THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Probability review 2

Lecture 4

STA 371G

Reading assignments

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- See handout on Canvas (under Files) for more information and examples


the muppets.

Who are these guys?







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Kermit the Frog

Miss Piggy

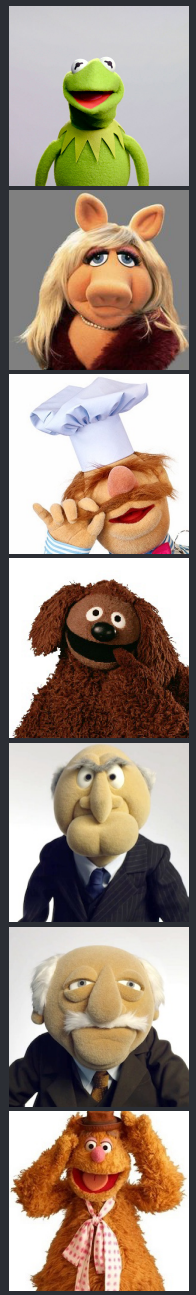
Swedish Chef

Rowlf the Dog

Statler

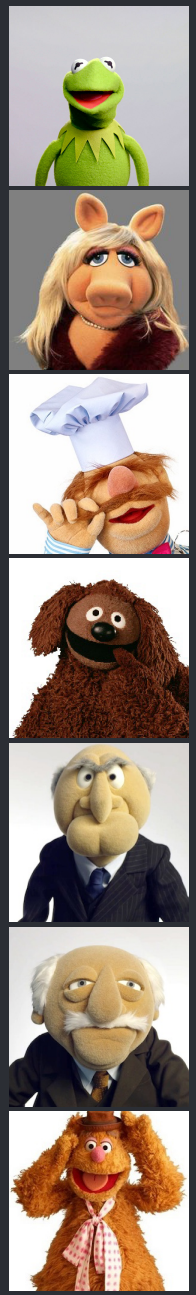
Waldorf

Fozzie Bear



Suppose we pick a Muppet at random. Each of these are events:

- $A =$ we select an animal (non-human?)



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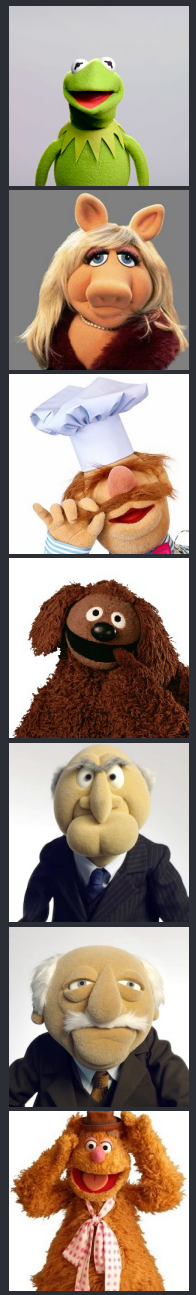
- $A =$ we select an animal (non-human?)
- $B =$ we select someone who is bald



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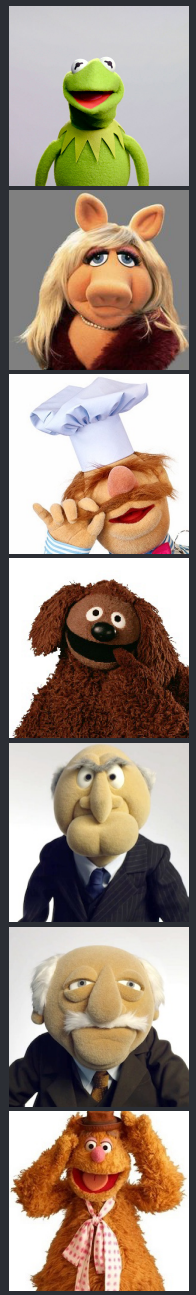

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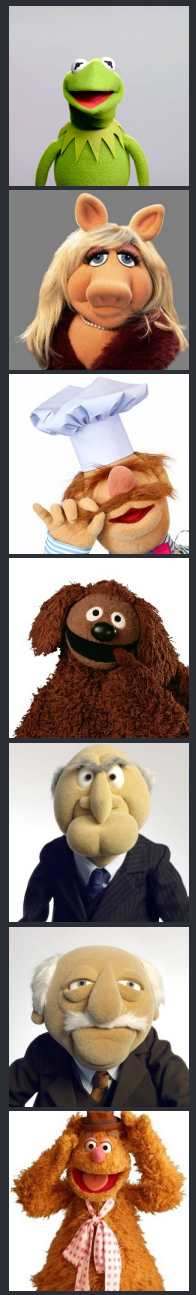
$P(A) =$



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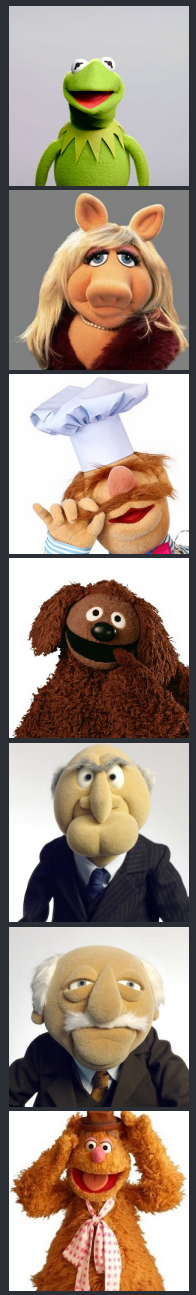
$P(A) = 4/7$ $\qquad$ $P(B) =$



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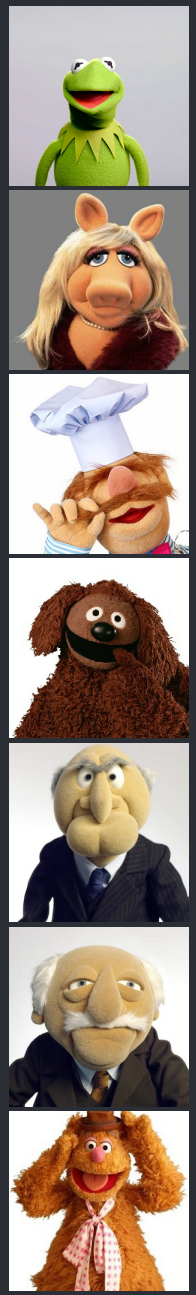

- $A|B =$ we select at random from among the bald characters, and get an animal
- $B|A =$ we select at random from among the animals, and get a bald Muppet

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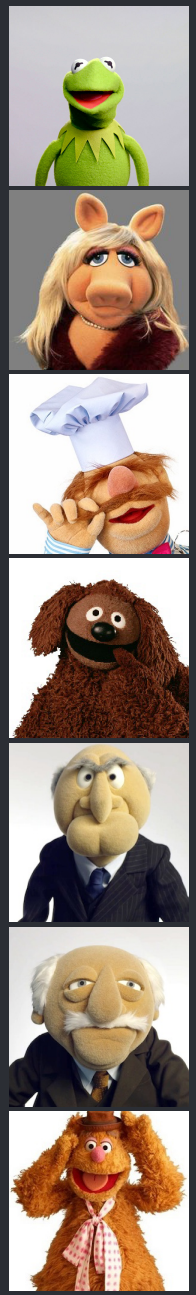

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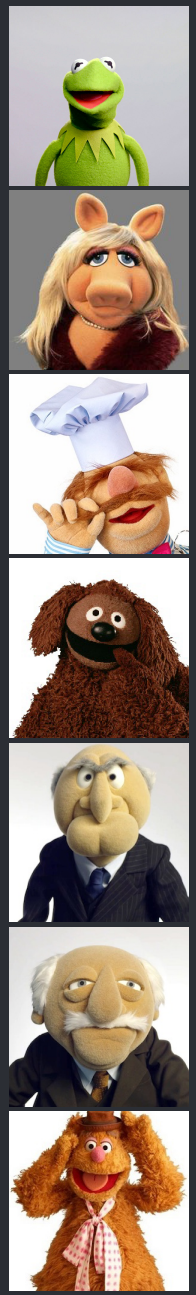
$$P(A) = 4/7 \qquad P(B) = 3/7$$

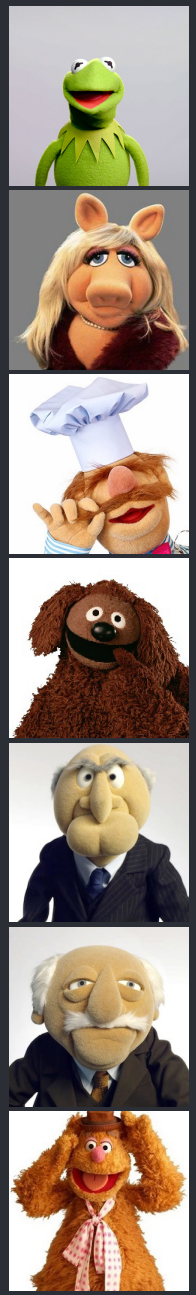
$$P(A|B) = \frac{\text{\# bald animals}}{\text{\# bald Muppets}} = \frac{P(A \text{ and } B)}{P(B)} = \frac{1}{3}$$

$$P(B|A) = \frac{\text{\# animals}}{\text{\# animal Muppets}} = \frac{P(A \text{ and } B)}{P(A)} = \frac{1}{4}$$



- $A|B$ = we select at random from among the bald characters, and get an animal
- $B|A$ = we select at random from among the animals, and get a bald character

$$P(A \text{ and } B) = P(\text{select bald animal from all Muppets})$$



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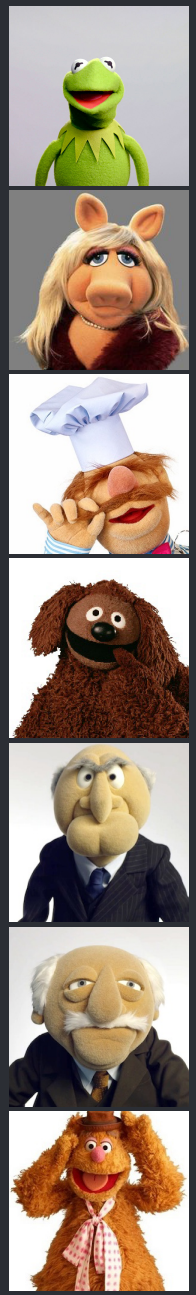

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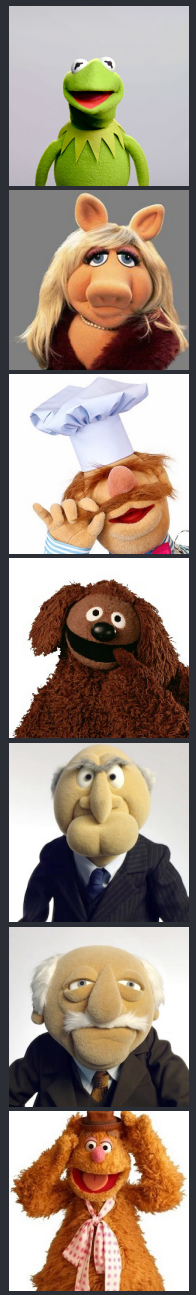
$$
\begin{aligned}
P(A \text{ and } B) &= P(\text{select bald animal from all Muppets}) \\
&= 1/7 \\
&= P(A)P(B|A) = \frac{4}{7} \cdot \frac{1}{4} \\
&= P(B)P(A|B) = \frac{3}{7} \cdot \frac{1}{3}
\end{aligned}
$$



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&\neq P(A)P(B)
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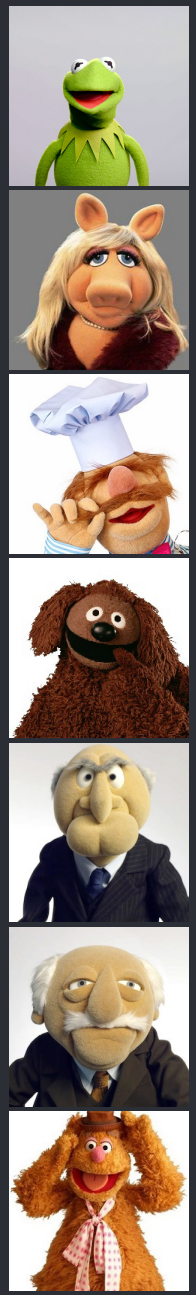
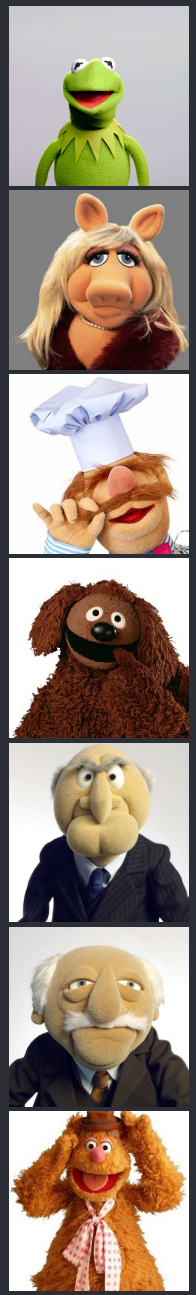
$$



The easy multiplication rule $P(A \text{ and } B) = P(A)P(B)$ does not work because baldness and animalness are *not* independent!



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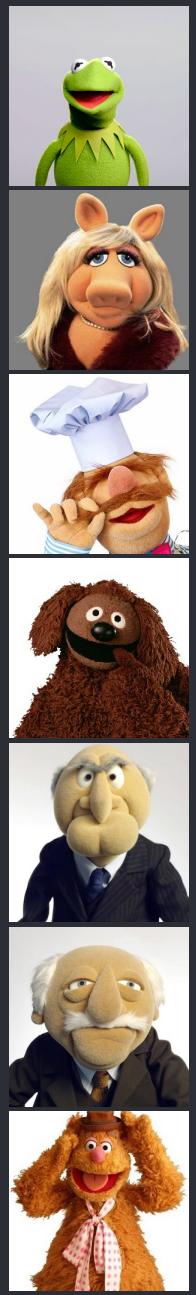
- If a Muppet is bald, they are less likely to be an animal



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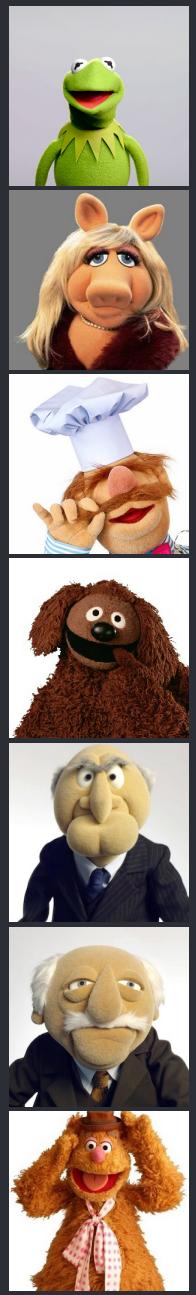

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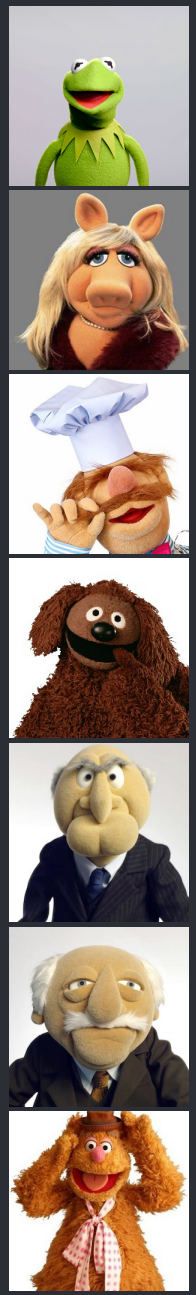

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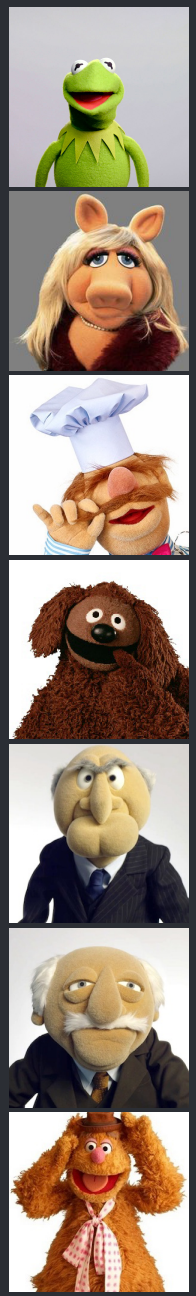
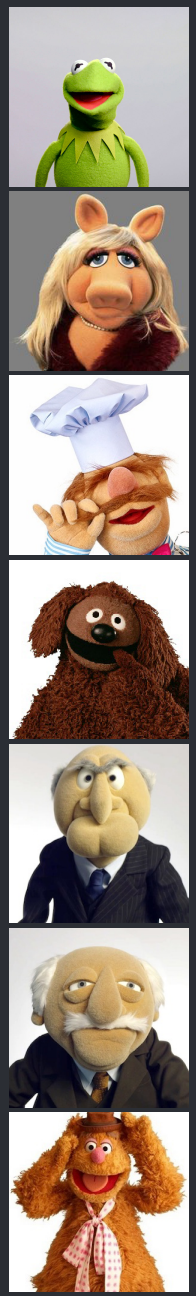
The more complex rule $P(A \text{ and } B) = P(A)P(B|A)$ will always work.



$P(A \text{ or } B)$ = probability of selecting an animal,

a bald Muppet, or both



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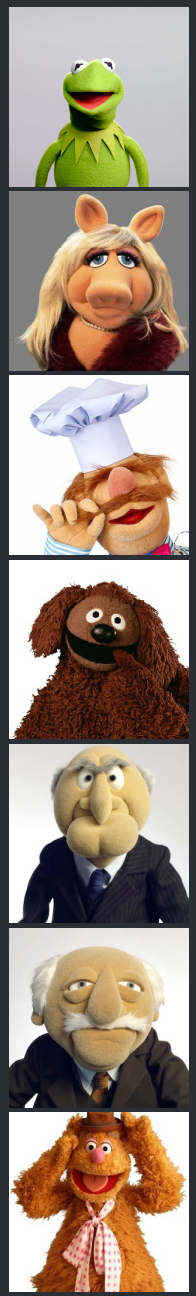
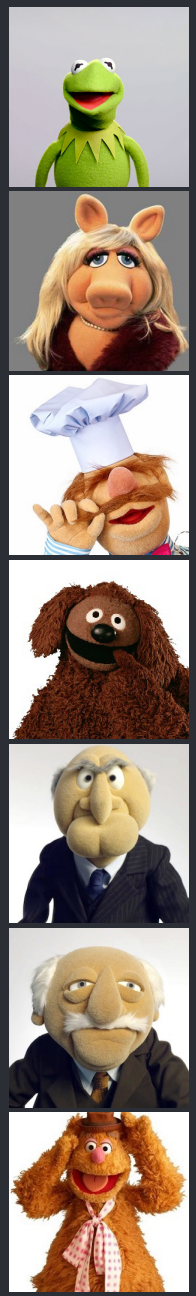
$= 6/7$



$$
\begin{aligned}
P(A \text{ or } B) &= \text{probability of selecting an animal,} \\
&\qquad \text{a bald Muppet, or both} \\
&= 6/7 \\
&= P(A) + P(B) - P(A \text{ and } B) \\
&= 4/7 + 3/7 - 1/7
\end{aligned}
$$



$$P(A \text{ or } B) = \text{probability of selecting an animal, a bald Muppet, or both}$$
$$= 6/7$$
$$= P(A) + P(B) - P(A \text{ and } B)$$
$$= 4/7 + 3/7 - 1/7$$

We have to subtract off $P(A \text{ and } B)$ because otherwise we are double-counting Kermit! (Poor Kermit: it's not easy being green.)

Conditional probability

When we say $P(A|B)$, what we mean is:

"In a world where we know B has already happened, how likely is it that A also happened?"


HOPE SHE WAS

Now that you've found a strange pair of underwear in your dresser, what is the probability that your partner is cheating on you?

$$P(\text{partner is cheating} \mid \text{underwear found})$$

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It's hard to know how to estimate this directly!

Let's come up with estimates for the following:

- $P(\text{underwear found} \mid \text{partner is cheating}) = P(U|C)$

Now that we found the underwear, we want to *update our estimate* of how likely it is that our partner is cheating, i.e., we want to know $P(C|U)$.

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Now that we found the underwear, we want to *update our estimate* of how likely it is that our partner is cheating, i.e., we want to know $P(C|U)$.
(Note: This is the *reverse* of the probability we know already!)

Let's figure it out!

Since $P(C \text{ and } U) = P(C|U)P(U)$,

$$P(C|U) = \frac{P(C \text{ and } U)}{P(U)}.$$

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Now we just need to figure out $P(C \text{ and } U)$ and $P(U)$!

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There are two mutually-exclusive ways you could find underwear in the drawer: if your partner is cheating, and if they aren't cheating.

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There are two mutually-exclusive ways you could find underwear in the drawer: if your partner is cheating, and if they aren't cheating. So:

$$P(U) = P(U \text{ and } C) + P(U \text{ and } \overline{C})$$