10. Maths - Matrix

Contents

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1. Matrix

- ✓ A Matrix is an array of numbers:

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$$

A Matrix

(This one has 2 Rows and 3 Columns)

2. Rows and columns

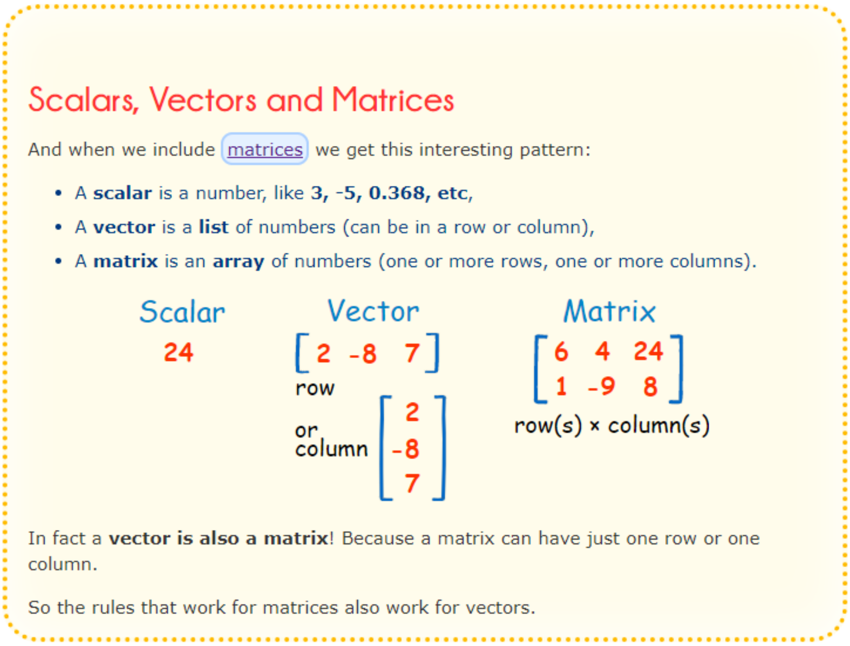
Rows and Columns

To show how many rows and columns a matrix has we often write **rows×columns**.

Example: This matrix is **2×3** (2 rows by 3 columns):

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$$

3. Scalar, Vector and Matrix



Scalars, Vectors and Matrices

And when we include matrices we get this interesting pattern:

- A **scalar** is a number, like **3, -5, 0.368, etc**,
- A **vector** is a **list** of numbers (can be in a row or column),
- A **matrix** is an **array** of numbers (one or more rows, one or more columns).

In fact a **vector is also a matrix**! Because a matrix can have just one row or one column.

So the rules that work for matrices also work for vectors.

4. Adding matrix

- ✓ We can add two matrices
- ✓ The two matrices must be the same size, i.e. the rows must match in size, and the columns must match in size.
- ✓ Example: a matrix with 3 rows and 5 columns can be added to another matrix of 3 rows and 5 columns.
- ✓ But it could not be added to a matrix with 3 rows and 4 columns (the columns don't match in size)

3+4=7

$$\begin{bmatrix}3 & 8\\4 & 6\end{bmatrix} + \begin{bmatrix}4 & 0\\1 & -9\end{bmatrix} = \begin{bmatrix}7 & 8\\5 & -3\end{bmatrix}$$

These are the calculations:

3+4=7	8+0=8
4+1=5	6−9=−3

5. Negative matrix

✓ The negative of a matrix is also simple:

$-(2)=-2$

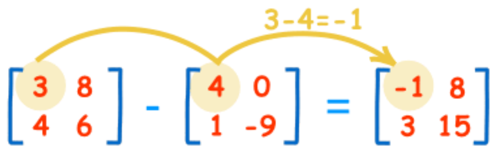
$$-\begin{bmatrix} 2 & -4 \\ 7 & 10 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ -7 & -10 \end{bmatrix}$$

These are the calculations:

$-(2)=-2$	$-(-4)=+4$
$-(7)=-7$	$-(10)=-10$

6. Subtracting matrix

- ✓ To subtract two matrices: subtract the numbers in the matching positions:
- ✓ Note: subtracting is actually defined as the addition of a negative matrix: A + (−B)



These are the calculations:

3−4=−1	8−0=8
4−1=3	6−(−9)=15

7. Multiply by a Constant

- ✓ We can multiply a matrix by a constant *(the value 2 in this case)*:
- ✓ We call the constant a scalar, so officially this is called "scalar multiplication".

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} \xrightarrow{2 \times 4 = 8} \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$$

These are the calculations:

2×4=8	2×0=0
2×1=2	2×−9=−18

8. Multiply Matrices

- ✓ Multiplying a matrix by another matrix we need to do the "dot product" of rows and columns

To work out the answer for the **1st row** and **1st column**:

"Dot Product"

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & \\ & \end{bmatrix}$$

The "Dot Product" is where we **multiply matching members**, then sum up:

$$(1, 2, 3) \bullet (7, 9, 11) = 1\times7 + 2\times9 + 3\times11$$
$$= 58$$

We match the 1st members (1 and 7), multiply them, likewise for the 2nd members (2 and 9) and the 3rd members (3 and 11), and finally sum them up.

Want to see another example? Here it is for the 1st row and **2nd column**:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ & \end{bmatrix}$$

$$(1, 2, 3) \bullet (8, 10, 12) = 1\times8 + 2\times10 + 3\times12$$
$$= 64$$

We can do the same thing for the **2nd row** and **1st column**:

$$(4, 5, 6) \bullet (7, 9, 11) = 4\times7 + 5\times9 + 6\times11$$
$$= 139$$

And for the **2nd row** and **2nd column**:

$$(4, 5, 6) \bullet (8, 10, 12) = 4\times8 + 5\times10 + 6\times12$$
$$= 154$$

And we get:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

DONE!

9. A real time example

Example: The local shop sells 3 types of pies.

- Apple pies cost **$3** each
- Cherry pies cost **$4** each
- Blueberry pies cost **$2** each

And this is how many they sold in 4 days:

	Mon	Tue	Wed	Thu
Apple	**13**	**9**	**7**	**15**
Cherry	**8**	**7**	**4**	**6**
Blueberry	**6**	**4**	**0**	**3**

Now think about this ... the **value of sales** for Monday is calculated this way:

Apple pie value + Cherry pie value + Blueberry pie value

$\$3\times13 + \$4\times8 + \$2\times6 = \$83$

So it is, in fact, the "dot product" of prices and how many were sold:

$$(\$3, \$4, \$2) \bullet (13, 8, 6) = \$3\times13 + \$4\times8 + \$2\times6 = \$83$$

We **match** the price to how many sold, **multiply** each, then **sum** the result.

In other words:

- The sales for Monday were: Apple pies: **\$3×13=\$39**, Cherry pies: **\$4×8=\$32**, and Blueberry pies: **\$2×6=\$12**. Together that is \$39 + \$32 + \$12 = **\$83**
- And for Tuesday: **\$3×9 + \$4×7 + \$2×4 = \$63**
- And for Wednesday: **\$3×7 + \$4×4 + \$2×0 = \$37**
- And for Thursday: **\$3×15 + \$4×6 + \$2×3 = \$75**

So it is important to match each price to each quantity.

Now you know why we use the "dot product".

And here is the full result in Matrix form:

$$\begin{bmatrix} \$3 & \$4 & \$2 \end{bmatrix} \times \begin{bmatrix} 13 & 9 & 7 & 15 \\ 8 & 7 & 4 & 6 \\ 6 & 4 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \$83 & \$63 & \$37 & \$75 \end{bmatrix}$$

\$3x13 + \$4x8 + \$2x6

They sold **\$83** worth of pies on Monday, **\$63** on Tuesday, etc.

Examples

In General:

To multiply an **m×n** matrix by an **n×p** matrix, the **n**s must be the same, and the result is an **m×p** matrix.

$$m \times n \; \times \; n \times p \rightarrow m \times p$$

So ... multiplying a **1×3** by a **3×1** gets a **1×1** result:

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1\times4+2\times5+3\times6 \end{bmatrix} = \begin{bmatrix} 32 \end{bmatrix}$$

But multiplying a **3×1** by a **1×3** gets a **3×3** result:

$$\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4\times1 & 4\times2 & 4\times3 \\ 5\times1 & 5\times2 & 5\times3 \\ 6\times1 & 6\times2 & 6\times3 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 12 \\ 5 & 10 & 15 \\ 6 & 12 & 18 \end{bmatrix}$$

10. Types of Matrix

A Matrix is an array of numbers:

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$$

A Matrix

(This one has 2 Rows and 3 Columns)

The **Main Diagonal** starts at the top left and goes down to the right:

$$\begin{bmatrix} 7 & 6 & 4 \\ 4 & 2 & -2 \\ 3 & 0 & 9 \end{bmatrix}$$

11. Transpose matrix

- ✓ To "transpose" a matrix, swap the rows and columns.
- ✓ We put a "T" in the top right-hand corner to mean transpose:

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 4 & -9 \\ 24 & 8 \end{bmatrix}$$

Square

A **square** matrix has the same number of rows as columns.

$$\begin{bmatrix} 2 & 0 \\ 1 & 8 \end{bmatrix}$$

A square matrix (2 rows, 2 columns)

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \\ 3 & 0 & 7 \end{bmatrix}$$

Also a square matrix (3 rows, 3 columns)

Identity Matrix

An **Identity Matrix** has **1**s on the main diagonal and **0**s everywhere else:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A 3×3 Identity Matrix

- It is square (same number of rows as columns)
- It can be large or small (2×2, 100×100, ... whatever)
- Its symbol is the capital letter **I**

It is the matrix equivalent of the number "1", when we multiply with it the original is unchanged:

$$A \times I = A$$

$$I \times A = A$$

Diagonal Matrix

A diagonal matrix has zero anywhere not on the main diagonal:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A diagonal matrix

Scalar Matrix

A scalar matrix has all main diagonal entries the same, with zero everywhere else:

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

A scalar matrix

Zero Matrix (Null Matrix)

Zeros just everywhere:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Zero matrix