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Development of an Algorithm Improving Label Arrangements in Offset Printing

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Abstract One of the most classic problems in the manufacturing industry is inventory processing. The best method to solve the problem is not to make any inventory. There is a way to effectively reduce inventory by merely changing the array of the pieces on the printing plates in the offset printing. It is setting an upper limit of acceptable for each plate and carrying out complete enumeration. These method reduce the accuracy, but dramatically reduces the operating time of the algorithm. The strength of this method lies in the fact that there is only change the arrangement of the pieces inside the plates.

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1 Introduction

A combinations with repetition is the number of cases, where k elements are selected from among different n elements allowing repetition [1]. It is indicated with the symbol ${}_nH_k$ and the following is established.

$$ {}_nH_k = {}_{n+k-1}C_k = \frac{(n+k-1)!}{(n-1)!k!} \tag{1} $$

For instance, the combinations with repetition ${}_2H_4$ to select four elements from among two elements A and B comprises the following five cases.

(1) [A, A, A, A] : a list consisting of four A's

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(2) [A, A, A, B] : a list consisting of three A's and one B
(3) [A, A, B, B] : a list consisting of two each of A's and B's
(4) [A, B, B, B] : a list consisting of one A and three B's
(5) [B, B, B, B] : a list consisting of four B's

Among the above cases, if we want to obtain three A's and nine B's, then we can choose [A, B, B, B] $\times$ 3. Let us consider the following.

$$[\text{A, A, B, B}] \times 1 + [\text{A, B, B, B}] \times 1 + [\text{B, B, B, B}] \times 1 \quad (2)$$

In this case, we can get the three A's and nine B's. However, the former case seems to be a 'Better' because making three different lists is inefficient. Let us examine another case.

$$[\text{A, A, A, A}] \times 1 + [\text{B, B, B, B}] \times 3 - [\text{A}] \times 1 - [\text{B}] \times 3 \quad (3)$$

[A, B, B, B] $\times$ 3 also seems to be 'Better' because there is no loss. Under the following conditions, [A, B, B, B] $\times$ 3 is the 'Best' method.

(1) Minimize the number of list.
(2) Minimize the loss of lists.

Offset printing, also called offset lithography, or litho-offset, in commercial printing, widely used printing technique in which the inked image on a printing plate is printed on a rubber cylinder and then transferred (i.e., offset) to paper or other material. The rubber cylinder gives great flexibility, permitting printing on wood, cloth, metal, leather, and rough paper (see Fig.1) [2].

Offset lithography is one of the most common ways of creating printed materials. A few of its common applications include: newspapers, magazines, brochures, stationery, and books. Compared to other printing methods, offset printing is best suited for economically producing large volumes of high quality prints in a manner that requires little maintenance [3]. Therefore, how to make the initial plates is an important issue. Above example means that what is the best arrangement in such print method.

In the past, production was based on ordering of products from companies. For instance, apparel companies stocked popular products in warehouses. If the companies didn't have inventory, then consumers could not obtain rare sized or unpopular products. However, as the Internet market has became popular, the production systems have been changed into systems. Factories do not produce products based on their prediction of consumption but do produce only ordered products.

World Komax is a company that produces labels using the offset printing. The labels refer to stickers containing bar-codes attached to garments or shoes as follows (see Fig.2). Each bar-code in the label contains fixed information such as product names and colors, and variable information such as the date of manufacture.

Since the program of World Komax is not suitable for small quantity batch productions, the number of labels loss increased compared to the past.

Section 2 of this paper will describe the process of label printing using offsets. The modeling of the problem will be carried out in Section 3, and examples to help the understanding of the problem will be prepared in Section 4. The final results of the algorithm will be described in Section 5.

2 Offset label printing process

2.1 Label printing process

Prior to describing the label printing process, we define the following terms first.

* **Plate** : A printing plate for the offset printing (see Fig.3)
* **Loss** : The number of labels printed in excess of the order-quantity

The offset label printing process is as follows. At first, we receive orders from customers. The order includes many types of labels to be printed and order-quantities by type (see Fig. 4). Thereafter, offset printing plates are made. Many types of labels are placed on each plate so that many labels are printed at one printing. When the plates have been made, the printing operation is carried out so that individual labels are produced in quantities not smaller than the order-quantities using individual plates. As the final process, the sheets are cut to the sizes of labels and labels are collected by type.

2.2 Major points for cost saving

The constraint conditions and major points that will be considered in this paper for cost saving are as follows. First, one type of label should be placed on one plate only. This is to prevent different types of labels from being mixed when collected by type after the printed sheets are cut. Meanwhile, the total number of labels placed on each plate is also constant because the sizes of individual plates are constant and the sizes of labels in one order are also constant. In addition, the number of plates should be minimized as little as possible because plates are made using molds and the costs are high. Finally, the losses of labels printed should be minimized because bar-codes which are used only one time are printed in the labels and if the inventory remains, they cannot be used and should be entirely discarded.

2.3 Sorting report output program

Plate fabrication and the use of printing paper incur costs. In order to reduce the costs, order details are inputted to output appropriate methods to place labels on the plate as sorting reports. The plate makers produce plates according to the instructions in the sorting reports (see Fig. 4).

Since the existing sorting report output method was not suitable for small quantity batch production systems, the algorithm should be improved. Therefore, this study was conducted to develop new algorithms suitable for small quantity batch production systems too.

3 Modeling and flowchart

To accurately formulate our problem as a mathematical optimization problem, we first need to define our notation.

- Let I be a set of products.
- For each product $i \in I$, let b_i be the number of order.
- Let $\mathbf{b} = (b_i | i \in I)$ be a vector of order-quantities.
- k is the total number of labels that can be placed in one Plate.
- π is a partition of I such that $1 \leq |P| \leq k$ for any $P \in \pi$.
- Let Γ_π be a set of matrix $A \in \mathrm{Mat}_{\pi \times I}(\mathbf{Z})$ satisfy the following:
  - For all $(P, i) \in \pi \times I$, $A_{P,i} \leq 0$, and $A_{P,i} = 0$ if and only if $i \notin P$.
  - For each $P \in \pi$, $\sum_{i \in P} A_{P,i} = k$

Using the above notation,

$$\left\lceil \max \left\{ \frac{b_i}{A_{P,i}} \middle| i \in P \right\} \right\rceil \tag{4}$$

is the number of printing of Plate P, where $\lceil\ \rceil$ means the ceiling. Assume that α is the cost to produce one Plate, and β is the cost of the loss of one label. Then, our goals is to obtain the following

$$\min_{\pi, A \in \Gamma_\pi} \{\alpha|\pi| + \beta E_{A,b} | A \in \Gamma_\pi\} \tag{5}$$

where

$$E_{A,b} = \sum_{P \in \pi} \sum_{i \in I} \left(\left\lceil \max \left\{ \frac{b_i}{A_{P,i}} \middle| i \in P \right\} \right\rceil \cdot A_{P,i} - b_i \right) \tag{6}$$

means the total number of losses of labels.

Naturally, the complete enumeration using combinations with repetition is the surest way. However, this method has a problem of taking too much time. For instance, when $n = 65$ and $k = 24$, the combination with repetition ${}_{65}H_{24}$ comprises 2.36×10^{21} cases, and the calculation of the cases takes more than 658 hours, that is, more than 27 days using a super computer that can calculate 10^{15} partitions per second. Given that there are limits of the time from the date of receipt of orders to the delivery date, this is a very long computation time.

In this algorithm, this problem was solved by introducing any positive integers as tolerances. Tolerances means the allowed amount of losses occurring in each plate. Adopting a partition that does not exceed the tolerance will dramatically reduce the time taken.

Based on the foregoing, the flowchart of the algorithm can be set forth as follows. This algorithm outputs matrix A containing the label of each product when I and $\mathbf{b}$ have been inputted for z, k, num (see Fig. 5).

Meanwhile, in the case of the $loop(k, z, num)$ function, the following flowchart should be followed. First, the $Part(k, num)$ function finds partitions using the combination with repetition ${}_{num}H_k$ (method to select k pieces of products from num pieces of products allowing repetition) and indicated in the form of a list. We set the result as *Part_list*. An appropriate P that has num pieces of products is selected from I and the loss is obtained using *Part_list* and the printing number.

If the loss exceeds the tolerance z, another *Part_list* will be selected and the foregoing will be repeated while adjusting num until the tolerance is not exceeded. As a result of this process, $A_{P,i}$ that does not exceed the tolerance is obtained (see Fig. 6).

4 Examples

This example was described to help the understanding of the problem. In this section, we assume that k is equal to 4.

Example 1 Let $I = \{A,B,C\}$ and $\mathbf{b} = (50,30,20)$.

We consider $\pi = \{\{A,B\},\{C\}\}$ is a partition for the order-quantity vector $\mathbf{b}$. Without loss of generality, assume that $P_1 = \{A,B\}, P_2 = \{C\}$. Since $k = 4$, the matrix A can be found as follows.

$$A = \begin{pmatrix} 2 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix} \tag{7}$$

In this case, the printing numbers (4) of P_1 and P_2 are as follows, respectively.

$$\left\lceil \max\left\{ \frac{b_i}{A_{P_1,i}} \middle| i \in P_1 \right\} \right\rceil = \left\lceil \max\left\{ \frac{50}{2}, \frac{30}{2} \right\} \right\rceil = 25 \tag{8}$$

and

$$\left\lceil \max\left\{ \frac{b_i}{A_{P_2,i}} \middle| i \in P_2 \right\} \right\rceil = \left\lceil \max\left\{ \frac{20}{4} \right\} \right\rceil = 5. \tag{9}$$

We can see the Fig.7 for more details. It can be seen that the total number of Loss $E_{A,\mathbf{b}}$ is 20.

Now, we consider a new partition $\pi = \{\{A,B,C\}\}$. In this case, $A = (\,2\;\;1\;\;1\,)$, and printing number (4) is

$$\left\lceil \max\left\{ \frac{b_i}{A_{P,i}} \middle| i \in P \right\} \right\rceil = \left\lceil \max\left\{ \frac{50}{2}, \frac{30}{1}, \frac{20}{1} \right\} \right\rceil = 30. \tag{10}$$

In addition, it can be easily seen that the total number of Loss $E_{A,\mathbf{b}}$ is 20 (see Fig.8). Fig.8 is a more efficient because its Loss is the same but its number of Plates is smaller.

Example 2 Assume that $I = \{A,B\}$, and the order-quantity vector $\mathbf{b} = (50,20)$.

We consider $\pi = \{\{A,B\}\}$ is a partition for the quantity vector $\mathbf{b}$. Since $k = 4$, the matrix $A = (\,2\;2\,)$, and the printing number (4) is

$$\left\lceil \max\left\{ \frac{b_i}{A_{P,i}} \middle| i \in P \right\} \right\rceil = \left\lceil \max\left\{ \frac{50}{2}, \frac{20}{2} \right\} \right\rceil = 25. \tag{11}$$

The total number of Loss $E_{A,\mathbf{b}}$ is 30 (see Fig.9).

For the same partition π, the matrix $A = (\,3\;\;1\,)$ can be considered. In this case, the printing number (4) is

$$\left\lceil \max\left\{ \frac{b_i}{A_{P,i}} \middle| i \in P \right\} \right\rceil = \left\lceil \max\left\{ \frac{50}{3}, \frac{20}{1} \right\} \right\rceil = 20. \tag{12}$$

and the total number of Loss $E_{A,\mathbf{b}}$ is 10 (see Fig.10). Fig.10 is more efficient because its number of Plate is the same but fewer losses occur.

5 Result

Each sorting report indicates the number of losses corresponding to one plate. Using this, the total cost can be obtained by replacing all plates with losses. We used 30 sorting report samples and it could be seen that the total cost decreased further when the algorithm in this study was applied than when the algorithm of the manufacturer was applied for all samples. It was identified that when the improved algorithm was used, the total cost was reduced by from minimum 0.4%(sample no. 15) to maximum 15.96%(sample no. 8) (see Fig. 11).

We used a paired t-test to verify the efficiency of the algorithm. The pared t-test is two sample t-test, and it is a test that verifies whether the two groups are different. The data were provided by the aforementioned company, and the two populations are as follows.

population1: total cost before applying the algorithm population2: total cost after applying the algorithm sample1: sample of 30 items from population1 sample2: sample of 30 items from population2

In order to proceed with the two-sample t-test, the two groups must first satisfy the normality and homoscedasticity. The number of samples extracted from the two populations is 30, which can be said to have normality based on the central limit theorem. In addition, we identified the homoscedasticity of the two samples through the var.test of R.

The null hypothesis (H_0) of var.test is that 'the variances of the two groups are equal', and the alternative hypothesis(H_1) is that 'the variances of the two groups are different'. If the p-value is below the significance level, the null hypothesis (H_0) will be rejected and if the p-value is not lower than the significance level, the alternative hypothesis (H_1) will be rejected. The result of var.test with the significance level to 0.05 is as follows.

Since the p-value is not lower than the significance level(0.05), the alternative hypothesis (H_1) is rejected and the null hypothesis (H_0) is adopted. That is, the variances of the two groups can be said to be equal.

Since the two groups satisfy normality and homoscedasticity, we verified whether the difference between the two groups is significant through paired t-test. In this test, the null hypothesis (H_0) is 'the total cost will be the same after applying the algorithm.' and the alternative hypothesis (H_1) is 'the total cost will be reduced after applying algorithm.' The result of the paired t-test with the significance level to 0.05 is as follows.

Since the p-value is below the significance level(0.05), the null hypothesis (H_0) is rejected and the alternative hypothesis (H_1) is adopted. That is, the difference between the two groups can be said to be significant.

Notice. Please note that the detailed idea of the algorithm cannot be described for confidentiality of the company.

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References

1. Author, Article title, Journal, Volume, page numbers (year)
2. Author, Book title, page numbers. Publisher, place (year)
3. R. A. Brualdi, Introductory Combinatorics, Pearson Prentice Hall (2004)
4. H. Kipphan, Handbook of Print Media: Technologies and Production Methods, Springer (2001)
5. ”Offset printing,” Encyclopædia Britannica, inc., 19 6 2016. Available: https://www.britannica.com/technology/offset-printing.