

# Edge Detection and Corner Detection

Computer Vision I

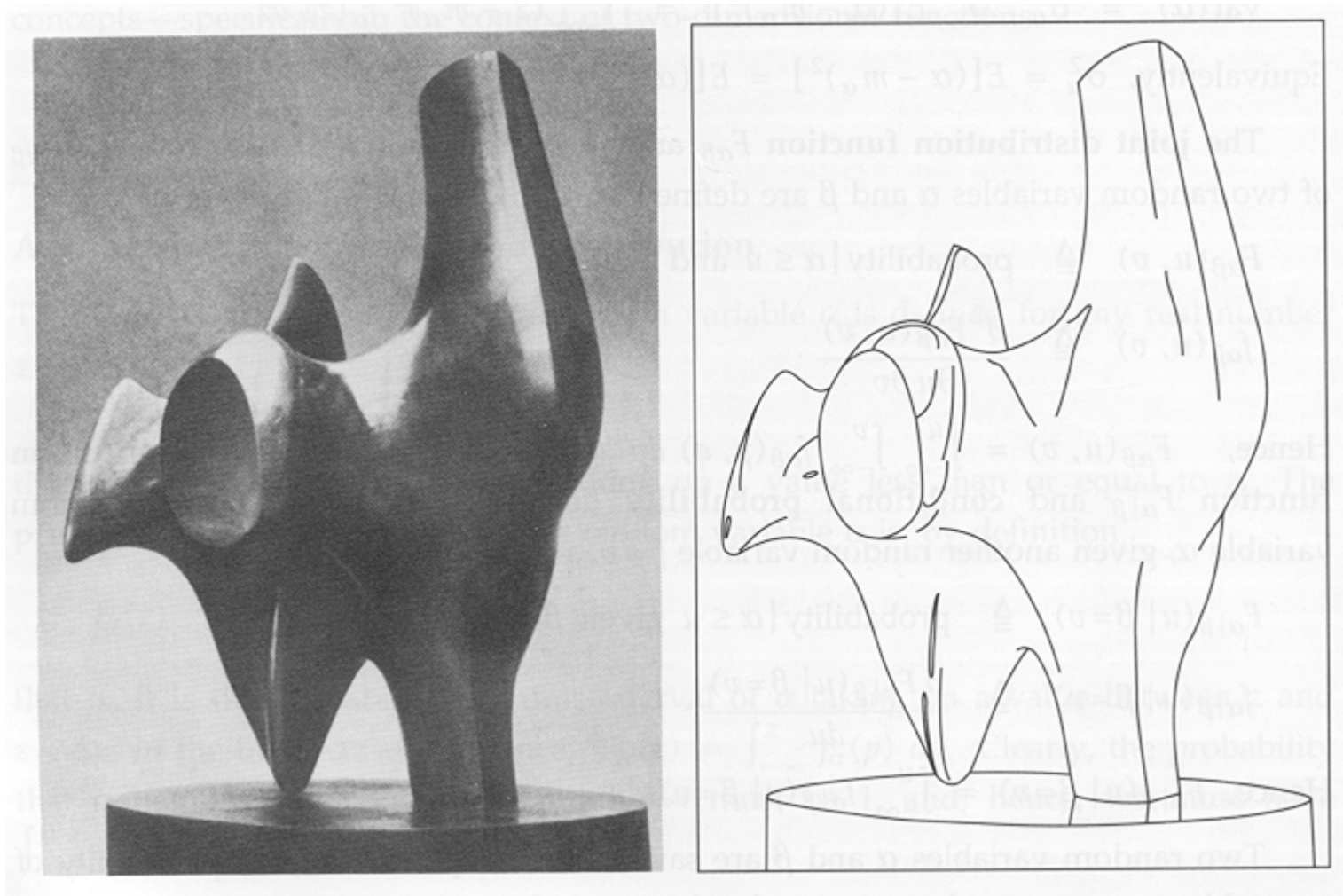
CSE 252A

Lecture 7

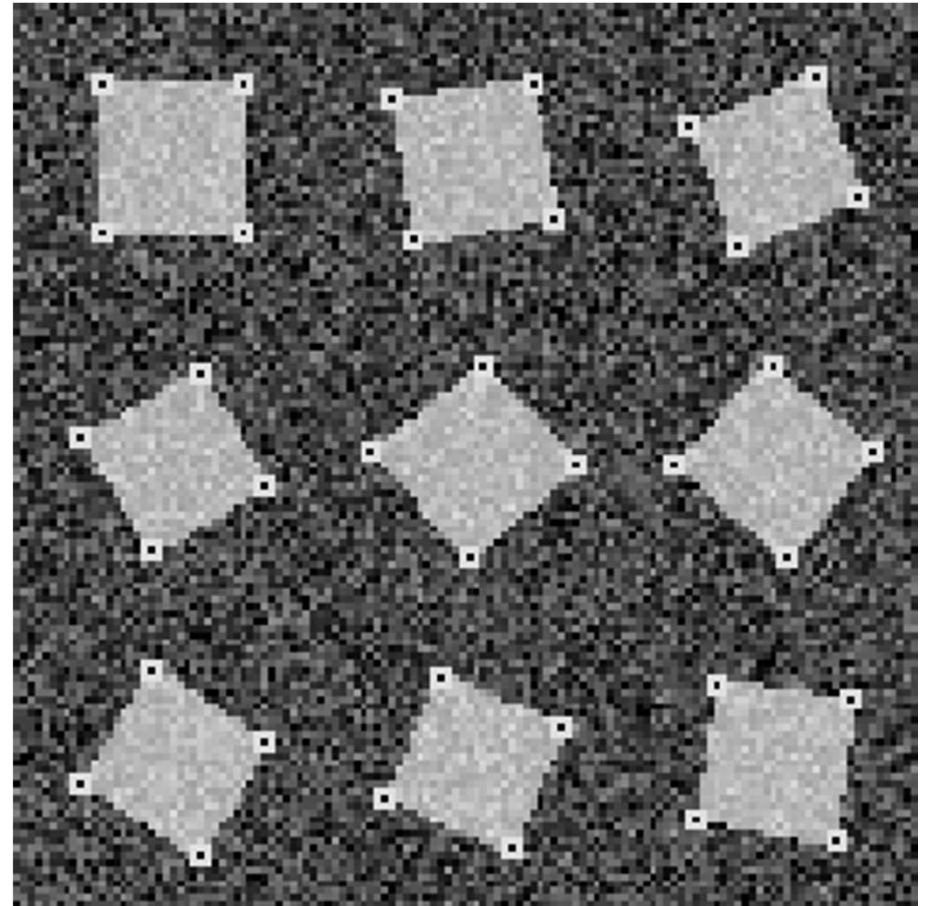
# Announcements

- Homework 2 is due Oct 22, 11:59 PM
- Homework 3 will be assigned on Oct 22
- Reading:
  - Chapter 5: Local Image Features

# Edges



# Corners



# Edges

What is an edge?

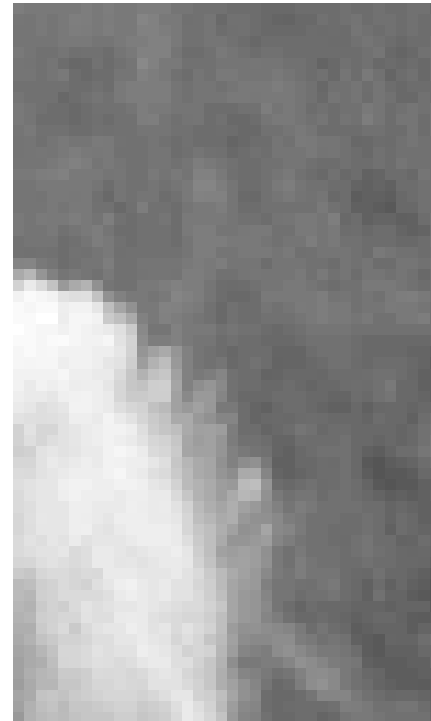
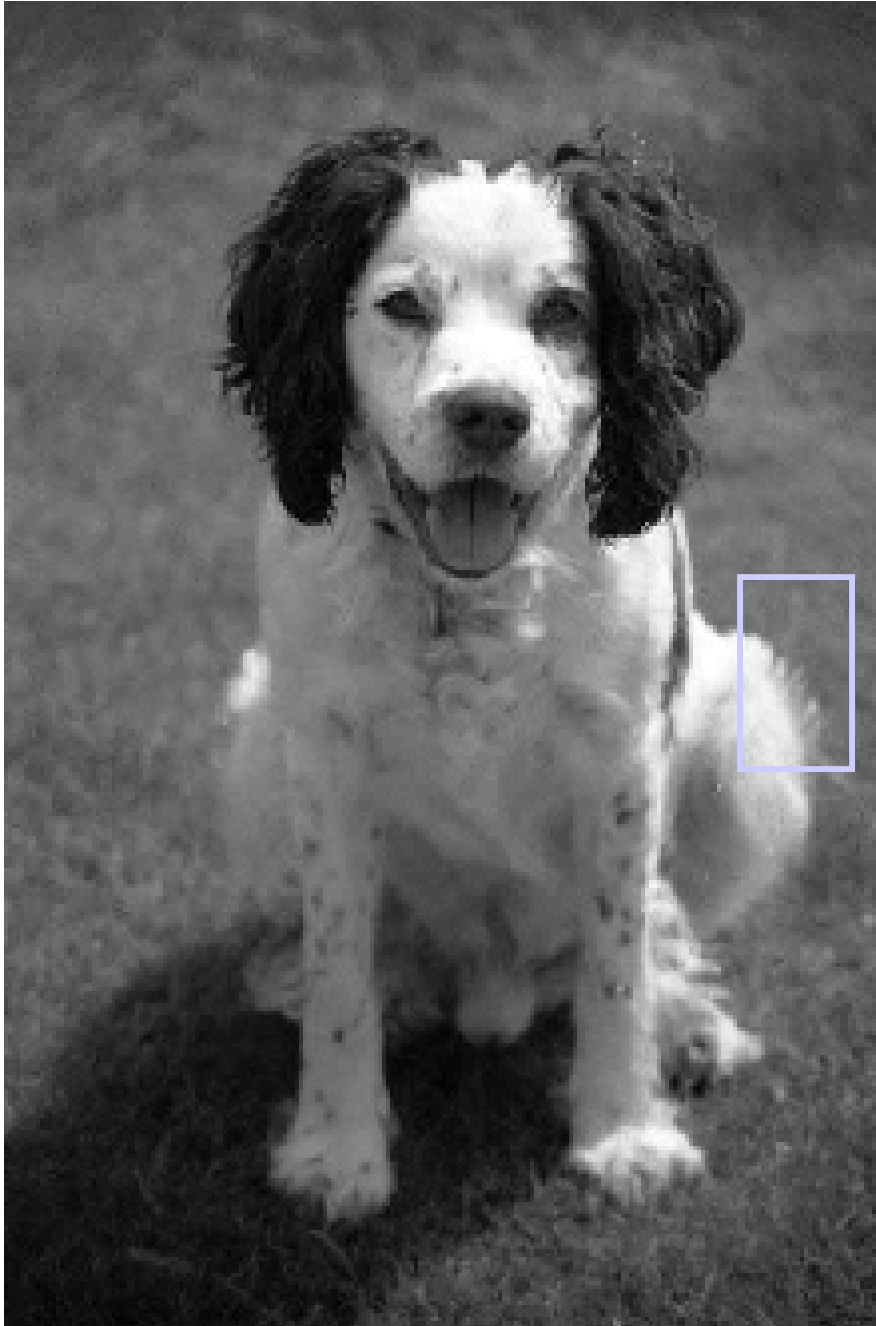
A discontinuity in image intensity.



Physical causes of edges

1. Object boundaries
2. Surface normal discontinuities
3. Reflectance (albedo) discontinuities
4. Lighting discontinuities (shadow boundaries)

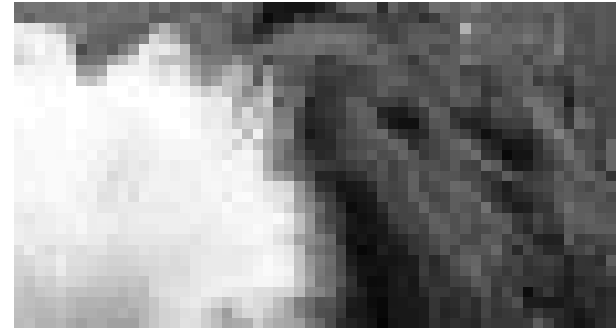
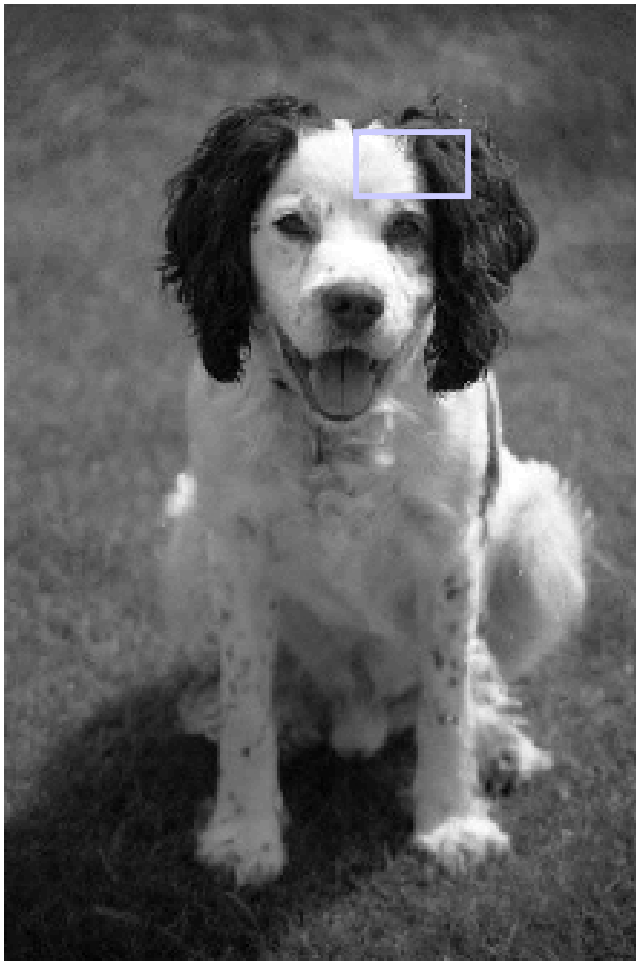
# Object Boundaries



# Surface normal discontinuities

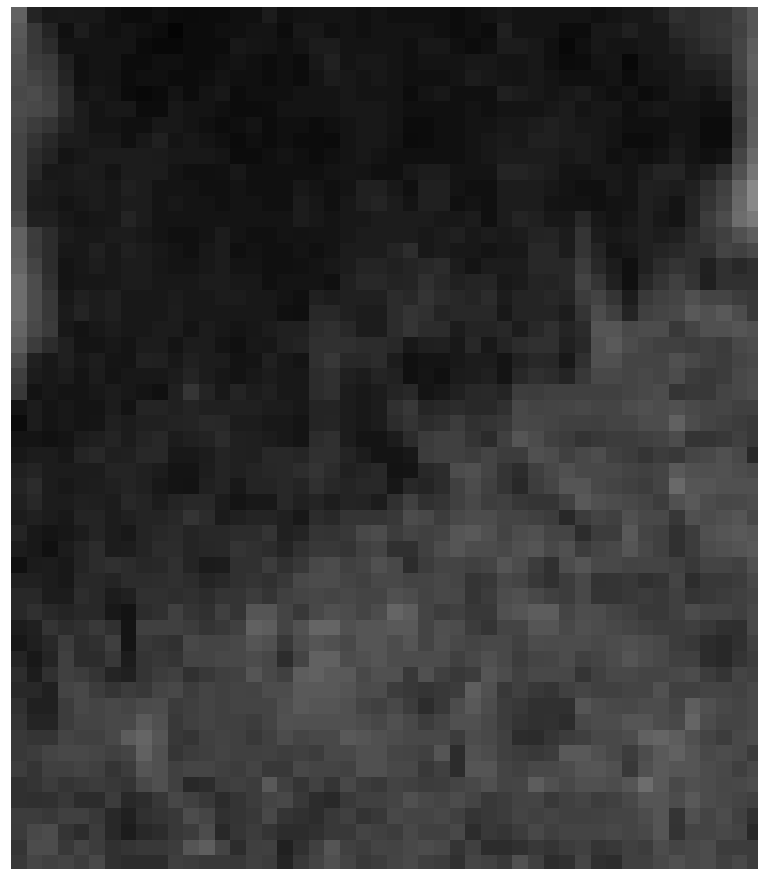
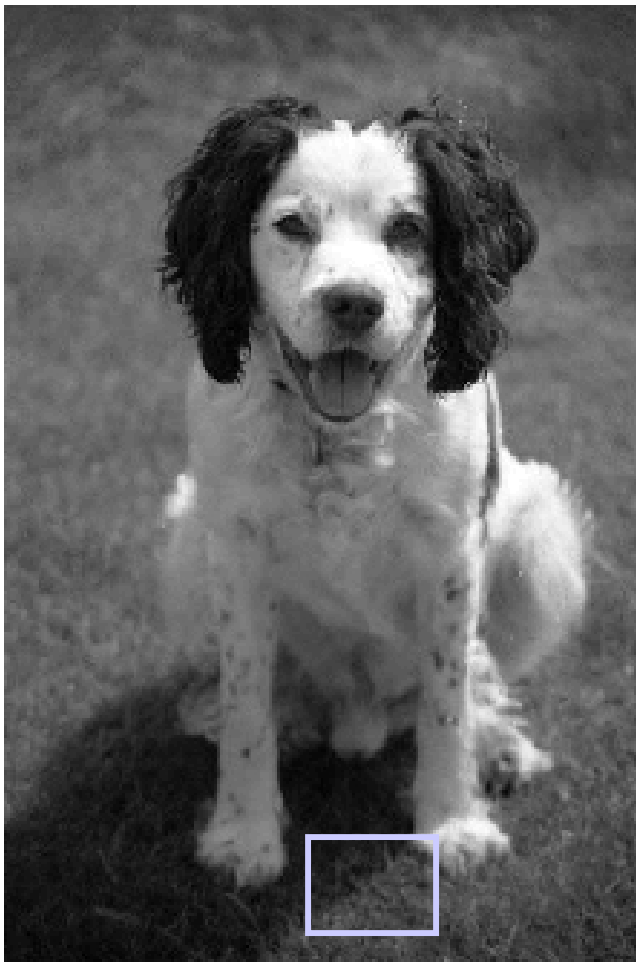


# Boundaries of materials properties

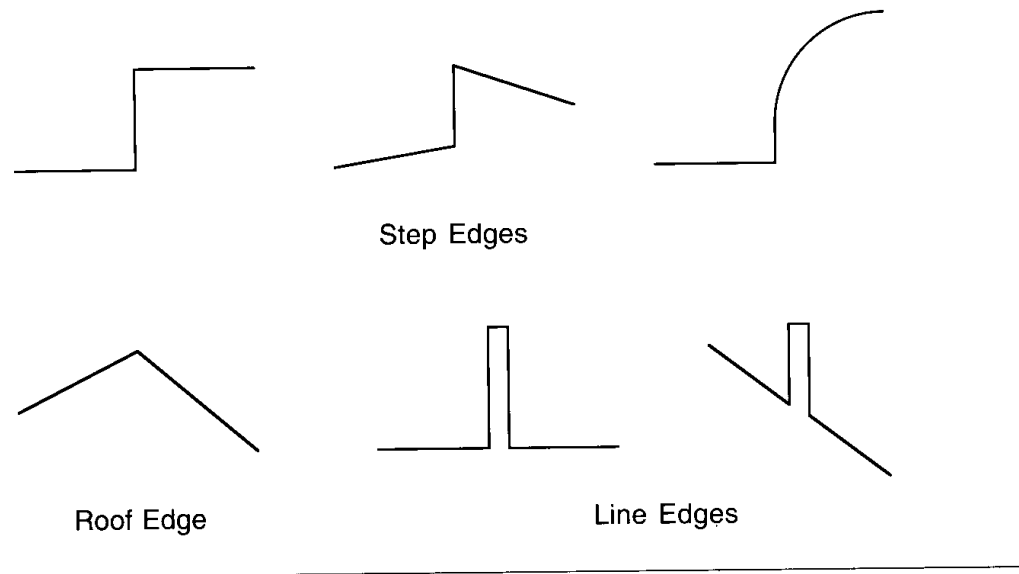




# Boundaries of lighting

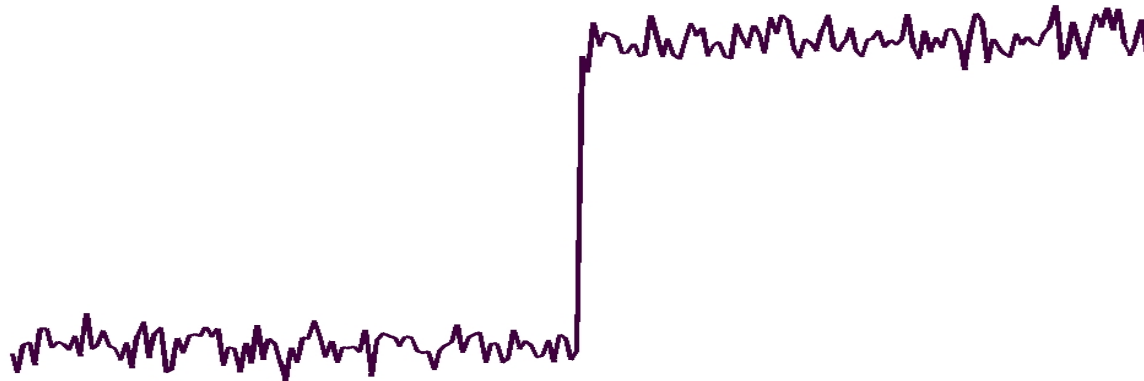


# Profiles of image intensity edges



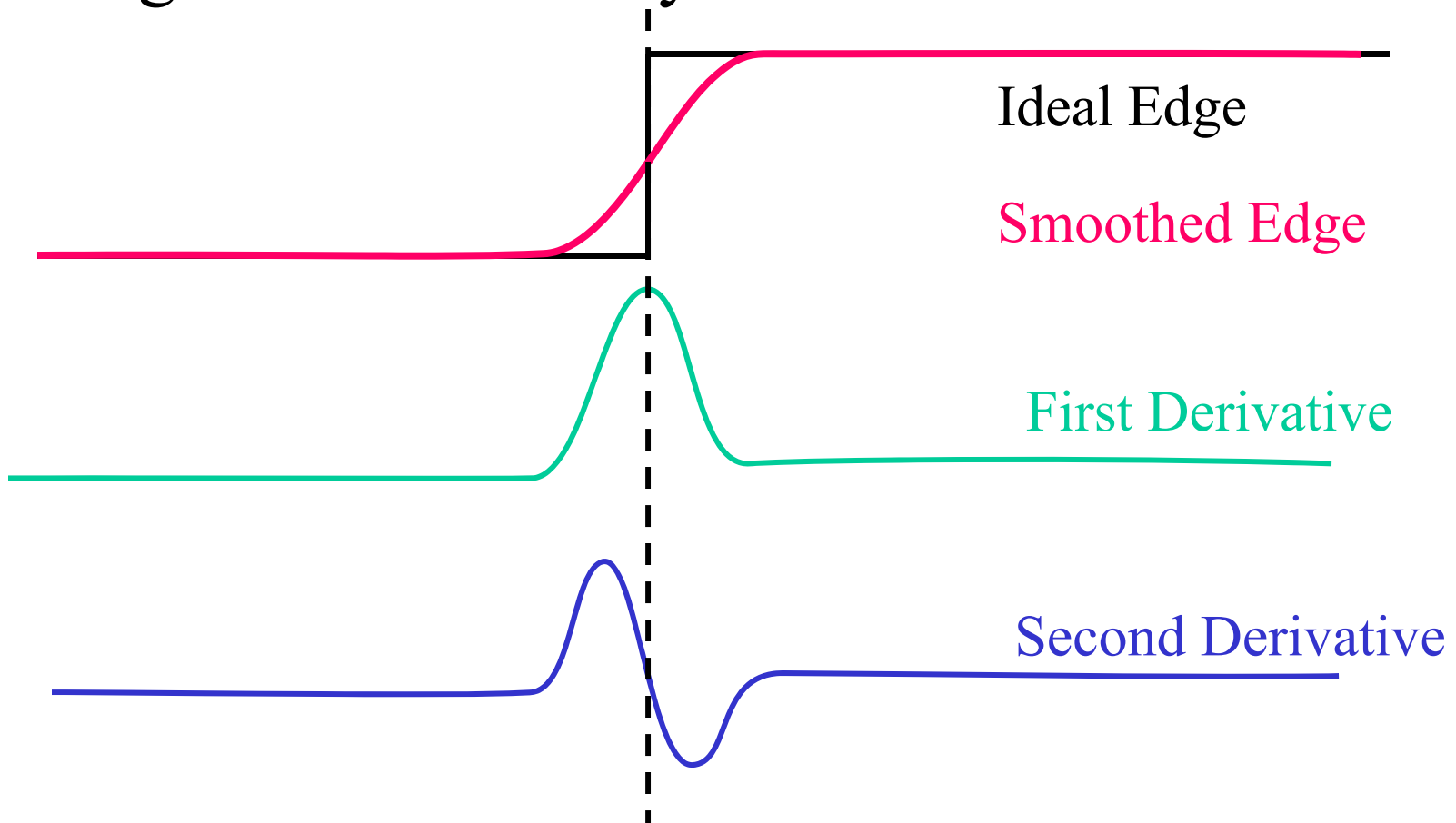
# Noisy Step Edge

- Derivative is high everywhere.
- Must smooth before taking gradient.



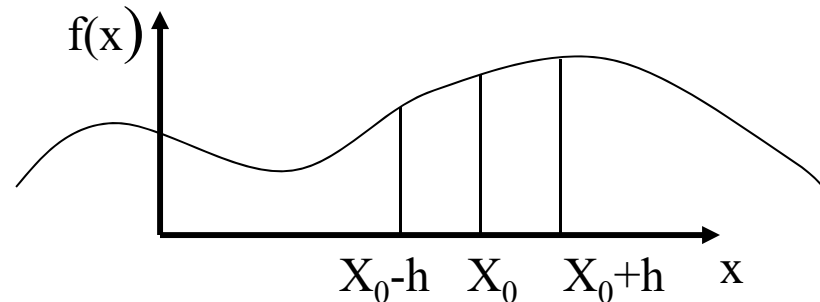
# Edge is Where Change Occurs: 1-D

- Change is measured by derivative in 1D



- Biggest change, derivative has maximum magnitude
- Or 2nd derivative is zero.

# Numerical Derivatives



Take Taylor series expansion of  $f(x)$  about  $x_0$

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{1}{2} f''(x_0)(x-x_0)^2 + \dots$$

Consider samples taken at increments of  $h$  and first two terms of the expansion, we have

$$f(x_0+h) = f(x_0) + f'(x_0)h + \frac{1}{2} f''(x_0)h^2$$

$$f(x_0-h) = f(x_0) - f'(x_0)h + \frac{1}{2} f''(x_0)h^2$$

Subtracting and adding  $f(x_0+h)$  and  $f(x_0-h)$  respectively yields

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$$

$$f''(x_0) = \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2}$$

Convolve with

First Derivative:  $[-1/2h \ 0 \ 1/2h]$

Second Derivative:  $[1/h^2 \ -2/h^2 \ 1/h^2]$

# Numerical Derivatives

Convolution kernel  
First Derivative:  $[-1/2h \ 0 \ 1/2h]$   
Second Derivative:  $[1/h^2 \ -2/h^2 \ 1/h^2]$

- With images, units of  $h$  is pixels, so  $h=1$ 
  - First derivative:  $[-1/2 \ 0 \ 1/2]$
  - Second derivative:  $[1 \ -2 \ 1]$
- When computing derivatives in the  $x$  and  $y$  directions, use these convolution kernels:

$$\frac{d}{dx} = [-1/2 \ 0 \ 1/2] \qquad \frac{d}{dy} = \begin{bmatrix} -1/2 \\ 0 \\ 1/2 \end{bmatrix}$$

# Implementing 1-D Edge Detection

1. Filter out noise: convolve with Gaussian
2. Take a derivative: convolve with  $[-1/2 \ 0 \ 1/2]$ 
  - We can combine 1 and 2.
3. Find the peak: Two issues:
  - Should be a local maximum.
  - Should be sufficiently high.

# 2D Edge Detection

## 1. Filter out noise

- Use a 2D Gaussian Filter.

## 2. Take a derivative

$$J = I \otimes G$$

- Compute the magnitude of the gradient:

$$\nabla J = (J_x, J_y) = \left( \frac{\partial J}{\partial x}, \frac{\partial J}{\partial y} \right) \text{ is the gradient}$$

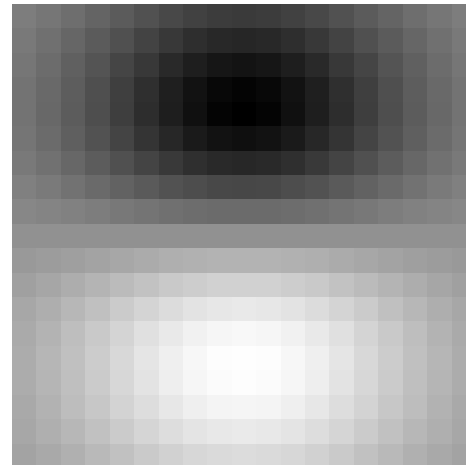
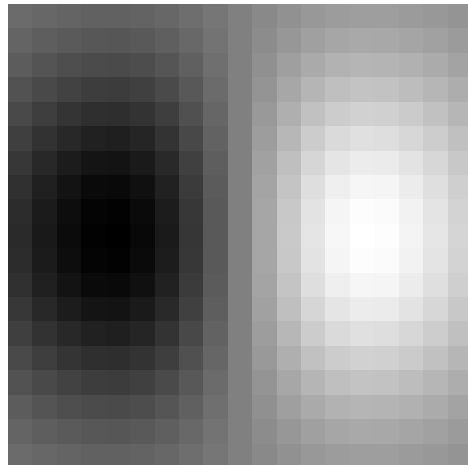
$$\|\nabla J\| = \sqrt{J_x^2 + J_y^2} \text{ is the magnitude of the gradient}$$

$$\tan^{-1}(J_y, J_x) \text{ the direction of the gradient}$$



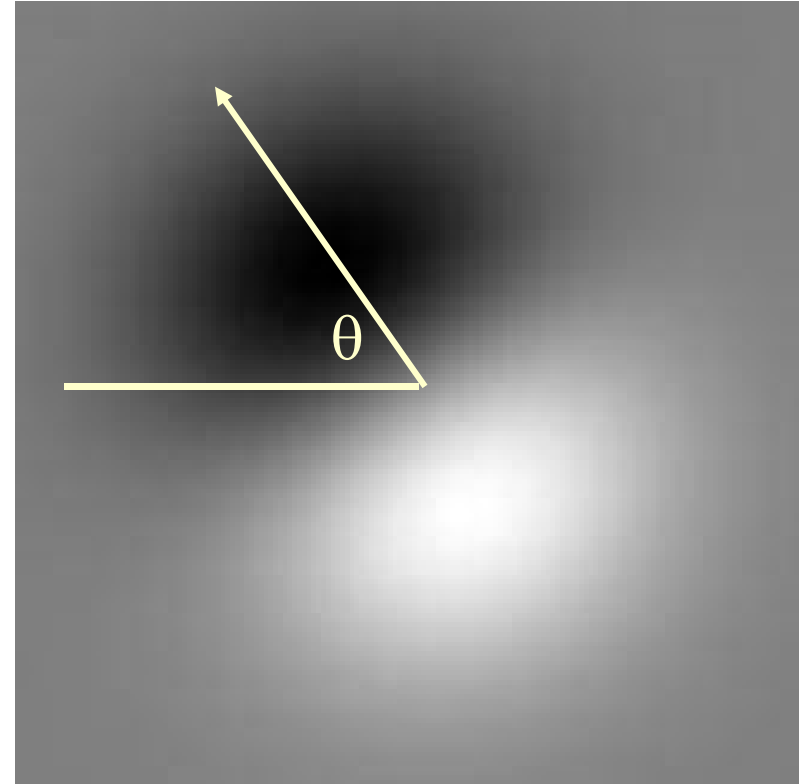
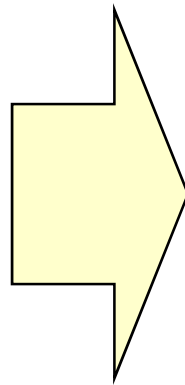
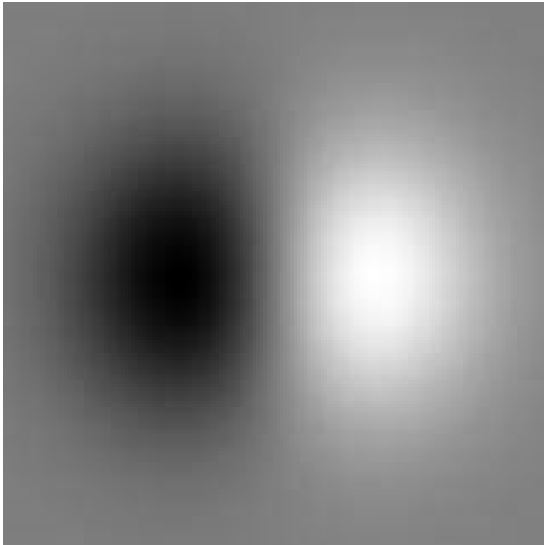
# Smoothing and Differentiation

- Need two derivatives, in x and y direction.
- Filter with Gaussian and then compute Gradient, OR
- Use a derivative of Gaussian filter
  - because differentiation is convolution, and convolution is associative

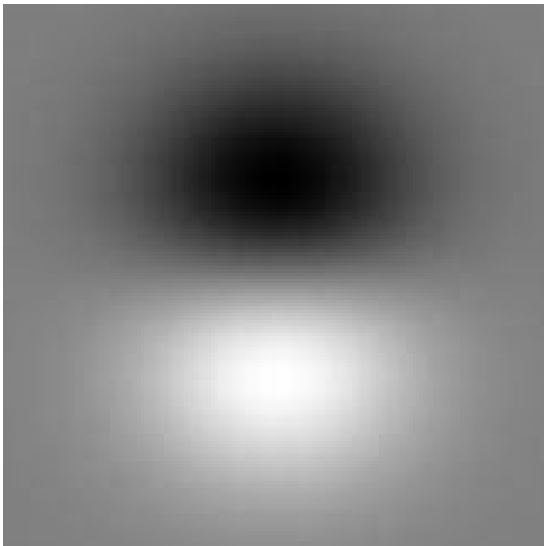


# Directional Derivatives

$$\frac{\partial G_\sigma}{\partial x}$$



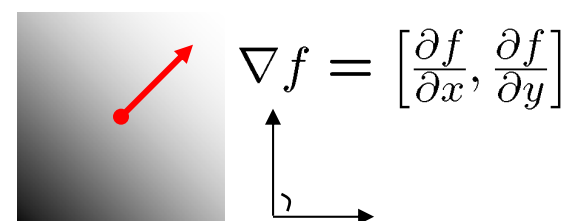
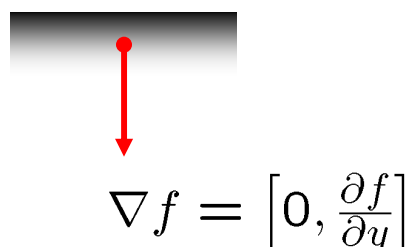
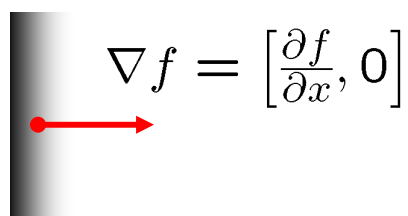
$$\frac{\partial G_\sigma}{\partial y}$$



$$\cos \theta \frac{\partial G_\sigma}{\partial x} + \sin \theta \frac{\partial G_\sigma}{\partial y}$$

# Gradient

- Given a function  $f(x,y)$  -- e.g., intensity is  $f$
- Gradient equation:  $\nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$
- Represents direction of most rapid change in intensity

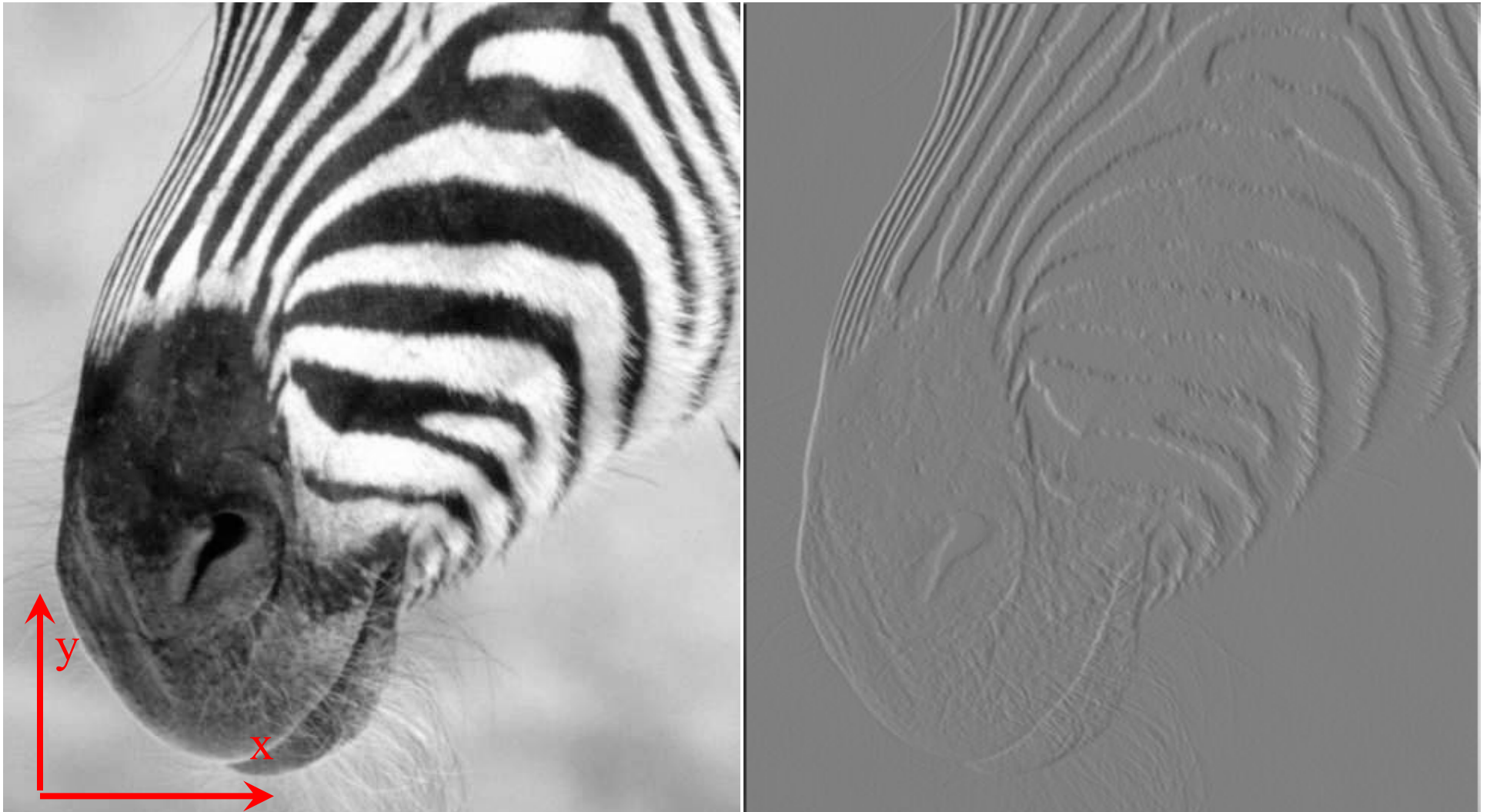


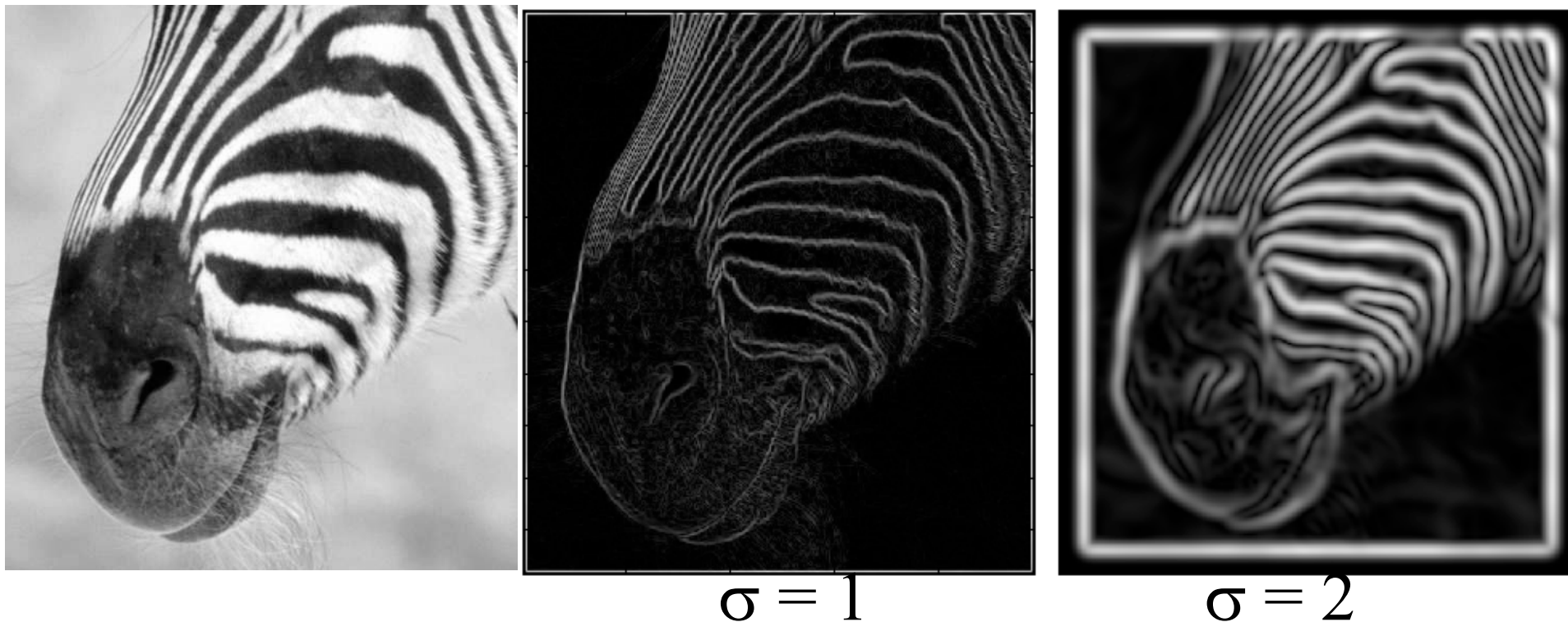
- Gradient direction:  $\theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$
- The *edge strength* is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

# Finding derivatives

Is this  $dI/dx$  or  $dI/dy$ ?





There are three major issues:

1. The gradient magnitude at different scales is different; which scale should we choose?
2. The gradient magnitude is large along a thick trail; how do we identify the significant points?
3. How do we link the relevant points up into curves?

There is ALWAYS a tradeoff between smoothing and good edge localization!

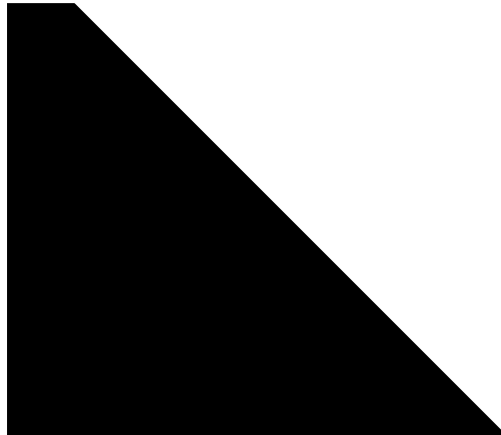
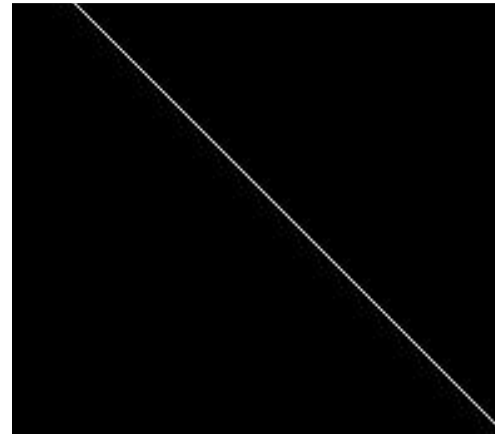


Image with Edge (No Noise)



Edge Location

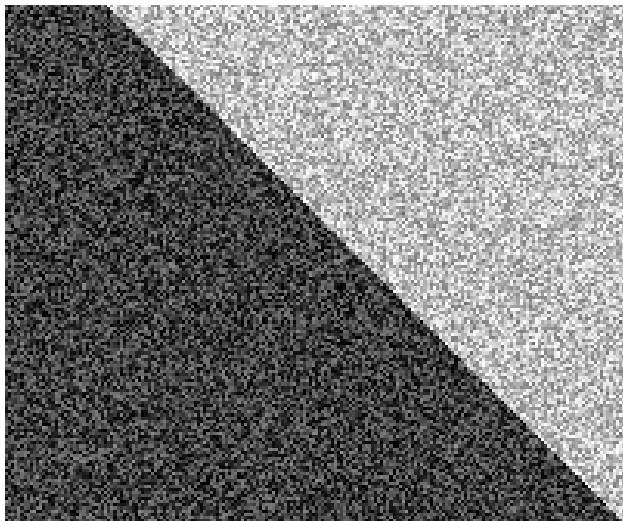
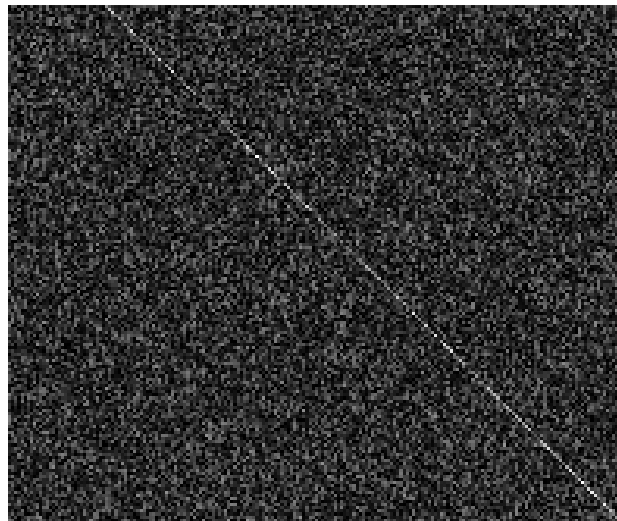
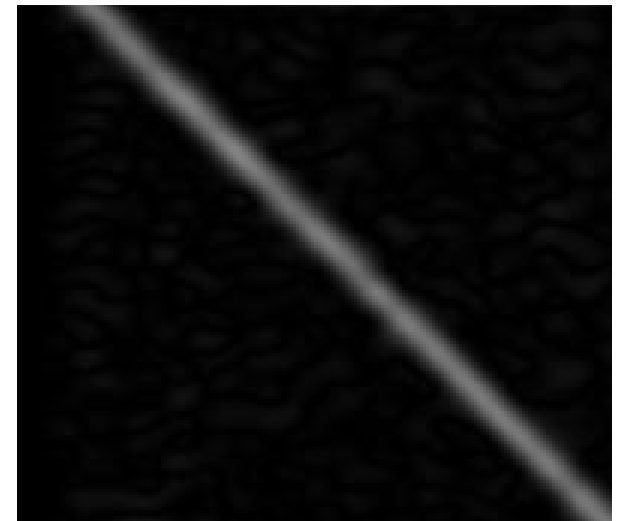


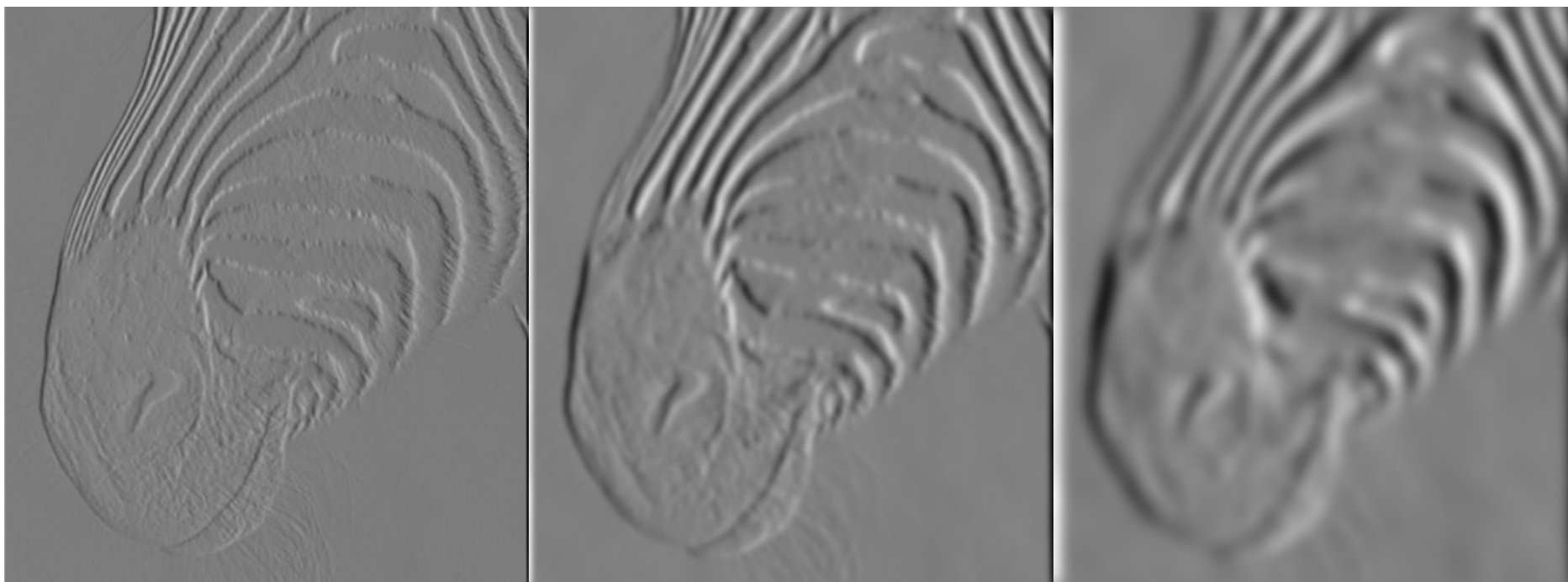
Image + Noise



Derivatives detect  
edge *and* noise



Smoothed derivative removes  
noise, but blurs edge

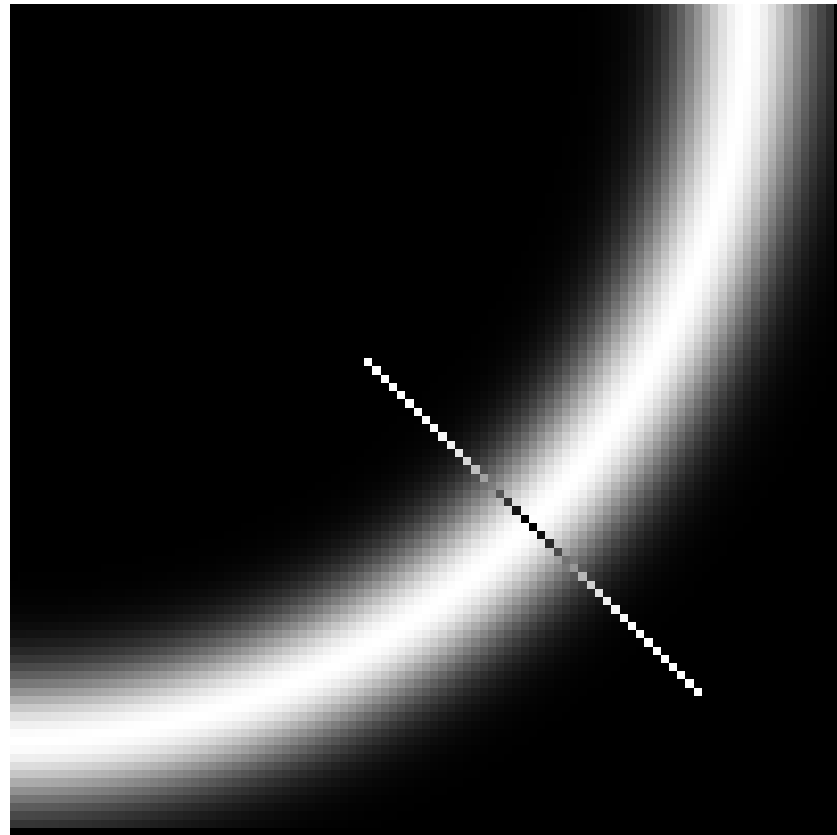
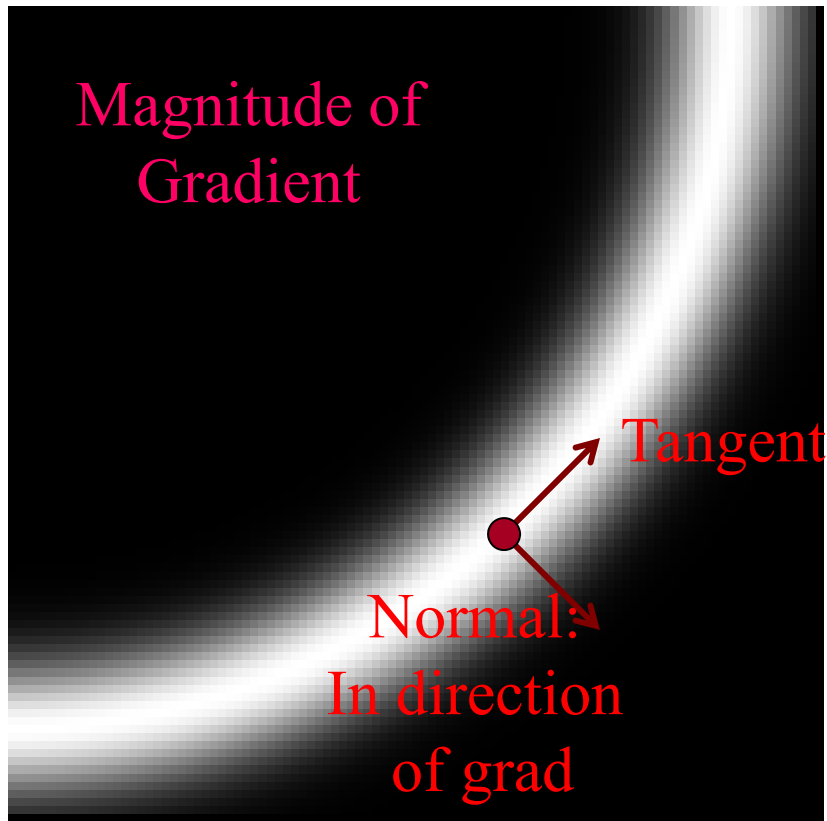


1 pixel

3 pixels

7 pixels

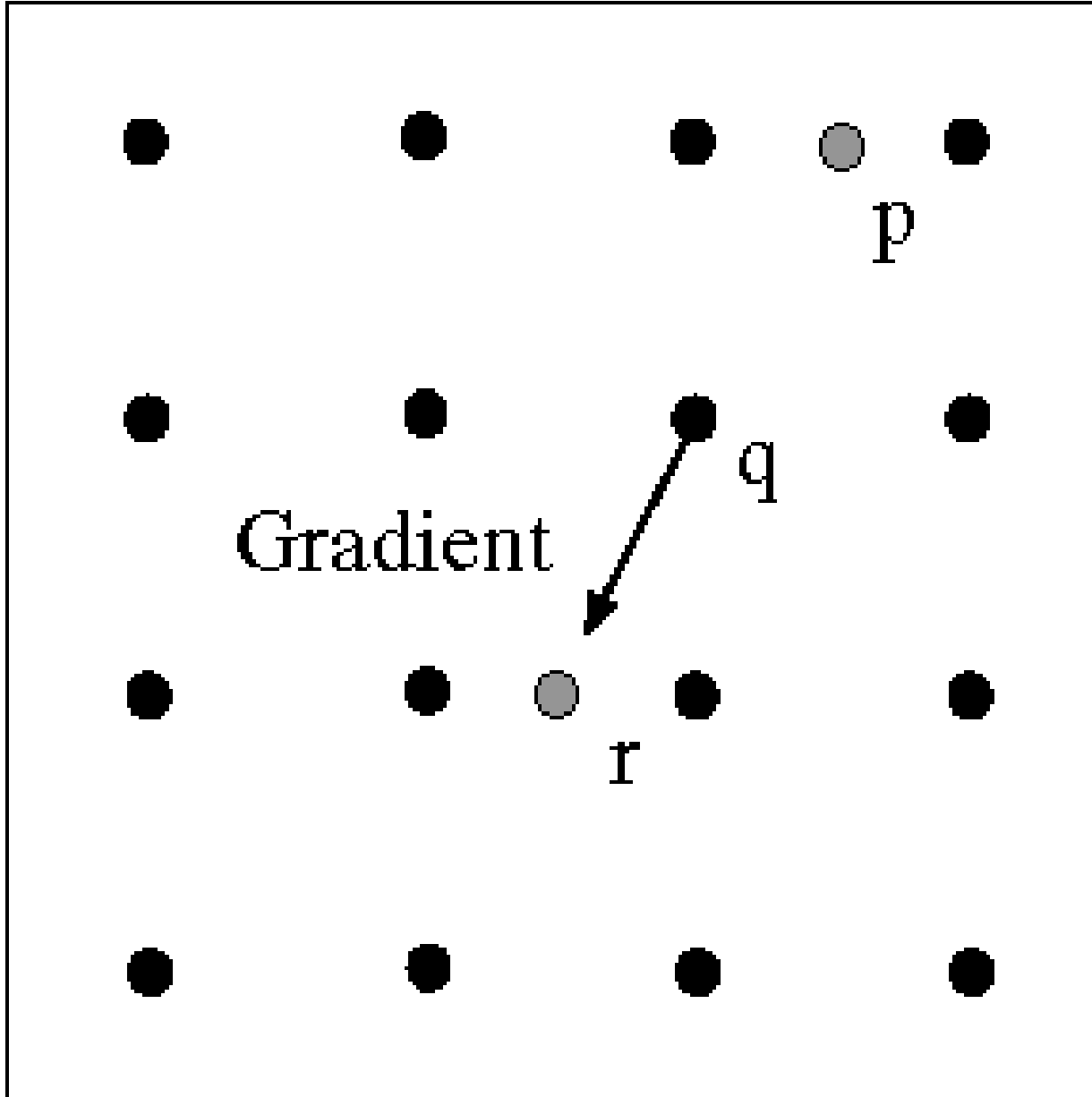
The scale of the smoothing filter affects derivative estimates



We wish to mark points along the curve where the magnitude is biggest. We can do this by looking for a maximum along a slice normal to the curve (non-maximum suppression). These points should form a curve. There are then two algorithmic issues: which point is the maximum, and where is the next point on the curve?



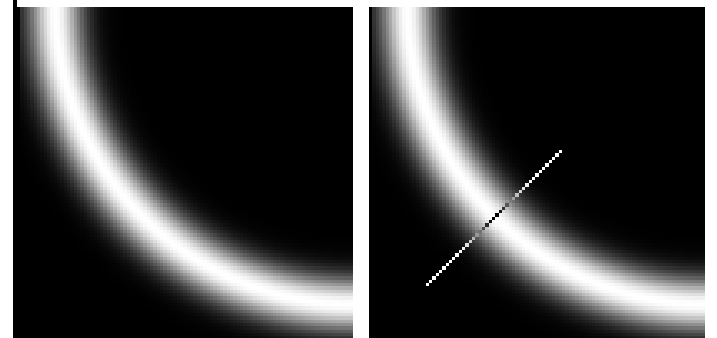
# Non-maximum suppression



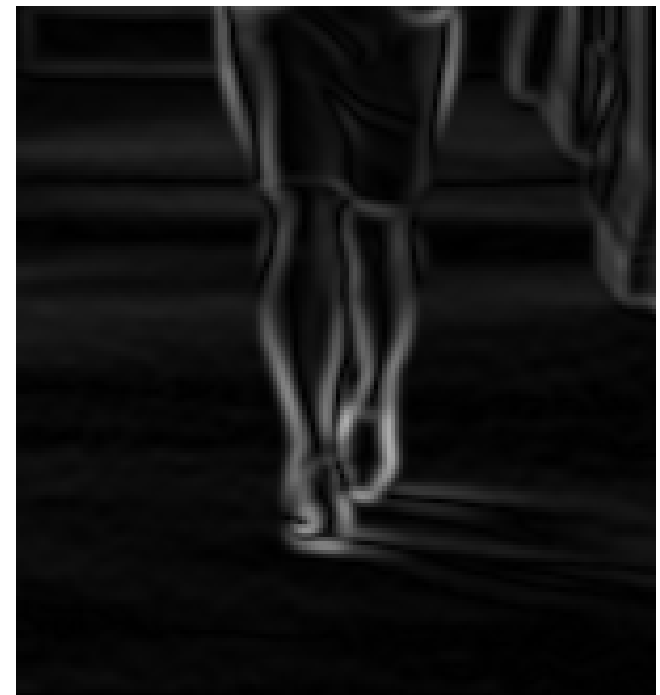
Using normal at  $q$ , find two points  $p$  and  $r$  on adjacent rows (or columns)

$q$  is a maximum if  $|\nabla J(q)|$  is larger than  $|\nabla J(p)|$  and  $|\nabla J(r)|$

Interpolate to get values

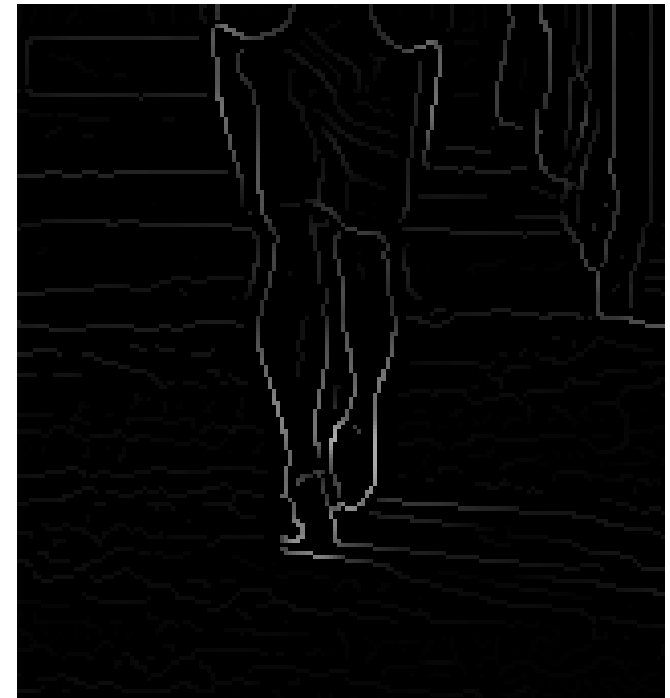


# Before Non-max Suppression



Gradient magnitude (x4 for visualization)

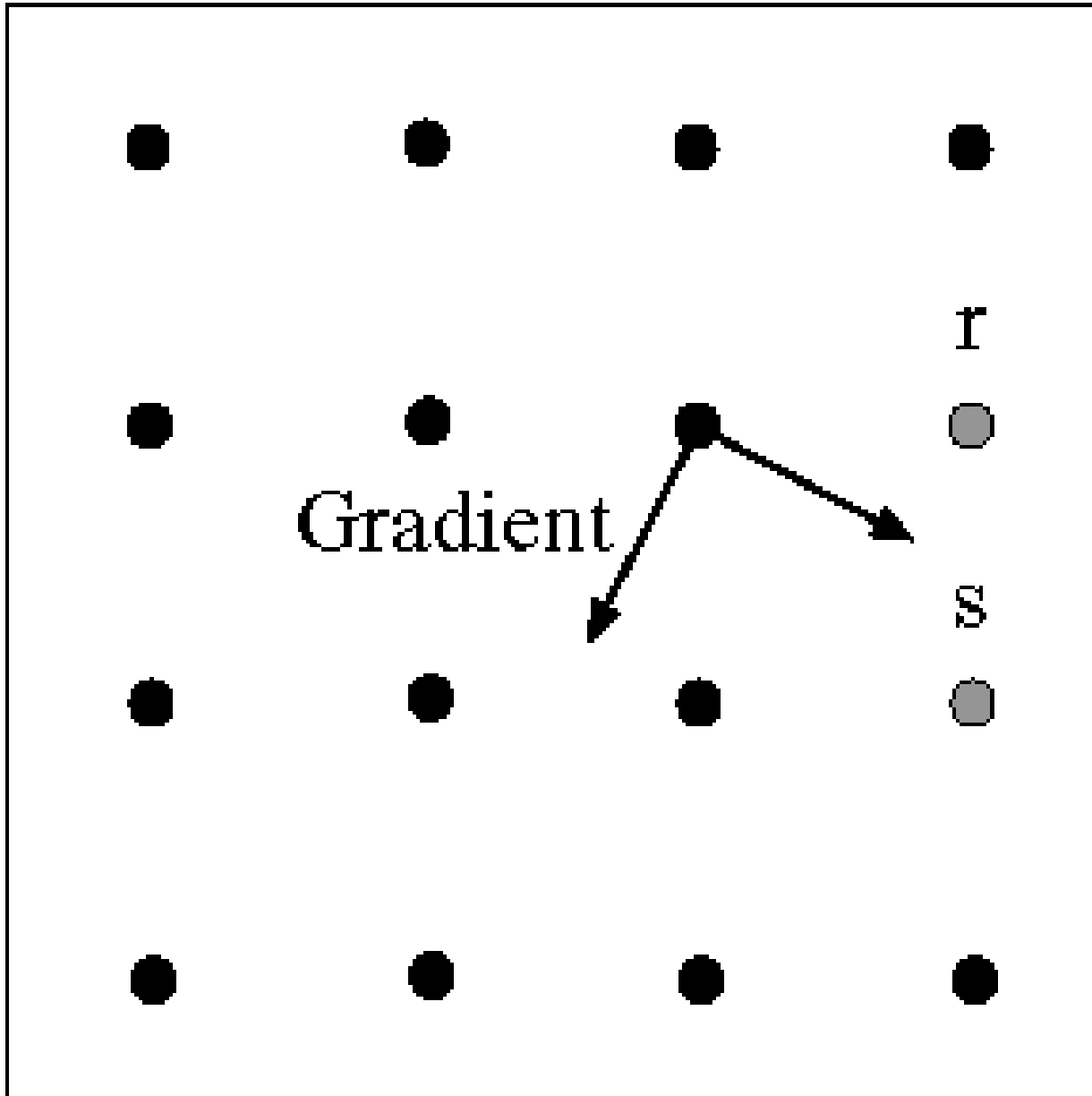
# After non-max suppression



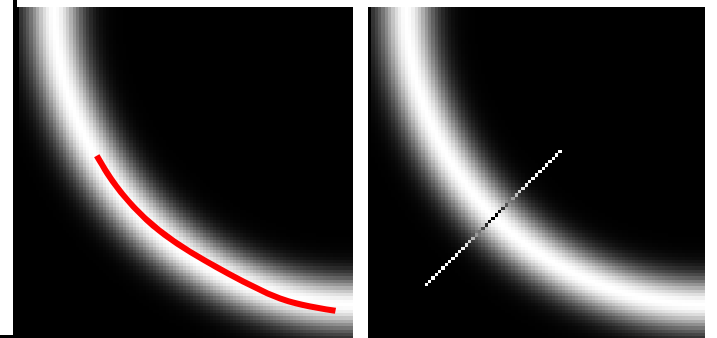
Gradient magnitude (x4 for visualization)

# Non-maximum suppression

Predicting the next edge point



- The marked point is an edge point.
- From edge tangent (normal to gradient), predict next point along edge curve (here either r or s)
- Link together to create edge curve

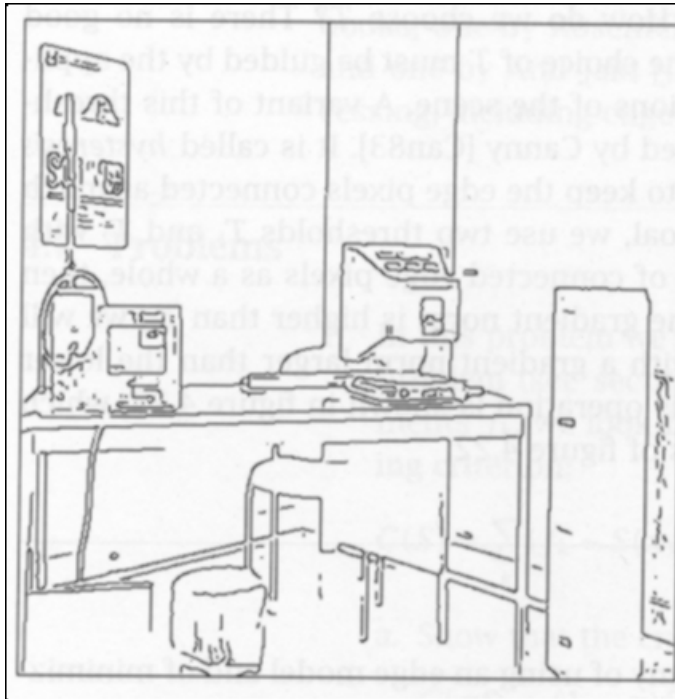


## Input image

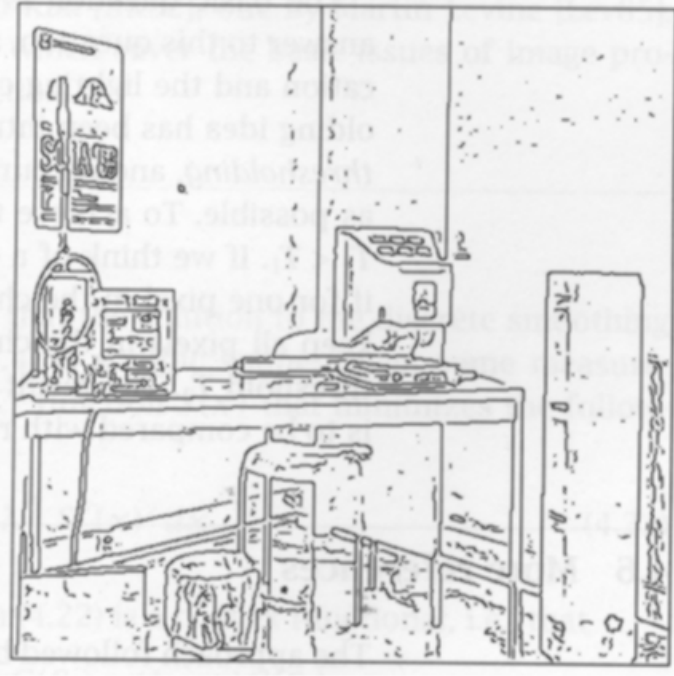


# Single Threshold

T=15



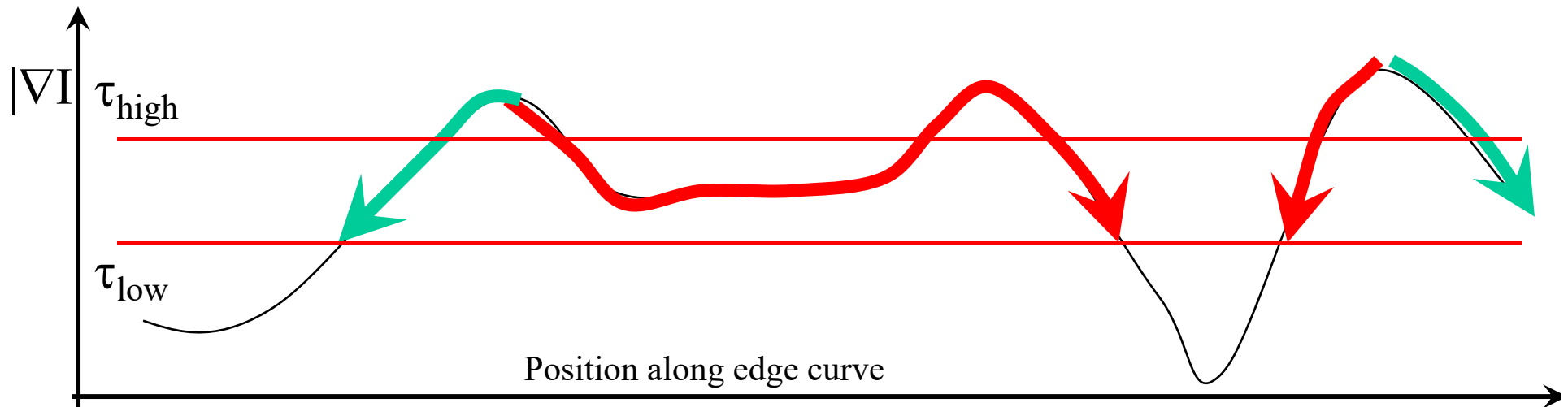
T=5



- When threshold is too high, important edges may be missed or be broken
- When threshold is too low, many extraneous edges, but none missed
- Hysteresis thresholding: Best of both

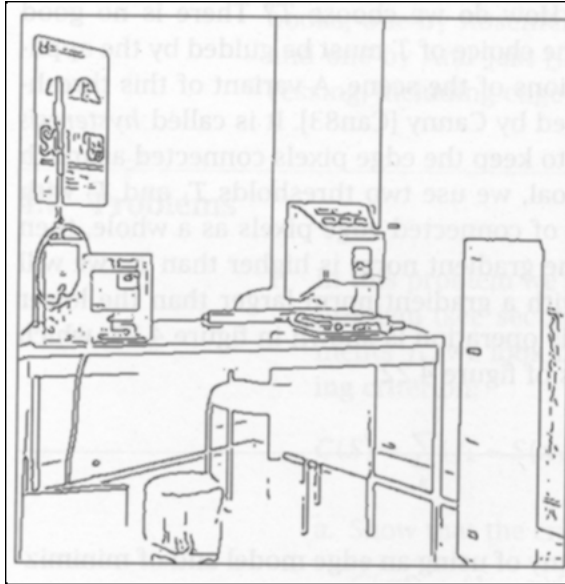
# Hysteresis Thresholding

- Start tracking an edge chain at pixel location that is local maximum of gradient magnitude where gradient magnitude  $> \tau_{\text{high}}$ .
- Follow edge in direction orthogonal to gradient.
- Stop when gradient magnitude  $< \tau_{\text{low}}$ .
  - i.e., use a high threshold to start edge curves and a low threshold to continue them.

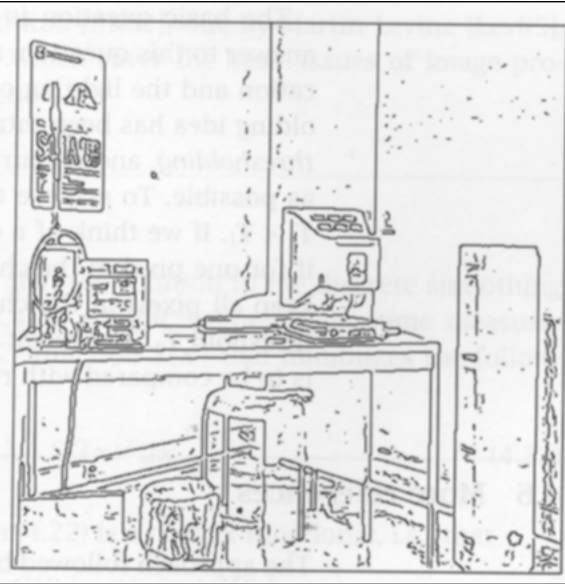


## Single Threshold

$T=15$

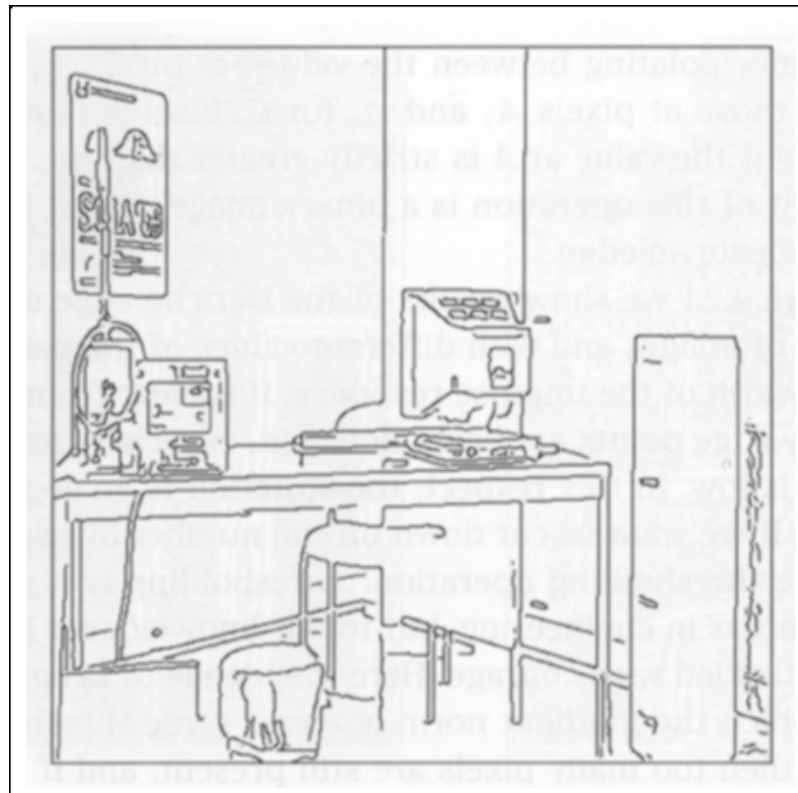


$T=5$



## Hysteresis thresholding

Hysteresis  
 $T_h=15$   $T_l=5$

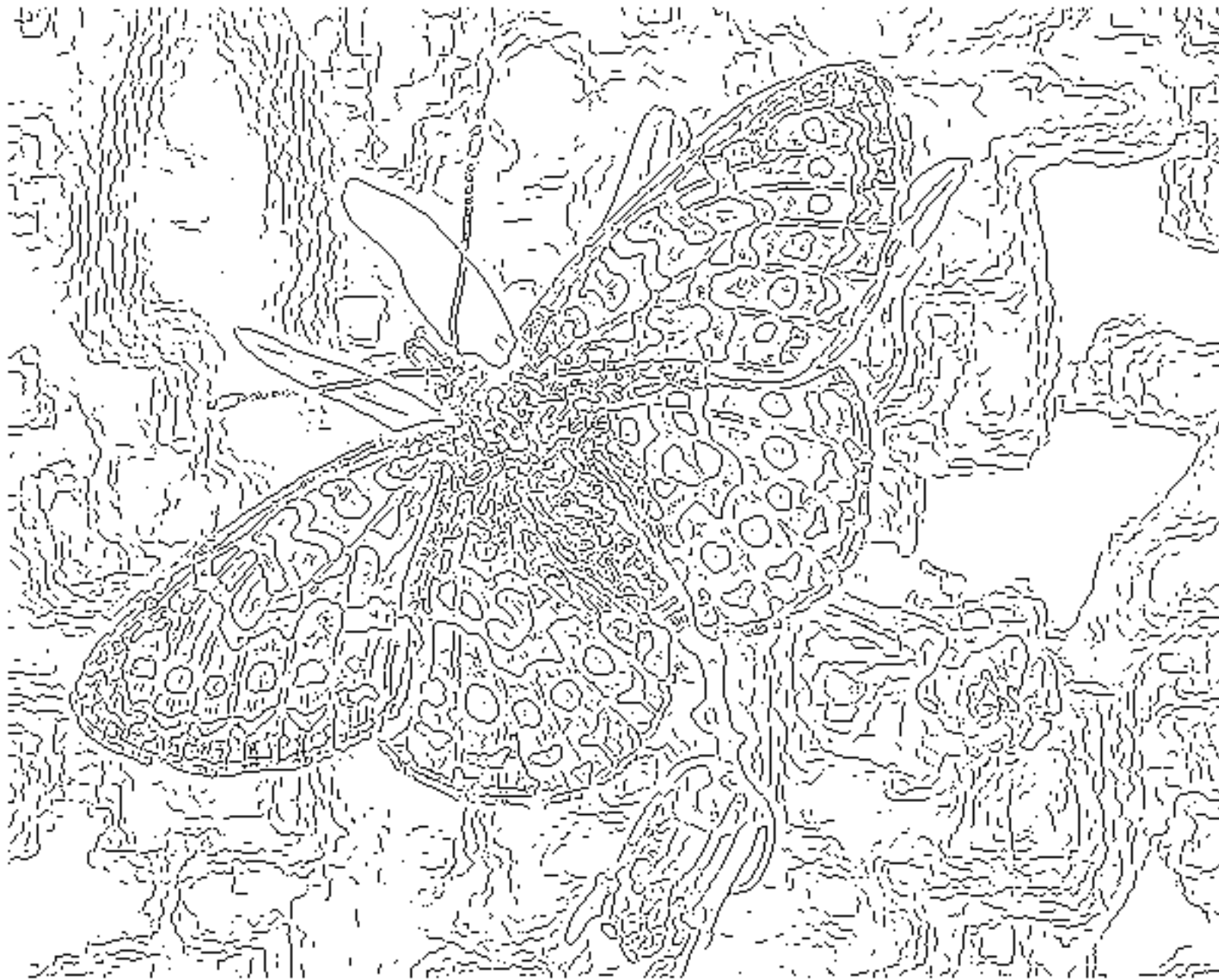




# Canny Edge Detection Algorithm

1. Three parameters  $\sigma$ ,  $\tau_{\text{high}}$ ,  $\tau_{\text{low}}$
2. Filter with symmetric Gaussian of width  $\sigma$
3. Computer gradient, magnitude, direction
4. Non-maximal suppression
5. Hysteresis thresholding using  $\tau_{\text{high}}$ ,  $\tau_{\text{low}}$





fine scale,  
high  
threshold



coarse  
scale,  
high high  
threshold

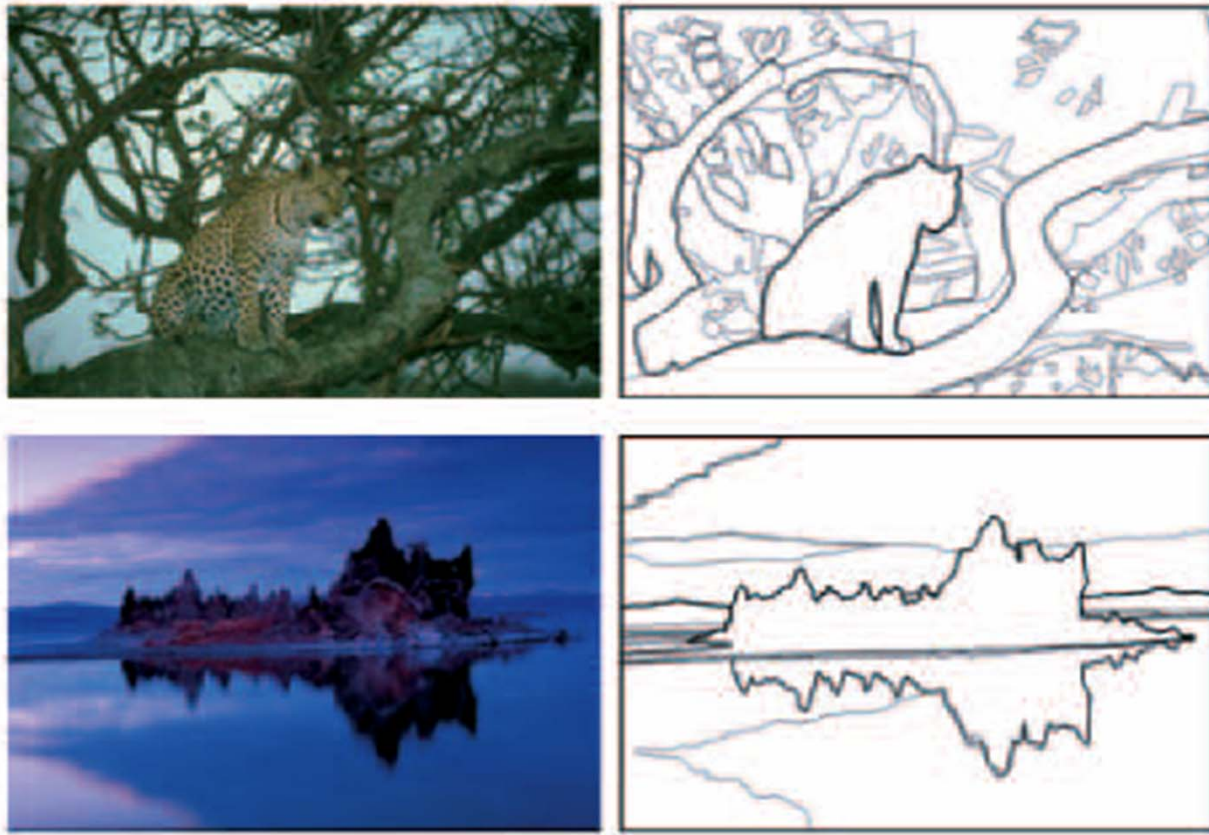


coarse  
scale,  
low high  
threshold

# Why is Canny so Dominant

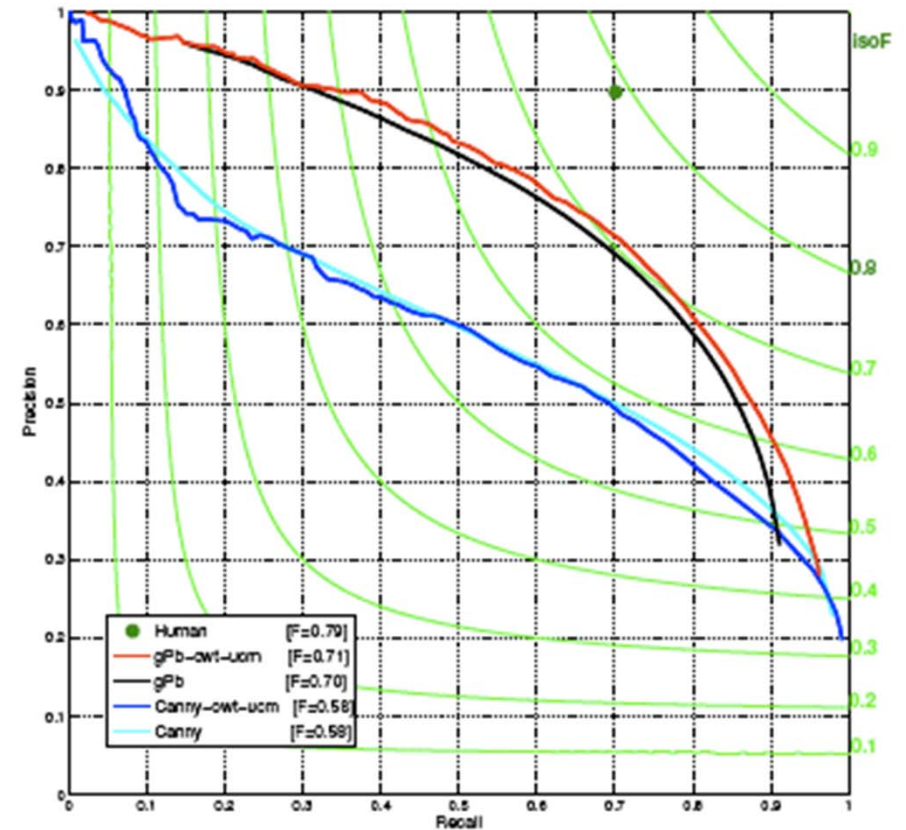
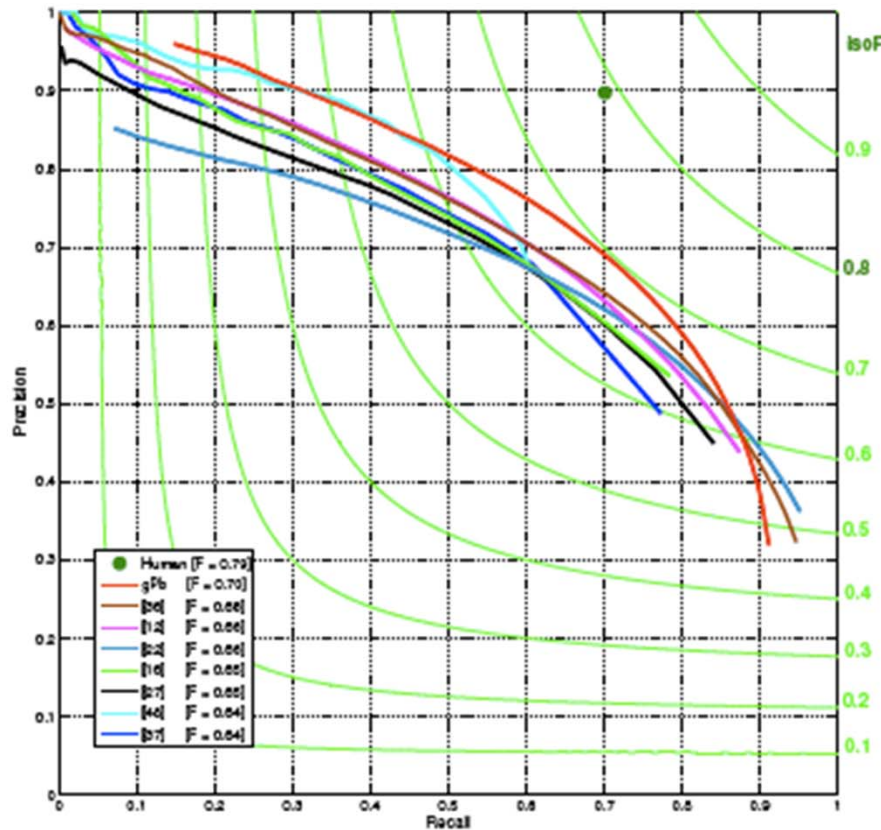
- Widely used for 30 years.
- Theory is nice
- Details are good
  - Magnitude of gradient,
  - Non-max suppression
  - Hysteresis thresholding
- Most subsequent detectors weren't much better until learning-based detectors came along
- Code was distributed

# Learning-based detectors: Not edges, but boundaries



- **Brightness**
- **Color**
- **Texture**
- **Subjective contours**
- **Grouping**
- **Multiscale**

# Boundary detection



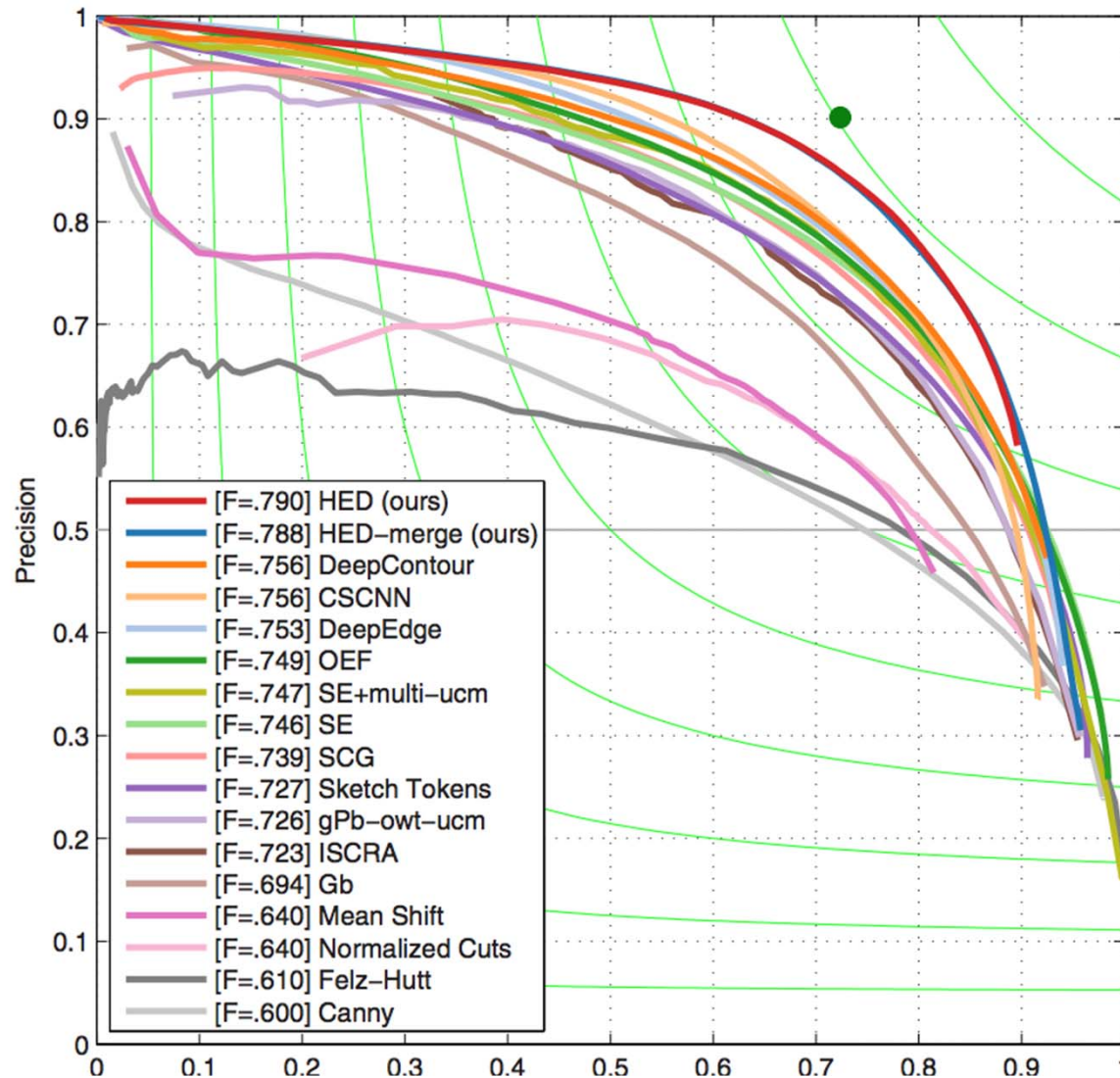
- Precision is the fraction of detections that are true positives rather than false positives, while recall is the fraction of true positives that are detected rather than missed.
- From Contours to Regions: An Empirical Evaluation, Arbelaez, M. Maire, C. Fowlkes, and J. Malik, CVPR 2008



# Learned Edge Detectors

- Dollar, Piotr, Zhuowen Tu, and Serge Belongie. "Supervised learning of edges and object boundaries." Computer Vision and Pattern Recognition, 2006 IEEE Computer Society Conference on. Vol. 2. IEEE, 2006
- Dollár, Piotr, and C. Lawrence Zitnick. "Structured forests for fast edge detection." Proceedings of the IEEE International Conference on Computer Vision. 2013.
- Xie, Saining, and Zhuowen Tu. ." Proceedings of the IEEE international conference on computer vision. 2015.
- Long, Jonathan, Evan Shelhamer, and Trevor Darrell. "Fully convolutional networks for semantic segmentation." Proceedings of the IEEE conference on computer vision and pattern recognition. 2015

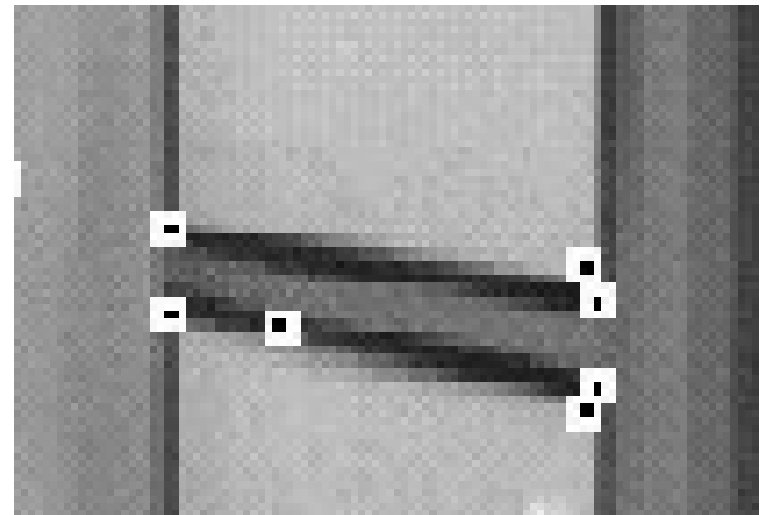
# HED Performance



Xie and Tu. "Holistically-nested edge detection." ICCV 2015

# Corner Detection

# Feature extraction: Corners



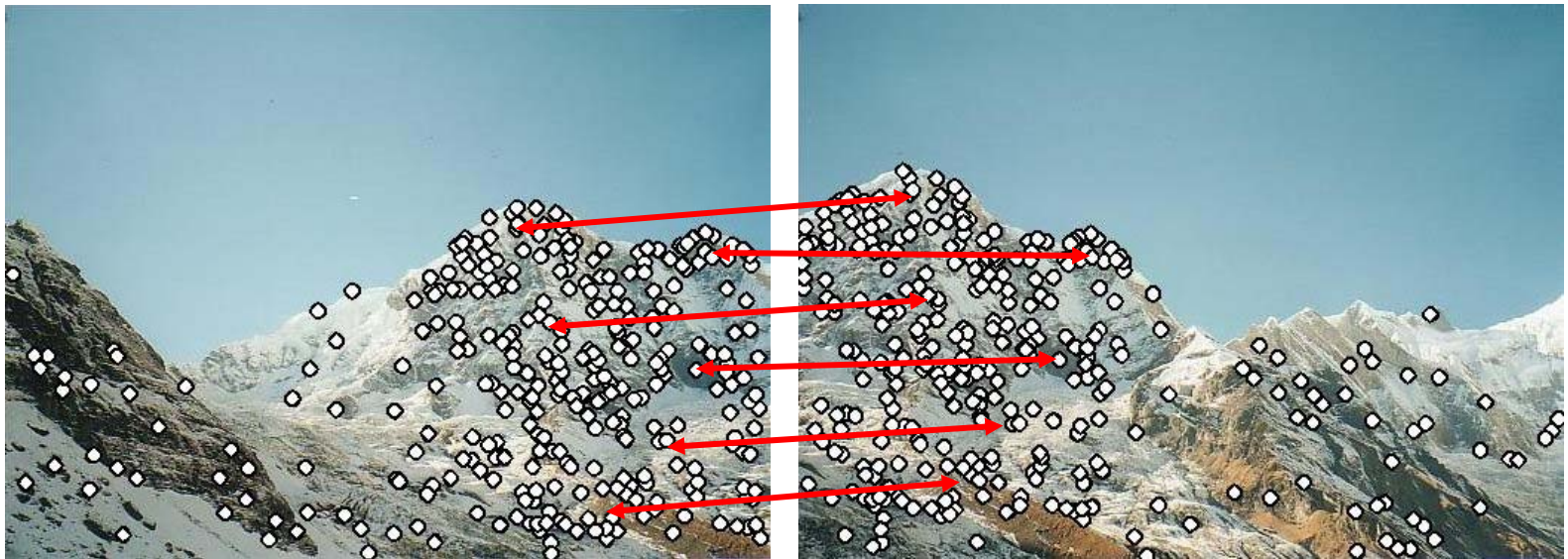
# Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?



# Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?



Step 1: extract features

Step 2: match features



# Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?



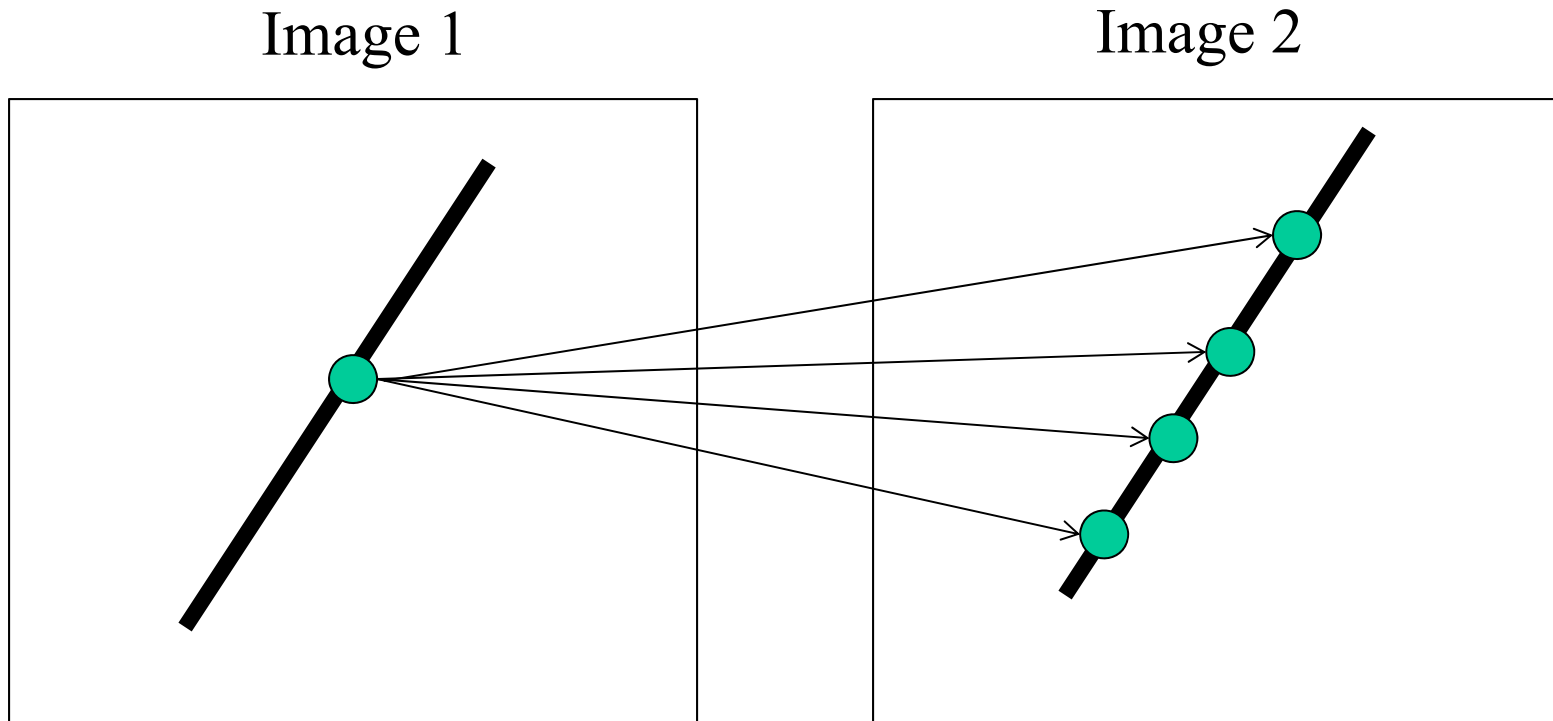
Step 1: extract features

Step 2: match features

Step 3: align images

# Corners contain more info than lines.

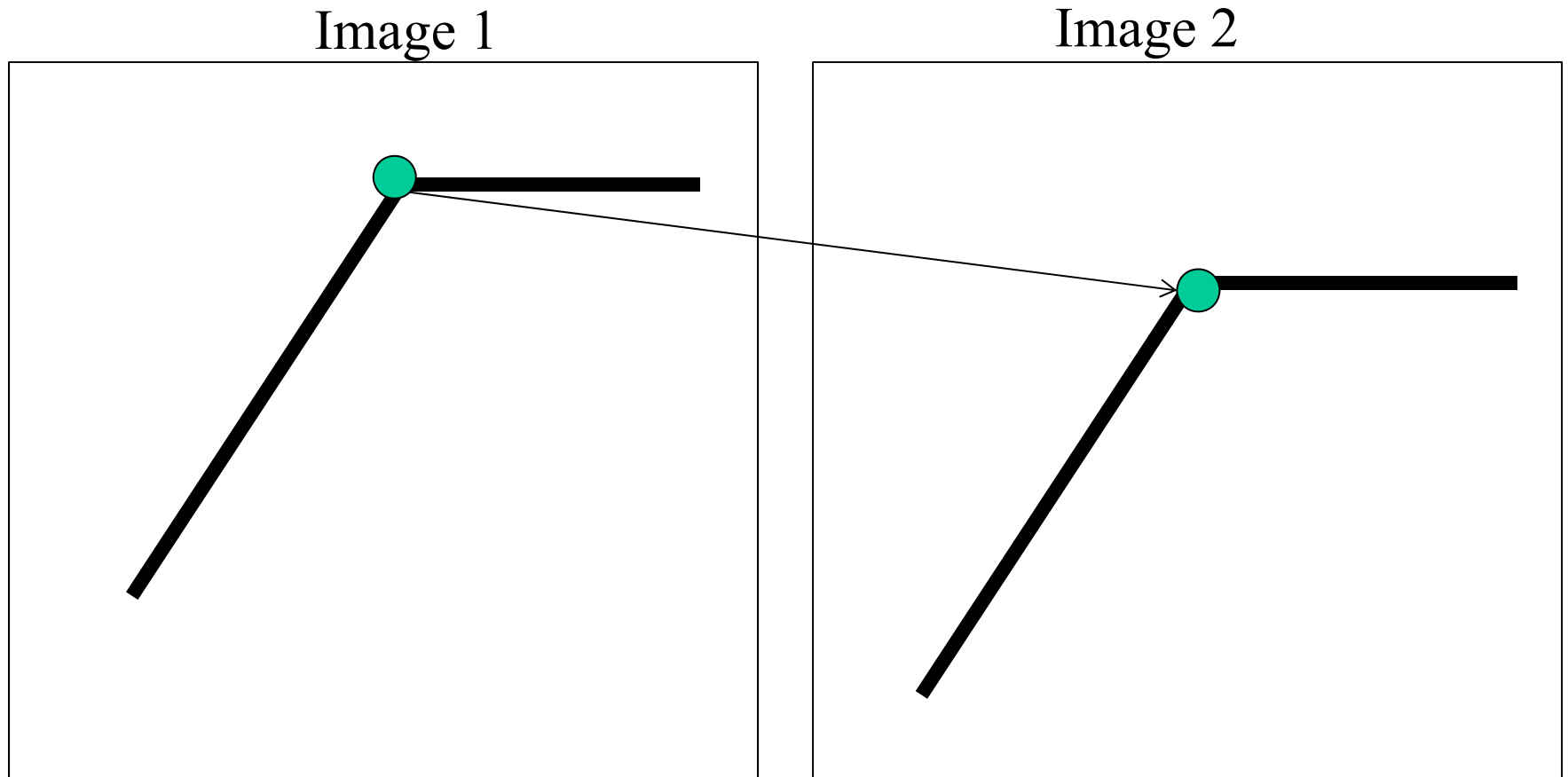
- A point on a line is hard to match.





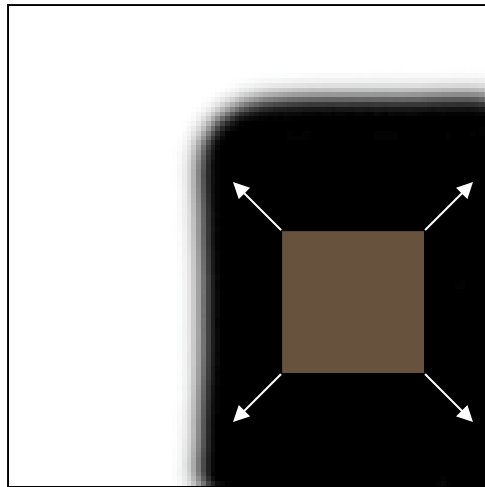
# Corners contain more info than lines.

- A corner is easier to match

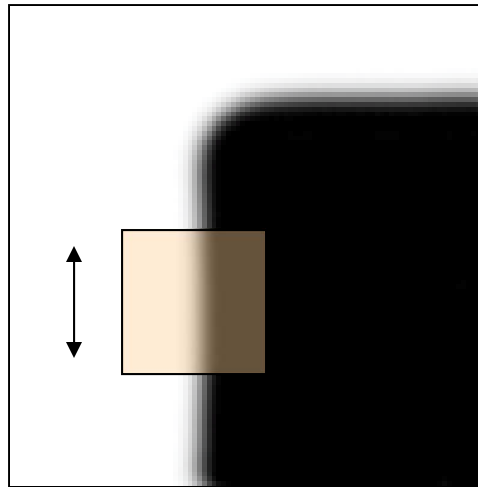


## The Basic Idea

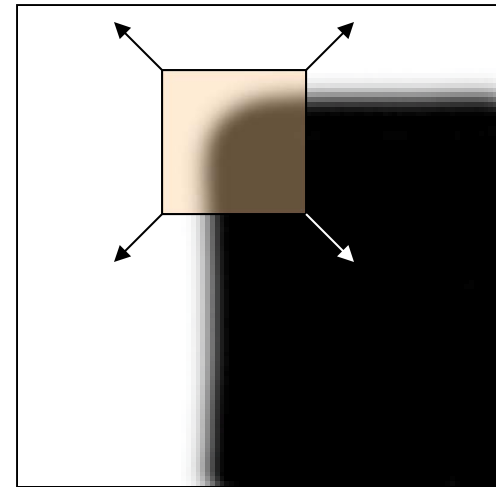
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



“flat” region:  
no change in  
all directions

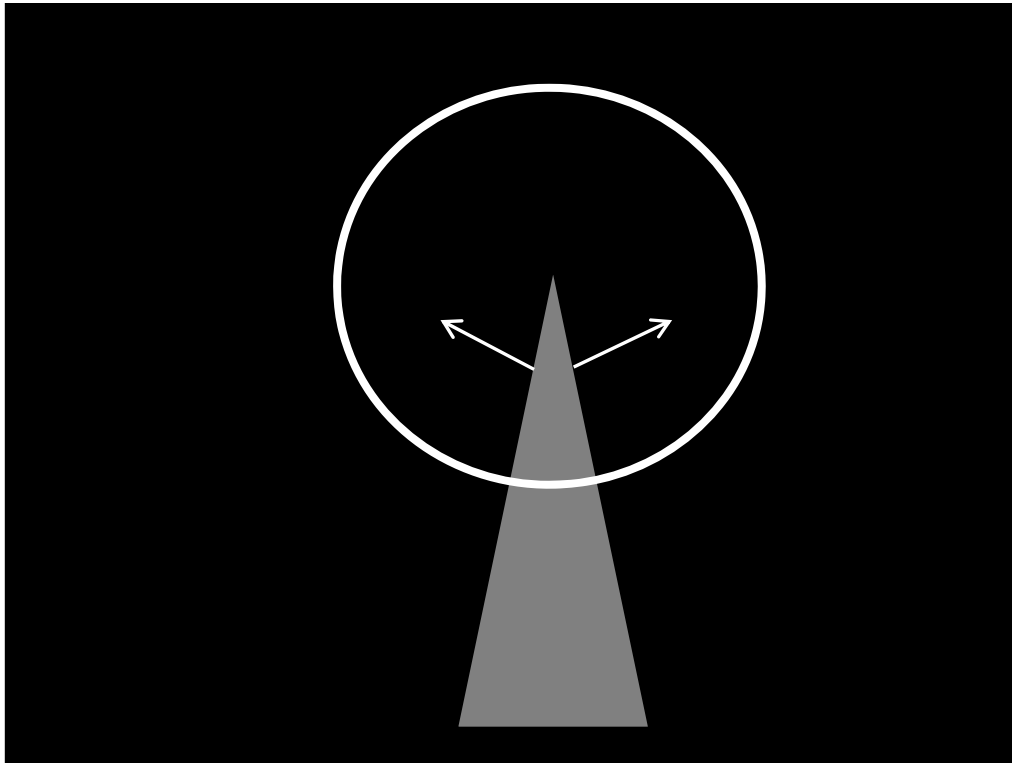


“edge”:  
no change  
along the edge  
direction



“corner”:  
significant  
change in all  
directions

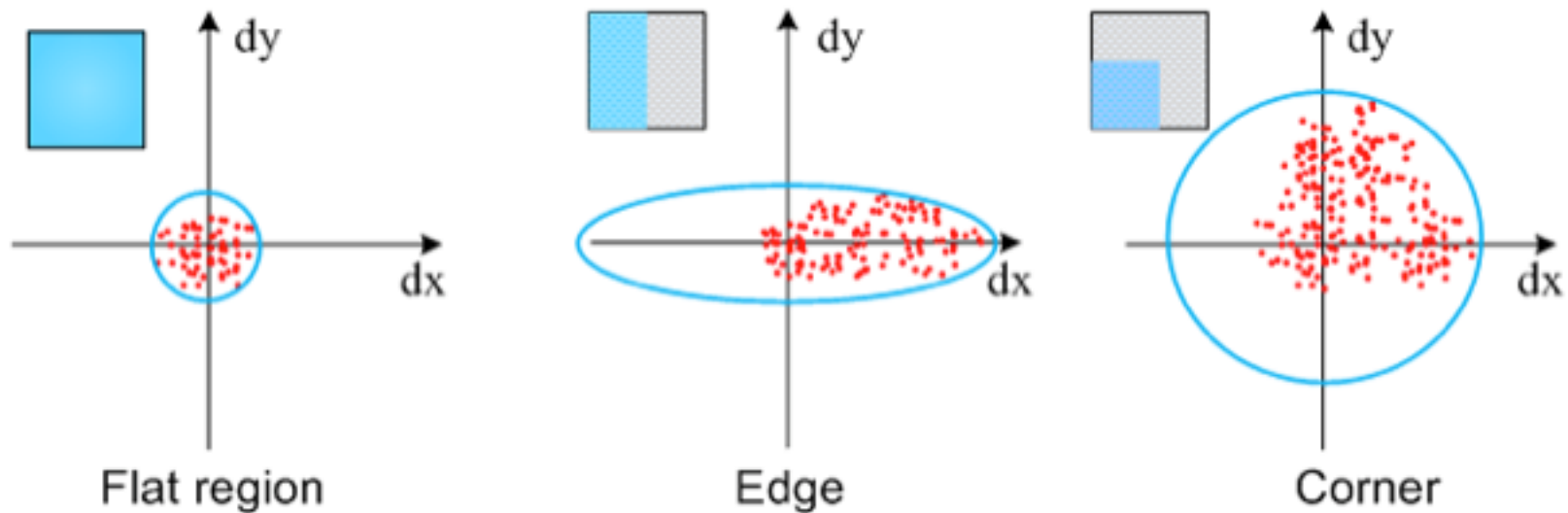
# Finding Corners



Intuition:

- Right at corner, gradient is ill-defined.
- Near corner, gradient has two different values.

# Distribution of gradients for different image patches



Derivative distribution of different regions

# Formula for Finding Corners

## Shi-Tomasi Detector

For each image location  $(x,y)$ , we create a matrix  $C(x,y)$ :

Sum over a small region

Gradient with respect to  $x$ ,  
times gradient with respect to  $y$

$$C(x, y) = \begin{bmatrix} \sum \sum I_x^2 & \sum \sum I_x I_y \\ \sum \sum I_x I_y & \sum \sum I_y^2 \end{bmatrix}$$

Matrix is symmetric

**WHY THIS?**

Because  $C$  is a symmetric positive semidefinite matrix, it can be factored as:

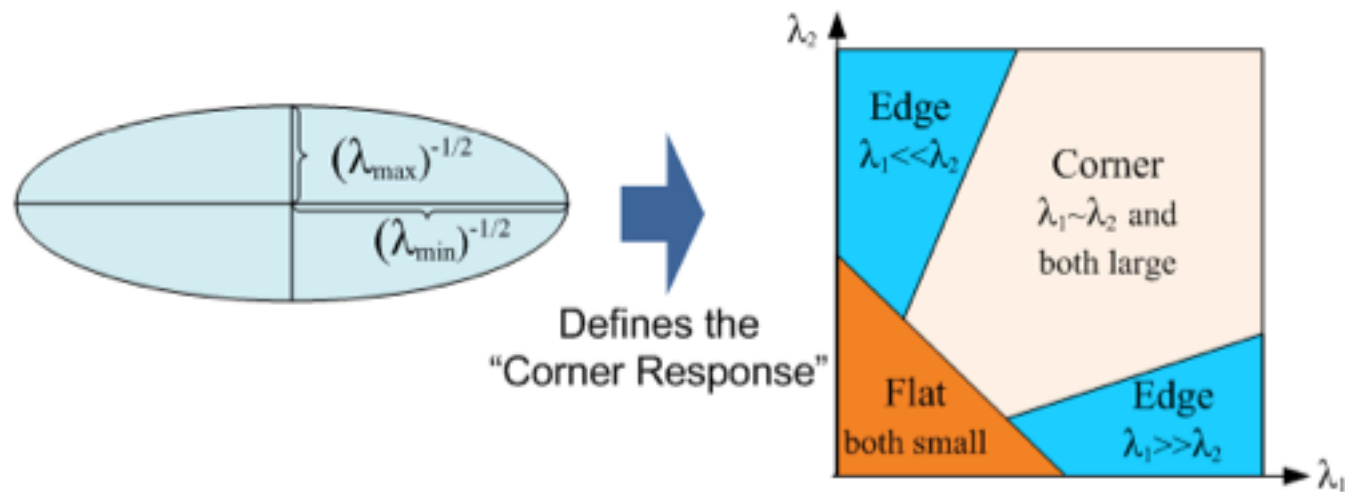
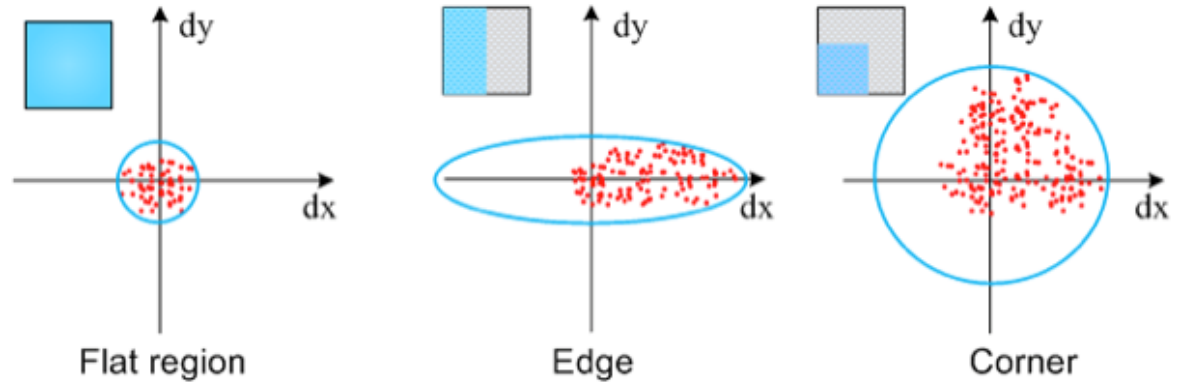
$$C = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R = R^T \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

where  $R$  is a 2x2 rotation matrix and  $\lambda_1$  and  $\lambda_2$  are non-negative.

1.  $\lambda_1$  and  $\lambda_2$  are the Eigenvalues of  $C$ .
2. The columns of  $R$  are the Eigenvectors of  $C$ .
3. Eigenvalues can be found by solving the characteristic equation  $\det(C - \lambda I) = 0$  for  $\lambda$ .

What is region like if:

1.  $\lambda_1 = 0, \lambda_2 > 0$ ?
2.  $\lambda_2 = 0, \lambda_1 > 0$ ?
3.  $\lambda_1 = 0$  and  $\lambda_2 = 0$ ?
4.  $\lambda_1 \gg 0$  and  $\lambda_2 \gg 0$ ?



# Shi-Tomasi Corner Detector

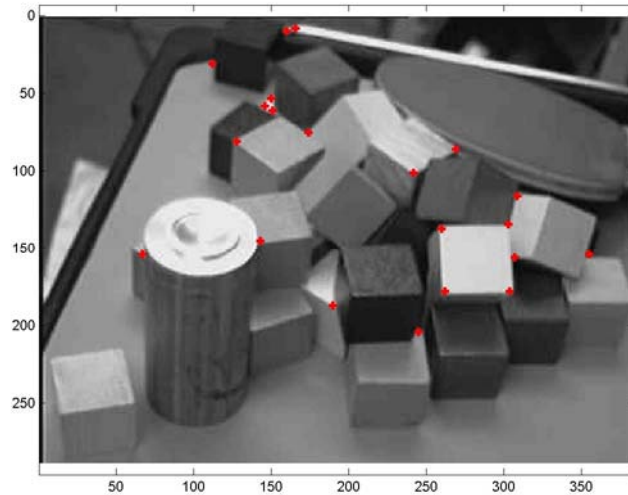
- Filter image with a Gaussian.
- Compute the gradient everywhere.
- Move window over image, and for each window location:
  1. Construct the matrix  $C$  over the window.
  2. Use linear algebra to find  $\lambda_1$  and  $\lambda_2$ .
  3. If they are both large, we have a corner.
    1. Let  $e(x,y) = \min(\lambda_1(x,y), \lambda_2(x,y))$
    2.  $(x,y)$  is a corner if it's local maximum of  $e(x,y)$  and  $e(x,y) > \tau$

Parameters: Gaussian std. dev, window size, threshold

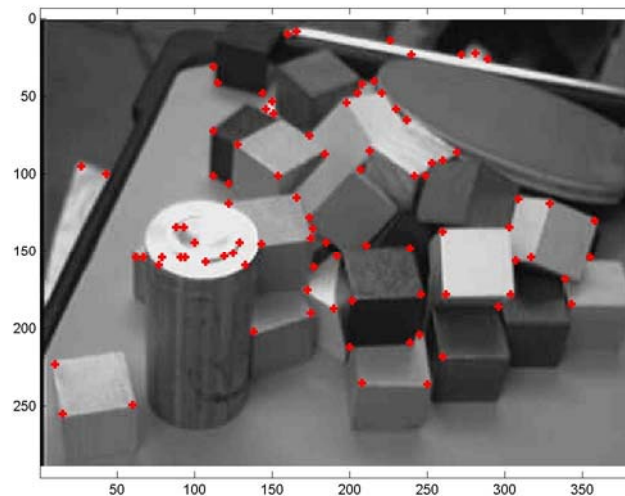
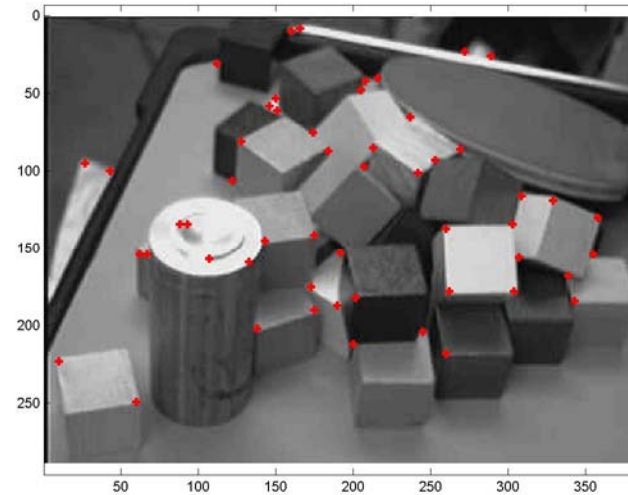


# Corner Detection Sample Results

*Threshold=25,000*



*Threshold=10,000*



*Threshold=5,000*

# Next Lecture

- Early vision: multiple images
  - Stereo
- Reading:
  - Chapter 7: Stereopsis