THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Probability review 2

Lecture 4

STA 371G

Reading assignments

- Reading assignments help you build (incomplete) mental structures that we can build on in class

Reading assignments

- Reading assignments help you build (incomplete) mental structures that we can build on in class
- Your assignment is to read and annotate: ask a question, answer a classmate's question, make an observation, try to explain something better than the text does

Reading assignments

- Reading assignments help you build (incomplete) mental structures that we can build on in class
- Your assignment is to read and annotate: ask a question, answer a classmate's question, make an observation, try to explain something better than the text does
- Make at least three high-quality annotations per document to get full credit (examples on Canvas)

Reading assignments

- Reading assignments help you build (incomplete) mental structures that we can build on in class
- Your assignment is to read and annotate: ask a question, answer a classmate's question, make an observation, try to explain something better than the text does
- Make at least three high-quality annotations per document to get full credit (examples on Canvas)
- Each assignment is graded on a 0-3 scale; you'll get full credit for a 2

Reading assignments

- Reading assignments help you build (incomplete) mental structures that we can build on in class
- Your assignment is to read and annotate: ask a question, answer a classmate's question, make an observation, try to explain something better than the text does
- Make at least three high-quality annotations per document to get full credit (examples on Canvas)
- Each assignment is graded on a 0-3 scale; you'll get full credit for a 2
- First assignment is due **Sunday, February 4 at 7:00 PM**

Reading assignments

- Reading assignments help you build (incomplete) mental structures that we can build on in class
- Your assignment is to read and annotate: ask a question, answer a classmate's question, make an observation, try to explain something better than the text does
- Make at least three high-quality annotations per document to get full credit (examples on Canvas)
- Each assignment is graded on a 0-3 scale; you'll get full credit for a 2
- First assignment is due **Sunday, February 4 at 7:00 PM**
- See handout on Canvas (under Files) for more information and examples


the muppets.

Who are these folks?







Who are these folks?

Kermit the Frog

Miss Piggy

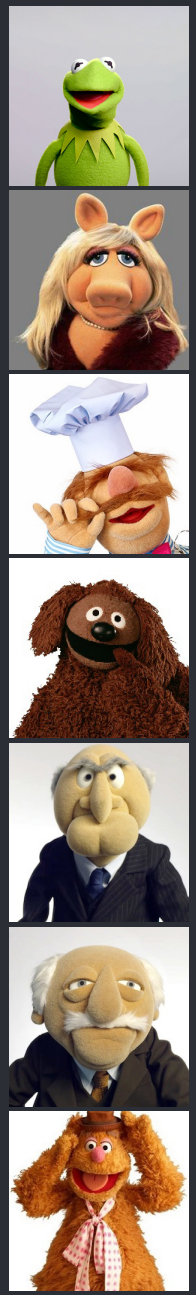
Swedish Chef

Rowlf the Dog

Statler

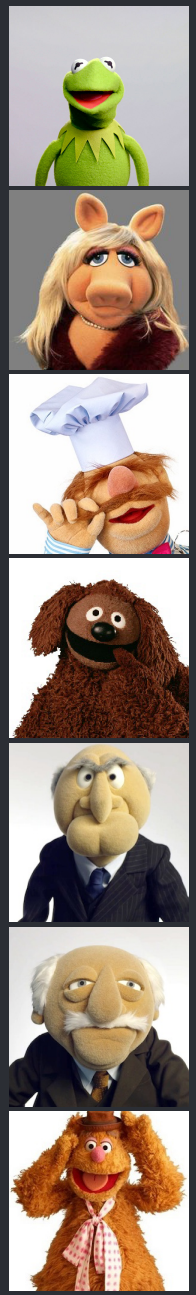
Waldorf

Fozzie Bear



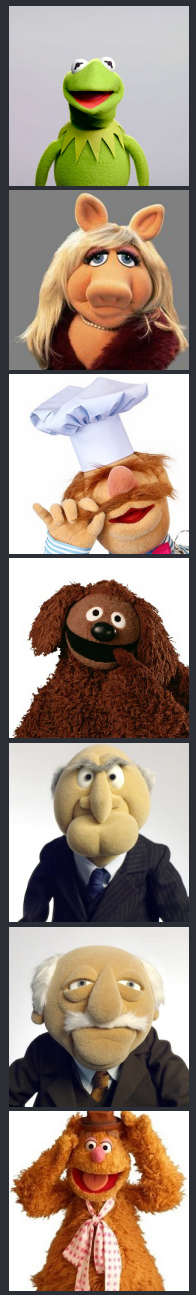
Suppose we pick a Muppet at random. Each of these are events:

- $A =$ we select an animal (non-human?)



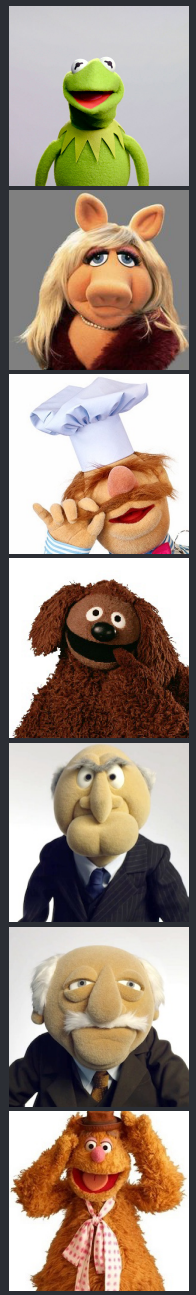
Suppose we pick a Muppet at random. Each of these are events:

- $A =$ we select an animal (non-human?)
- $B =$ we select someone who is bald



Suppose we pick a Muppet at random. Each of these are events:

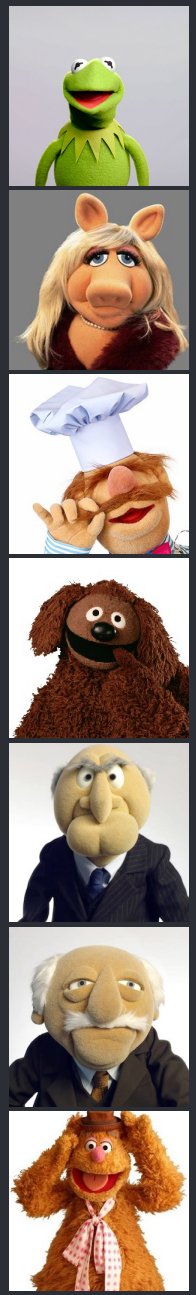
- $A =$ we select an animal (non-human?)
- $B =$ we select someone who is bald



Suppose we pick a Muppet at random. Each of these are events:

- $A =$ we select an animal (non-human?)
- $B =$ we select someone who is bald

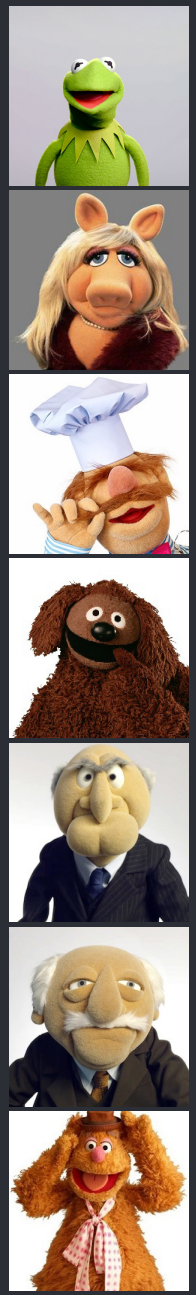
$P(A) =$



Suppose we pick a Muppet at random. Each of these are events:

- $A =$ we select an animal (non-human?)
- $B =$ we select someone who is bald

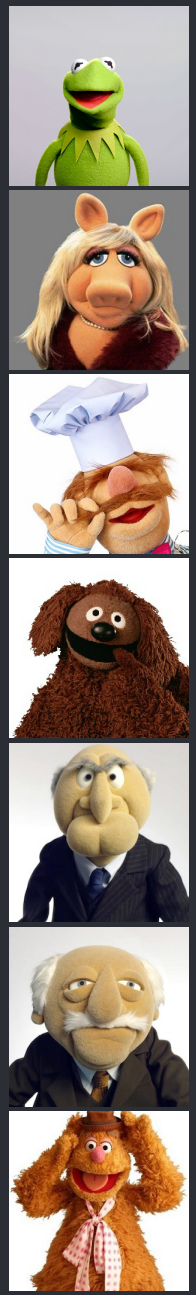
$P(A) = 4/7$ $\qquad$ $P(B) =$



Suppose we pick a Muppet at random. Each of these are events:

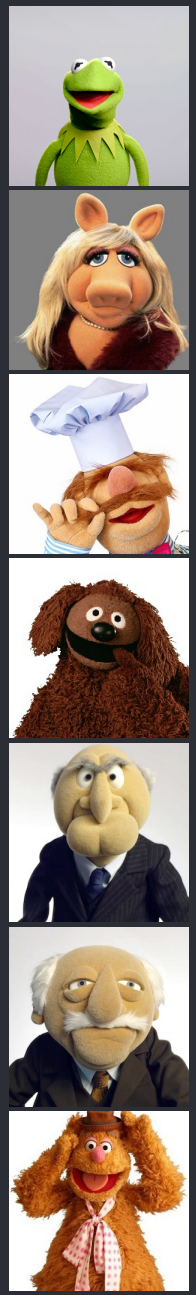
- $A =$ we select an animal (non-human?)
- $B =$ we select someone who is bald

$P(A) = 4/7 \qquad P(B) = 3/7$



- $A|B =$ we select at random from among the bald characters, and get an animal
- $B|A =$ we select at random from among the animals, and get a bald Muppet

$$P(A) = 4/7 \qquad P(B) = 3/7$$

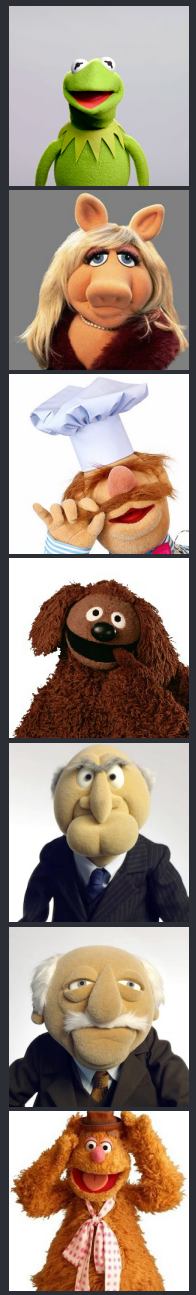


- $A|B$ = we select at random from among the bald characters, and get an animal
- $B|A$ = we select at random from among the animals, and get a bald Muppet

$$P(A) = 4/7 \qquad P(B) = 3/7$$

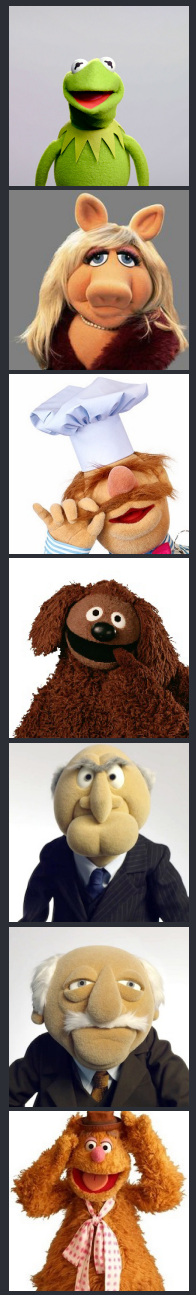
$$P(A|B) = \frac{\text{\# bald animals}}{\text{\# bald Muppets}} = \frac{P(A \text{ and } B)}{P(B)} = \frac{1}{3}$$

$$P(B|A) = \frac{\text{\# animals}}{\text{\# animal Muppets}} = \frac{P(A \text{ and } B)}{P(A)} = \frac{1}{4}$$



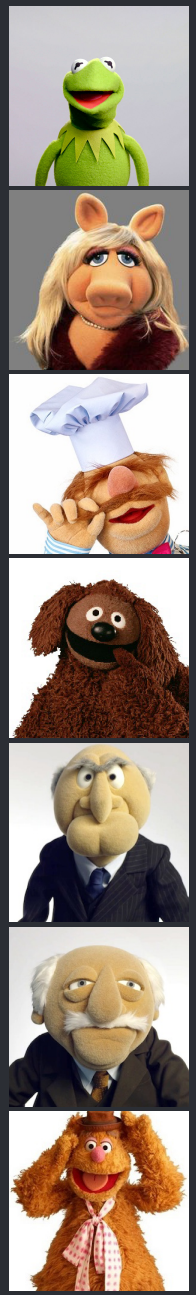
- $A|B$ = we select at random from among the bald characters, and get an animal
- $B|A$ = we select at random from among the animals, and get a bald character

$$P(A \text{ and } B) = P(\text{select bald animal from all Muppets})$$



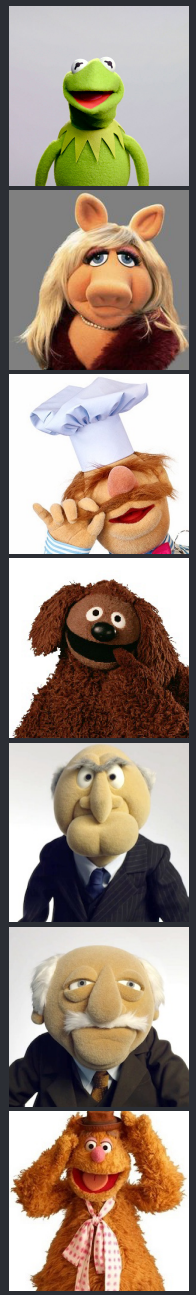
- $A|B$ = we select at random from among the bald characters, and get an animal
- $B|A$ = we select at random from among the animals, and get a bald character

$$P(A \text{ and } B) = P(\text{select bald animal from all Muppets})$$
$$= 1/7$$



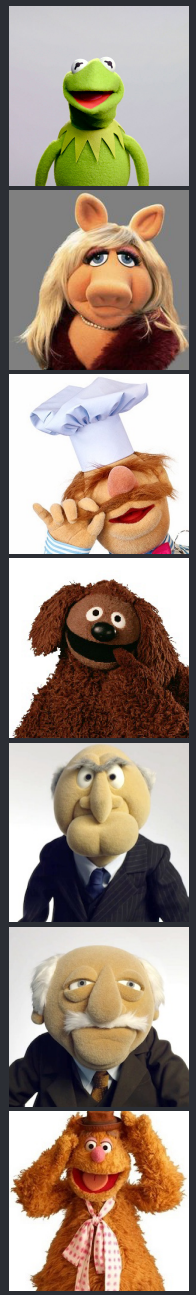
- $A|B$ = we select at random from among the bald characters, and get an animal
- $B|A$ = we select at random from among the animals, and get a bald character

$$
\begin{aligned}
P(A \text{ and } B) &= P(\text{select bald animal from all Muppets}) \\
&= 1/7 \\
&= P(A)P(B|A) = \frac{4}{7} \cdot \frac{1}{4} \\
&= P(B)P(A|B) = \frac{3}{7} \cdot \frac{1}{3}
\end{aligned}
$$

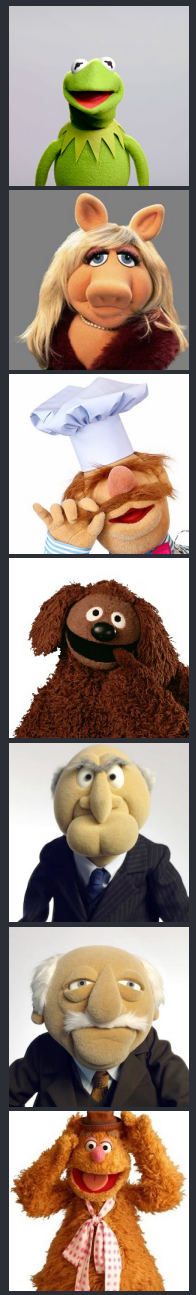


- $A|B$ = we select at random from among the bald characters, and get an animal
- $B|A$ = we select at random from among the animals, and get a bald character

$$
\begin{aligned}
P(A \text{ and } B) &= P(\text{select bald animal from all Muppets}) \\
&= 1/7 \\
&= P(A)P(B|A) = \frac{4}{7} \cdot \frac{1}{4} \\
&= P(B)P(A|B) = \frac{3}{7} \cdot \frac{1}{3} \\
&\neq P(A)P(B)
\end{aligned}
$$

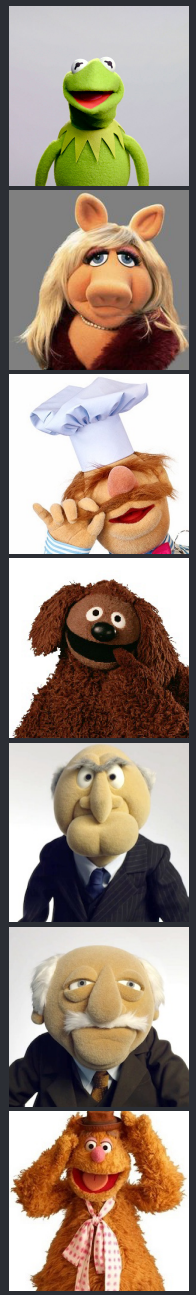


The easy multiplication rule $P(A \text{ and } B) = P(A)P(B)$ does not work because baldness and animalness are *not* independent!



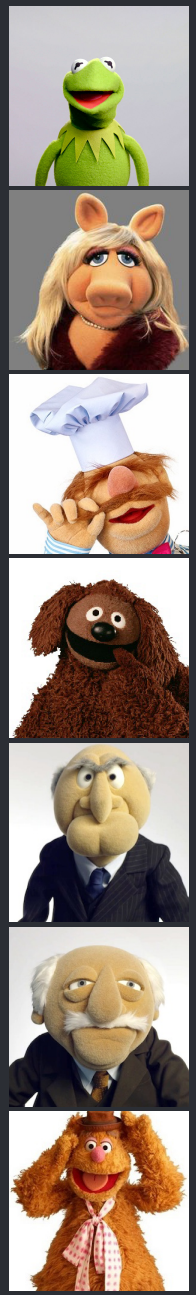
The easy multiplication rule $P(A \text{ and } B) = P(A)P(B)$ does not work because baldness and animalness are *not* independent!

- If a Muppet is bald, they are less likely to be an animal



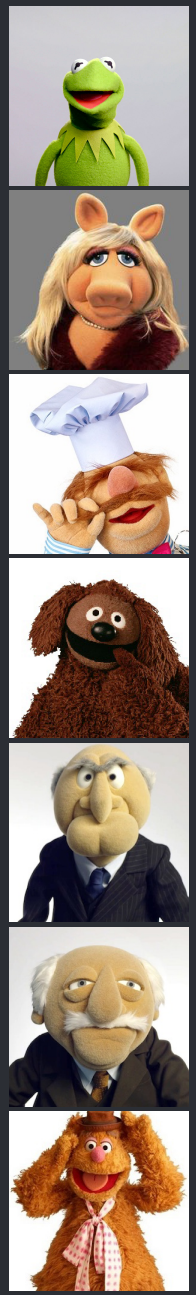
The easy multiplication rule $P(A \text{ and } B) = P(A)P(B)$ does not work because baldness and animalness are *not* independent!

- If a Muppet is bald, they are less likely to be an animal
- If a Muppet is an animal, they are less likely to be bald



The easy multiplication rule $P(A \text{ and } B) = P(A)P(B)$ does not work because baldness and animalness are *not* independent!

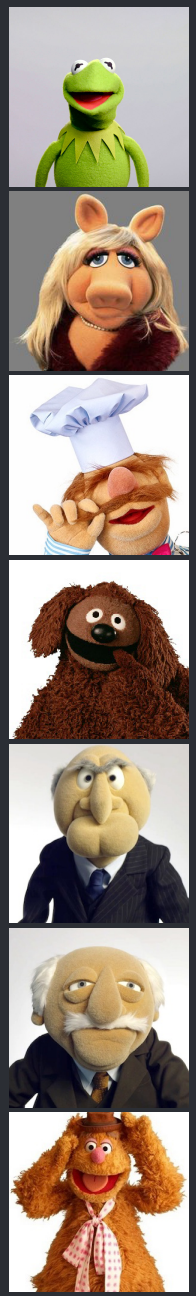
- If a Muppet is bald, they are less likely to be an animal
- If a Muppet is an animal, they are less likely to be bald



The easy multiplication rule $P(A \text{ and } B) = P(A)P(B)$ does not work because baldness and animalness are *not* independent!

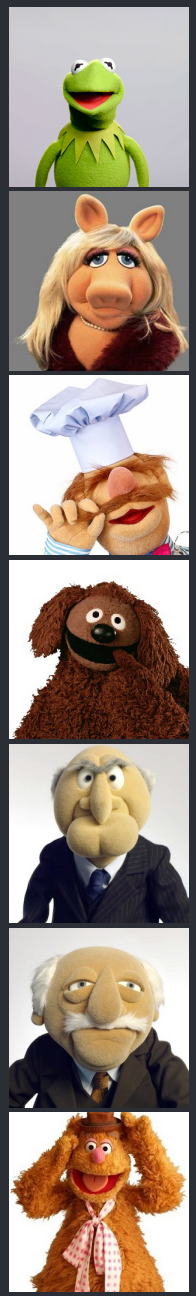
- If a Muppet is bald, they are less likely to be an animal
- If a Muppet is an animal, they are less likely to be bald

The more complex rule $P(A \text{ and } B) = P(A)P(B|A)$ will always work.



$P(A \text{ or } B)$ = probability of selecting an animal,

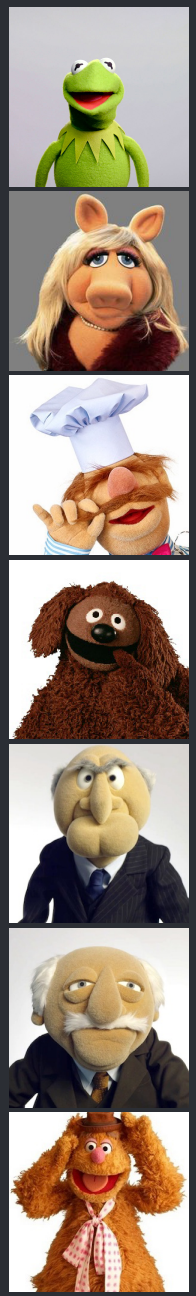
a bald Muppet, or both



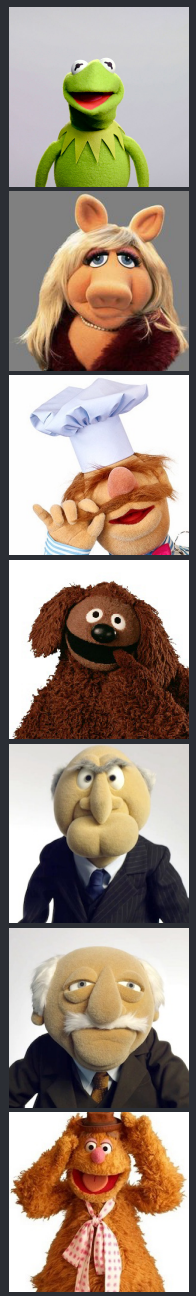
$P(A \text{ or } B)$ = probability of selecting an animal,

a bald Muppet, or both

$= 6/7$



$$
\begin{aligned}
P(A \text{ or } B) &= \text{probability of selecting an animal,} \\
&\qquad \text{a bald Muppet, or both} \\
&= 6/7 \\
&= P(A) + P(B) - P(A \text{ and } B) \\
&= 4/7 + 3/7 - 1/7
\end{aligned}
$$



$$P(A \text{ or } B) = \text{probability of selecting an animal, a bald Muppet, or both}$$
$$= 6/7$$
$$= P(A) + P(B) - P(A \text{ and } B)$$
$$= 4/7 + 3/7 - 1/7$$

We have to subtract off $P(A \text{ and } B)$ because otherwise we are double-counting Kermit! (Poor Kermit: it's not easy being green.)

Conditional probability

When we say $P(A|B)$, what we mean is:

"In a world where we know B has already happened, how likely is it that A also happened?"


HOPE SHE WAS

Now that you've found a strange pair of underwear in your dresser, what is the probability that your partner is cheating on you?

$$P(\text{partner is cheating} \mid \text{underwear found})$$

Now that you've found a strange pair of underwear in your dresser, what is the probability that your partner is cheating on you?

$$P(\text{partner is cheating} \mid \text{underwear found})$$

It's hard to know how to estimate this directly!

Let's come up with estimates for the following:

- $P(\text{underwear found} \mid \text{partner is cheating}) = P(U|C)$

Let's come up with estimates for the following:

- $P(\text{underwear found} \mid \text{partner is cheating}) = P(U|C)$
- $P(\text{underwear found} \mid \text{partner is not cheating}) = P(U|\overline{C})$

Let's come up with estimates for the following:

- $P(\text{underwear found} \mid \text{partner is cheating}) = P(U|C)$
- $P(\text{underwear found} \mid \text{partner is not cheating}) = P(U|\overline{C})$
- $P(\text{partner is cheating}) = P(C)$ (before we found the underwear!)

Let's come up with estimates for the following:

- $P(\text{underwear found} \mid \text{partner is cheating}) = P(U|C)$
- $P(\text{underwear found} \mid \text{partner is not cheating}) = P(U|\overline{C})$
- $P(\text{partner is cheating}) = P(C)$ (before we found the underwear!)

Let's come up with estimates for the following:

- $P(\text{underwear found} \mid \text{partner is cheating}) = P(U|C)$
- $P(\text{underwear found} \mid \text{partner is not cheating}) = P(U|\overline{C})$
- $P(\text{partner is cheating}) = P(C)$ (before we found the underwear!)

Now that we found the underwear, we want to *update our estimate* of how likely it is that our partner is cheating, i.e., we want to know $P(C|U)$.

Let's come up with estimates for the following:

- $P(\text{underwear found} \mid \text{partner is cheating}) = P(U|C)$
- $P(\text{underwear found} \mid \text{partner is not cheating}) = P(U|\overline{C})$
- $P(\text{partner is cheating}) = P(C)$ (before we found the underwear!)

Now that we found the underwear, we want to *update our estimate* of how likely it is that our partner is cheating, i.e., we want to know $P(C|U)$.
(Note: This is the *reverse* of the probability we know already!)

Let's figure it out!

Since $P(C \text{ and } U) = P(C|U)P(U)$,

$$P(C|U) = \frac{P(C \text{ and } U)}{P(U)}.$$

Let's figure it out!

Since $P(C \text{ and } U) = P(C|U)P(U)$,

$$P(C|U) = \frac{P(C \text{ and } U)}{P(U)}.$$

Now we just need to figure out $P(C \text{ and } U)$ and $P(U)$!

Let's figure it out!

There are two mutually-exclusive ways you could find underwear in the drawer: if your partner is cheating, and if they aren't cheating.

Let's figure it out!

There are two mutually-exclusive ways you could find underwear in the drawer: if your partner is cheating, and if they aren't cheating. So:

$$P(U) = P(U \text{ and } C) + P(U \text{ and } \overline{C})$$