Image Formation: Geometric Camera Models

Computer Vision I

CSE 252A

Lecture 2

Announcements

- Course website

  https://cseweb.ucsd.edu/classes/fa19/cse252A-a/

- Piazza and Gradescope
- Homework 1 will be assigned today
  - Python
  - Due Tue, Oct 8, 11:59 PM
- Wait list
- Reading:
  - Chapters 1: Geometric camera models

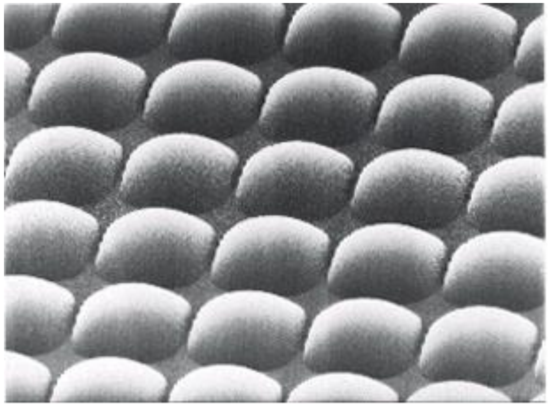
Earliest Surviving Photograph

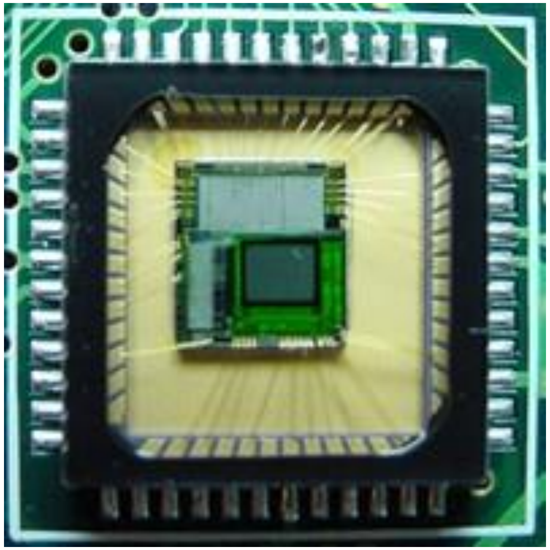
- First photograph on record, “la table service” by Nicephore Niepce in 1822.
- Note: First photograph by Niepce was in 1816.

How Cameras Produce Images

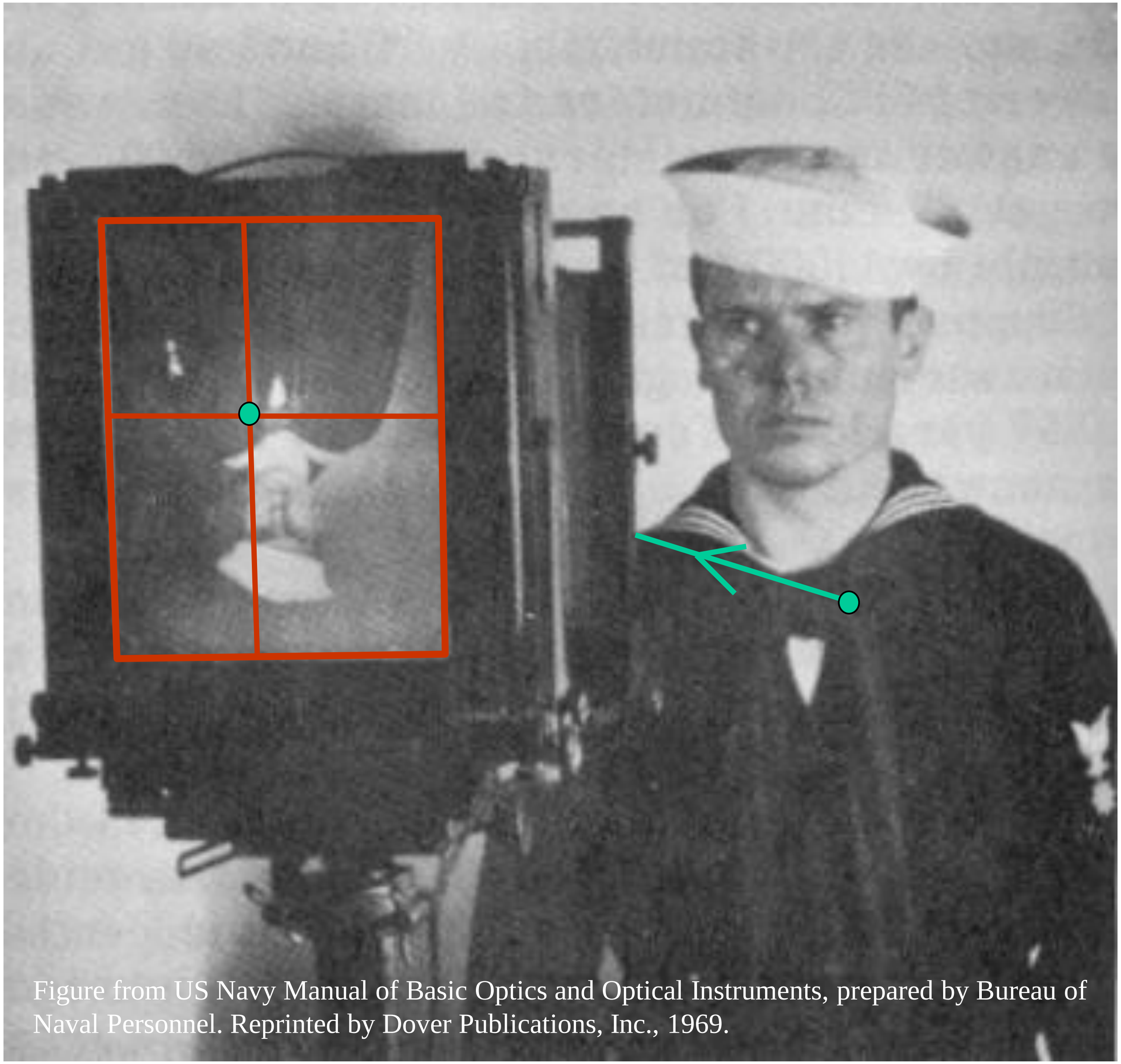
- Basic process:
  - photons hit a detector
  - the detector becomes charged
  - the charge is read out as brightness

- Sensor types:
  - CCD (charge-coupled device)
    - high sensitivity
    - high power
    - cannot be individually addressed
    - blooming
  - CMOS
    - simple to fabricate (cheap)
    - lower sensitivity, lower power
    - can be individually addressed



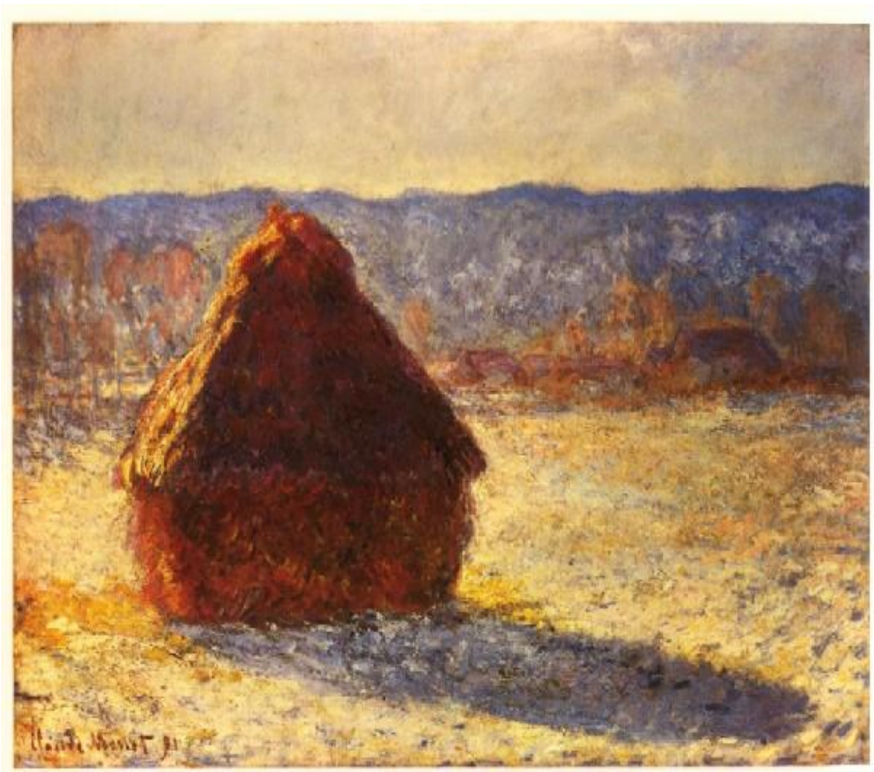


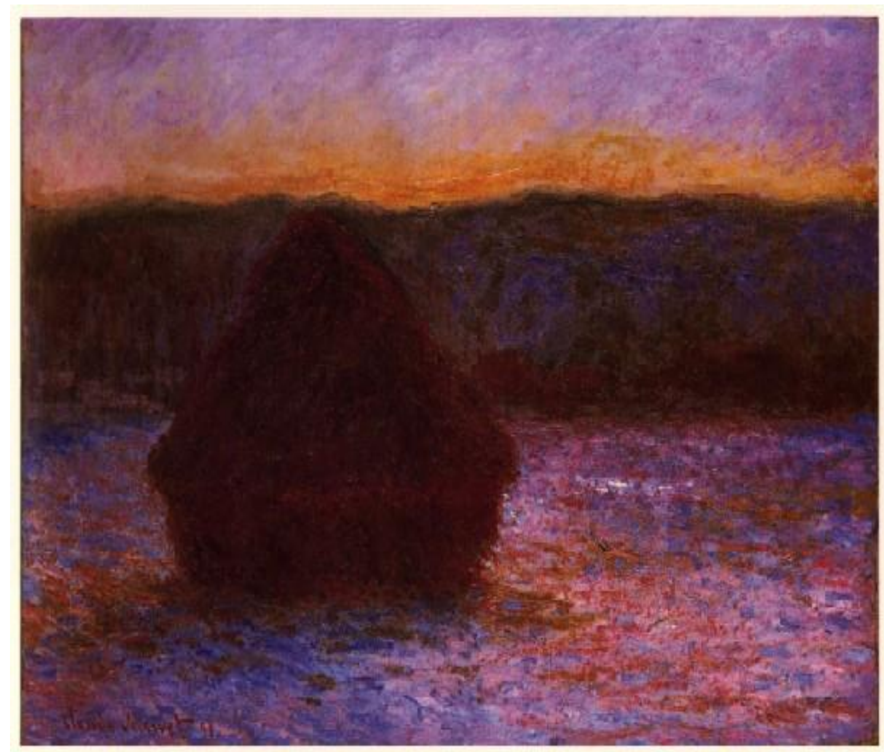
Images are two-dimensional patterns of brightness values.



They are formed by the projection of 3D objects.

Effect of Lighting: Monet





Change of Viewpoint: Monet

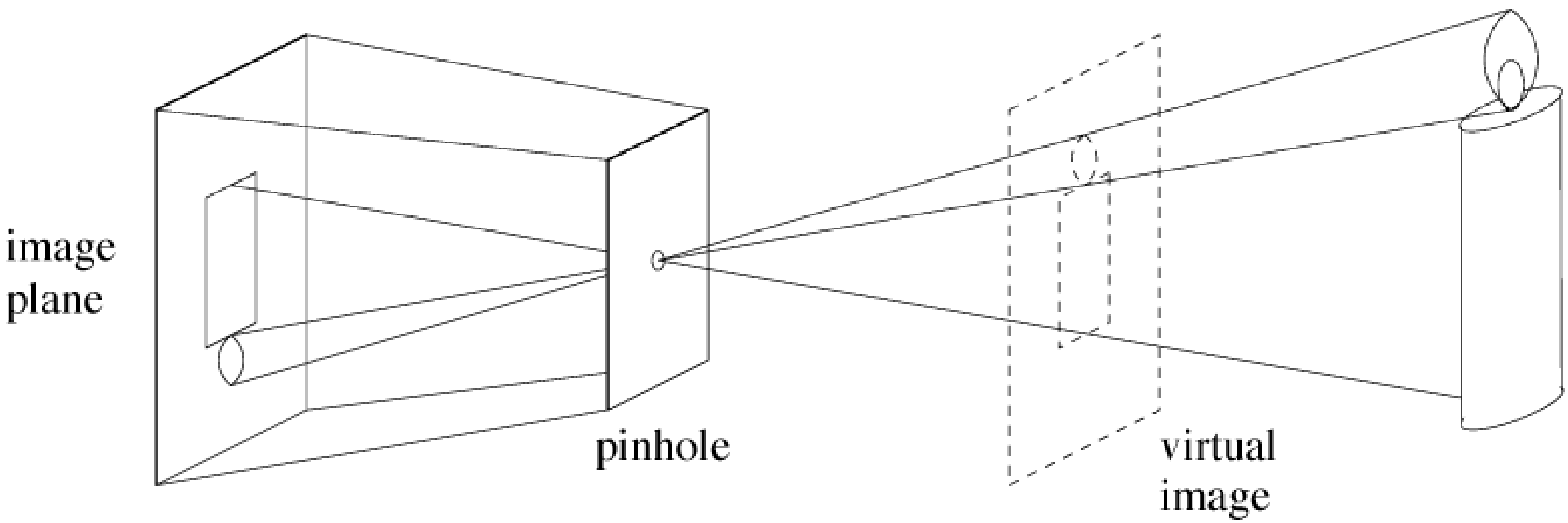
Haystack at Chailly at sunrise (1865)

Image Formation: Outline

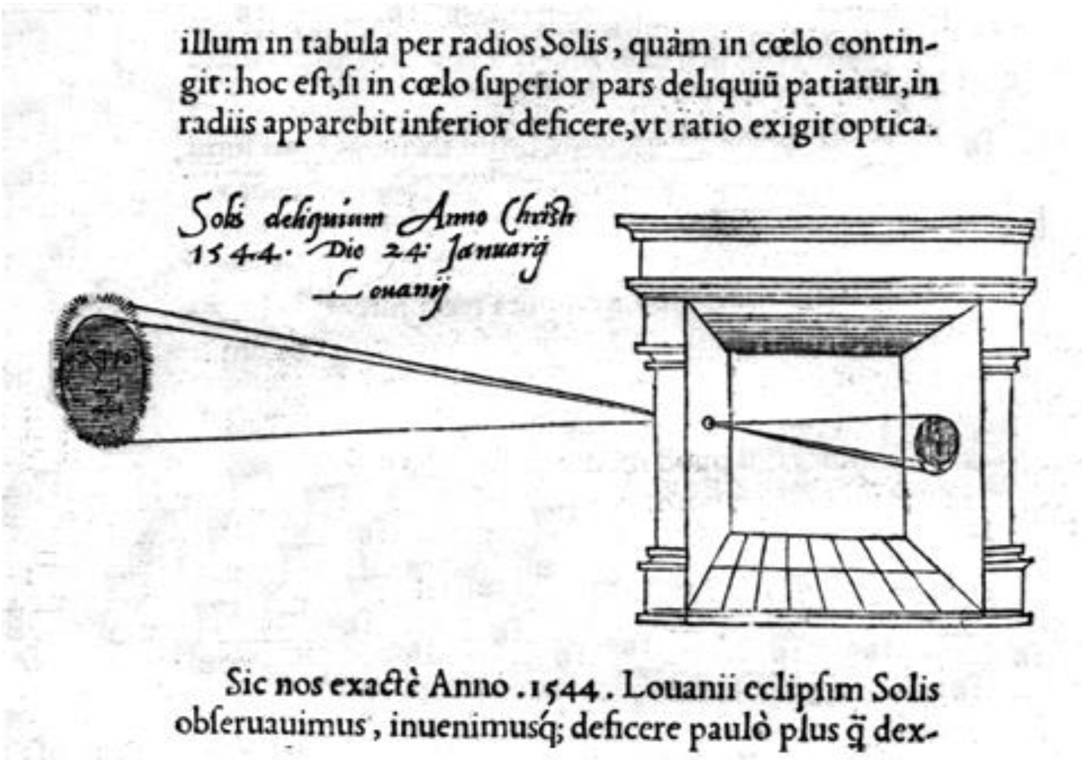
- Geometric camera models
- Light and shading
- Color

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it



Camera Obscura

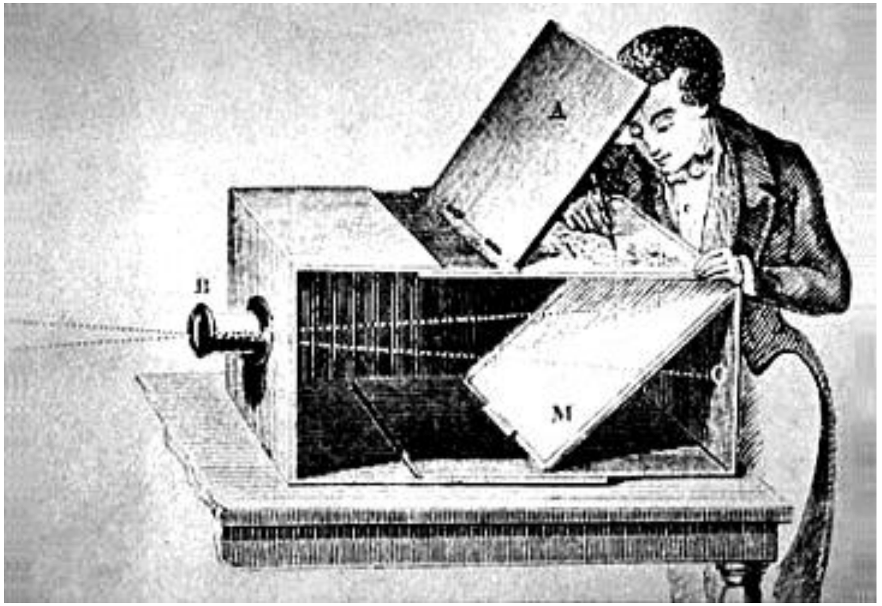

illum in tabula per radios Solis, quàm in cœlo contin-
git: hoc eſt, ſi in cœlo ſuperior pars deliquiũ patiatur, in
radiis apparebit inferior deficere, vt ratio exigit optica.

Sic nos exactè Anno .1544. Louanii eclipſim Solis
obſeruauimus, inuenimusq; deficere paulò plus q̃ dex-

"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays”. --- Leonardo *Da Vinci*

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)

Camera Obscura



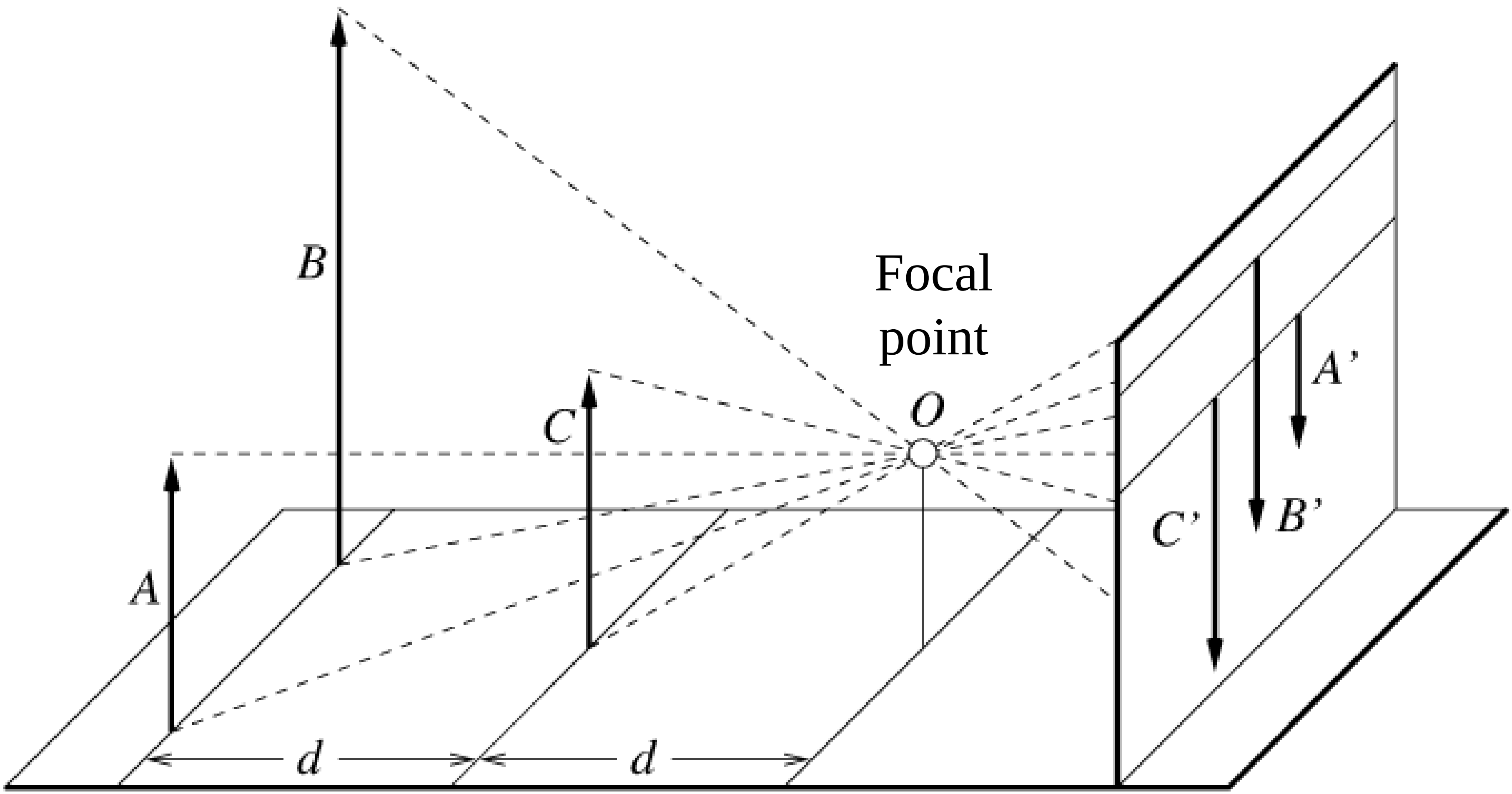
- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer).

Camera Obscura

Jetty at Margate England, 1898.

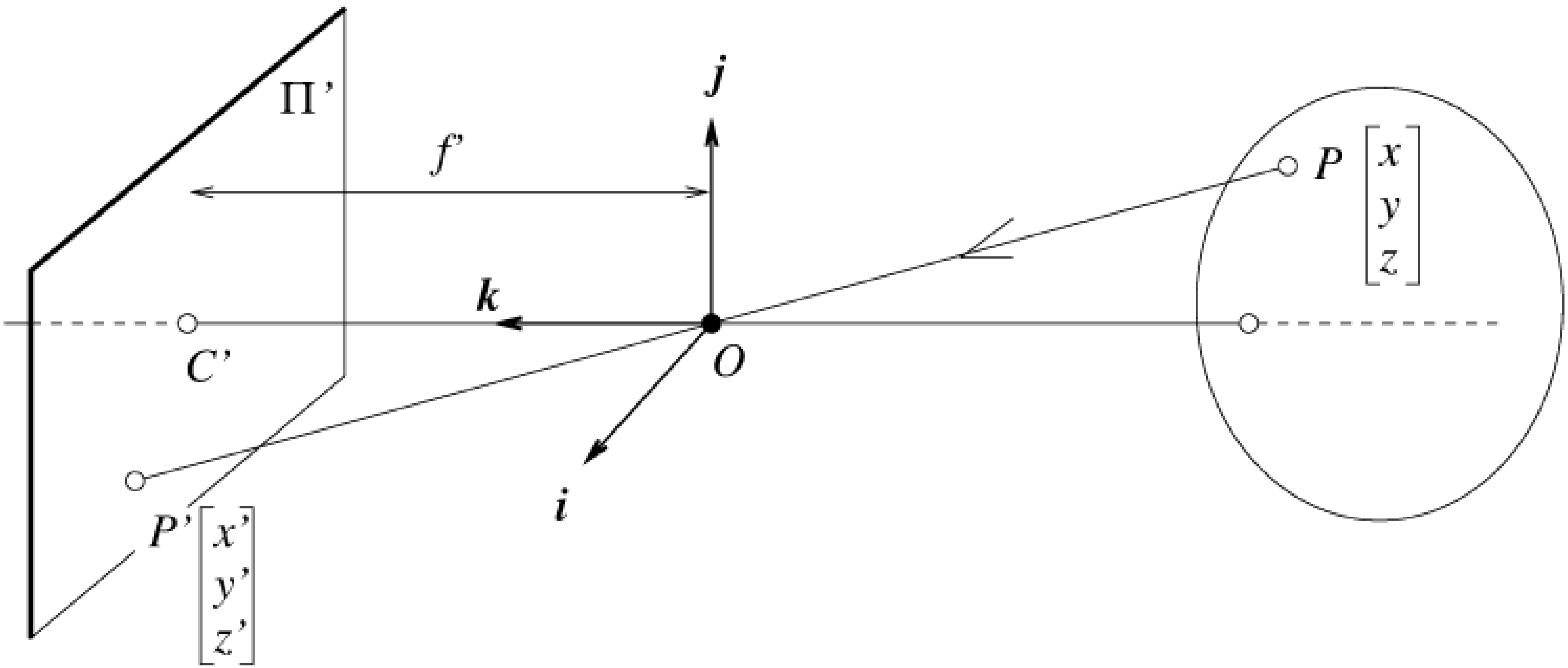
http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)

A and C are same size, but A is further from camera, so its image A’ is smaller



(Forsyth & Ponce)

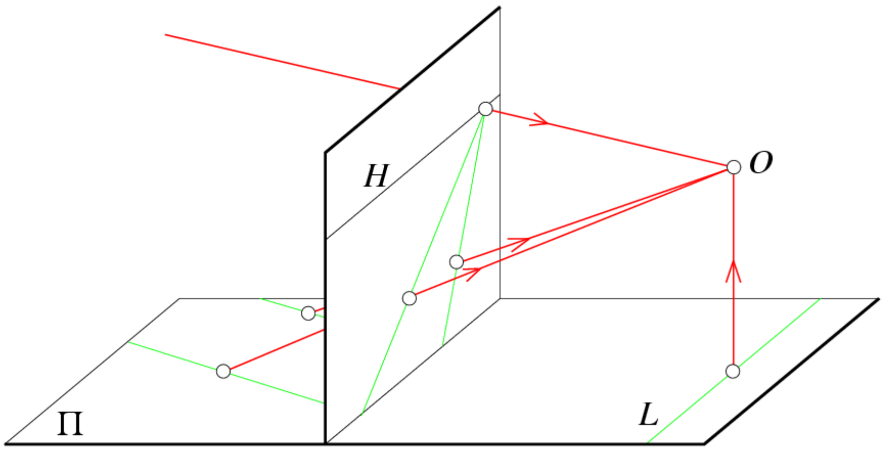
Purely Geometric View of Perspective



The projection of the point **P** on the image plane **Π**' is given by the point of intersection **P**' of the ray defined by **PO** with the plane **Π**'.

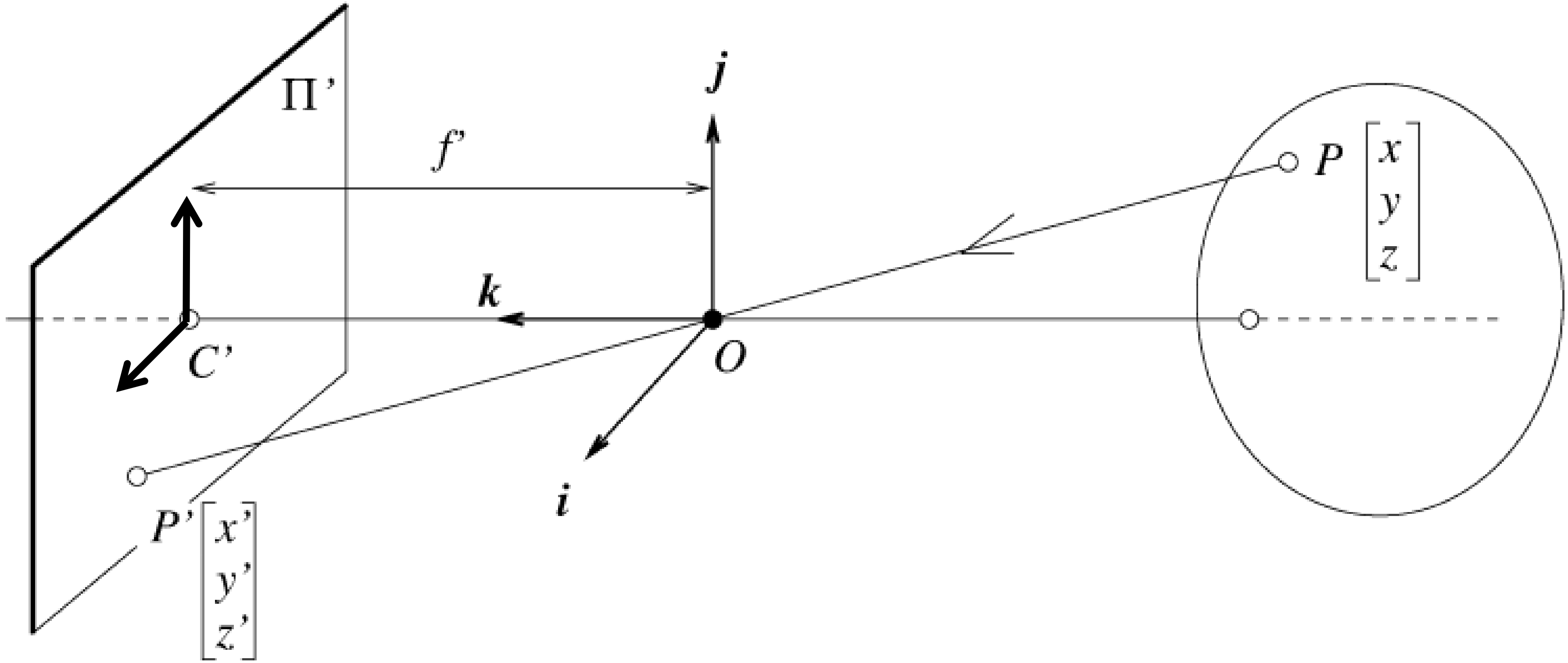
Geometric properties of projection

- 3-D points map to points
- 3-D lines map to lines
- Planes map to whole image or half-plane
- Polygons map to polygons



- Important point to note: Angles & distances not preserved, nor are inequalities of angles & distances.
- Degenerate cases:
  - line through focal point project to point
  - plane through focal point projects to a line

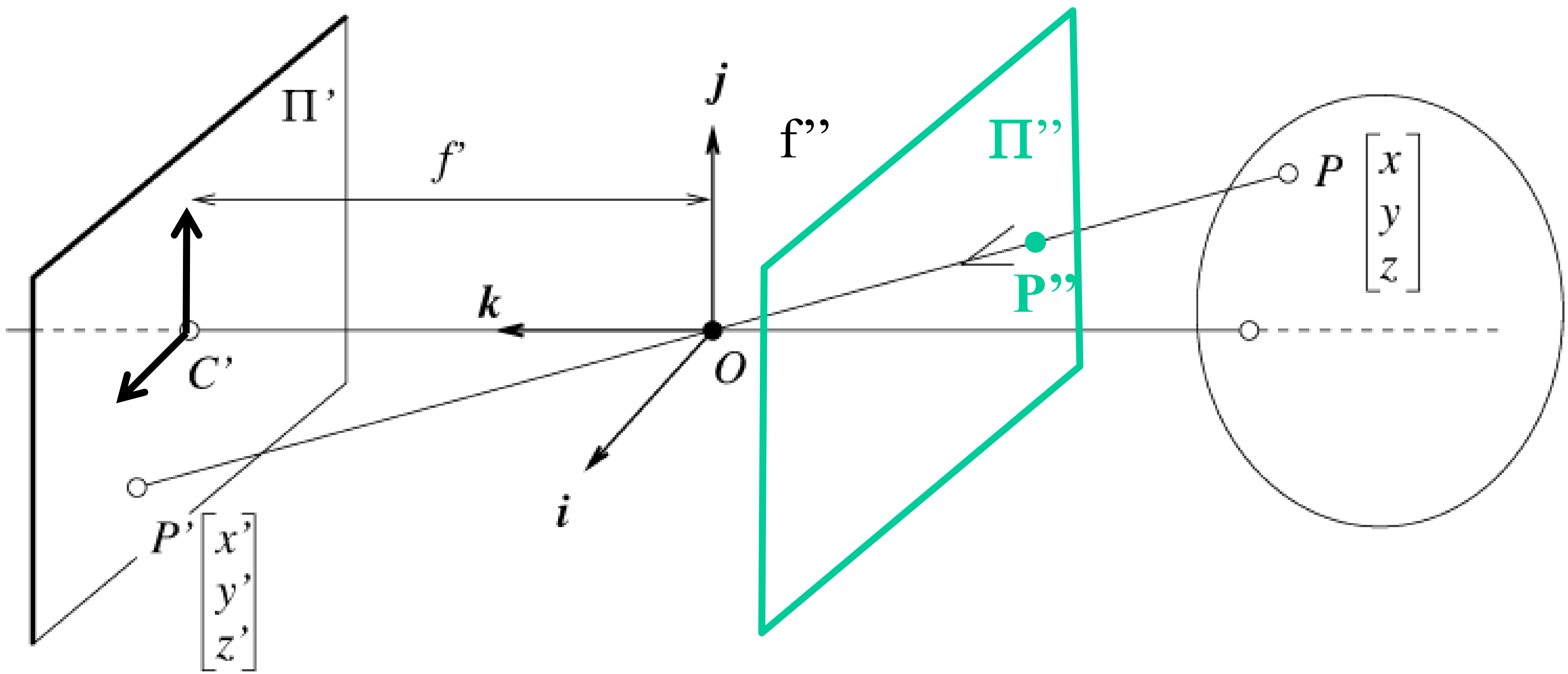
Equation of Perspective Projection



Cartesian coordinates:

- We have, by similar triangles, that for **P**=(x, y, z), the intersection of OP with **Π**' is (f' x/z, f' y/z, f')
- Establishing an image plane coordinate system at C' aligned with i and j, we get $(x, y, z) \rightarrow (f'\frac{x}{z}, f'\frac{y}{z})$

Virtual Image Plane



- Virtual image plane in front of optical center.
- Image is 'upright'

$$(x, y, z) \rightarrow (f''\frac{x}{z}, f''\frac{y}{z})$$

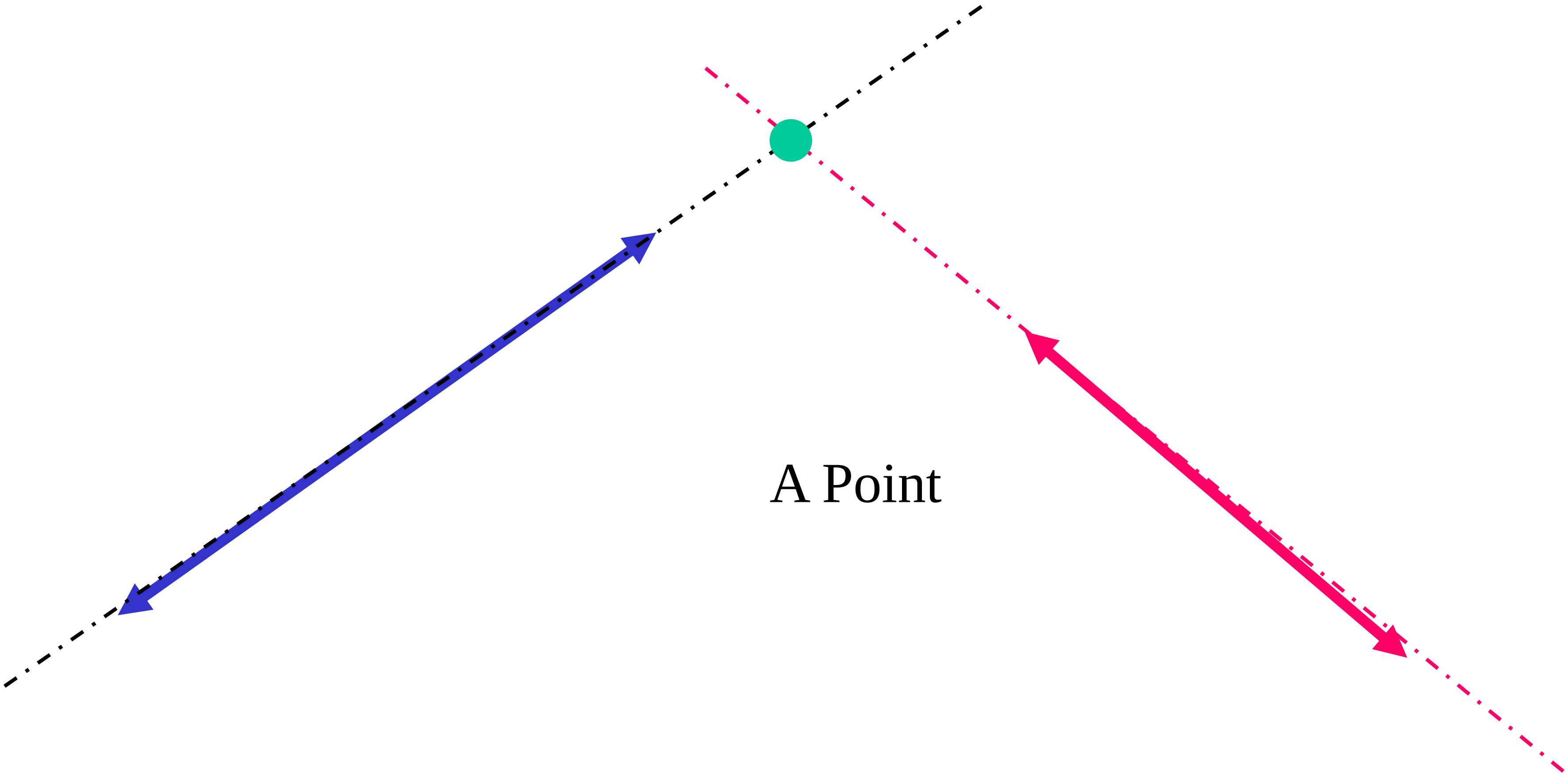
A Digression

Projective Geometry

and

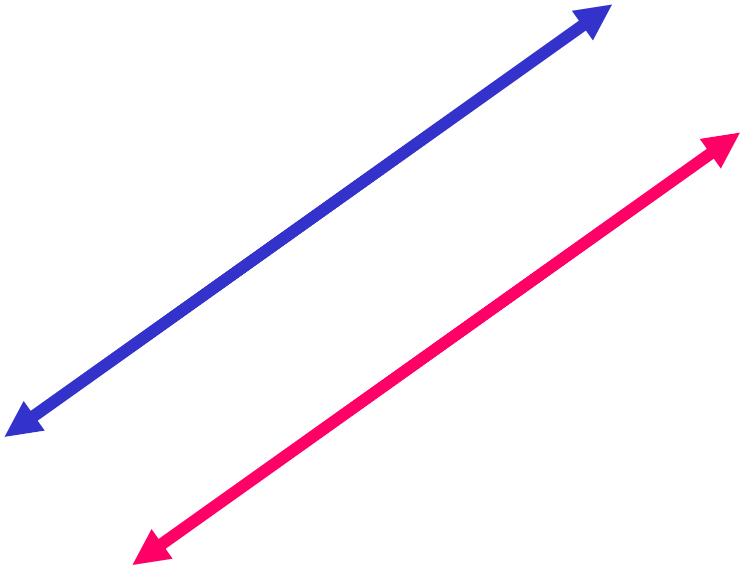
Homogenous Coordinates

What is the intersection of two lines in a plane?



A Point

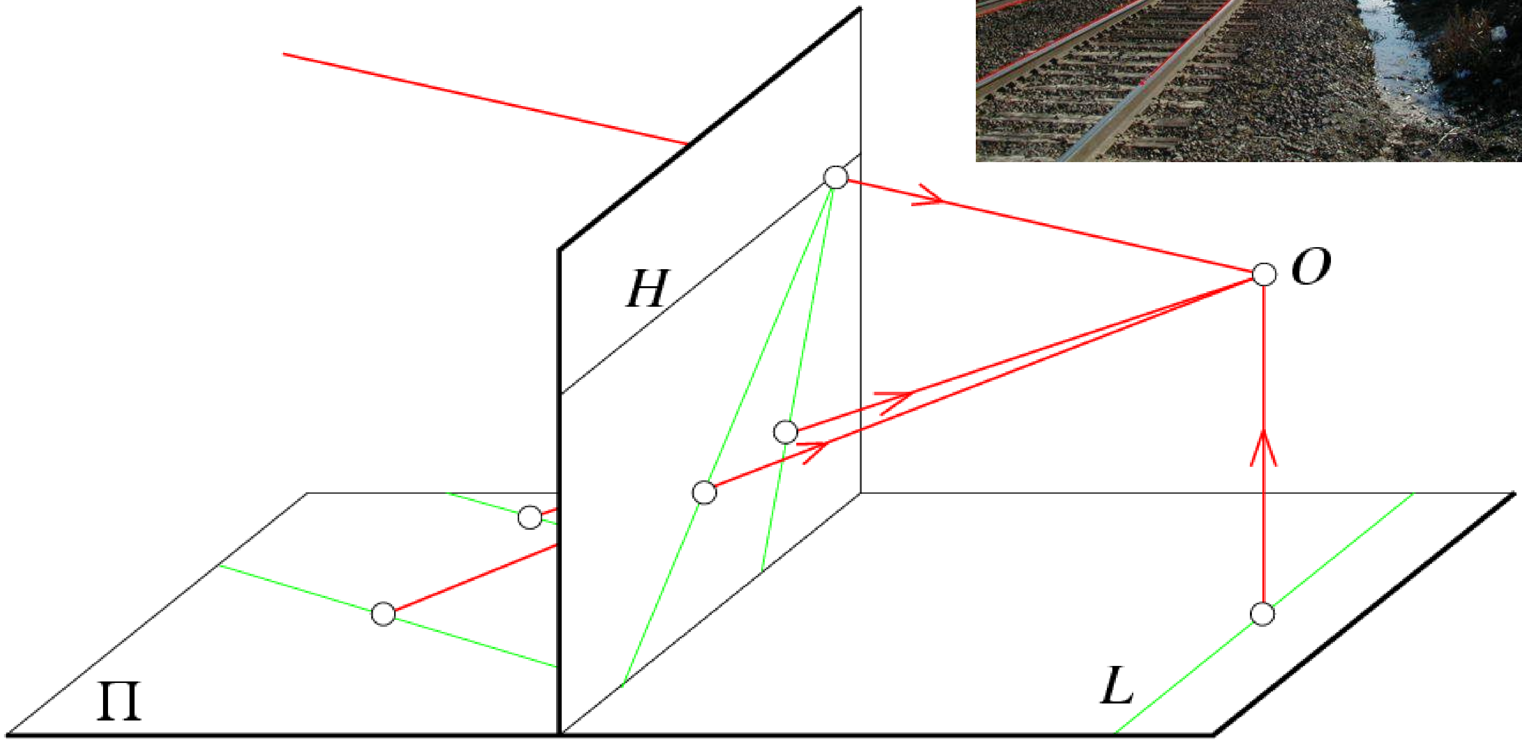
Do two lines in the plane always intersect at a point?



No, Parallel lines don't meet at a point.

Can the perspective image of two parallel lines meet at a point?

YES



Projective geometry provides an elegant means for handling these different situations in a unified way and **homogenous coordinates** are a way to represent entities (points & lines) in projective spaces.

Projective Geometry

- Axioms of Projective Plane
  1. Every two distinct points define a line
  2. Every two distinct lines define a point (intersect at a point)
  3. There exists three points, A,B,C such that C does not lie on the line defined by A and B.
- Different than Euclidean and affine geometry
- Projective plane is “bigger” than affine plane – includes “line at infinity”

Homogeneous coordinates

- Boardwork
  - 2D points and lines
  - Point at infinity
  - Line at infinity

Homogeneous coordinates

- 3D point using inhomogeneous coordinates as 3-vector

$$\tilde{\mathbf{X}} = \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix}$$

- 3D point using affine homogeneous coordinates as 4-vector

$$\mathbf{X} = \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \\ 1 \end{bmatrix}$$

Homogeneous coordinates

- 3D point using *affine* homogeneous coordinates as 4-vector

$$\mathbf{X} = \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \\ 1 \end{bmatrix}$$

- 3D point using *projective* homogeneous coordinates as 4-vector (**up to scale**)

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix}$$

Homogeneous coordinates

- Projective homogeneous 3D point to affine homogeneous 3D point

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = \frac{1}{W} \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = \begin{bmatrix} \frac{X}{W} \\ \frac{Y}{W} \\ \frac{Z}{W} \\ 1 \end{bmatrix} = \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \\ 1 \end{bmatrix}$$

- Dehomogenize 3D point

$$\tilde{\mathbf{X}} = \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix} = \begin{bmatrix} \frac{X}{W} \\ \frac{Y}{W} \\ \frac{Z}{W} \end{bmatrix}$$

Homogeneous coordinates

- Homogeneous points are defined up to a nonzero scale

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = \lambda \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = \begin{bmatrix} \lambda X \\ \lambda Y \\ \lambda Z \\ \lambda W \end{bmatrix}$$

$$\tilde{\mathbf{X}} = \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix} = \begin{bmatrix} \frac{\lambda X}{\lambda W} \\ \frac{\lambda Y}{\lambda W} \\ \frac{\lambda Z}{\lambda W} \end{bmatrix} = \begin{bmatrix} \frac{X}{W} \\ \frac{Y}{W} \\ \frac{Z}{W} \end{bmatrix}$$

Homogeneous coordinates

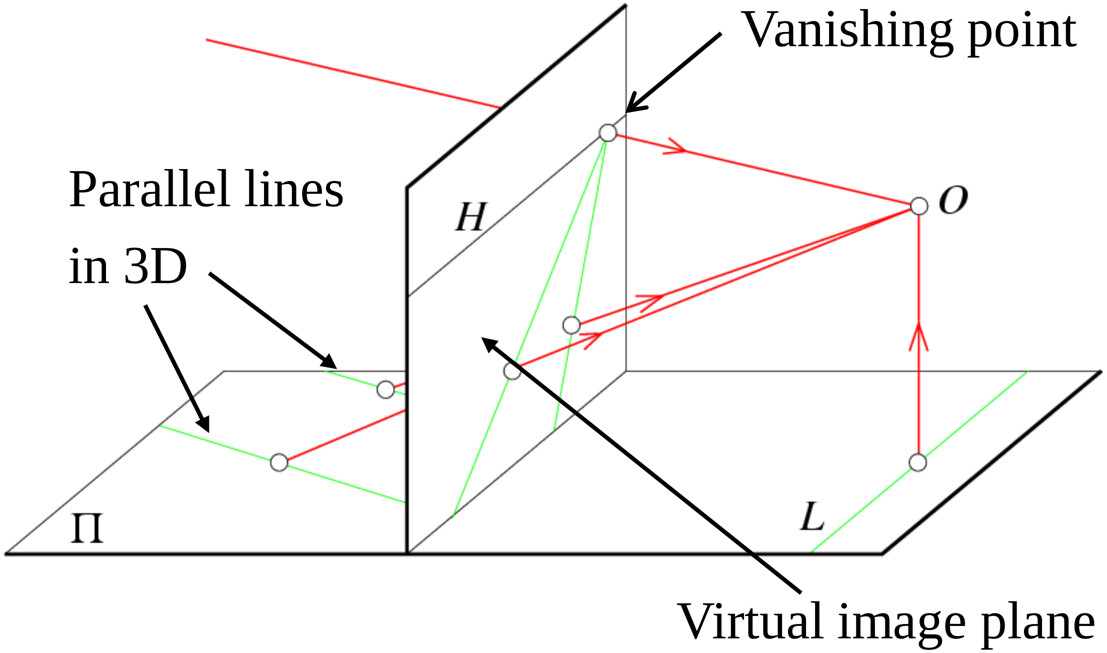
- When $W = 0$, then it is a point at infinity
- Affine homogeneous coordinates are projective homogeneous coordinates where $W = 1$
- When not differentiating between affine homogeneous coordinates and projective homogeneous coordinates, simply call them homogeneous coordinates

End of the Digression

In a perspective image, parallel lines meet at a point, called the vanishing point

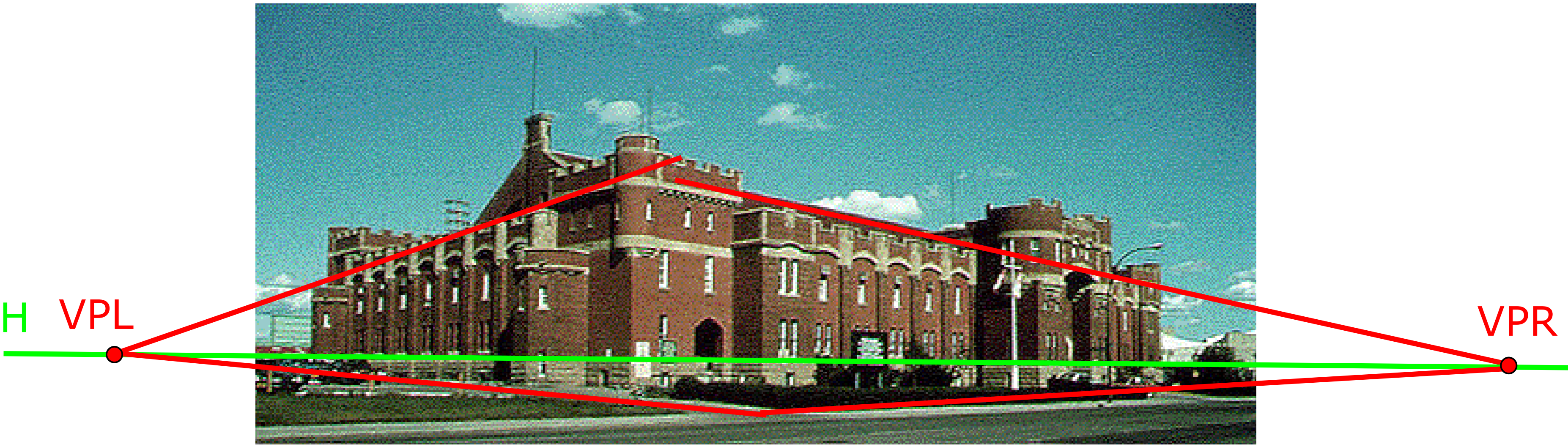
Doesn’t need to be near the center of the image

Parallel lines meet in the image

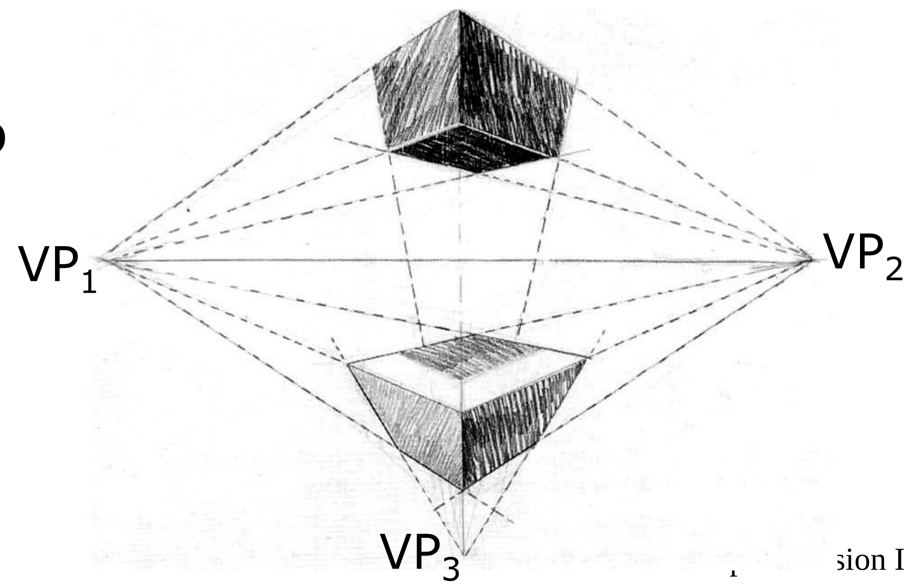


- Vanishing point location: Intersection of 3-D line through O parallel to given line(s)
- A single line can have a vanishing point

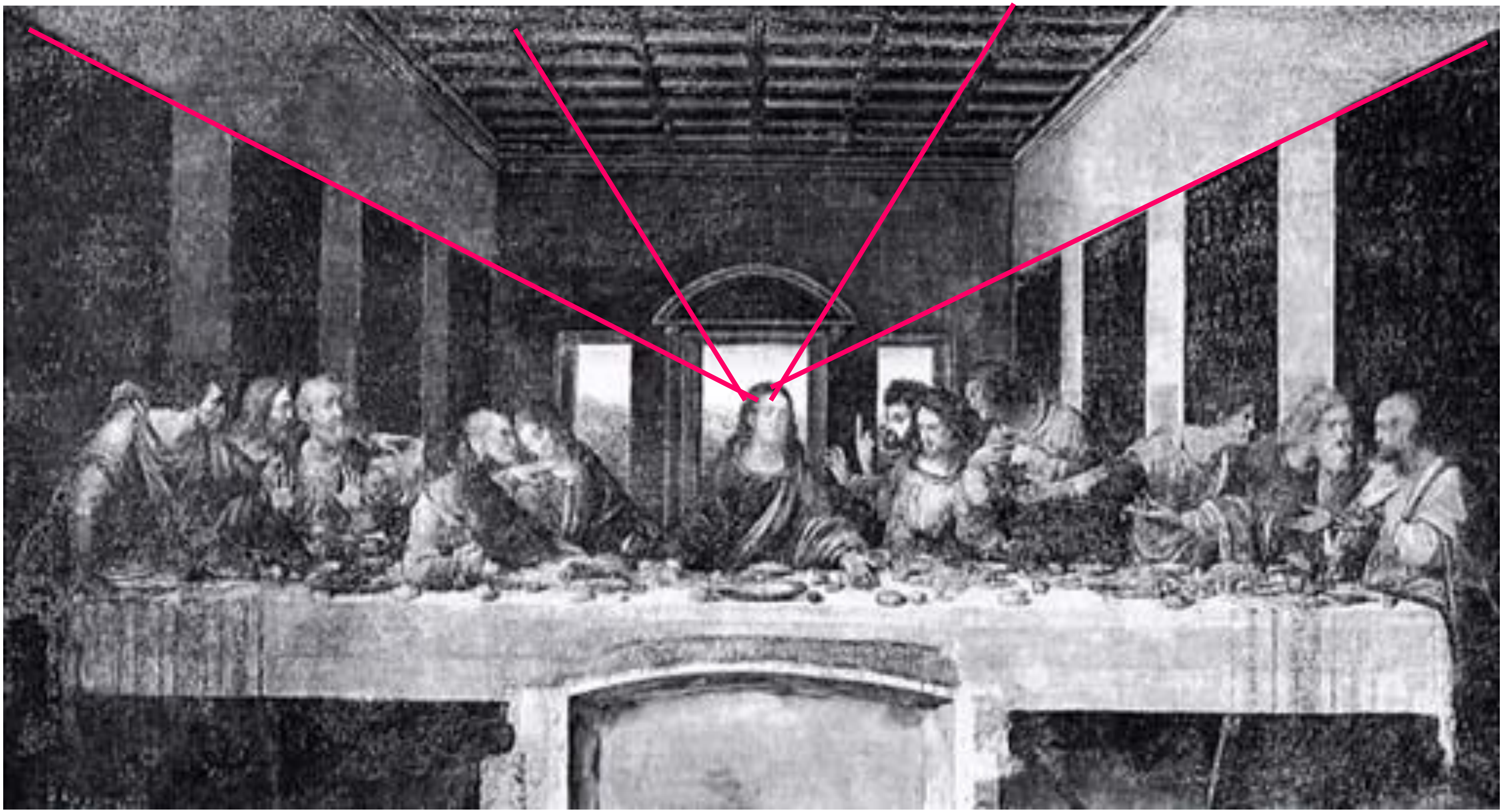
Vanishing points



Different directions correspond to different vanishing points



Vanishing Points



Vanishing Point

- In the **projective plane**, parallel lines meet at a point at infinity.

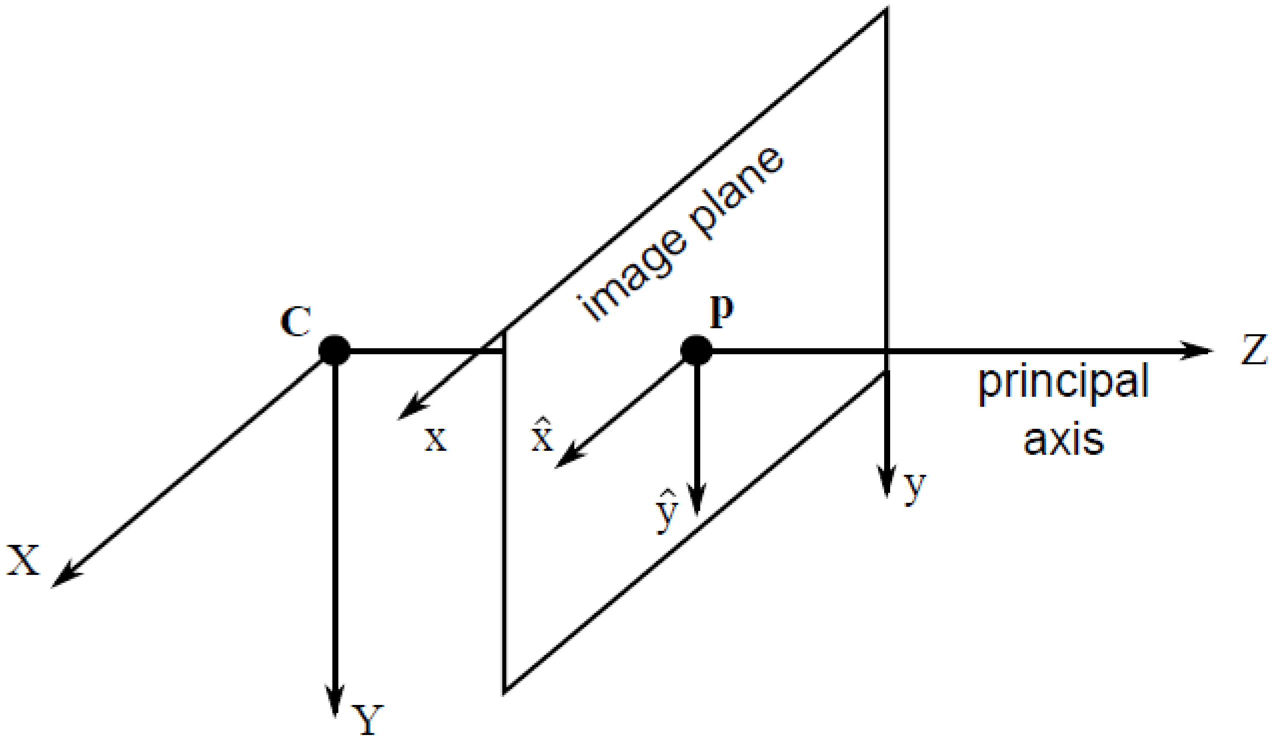
- The 2D vanishing point in the image is the perspective projection of this 3D point at infinity

What is a Camera?

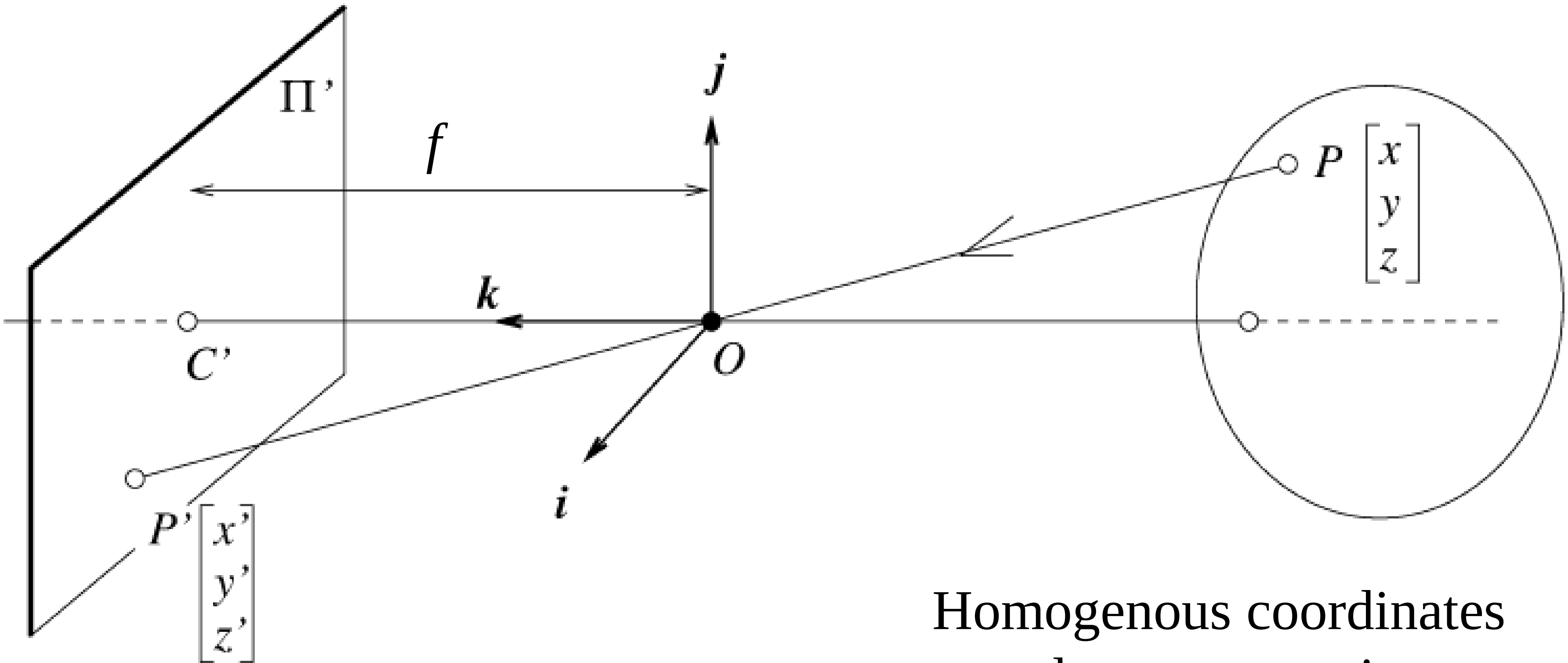
- An mathematical expression that relates points in 3D to points in an image for different types of physical cameras or imaging situations

Geometry

- How do 3D world points project to 2D image points?



The equation of projection



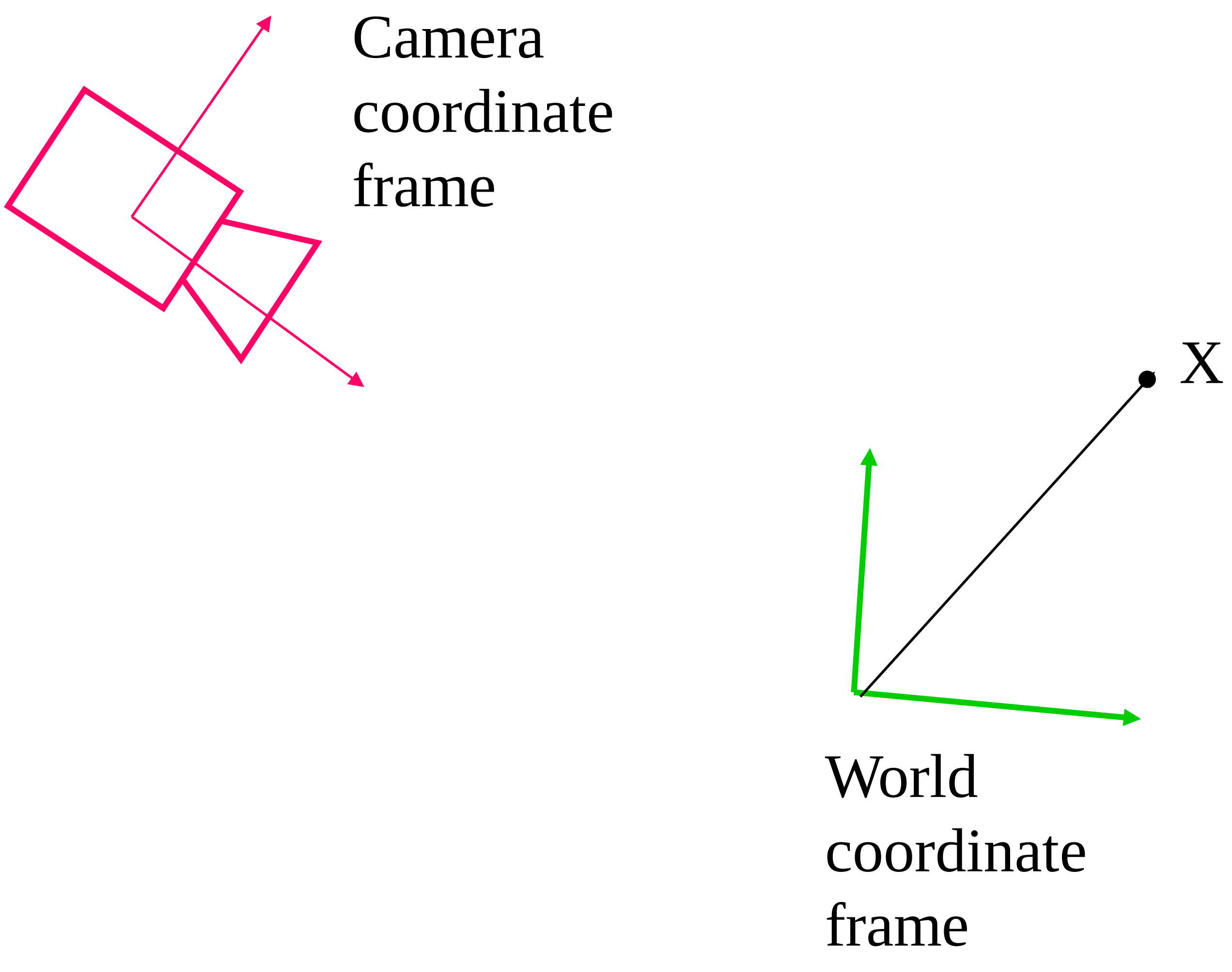
Homogenous coordinates and camera matrix

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Cartesian coordinates:

$$(x, y, z) \rightarrow \left(f\frac{x}{z}, f\frac{y}{z}\right)$$

What if camera coordinate system differs from world coordinate system?

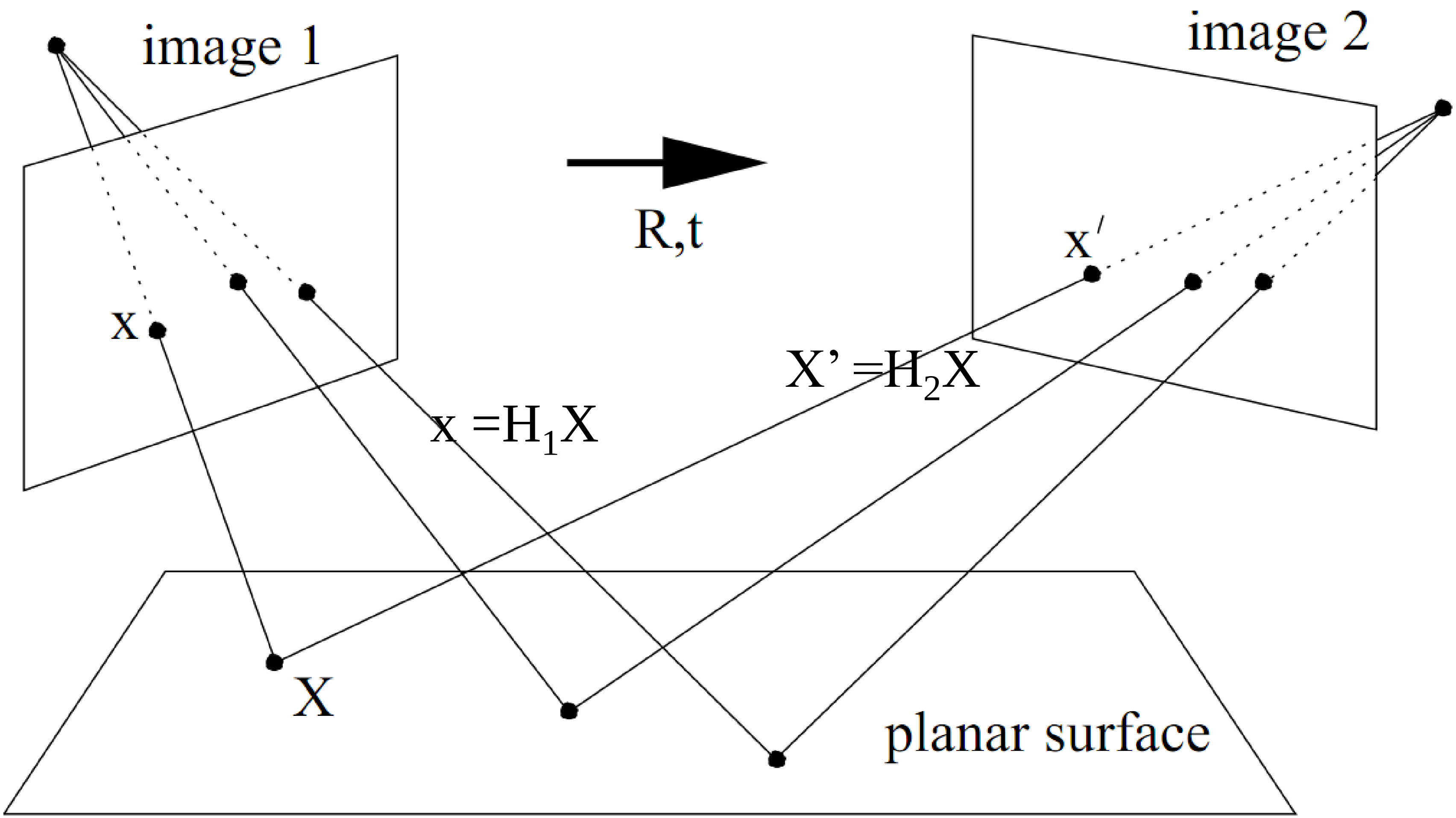


Special cases

- Imaging a plane
- Only camera rotation (no translation)
- In both cases, mapping between images is a planar projective transformation

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{21} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

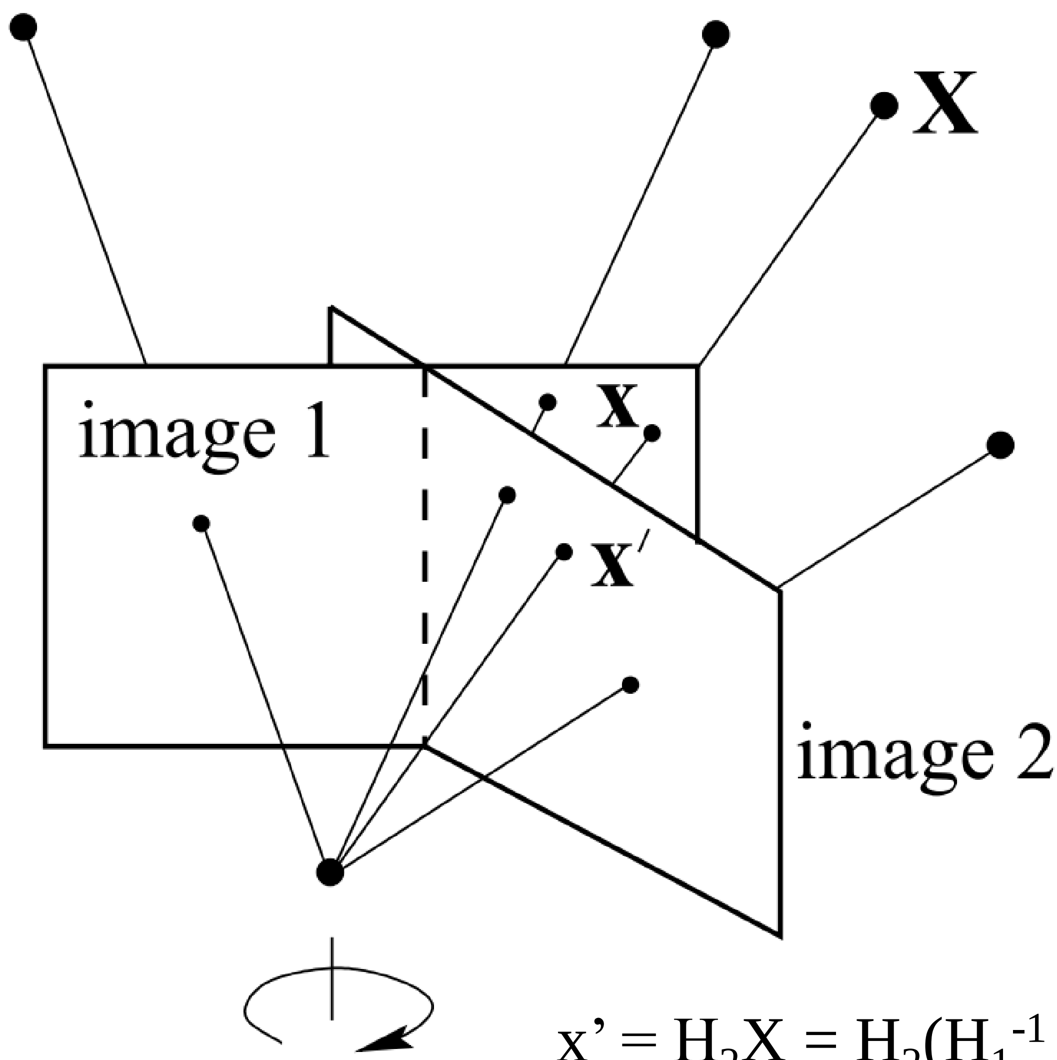
Planar Homography



$$x' = H_2X = H_2(H_1^{-1}x) = (H_2H_1^{-1})x$$

Application: Two photos of a white board

Planar Homography: Pure Rotation



$$x' = H_2X = H_2(H_1^{-1}\,x) = (H_2H_1^{-1})x$$

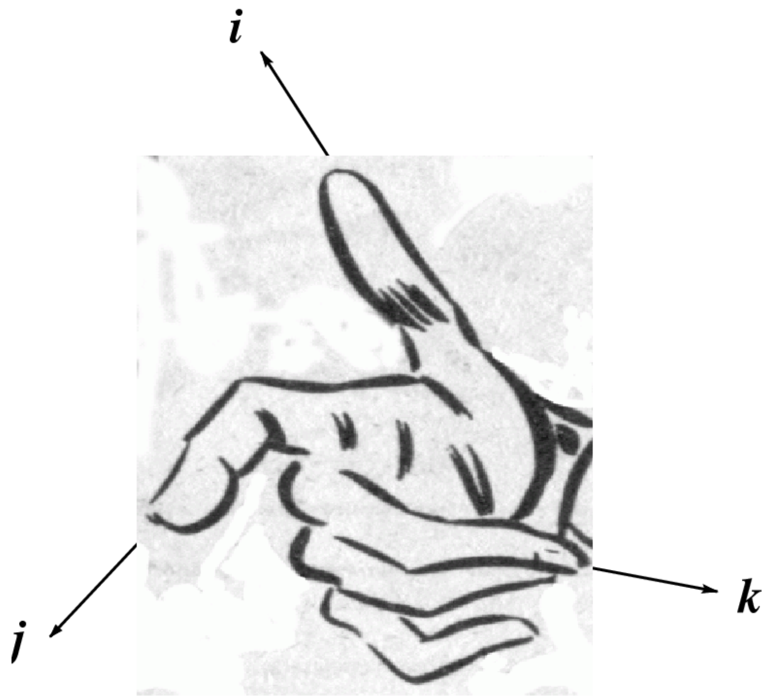
Application: Panoramas

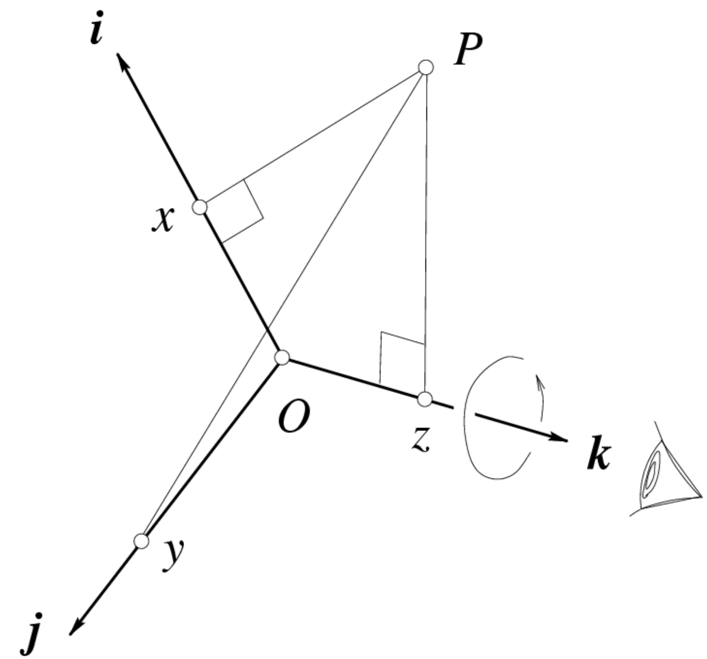
 Figure borrowed from Hartley and Zisserman “Multiple View Geometry in computer vision”

Application: Panoramas and image stitching

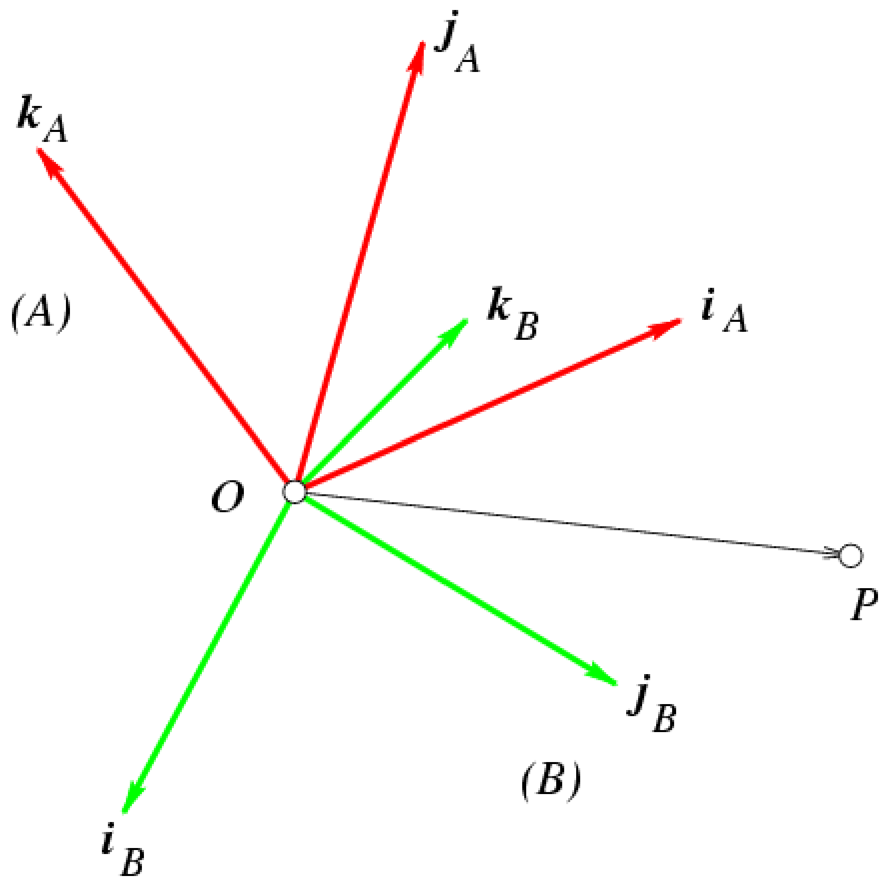
All images are warped to central image

Euclidean Coordinate Systems



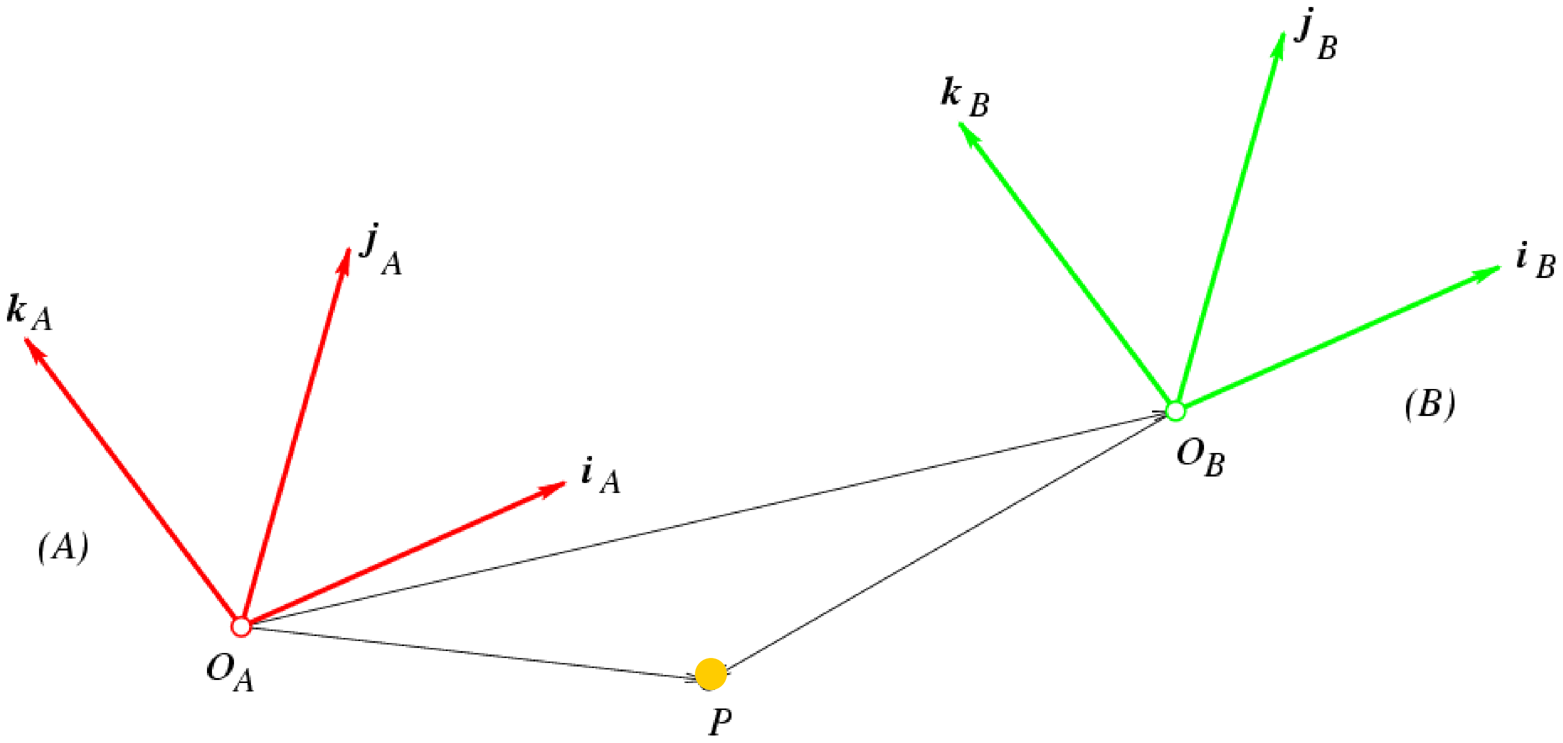


Coordinate Change: Rotation Only



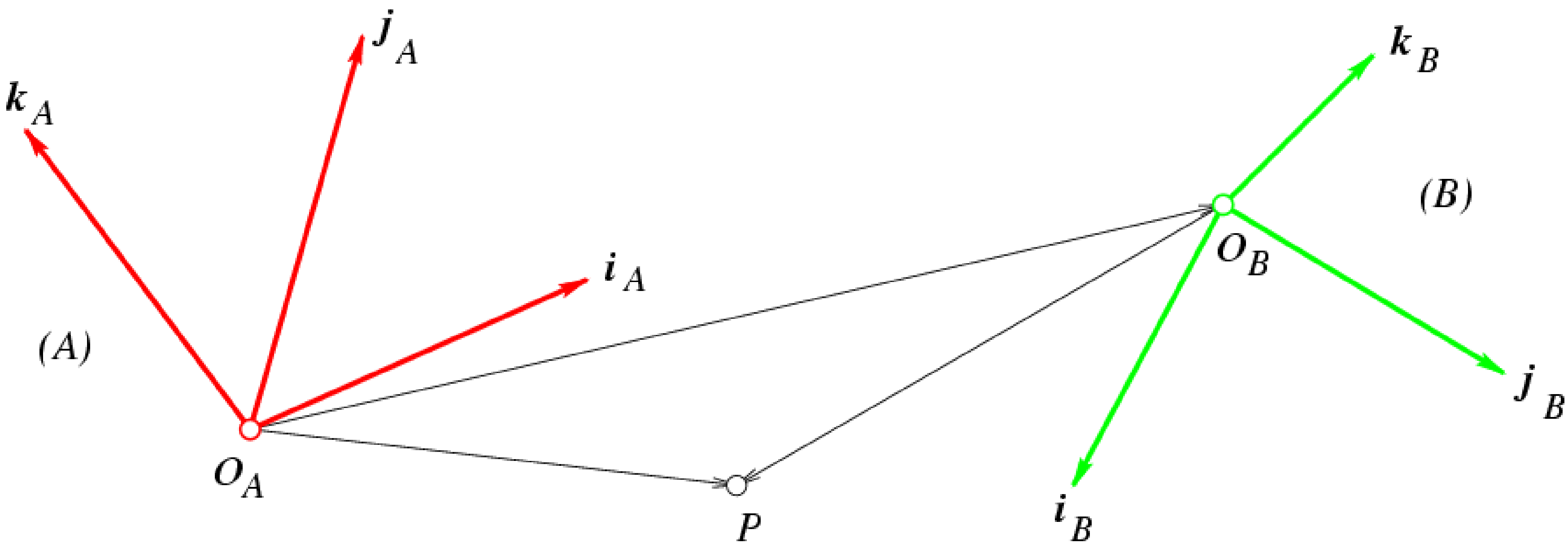
$$X' = R\,X$$

Coordinate Change: Translation Only



$$X' = X + t$$

Coordinate Changes: Rotation and Translation



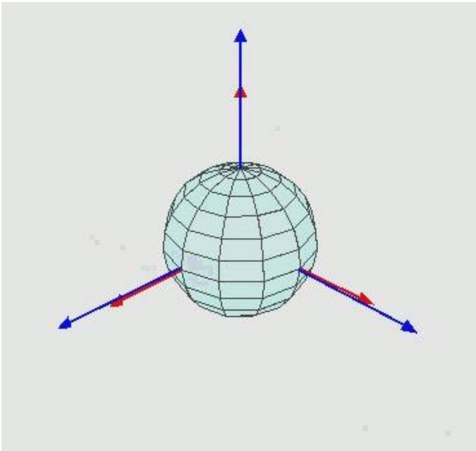
$$X' = R\,X + t$$

Some points about SO(n)

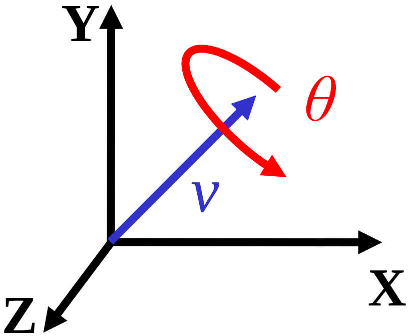
- $SO(n) = \{ R \in \Re^{nxn} : R^T R = I, \det(R) = 1\}$
  - SO(2): rotation matrices in plane $\Re^2$
  - SO(3): rotation matrices in 3-space $\Re^3$
- Forms a Group under matrix product operation:
  - Identity
  - Inverse
  - Associative
  - Closure
- Closed (finite intersection of closed sets)
- Bounded $R_{i,j} \in [-1, +1]$
- Does not form a vector space.
- Manifold of dimension n(n-1)/2
  - Dim(SO(2)) = 1
  - Dim(SO(3)) = 3

Parameterizations of SO(3)

–Even though a rotation matrix is 3x3 with nine numbers, it only has three degrees of freedom. It can be parameterized with three numbers. There are many parameterizations.

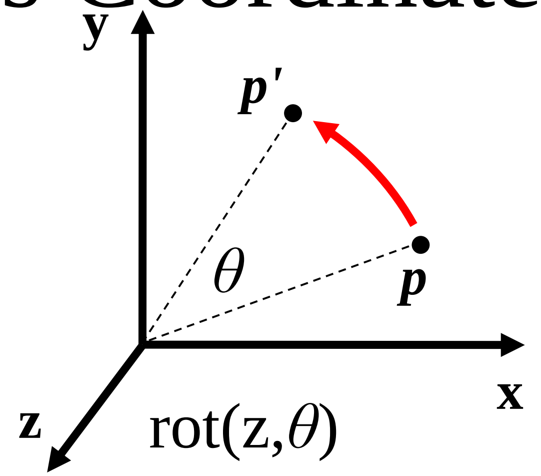


- Other common parameterizations
  - Euler Angles
  - Axis Angle
  - Quaternions
    - four parameters; homogeneous



Rotation: Homogenous Coordinates

- About z axis



$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Rotation

- About x axis:

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

- About y axis:

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

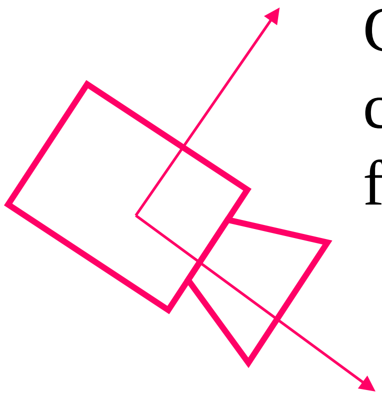
Euler Angles: Roll-Pitch-Yaw

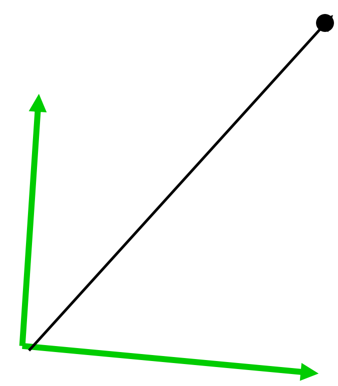
- Composition of rotations

$$R = R_Z(\gamma)\, R_Y(\beta)\, R_X(\alpha)$$

$$R = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix}$$

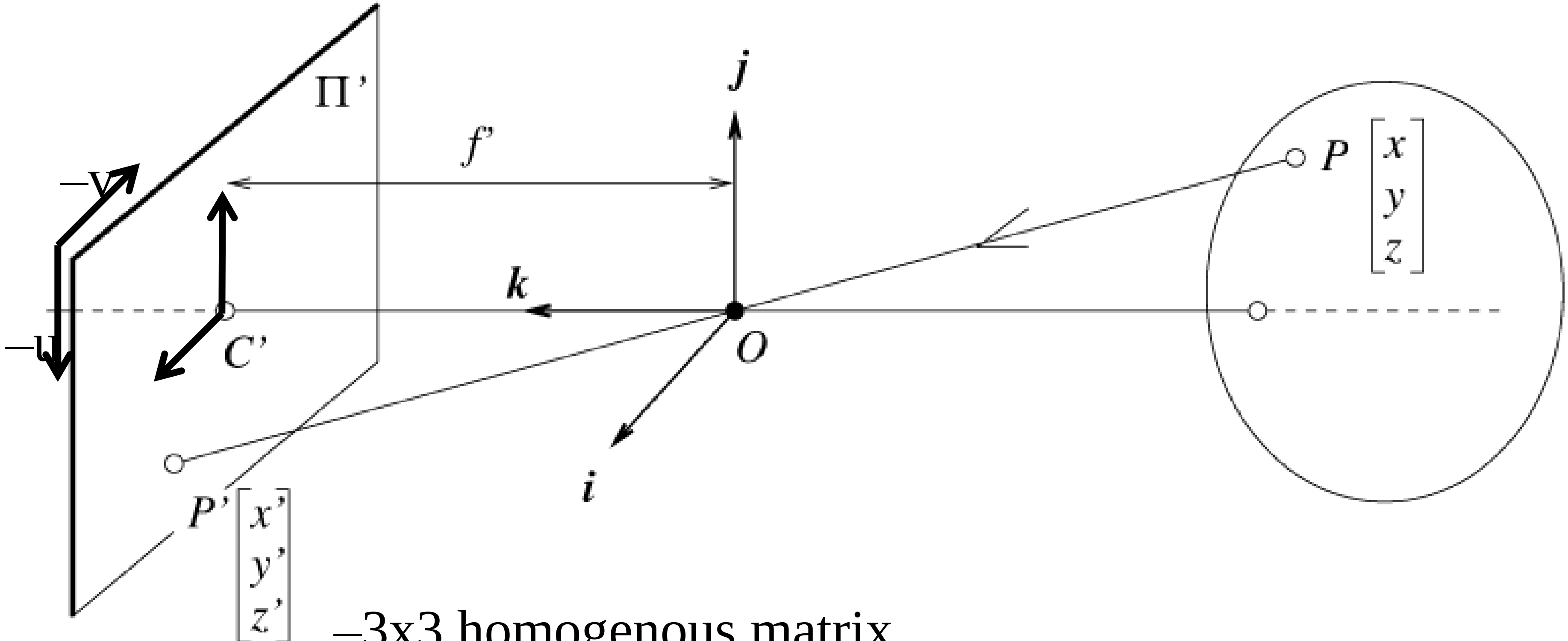
What if camera coordinate system differs from world coordinate system?





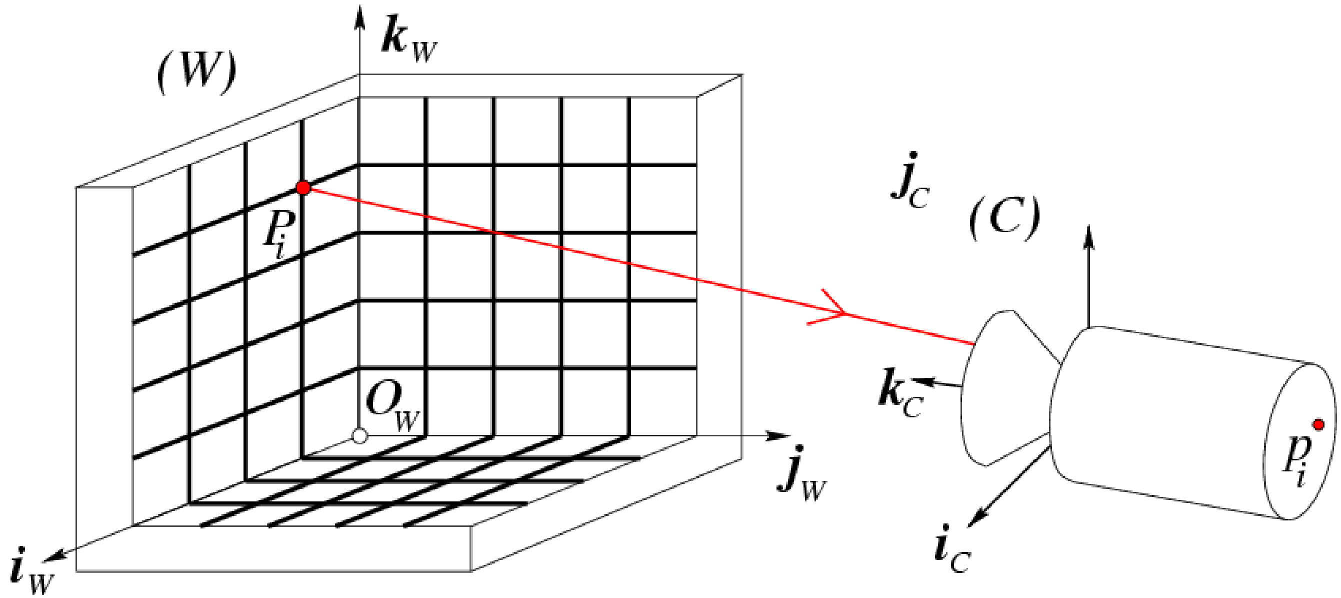
$$X_{Camera} = R\,X_{World} + t$$

Intrinsic parameters



- –3x3 homogenous matrix
- –Focal length
- –Principal Point
- –Units (e.g. pixels)
- –Pixel Aspect ratio

Camera Calibration



Given n points $P_1, \ldots, P_n$ with *known* positions and their images $p_1, \ldots, p_n$ –, estimate intrinsic and extrinsic camera parameters

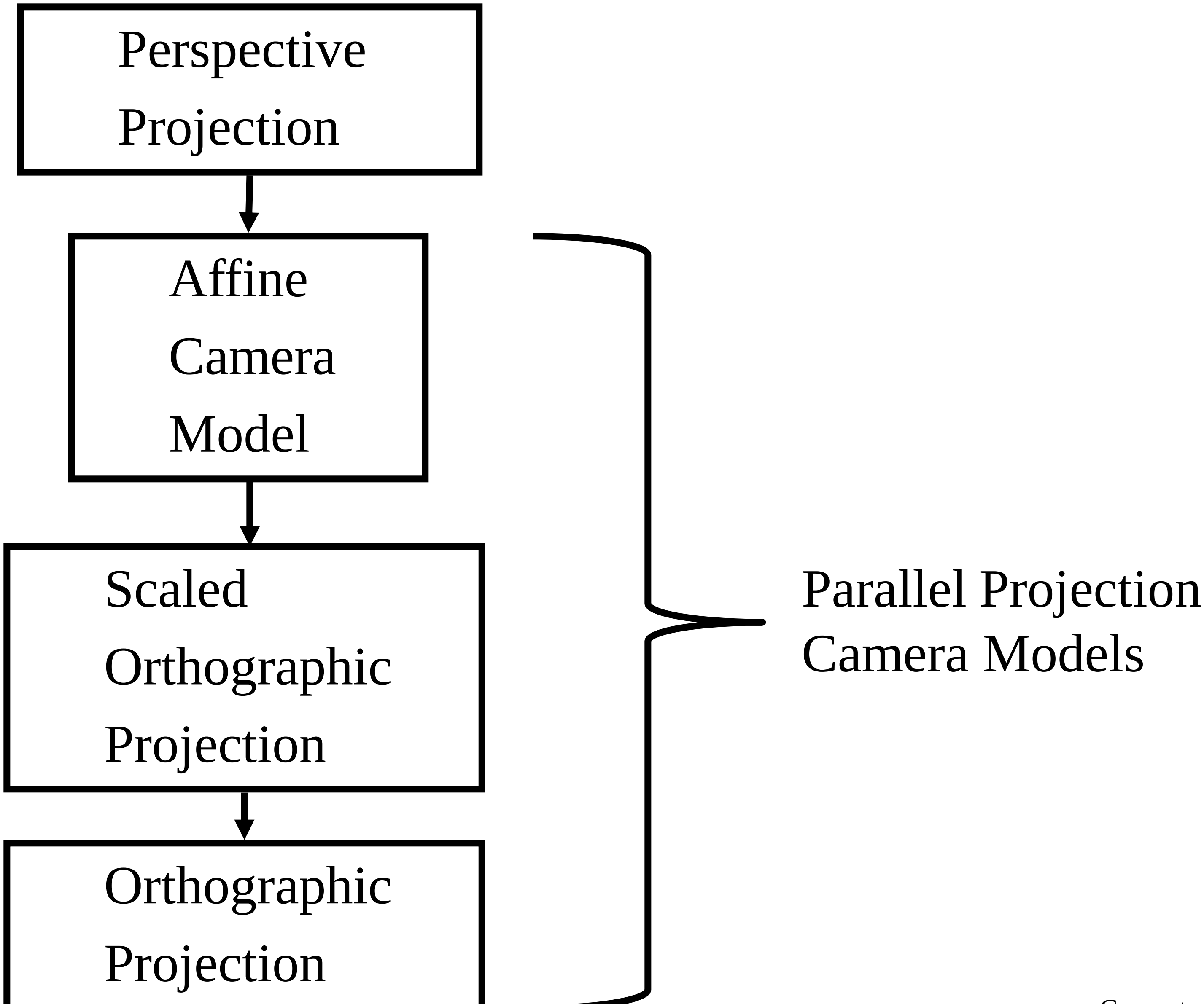
- See Textbook for how to do it.
- Camera Calibration Toolbox for Matlab (Bouguet)
  – http://www.vision.caltech.edu/bouguetj/calib_doc/

Camera parameters

- Extrinsic Parameters: Since camera may not be at the origin, there is a rigid transformation between the world coordinates and the camera coordinates
- Intrinsic parameters: Since scene units (e.g., cm) differ image units (e.g., pixels) and coordinate system may not be centered in image, we capture that with a 3x3 transformation comprised of focal length, principal point, pixel aspect ratio, and skew

$$\begin{pmatrix} x \\ y \\ w \end{pmatrix} = \underbrace{\begin{pmatrix} \text{Transformation} \\ \text{represented by} \\ \text{intrinsic parameters} \end{pmatrix}}_{3 \text{ x } 3} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} \text{Rigid Transformation} \\ \text{represented by} \\ \text{extrinsic parameters} \end{pmatrix}}_{4 \text{ x } 4} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

Camera Models

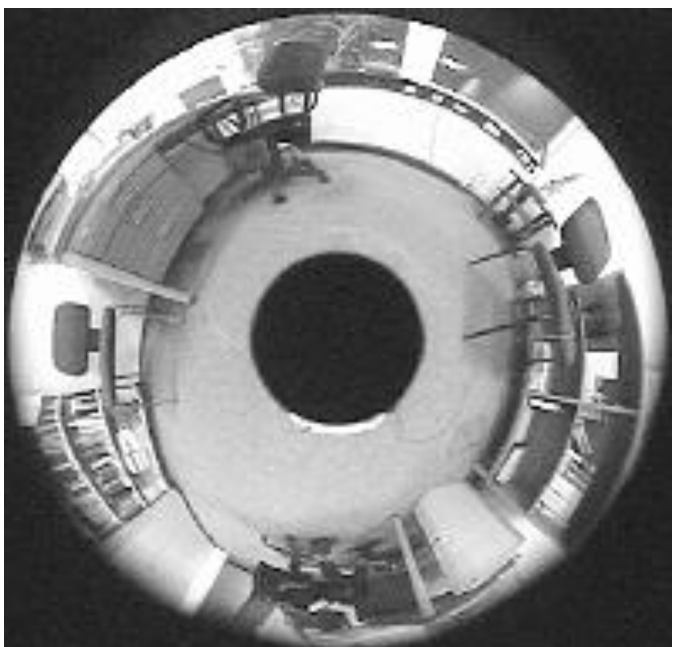


For all cameras?

Other camera models

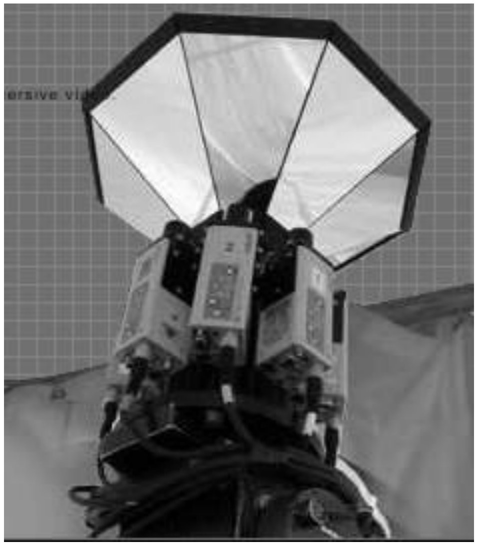
- Generalized camera – maps points lying on rays and maps them to points on the image plane.

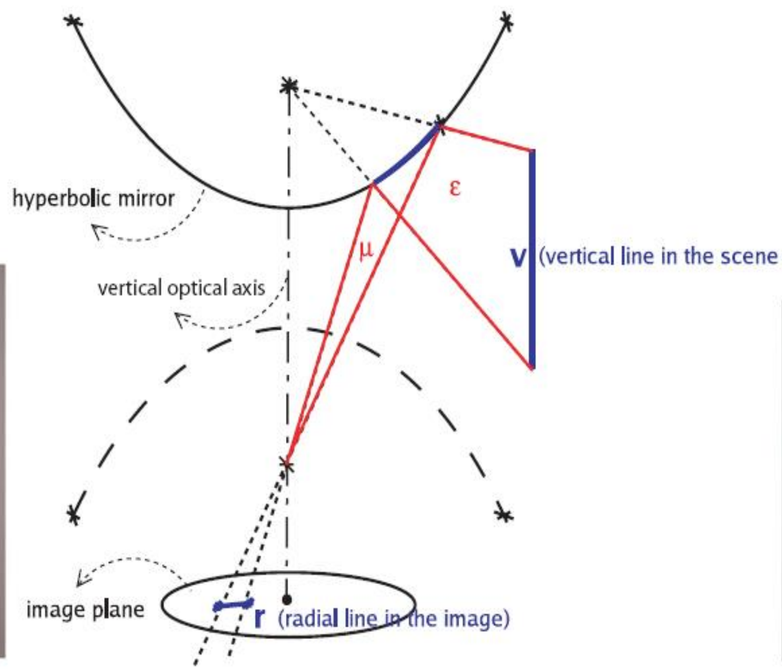
Omnicam (hemispherical)

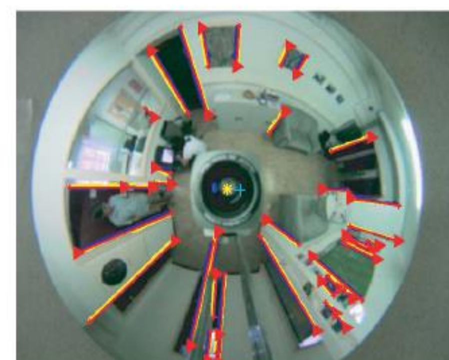


Light Probe (spherical)

Some Alternative "Cameras"

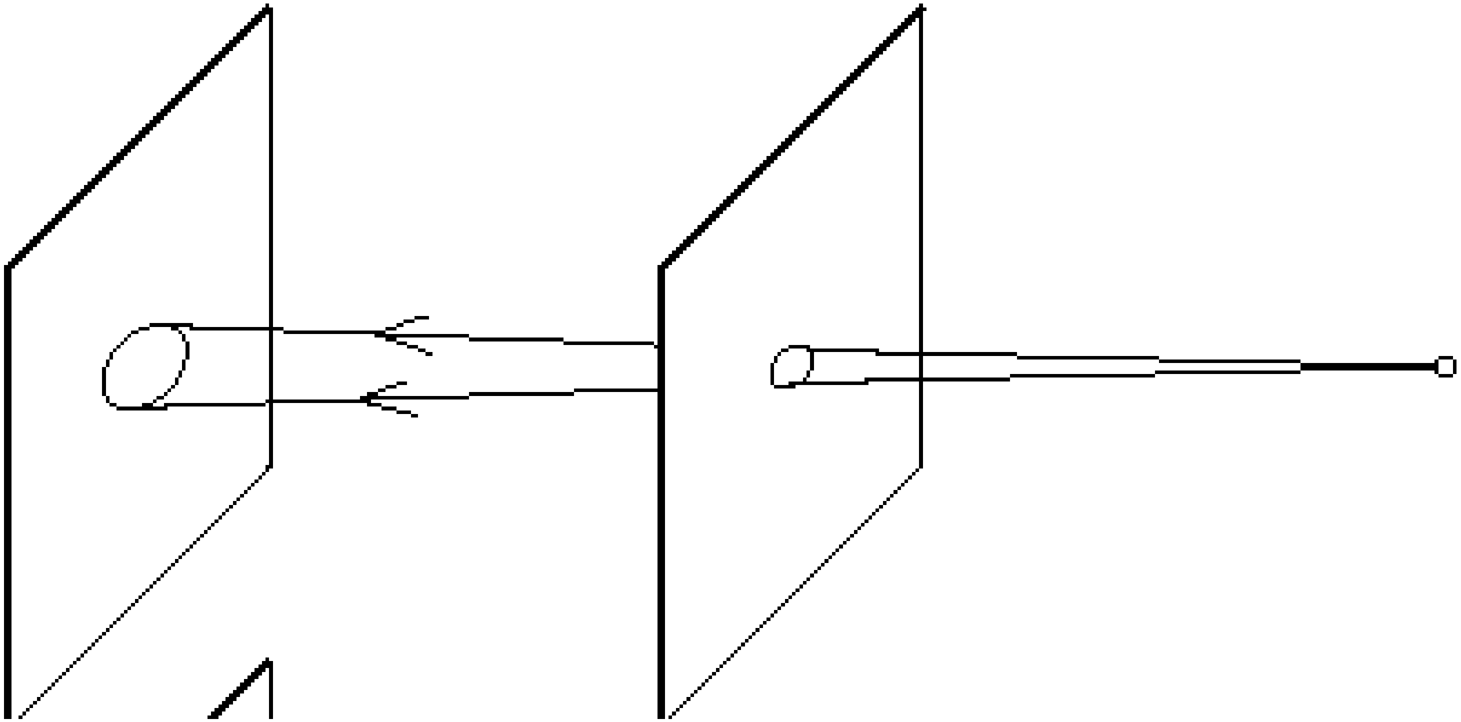






Lenses

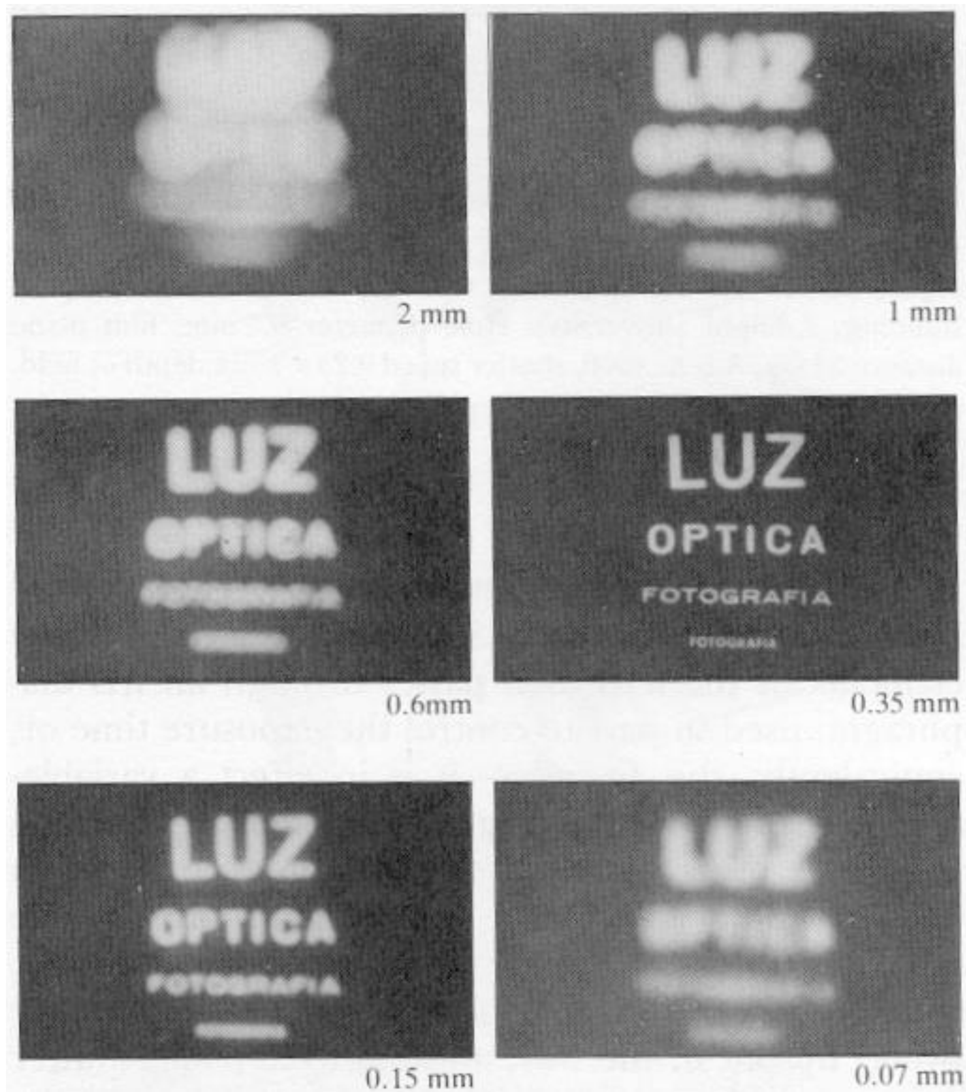
Beyond the pinhole Camera
Getting more light – Bigger Aperture



Pinhole Camera Images with Variable Aperture

2 mm

1mm



.6 mm

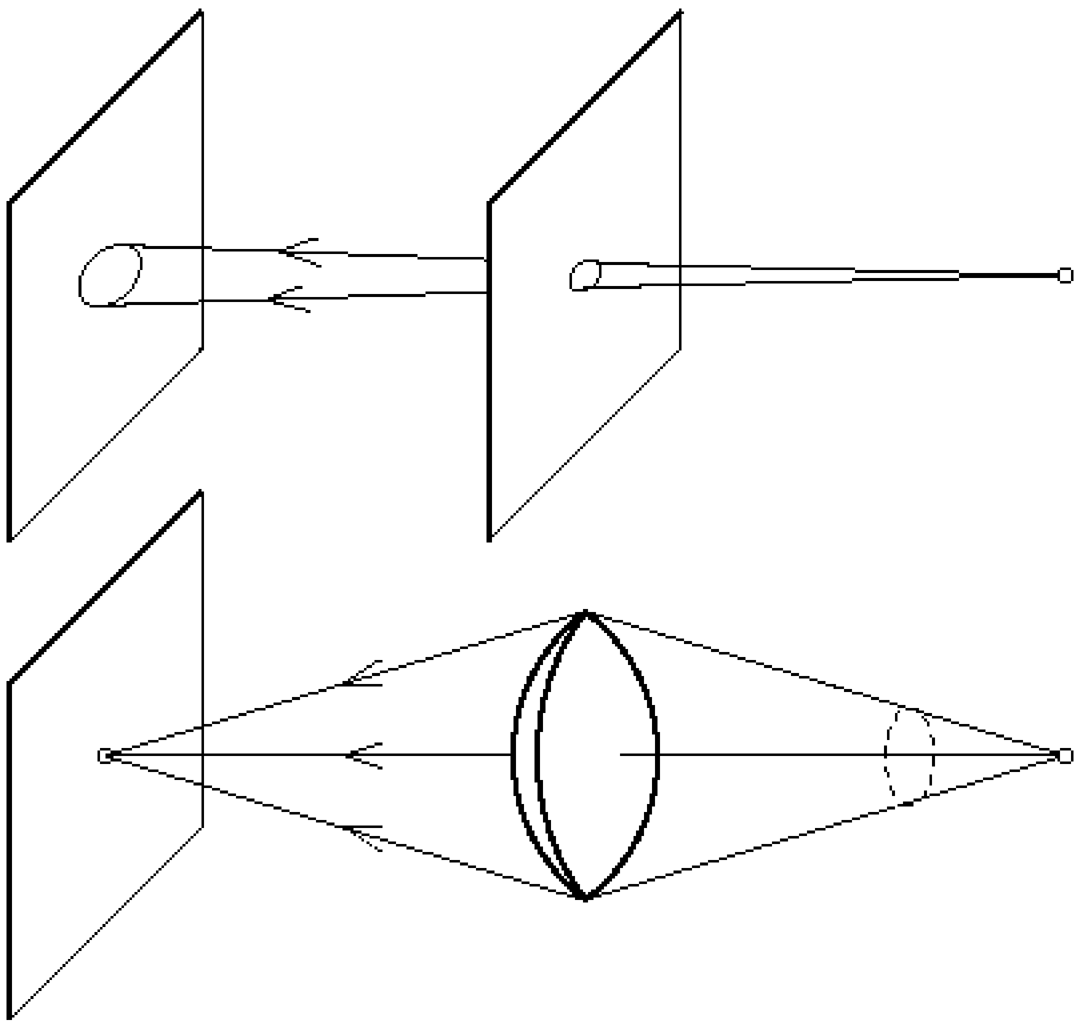
.35 mm

.15 mm

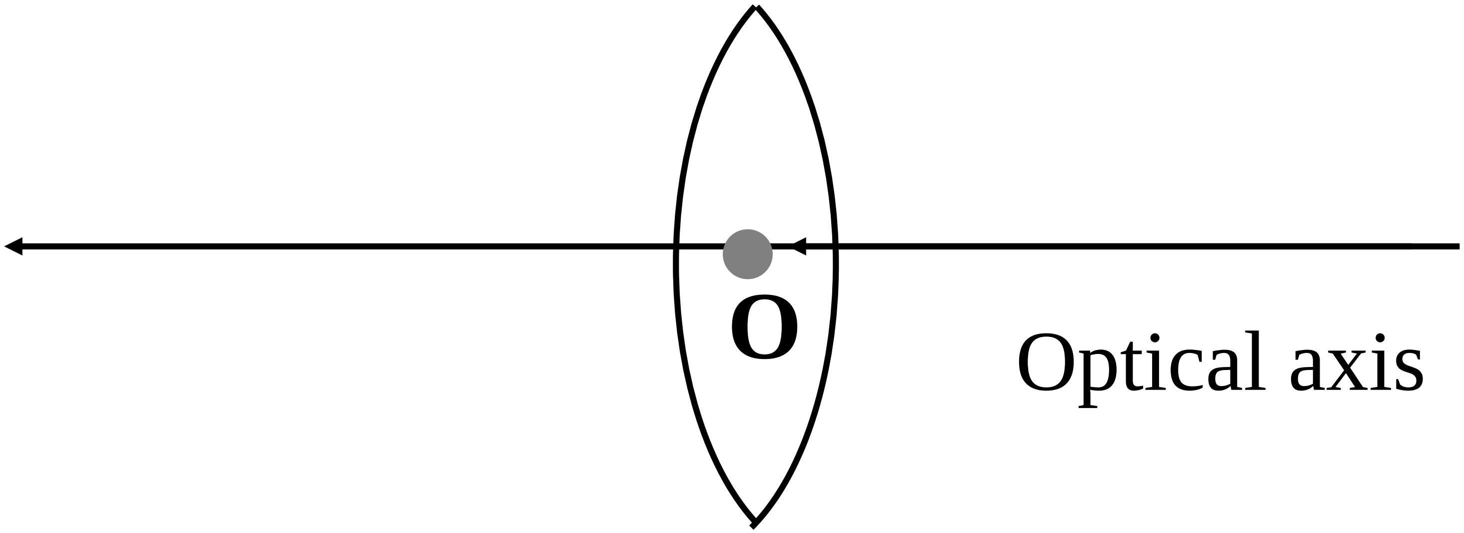
.07 mm

The reason for lenses

We need light, but big pinholes cause blur.

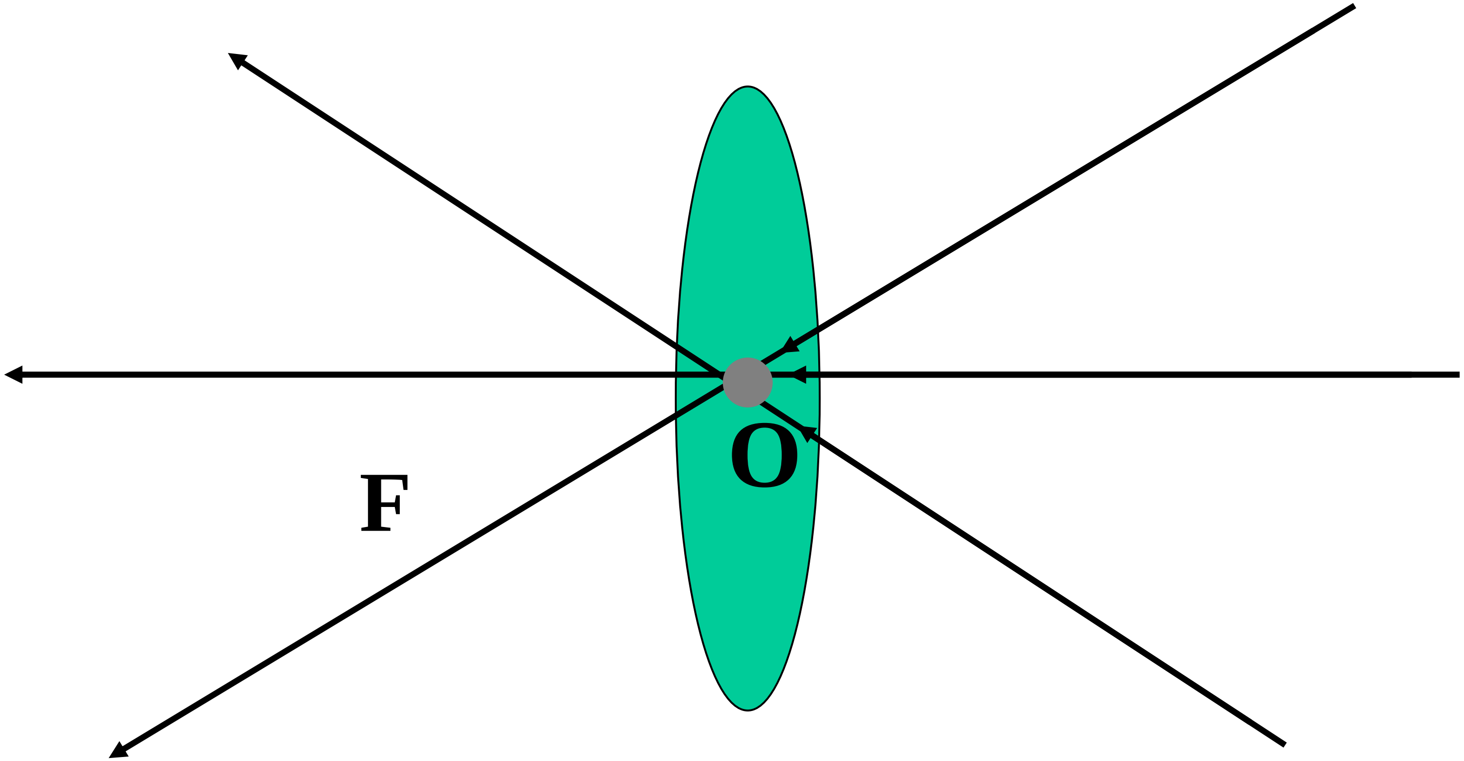


Thin Lens



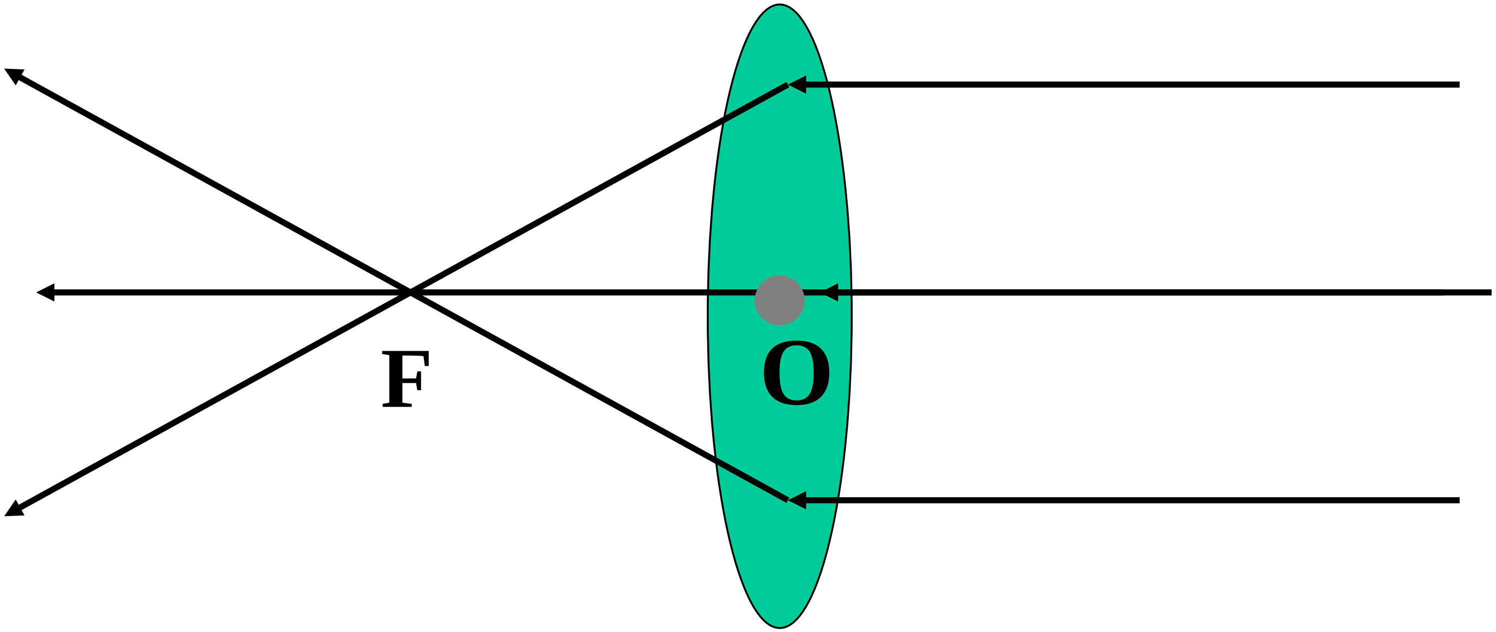
- Rotationally symmetric about optical axis.
- Spherical interfaces.

Thin Lens: Center



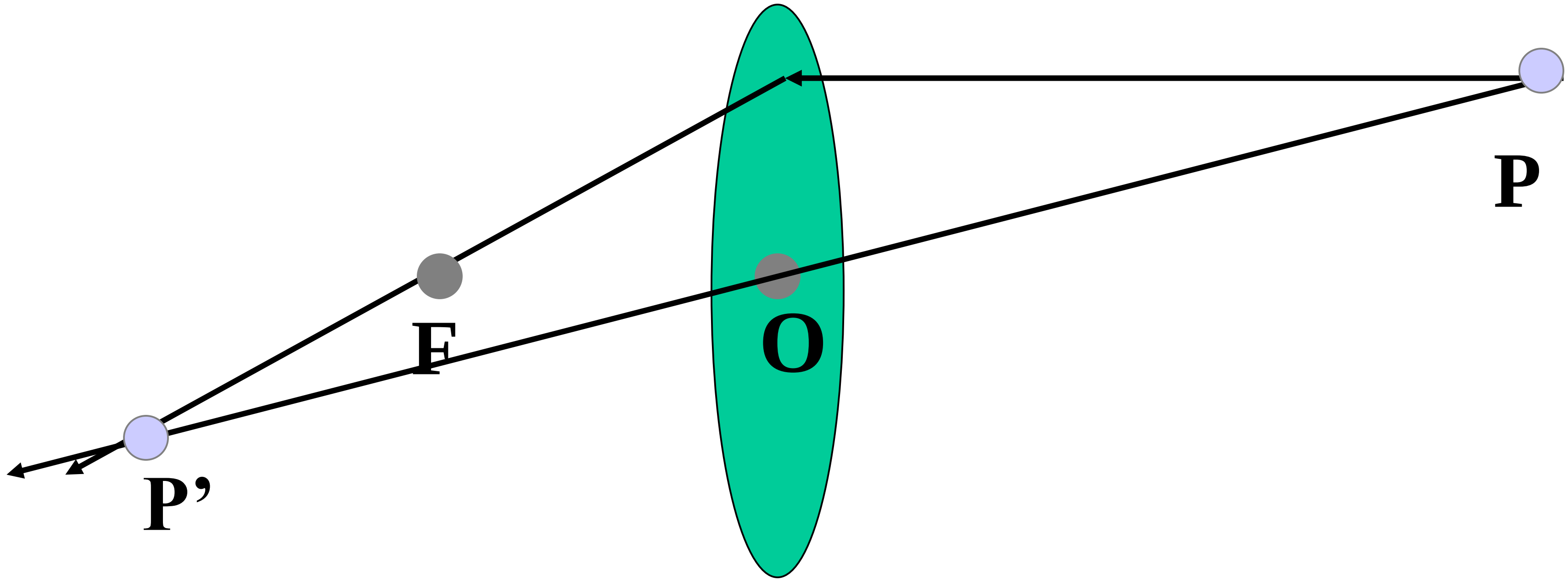
- All rays that enter lens along line pointing at **O** emerge in same direction.

Thin Lens: Focus



Parallel lines pass through the focus, F

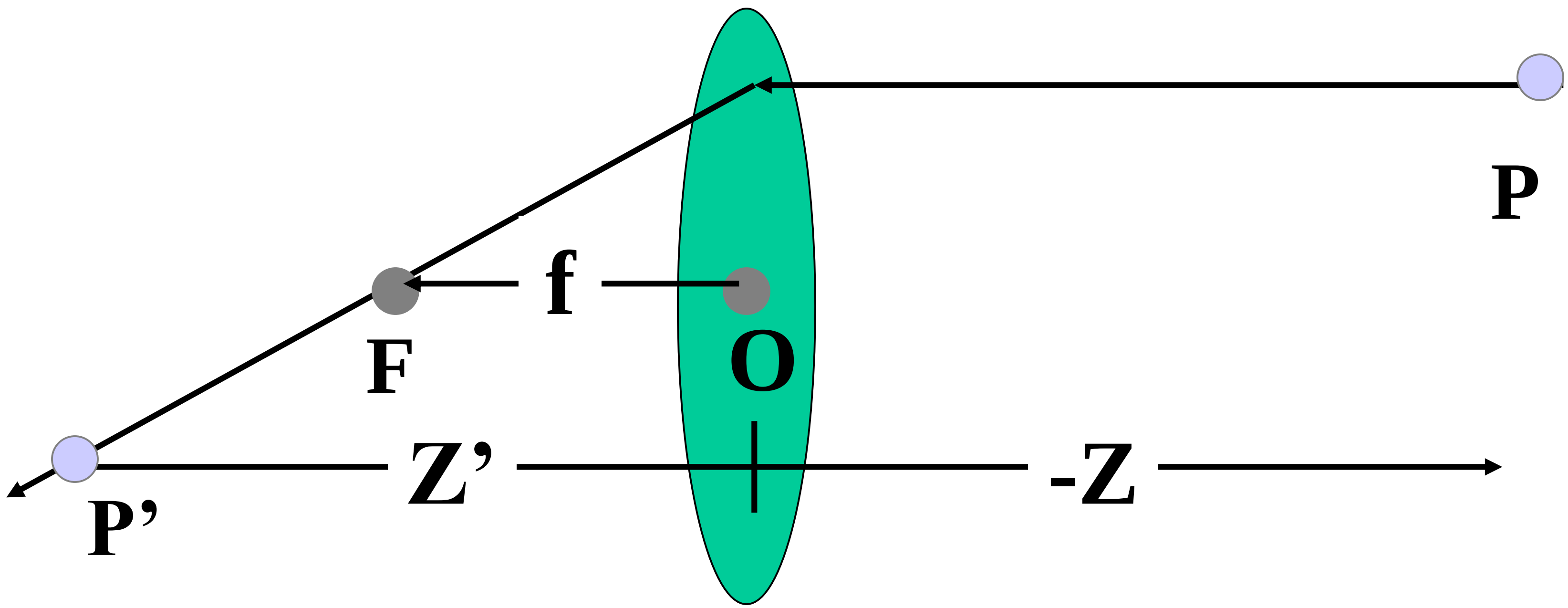
Thin Lens: Image of Point



– All rays passing through lens and starting at **P** converge upon **P’**

– So light gather capability of lens is given the area of the lens and all the rays focus on P’ instead of become blurred like a pinhole

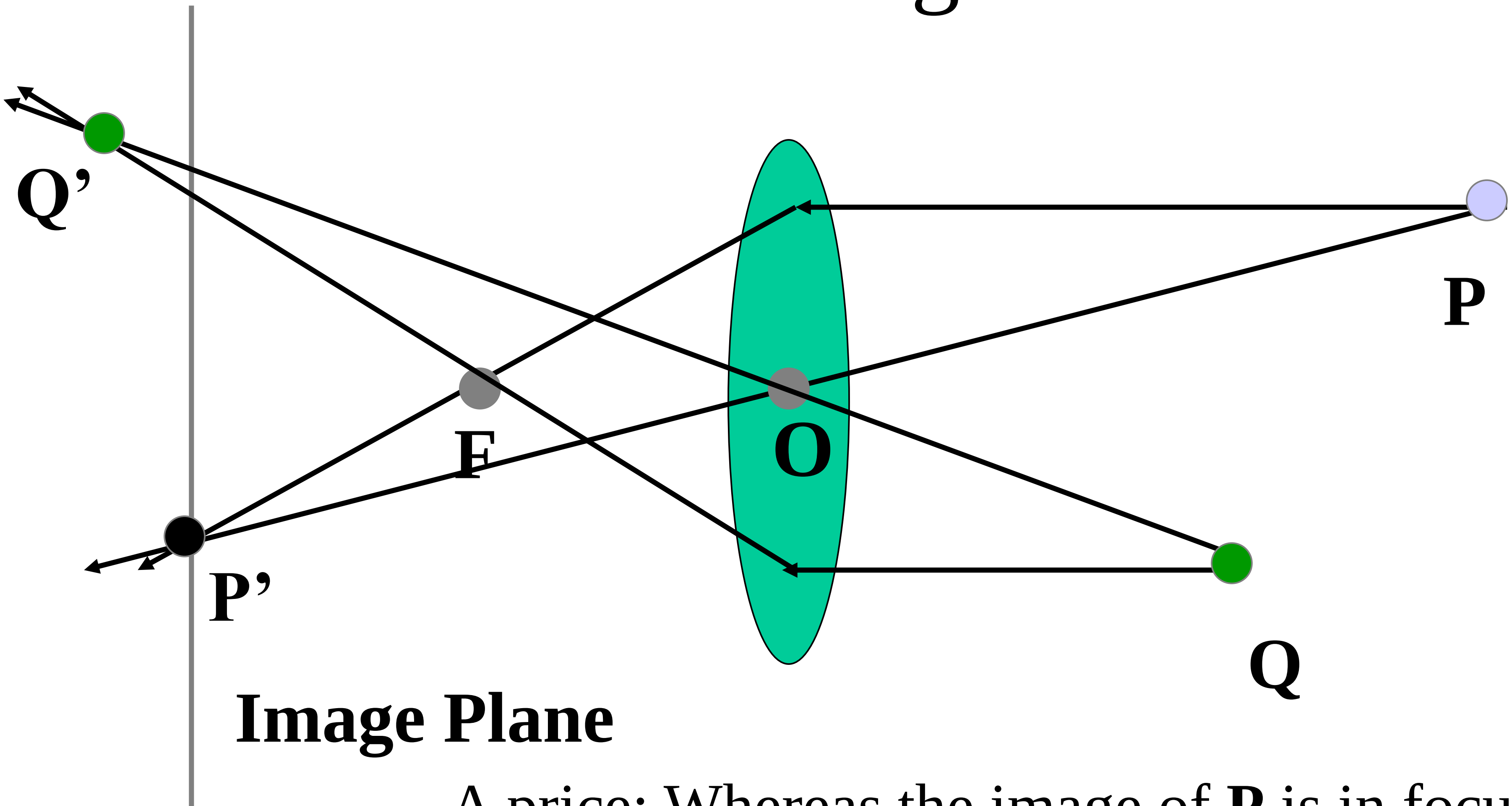
Thin Lens: Image of Point



$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

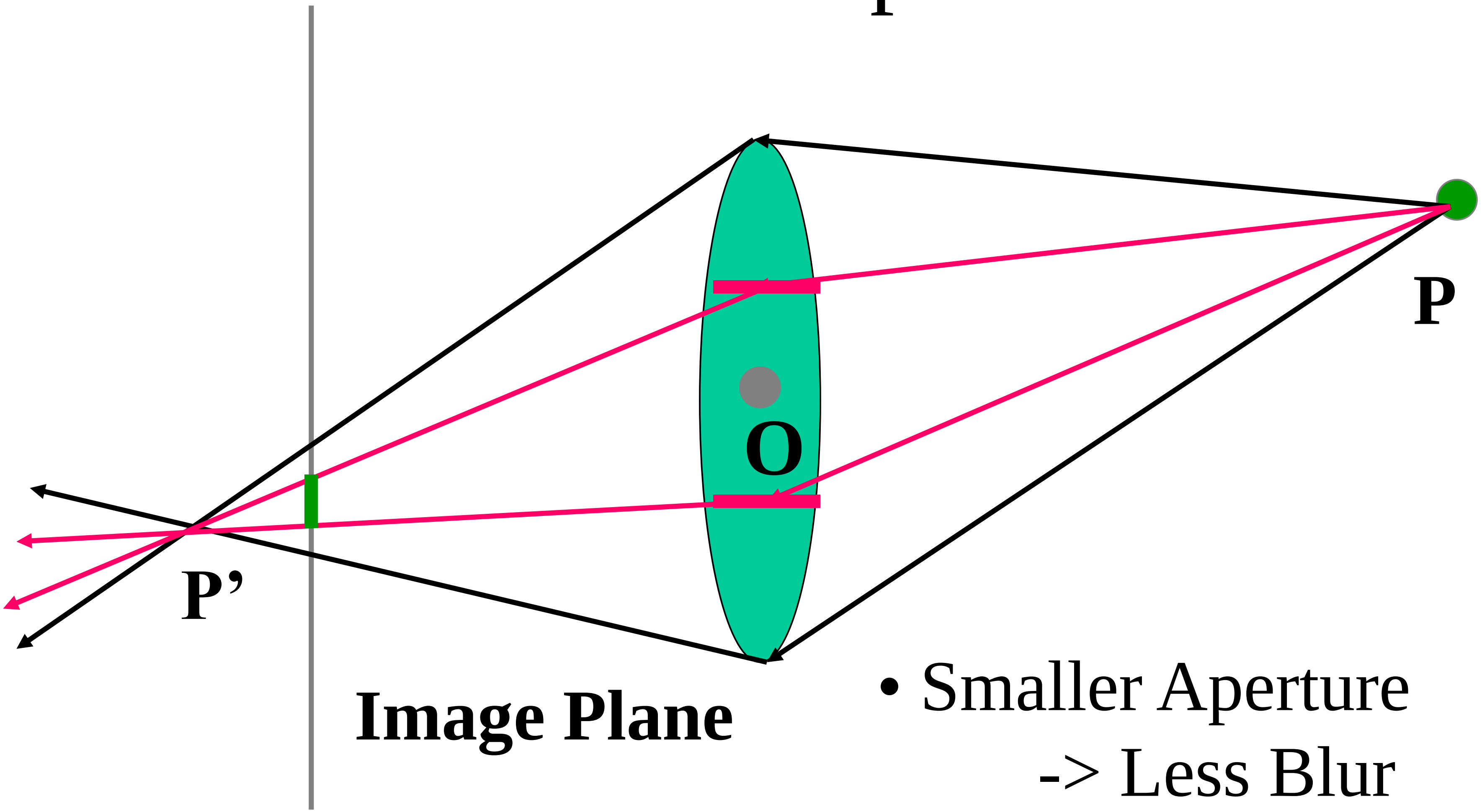
Relation between depth of Point (-Z) and the depth where it focuses (Z’)

Thin Lens: Image Plane



A price: Whereas the image of **P** is in focus, the image of **Q** isn't.

Thin Lens: Aperture



- Smaller Aperture
  -> Less Blur
- Pinhole -> No Blur

Next Lecture

- Image Formation: Light and Shading
- Reading:
  – Chapter 2: Light and Shading