

# Image Formation: Light and Shading

Computer Vision I

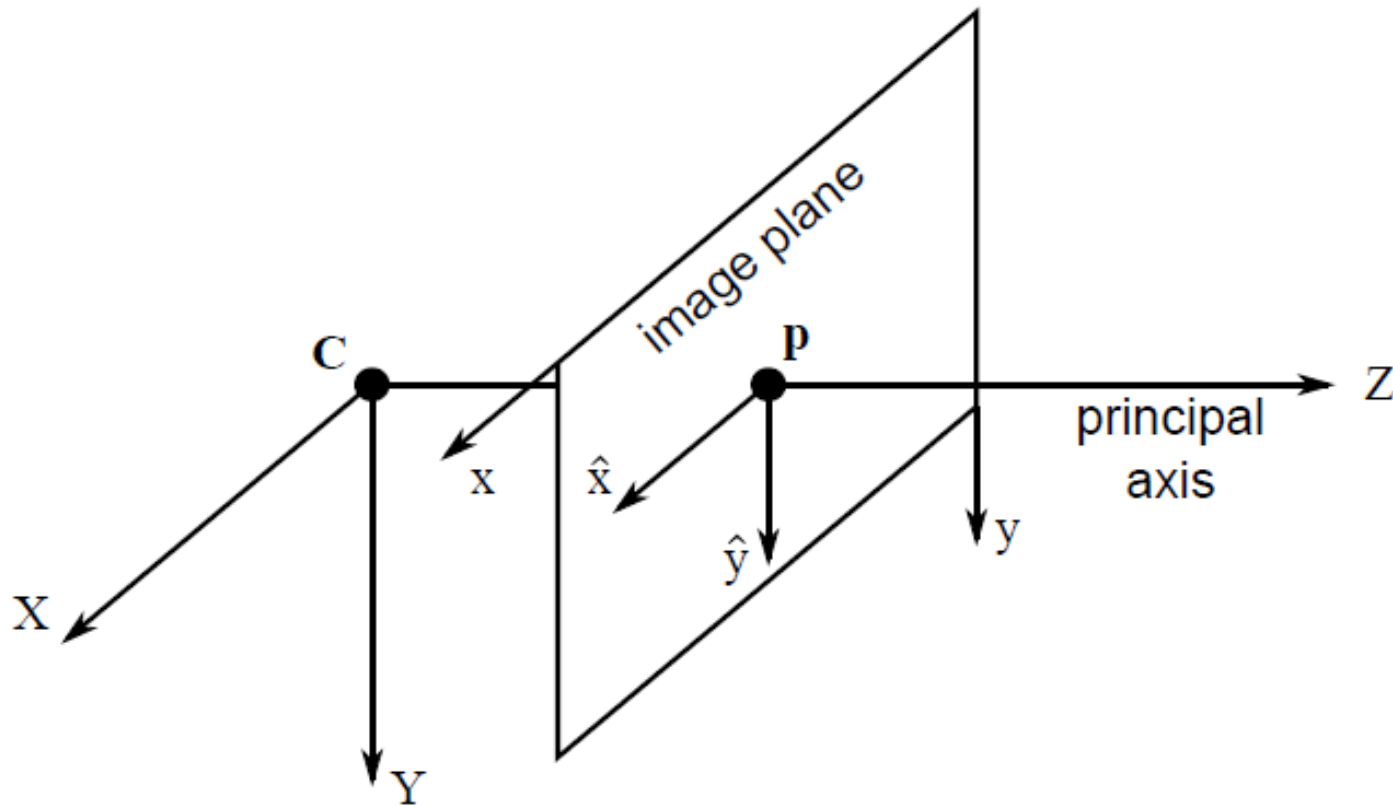
CSE 252A

Lecture 3

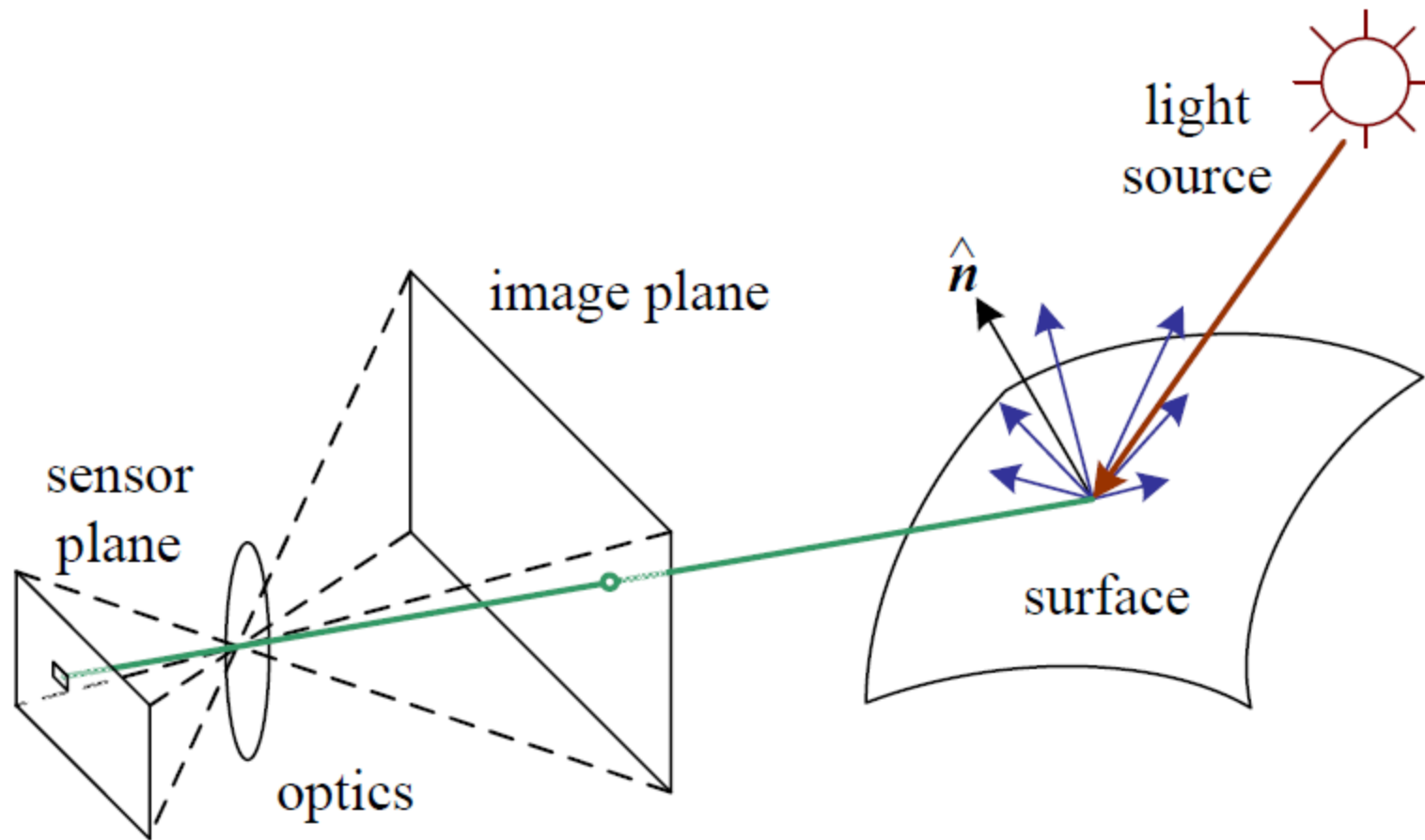
# Announcements

- Homework 1 is due Oct 8, 11:59 PM
- Homework 2 will be assigned on Oct 8
- Reading:
  - Chapter 2: Light and Shading

# Geometric image formation



# Photometric image formation



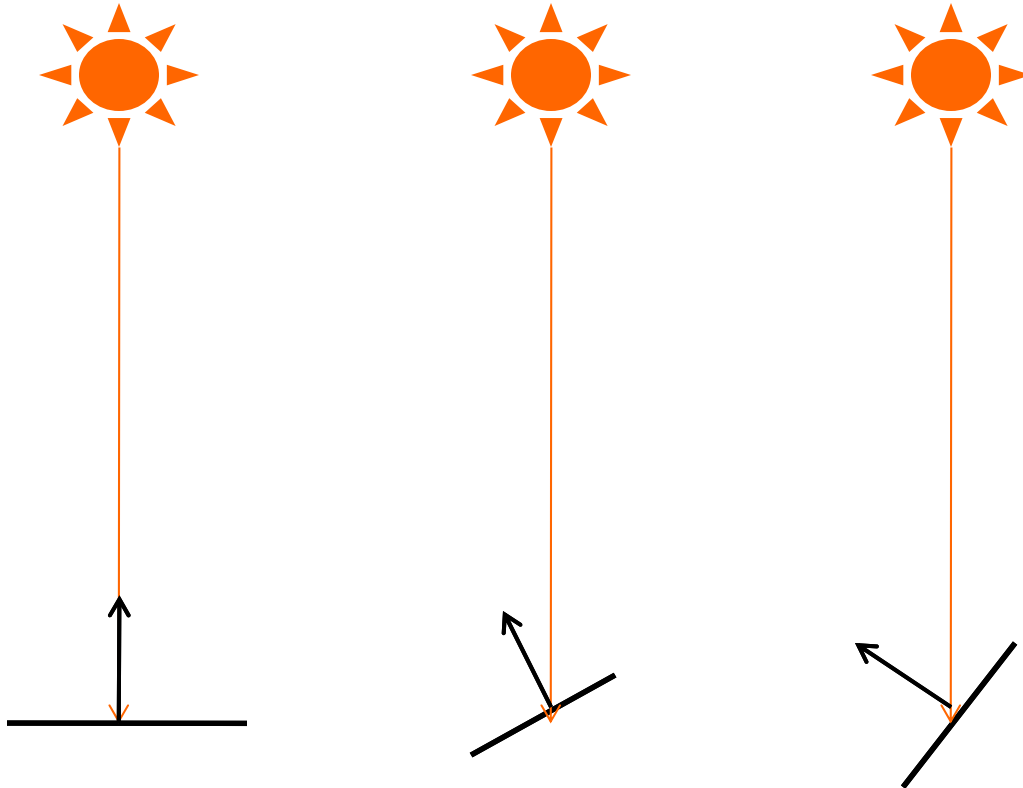
# Radiometry

- Solid Angle
- Irradiance
- Radiance
- Bidirectional Reflectance Distribution Function (BRDF)

# Appearance: lighting, surface reflectance, and shading



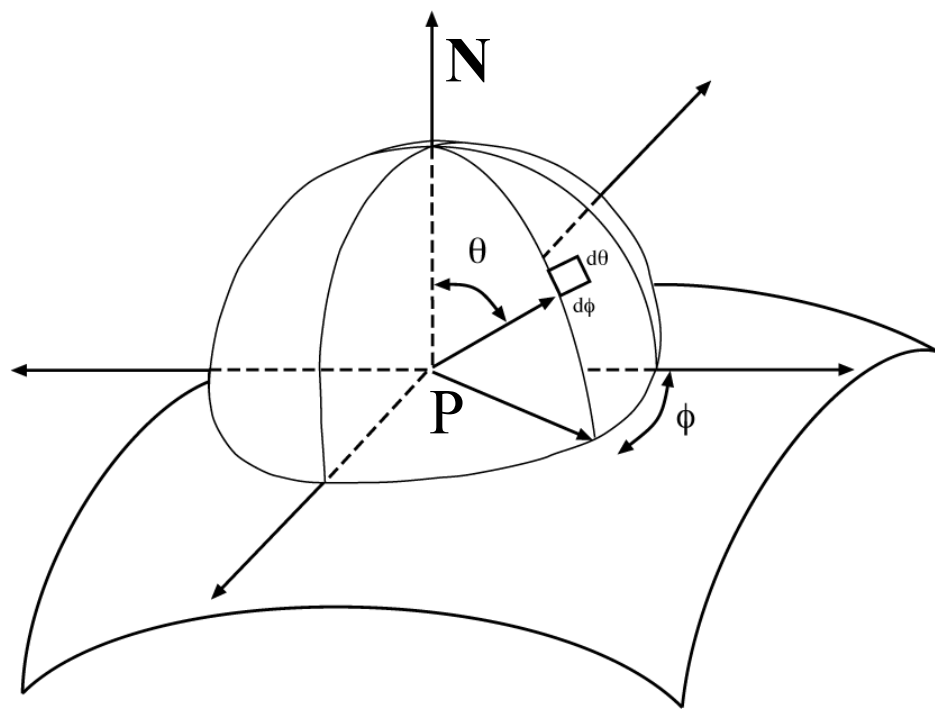
# Foreshortening



–The surface is foreshortened by the cosine of the angle between the normal and direction to the light.

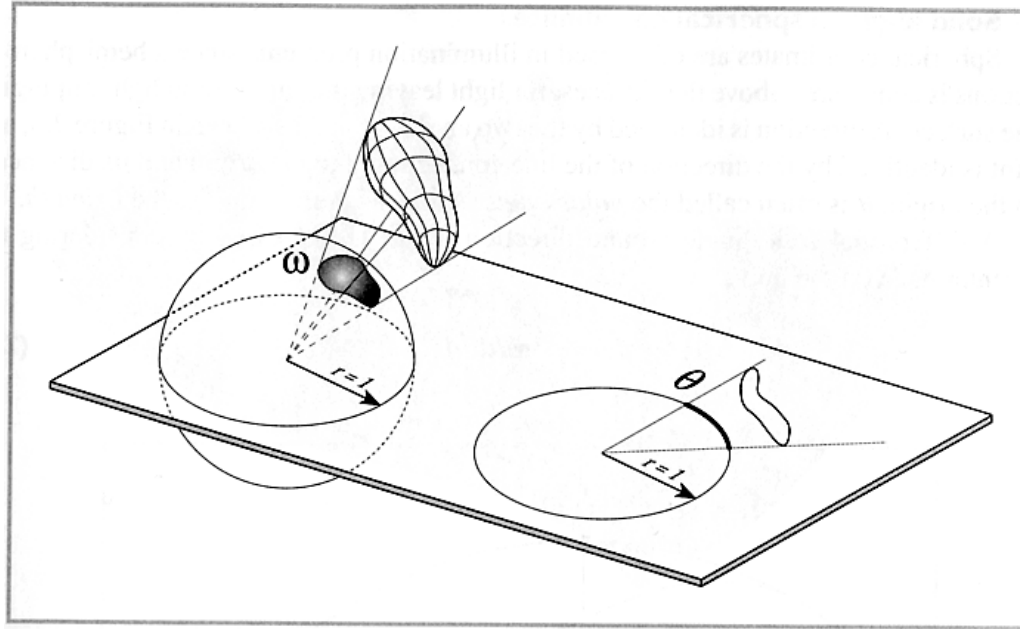
# A local coordinate system on a surface

- Consider a point  $\mathbf{P}$  on the surface
- Light arrives at  $\mathbf{P}$  from a hemisphere of directions defined by the surface normal  $\mathbf{N}$
- We can define a local coordinate system whose origin is  $\mathbf{P}$  and with one axis aligned with  $\mathbf{N}$
- Convenient to represent in spherical angles.





# Measuring Angle

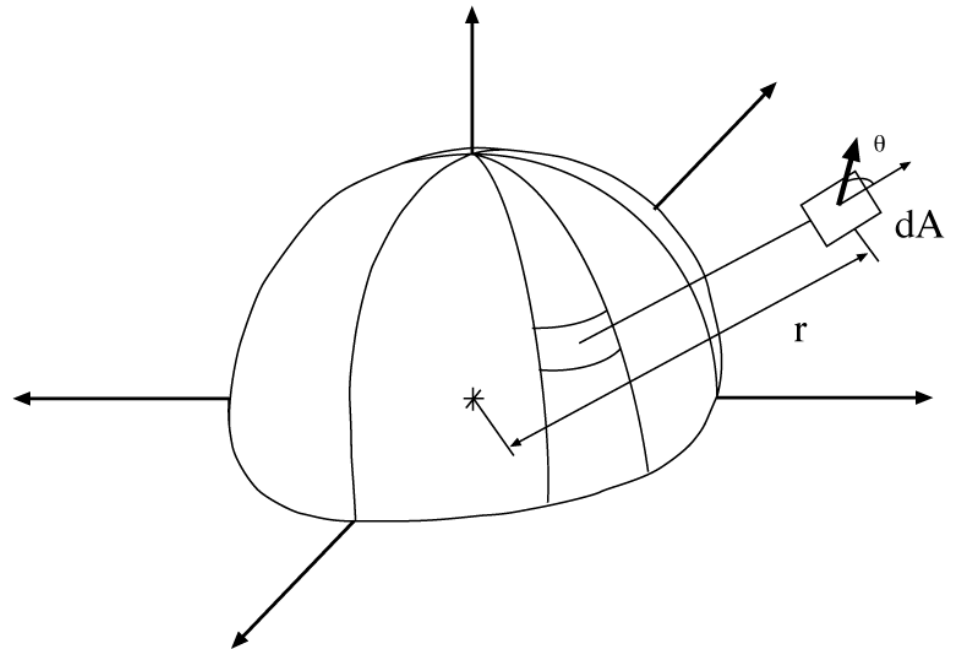


- The **solid angle** subtended by an object from a point P is the area of the projection of the object onto the unit sphere centered at P.
- Definition is analogous to projected angle in 2D
- Measured in *steradians*, sr
- If I'm at P, and I look out, solid angle tells me how much of my view is filled with an object

# Solid Angle

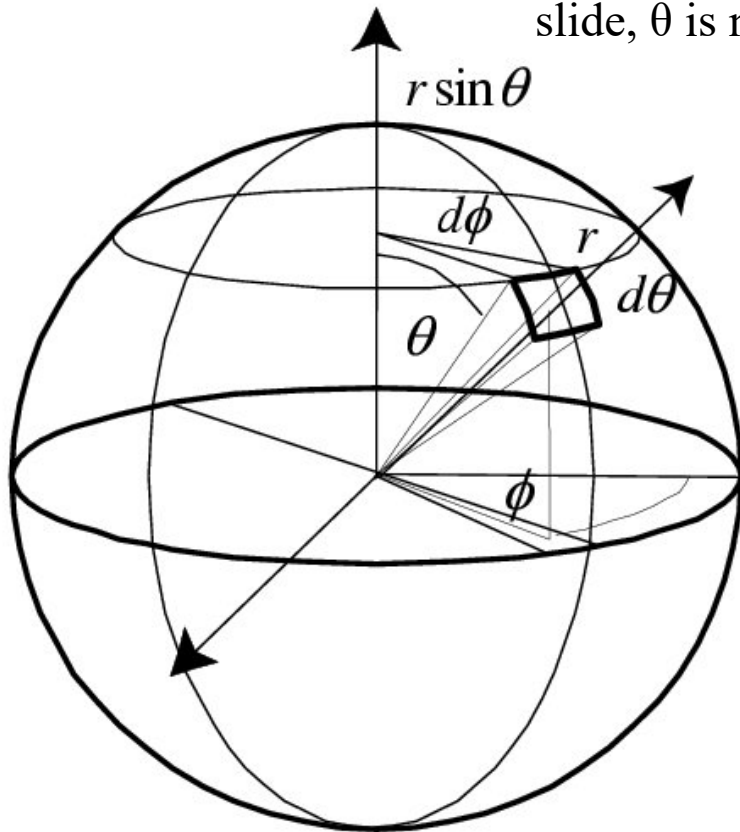
- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point
- The solid angle subtended by a patch area  $dA$  is given by

$$d\omega = \frac{dA \cos \theta}{r^2}$$



# Differential Solid Angles

Note:  $(\theta, \phi)$  are spherical angles in this slide,  $\theta$  is not foreshortening.



$$\begin{aligned}dA &= (r d\theta)(r \sin \theta d\phi) \\ &= r^2 \sin \theta d\theta d\phi\end{aligned}$$

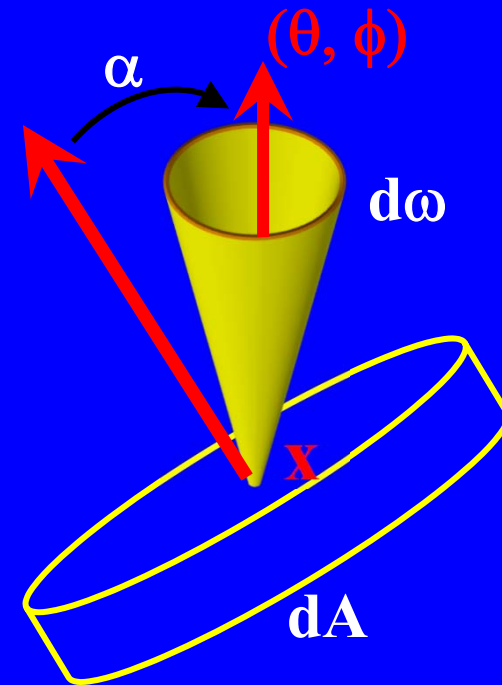
Differential solid angle when representing point on sphere in spherical coordinates

$$d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi$$

$$S = \int_0^\pi \int_0^{2\pi} \sin \theta d\theta d\phi = 4\pi$$

# Radiance

- Power is energy per unit time (watts)
- **Radiance:** Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle
- Symbol:  $L(\mathbf{x}, \theta, \phi)$
- Units: watts per square meter per steradian :  $\text{W}/\text{m}^2/\text{sr} = \text{W m}^{-2} \text{sr}^{-1}$



$$L = \frac{P}{(dA \cos \alpha) d\omega}$$

Power emitted from patch, but radiance in direction different from surface normal

# Irradiance

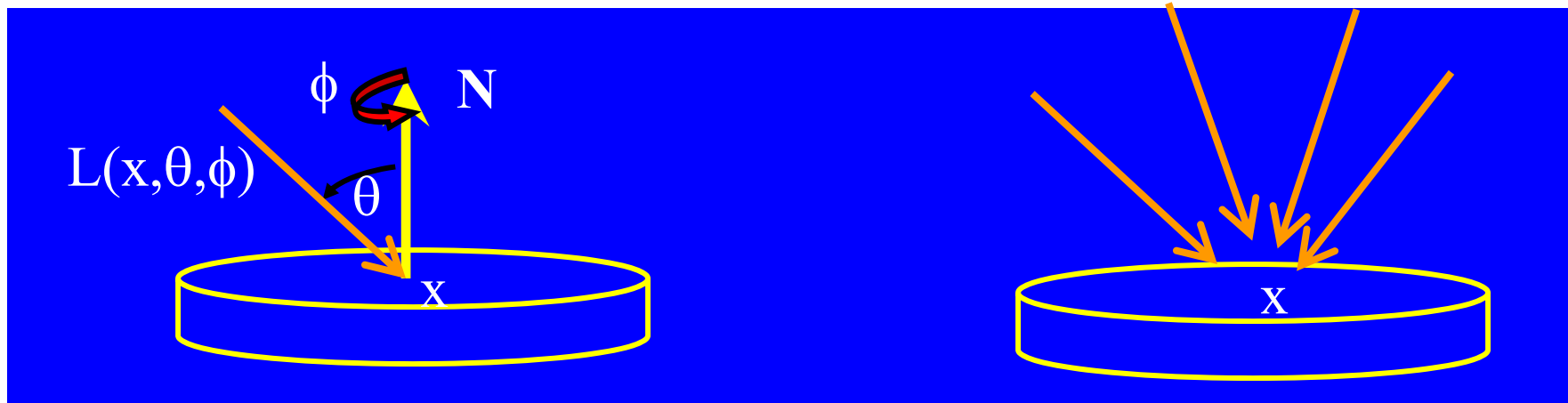
- How much light is arriving at a surface?
- Units of irradiance:  $\text{W}/\text{m}^2 = \text{W m}^{-2}$
- This is a function of incoming angle.
- A surface experiencing radiance  $L(\underline{x}, \theta, \phi)$  coming in from solid angle  $d\omega$  experiences **irradiance**:

$$E(\underline{x}) = \int_{\text{hemisphere}} L(\underline{x}, \theta, \phi) \cos \theta d\omega$$

- Crucial property:  
Total **Irradiance** arriving at the surface is given by adding irradiance over all incoming angles  
Total irradiance is

$$\int_{\text{hemisphere}} L(\underline{x}, \theta, \phi) \cos \theta d\omega$$

$$= \int_0^{2\pi} \int_0^{\pi/2} L(\underline{x}, \theta, \phi) \cos \theta \sin \theta d\theta d\phi$$



# Camera sensor

- Measured pixel intensity is a function of irradiance integrated over
  - Pixel's area
  - over a range of wavelengths
  - for some period of time



$$I = \int_t \int_\lambda \int_x \int_y E(x, y, \lambda, t) s(x, y) q(\lambda) dx dy d\lambda dt$$

- Ideally, it's linear to the radiance, but the camera response  $C(.)$  may not be linear

$$I = R \left( \int_t \int_\lambda \int_x \int_y E(x, y, \lambda, t) s(x, y) q(\lambda) dx dy d\lambda dt \right)$$

# Color Cameras



We consider 3 concepts:

1. Prism (with 3 sensors)
2. Filter mosaic
3. Filter wheel

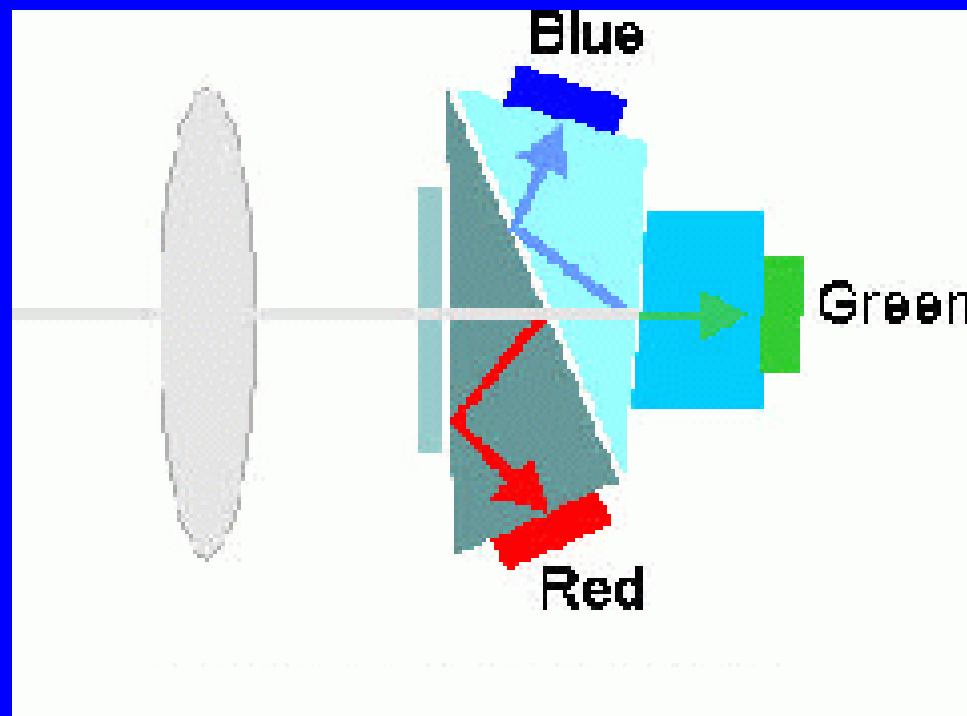
... and X3

# Prism color camera

Separate light in 3 beams using dichroic prism

Requires 3 sensors & precise alignment

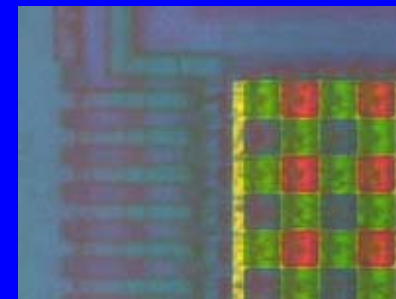
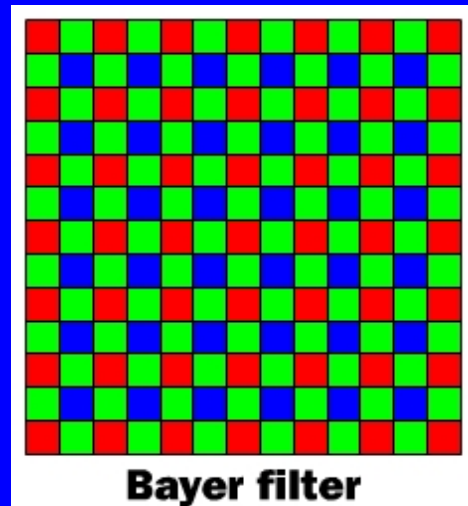
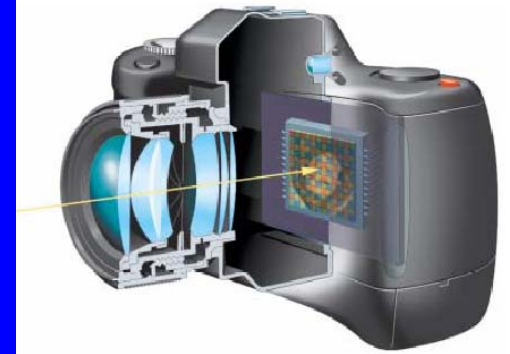
Good color separation



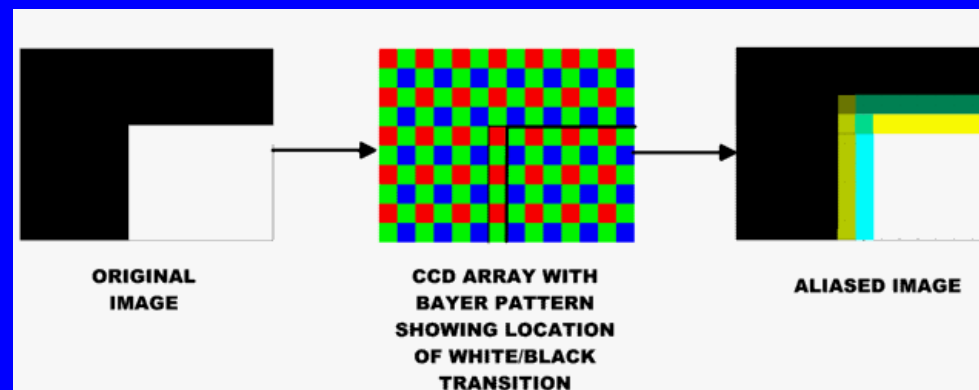


# Filter mosaic

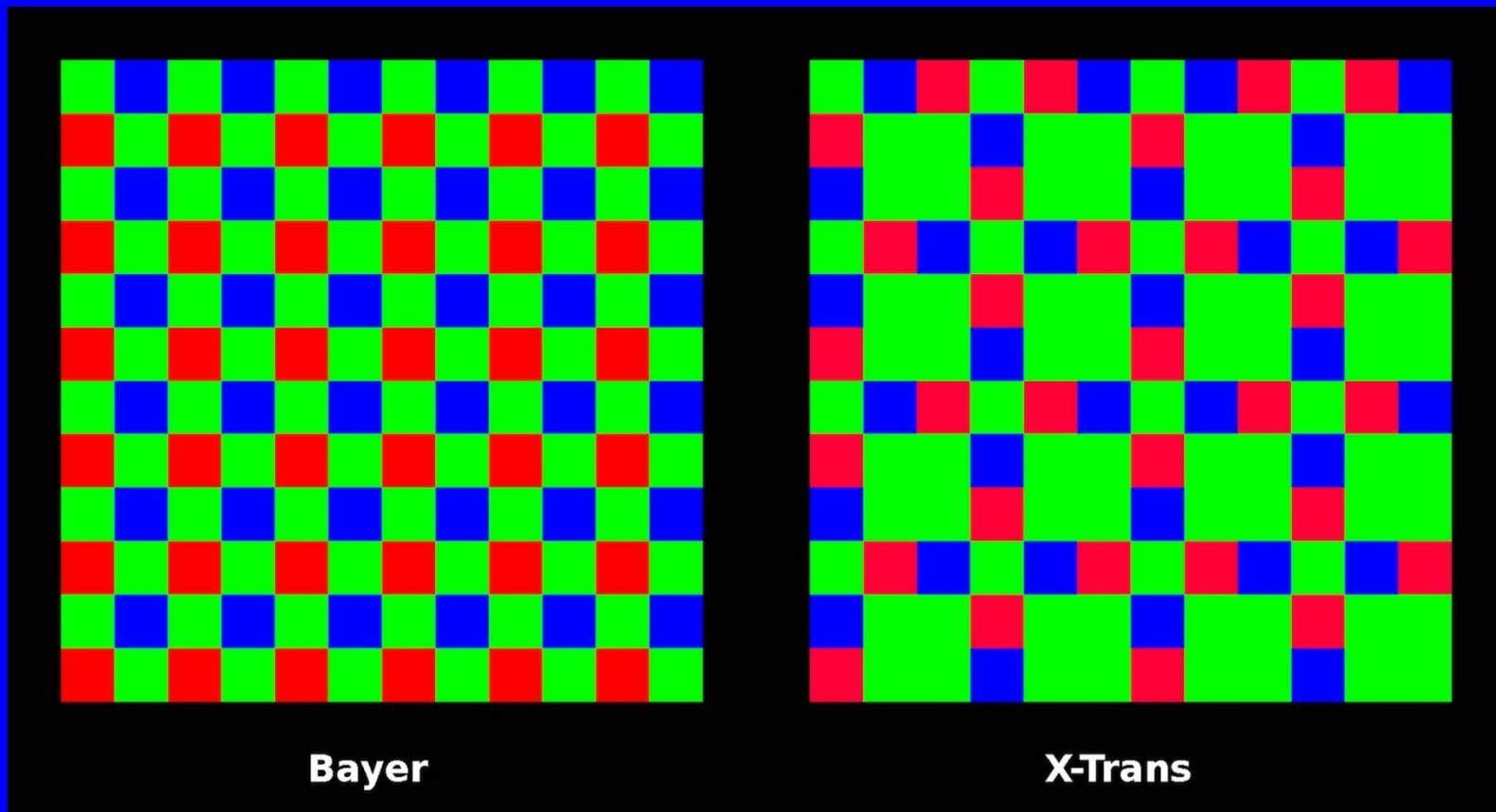
Coat filter directly on sensor



Demosaicing (obtain full color & full resolution image)



# FujiFilm X-Trans

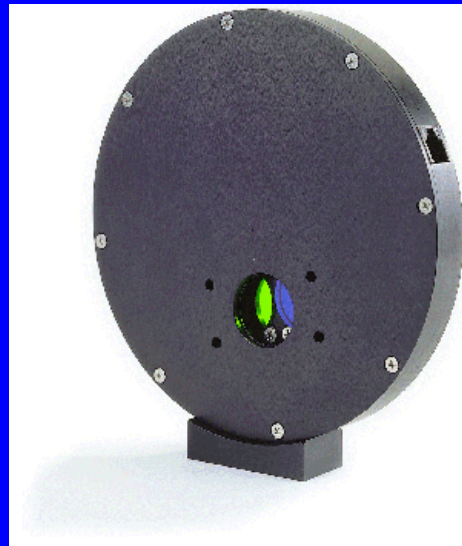


And rumor mill says that Canon may come out with a 120MP sensor with X3-like technology in 2018.

# Filter wheel

Rotate multiple filters in front of lens

Allows more than 3 color bands



Only suitable for static scenes

# Prism vs. mosaic vs. wheel

## approach

# sensors

Separation

Cost

Framerate

Artifacts

Bands

## Prism

3

High

High

High

Low

3

High-end  
cameras

## Mosaic

1

Average

Low

High

Aliasing

3

Low-end  
cameras

## Wheel

1

Good

Average

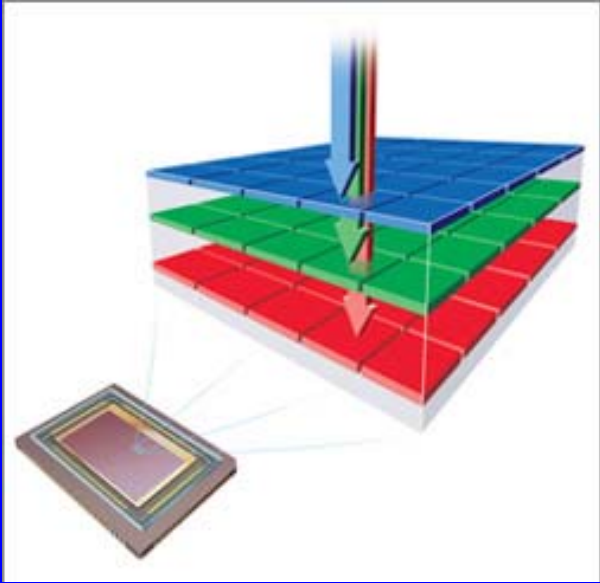
Low

Motion

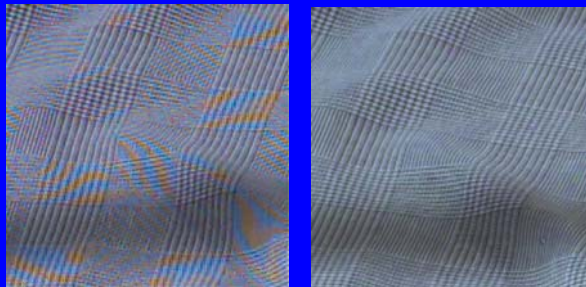
3 or more

Scientific  
applications

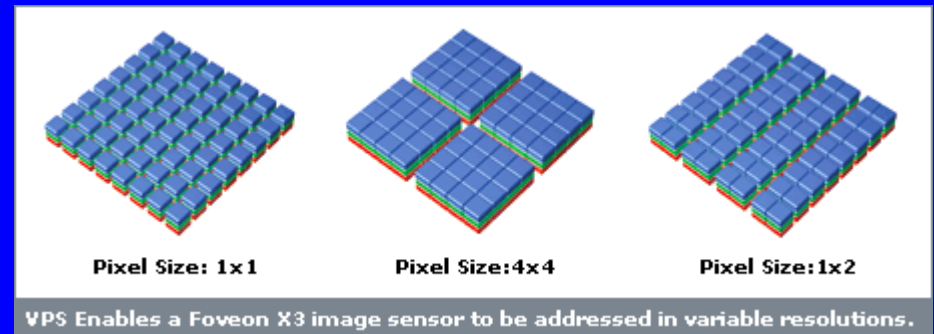
# “newer” color CMOS sensor Foveon’s X3 – Sigma, Fujifilm



better image quality



smarter pixels



Canon is rumored to be coming out with a 120MP sensor with similar technology

# Light at surfaces

Many effects when light strikes a surface -- could be:

- transmitted
  - Skin, glass
- reflected
  - mirror
- scattered
  - milk
- travel along the surface and leave at some other point
- absorbed
  - sweaty skin

## **Assume that**

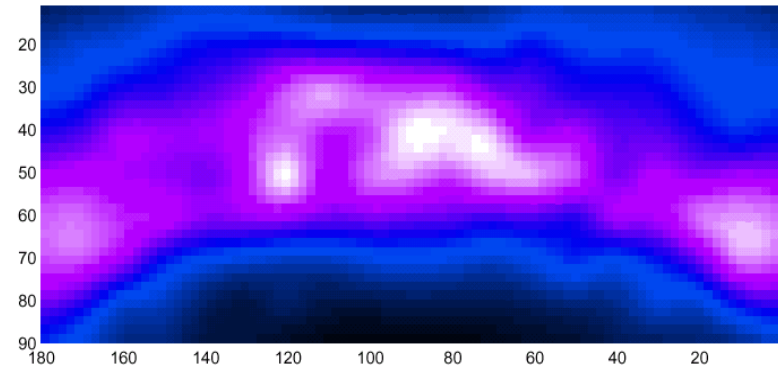
- surfaces don't fluoresce
  - e.g., scorpions, detergents
- surfaces don't emit light (i.e., are cool)
- all the light leaving a point is due to that arriving at that point

# Surface Reflectance Models

## Common Models

- Lambertian
- Phong
- Physics-based
  - Specular  
[Blinn 1977], [Cook-Torrance 1982], [Ward 1992]
  - Diffuse  
[Hanrahan, Kreuger 1993]
  - Generalized Lambertian  
[Oren, Nayar 1995]
  - Thoroughly Pitted Surfaces  
[Koenderink et al 1999]
- Phenomenological
  - [Koenderink, Van Doorn 1996]

## Arbitrary Reflectance

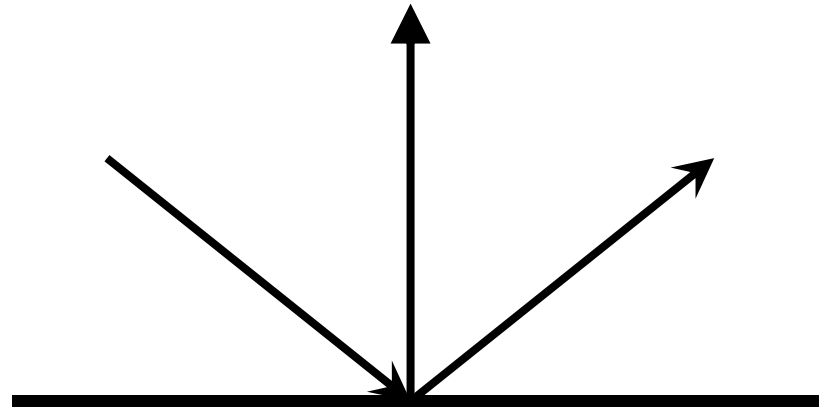


- Non-parametric model
- Anisotropic
- Non-uniform over surface
- BRDF Measurement [Dana et al, 1999], [Marschner]

## Specialized

- Hair, skin, threads, paper [Jensen et al]

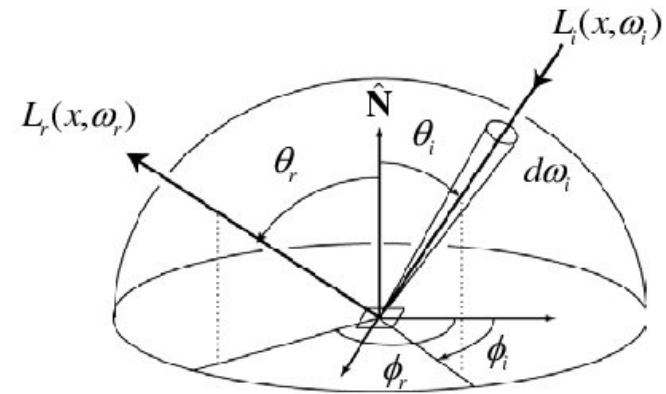
# Specular Reflection: Smooth Surface





# Lambertian (Diffuse) Surface

- BRDF is a constant called the albedo.  $\rho(\underline{x}; \theta_{in}, \phi_{in}; \theta_{out}, \phi_{out}) = K$
- Emitted radiance is NOT a function of outgoing direction – i.e., constant in all directions.
- For lighting coming in single direction  $\omega_i$ , emitted radiance is proportional to cosine of the angle between normal and light direction



$$L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i \rightarrow \omega_r) L_i(x, \omega_i) \cos \theta_i d\omega_i$$

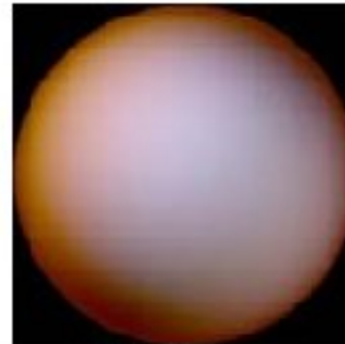
$$L_r = KN \cdot \omega_i$$

# Phong Model

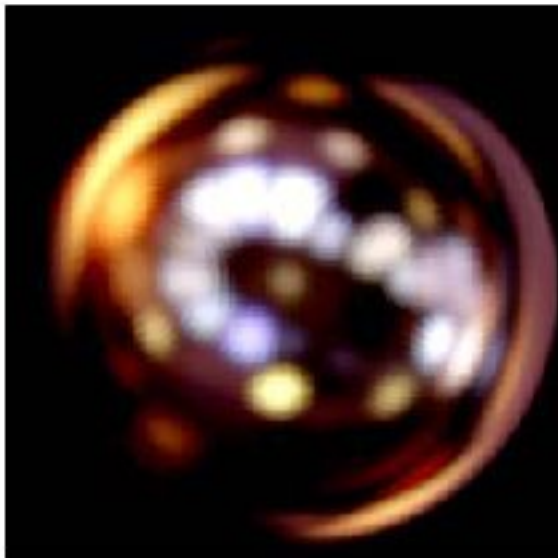
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**Mirror**



**Diffuse**



S

**CS348B Lecture 10**

**Pat Hanrahan, Spring 2002**

# Non-Lambertian reflectance



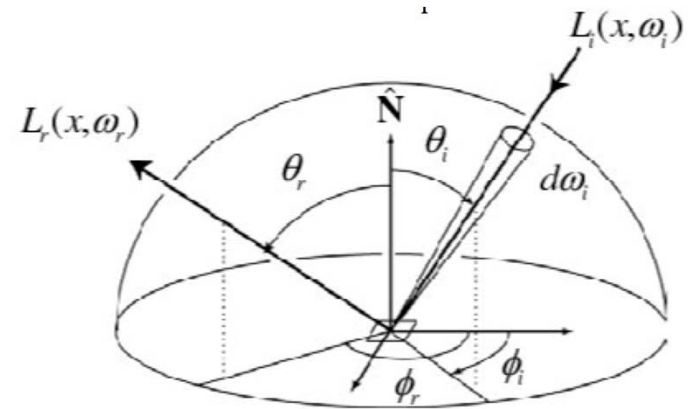
# General BRDF: e.g., Velvet



*Portrait of Sir Thomas Morre, Hans Holbein the Younger, 1527*

# Isotropic BRDF

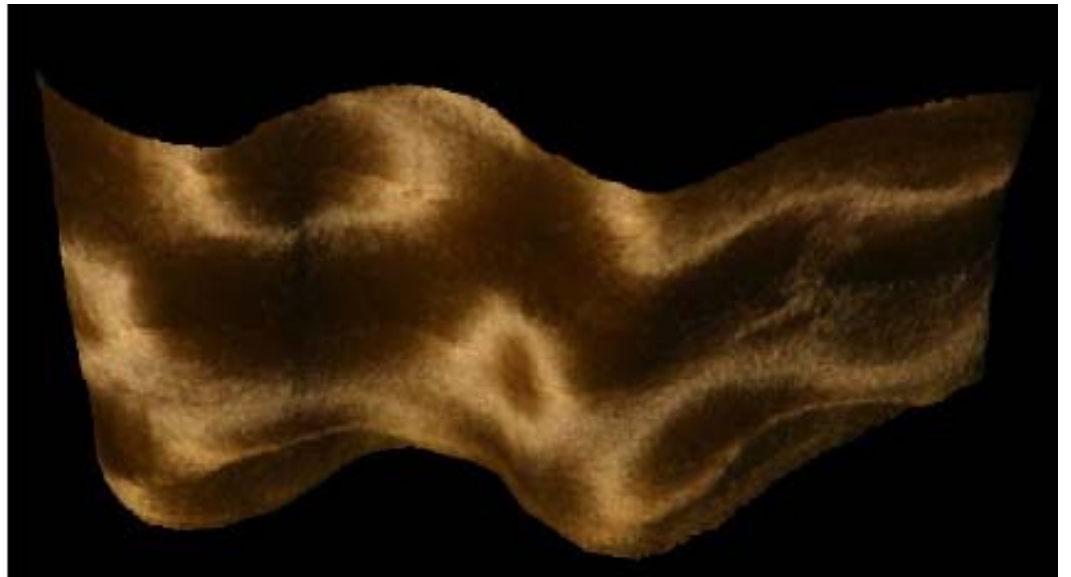
$$\rho(\theta_i, \phi_i; \theta_o, \phi_o) = \rho_r(\theta_i, \theta_o, \phi_i - \phi_o)$$



Isotropic BRDF's are symmetric about the surface normal. If the surface is rotated about the normal for the same incident and emitting directions, the value of the BRDF is the same.



# Anisotropic BRDF



# BRDF

With assumptions in previous slide

- **Bi-directional Reflectance Distribution Function**

$$\rho(\theta_{\text{in}}, \phi_{\text{in}}; \theta_{\text{out}}, \phi_{\text{out}})$$

- **Ratio of emitted radiance to incident irradiance (units:  $\text{sr}^{-1}$ )**

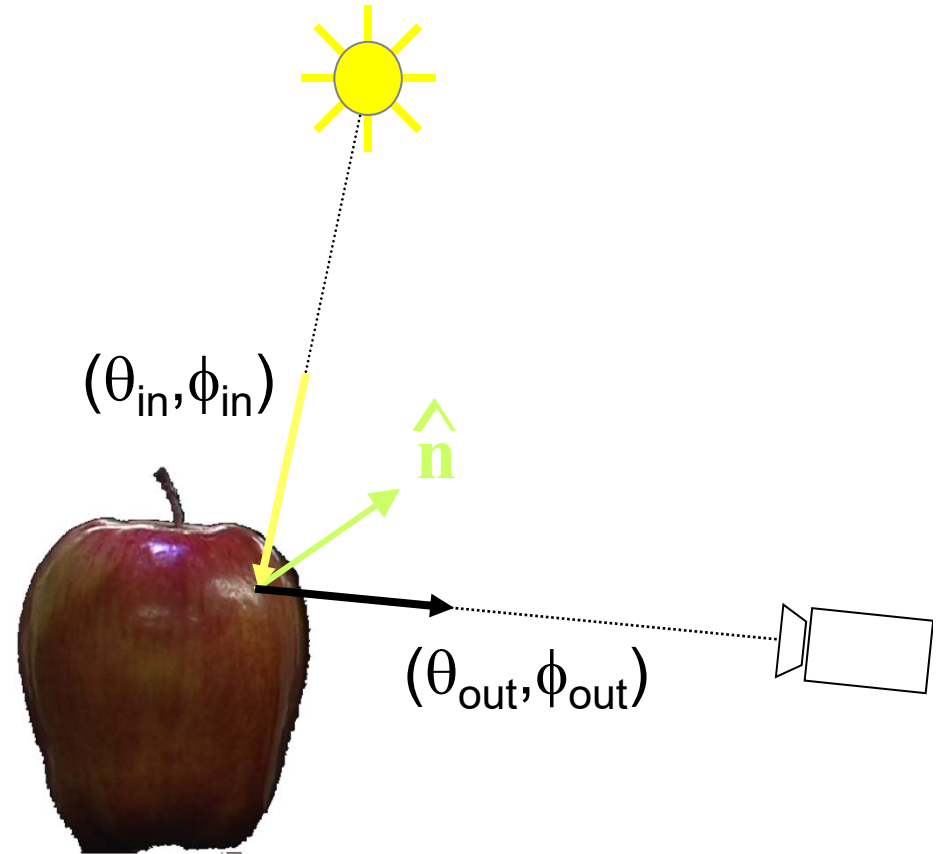
- **Function of**

- **Incoming light direction:**

$$\theta_{\text{in}}, \phi_{\text{in}}$$

- **Outgoing light direction:**

$$\theta_{\text{out}}, \phi_{\text{out}}$$



$$\rho(\underline{x}; \theta_{\text{in}}, \phi_{\text{in}}; \theta_{\text{out}}, \phi_{\text{out}}) = \frac{L_o(\underline{x}; \theta_{\text{out}}, \phi_{\text{out}})}{L_i(\underline{x}; \theta_{\text{in}}, \phi_{\text{in}}) \cos \theta_{\text{in}} d\omega}$$

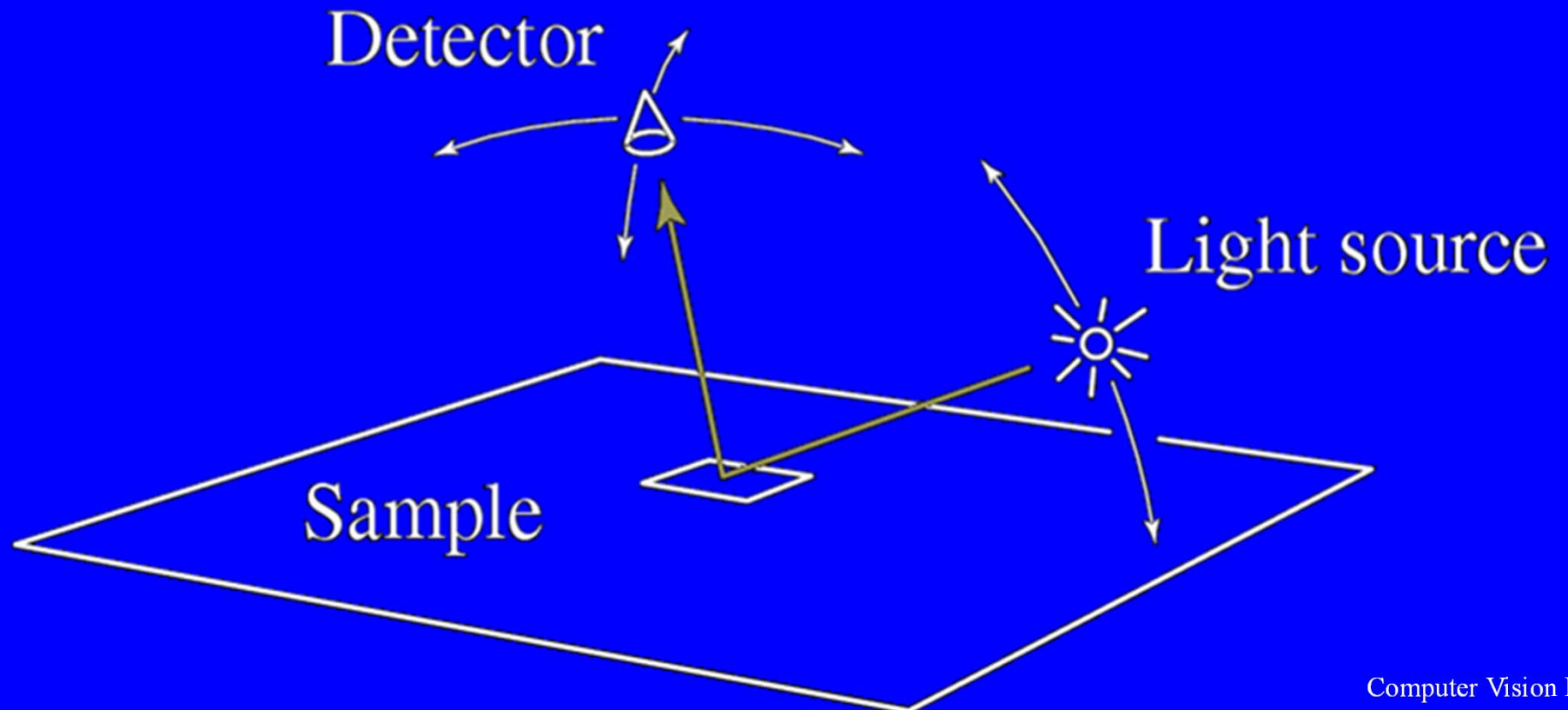
Where  $\rho$  is sometimes denoted  $f_r$ .

# Ways to measure BRDFs



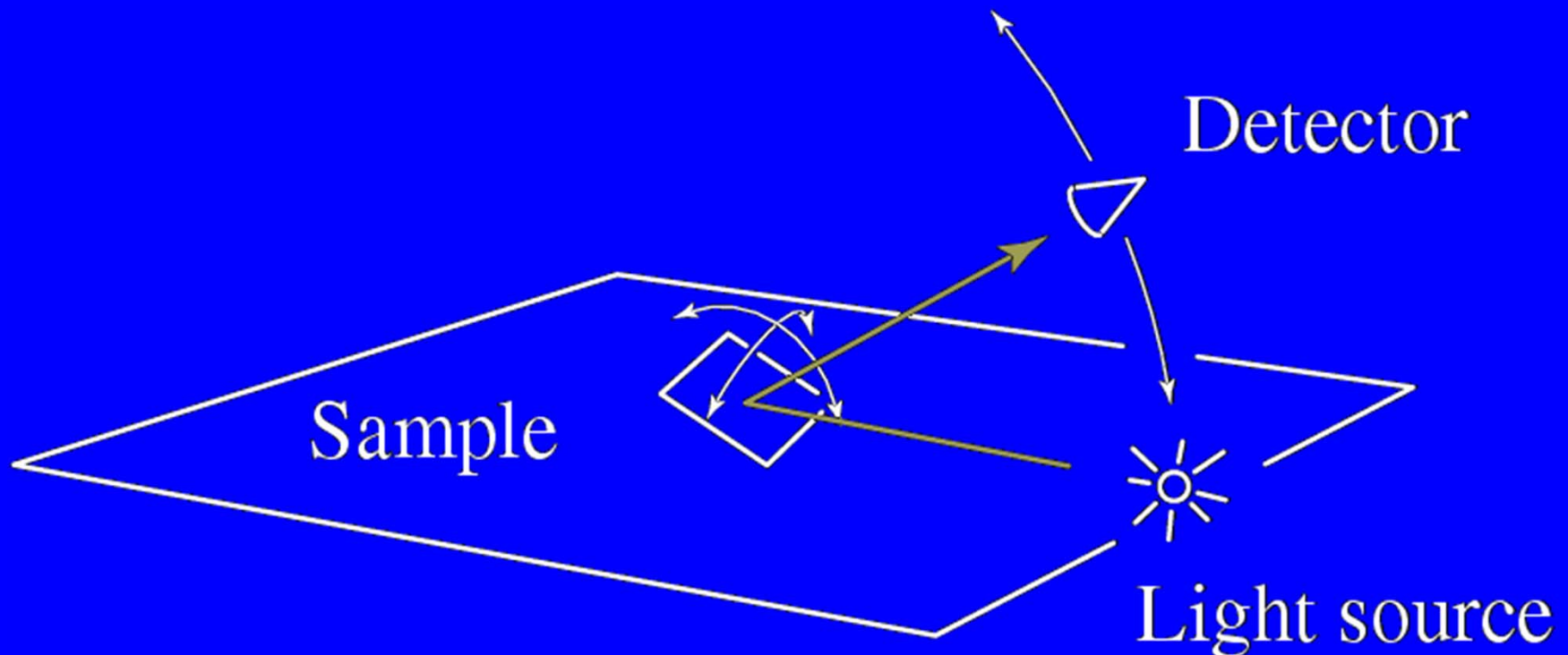
# Gonioreflectometers

- Three degrees of freedom spread among **light source**, **detector**, and/or sample



# Gonioreflectometers

- Three degrees of freedom spread among light source, **detector**, and/or **sample**



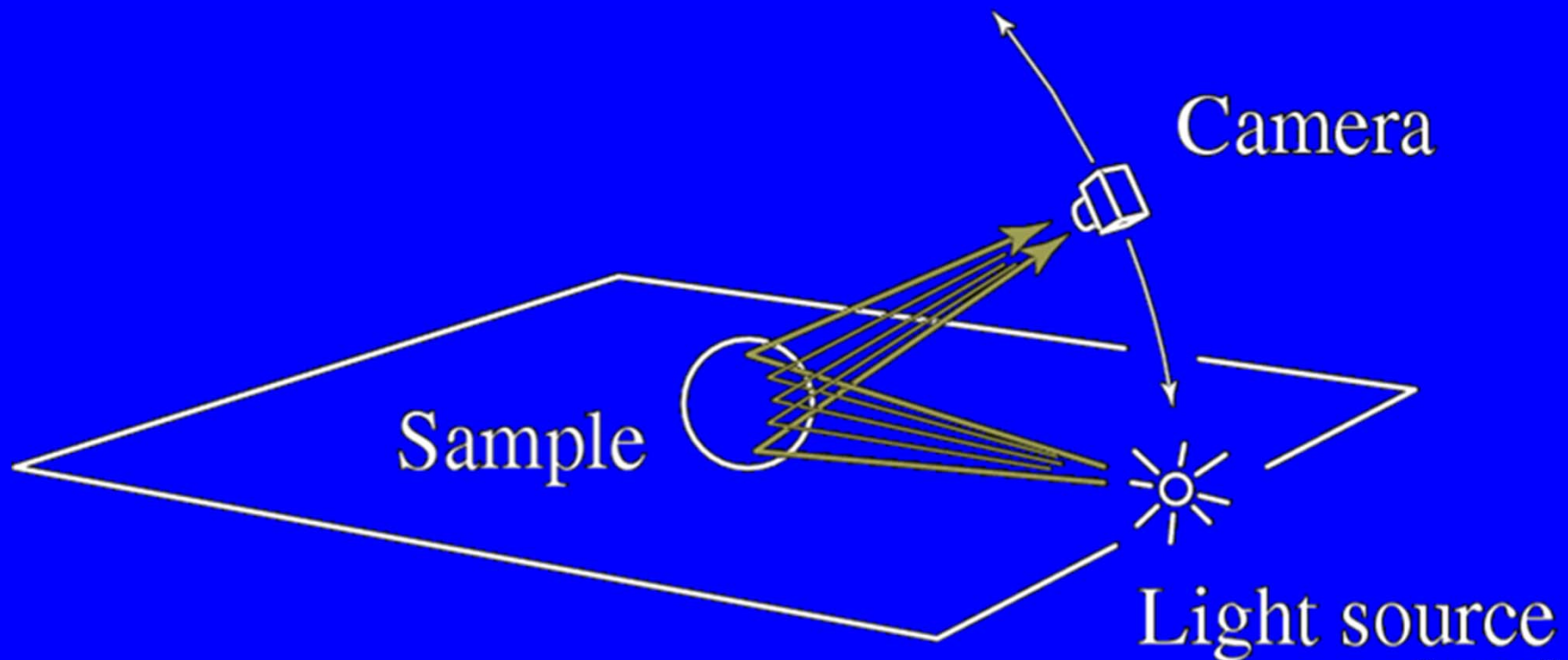
# Gonioreflectometers

- Can add fourth degree of freedom to measure anisotropic BRDFs



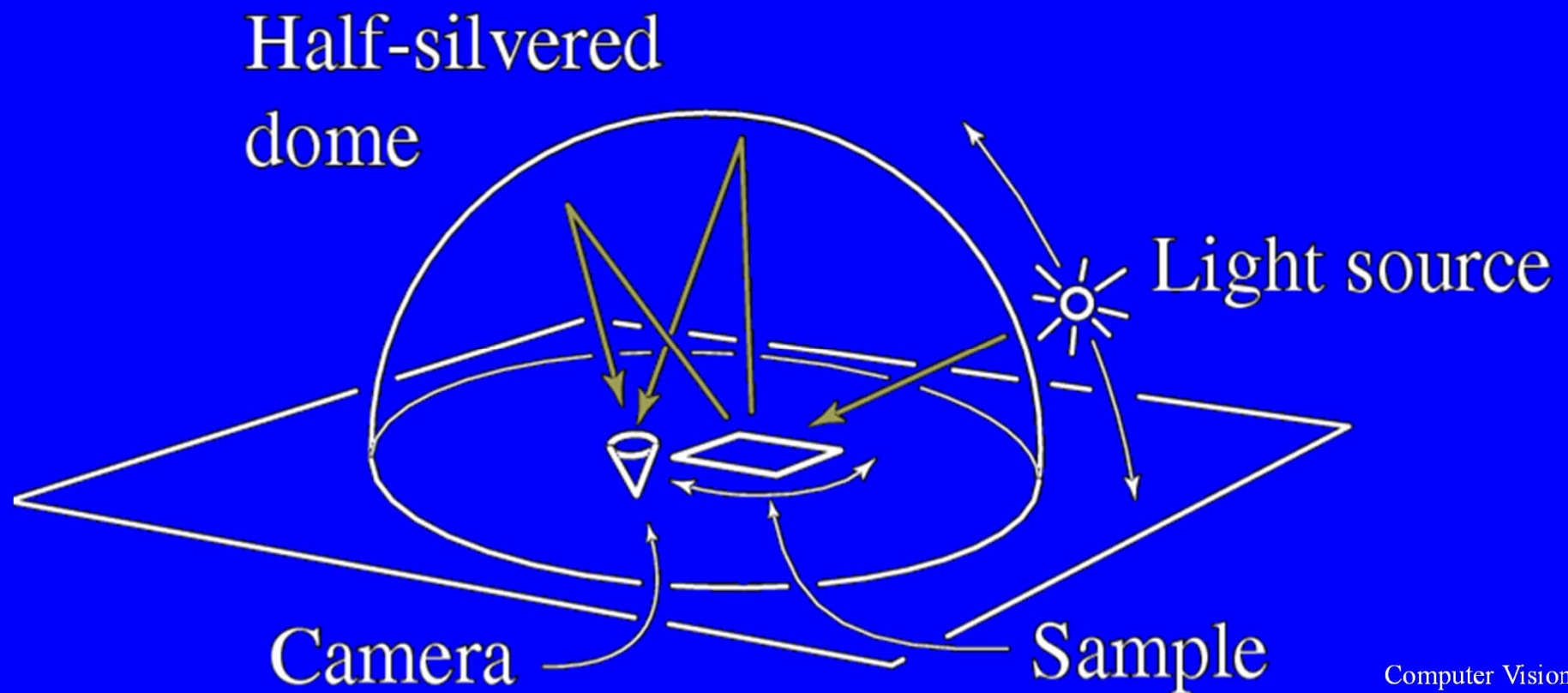
# Marschner's Image-Based BRDF Measurement

- For uniform BRDF, capture 2-D slice corresponding to variations in normals



# Ward's BRDF Measurement Setup

- Collect reflected light with hemispherical (should be ellipsoidal) mirror [SIGGRAPH 92]



# Ward's BRDF Measurement Setup

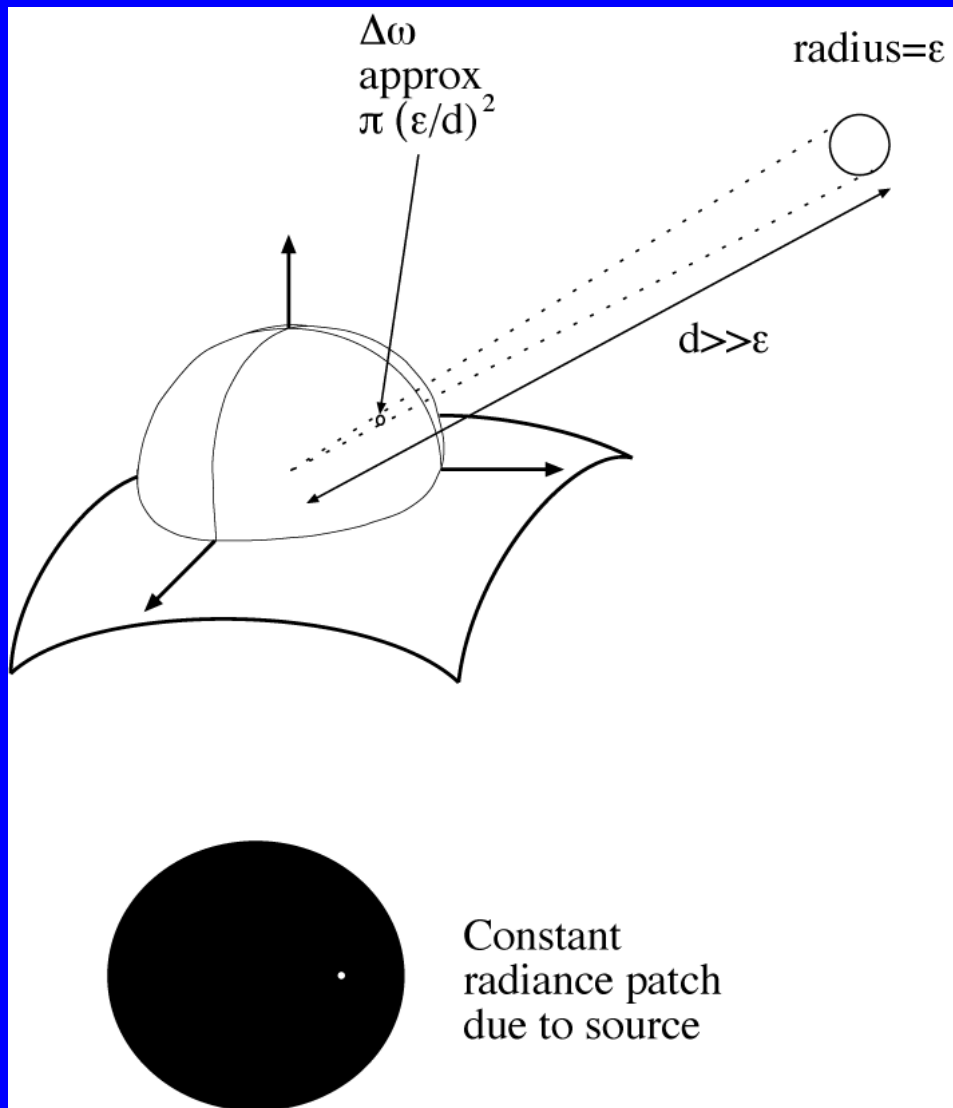
- Result: each image captures light at all exitant angles



# Light sources and shading

- How bright (or what color) are objects?
- One more definition: Exitance of a source is
  - the internally generated power radiated per unit area on the radiating surface
- Also referred to as radiant emittance
- Similar to irradiance
  - Same units,  $\text{W}/\text{m}^2 = \text{W m}^{-2}$

# Radiosity due to a point source



- small, distant sphere radius  $\epsilon$  and exitance  $E$ , which is far away subtends solid angle of about

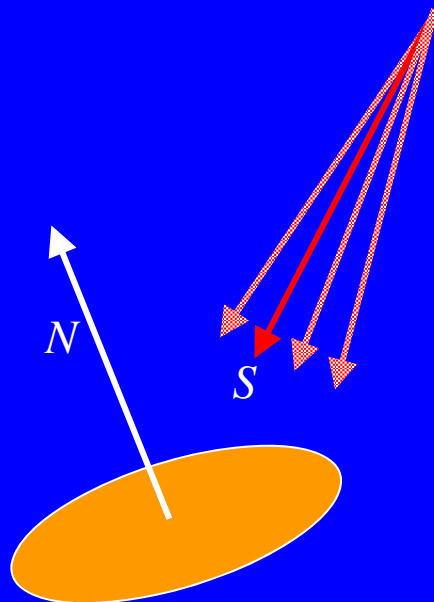
$$\pi \left( \frac{\epsilon}{d} \right)^2$$



# Standard nearby point source model

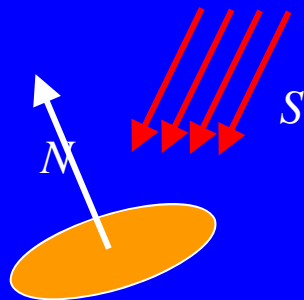
$$\rho_d(x) \left( \frac{N(x)^T S(x)}{r(x)^2} \right)$$

- $N$  is the surface normal
- $\rho$  is diffuse (Lambertian) albedo
- $S$  is source vector - a vector from  $x$  to the source, whose length is the intensity term
  - works because a dot-product is basically a cosine



# Standard distant point source model

- Issue: nearby point source gets bigger if one gets closer
  - the sun doesn't for any reasonable meaning of closer
- Assume that all points in the model are close to each other with respect to the distance to the source. Then the source vector doesn't vary much, and the distance doesn't vary much either, and we can roll the constants together to get:



$$\rho_d(x) \left( N(x)^T S(x) \right)$$

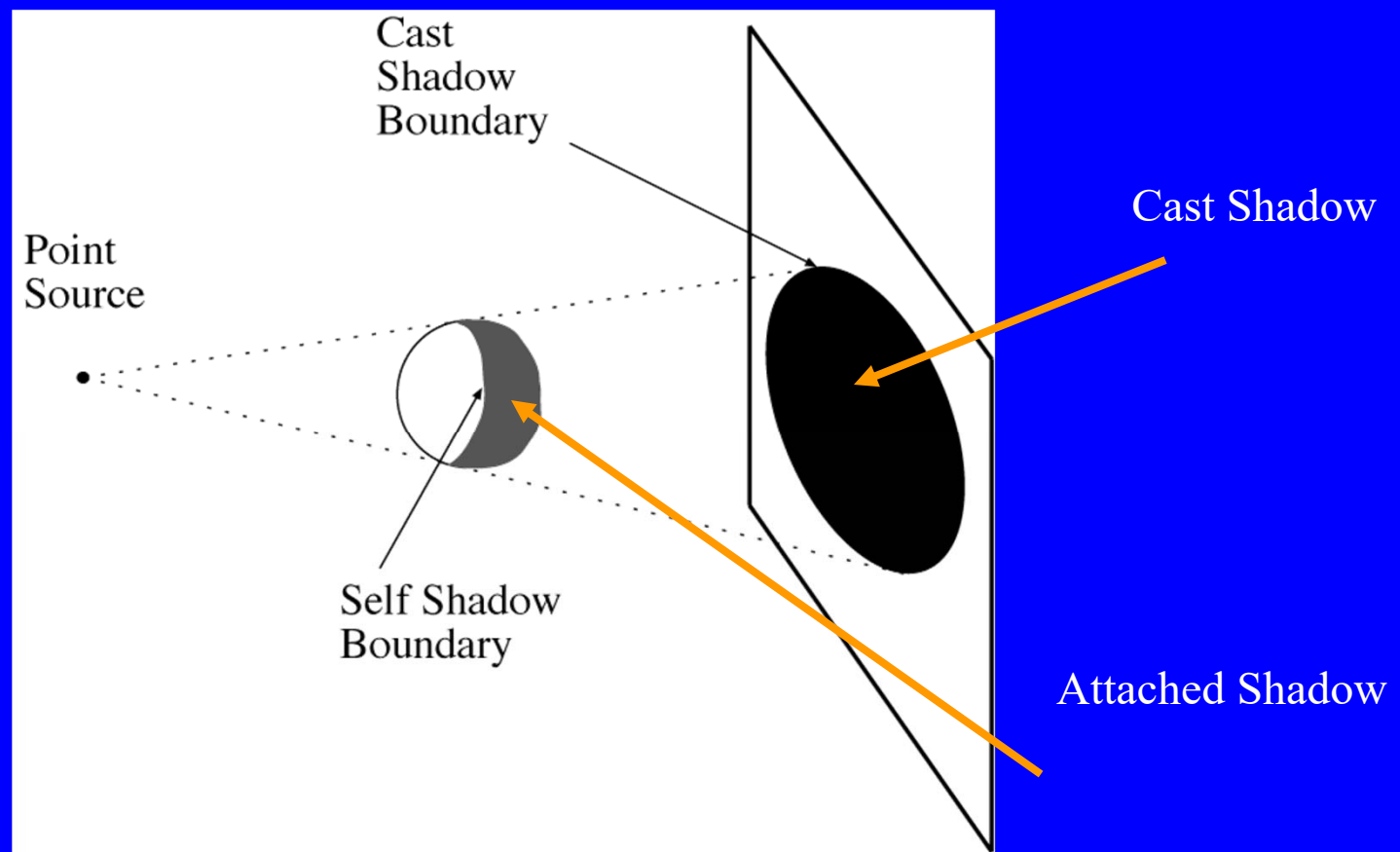
# Lighting at infinity

- Direction is a three vector  $\mathbf{s}$ , with  $|\mathbf{s}| = 1$ .
- Described as function on a sphere: radiance as a function of direction  $r(\mathbf{s})$
- Single point source is a delta function at some direction
- Multiple point sources: sum of delta functions



# Shadows cast by a point source

- A point that can't see the source is in shadow
- For point sources, the geometry is simple



# Shading models

## Local shading model

- Surface has incident radiance due only to sources visible at each point
- Advantages:
  - often easy to manipulate, expressions easy
  - supports quite simple theories of how shape information can be extracted from shading
- Used in vision & real time graphics

## Global shading model

- surface radiosity is due to radiance reflected from other surfaces as well as from surfaces
- Advantages:
  - usually very accurate
- Disadvantage:
  - extremely difficult to infer anything from shading values
- Rarely used in vision, often in photorealistic graphics

# Next Lecture

- Photometric Stereo
- Reading:
  - Section 2.2.4: Photometric Stereo
    - Shape from Multiple Shaded Images