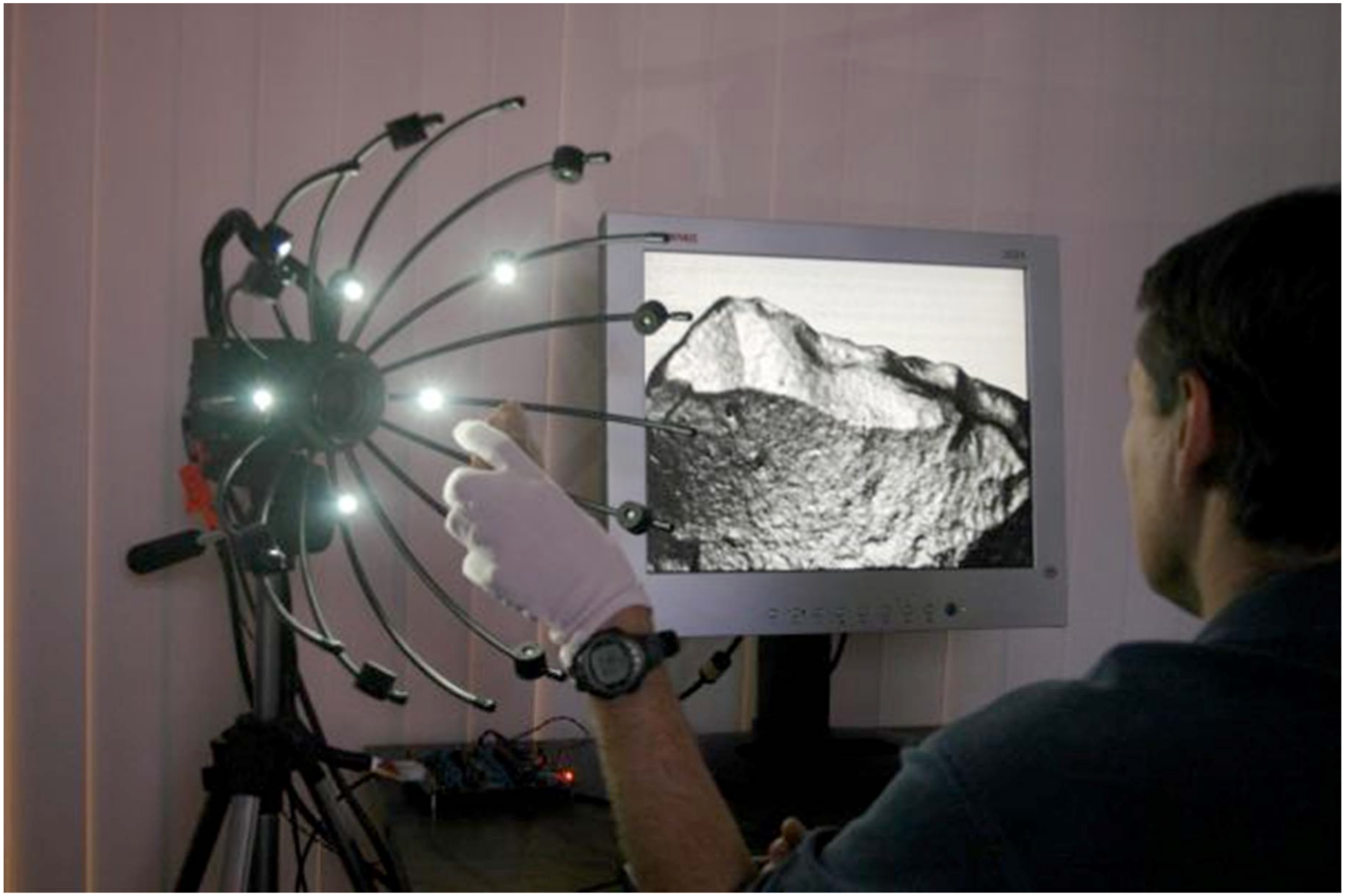
Two shape-from-X methods that use shading

- Shape-from-shading: Use just one image to recover shape. Requires knowledge of light source direction and BRDF everywhere. Too restrictive to be useful.

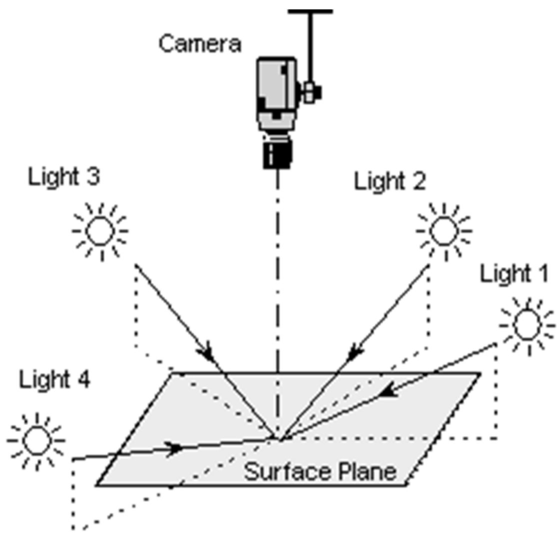
- Photometric stereo: Single viewpoint, multiple images under different lighting.



Photometric Stereo Rigs: One viewpoint, changing lighting



An example of photometric stereo

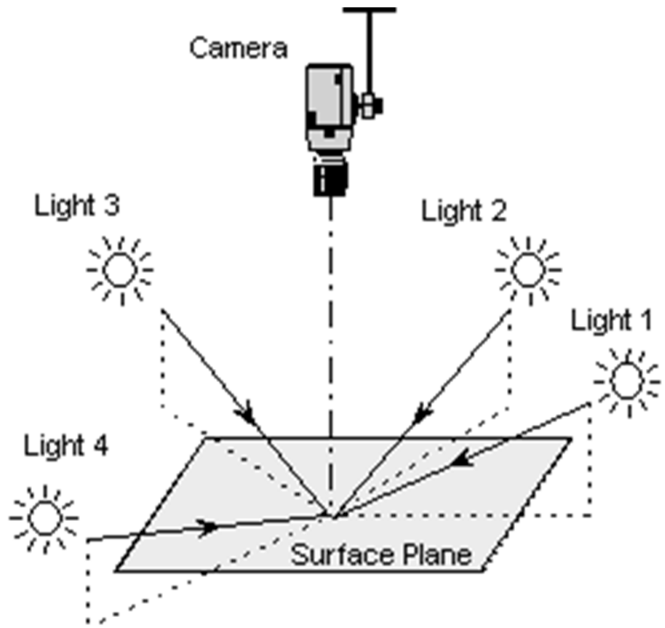


surface
(albedo texture map)

+ surface normals

albedo

Photometric stereo



- Single viewpoint, multiple images under different lighting.
  1. Arbitrary known BRDF, known lighting
  2. Lambertian BRDF, known lighting
  3. Lambertian BRDF, unknown lighting

1. Photometric Stereo: General BRDF and Reflectance Map

BRDF

- Bi-directional Reflectance Distribution Function

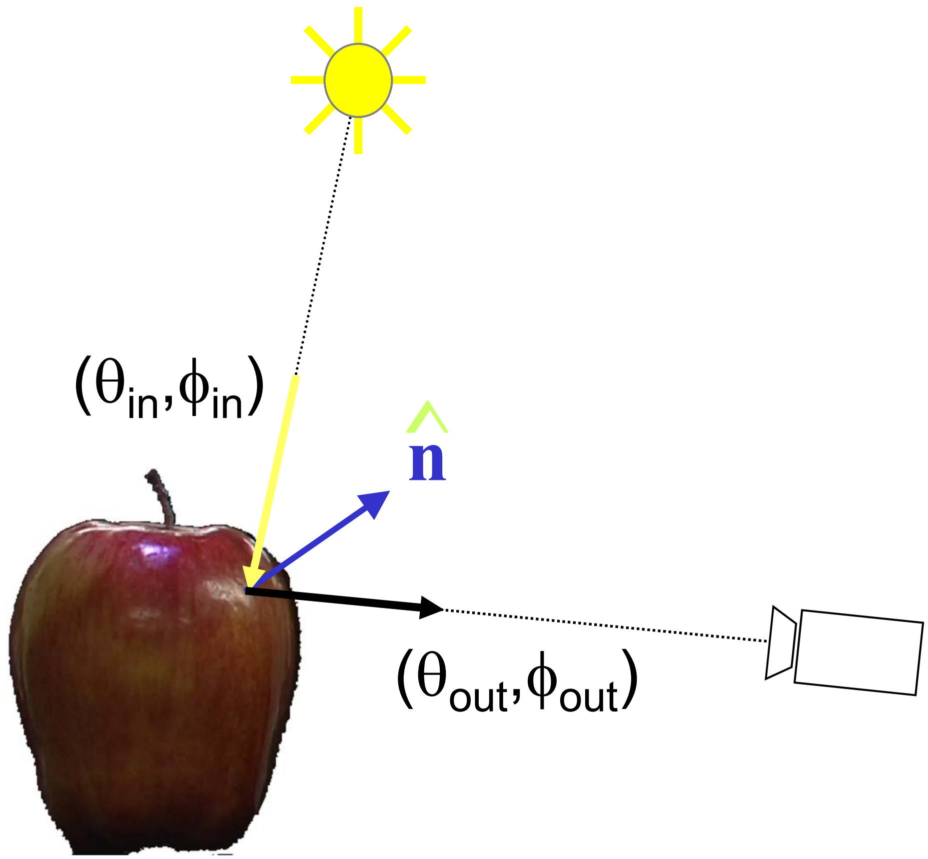
  $$\rho(\theta_{in}, \phi_{in} ; \theta_{out}, \phi_{out})$$

- Function of
  - Incoming light direction:

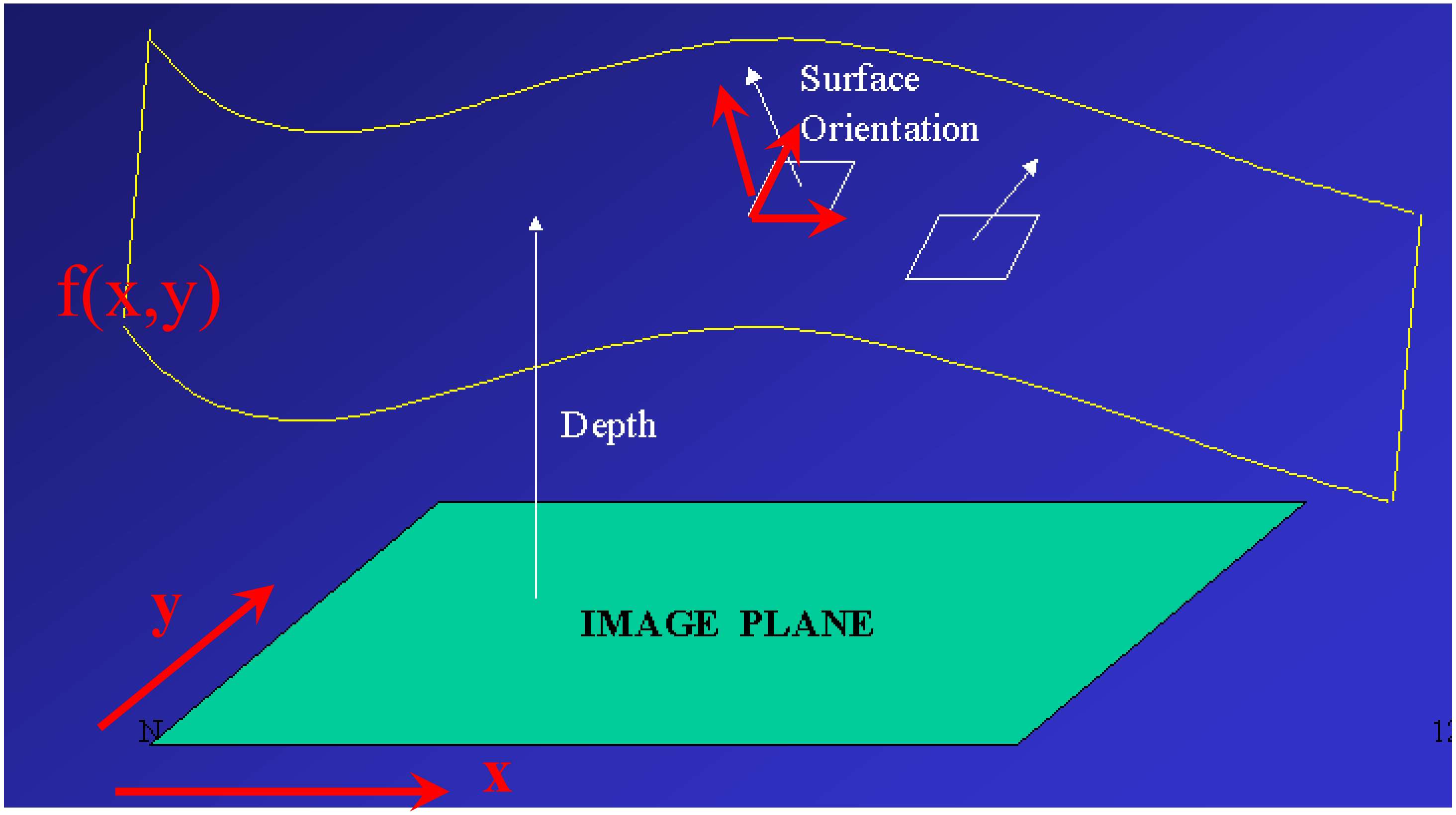
    $$\theta_{in} , \phi_{in}$$
  - Outgoing light direction:

    $$\theta_{out} , \phi_{out}$$

- Ratio of incident irradiance to emitted radiance



Coordinate system



Surface: $\mathbf{s}(x,y) = (x, y, f(x,y))$

Tangent vectors: $$\frac{\partial s(x,y)}{\partial x} = \left(1, 0, \frac{\partial f}{\partial x}\right)$$

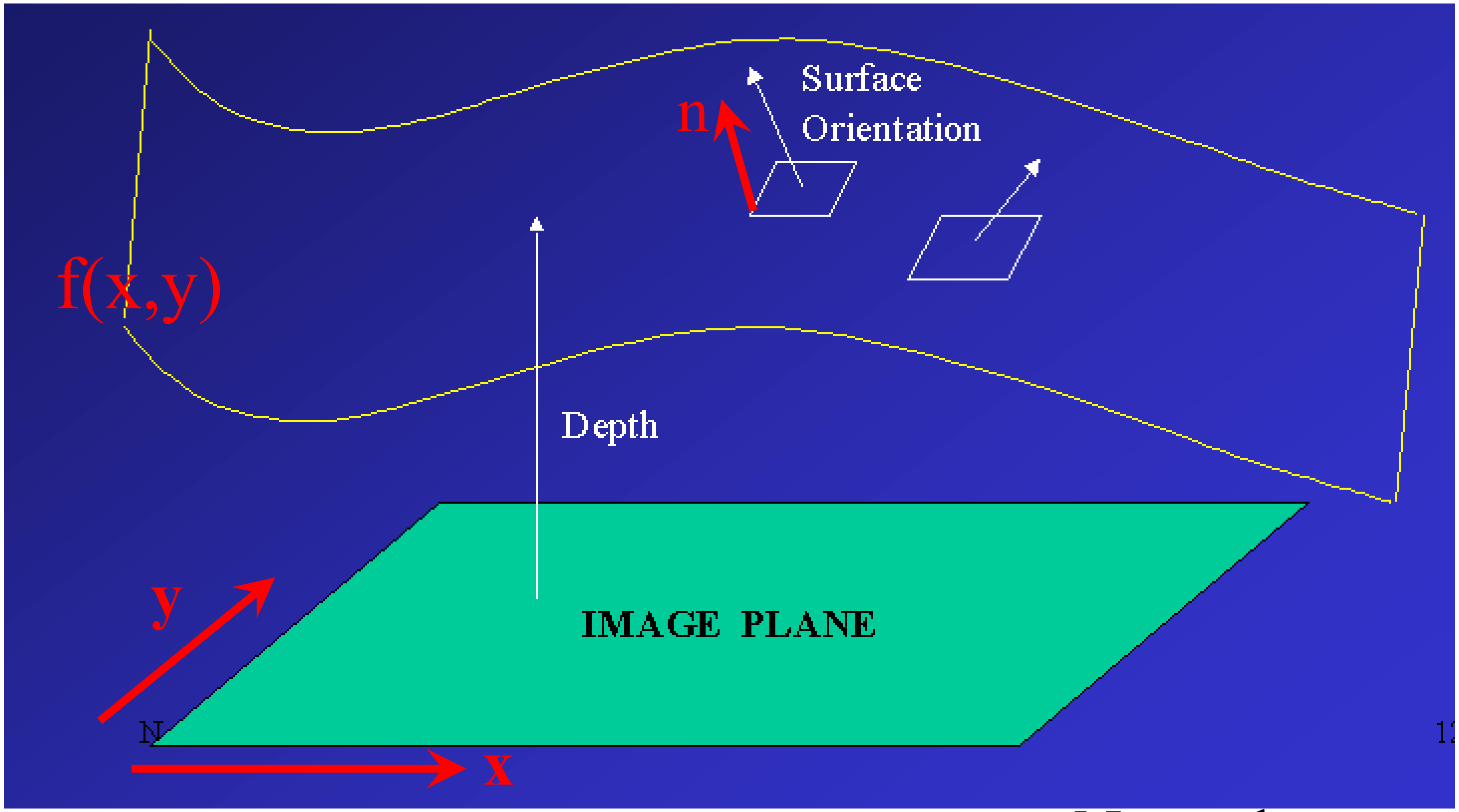
$$\frac{\partial s(x,y)}{\partial y} = \left(0, 1, \frac{\partial f}{\partial y}\right)$$

Normal vector

$$\mathbf{n} = \boxed{\frac{\partial s}{\partial x} \times \frac{\partial s}{\partial y}}$$

$$= \left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1\right)$$

Gradient Space (p,q)



Normal vector

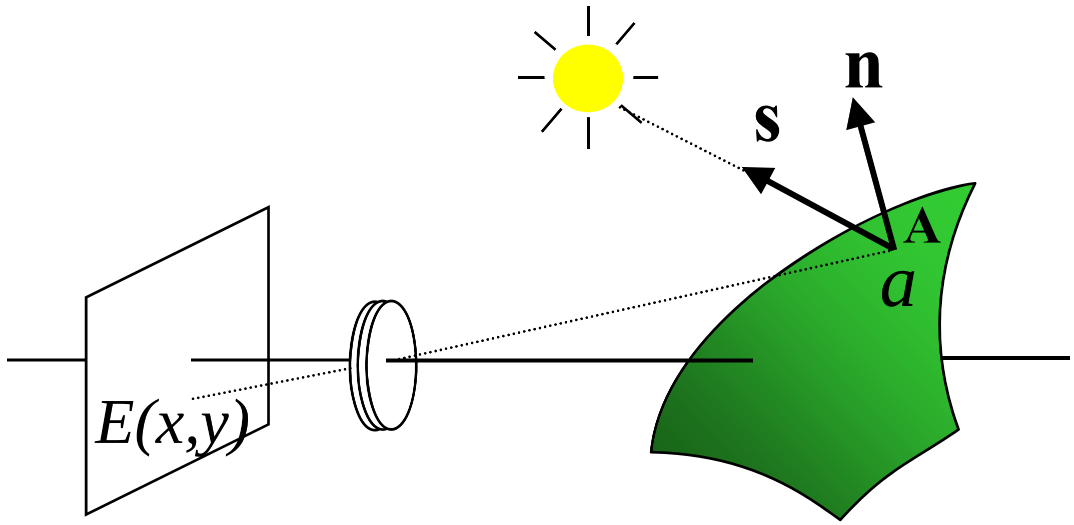
Gradient Space : (p,q)

$$p = \frac{\partial f}{\partial x}, \quad q = \frac{\partial f}{\partial y}$$

$$\mathbf{n} = \frac{\partial s}{\partial x} \times \frac{\partial s}{\partial y} = \left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1 \right)^T$$

$$\hat{\mathbf{n}} = \frac{1}{\sqrt{p^2 + q^2 + 1}} (-p, -q, 1)^T$$

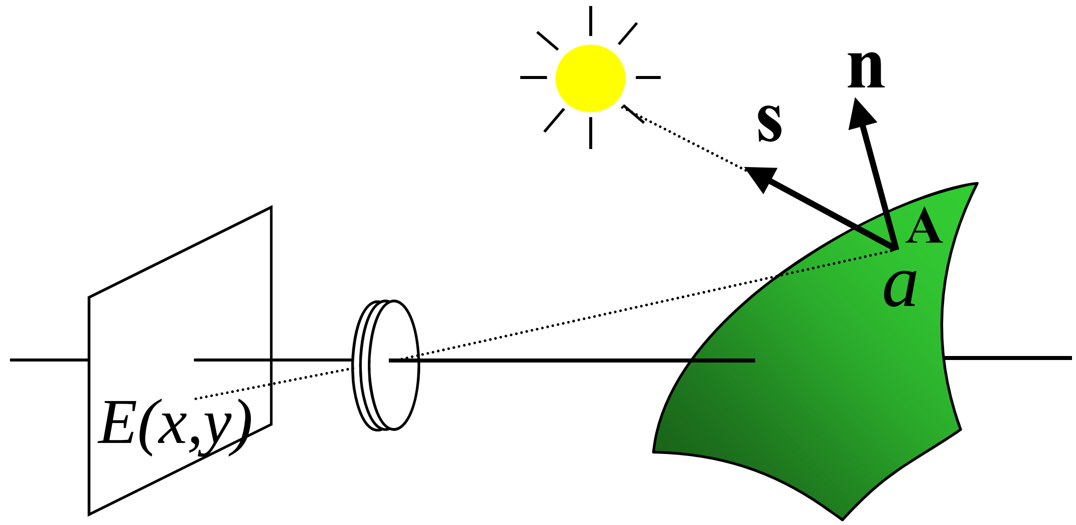
Image Formation



For a given point A on the surface *a*, the image irradiance E(x,y) is a function of

1. The BRDF at A
2. The surface normal at A
3. The direction of the light source

Reflectance Map

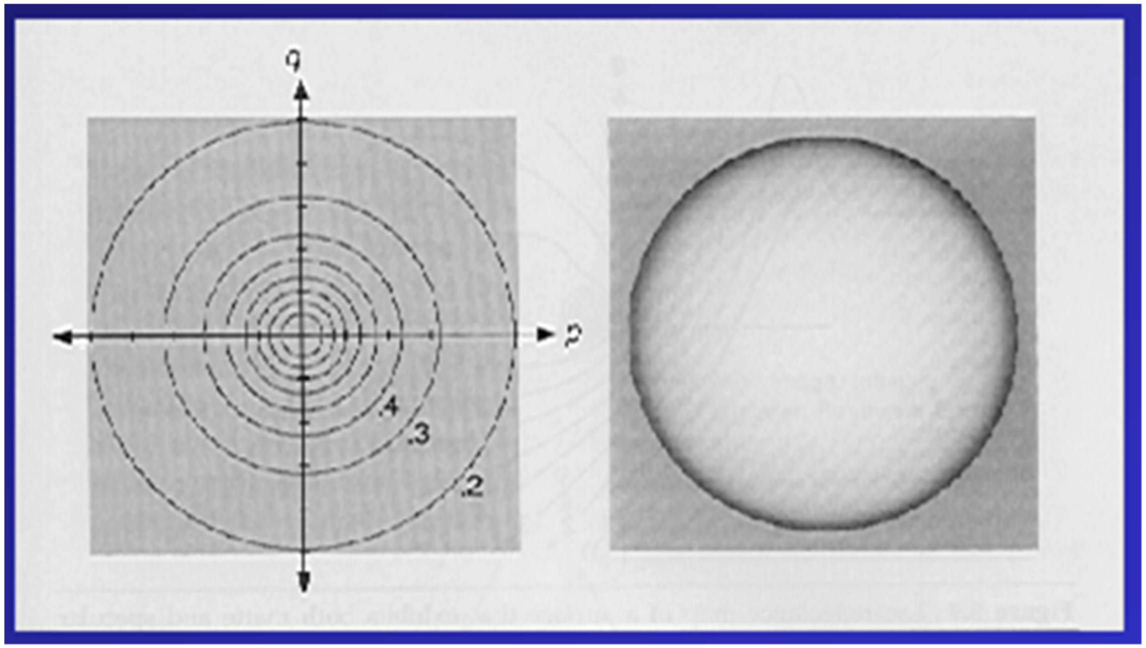


Let the BRDF be the same at all points on the surface, and let the light direction **s** be a constant.

1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have E(p,q).

Example Reflectance Map: Lambertian surface

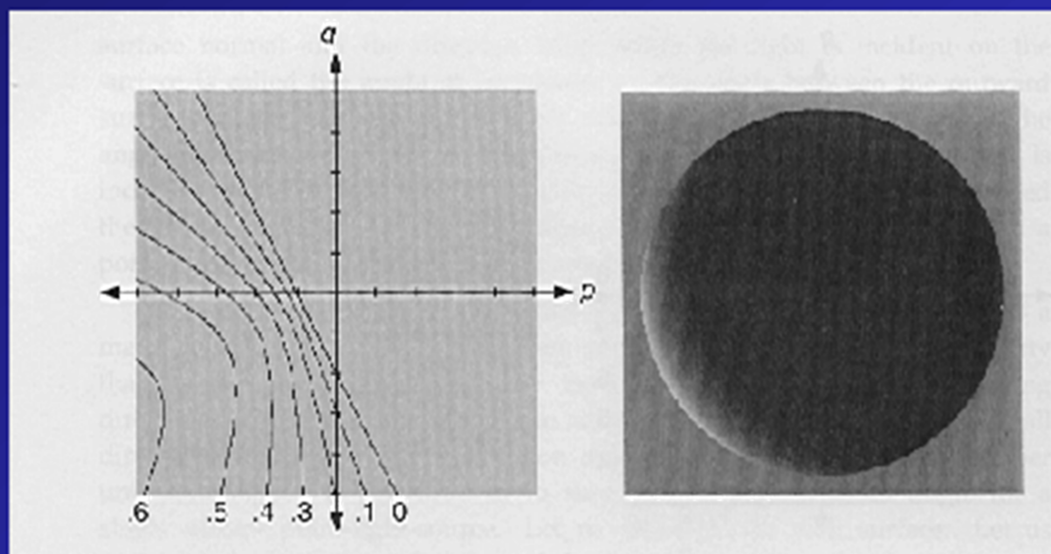
E(p,q)



For lighting from front

LAMBERTIAN REFLECTANCE MAP

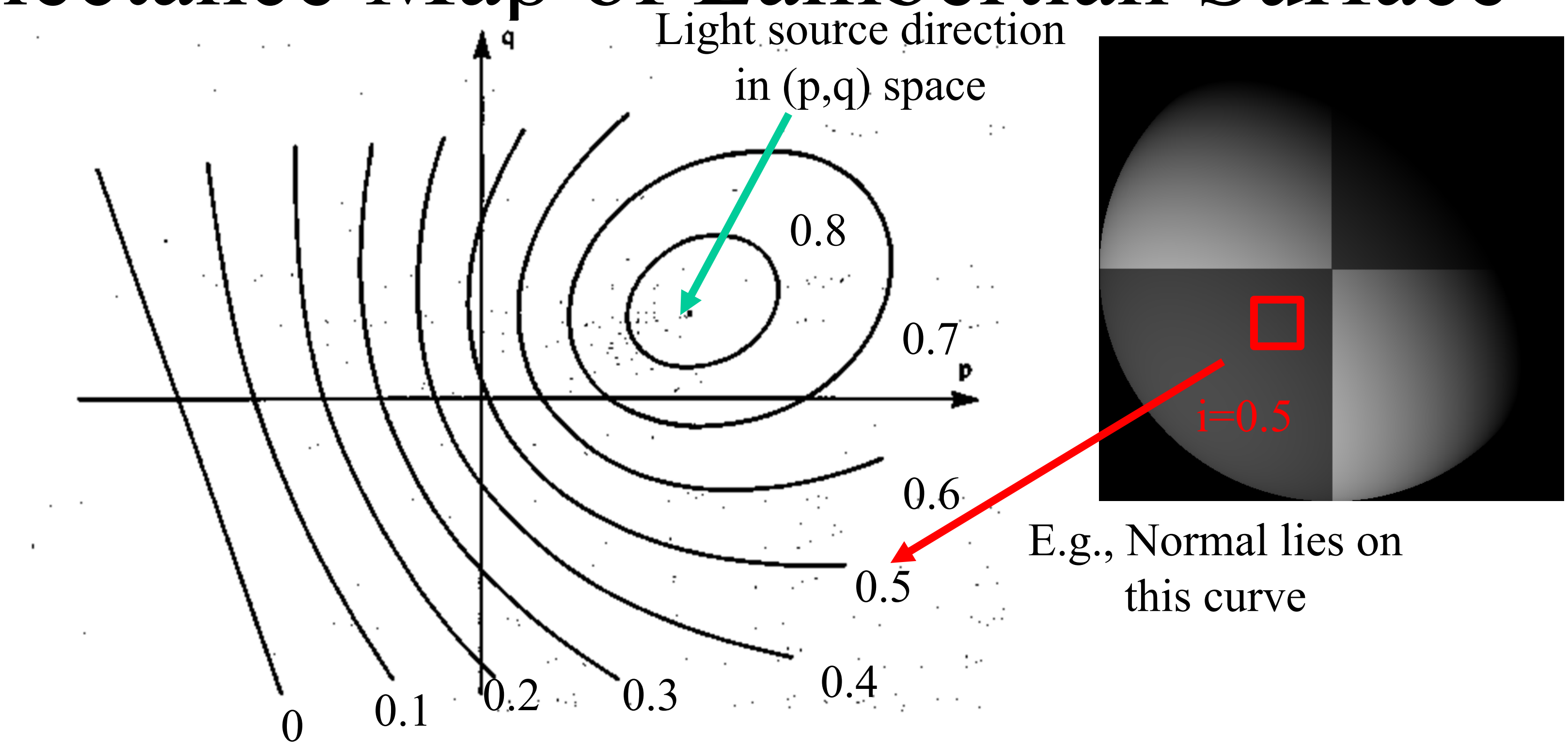
$$E = L\rho \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2}\,\sqrt{1 + p_s^{\,2} + q_s^{\,2}}}$$



$p_s = -2$ $q_s = -1$

Light Source Direction,
expressed in gradient space.

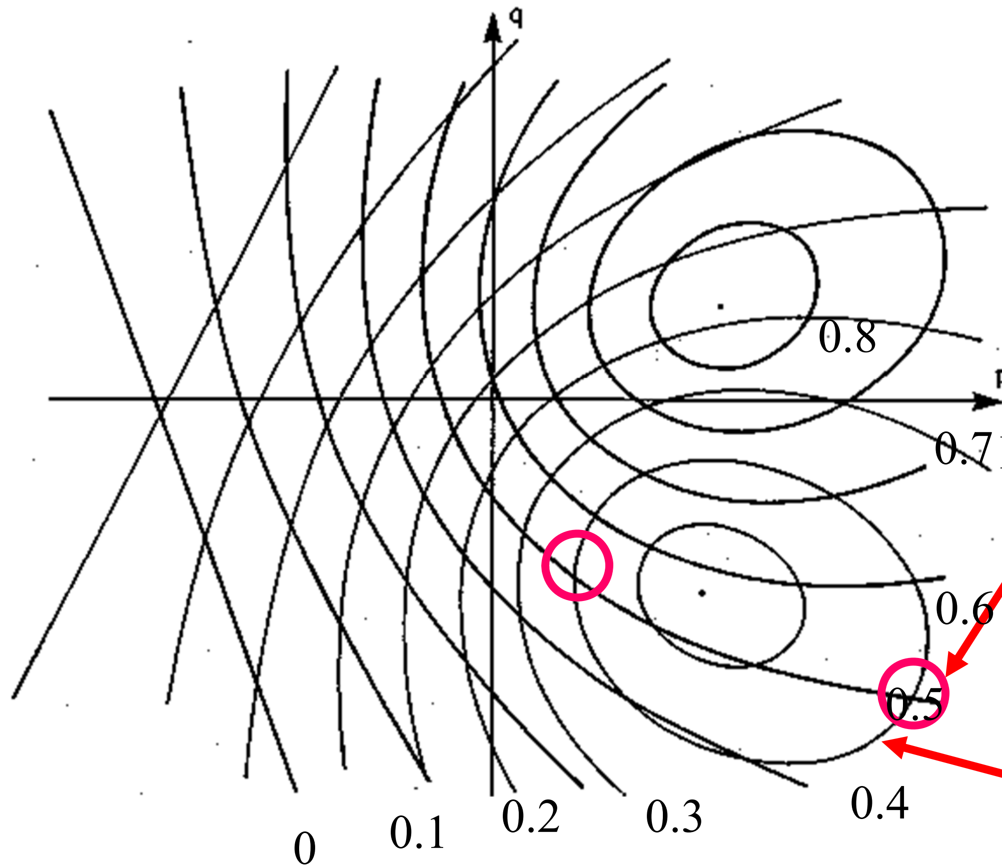
Reflectance Map of Lambertian Surface



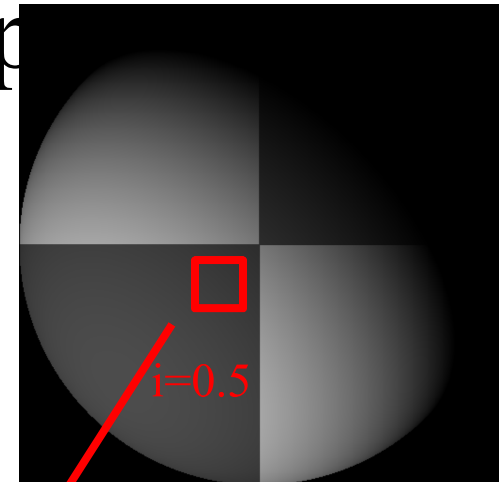
What does the intensity (irradiance) of one pixel in one image tell us?

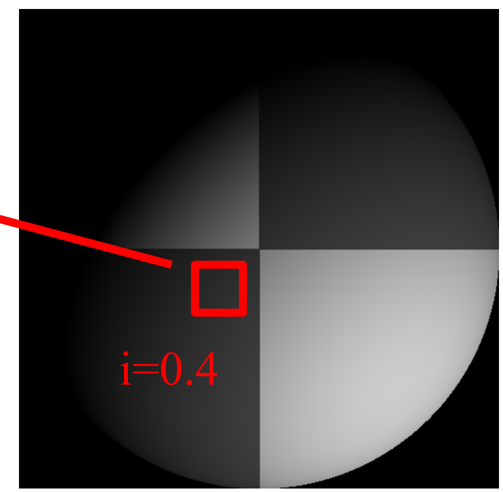
It constrains the surface normal projecting to that point to a curve

Two Light Sources
Two reflectance map



E.g., Normal lies on this curve





A third image would disambiguate match

Three Source Photometric stereo: Step1

Offline:

Using source directions & BRDF, construct reflectance map for each light source direction. $R_1(p,q)$, $R_2(p,q)$, $R_3(p,q)$

Online:

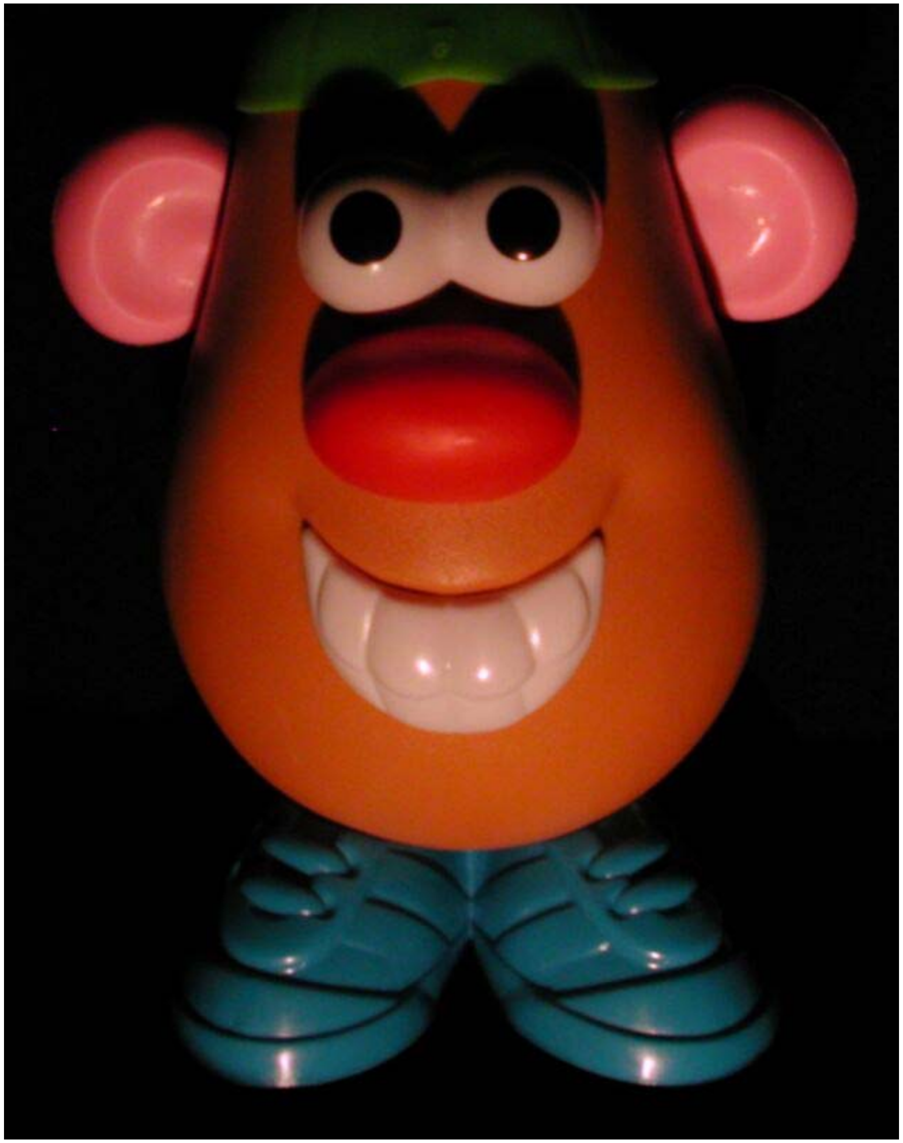
1. Acquire three images with known light source directions. $E_1(x,y)$, $E_2(x,y)$, $E_3(x,y)$
2. For each pixel location (x,y), find (p,q) as the intersection of the three curves

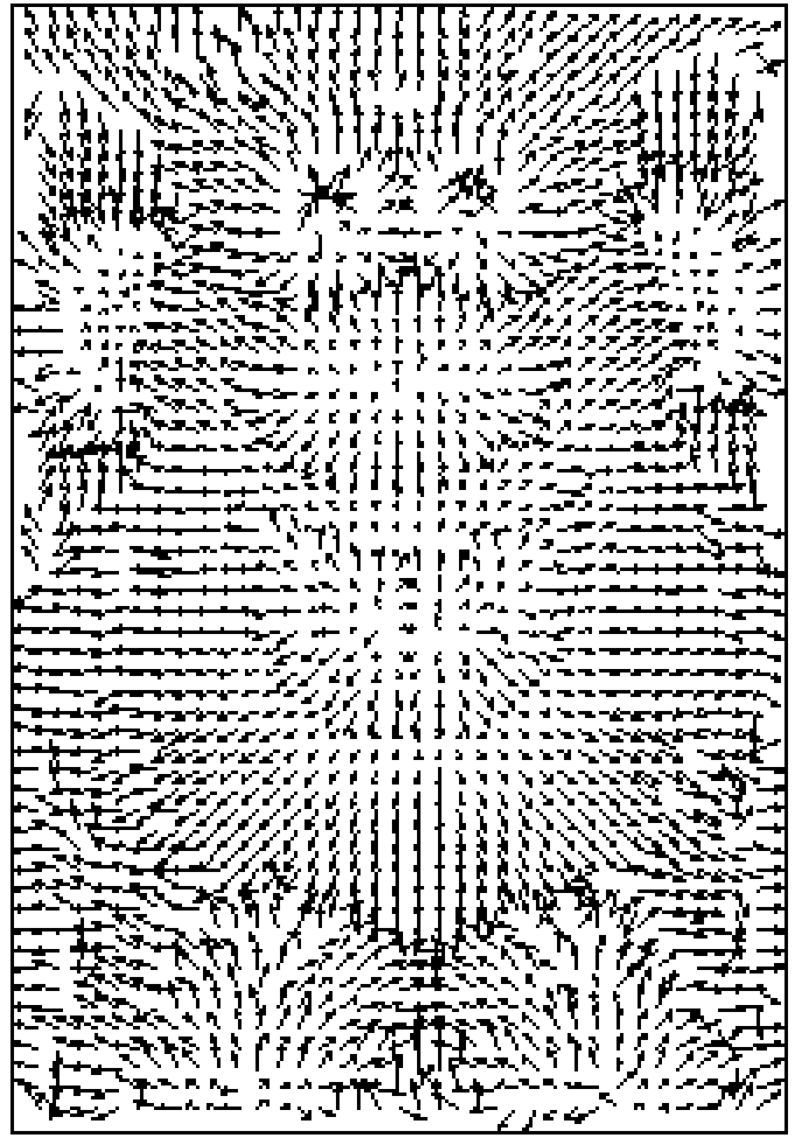
   $R_1(p,q)=E_1(x,y)$

   $R_2(p,q)=E_2(x,y)$

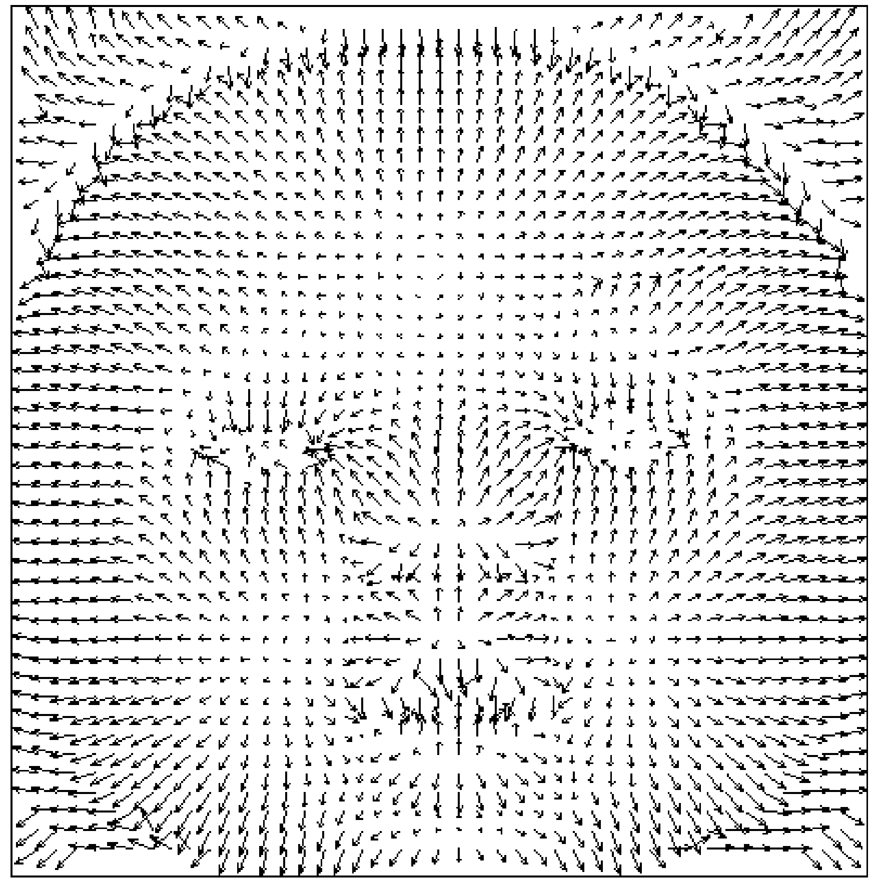
   $R_3(p,q)=E_3(x,y)$
3. This is the surface normal at pixel (x,y). Over image, the normal field is estimated

Normal Field

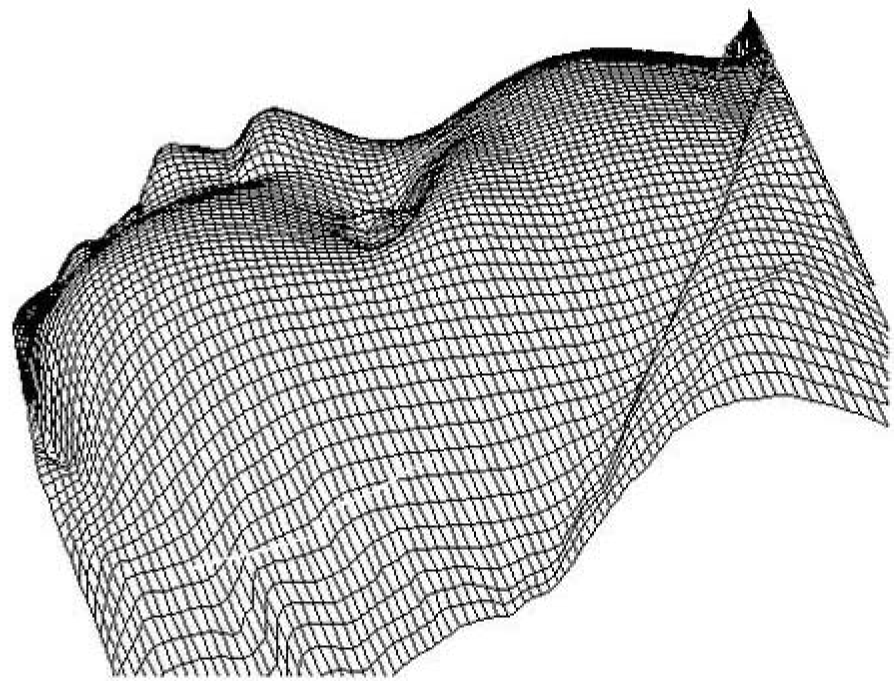




Plastic Baby Doll: Normal Field



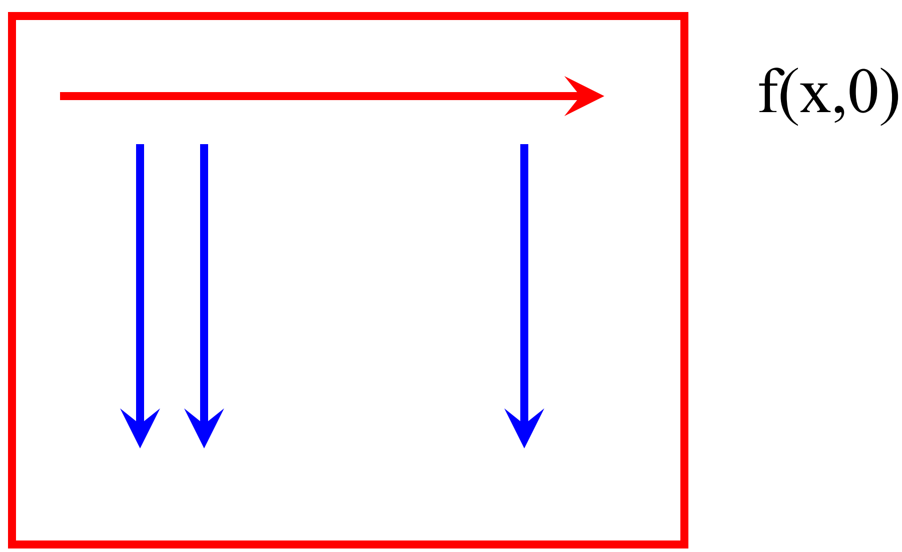
Next step:
Go from normal field to surface



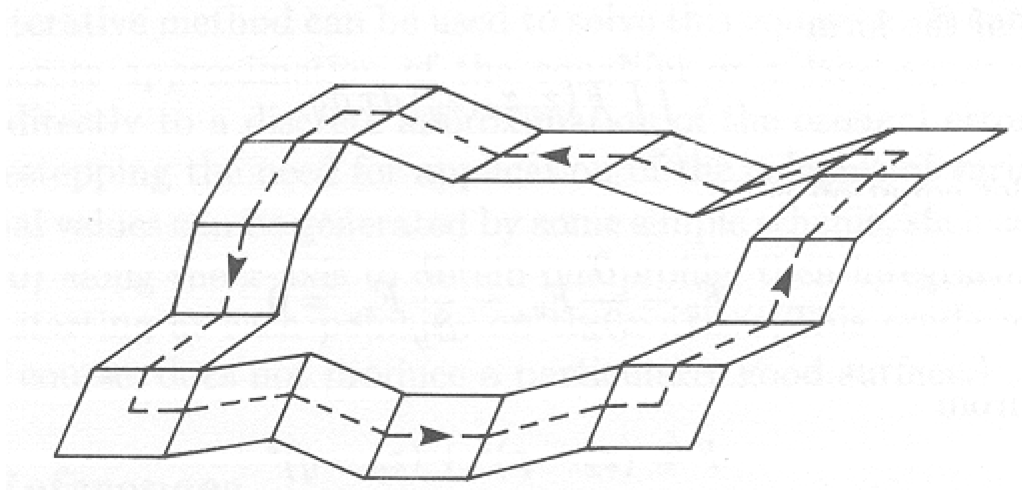
Recovering the surface f(x,y)

Many methods: Simplest approach

1. From estimate $\mathbf{n} = (n_x, n_y, n_z)$, $p = n_x/n_z$, $q = n_y/n_z$
2. Integrate $p = df/dx$ along a row $(x,0)$ to get $f(x,0)$
3. Then integrate $q = df/dy$ along each column starting with value of the first row



What might go wrong?



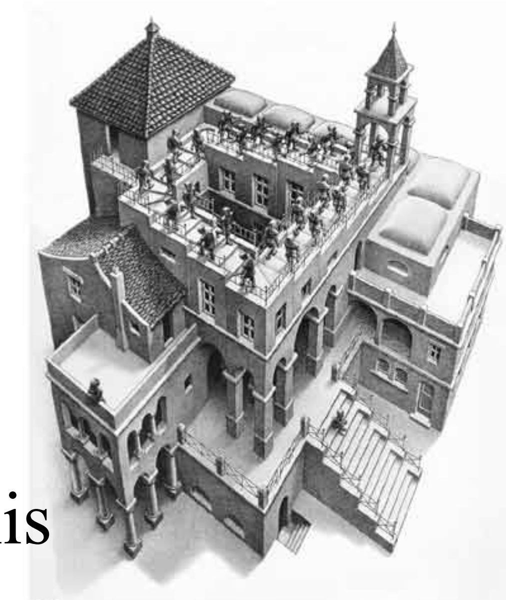
• Height z(x,y) is obtained by integration along a curve from (x_0, y_0).

$$z(x,y) = z(x_0, y_0) + \int_{(x_0,y_0)}^{(x,y)} (p\,dx + q\,dy)$$

• If one integrates the derivative field along any closed curve, on expects to get back to the starting value.

• Might not happen because of noisy estimates of (p,q)

What might go wrong?



Integrability. If f(x,y) is the height function, we expect that

$$\frac{\partial}{\partial y}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\frac{\partial f}{\partial y}$$

In terms of estimated gradient space (p,q), this means:

$$\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}$$

But since p and q were estimated indpendently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold

Horn’s Method

[“Robot Vision, B.K.P. Horn, 1986]

- Formulate estimation of surface height z(x,y) from gradient field by minimizing cost functional:

$$\iint_{\text{Image}} (z_x - p)^2 + (z_y - q)^2 \, dxdy$$

where (p,q) are estimated components of the gradient while z_x and z_y are partial derivatives of best fit surface

- Solved using calculus of variations – iterative updating
- z(x,y) can be discrete or represented in terms of basis functions.
- Integrability is naturally satisfied.

What if the BRDF unknown

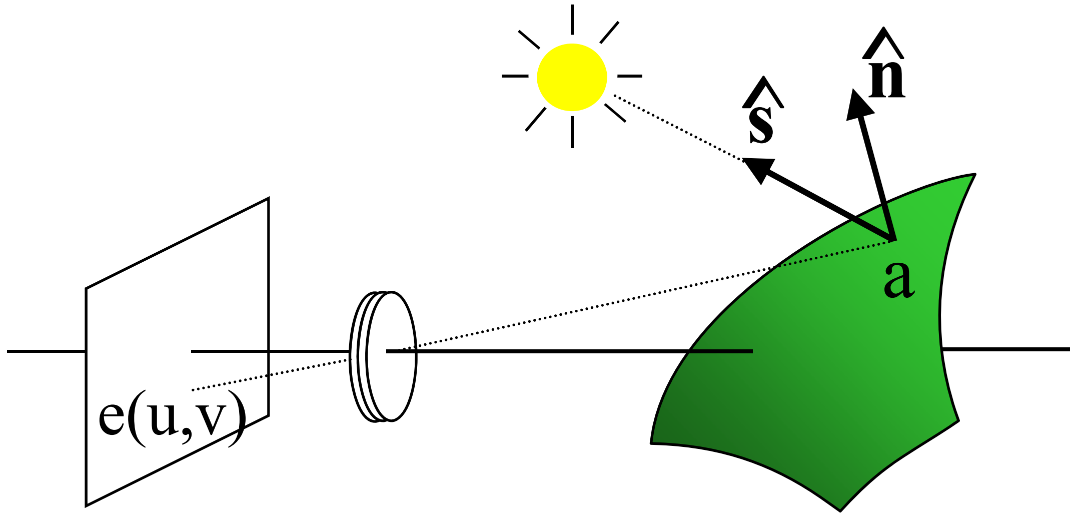
Simultaneous recovery of shape and spatially varying reflectance of a surface from photometric stereo images

Input image Projected 3D reconstruction with texture

[Alldrin, Zickler, Kriegman, "Photometric Stereo With Non-Parametric and Spatially-Varying Reflectance", CVPR 2008]
See also [Santo, Samejima, Sugano, Shi, Matsushita, "Deep Photometric Stereo Network," ICCV 2017]

2. Photometeric Stereo: Lambertian Surface, Known Lighting

Lambertian Surface



At image location (u,v), the intensity of a pixel x(u,v) is:

$$e(u,v) = [a(u,v)\,\hat{\mathbf{n}}(u,v)] \cdot [s_0 \hat{\mathbf{s}}]$$
$$= \mathbf{b}(u,v) \cdot \mathbf{s}$$

where

- a(u,v) is the albedo of the surface projecting to (u,v).
- $\hat{\mathbf{n}}$(u,v) is the direction of the surface normal.
- s_0 is the light source intensity.
- $\hat{\mathbf{s}}$ is the direction to the light source.

Lambertian Photometric stereo

- If the light sources $\mathbf{s}_1$, $\mathbf{s}_2$, and $\mathbf{s}_3$ are known, then we can recover $\mathbf{b}$ from as few as three images. (Photometric Stereo: Silver 80, Woodham81).

$$[e_1 \ \ e_2 \ e_3\,] = \mathbf{b}^{\mathrm{T}}[\mathbf{s}_1 \ \ \mathbf{s}_2 \ \mathbf{s}_3\,]$$

- i.e., we measure e_1, e_2, and e_3 and we know $\mathbf{s}_1$, $\mathbf{s}_2$, and $\mathbf{s}_3$. We can then solve for $\mathbf{b}$ by solving a linear system.

$$\mathbf{b}^{\mathbf{T}} = [e_1 \quad e_2 \quad e_3][\mathbf{s_1} \quad \mathbf{s_2} \quad \mathbf{s_3}]^{-1}$$

- Normal $\hat{\mathbf{n}} = \mathbf{b}/|\mathbf{b}|$ and albedo a = $|\mathbf{b}|$

What if we have more than 3 Images? Linear Least Squares

$[e_1 \; e_2 \; e_3 \ldots e_n] =$
$\mathbf{b}^T[\mathbf{s}_1 \; \mathbf{s}_2 \; \mathbf{s}_3 \ldots \mathbf{s}_n]$

Rewrite as

$$\mathbf{e} = \mathbf{Sb}$$

where

- $\mathbf{e}$ is n by 1
- $\mathbf{b}$ is 3 by 1
- $\mathbf{S}$ is n by 3

Solving for b gives

$$\mathbf{b} = (\mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T\mathbf{e}$$

Let the residual be

$r = e - Sb$

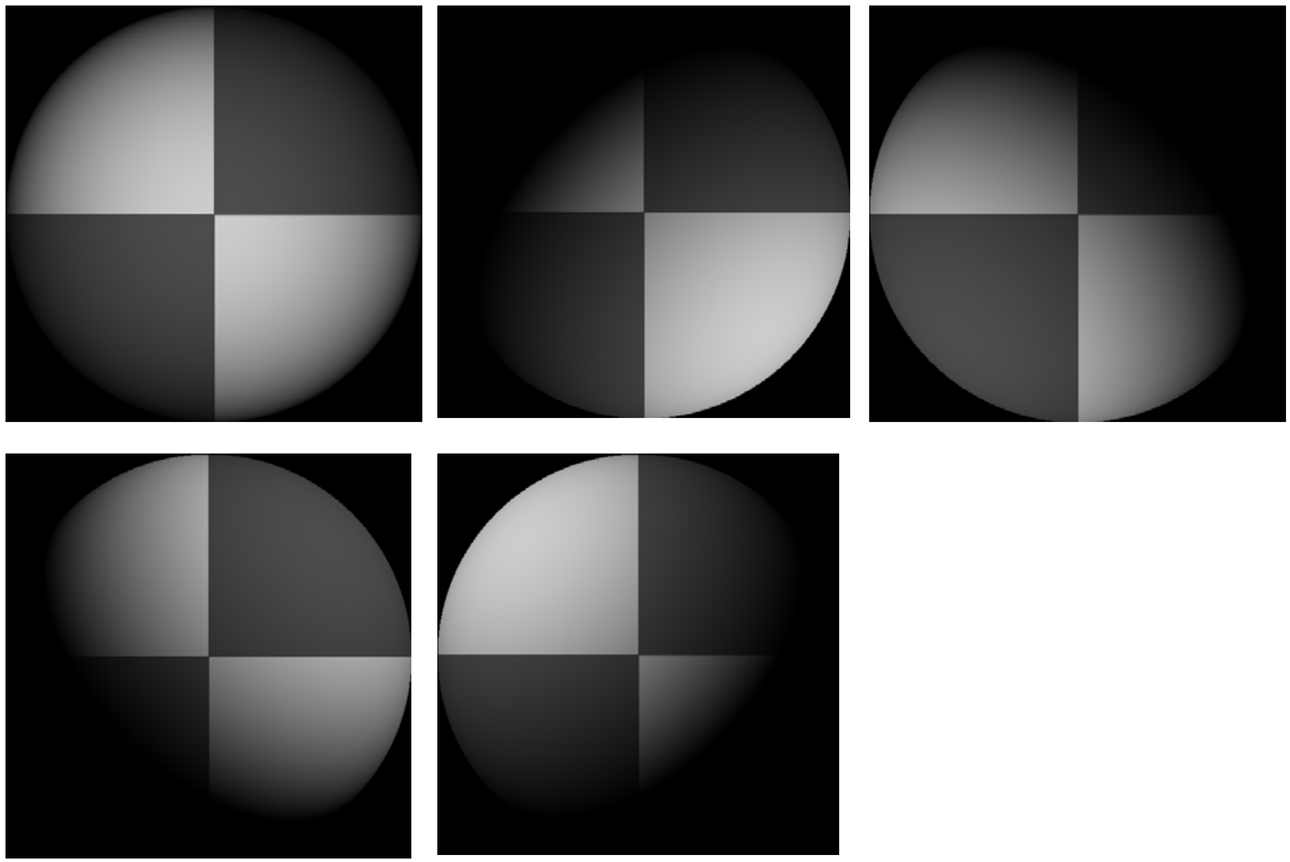
Squaring this:

$r^2 = r^T r = (e-Sb)^T (e-Sb)$
$= e^T e - 2b^T S^T e + b^T S^T S b$

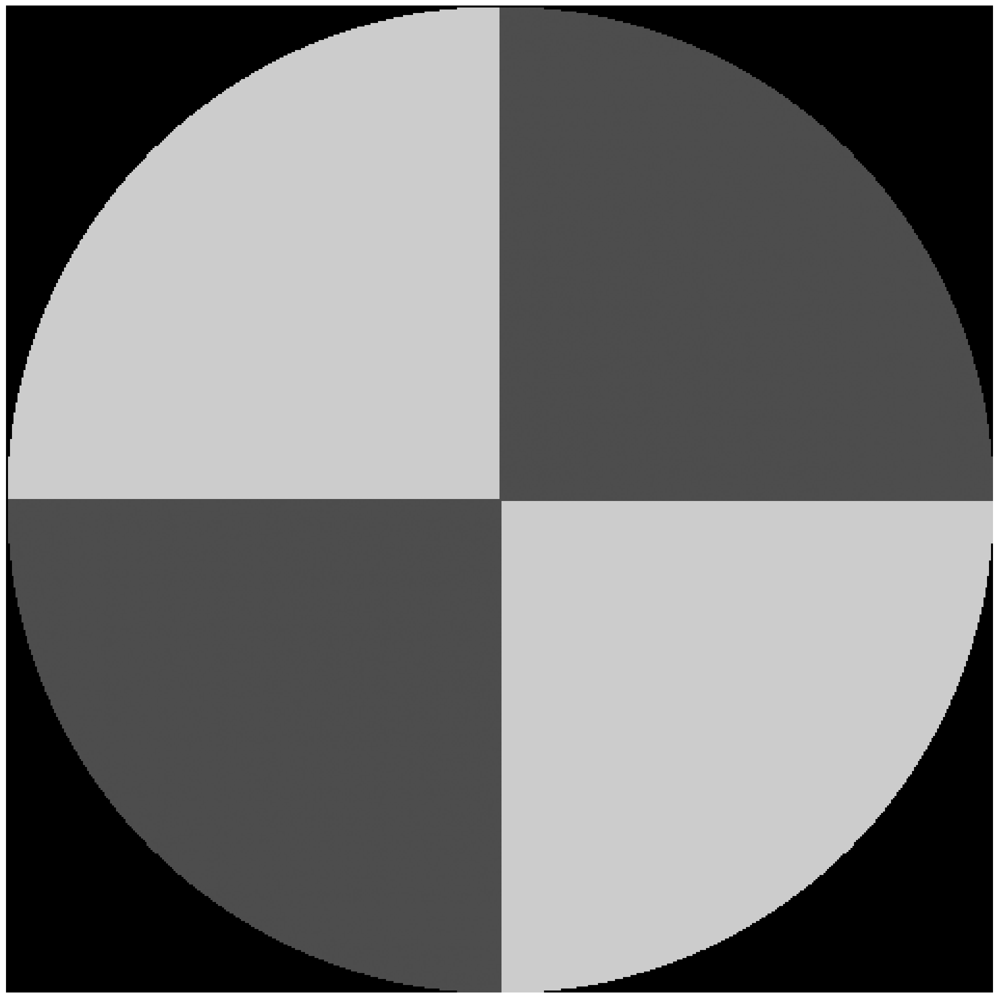
$(r^2)_b = 0$ - zero derivative is a necessary condition for a minimum, or

$-2S^T e + 2S^T S b = 0;$

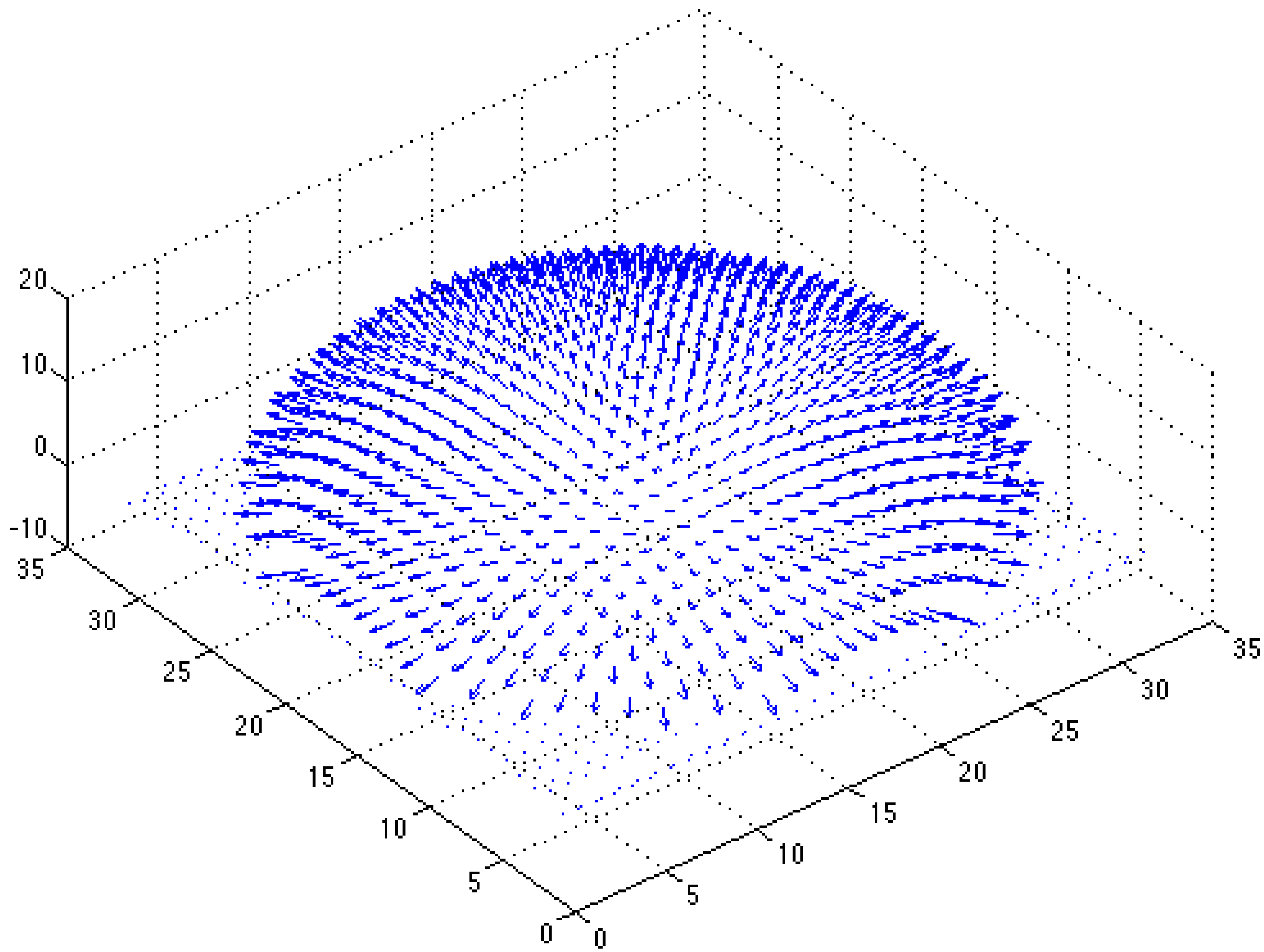
Input Images



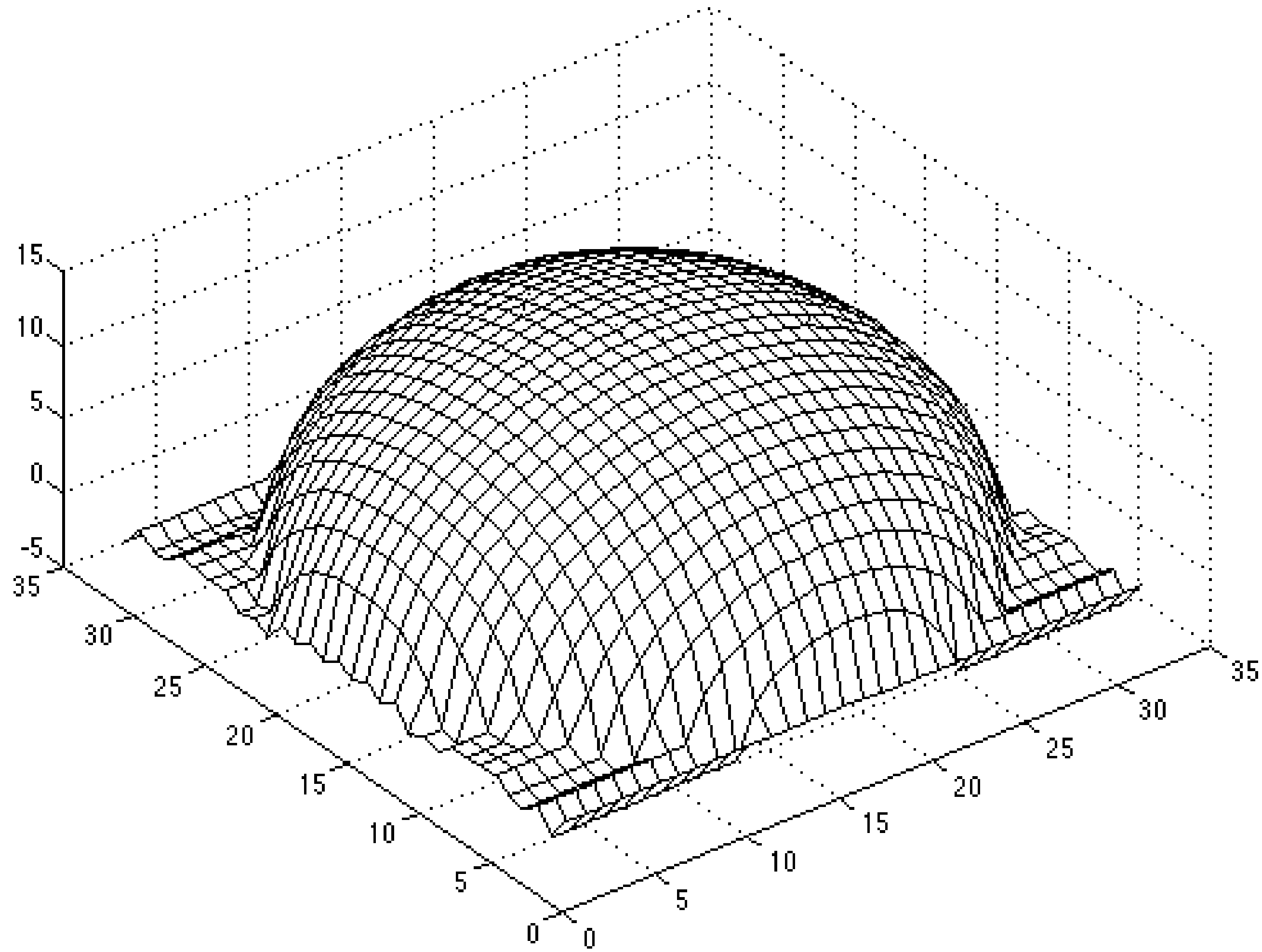
Recovered albedo



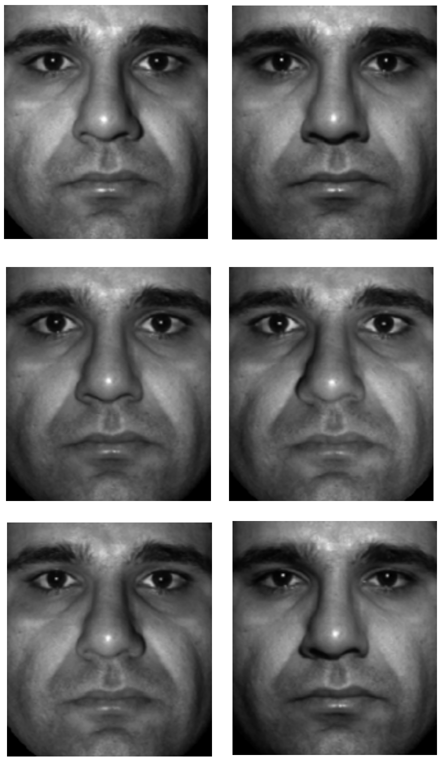
Recovered normal field



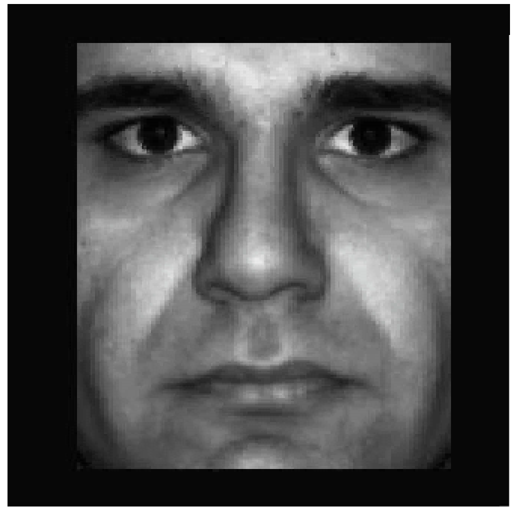
Surface recovered by integration



An example of photometric stereo



Images with known associated light sources



Albedo

Surface (from normals)

Surface
(albedo texture map)

Next Lecture

- Illumination cones
  - Photometric Stereo with unknown lighting and Lambertian surfaces
- Reading:
  - What Is the Set of Images of an Object under All Possible Illumination Conditions?