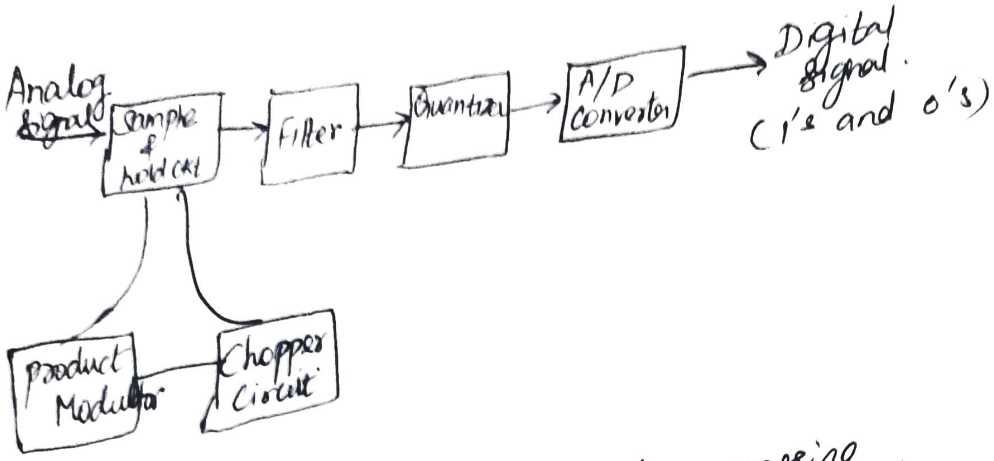
UNIT-1

DISCRETE-TIME SIGNALS AND SYSTEMS



Advantages of digital signal processing.

* Digital signal processess does not get affected easily by noise signal
* The efficiency & accuracy of the DSP are very high when compared to the analog processors.
* It is easy to store the data.

Discrete Fourier transform (DFT)

It is one of the Mathematical tool which converts time domain to the frequency domain

$x(n) \longrightarrow X(k)$

function of time — function of frequency

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi kn}{N}} \qquad K = 0 \text{ to } N-1$$

The relationship between Z-transform & fourier transform is given by

$$X(Z) = \sum_{n=0}^{N-1} x(n)\, z^{-n}.$$

The relation is $Z = e^{j\omega}$.

$$X(\omega) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}.$$

The relation between fourier transform & discrete fourier transform is given by

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}$$

$$X(\omega) = \sum_{n=0}^{N-1} x(n)\, e^{-j\omega n}$$

The relation is $\omega = \frac{2\pi k}{N}$.

Condition:

1. The input length must satisfy 2^n
2. The first sequence must be an origin sequence.

Inverse Discrete Fourier Transform (IDFT)

It is one of the Mathematical tool which converts frequency domain signal to the time domain signal.

$X(k) \longrightarrow x(n)$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi kn}{N}} \; ; n = 0 \text{ to } N-1$$

Problem:

1. Determine the DFT for the following sequence. $x(n) = \{1, 2, 3, 4\}$ and also sketch magnitude and phase spectrum

Given:

$$x(n) = \{\underset{x(0)}{1}, \underset{x(1)}{2}, \underset{x(2)}{3}, \underset{x(3)}{4}\}.$$

Solution

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi kn}{N}}$$

$N = 4.$

$$X(K) = \sum_{n=0}^{3} x(n)\, e^{-\frac{j\cancel{2}\pi kn}{\cancel{4}\,2}}$$

$$X(K) = x(0)\, e^{0} + x(1)\, e^{-\frac{j\pi k}{2}} + x(2)\, e^{-\frac{j2\pi k}{2}} + x(3)\, e^{-\frac{3j\pi k}{2}}$$

$$\boxed{X(K) = 1 + 2\, e^{-\frac{j\pi k}{2}} + 3\, e^{-j\pi k} + 4\, e^{-\frac{3j\pi k}{2}}}$$

$K = 0 \text{ to } N-1$

$K = 0 \text{ to } 3.$

Sub $k=0$

$$X(0) = 1+2e^0+3e^0+4e^0$$

$$= 1+2+3+4$$

$$\boxed{X(0)=10}$$

Sub $k=1$

$$X(1) = 1+2e^{-j\pi/2}+3e^{-j\pi}+4e^{-\frac{3}{2}j\pi}$$

$$\because \left(e^{-j\theta} = \cos\theta - j\sin\theta\right)$$

$$X(1) = 1+2\left(\cos\frac{\pi}{2} - j\sin\pi/2\right)+3\left(\cos\pi - j\sin\pi\right)+ 4\left(\cos\frac{3\pi}{2} - j\sin 3\pi/2\right)$$

$$= 1+2\left[0-j(1)\right]+3\left[-1-j(0)\right]+ 4\left[0-j(-1)\right]$$

$$= 1+2(-j)+3(-1)+4(j)$$

$$= 1-2j-3+4j$$

$$\boxed{X(1) = -2+2j}$$

$\cos\frac{\pi}{2}=0$

$\sin\pi/2 = 1$

$\cos\pi = -1$

$\sin\pi = 0$

$\cos\frac{3\pi}{2}=0$

$\sin\frac{3\pi}{2}=-1$

0
+2

$\underline{\underline{K=2}}$

$X(2) = 1 + 2e^{-\frac{j\pi 2}{2}} + 3e^{-2\pi j} + 4e^{-\frac{j\pi 6 \cdot 3}{7}}$

$= 1 + 2e^{-j\pi} + 3e^{-2\pi j} + 4e^{-3j\pi}$

$= 1 + 2(\cos\pi - j\sin\pi) + 3(\cos 2\pi - j\sin 2\pi) + 4(\cos 3\pi - j\sin 3\pi)$

$= 1 + 2(-1 - j(0)) + 3(1 - j(0)) + 4(-1 - j(0))$

$= 1 - 2 + 3 - 4$

$= 4 - 6$

$\boxed{X(2) = -2}$

$\underline{\underline{K=3}}$

$X(3) = 1 + 2e^{-j\pi\frac{3}{2}} + 3e^{-j\pi 3} + 4e^{-j\pi 9/2}$

$= 1 + 2\left(\cos\frac{3\pi}{2} - j\sin 3\pi/2\right) + 3(\cos 3\pi - j\sin 3\pi) +$
$4\left(\cos\frac{9\pi}{2} - j\sin\frac{9\pi}{2}\right)$

$= 1 + 2[0 - j(-1)] + 3[-1 - j(0)] + 4[0 - j(1)]$

$= 1 + 2j - 3 - 4j$

$\boxed{X(3) = -2 - 2j}$

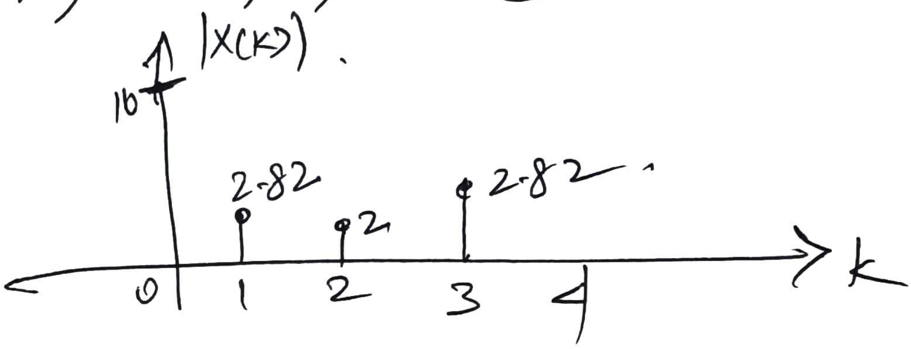
$$X(K) = \{10, -2+2j, -2, -2-2j\}$$

Magnitude Spectrum

$$|X(K)| = \left\{ \sqrt{10^2+0^2}, \sqrt{(-2)^2+2^2} + \sqrt{(-2)^2+0^2} + \sqrt{(-2)^2+(-2)^2} \right\}$$

$$= \left\{ \sqrt{100}, \sqrt{4+4}, \sqrt{4}, \sqrt{4+4} \right\}$$

$$|X(K)| = \{10, 2.82, 2, 2.82\}.$$



Phase Spectrum

$$\angle X(K) = \left\{ \tan^{-1}(0/10), \tan^{-1}\left(\frac{2}{-2}\right), \tan^{-1}(0/-2), \tan^{-1}(-2/-2) \right\}$$

$$\angle X(K) = \{0, -0.78, 0, 0.78\}$$

Inverse Discrete Fourier Transform (IDFT)

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi kn}{N}}$$

N → Length of input

k, n → 2 variables, varies from 0 → N-1

X(k) → DFT sequences.

x(n) → IDFT sequences.

k → varies from 0 - N-1

n → varies from 0 - N-1

(2) Determine the DFT for the following sequence $x(n) = \{1, -1, 2, -2\}$ and also sketch magnitude and phase spectrum.

(3) Determine 4 point IDFT for the following sequence

$$X(k) = \{15, 3+j, -5, 3-j\}$$

Given:

$$X(k) = \{15, 3+j, -5, 3-j\}$$

Solution:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}}$$

$N=4$

$$x(n) = \frac{1}{4} \sum_{k=0}^{3} X(k) e^{\frac{j2\pi kn}{\cancel{4}2}}$$

$$= \frac{1}{4}\left[X(0)e^{0} + X(1)e^{\frac{j\pi n}{2}} + X(2)e^{\frac{j\cancel{2}\pi n\cancel{2}}{\cancel{2}}} + X(3)e^{jn\pi \frac{3}{2}}\right]$$

$$= \frac{1}{4}\left[15 + (3+j)e^{jn\pi/2} - 5e^{j\pi n} + (3-j)e^{jn\pi 3/2}\right]$$

$n = 0$ to $N-1$

$n = 0$ to 3

Sub $n=0$

$$x(0) = \frac{1}{4}\left[15 + 3+j(e^0) - 5(e^0) + (3-j)(e^0)\right]$$

$$= \frac{1}{4}\left[15 + 3 + \cancel{j} - 5 + 3 - \cancel{j}\right]$$

$$= \frac{1}{\cancel{4}}(\cancel{16})\ 4$$

$$\boxed{x(0) = 4}$$

$n=1$

$$x(1) = \frac{1}{4}\left[15 + (3+j)e^{j\pi/2} - 5e^{j\pi} + (3-j)e^{j\pi\frac{3}{2}}\right]$$

$$= \frac{1}{4}\left[15 + (3+j)\left(\cos\frac{\pi}{2} + j\sin\pi/2\right) - 5\left[\cos\pi + j\sin\pi\right) + (3-j)\left(\cos\frac{3\pi}{2} + j\sin 3\pi/2\right)\right]$$

$$= \frac{1}{4}\left[15 + (3+j)\left[0 + j(1)\right] - 5\left[-1\right] + (3-j)\left[0 + j(-1)\right]\right]$$

$$= \frac{1}{4}\left[15 + \cancel{3j} - 1 + 5 - \cancel{3j} - 1\right]$$

$$= \frac{1}{4}\left[18\right]$$

$$\boxed{x(1) = 4.5}$$

$\underline{n=2}$

$$x(2) = \frac{1}{4}\left[15 + (3+j)e^{j\pi} - 5e^{j2\pi} + (3-j)e^{j\pi\frac{6}{2}}\right]$$

$$= \frac{1}{4}\left[15 + (3+j)(\cos\pi + j\sin\pi) - 5\left[\cos 2\pi + j\sin 2\pi\right] + (3-j)(\cos 3\pi + j\sin 3\pi)\right]$$

$$= \frac{1}{4}\left[15 + (3+j)(-1) - 5(1) + (3-j)(-1)\right]$$

$$= \frac{1}{4}\left[15 - 3 - \cancel{j} - 5 - 3 + \cancel{j}\right] \Rightarrow \frac{1}{4}\left[15 - 3 - 5 - 3\right]$$

$$= \frac{1}{4}[4]$$

$$\boxed{x(2) = 1}$$

$\underline{n=3}$

$$x(3) = \frac{1}{4}\left[15 + (3+j)e^{j\pi 3/2} - 5e^{j\pi 3} + (3-j)e^{j\pi 9/2}\right]$$

$$= \frac{1}{4}\left[15 + (3+j)(\cos 3\pi/2 + j\sin 3\pi/2) - 5(\cos 3\pi + j\sin 3\pi) + (3-j)\left(\cos\frac{9\pi}{2} + j\sin\frac{9\pi}{2}\right)\right]$$

$$= \frac{1}{4}\left[15 + (3+j)\left[0 + j(-1)\right] - 5(-1) + (3-j)\left[(0) + j(1)\right]\right]$$

$$= \frac{1}{4}\left[15 + (3+j)(-j) + 5 + (3-j)(0+j)\right]$$

$$= \frac{1}{4}\left[15 - 3\cancel{j} + 1 + 5 + 0 + 3\cancel{j} - j^2\right]$$

$$= \frac{1}{4}\left[15 + 5 + 1 + 0 + 1\right]$$

$$x(3) = \frac{1}{4}(22) = \frac{11}{2} = 5.5$$

$$x(n) = \{4, 4.5, 1, 5.5\}$$

Convolution.

The process of combining two different inputs to get a single output is called as convolution.

There are three stages involved in finding the output of convolution. They are:

1. Shifting
2. Multiplication.
3. Addition.

Convolution is classified into two types. They are:

Convolution → Linear Convolution, Circular convolution.

Linear Convolution	Circular Convolution.
1. The number of sequences in both inputs need not be equal.	1. The number of sequences in both inputs must be equal.
2. 0^{th} padding is not required.	2. Zero padding is required
3. The number of output is given by the relation $y(n) = N_1 + N_2 - 1$ N_1 - No. of sequence in first input N_2 - No. of sequence in second input	3. The number of output is given by the relation $y(n) = N_1 = N_2 = N$
4. Formula is given by, $y(n) = \sum_{K=-\infty}^{\infty} x(k) h(n-k)$	4. Formula is given by, $y(n) = \sum_{K=0}^{N-1} x(k) h(n-k)$.
5. Linear convolution is applicable to infinite length of sequence and aperiodic signal.	5. Circular Convolution is applicable only for finite length of sequence and periodic signal

6. Origin point may lie any where in the given input sequence.	6. Origin point must lie in the starting point of the input sequence.
7. Results can be verified with the help of tabulation method.	7. Results can be verified with the help of matrix method.
8. Memory required is very large.	8. Memory required is less
9. Linear output can be obtained through circular convolution.	9. Circular convolution output cannot be achieved through linear convolution.
10. If the product becomes zero linear convolution gets stopped.	10. If one complete rotation is over then circular convolution gets stopped

1. Determine the output using linear convolution from the given input sequence $x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$h(n) = \{1, -1, \underset{\uparrow}{2}, -2\}$

time to frequency

for Graph (k)

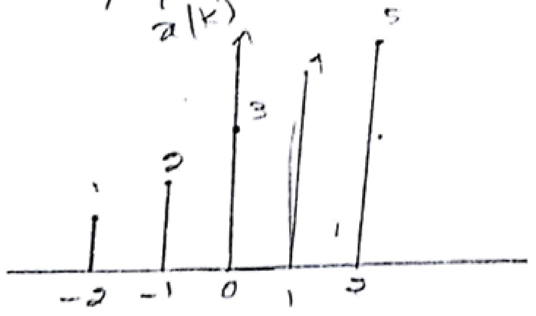
Given:

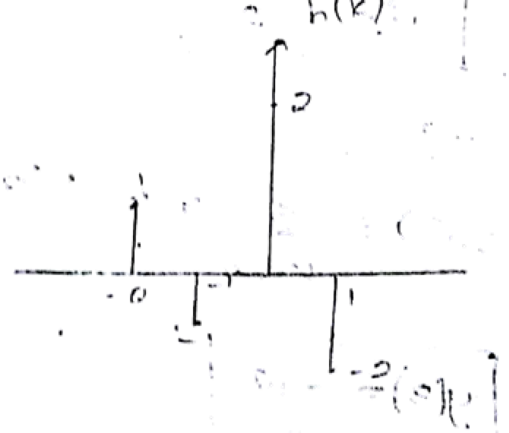
$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$h(n) = \{1, -1, \underset{\uparrow}{2}, -2\}$

Solution:

1. Graphical representation.





(3) Determine the output using circular convolution for the given sequence

$x_1(n) = \{2, 1, 2, 1\}$ & $h(n) = \{1, 2, 3, 4\}$.

Solution :-

Circular convolution

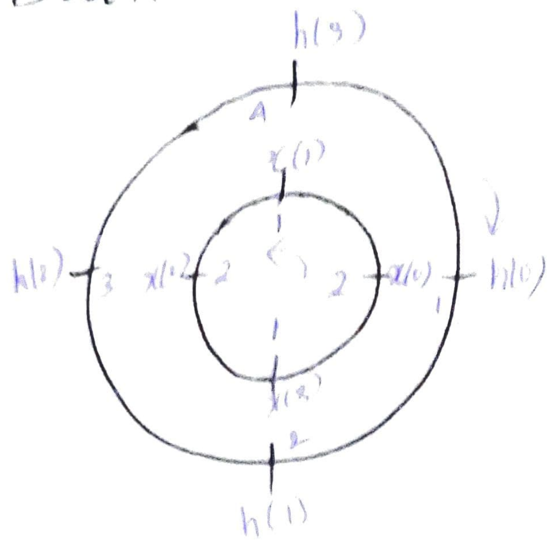
$$y(n) = \sum_{K=0}^{N-1} x(K)\, h(n-K)$$

No. of samples in the output $= N_1 = N_2 = N$

$N = 4$

Stockham's Method

(i)



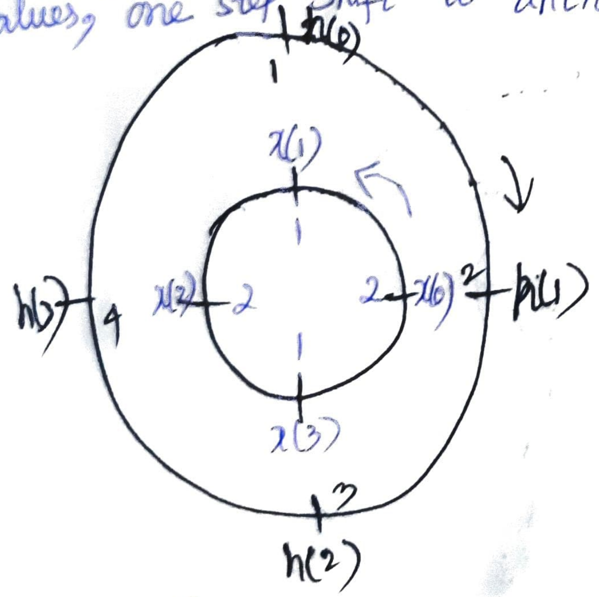
$$y(0) = \sum_{k=0}^{3} x(k)\, h(-k)$$

$$= (2\times1) + (1\times4) + (2\times3) + (1\times2)$$

$$= 2 + 4 + 6 + 2 = 14$$

$$\boxed{y(0) = 14}$$

(ii) Write the inner circle as it is, to write the outer circle values, one step shift to anticlockwise direction.



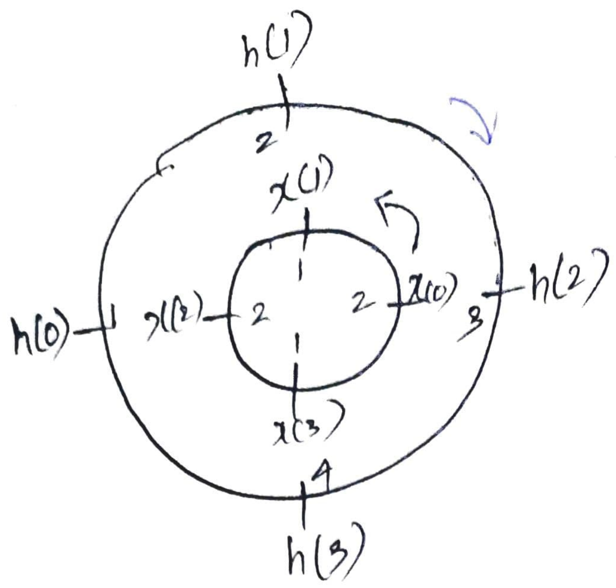
$$y(1) = \sum_{k=0}^{3} x(k)\, h(1-k)$$

$$= (1\times1) + (2\times2) + (1\times3) + (2\times4)$$

$$= 1 + 4 + 3 + 8$$

$$y(1) = 16$$

(iii)



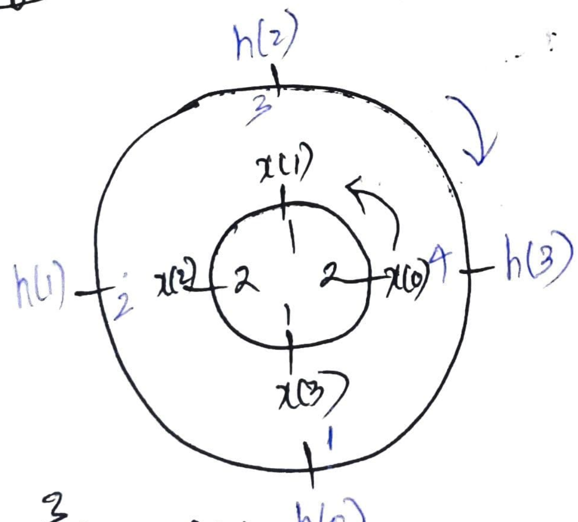
$$y(2) = \sum_{k=0}^{3} x(k)\, h(2-k)$$

$$= (1\times2)+(2\times1)+(3\times2)+(1\times4)$$

$$= 2+2+6+4$$

$$\boxed{y(2) = 14}$$

(iv)



$$y(3) = \sum_{k=0}^{3} x(k)\, h(3-k)$$

$$= (1\times1)+(2\times2)+(3\times1)+(4\times2)$$

$$= 1+4+3+8$$

$$\boxed{y(3) = 16}$$

$$y(n) = \{\underset{\uparrow}{14}, 16, 14, 16\}$$

Method 2 [Matrix Method

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} (2\times 1)+(4\times 1)+(3\times 2)+(2\times 1) \\ (2\times 2)+(1\times 1)+(4\times 2)+(3\times 1) \\ (3\times 2)+(2\times 1)+(1\times 2)+(4\times 1) \\ (4\times 2)+(3\times 1)+(2\times 2)+(1\times 1) \end{bmatrix}$$

Matrix form of 2nd sequences $h(n)$.

Matrix form of 1st sequence

$$= \begin{bmatrix} 2+4+6+2 \\ 4+1+8+3 \\ 6+2+2+4 \\ 8+3+4+1 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

(2) Determine the output using circular convolution for the given sequences

$x(n) = \{1, 3, 5, 7\}$

$h(n) = \{2, 4, 6, 8\}$

DFT for Linear filtering of Long Duration Sequences:

① Overlap Save Method:

→ $h(n)$ of Length 'M' & $x(n)$ → segmented into blocks of 'L'.

Step 1: Select value of $N = 2^M$

Step 2: The length of $h(n)$ is made 'N' by padding $L-1$ zeroes $[N = M+L-1]$

$$h(n) = \{h(0), h(1) \ldots h(M-1), 0, 0 \ldots 0\}.$$

Step 3: The sequence $x(n)$ is divided into sub sequences of long 'N' as:

$$x_1(n) = \{\underbrace{0, 0 \cdots 0}_{M-1 \text{ zeros}}, x(0), x(1) \cdots x(L-1)\}$$

$$x_2(n) = \{\underbrace{x(L-M+1) \cdots, x(L-1)}_{M-1 \text{ samples of } x_2(n)}, x(L), x(L+1) \cdots x(2L-1)\},$$

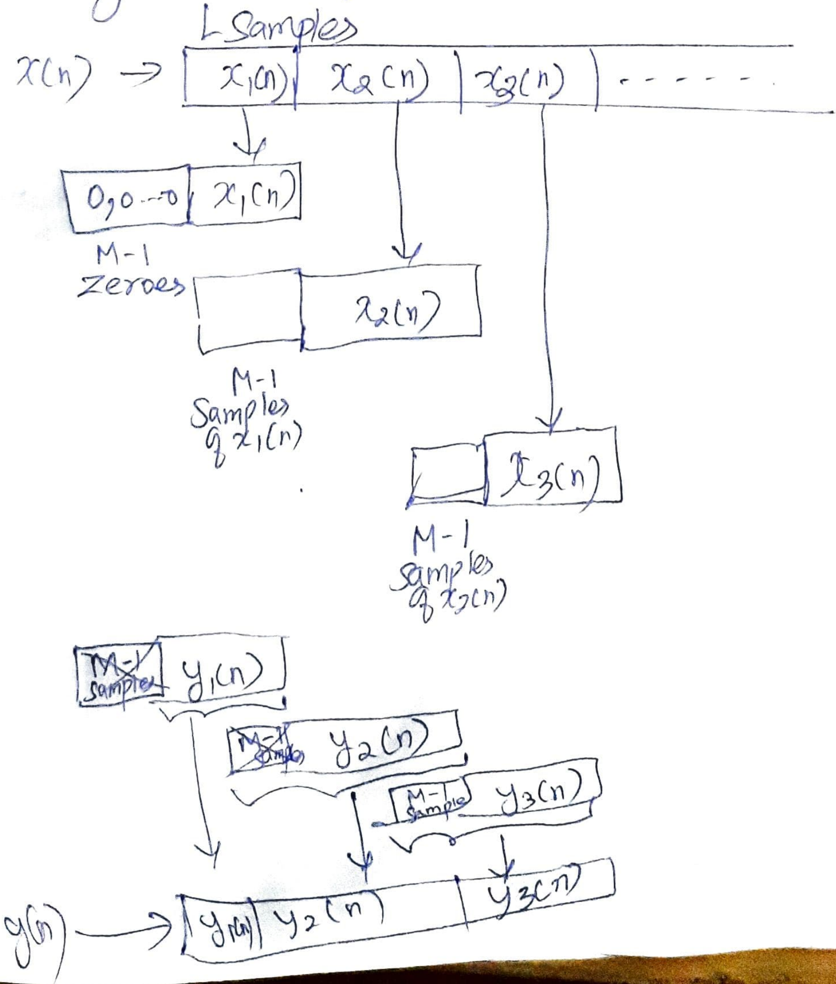
Step 4: Calculate $Y_1(k) = X_1(k) H(k)$

$y_1(n) = x_1(n) \circledast_N h(n)$

$y_1(n) = IDFT\{Y_1(k)\}$

Step 5: Repeat step 4 to obtain $y_2(n), y_3(n) \ldots$

Step 6: First $M-1$ samples of $y_1(n), y_2(n)$, $y_3(n)$ are discarded & remaining samples are fitted one after the other to get the final sequence.



(1) Find $y(n)$ for $h(n) = \{1, 1, 1\}$ and $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$ using Overlap - save Method.

$h(n) = \{1, 1, 1\}$ $\therefore M = 3$

Step 1:

$$N = 2^M = 2^3 = 8$$

$$\boxed{N = 8}$$

$$N = M + L - 1 \Rightarrow L = N - M + 1$$
$$= 8 - 3 + 1 \Rightarrow \boxed{L = 6}$$

Step 2:

$$h(n) = \{1, 1, 1, \underbrace{0, 0, 0, 0, 0}_{L-1 \text{ zeroes}}\}$$

$L = 6$
$L - 1 = 5$

Step 3: $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$

$$x_1(n) = \{\underbrace{0, 0}_{M-1 \text{ zeros}}\ 3, -1, 0, 1, 3, 2\}$$

Block length = 8 samples

$M - 1 =$
$3 - 1 = 2$

$$x_2(n) = \{\underbrace{3, 2}_{\substack{M-1 \text{ samples} \\ \text{of } x_1(n)}}, 0, 1, 2, 1, 0, 0\}.$$

$M - 1 \Rightarrow 3 - 1 = 2$

$M - 1$ $x_1(n)$

Step 4:

$$y_1(n) = x_1(n) \, (N) \, h(n)$$

$$\begin{bmatrix} 0 & 2 & 3 & 1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 2 & 3 & 1 & 0 & -1 & 3 \\ 3 & 0 & 0 & 2 & 3 & 1 & 0 & -1 \\ -1 & 3 & 0 & 0 & 2 & 3 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 2 & 3 & 1 \\ 1 & 0 & -1 & 3 & 0 & 0 & 2 & 3 \\ 3 & 1 & 0 & -1 & 3 & 0 & 0 & 2 \\ 2 & 3 & 1 & 0 & -1 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+2+3+0+0+0+0+0 \\ 3+0+2+0+0+0+0+0 \\ 3+0+0+0+0+0+0+0 \\ -1+3+0+0+0+0+0+0 \\ 0-1+3+0+0+0+0+0 \\ 1+0-1+0+0+0+0+0 \\ 3+1+0+0+0+0+0+0 \\ 2+3+1+0+0+0+0+0 \end{bmatrix}$$

8x8 Matrix

$$= \begin{bmatrix} 5 \\ 2 \\ 3 \\ 2 \\ 2 \\ 0 \\ 4 \\ 6 \end{bmatrix}$$

$y_1(n) = \{ \underbrace{5, 2}, 3, 2, 2, 0, 4, 6 \}$

Step 5 :- $y_2(n) = x_2(n) \; (N) \; h(n)$

$$\begin{bmatrix} 3 & 0 & 0 & 1 & 2 & 1 & 0 & 2 \\ 2 & 3 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 2 & 3 & 0 & 0 & 1 & 2 & 1 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 2 \\ 2 & 1 & 0 & 2 & 3 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 2 & 3 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 2 & 3 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 5 \\ 3 \\ 3 \\ 4 \\ 3 \\ 1 \end{bmatrix}$$

$y_2(n) = \{ 3, 5, 5, 3, 3, 4, 3, 1 \}$

Discard
$3 - 1 = 2$

Step 6 : $y_1(n) = [\boxed{5, 2}, 3, 2, 2, 0, 4, 6]$, $y_2(n) = [\boxed{3, 5}, 5, 3, 3, 4, 3, 1]$

$y(n) = \{ 3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1 \}$

$y(n)$ = 12 samples
$h(n)$ = 3 sample
$x(n)$ = 10 samples

$3 + 10 - 1$
$= 12$

(2) Determine the output for the given sequences $x(n) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$h(n) = \{1, 2, -1\}$ using overlap save Method & Overlap add Method.

2. Overlap Add Method :-

$h(n) \longrightarrow$ length 'M' & $x(n) \rightarrow$ Segmented into blocks

Step 1: Select $N = 2^M$, $h(n) = \{h(0), h(1) \ldots \underbrace{0, 0 \ldots 0}\}$

step 2: Length of $h(n)$ is made 'N' by padding $L-1$ zeros $[N = M+L-1]$

Step 3: The seq. $x(n)$ is divided into sub seq. of length 'N'.

$$x_1(n) = \{x(0), x(1) \ldots x(L-1), \underbrace{0, 0, \ldots 0}\}$$

$$x_2(n) = \{x(L), x(L+1) \ldots x(2L-1), \underbrace{0, 0 \ldots 0}\}$$

$$x_3(n) = \{x(2L), x(2L+1), \ldots x(3L-1), \underbrace{0, 0 \ldots 0}\}$$

Step 4: Calculate $Y_1(K) = X_1(K) \cdot H(K)$

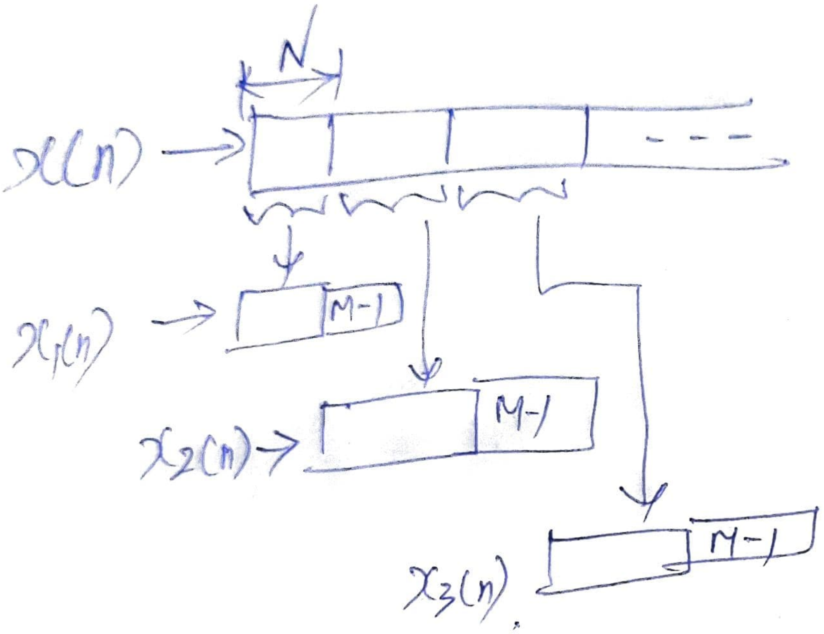
$$y_1(n) = \text{IDFT}\{Y_1(K)\}.$$

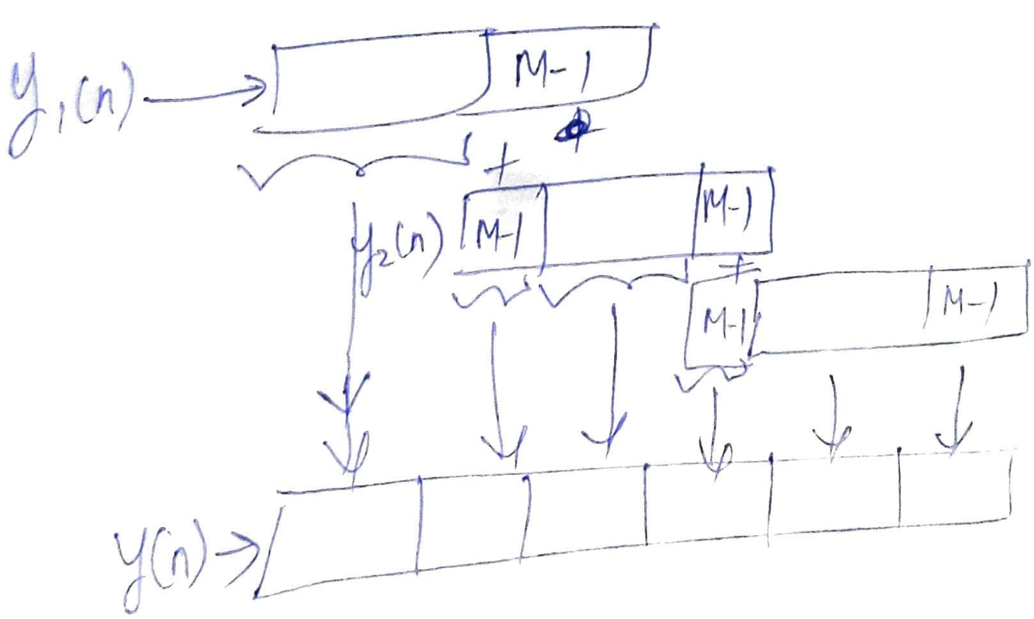
(or)

$$y_1(n) = x_1(n) \, (N) \, h(n)$$

Step 5: Repeat step 4 to obtain $y_2(n), y_3(n)$ ---

Step 6: Add all M-1 samples of each o/p sequence to first M-1 samples of succeeding o/p seq. Such seq are fitted one after another to get final seq.





① For $h(n) = \{3, 2, 1, 1\}$ & $x(n) = \{1, 2, 3, 3, 2, 1, -1, -2, -3, 5, 6, -1, 2, 0, 2, 1\}$

Find the o/p using overlap Add Method assume block length as 7.

Solution:

$h(n) = \{3, 2, 1, 1\} \quad \therefore M = 4$

$\boxed{\cancel{N = 2^M}}$ → Already given in the question.

Step 1: $N = 7$

$N = M + L - 1 \Rightarrow 7 = 4 + L - 1 \Rightarrow L = 4.$

Step 2: $h(n) = \{3, 2, 1, 1, \underbrace{0, 0, 0}_{L-1}\}$

Step 3: $x_1(n) = \{1, 2, 3, 3, \underbrace{0, 0, 0}_{M-1}\}$

$x_2(n) = \{2, 1, -1, -2, \underbrace{0, 0, 0}_{M-1}\}$

$x_3(n) = \{-3, 5, 6, -1, \underbrace{0, 0, 0}_{M-1}\}$

$x_4(n) = \{2, 0, 2, 1, \underbrace{0, 0, 0}_{M-1}\}$

$L - 1 = 4 -$

$\boxed{L - 1 = 3}$ zero

Since $L = 4$ take 4 samples

$4 - 1 = 3$

Step 4: $y_1(n) = x_1(n) \, ⓝ \, h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 3 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 3 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 & 3 \\ 3 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 3 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 3 & 2 & 1 & 0 \\ 0 & 0 & 0 & 3 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 14 \\ 18 \\ 11 \\ 6 \\ 3 \end{bmatrix}$$

Step 5:

$y_2(n) = x_2(n) \, ⓝ \, h(n)$

$= \{2, 1, -1, -2, 0, 0, 0\} \, ⓝ \, \{3, 2, 1, 1, 0, 0, 0\}$

$y_2(n) = \{6, 7, 1, -5, -4, -3, -2\}$

$y_3(n) = x_3(n) \, ⓝ \, h(n)$.

$= \{-3, 5, 6, -1, 0, 0, 0\} \, ⓝ \, \{3, 2, 1, 1, 0, 0, 0\}$

$y_3(n) = \{-9, 9, 25, 11, 9, 5, -1\}$

$y_4(n) = x_4(n) \, ⓝ \, h(n)$

$= \{2, 0, 2, 1, 0, 0, 0\} \, ⓝ \, \{3, 2, 1, 1, 0, 0, 0\}$

$y_4(n) = \{6, 4, 8, 9, 4, 3, 1\}$.

Step 6:

$M = 4$
$M - 1 = 3$
Last 3 samples of $y_1(n)$ will be added with initial 3 samples of $y_2(n)$

$y_1(n) =$	3	8	14	18	11	6	3												
$y_2(n) =$					6	7	1	-5	-4	-3	-2								
$y_3(n) =$									-9	9	25	11	9	5	-1				
$y_4(n)$													6	4	8	9	4	3	1
$y(n) =$	3	8	14	18	17	13	4	-5	-13	6	23	11	15	9	7	9	4	3	1

Fast Fourier Transform:

It is one of the Mathematical tool which converts time domain signal to the frequency domain signal as similar as DFT.

In the case of DFT if the length of input is too large then it takes more time to convert time domain to the frequency domain,

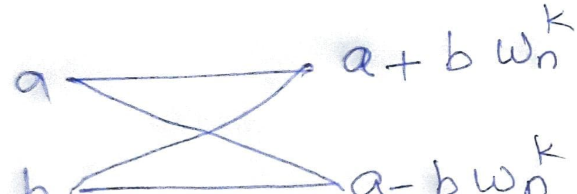
whereas, in the case of FFT it takes only lesser number of computations to convert time to frequency domain

FFT is classified into two types. They are

1. Decimation in time (DIT)
2. Decimation in frequency (DIF)

Fast Fourier Transform

DIT

i) 

ii) Assume 8 bit

Normal order

0 1 2 3 4 5 6 7

↓ (to)

Bit Reversal order

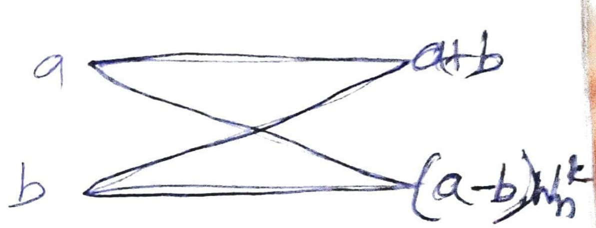
0 4 2 6 1 5 3 7

Stage 1:

2 point N=2, k=0

(1, 2) (3, 4) (5, 6) (7, 8).

DIF



(ii) Normal order

0 1 2 3 4 5 6 7

↓

Stage 1:

8 point N=8, k=0,1,2,3

(1,5) (2,6) (3,7) (4,8)

Stage 2:

(1, 2) (3, 4) (5, 6) (7, 8).

Stage 2:

4 point $N=4$, $k=0, 1$.

(1, 3) (2, 4), (5, 7), (6, 8).

Stage 3:

8 point $N=8$, $k=0, 1, 2, 3$.

(1, 5) (2, 6), (3, 7), (4, 8)

0 1 2 3 4 5 6 7

Stage 2:

4 point $N=4$, $k=0, 1$

(1, 3) (2, 4), (5, 7) (6, 8)

Stage 3:

2 point $N=2$, $k=0$

(1, 2) (3, 4) (5, 6) (7, 8)

Twiddle factor:

$$W_N^k = e^{-j\frac{2\pi k}{N}}$$

Stage 1: $N=2, k=0$

$$W_2^0 = e^{-j\frac{2\pi(0)}{2}}$$

$$W_2^0 = 1$$

Twiddle factor

$$W_N^k = e^{-j\frac{2\pi k}{N}}$$

Stage 1: $N=2, k=0$

$$W_2^0 = e^0$$

$$\boxed{W_2^0 = 1}$$

Stage 2: $N=4, k=0,1$

$$W_4^0 = e^0$$

$$\boxed{W_4^0 = 1}$$

$$W_4^1 = e^{-j\frac{2\pi}{4}}$$

$$W_4^1 = e^{-j\frac{\pi}{2}}$$

$\because e^{-j\theta} = \cos\theta - j\sin\theta$

$$e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2}$$

$$= 0 - j(1)$$

$$\boxed{W_4^1 = -j}$$

$$\boxed{W_4^1 = -j}$$

Stage 3: $N=8, k=0,1,2,3$

$$W_8^0 = e^0 = 1$$

$$\boxed{W_8^0 = 1}$$

$$W_8^1 = e^{-j\frac{2\pi}{8}} = e^{-j\frac{\pi}{4}}$$

$$= \cos\frac{\pi}{4} - j\sin\frac{\pi}{4}$$

$$\boxed{W_8^1 = 0.707 - j0.707}$$

$$W_8^2 = e^{-j\frac{2\pi 2}{8}} = e^{-j\frac{4\pi}{8}}$$

$$= e^{-j\frac{\pi}{2}} \Rightarrow \cos\pi/2 - j\sin\pi/2$$

$$= 0 - j(1)$$

$$\boxed{W_8^2 = -j}$$

$$W_8^3 = e^{-j\frac{2\pi 3}{8}}$$

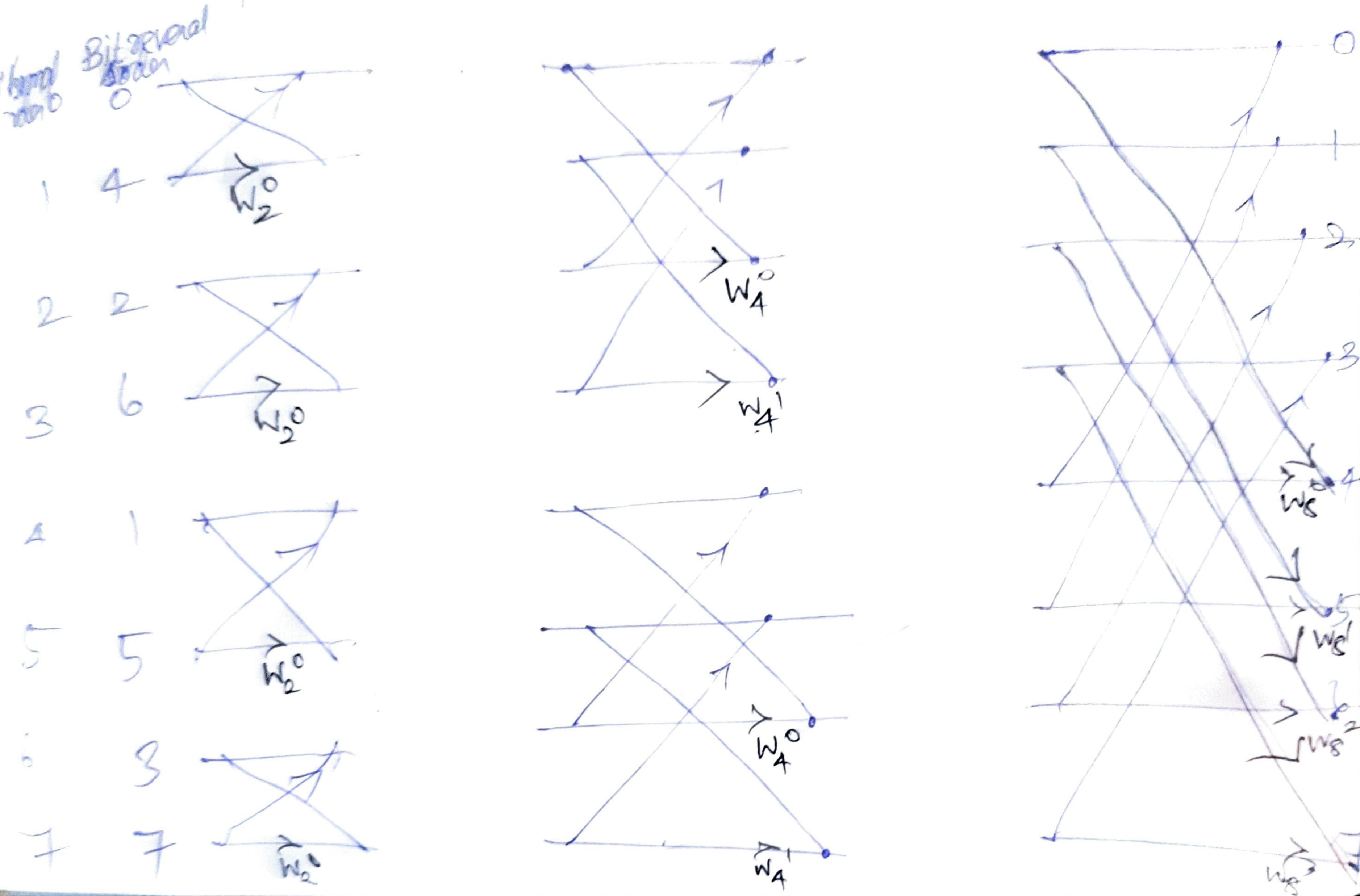
$$= \cos\frac{3\pi}{4} - \sin\frac{3\pi}{2}$$

$$\boxed{W_8^3 = -0.707 - j0.707}$$

Stage 1 (2 point

Stage 2 (4 point)

Stage 3 (8 point)

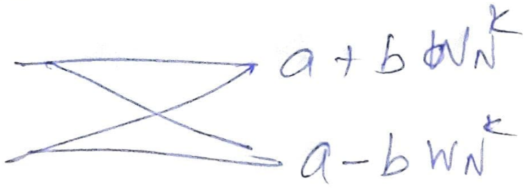


① Determine 8 point DFT for the following sequence $x(n) = \{1, -1, 2, -2, 3, -3, 4, -4\}$ using DIT & DIF Method.

Given:

$x(n) = \{1, -1, 2, -2, 3, -3, 4, -4\}$

DIT Method:



Twiddle factor

$W_2^0 = 1$

$W_4^0 = 1$

$W_4^1 = -j$

$W_8^0 = 1$

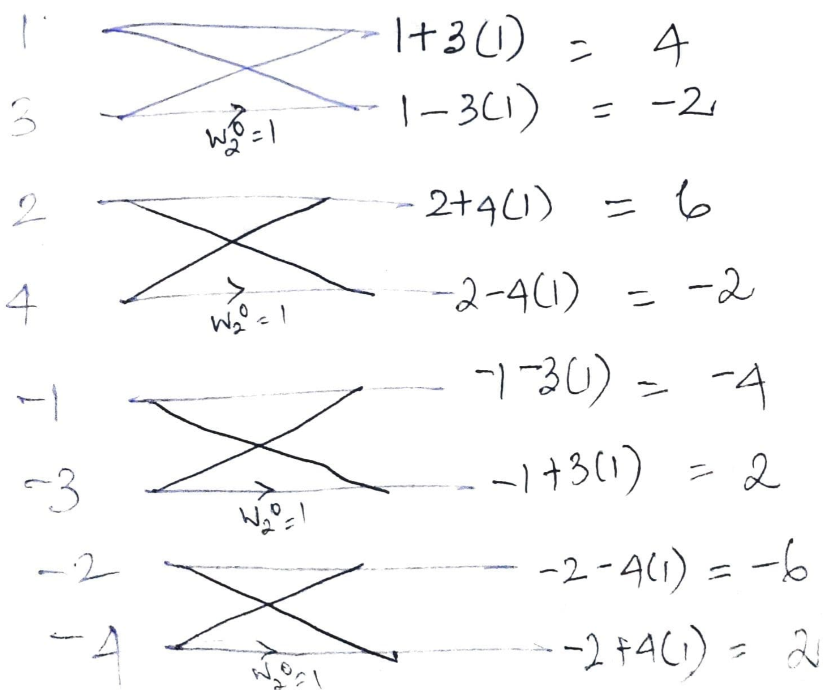
$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

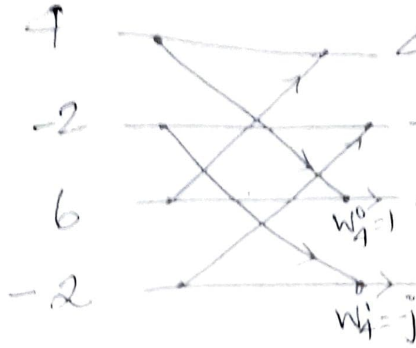
$W_8^3 = -0.707 - j0.707$

Normal order	Bit Reversal order
$x(0) = 1$	$x(0) = 1$
$x(1) = -1$	$x(4) = 3$
$x(2) = 2$	$x(2) = 2$
$x(3) = -2$	$x(6) = 4$
$x(4) = 3$	$x(1) = -1$
$x(5) = -3$	$x(5) = -3$
$x(6) = 4$	$x(3) = -2$
$x(7) = -4$	$x(7) = -4$

2 point



4 point

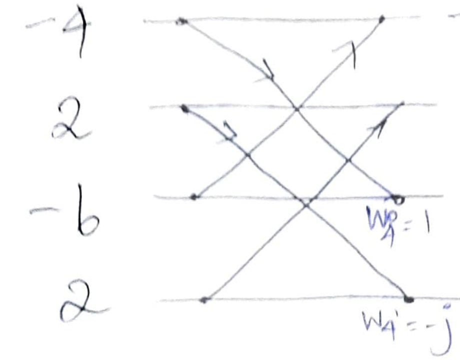


$4+6W_4^0 = 4+6(1) = 10$

$-2+(-2)W_4^1 = -2-2(-j) = -2+2j$

$4-6W_4^0 = 4-6(1) = -2$

$-2+2W_4^1 = -2+2(-j) = -2-2j$



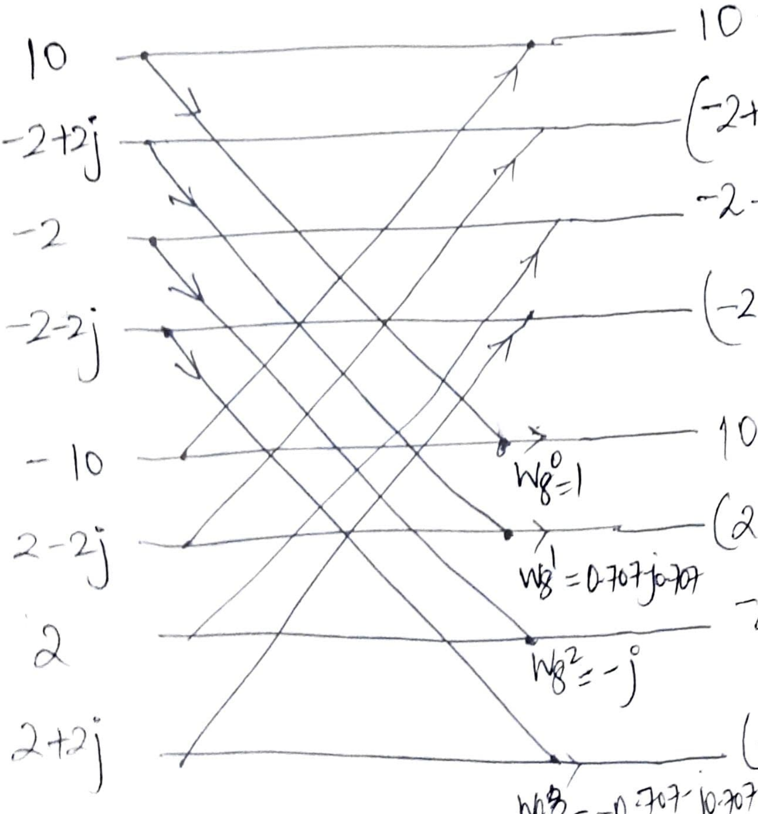
$-4-6W_4^0 = -4-6(1) = -10$

$2+2W_4^1 = 2+2(-j) = 2-2j$

$-4+6W_4^0 = -4+6(1) = 2$

$2-2W_4^1 = 2-2(-j) = 2+2j$

8 point



$10-10W_8^0 = \cancel{10}$

$(-2+2j)+(2-2j)W_8^1$

$-2+2W_8^2$

$(-2-2j)+(2+2j)W_8^3$

$10+10W_8^0$

$(2-2j)-(2-2j)W_8^1$

$-2-2W_8^2$

$(-2-2j)-(2+2j)W_8^3$

$X(0) = 10 - 10W_8^0 = 10 - 10(1)$

$$\boxed{X(0) = 0}$$

$X(1) = (-2+2j) + (2-2j)W_8^1$

$= (-2+2j) + (2-2j)0.707 - j0.707$

$= (-2+2j) + 1.414 - 1.414j - 1.414j - 1.414$

$= -2+2j + 1.\cancel{414} - 1.414j - 1.414j - 1.\cancel{414}$

$$\boxed{X(1) = -2 - 0.828j}$$

$X(2) = -2+2W_8^2 = -2+2(-j)$

$$\boxed{X(2) = -2-2j}$$

$$X(3) = (-2-2j) + (2+2j) W_8^3$$

$$= (-2-2j) + (2+2j)(-0.707 - j0.707)$$

$$= -2 - 2j - \cancel{1.414} - 1.414j - 1.414j + \cancel{1.414}$$

$$\boxed{X(3) = -2 - 4.828j}$$

$$X(4) = 10 + 10 W_8^0 \Rightarrow 10 + 10 = 20$$

$$\boxed{X(4) = 20}$$

$$X(5) = (-2+2j) - (2-2j) W_8^1$$

$$= -2 + 2j - (2-2j)(0.707 - j0.707)$$

$$= -2 + 2j - \cancel{1.414} + 1.414j + 1.414j + \cancel{1.414}$$

$$\boxed{X(5) = -2 + 4.828j}$$

$$X(6) = -2 - 2 W_8^2 = -2 - 2(-j)$$

$$\boxed{X(6) = -2 + 2j}$$

$$X(7) = (-2-2j) - (2+2j) W_8^3$$

$$= -2 - 2j - (2+2j)(-0.707 - j0.707)$$

$$= -2 - 2j + \cancel{1.414} + 1.414j + 1.414j - \cancel{1.414}$$

$$\boxed{X(7) = -2 + 0.828j}$$

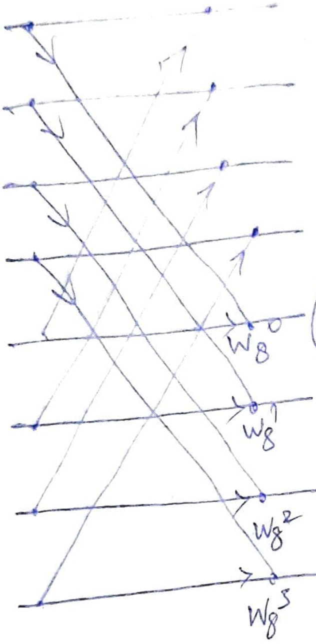
$$X(k) = \{0, -2-2j, -2-4.828j, 20, -2+4.828j, -2+2j, -2+0.828j\}$$

(2) Given $x(n) = \{1, -1, 2, -2, 3, -3, 4, -4\}$

Solution:

8 point $N=8$, $k: 0, 1, 2, 3$

$x(0) = 1$
$x(1) = -1$
$x(2) = 2$
$x(3) = -2$
$x(4) = 3$
$x(5) = -3$
$x(6) = 4$
$x(7) = -4$



$1+3 = 4$

$-1-3 = -4$

$2+4 = 6$

$-2-4 = -6$

$(1-3)W_8^0 = (-2)1 = -2$

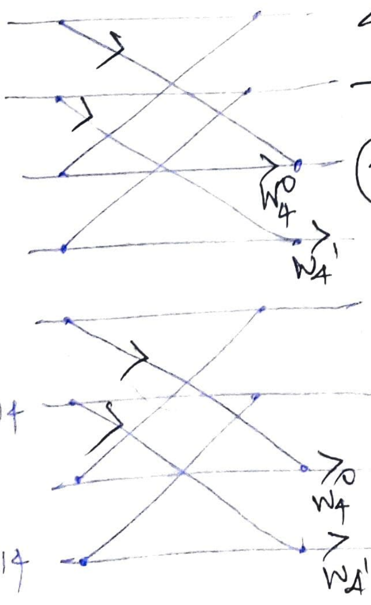
$(-1+3)W_8^1 = (2)\,0.707-j0.707 = 1.414-j1.414$

$(+2-4)W_8^2 = (-2)-j = +2j$

$(-2+4)W_8^3 = (2)-0.707-j0.707 = -1.414-j1.414$

4 point $N=4$, $k=0,1$.

4
-4
6
-6
-2
$1.414-j1.414$
$+2j$
$-1.414-j1.414$



$4+6 = 10$

$-4-6 = -10$

$(4-6)W_4^0 = -2$

$(-4+6)W_4^1 = 2(-j) = -2j$

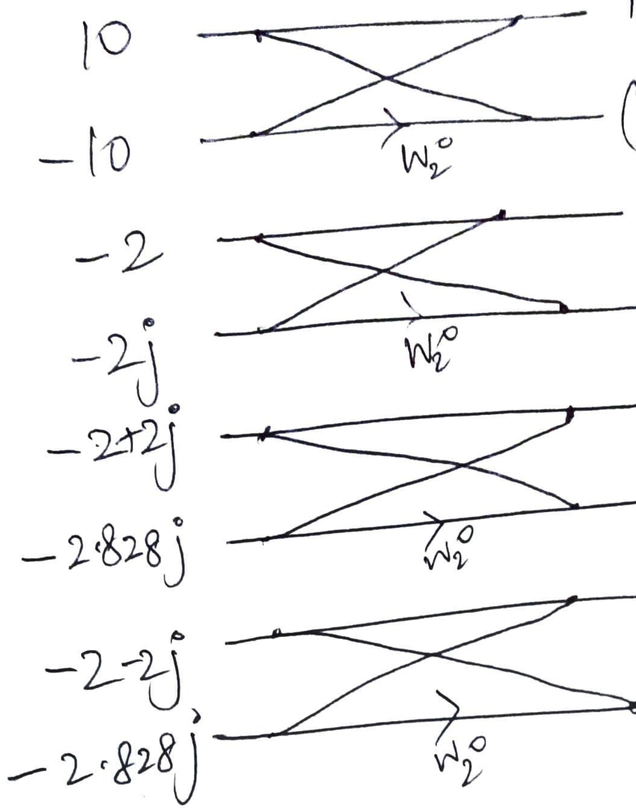
$-2+2j = -2+2j$

$(1.414-j1.414)+(-1.414-j1.414) = -2.828j$

$(-2-2j)W_4^0 = -2-2j$

$(1.414-j1.414)-(1.414-j1.414)$
$= -2.828j$

2 point $N=2, k=0$



$10-10 = 0 = X(0)$

$(10+10)W_2^0 = 20 = X(4)$

$-2-2j = -2+2j = X(2)$

$(-2+2j)W_2^0 = -2+2j = X(6)$

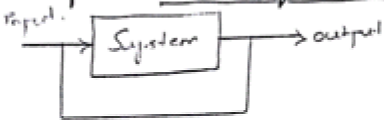
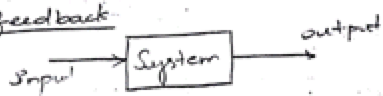
$(-2+2j)-2.828j = -2-0.828j = X(1)$

$[(-2+2j)+2.828j)]W^0 = -2+4.828j = X(5)$

$-2-2j-2.828j = -2-4.828j = X(3)$

$-2-2j+2.828j = -2+0.828j = X(7)$

$$X(k) = \{0, -2-0.828j, -2-2j, -2-4.828j, 20, -2+4.828j, -2+2j, -2+0.828j\}$$

IIR	FIR
1. The expansion of IIR is Infinite Impulse Response	1. The expansion of FIR is Finite Impulse Response
2. IIR is difficult to implement because IIR filter receives only analog inputs	2. FIR is easy to implement because it receives digital inputs directly.
3. IIR filters are also known as non-recursive system. Present output → past output, present input, past input	3. FIR filters are also known as recursive systems. Present output → Present input + past input.
4. IIR filters have feedback 	4. FIR filter doesnot have feedback 
5. IIR filters may or may not be stable	5. FIR filters are always stable
6. IIR filters are less affected by noise so accuracy and efficiency will be high.	6. FIR filters are easily affected by noise so the efficiency and accuracy may become poor.
7. IIR filters have both poles and zeros.	7. FIR filters have only zeros.

STRUCTURES OF IIR

It is classified into four types. They are

1. Direct form - I
2. Direct form - II.
3. Cascade Realization
4. Paralal Realization.

(i) Direct form structure

(a) Direct form - I

(b) Direct form - II

~~Prob~~

(1) Realize direct form I, II for the given differential equation

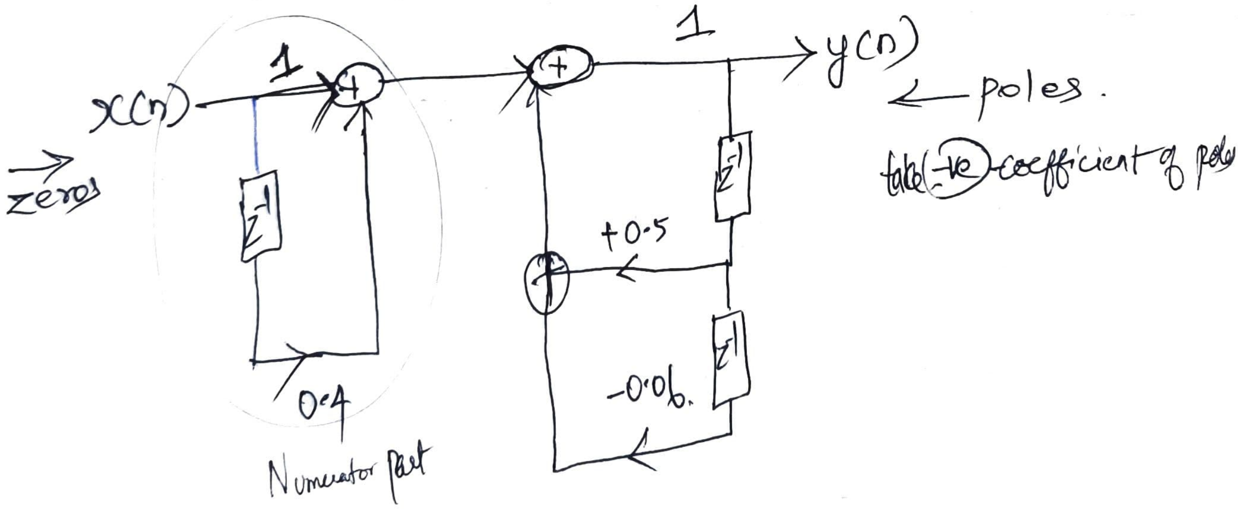
$$H(z) = \frac{1+0.4z^{-1}}{1-0.5z^{-1}+0.06z^{-2}}$$

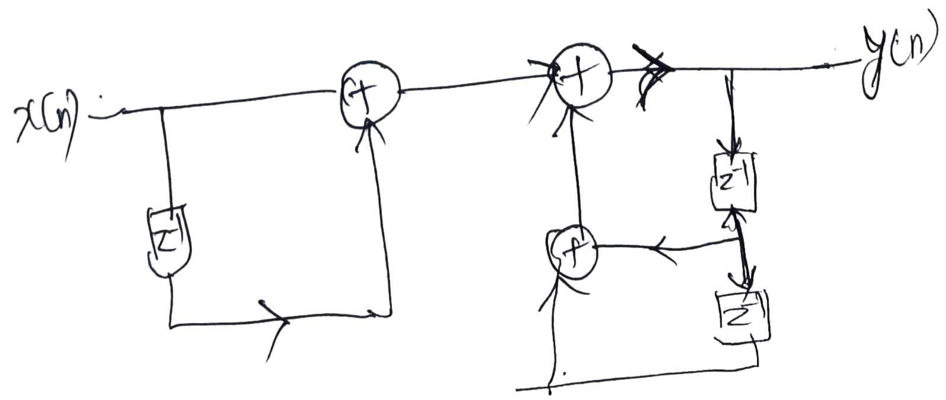
Given:

$$H(z) = \frac{1+0.4z^{-1}}{1-0.5z^{-1}+0.06z^{-2}}$$ zeros / poles.

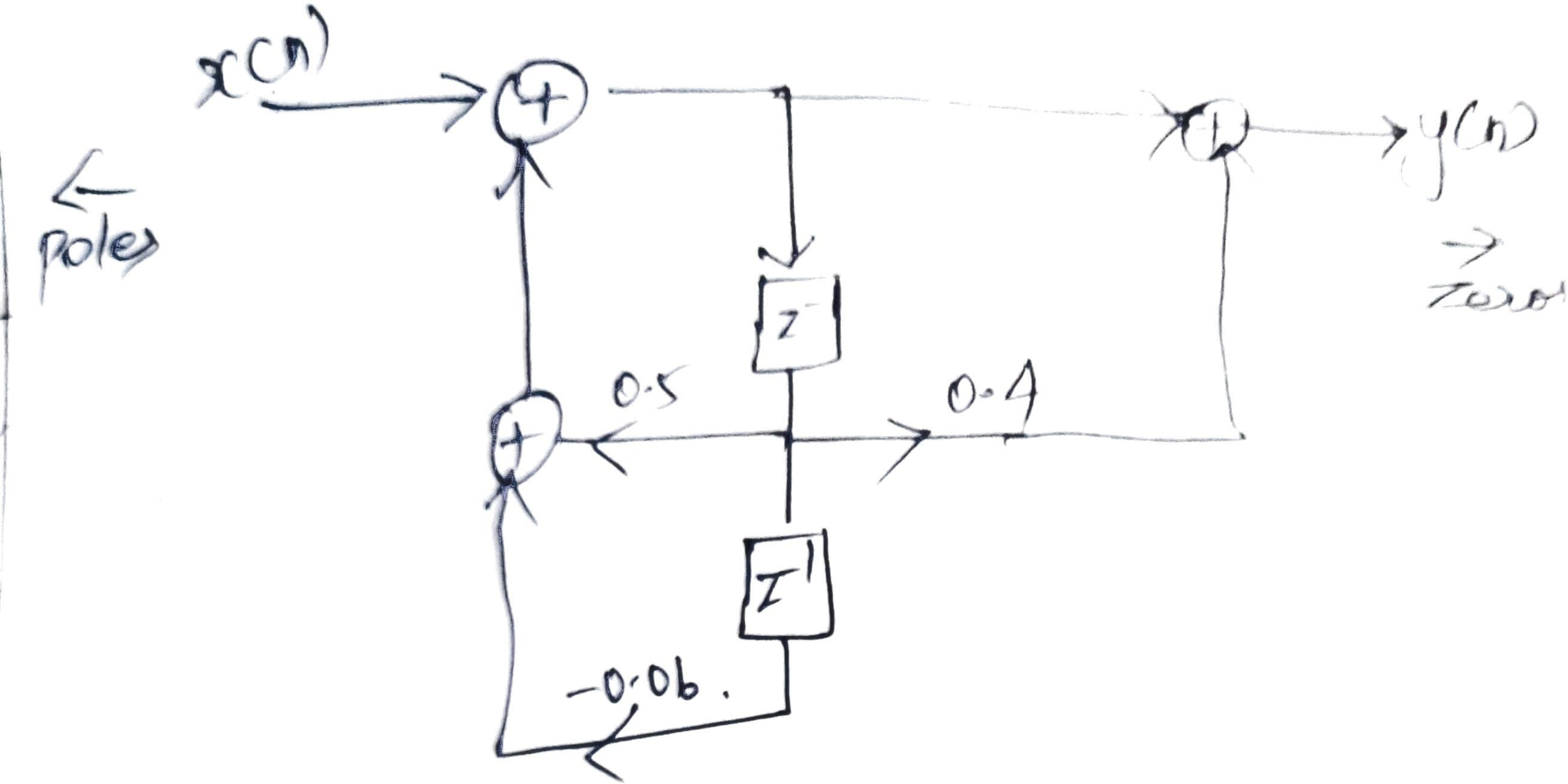
z^{-n} → delay

Direct form - I.





Direct form - II



② $$H(z) = \frac{z^{-1} - 3z^{-2}}{(10 - z^{-1})(1 + 0.5z^{-1} + 0.5z^{-2})}$$

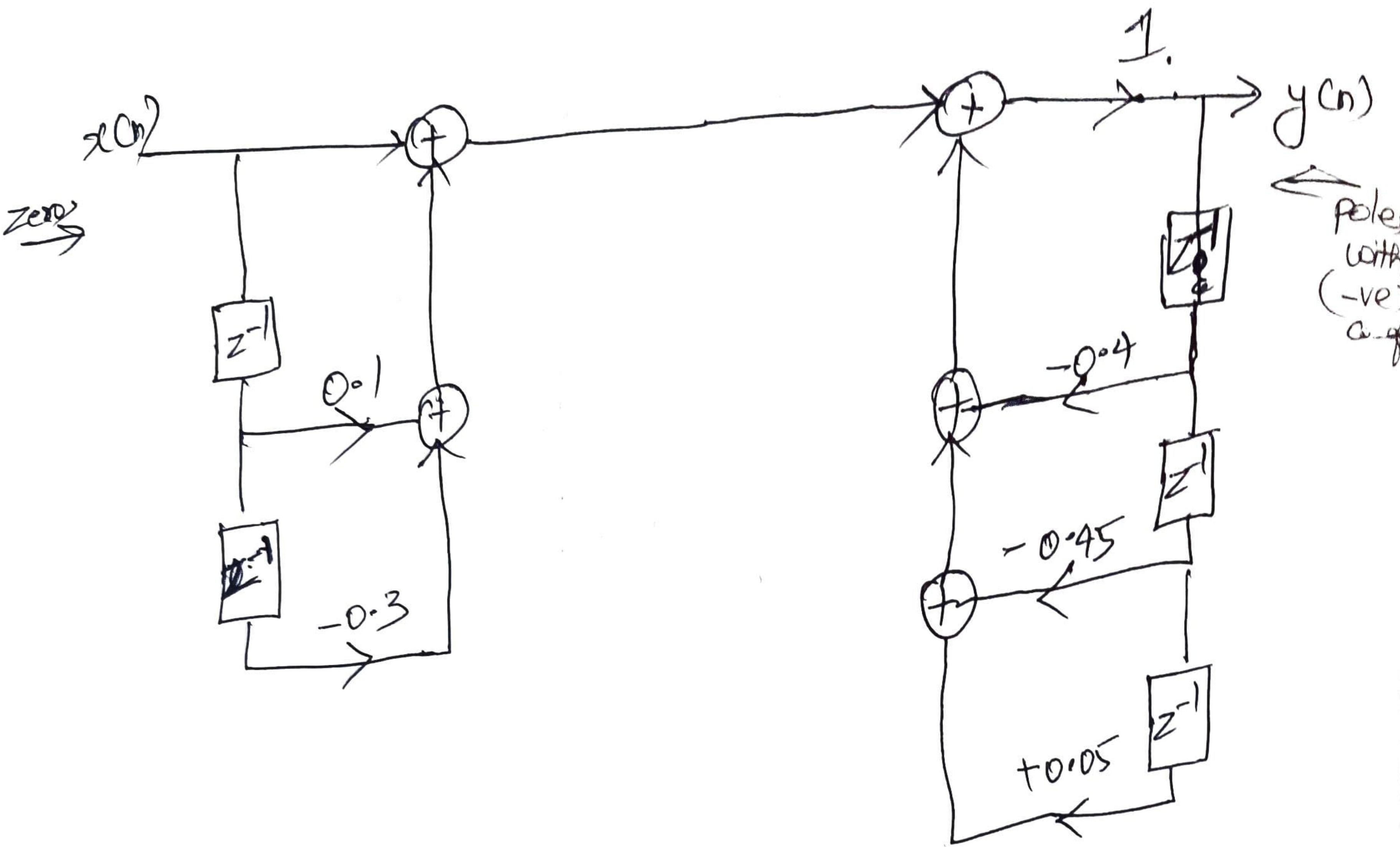
$$= \frac{z^{-1} - 3z^{-2}}{10 + 5z^{-1} + 5z^{-2} - z^{-1} - 0.5z^{-2} - 0.5z^{-3}}$$

$$H(z) = \frac{z^{-1} - 3z^{-2}}{10 + 4z^{-1} + 4.5z^{-2} - 0.5z^{-3}}$$

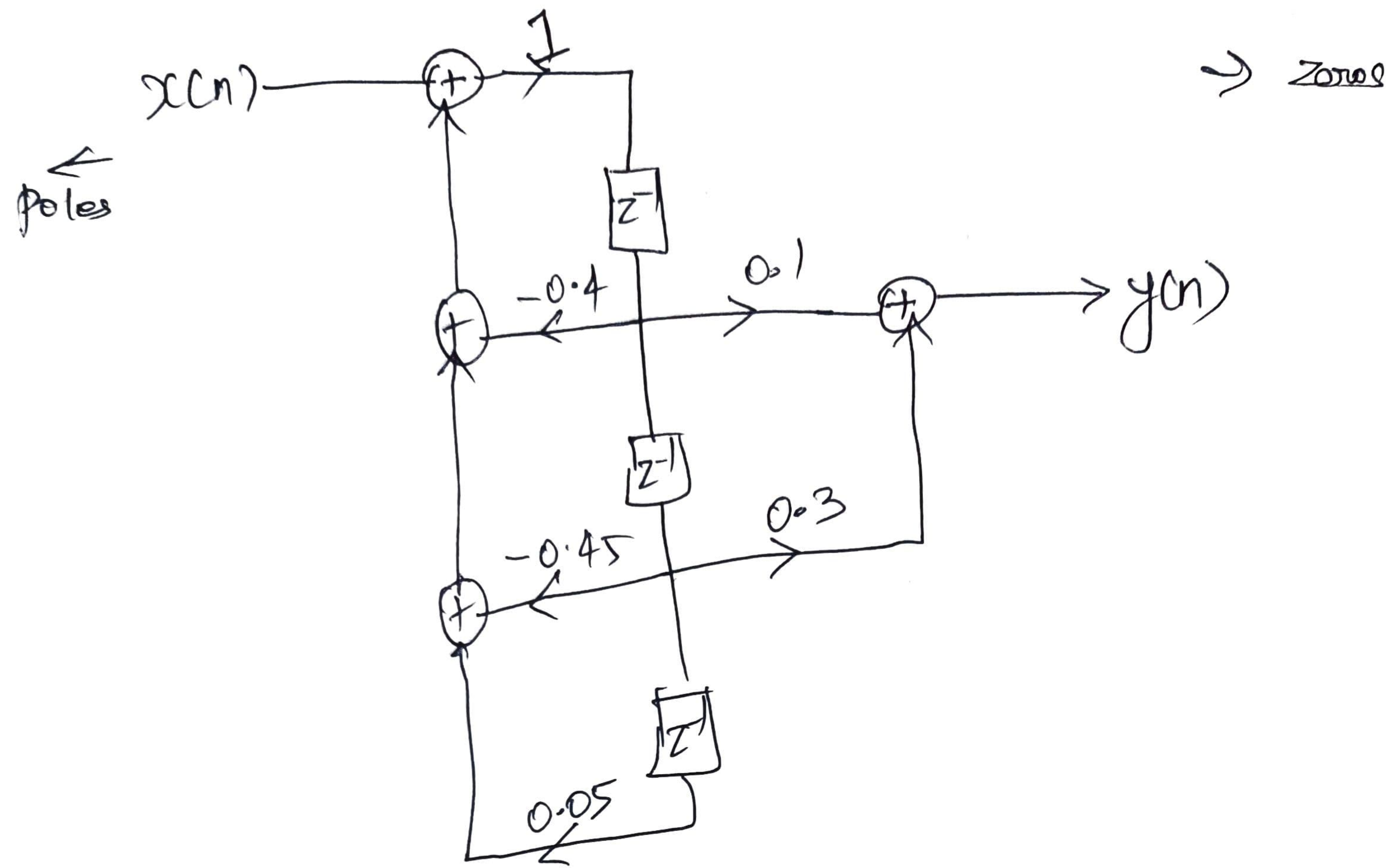
$$H(z) = \frac{(0.1)z^{-1} - 0.3z^{-2}}{1 + 0.4z^{-1} + 0.45z^{-2} - 0.05z^{-3}}.$$

(3) computes the

Direct form -I :-



Direct form - II



(3) $y(n) - \frac{1}{4} y(n-1) + \frac{1}{8} y(n-2) = x(n) + \frac{1}{2} x(n-2)$

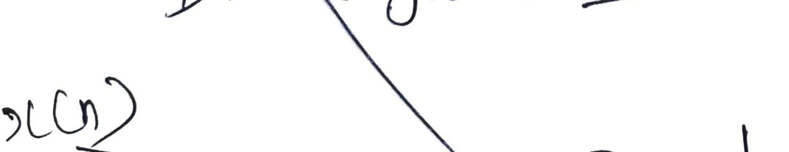
Take Z-transform.

$$Y(z) - \frac{1}{4} z^{-1} Y(z) + \frac{1}{8} z^{-2} Y(z) =$$

$$X(z) + \frac{1}{2} z^{-2} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2} z^{-2}}{1 - \frac{1}{4} z^{-1} + \frac{1}{8} z^{-2}}.$$

Direct form-II.



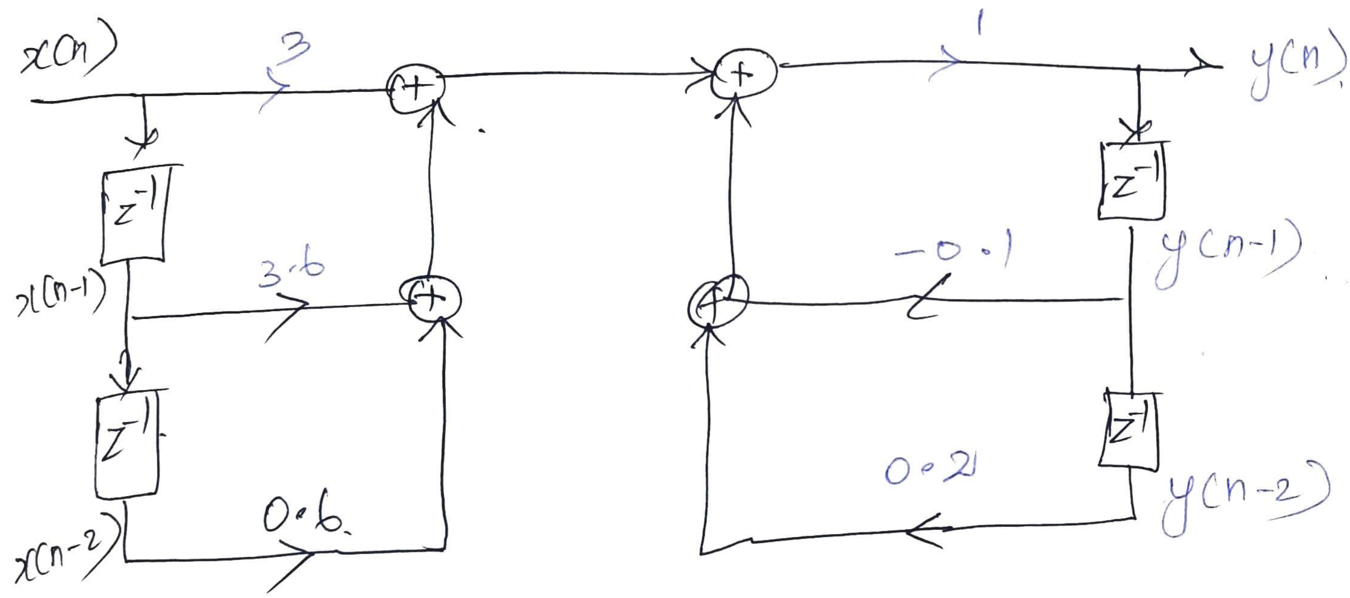
③ Obtain the direct form-I, direct form-II, cascade & parallel form realization for the system

$$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2).$$

Soln :

Direct form -I

$$y(n) + 0.1\,y(n-1) - 0.2\,y(n-2) = 3x(n) + 3.6x(n-1) + 0.6x(n-2)$$



Direct form -II

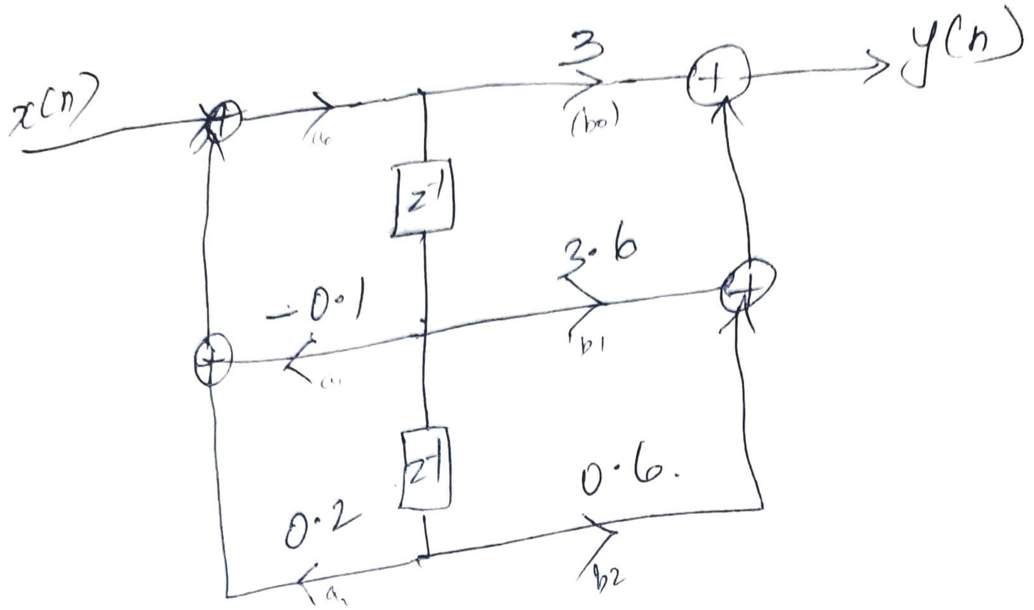
$$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

Taking z-transform :

$$Y(z) = -0.1z^{-1}Y(z) + 0.2z^{-2}Y(z) + 3X(z) + 3.6z^{-1}X(z) + 0.6z^{-2}X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$



cascade form

$$\frac{Y(z)}{X(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

$$= \frac{3+3z^{-1}+0.6z^{-1}+0.6z^{-2}}{(1+0.5z^{-1})(1-0.4z^{-1})}$$

Product value -0.2

$0.5 \times 0.4 = 0.2$

$(1+0.5z^{-1})(1-0.4z^{-1})$

$1-0.4z^{-1}+0.5z^{-1}-0.2z^{-2}$

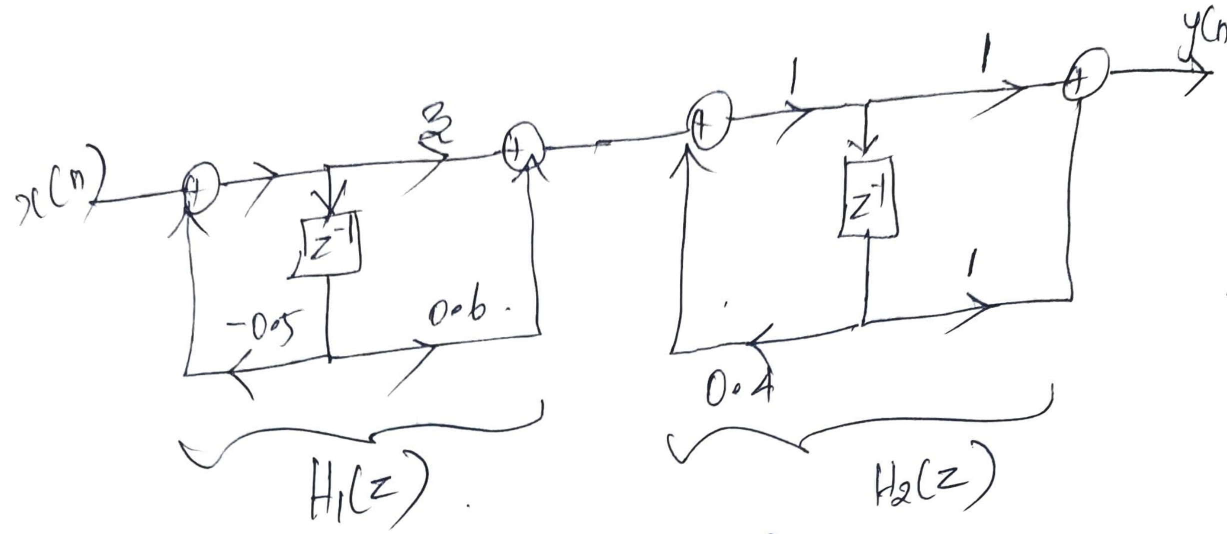
$=1+0.1z^{-1}-0.2z^{-2}$

$$H(z) \longrightarrow \underbrace{H_1(z)}_{DF\text{-}II} \cdot \underbrace{H_2(z)}_{DF\text{-}II}$$

$$= \frac{3(1+z^{-1})+0.6z^{-1}(1+z^{-1})}{(1+0.5z^{-1})\ (1-0.4z^{-1})}$$

$$= \frac{(3+0.6z^{-1})(1+z^{-1})}{(1+0.5z^{-1})(1-0.4z^{-1})}$$

$$H_1(z) = \frac{3+0.6z^{-1}}{1+0.5z^{-1}} \qquad H_2(z) = \frac{(1+z^{-1})}{(1-0.4z^{-1})}$$



Cascade Realization

Parallel form

$$H(z) = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$

→ To find parallel form we use partial fraction method.

Using long Division Method.

$$\begin{array}{r|l} & \quad -3 \\ \hline -0.2z^{-2}+0.1z^{-1}+1 & 0.6z^{-2}+3.6z^{-1}+3 \\ & +0.6z^{-2}-0.3z^{-1}-3 \\ & (-) \quad (+) \quad (+) \\ \hline & \quad 3.9z^{-1}+6 \end{array}$$

$$= -3 + \frac{3.9z^{-1}+6}{(1+0.5z^{-1})(1-0.4z^{-1})}$$

$$\frac{3.9z^{-1}+6}{(1+0.5z^{-1})(1-0.4z^{-1})}$$

Multiply z

$$= \frac{3.9+6z}{(z+0.5)(z-0.4)}$$

$$= \frac{3.9+6z}{(z+0.5)(z-0.4)} = \frac{A}{z+0.5} + \frac{B}{z-0.4}$$

$$A = \cancel{(z+0.5)} \cdot \frac{3.9+6z}{\cancel{(z+0.5)}(z-0.4)} \Bigg|_{z=-0.5}$$

$$= \frac{3.9+6(-0.5)}{-0.5-0.4}$$

$$= \frac{3.9 - 3}{-0.9} = \frac{-0.9}{0.9} = -1$$

$$\boxed{A = -1}$$

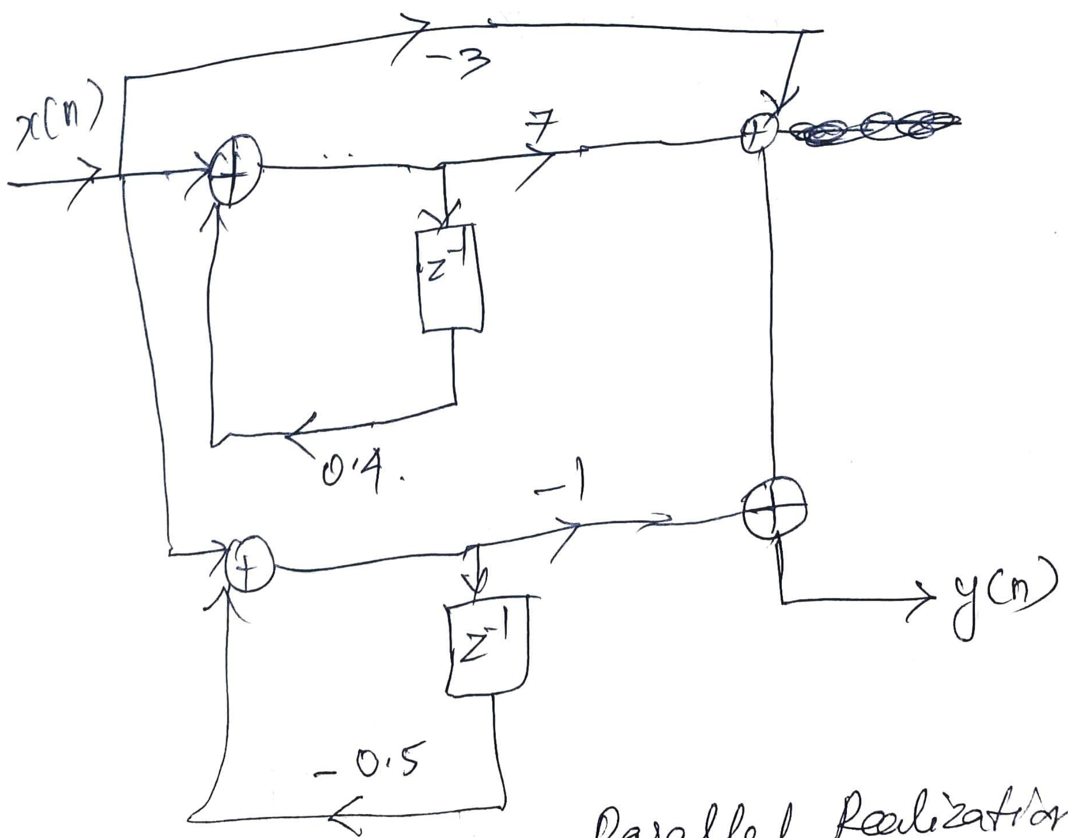
$$B = \cancel{(z - 0.4)} \cdot \frac{3.9 + 6z}{(z+0.5)\cancel{(z-0.4)}} \Bigg|_{z = +0.4}$$

$$= \frac{3.9 + 6(0.4)}{0.4 + 0.5} = \frac{3.9 + 2.4}{0.9}$$

$$= \frac{6.3}{0.9} = 7$$

$$\boxed{B = 7}$$

$$H(z) = -3 + \frac{7}{1 - 0.4z^{-1}} - \frac{1}{1 + 0.5z^{-1}}$$



Parallel Realization

(2) Cascade form.

$$y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n) + \frac{1}{3} x(n-1)$$

Soln. Taking Z-transform

$$Y(z) = \frac{3}{4} z^{-1} Y(z) - \frac{1}{8} z^{-2} Y(z) + X(z) + \frac{1}{3} z^{-1} X(z)$$

$$Y(z) - \frac{3}{4} z^{-1} Y(z) + \frac{1}{8} z^{-2} Y(z) = X(z) + \frac{1}{3} z^{-1} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z)\left[1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right] = X(z)\left[1 + \frac{1}{3} z^{-1}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

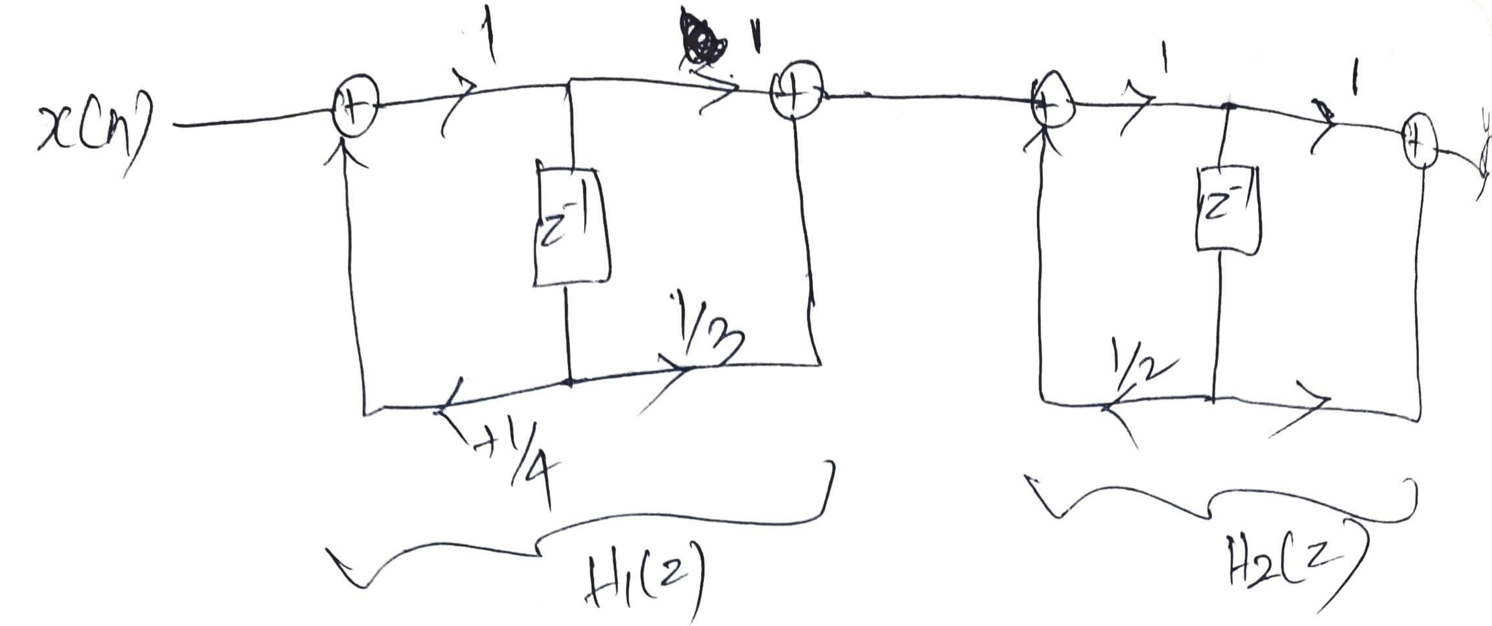
product value is $\frac{1}{8}$. $\frac{1}{4}, \frac{1}{2}$ producty this two term is $\frac{1}{8}$

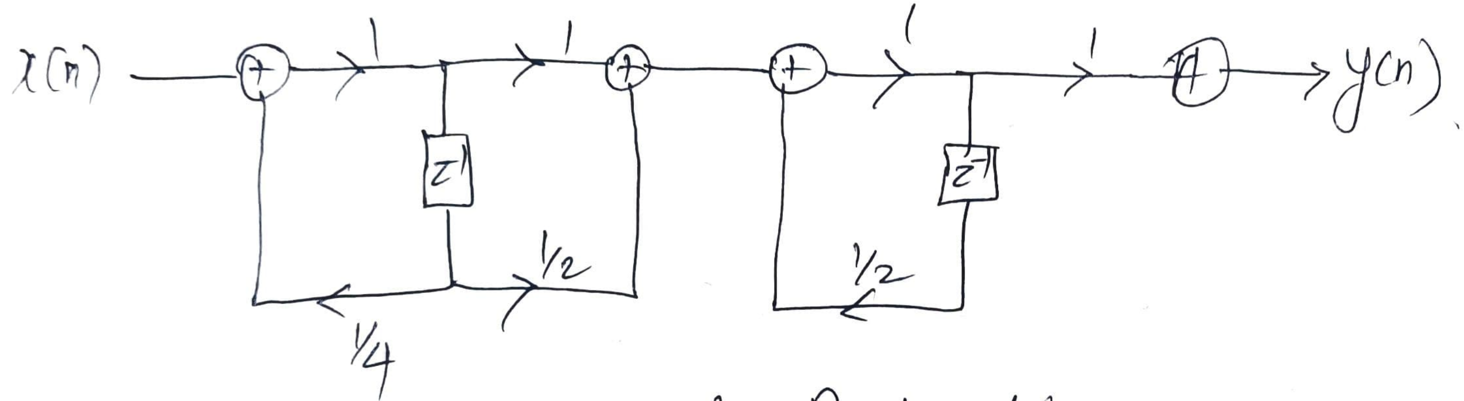
Denominator term can be split has a.

$$= \frac{1 + \frac{1}{3} z^{-1}}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{2} z^{-1}\right)}$$

$$H(z) \longrightarrow H_1(z) \cdot H_2(z)$$

$$H_1(z) = \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{1}{4} z^{-1}}, \quad H_2(z) \rightarrow \frac{1}{1 - \frac{1}{2} z^{-1}}.$$





Cascade Realization

IIR filter Design

IIR filter
- Butterworth filter
- Chebyshev filter

Design procedure for Butterworth filter

Step 1: Determine the order of the filter.

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

where, α_p - pass band attenuation, α_s - stop band attenuation } (in practical application)

Ω_p - pass band frequency, Ω_s - stop band frequency } rad/sec

If α_p and α_s are not given in decibel then take $-20\log$[magnitude value] which will give the value in terms of decibel.

If Ω_p and Ω_s are not given in radian/second

If Ω_p and Ω_s are not given

then convert the given values into radian/second with the help of impulse invariant transformation and bilinear transformation

Impulse Invariant transformation	Bilinear transformation
1. Formula for converting Hz to radian/sec.	Formula for converting Hz to radian/sec.
$\left.\begin{matrix} f_p \\ f_s \end{matrix}\right\}$ = 'Hz'	$\left.\begin{matrix} f_p \\ f_s \end{matrix}\right\}$ = 'Hz'
$\left.\begin{matrix} \omega_p = 2\pi f_p \\ \omega_s = 2\pi f_s \end{matrix}\right\}$ radian/sec (analog)	$\left.\begin{matrix} \omega_p = 2\pi f_p \\ \omega_s = 2\pi f_s \end{matrix}\right\}$ radian/sec (analog)
$\left.\begin{matrix} \Omega_p = \dfrac{\omega_p}{T} \\ \Omega_s = \dfrac{\omega_s}{T} \end{matrix}\right\}$ radian/sec (digital)	$\left.\begin{matrix} \Omega_p = \dfrac{2}{T}\tan\left(\dfrac{\omega_p}{2}\right) \\ \Omega_s = \dfrac{2}{T}\tan\left(\dfrac{\omega_s}{2}\right) \end{matrix}\right\}$ radian/sec (digital)

2. Round off to next higher integer. Step 2: Round off to next higher integer.

Step 3: Determine the analog transfer function.

$$H_a(s) = \frac{1}{N}$$

where, N = order of filter

$$s = e^{j\left(\pi/2 + \frac{(2K-1)\pi}{2N}\right)} \; ; K = 1 \text{ to } N$$

Step 4: Determine the cut-off frequency

$$\Omega_c = \frac{\Omega_p}{\left(10^{0.1\alpha_p} - 1\right)^{1/2N}}$$

Step 5: Low pass filter transfer function is given by,

$$H(s) = H_a(s) \Big|_{s \to \frac{s}{\Omega_c}}$$

High pass filter transfer function is given by,

$$H(s) = H_a(s) \Big|_{s \to \frac{\Omega_c}{s}}$$

Step 6: Convert the transfer function H(s) to digital transfer function H(z).

Impulse Invariant transformation	Bilinear Transformation
1. Apply partial fraction method for H(s) obtained in step 5. $\frac{\text{Constant}}{s+P} \Rightarrow \frac{\text{Constant}}{1 - e^{-PT} z^{-1}}$	1. Apply in step 5. $H(z) = H(s) \Big\vert_{s \to \frac{2}{T}\left(\frac{z-1}{z+1}\right)}$ where T – Sampling time period. If not given assume T = 1 sec.

1) Design a digital low pass butterworth filter for the following specification $\alpha_p = 3dB$; $\alpha_s = 15dB$; $\Omega_p = 500$ rad/sec; $\Omega_s = 1000$ rad/sec using bilinear transformation and also draw direct form-I.

Given:

$\alpha_p = 3dB$ $\qquad \Omega_p = 500$ rad/sec

$\alpha_s = 15dB$ $\qquad \Omega_s = 1000$ rad/sec

Solution:

Step 1: Determine the order of the filter

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}.$$

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\times 15} - 1}{10^{0.1\times 3} - 1.}}}{\log\left(\dfrac{1000}{500}\right)}$$

$$N \geq \frac{\log \sqrt{\dfrac{30.62}{0.99}}}{\log(2)}$$

$$N \geq \frac{0.74}{0.30}$$

$$\boxed{N \geq 2.46}$$

Step 2: Round off next highest Integer

$$\boxed{N = 3}$$

Step 3: Determine the transfer function

$$S = e^{j\left(\pi/2 + \frac{(2k-1)\pi}{2N}\right)} \quad k = 1 \text{ to } N$$

$$N = 3$$

$$S = e^{j\left(\pi/2 + \frac{(2k-1)\pi}{6}\right)} \quad k = 1 \text{ to } 3$$

$\underline{k=1}$

$$S = e^{j\left(\frac{\pi}{2} + \frac{\pi}{6}\right)}$$

$$= e^{j\left(\frac{6\pi + 2\pi}{12}\right)} = e^{j\left(\frac{8\pi}{12}\right)} = e^{j\left(\frac{2\pi}{3}\right)}$$

$$= \cos\left(\frac{2\pi}{3}\right) + j\sin\left(\frac{2\pi}{3}\right)$$

$$\boxed{S = -0.5 + j0.86}$$

$\underline{k=2}$

$$S = e^{j(\pi/2 + 3\pi/6)} = e^{j\left(\frac{6\pi + 6\pi}{12}\right)} = e^{j\left(\frac{12\pi}{12}\right)} = e^{j\pi}$$

$$= \cos\pi + j\sin\pi$$

$$\boxed{S = -1}$$

$\underline{k=3}$

$$S = e^{j(\pi/2 + 5\pi/6)} = e^{j\left(\frac{6\pi + 10\pi}{12}\right)} = e^{j\left(\frac{16\pi}{12}\right)}$$

$$\Rightarrow e^{j(4\pi/3)}$$

$$\Rightarrow \cos(4\pi/3) + j\sin(4\pi/3)$$

$$\boxed{S = -0.5 - j0.86}$$

$$H_a(S) = \frac{1}{N}$$

$$= \frac{1}{(S+0.5-j0.86)(S+1)(S+0.5+j0.86)}$$

$$= \frac{1}{\left[(S+0.5)^2 - (j0.86)^2\right] S+1}$$

$$= \frac{1}{\left[(S^2+0.25+S) - j^2 0.739\right] S+1}$$

$$= \frac{1}{(S^2+0.25+S+0.739)(S+1)}$$

$$\boxed{H_a(S) = \frac{1}{(S^2+S+1)(S+1)}}$$

Step A: Determine the cut-off frequency.

$$\Omega_c = \frac{\Omega_p}{\left(10^{0.1\alpha_p} - 1\right)^{1/2N}}$$

$$= \frac{500}{\left(10^{0.1 \times 3} - 1\right)^{\frac{1}{6}}} \Rightarrow \frac{500}{0.99}$$

$$\boxed{\Omega_c = 505.05}$$

Step 5: Low pass filter transfer function is given by

$$H(s) = H_a(s) \Big|_{s \to \frac{s}{\Omega_c}}$$

$$= \frac{1}{(s^2+s+1)(s+1)} \Bigg|_{s \to \frac{s}{505.05}}$$

$$= \frac{1}{\left(\left(\frac{s}{505.05}\right)^2 + \left(\frac{s}{505.05}\right) + 1\right)\left(\frac{s}{505.05} + 1\right)}$$

$$= \frac{1}{\left(\frac{s^2 + s(505.05) + (505.05)^2}{(505.05)^2}\right)\left(\frac{s+505.05}{505.05}\right)}$$

$$H(s) = \frac{(505.05)^3}{\left[s^2 + 505.05s + (505.05)^2\right]\left[s+505.05\right]}$$

Step 6: Convert the analog transfer function to digital transfer function.

By Bilinear transformation.

$$H(z) = H(s) \Big|_{s \to \frac{2}{T}\left(\frac{z-1}{z+1}\right)} \qquad T = 1 \text{ sec}.$$

$$H(z) = \frac{(505.05)^3}{\left[s^2 + 505.05s + (505.05)^2\right]\left[s+505.05\right]} \Bigg|_{s \to 2\left(\frac{z-1}{z+1}\right)}$$

$$= \frac{(505.05)^3}{\left[\frac{4(z-1)^2}{(z+1)^2} + 505.05\left(\frac{2(z-1)}{(z+1)}\right) + (505.05)^2\right]\left[2\left(\frac{z-1}{z+1}\right) + 505.05\right]}$$

$$= \frac{(505.05)^3}{\left(\frac{4(z-1)^2 + 1010.1(z-1)(z+1) + (505.05)^2(z+1)^2}{(z+1)^2}\right)\left(\frac{2(z-1)+505.05(z+1)}{(z+1)}\right)}$$

$$= \frac{(505.05)^3 (z+1)^3}{\{4(z^2+1-2z) + 1010.1(z^2-1) + (505.05)^2(z^2+1+2z)\}\{(2z-2)+505.05z+505.05\}}$$

$$= \frac{128825882.5\,(z^3+1+3z^2+3z)}{(4z^2+4-8z+1010.1z^2-1010.1+255075.5z^2+255075.5+510151z)(507.05z+503.05)}$$

$$= \frac{128825882.5z^3 + 128825882.5 + 386477647.5z^2 + 386477647.5z}{(256089.6z^2+510143z+254069.4)(507.05z+503.05)}$$

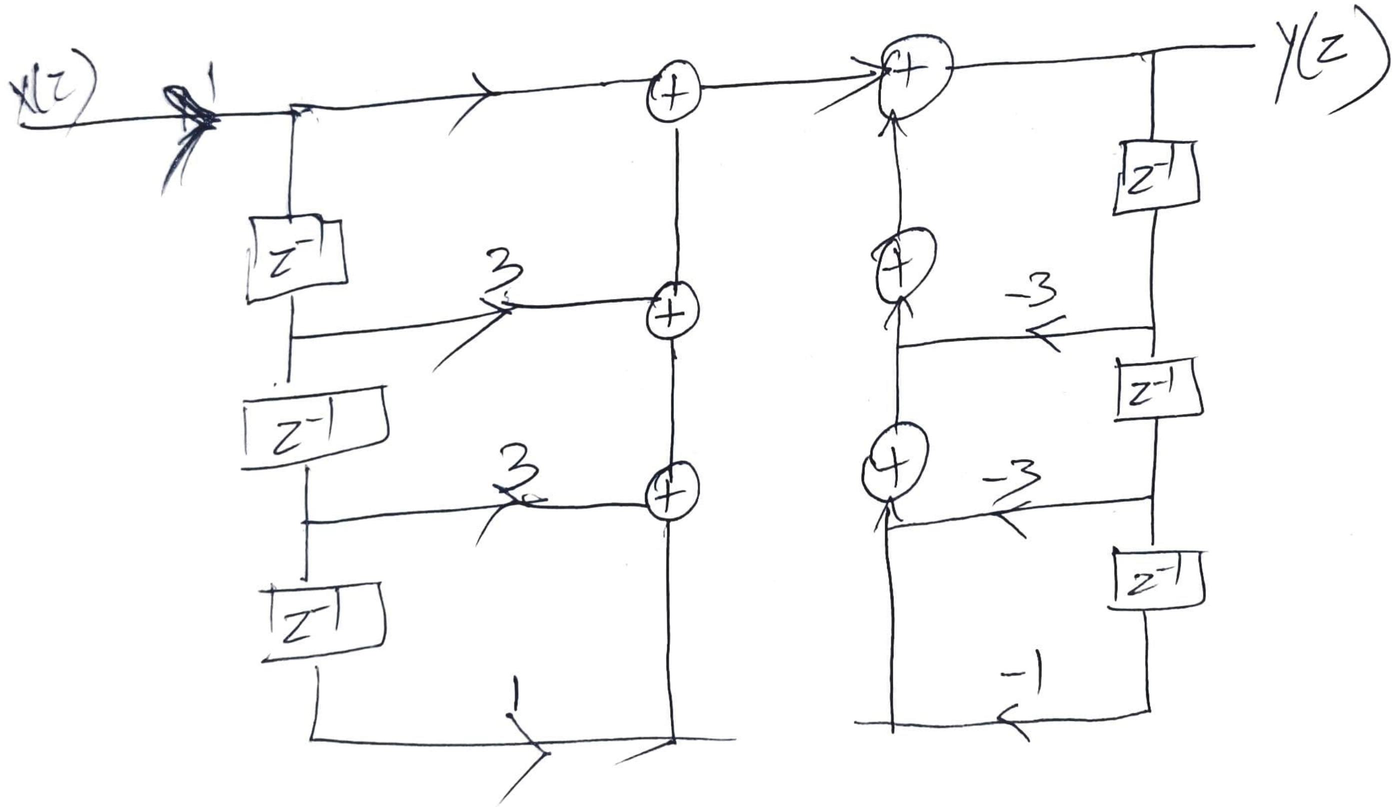
$$= \frac{128825882.5z^3 + 128825882.5 + 386477647.5z^2 + 386477647.5z}{129850231.7z^3 + 128825873.z^2 + 258668008.2z^2 + 256627478.22 + 128825889.3z + 127809611.7}$$

$$= \frac{128825882.5\cancel{z^3}(1+z^{-3}+3z^{-1}+3z^{-2})}{129850231.7\cancel{z^3}(1+z^{-1}+2z^{-1}+2z^{-2}+z^{-2}+z^{-3})}$$

$$H(z) = \frac{1+3z^{-1}+3z^{-2}+z^{-3}}{1+3z^{-1}+3z^{-2}+z^{-3}}$$

$$\frac{Y(z)}{X(z)} = \frac{1+z^{-3}+3z^{-2}+3z^{-1}}{1+z^{-3}+3z^{-2}+3z^{-1}}$$

$$Y(z) = X(z) + X(z)z^{-3} + 3z^{-2}X(z) + 3z^{-1}X(z) - z^{-3}Y(z) - 3z^{-2}Y(z) - 3z^{-1}Y(z)$$



(2) Design a digital low pass butterworth filter for the following specification

$$0.8 \le |H(e^{j\omega})| \le 1 \; ; \; 0 \le \omega \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0.2 \; ; \; 0.6\pi \le \omega \le \pi.$$ using Impulse Invariant transformation.

<u>Given:-</u>

$$0.8 \le |H(e^{j\omega})| \le 1 \; ; \; 0 \le \omega \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0.2 \; ; \; 0.6\pi \le \omega \le \pi$$

Solution:

$$\alpha_p = -20 \log 0.8 = 1.93 \text{ dB}$$

$$\alpha_s = -20 \log 0.2 = 13.97 \text{ dB.}$$

$$\omega_p = 0.2\pi$$

$$\Omega_p = \frac{\omega_p}{T} = \frac{0.2\pi}{T} = 0.62 \text{ rad/sec}$$

$$\omega_s = 0.6\pi$$

$$\Omega_s = \frac{\omega_s}{T} = \frac{0.6\pi}{T} = 1.88 \text{ rad/sec.}$$

Step 1: Determine the order of filter.

$$N \ge \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

$$N \geq \frac{\log\sqrt{\dfrac{10^{0.1\times13.97}-1}{10^{0.1\times1.93}-1}}}{\log\left(\dfrac{1.88}{0.62}\right)}$$

$$N \geq \frac{\log\sqrt{\dfrac{24.97-1}{1.55-1}}}{0.48}$$

$$N \geq \log\left(\frac{6.59}{0.48}\right)$$

$$N \geq 1.70$$

Step2: Round off to next higher integer

$$N=2.$$

Step3: Determine the transfer function

$$H_a(s) = \frac{1}{N}$$

$$S = e^{j\left(\frac{\pi}{2} + \frac{(2K-1)\pi}{2N}\right)} \qquad K = 1 \text{ to } 2$$

K=1

$$S = e^{j\left(\frac{\pi}{2} + \frac{\pi}{4}\right)} \Rightarrow e^{j\left(\frac{4\pi+2\pi}{8}\right)} \Rightarrow e^{j\left(\frac{6\pi}{8}\right)}$$

$$S = e^{j(3\pi/4)}$$

$$S = \cos\left(\frac{3\pi}{4}\right) + j\sin\left(\frac{3\pi}{4}\right)$$

$$\boxed{S = -0.70 + j0.70}$$

$\underline{\underline{K=2}}$

$$S = e^{j(\pi/2 + \frac{3\pi}{4})} \Rightarrow e^{j\left(\frac{4\pi + 6\pi}{8}\right)} \Rightarrow e^{j\left(\frac{10\pi}{8}\right)} = e^{j\left(\frac{5\pi}{4}\right)}$$

$$S = \cos\left(\frac{5\pi}{4}\right) + j\sin\left(\frac{5\pi}{4}\right)$$

$$\boxed{S = -0.70 - j0.70}$$

$$H_a(S) = \frac{1}{(S + 0.70 - j0.70)(S + 0.70 + j0.70)}$$

$$\boxed{H_a(S) = \frac{1}{(S + 0.70)^2 - (j0.70)^2}}$$

step 4: Determine the cut-off frequency

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}}$$

$$= \frac{\Omega_p}{(10^{0.1 \times 1.93} - 1)^{1/4}} \Rightarrow \frac{0.62}{(0.55)^{1/4}} \Rightarrow \frac{0.62}{0.86}$$

$$\boxed{\Omega_c = 0.72}$$

Step 5: Low pass filter transfer function.

$$H(S) = H_a(S) \Big|_{S \to \frac{S}{\Omega_c}} = \frac{S}{0.72}$$

$$H(S) = \frac{1}{(S + 0.70)^2 + 0.49} \Bigg|_{S \to \frac{S}{0.72}}$$

$$= \frac{1}{\left(\frac{s}{0.72} + 0.70\right)^2 + 0.49}$$

$$= \frac{1}{\frac{(s + 0.70 \times 0.72)^2 + 0.49 \times 0.72}{(0.72)^2}}$$

$$= \frac{0.51}{(s + 0.504)^2 + 0.3528} \Rightarrow \frac{0.51}{s^2 + 1.008 s + 0.25 + 0.35}$$

$$\boxed{H(s) = \frac{0.51}{s^2 + s + 0.6}} \Rightarrow \begin{matrix} s = -0.5 + 0.5j \\ s = -0.5 - 0.5j \end{matrix}$$

Step 6: $H(s) = H(z)$

Take Partial fraction.

$$H(s) = \frac{0.51}{s^2 + s + 0.6} = \frac{0.51}{(s + 0.5 - 0.5j)(s + 0.5 + 0.5j)}$$

$$\frac{0.51}{(s + 0.5 - 0.5j)(s + 0.5 + 0.5j)} = \frac{A}{(s + 0.5 - 0.5j)} + \frac{B}{s + 0.5 + 0.5j}$$

$$= \frac{A(s + 0.5 + 0.5j) + B(s + 0.5 - 0.5j)}{(s + 0.5 - 0.5j)(s + 0.5 + 0.5j)}$$

$$0.51 = A(s + 0.5 + 0.5j) + B(s + 0.5 - 0.5j)$$

put $s = -0.5 - 0.5j)$

$$0.51 = B(-\cancel{0.5} - 0.5j + \cancel{0.5} - 0.5j)$$

$$0.51 = B(-j)$$

$$\boxed{B = 0.51j}$$

put $s = -0.5 + 0.5j$

$$0.51 = A(-\cancel{0.5} + 0.5j + \cancel{0.5} + 0.5j)$$

$$0.51 = A(1j)$$

$$\boxed{A = -0.51j}$$

$$H(s) = \frac{-0.51j}{s + \underbrace{(0.5 - 0.5j)}_{P}} + \frac{0.51j}{s + \underbrace{0.5 + 0.5j}_{P}}$$

Step 6: $H(z) = \frac{\text{Constant}}{s+P} = \frac{\text{Constant}}{1 - e^{-PT} z^{-1}}$

$$\boxed{H(z) = \frac{-0.51j}{1 - e^{-(0.5-0.5j)} z^{-1}} + \frac{0.51j}{1 - e^{-(0.5+0.5j)} z^{-1}}}$$

$$= \frac{-0.51j}{1 - 0.6 \cdot e^{0.5j} z^{-1}} + \frac{0.51j}{1 - 0.6 e^{-0.5j} z^{-1}}$$

$$= \frac{-0.51j(1 - 0.6e^{-0.5j} z^{-1}) + 0.51j(1 - 0.6e^{0.5j} z^{-1})}{(1 - 0.6e^{0.5j} z^{-1})(1 - 0.6e^{-0.5j} z^{-1})}$$

$$= \frac{-\cancel{0.5 j} + 0.8 j e^{-0.5 j} z^{-1} + \cancel{0.5 j} - 0.3 j e^{0.5 j} z^{-1}}{1 - 0.6 e^{-0.5 j} z^{-1} - 0.6 e^{0.5 j} z^{-1} + 0.36 z^{-2}}$$

$$= \frac{-0.3 j z^{-1} \left(e^{0.5 j} - e^{-0.5 j}\right)}{1 - 0.6 z^{-1} \left(e^{0.5 j} + e^{-0.5 j}\right) + 0.36 z^{-2}}$$

$$= \frac{-0.3 j z^{-1} (2 j \sin 0.5)}{1 - 0.6 z^{-1} \cdot 2 \cos 0.5 + 0.36 z^{-2}}$$

$e^{j\theta} + e^{-j\theta} = 2\cos\theta$

$$\boxed{H(z) = \frac{0.28 z^{-1}}{1 - 1.044 z^{-1} + 0.36 z^{-2}}}$$

Design of Chebyshev filter

Design Procedure

Step 1: Determine the order of the filter

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

Step 2: Round off to the next higher integer.

Step 3: Determine the minor axis (a) and major axis (b) of the ellipse

$$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right]$$

$$b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

$$\mu = \varepsilon^{-1} + \sqrt{\varepsilon^{-2}+1}$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p}-1}$$

Step 4: Determine the denominator polynomial of the transfer function.

$$S = a\cos\left(\frac{\pi}{2} + \frac{(2k-1)\pi}{2N}\right) + jb\sin\left(\frac{\pi}{2} + \frac{(2k-1)\pi}{2N}\right) \quad ; k = 1 \text{ to } N$$

Step 5: Determine the numerator value of the transfer function.

If N is odd, Substitute S=0 in step 4

If N is even, Substitute S=0 in step 4 and divide by $\sqrt{1+\varepsilon^2}$

$$\frac{\text{Sub } S=0 \text{ in Step 4}}{\sqrt{1+\varepsilon^2}}$$

Step 6: Determine the transfer function

$$H(s) = \frac{\text{Step 5}}{\text{Step 4}}$$

Step 7: Convert the analog transfer function into digital filter

$H(s) \rightarrow H(z)$ Similar as butterworth

Chebyshev filter

Design Procedure

~~step 1~~

① Design the digital low pass Chebyshev filter for the following specification:

$\alpha_p = 3dB \quad \Omega_p = 500$ rad/sec

$\alpha_s = 15dB \quad \Omega_s = 1000$ rad /sec using bilinear transformation.

Given:

$\alpha_p = 3dB \quad \Omega_p = 500$ rad/sec

$\alpha_s = 15dB \quad \Omega_s = 1000$ rad/sec.

Solution:

Step 1: Determine the order of the filter.

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\left(\dfrac{\Omega_s}{\Omega_p}\right)} = \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\times 15}-1}{10^{0.1\times 3}-1}}}{\cosh^{-1}\left(\dfrac{1000}{500}\right)}$$

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{30.62}{0.99}}}{1.31} \Rightarrow N \geq \frac{1.50}{1.31}$$

$$N \geq 1.14$$

Step 2: Round off to Next higher integer

$$N = 2$$

Step 3: Determine minor axis (a) and major axis (b).

$$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$$

$$\varepsilon = \sqrt{10^{0.1 \times 3} - 1}$$

$$\boxed{\varepsilon = 1}$$

$$\mu = \varepsilon^{-1} + \sqrt{\varepsilon^{-2} + 1}$$

$$= 1^{-1} + \sqrt{1^{-2} + 1}$$

$$= 1 + \sqrt{1+1}$$

$$= 1 + \sqrt{2} \Rightarrow 1 + 1.41$$

$$\boxed{\mu = 2.41}$$

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2} \right]$$

$$= 500 \left[\frac{(2.41)^{1/2} - (2.41)^{-1/2}}{2} \right]$$

$$= \frac{500}{2} (1.55 - 0.64) \Rightarrow 250 (0.91)$$

$$\boxed{a = 227.5}$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2} \right].$$

$$\Rightarrow 500 \left[\frac{(2.41)^{1/2} + (2.41)^{-1/2}}{2} \right] \Rightarrow \frac{500}{2} (1.55 + 0.64)$$

$$\boxed{b = 547.5}$$

Step 4: Determine the denominator polynomial of the transfer function

$$S = a\cos\left(\frac{\pi}{2} + \frac{(2K-1)\pi}{2N}\right) + jb\sin\left(\frac{\pi}{2} + \frac{(2K-1)\pi}{2N}\right) \quad K = 1 \text{ to } N$$

$K=1$

$$S = 227.5\cos\left(\frac{\pi}{2} + \frac{\pi}{4}\right) + j547.5\sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$$

$$= 227.5\cos\left(\frac{3\pi}{4}\right) + j547.5\sin\left(\frac{3\pi}{4}\right)$$

$$\boxed{S = -160.36 + j387.14}$$

$K=2$

$$S = 227.5\cos\left(\frac{\pi}{2} + \frac{3\pi}{4}\right) + j547.5\sin\left(\frac{\pi}{2} + \frac{3\pi}{4}\right)$$

$$= 227.5\cos\left(\frac{2\pi + 3\pi}{4}\right) + j547.5\sin\left(\frac{2\pi+3\pi}{4}\right)$$

$$= 227.5\cos\left(\frac{5\pi}{4}\right) + j547.5\sin\left(\frac{5\pi}{4}\right)$$

$$\boxed{S = -160.86 - j387.14}$$

$$(S + 160.86 - j387.14)(S + 160.86 + j387.14)$$

Step 5: Determine the numerator value of the transfer function:

If N is even

$$\frac{\text{sub } S = 0 \text{ in Step 4}}{\sqrt{1+\varepsilon^2}}$$

$$= \frac{(160.86 - j387.14)(160.86 + j387.14)}{\sqrt{2}}$$

$$= \frac{175753.31}{\sqrt{2}}$$

$$\Rightarrow 124276.36$$

Step 6: Determine the transfer function

$$H(s) = \frac{124276.36}{(s+160.86-j387.14)(s+160.86+j387.14)}$$

$$= \frac{124276.36}{(s+160.86)^2 + (387.14)^2}$$

$$\boxed{H(s) = \frac{124276.36}{s^2+321.72s+175753.31}}$$

Step 7: Determine the digital transfer fn.

$$H(z) = H(s)\Big|_{s \to \frac{2}{T}\left(\frac{z-1}{z+1}\right)}$$

$$H(z) = \frac{124276.36}{s^2+321.72s+175753.31}\Bigg|_{s \to 2\left(\frac{z-1}{z+1}\right)}$$

$$H(z) = \frac{124276.36}{\left[\frac{2(z-1)}{z+1}\right]^2 + 321.72\left(\frac{2(z-1)}{z+1}\right) + 175753.31}$$

$$H(z) = \frac{124276.36}{\left[\frac{2(z-1)}{z+1}\right]^2 + 321.72\left(\frac{2(z-1)}{z+1}\right) + 175753.31}$$

$$\boxed{H(z) = \frac{124276.36}{4\left(\frac{z-1}{z+1}\right) + 643.44\left(\frac{z-1}{z+1}\right) + 175753.31}}$$

(2) Design a digital low pass chebyshev filter for the following specification using Impulse Invariant transformation

$\alpha_p = 0.5 dB$ $\quad f_p = 0.10 Hz$

$\alpha_s = 15 dB$ $\quad f_s = 0.15 Hz$

Soln :-

$\omega_p = 2\pi f_p = 2\pi \times 0.10 = 0.2\pi$

$\omega_s = 2\pi f_s = 2\pi \times 0.15 = 0.3\pi$

$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = 0.64$ rad/sec

$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = 1.01$ rad/sec .

17/8/17

Unit - II

FIR Filter design.

Windowing Technique:

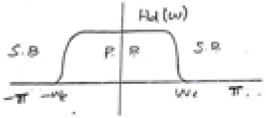
Design Procedure:

Step 1: Write down the type of filter, type of window, order of the filter and cut off frequency from the given problem.

Type of Filter:

(i) Low pass filter: The filter which allows below the cut off frequency and suppress the above is called as low pass filter.

The general frequency response of low pass filter is given by,



Mathematically the frequency response is,

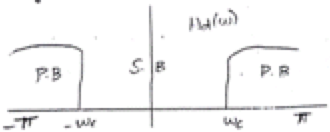
$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} ; & -\omega_c \le \omega \le \omega_c \\ 0 ; & \text{elsewhere} \end{cases}$$

(or)

$$H_d(\omega) = e^{-j\omega\alpha} ; \ |\omega| \le \omega_c$$

(ii) High Pass filter: The filter which allows above the cut off frequency and suppress the below is called as high pass filter.

The general frequency response of high pass filter is given by,



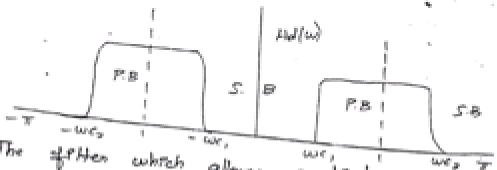
$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} ; & -\pi \le \omega \le -\omega_c \\ e^{-j\omega\alpha} ; & \omega_c \le \omega \le \pi \end{cases} \quad (\text{or}) \quad = e^{-j\omega\alpha} ; \ \omega_c \le |\omega| \le \pi$$

(or)

$$H_d(\omega) = e^{-j\omega\alpha} ; \ |\omega| \ge \omega_c$$

(iii) Band pass filter: It is a combination of high pass with low pass filter.

The general frequency response of band pass filter is given by,

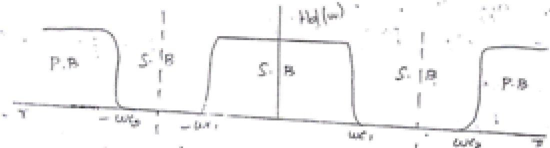


The filter which allows particular range of frequency and suppress all other frequencies is called as band pass filter.

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} ; & -\omega_{c2} \le \omega \le -\omega_{c1} \\ e^{-j\omega\alpha} ; & \omega_{c1} \le |\omega| \le \omega_{c2} \end{cases} \quad (\text{or}) \quad e^{-j\omega\alpha} ; \ \omega_{c1} \le |\omega| \le \omega_{c2}$$

(iv) Band Stop filter: It is a combination of low pass with high pass filter.

The general frequency response of band stop filter is given by,



The filter which suppress particular range of frequency and allow all other frequencies is called band stop filter.

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha}; & -\pi \le \omega \le -\omega c_2 \\ e^{-j\omega\alpha}; & -\omega c_1 \le \omega \le \omega c_1 \\ e^{-j\omega\alpha}; & \omega c_2 \le \omega \le \pi \end{cases} \quad (\text{or}) = \begin{cases} e^{-j\omega\alpha}; & \omega c_2 \le |\omega| \le \pi \\ e^{-j\omega\alpha}; & |\omega| \le \omega c_1 \end{cases}$$

(or)

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha}; & |\omega| \ge \omega c_2 \\ e^{-j\omega\alpha}; & |\omega| \le \omega c_1 \end{cases}$$

If the cutoff frequencies are given in terms of hertz then, convert it into radian/second by using the formula, $\omega = 2\pi f T$.

where T – Sampling time period

$$T = \frac{1}{\text{Sampling frequency}} = \frac{1}{F}$$

where, $\alpha = \frac{N-1}{2}$

Types of Window:

(i) Rectangular window:

$$w(n) = 1 \; ; \; 0 \le n \le N-1$$

(ii) Hamming window:

$$w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) \; ; \; 0 \le n \le N-1$$

(iii) Hanning window:

$$w(n) = 0.5 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right) \; ; \; 0 \le n \le N-1$$

(iv) Blackman Window:

$$w(n) = 0.42 - 0.5 \cos\left(\frac{2\pi n}{N-1}\right) + 0.08 \cos\left(\frac{4\pi n}{N-1}\right) \; ; \; 0 \le n \le N-1$$

Step 2: Determine the inverse fourier transform.

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega .$$

(i) Low pass filter:

The inverse fourier transform for low pass filter is given by,

$$h_d(n) = \frac{\sin \omega_c (n-\alpha)}{\pi (n-\alpha)} \; ; \; n \ne \alpha$$

If $n = \alpha$;

$$hd(n) = \frac{0}{0} \text{ Indeterminant form} \qquad \frac{\sin A\theta}{\theta} = A$$

$$hd(n) = \frac{\omega c}{\pi} \; ; \; n = \alpha$$

(ii) High pass filter:

The inverse fourier transform of high pass filter is given by,

$$\frac{\sin A\theta}{\theta} = A$$

$$hd(n) = \left[\frac{\sin \pi(n-\alpha) - \sin \omega_c (n-\alpha)}{\pi (n-\alpha)}\right] ; \; n \ne \alpha$$

If $n = \alpha$;

$$hd(n) = \frac{0}{0} \text{ Indeterminant form.}$$

$$hd(n) = \frac{\pi - \omega_c}{\pi} \; ; \; n = \alpha$$

(iii) Band Pass filter:

The inverse fourier transform of band pass filter is given by,

$$hd(n) = \left[\frac{\sin \omega c_2 (n-\alpha) - \sin \omega c_1 (n-\alpha)}{\pi (n-\alpha)}\right] ; \; n \ne \alpha$$

If $n = \alpha$;

$$hd(n) = \frac{0}{0} \text{ Indeterminant form}$$

$$hd(n) = \frac{\omega c_2 - \omega c_1}{\pi} \; ; \; n = \alpha$$

(iv) Band stop filter:

The inverse fourier transform of band stop filter is given by,

$$h_d(n) = \left[\frac{\sin \pi(n-\alpha) + \sin \omega_{c_1}(n-\alpha) - \sin \omega_{c_2}(n-\alpha)}{\pi(n-\alpha)}\right] ; n \neq \alpha$$

If $n = \alpha$;

$h_d(n) = \frac{0}{0}$ Indeterminant form

$$h_d(n) = \frac{\pi + \omega_{c_1} - \omega_{c_2}}{\pi} ; n = \alpha$$

Step 3: Determine the impulse response of the system.

$$h(n) = h_d(n)\, \omega(n)$$

Step 4: Determine the transfer function

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

Step 5: Realize the transfer function using FIR structu

1. Design a low pass filter with cutoff frequency 1.2 rad/sec using hamming window for N=7.

Given:

Low pass filter

Hamming window

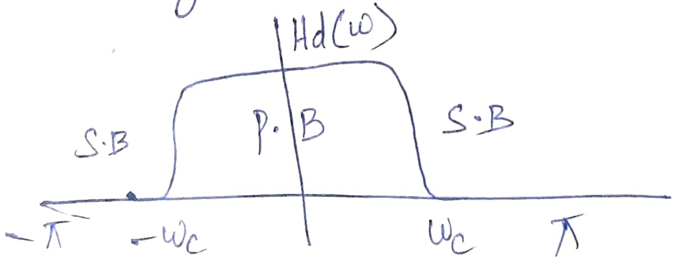
$\omega_c = 1.2$ rad/sec

$N = 7$

Solution:

Step 1: The filter which allows below the cutoff frequency and suppress the above is called as low pass filter.

The general frequency response of low pass filter is given by.



Mathematically,

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & ; -\omega_c \le \omega \le \omega_c \\ 0 & ; \text{elsewhere.} \end{cases}$$

(or)

$$H_d(\omega) = e^{-j\omega\alpha} \quad ; |\omega| \le \omega_c$$

step2: Hamming Window

$$w(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right) ; \quad 0 \le n \le N-1$$

$$\alpha = \frac{N-1}{2}$$

$$N = 7$$

$$\alpha = \frac{7-1}{2} = 6/2$$

$$\boxed{\alpha = 3}$$

Inverse fourier transform of low pass filter is given by,

$$h_d(n) = \frac{\sin w_c (n-\alpha)}{\pi(n-\alpha)} ; \quad n \neq \alpha$$

$$h_d(n) = \frac{w_c}{\pi} ; \quad n = \alpha .$$

Step 3: Determine the impulse response of the system.

$$h(n) = h_d(n)\, w(n)$$

$$h(n) = \left[\frac{\sin w_c (n-\alpha)}{\pi(n-\alpha)}\right]\left[0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right)\right]$$

$$0 \le n \le 6$$

$n \neq \alpha$

$n \neq 3$ (This condition does not exist)

$$\left.\begin{matrix} 0 & 1 & 2 & \cancel{3} & 4 & 5 & 6 \end{matrix}\right\}$$ They are symmetry

$n = 0$

$$h(0) = \left[\frac{\sin 1.2(0-3)}{\pi(0-3)}\right](0.54 - 0.46)$$

$$\boxed{h(0) = -3.75 \times 10^{-3} = h(6)}$$

$n = 1$

$$h(1) = \left[\frac{\sin 1.2(1-3)}{\pi(1-3)}\right]\left[0.54 - 0.46 \cos\left(\frac{2\pi}{6}\right)\right]$$

$$\boxed{h(1) = 0.033 = h(5)}$$

$n = 2$

$$h(2) = \left[\frac{\sin 1.2(2-3)}{\pi(2-3)}\right]\left[0.54 - 0.46 \cos\left(\frac{4\pi}{6}\right)\right]$$

$$h(2) = 0.228 = h(4)$$

$n = \alpha$

$n = 3$

$$h(n) = \left[\left(\frac{w_c}{\pi}\right)\left(0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)\right)\right]$$

$$h(3) = \frac{1.2}{\pi}\left[0.54 - 0.46\cos\left(\frac{6\pi}{6}\right)\right]$$

$$\boxed{h(3) = 0.38}$$

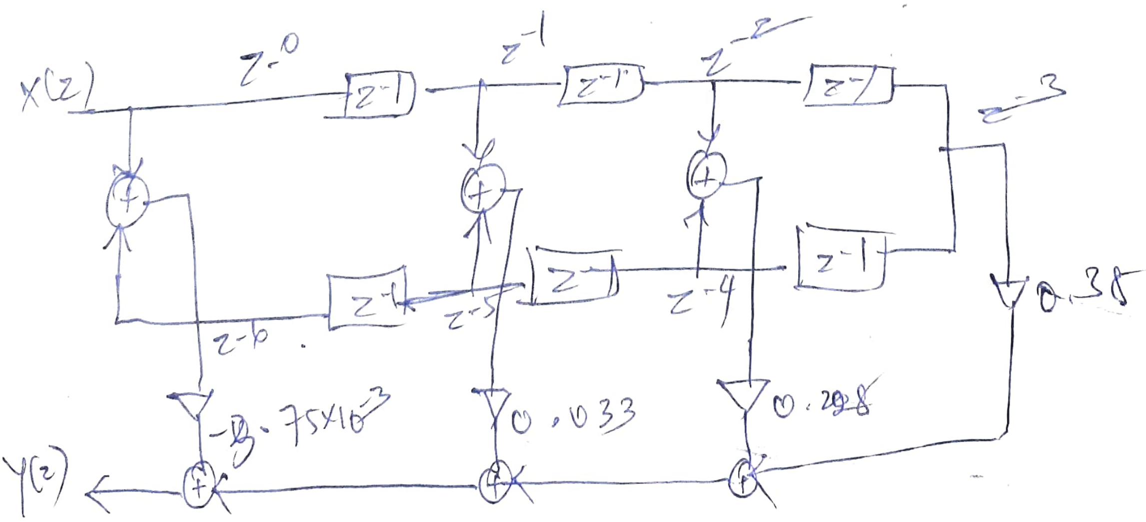
step 4: $H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$

$$H(z) = \sum_{n=0}^{6} h(n) z^{-n}$$

$$= h(0) z^{-0} + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4} + h(5) z^{-5} + h(6) z^{-6}.$$

$$H(z) = \frac{Y(z)}{X(z)} = -3.75 \times 10^{-3} (z^{-0} + z^{-6}) + 0.033(z^{-1} + z^{-5}) + 0.228 (z^{-2} + z^{-4}) + 0.38 z^{-3}$$

steps: Realization using FIR structure



2. Design a band pass filter with cutoff frequency 1.2 rad/sec to 1.7 rad/sec using Rectangular window for $N=5$.

Given:-

$\omega_{c_1} = 1.2$ rad/sec

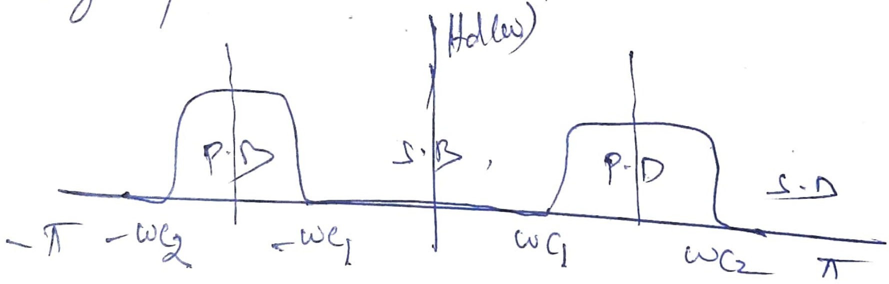
$\omega_{c_2} = 1.7$ rad/sec

Solution:-

step 1:- $\omega_{c_1} = 1.2$ rad/sec

$\omega_{c_2} = 1.7$ rad/sec

Band pass filter - It is combination of high pass with Low pass filter.



$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & ; -\omega_{c_2} \le \omega \le -\omega_{c_1} \\ e^{-j\omega\alpha} & ; \omega_{c_1} \le \omega \le \omega_{c_2} \end{cases}$$

Rectangular window:

$$w(n) = 1 \quad ; \quad 0 \le n \le N-1$$

Step 2 : Determine the inverse fourier transform

$$\alpha = \frac{N-1}{2} = \frac{5-1}{2} = \frac{4}{2} = 2$$

$$\boxed{\alpha = 2}$$

Step 3 : Determine the impulse response of the system

$$h(n) = h_d(n)\,\omega(n)$$

$$h(n) = \left[\frac{\sin \omega_{c2}(n-\alpha) - \sin \omega_{c1}(n-\alpha)}{\pi(n-\alpha)}\right][1] \; ; n \neq \alpha$$

$n \neq 2$ The condition.

$$0 \quad 1 \quad \cancel{2} \quad 3 \quad 4$$

$\underline{n=0}$

$$h(0) = \frac{\sin 1.7(-2) - \sin 1.2(-2)}{\pi(-2)}$$

$$\boxed{h(0) = -0.14 = h(4)}$$

$\underline{n=1}$

$$h(1) = \frac{\sin 1.7(-1) - \sin(1.7)(-1)}{\pi(-1)}$$

$$\boxed{h(1) = 0.018 = h(3)}$$

$$h_d(n) = \frac{\omega_{c2} - \omega_{c1}}{\pi} \quad n = \alpha \Rightarrow n = 2$$

$$h_d(n) = \left(\frac{1.7 - 1.2}{\pi}\right)$$

$$\boxed{h(2) = 0.159}$$

Step 4: Determine the transfer function.

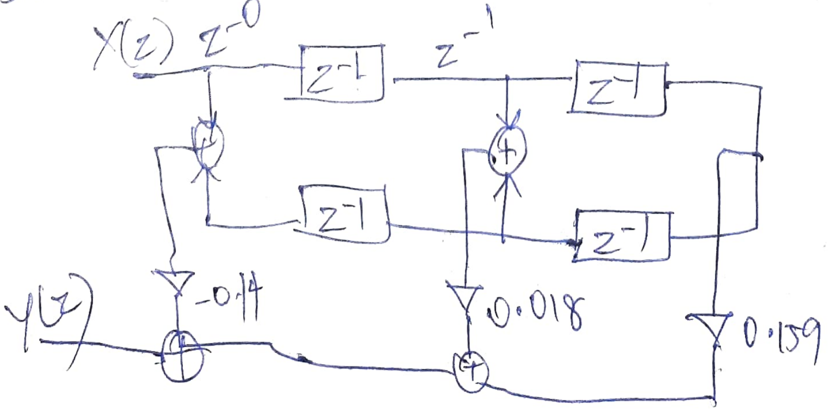
$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= \sum_{n=0}^{4} h(n) z^{-n}$$

$$= h(0)z^{-0} + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4}$$

$$H(z) = -0.14(z^{-0} + z^{-4}) + 0.018(z^{-1} + z^{-3}) + 0.159z^{-2}$$

Step 5: Realization.



(B) Design a high pass filter with cut off frequency 3000 Hz and sampling freq 8000 Hz using blackman window for N = 8

Given: High Pass filter

$f_C = 3000$ Hz

$f_s = 8000$ Hz

$N = 8$

Blackman window

Soln:- $\omega = 2\pi f_c T \Rightarrow T = \frac{1}{f_s} = \frac{1}{8000}$

$= 2\pi \times 3000 \times \frac{1}{8000}$

$\alpha = \frac{N-1}{2}$

(A) Design a band stop filter with cut-off frequency 1000 Hz and 5000 Hz and the sampling frequency is 9000 Hz using Rectangular window for N = 6.

Given $f_{c_1} = 1000$ Hz

$f_{c_2} = 5000$

$f_s = 9000$ Hz

Soln: $\omega c_1 = 2\pi f_{c_1} T$ $\quad T = \frac{1}{f_s}$

$= 2\pi \times 1000 \times \frac{1}{9000}$.

4) Design a filter whose frequency response is given by $H_d(\omega) = e^{-j\omega 3}$; $|\omega| \le \pi/4$ using hamming window.

Given:

$$H_d(\omega) = e^{-j3\omega} \; ; |\omega| \le \pi/4$$

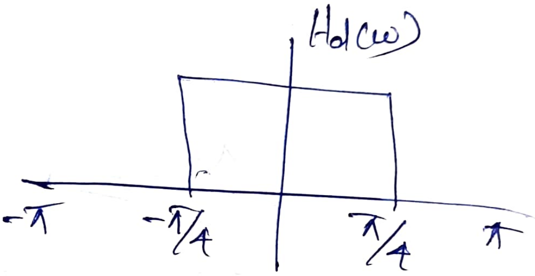
Solution:

$\alpha = 3 \qquad \alpha = \frac{N-1}{2}$

$\frac{N-1}{3} = 3$

$N-1 = 6$

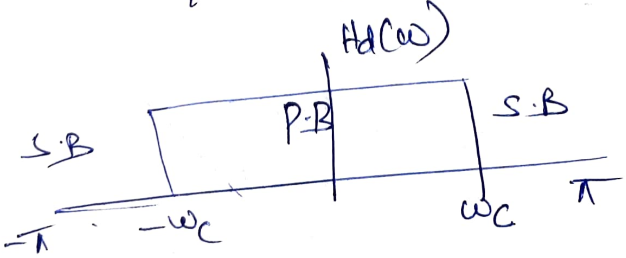
$\boxed{N = 7}$



$\omega_c = \pi/4$

$\boxed{\omega_c = 0.78 \text{ rad/sec}}$

Step 1: Low pass filter



$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & ; -\omega_c \le \omega \le \omega_c \\ 0 & ; \text{elsewhere} \end{cases}$$

(or)

$e^{-j\omega\alpha}$; $|\omega| \le \omega_c$

Hamming window

$$\omega(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right); \quad 0 \le n \le N-1$$

step 2: Determine the impulse response of the system

$$h(n) = h_d(n)\,\omega(n)$$

$$h(n) = \left[\frac{\sin \omega_c (n-\alpha)}{\pi(n-\alpha)}\right]\left[0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right)\right]$$

$n \neq \alpha$

$n \neq 3$

$$h(n) = \left[\frac{\sin(0.78)(n-3)}{\pi(n-3)}\right]\left[0.54 - 0.46 \cos\left(\frac{2\pi n}{6}\right)\right]$$

$0 \quad 1 \quad 2 \quad \cancel{3} \quad 4 \quad 5 \quad 6$

$\underline{n=0}$

$$h(0) = \left[\frac{\sin 0.78(-3)}{\pi(-3)}\right]\left[0.54 - 0.46 \cos(0)\right]$$

$$\boxed{h(0) = 6.09 \times 10^{-3} = h(6)}$$

$\underline{n=1}$

$$h(1) = \left[\frac{\sin 0.78(-2)}{\pi(-2)}\right]\left[0.54 - 0.46 \cos\left(\frac{2\pi}{6}\right)\right]$$

$$\boxed{h(1) = 0.049 = h(5)}$$

$\underline{n=2}$

$$h(2) = \left[\frac{\sin 0.78(-1)}{\pi(-1)}\right]\left[0.54 - 0.46\cos\left(\frac{4\pi}{6}\right)\right]$$

$$\boxed{h(2) = 0.17 = h(4)}$$

Step 3: Determine the transfer function.

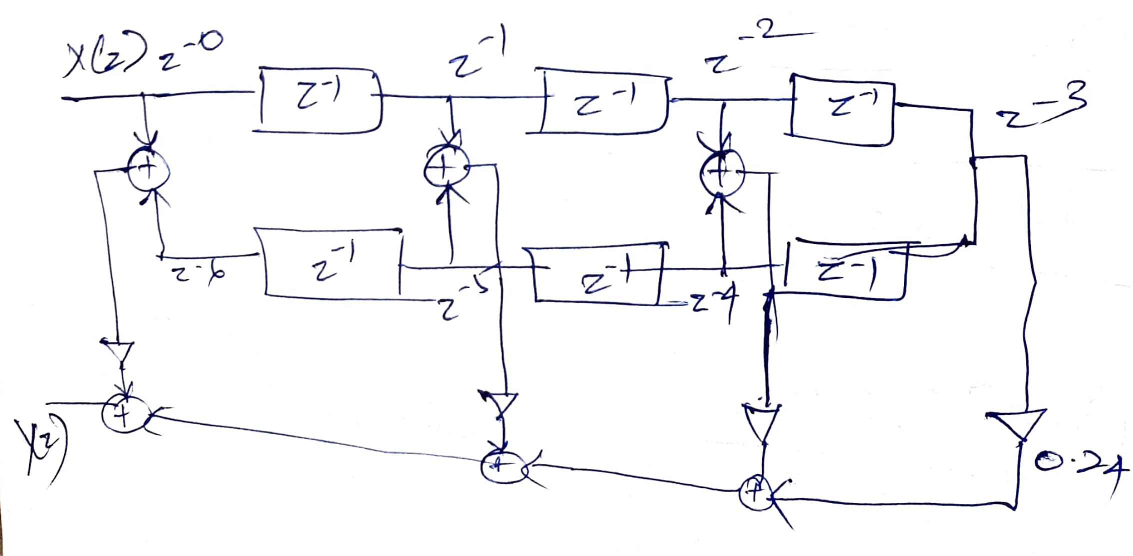
$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= \sum_{n=0}^{6} h(n) z^{-n}$$

$$= h(0)z^{-0} + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}.$$

$$H(z) = 6.09\times10^{-3}\left(z^{-0} + z^{-6}\right) + 0.049\left(z^{-1} + z^{-5}\right) + 0.17\left(z^{-2} + z^{-4}\right) + 0.24z^{-3}$$

Step 4: Realization.



Frequency Sampling Method

Design procedure:

Step 1: Determine the normalized cut-off frequency

$\omega = 2\pi f t$ rad/sec.

$$T = \frac{1}{\text{sampling frequency}} = \frac{1}{F}$$

Step 2: Determine the frequency samples.

$$H(K) = e^{-jK\left(\frac{N-1}{N}\right)\pi}$$

Step 3: Determine the impulse response of the system

$$h(n) = \frac{1}{N}\left\{ H(0) + 2 \sum_{K=1}^{\left(\frac{N-1}{2}\right)} Re\left[H(K)\, e^{\frac{j2\pi kn}{N}}\right]\right\}$$

N is odd

$$h(n) = \frac{1}{N}\left\{ H(0) + 2 \sum_{K=1}^{\left(\frac{N}{2}-1\right)} Re\left[H(K)\, e^{\frac{j2\pi kn}{N}}\right]\right\}$$

N is even.

$0 \leq n \leq N-1$

Step 4: Determine the transfer function

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

Step 5: Realize the transfer function using FIR structure.

① Design a low pass filter with cutoff freq. 1000 Hz and sampling frequency 8000 Hz using frequency sampling method for N=15

Given:

$f = 1000$ Hz

$f_s = 8000$ Hz

$N = 15$

Solution:

Step 1: $\omega = 2\pi f t$

$= 2\pi \times 1000 \times \frac{1}{8000}$

$= \pi/4$.

$\boxed{\omega = 0.78 \text{ rad/sec}}$

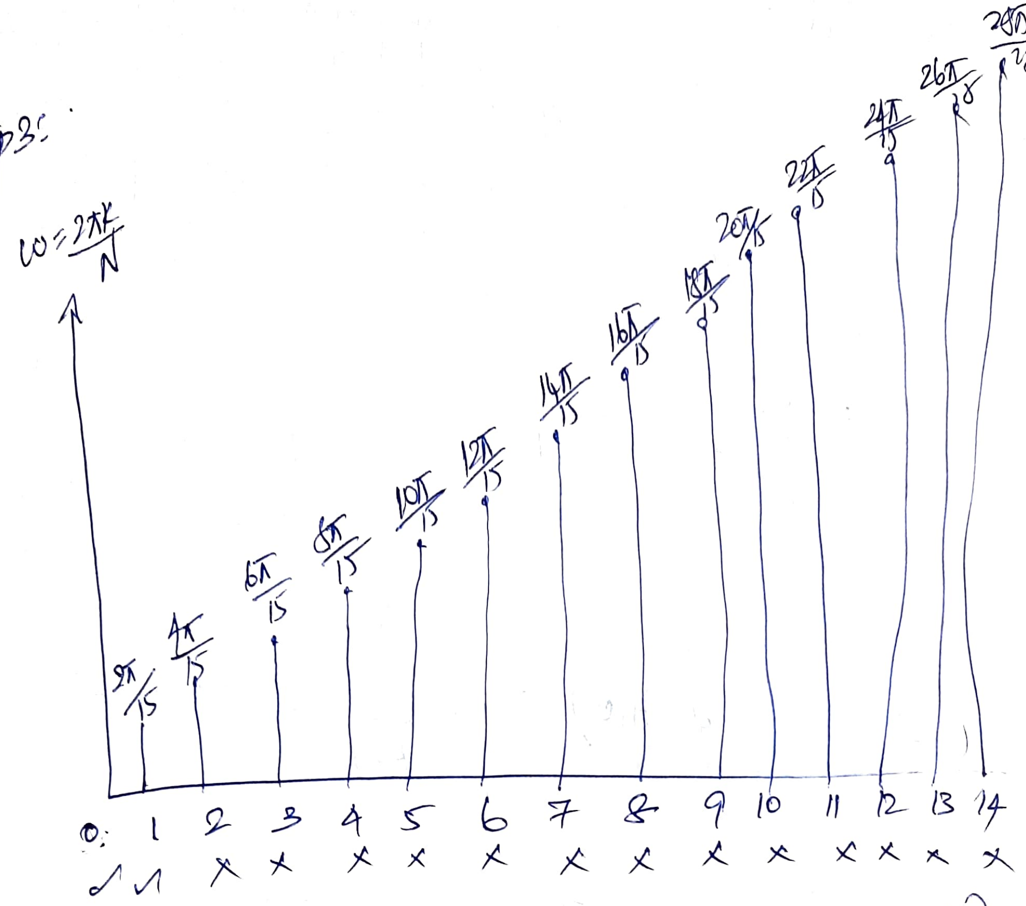
Step 2: $H(k) = e^{-jk\left(\frac{N-1}{N}\right)\pi}$

$= e^{-jk\left(\frac{14}{15}\right)\pi}$

$\boxed{H(k) = e^{-j\frac{14\pi k}{15}}}$

$\omega = \frac{2\pi k}{N}$.

Step 3:



$$h(n) = \frac{1}{N}\left\{ H(0) + 2 \sum_{k=1}^{\left(\frac{N-1}{2}\right)} Re\left(H(k)\, e^{\frac{j2\pi k n}{N}} \right) \right\}$$

$$= \frac{1}{15}\left\{ 1 + 2 \sum_{k=1}^{7} Re\left(e^{-\frac{j14\pi k}{15}} \cdot e^{\frac{j2\pi k n}{N}} \right) \right\}$$

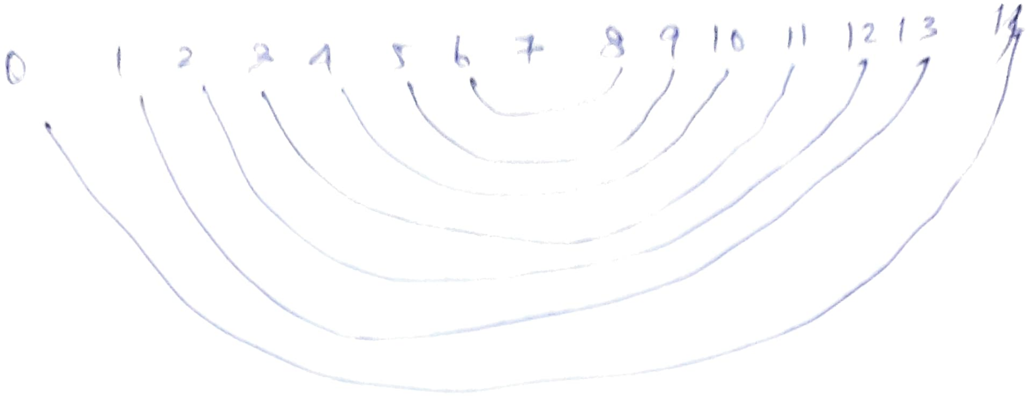
$= \cancel{\ }$ * only if Allows 1 other frequency are not allowed

$$= \frac{1}{15}\left\{ 1 + 2\, Re\; e^{\frac{j2\pi}{15}(n-7)} \right\}$$

$$h(n) = \frac{1}{15}\left\{ 1 + 2\cos\left[\frac{2\pi}{15}(n-7) \right] \right\}$$

$n = 0$ to $N-1$

$n = 0$ to 14



$n=0$

$$h(0) = \frac{1}{15}\left[1 + 2\cos\frac{2\pi}{15}(-7)\right]$$

$$\boxed{h(0) = -0.06 = h(14)}$$

$n=1$

$$h(1) = \frac{1}{15}\left[1 + 2\cos\left(\frac{2\pi}{15}(-6)\right)\right]$$

$$\boxed{h(1) = -0.04 = h(13)}$$

$n=2$

$$h(2) = \frac{1}{15}\left[1 + 2\cos\frac{2\pi}{15}(-5)\right]$$

$$\boxed{h(2) = 0 = h(12)}$$

$n=3$

$$h(3) = \frac{1}{15}\left[1 + 2\cos\frac{2\pi}{15}(-4)\right]$$

$$\boxed{h(3) = 0.05 = h(11)}$$

$n=4$

$$h(4) = \frac{1}{15}\left[1 + 2\cos\frac{2\pi}{15}(-3)\right]$$

$$\boxed{h(4) = 0.10 = h(10)}$$

$n=5$

$$h(5)=\frac{1}{15}\left[1+2\cos\frac{2\pi}{15}(-2)\right]$$

$$\boxed{h(5)=0.15=h(9)}$$

$n=6$

$$h(6)=\frac{1}{15}\left[1+2\cos\frac{2\pi}{15}(-1)\right]$$

$$\boxed{h(6)=0.18=h(8)}$$

$$h(7)=\frac{1}{15}\left[1+2\cos\left(\frac{2\pi}{15}\right)(0)\right]$$

$$\boxed{h(7)=0.2}$$

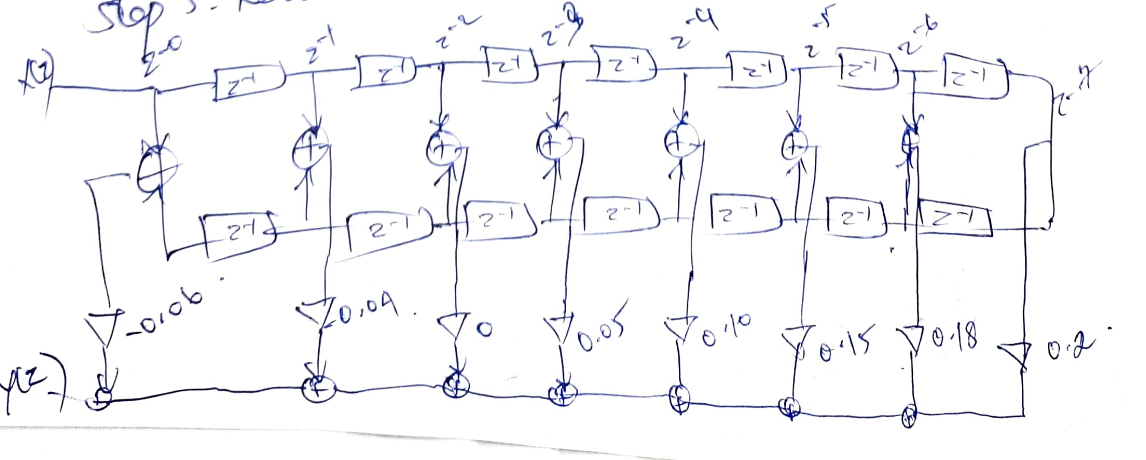
Step 4: Determine the transfer function.

$$H(z)=\sum_{n=0}^{N-1} h(n)z^{-n}$$

$$=\sum_{n=0}^{14} h(n)z^{-n}$$

$$H(z)=-0.06(z^{-0}+z^{-14})-0.04(z^{-1}+z^{-13})+0(z^{-2}+z^{-12})$$
$$+0.05(z^{-3}+z^{-11})+0.10(z^{-4}+z^{-10})+0.15(z^{-5}+z^{-9})$$
$$+0.18(z^{-6}+z^{-8})+0.2z^{-7}$$

Step 5: Realization



3. Design a band pass filter with cut off frequency 1000 Hz and 3000 Hz and sampling frequency 8000Hz using frequency sampling method $N=10$.

Given:

$f_1 = 1000$ Hz

$f_2 = 3000$ Hz

$f_s = 8000$ Hz

$N = 10$

Solution:

Step 1: $\omega = 2\pi f_1 T$

$\omega_{c1} = 2\pi \times 1000 \times \frac{1}{8000}$

$\boxed{\omega_{c1} = 0.78 \text{ rad/sec}}$

$\omega_{c2} = 2\pi \times 3000 \times \frac{1}{8000}$

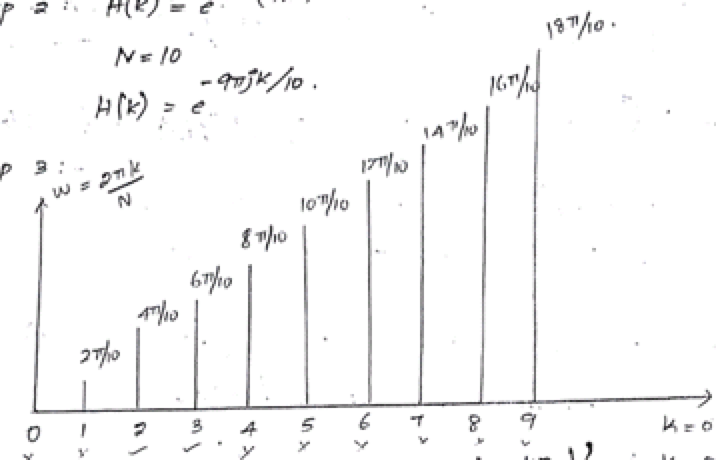
$\boxed{\omega_{c2} = 2.35 \text{ rad/sec}}$

Step 2: $H(k) = e^{-jk\left(\frac{N-1}{N}\right)\pi}$

$N = 10$

$H(k) = e^{-9\pi jk/10}$.

Step 3: $\omega = \frac{2\pi k}{N}$



$k = 0$ to $N-1$

$k = 0$ to 9

$N = 10$

$$h(n) = \frac{1}{N}\left\{H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left(H(k)\, e^{\frac{j2\pi kn}{N}}\right)\right\}$$

$$= \frac{1}{10}\left\{0 + 2\sum_{k=1}^{4} Re\left(e^{-9\pi kj/10}\, e^{j2\pi kn/10}\right)\right\}$$

$$= \frac{2}{10}\left\{\sum_{k=2}^{3} Re\left(e^{-2\pi kj/10\,(n-9/2)}\right)\right\}$$

$$= \frac{1}{5} Re\left[\left(e^{-\pi 2j/5\,(n-9/2)}\right) + e^{-j\pi 3/5\,(n-9/2)}\right]$$

4. Design a band stop filter with cut off frequency 1000 Hz and 3000 Hz and sampling frequency 8000 Hz using frequency sampling method for $N=7$.

Given

$f_{c1} = 1000\,Hz$

$f_{c2} = 3000\,Hz$

$f_s = 8000\,Hz$

Solution:

$\omega = 2\pi f T$

$$\omega_{c1} = 2\pi \times 1000 \times \frac{1}{8000}$$

$$\boxed{\omega_{c1} = 0.78\ rad/sec}$$

$$\omega_{c2} = 2\pi \times 3000 \times \frac{1}{8000}$$

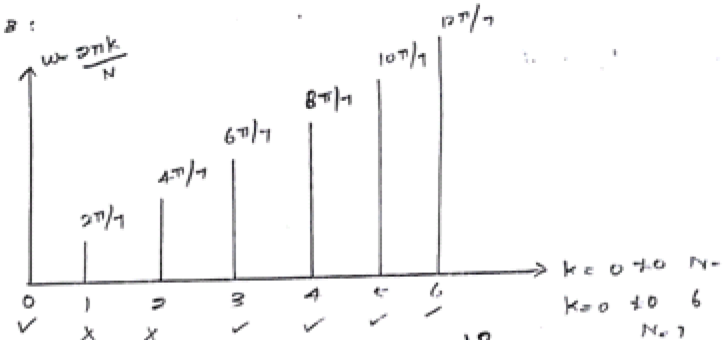
$$\boxed{\omega_{c2} = 2.35\ rad/sec}$$

Step 2: $H(k) = e^{-jk\left(\frac{N-1}{N}\right)\pi}$

$N = 7$

$$H(k) = e^{-\frac{6jk\pi}{7}}$$

Step 3:



$$h(n) = \frac{1}{N}\left\{H(0) + 2\sum_{k=1}^{\left(\frac{N-1}{2}\right)} Re\left(H(k)\, e^{\frac{j2\pi kn}{N}}\right)\right\}$$

$$= \frac{1}{7}\left\{1 + 2\sum_{k=1}^{3} Re\left(e^{-\frac{j6k\pi}{7}}\, e^{\frac{j2\pi kn}{7}}\right)\right\}$$

$$= \frac{1}{7}\left\{1 + 2\sum_{k=3} Re\left(e^{\frac{2k\pi}{7}(n-3)}\right)\right\}$$

$$= \frac{1}{7}\left\{1 + 2\ Re\left(e^{\frac{6\pi}{7}(n-3)}\right)\right\}$$

0 1 2 3 4 5 6.

19/9/17 Unit - IV Finite word length affects of IIR
or Digital filter.

Digital Signal processing algorithms are realized either with special purpose digital hardware or programs for a general purpose digital computer. In both cases the numbers and co-efficients are stored in finite length registers. Therefore, co-efficients and numbers are quantized by truncation and rounding methods when they are stored.

The following errors arise due to quantization of numbers. They are:

1. Input quantization error
2. Product quantization error
3. Co-efficient quantization error.

1. Input quantization error:

The convertion of continuous time input signal to the digital value produces an error is called as input quantization error.

2. Product quantization error.

The product quantization error arise at the output of multiplier. Multiplication of b bit data with a b bit data results a product of two b bits

3/ Co-efficient quantization error:

The filter co-efficients are computed to infinite precision in theory. If they are quantized then the frequency response of a resulting filter may differ from desired response and sometimes filter may fail to need the desired specification. If the poles of the desired filter are closer to the unit circle then those of the filter with quantized co-efficients may lie just outside the unit circle leads to instability.

Types of number representation:

There are ~~three~~ two types of number representation. They are:

1. Fixed point number representation.
2. Floating point number

Finite word length effect in DSP.

Number Representation

1. Fixed point Number Representation
2. Floating point Number Representation.

Fixed point Number Representation.

In a fixed point Arithmetic the position of binary bit is fixed.

(eg) $(01.1100)_2 \to (1.75)_{10}$

01.1100

- $0 \times 2^{-4} = 0$
- $0 \times 2^{-3} = 0$
- $1 \times 2^{-2} = 0.25$
- $1 \times 2^{-1} = 0.5$
- $1 \times 2^{0} = 1$
- $0 \times 2^{1} = 0$

$\underline{1.75}$

The fixed point Representation is classified into three types. They are

1. Sign magnitude form.
2. 1's complement form.
3. 2's complement form.

1. Sign magnitude form.

In this representation, if the MSB bit is 0 then it is positive number & if the MSB bit is 1 then it is negative number

eg:

$\underline{0}1.11000 \Rightarrow +1.75$

$\underline{1}1.11000 \Rightarrow -1.75$

2. 1's complement

The 1's complement for the negative number is obtained by complementing all bits of positive number

eg: $+0.875 \rightarrow 0.111000$

$-0.875 \rightarrow 1.000111$

complement

3. 2's complement

The 2's complement of the negative number is obtained by complementing all bits of positive number & adding 1 to the LSB

eg: $(-0.875) \rightarrow (+0.875) \rightarrow 0.111000$

$(-0.875) \rightarrow 1.000111$

$\qquad\qquad\qquad\quad 1$

$\qquad\qquad\quad 1.001000$

$-0.875 \rightarrow 1.001000$

Examples on Fixed point Representation

Problem 1:- Arithmetic fixed point,

Compute $0.75 + (-0.625)$ using fixed point numbers

$0.75 \times 2 = 1.5$ 1
$0.5 \times 2 = 1.0$ 1
$0.0 \times 2 = 0$ 0
0

Step 1: $0.75 \Rightarrow 0000.1100$

Step 2: -0.625

$0.625 = 0000.1010$

$0.625 \times 2 = 1.25$ 1
$0.25 \times 2 = 0.5$ 0
$0.5 \times 2 = 1.0$ 1
$0.0 \times 2 = 0$ 0

We want -0.625

$-0.625 \Rightarrow$ 2's complement

$$
\begin{array}{rr}
-0.625 = & 1111.0101 \\
 & +1 \\
\hline
-0.625 = & 1111.0110
\end{array}
$$

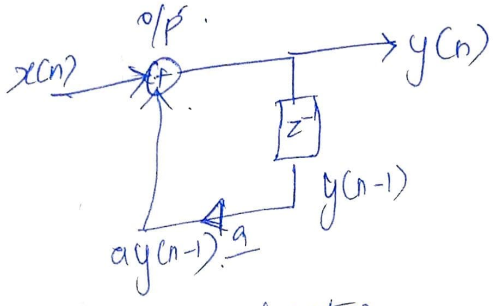
Step 3:

$$
\begin{array}{rr}
 & 0000.1100 \\
 & 1111.0110 \\
\hline
1 & 0000\ 0010
\end{array}
$$

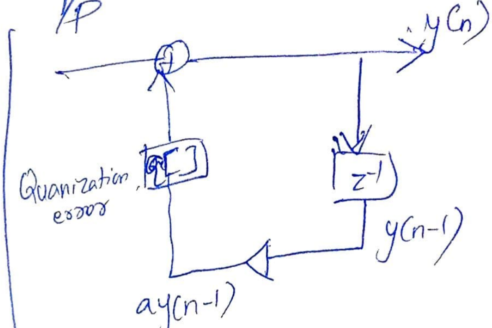
LIMIT CYCLE OSCILLATION

Consider a 1st order difference equation.

$$\underbrace{y(n)}_{o/p} = a\,y(n-1) + \underbrace{x(n)}_{I/P}$$



Ideal system



Non-linear system with oscillator.

Problems on "characteristics of Limit cycle oscillation" & dead band.

$$\text{Dead band} = \frac{2^{-B}}{2(1-|a|)}$$

Problem

① Determine the characteristics of Limit cycle oscillation w.r.t system described by the equation

$y(n) = 0.95\,y(n-1) + x(n)$. Find the dead band of the filter. Assume bit length $= 4$ bits & $x(n) = \begin{cases} 0.75, & n=0 \\ 0, & n\neq 0 \end{cases}$

Soln :-

$$y(n) = 0.95\,y(n-1) + x(n)$$

$$y'(n) = Q[0.95\,y(n-1)] + x(n)$$

$n=0$

$$y'(0) = Q[0.95\,y(-1)] + x(0)$$

$$= Q[0.95\,(0)] + 0.75$$

1
2
2
2

Let $y(n) \neq 0$
$n > 0$
$n < 0$
ve $y(n) = 0$

$= 0.75_{10}$.

Find Binary value

0.75×2

1.50

0.5×2

1.0

$0 \times 0 \times 2 = 0$.

$0.75_{10} \rightarrow 0.11_2$.

0.11

$\rightarrow 1 \times 2^{-1} = 0.5$

$\rightarrow 1 \times 2^{-2} = 0.25$

0.75.

$0.11_2 \rightarrow 0.75_{10}$.

$\underline{n=1}$

$y'(1) = Q[0.95 y(0)] + x(1)$

$= Q[0.95 \times 0.75] + 0$.

$= Q[0.7125]$

$= 0.7125_{10}$.

↓

convert to Binary

0.101102.

0.7125×2

1.4250

0.4250×2

0.850×2

1.700

0.7×2

1.4

0.10110_2

$\downarrow$ Rounding 4 bits

0.1011_2

$\downarrow$

Convert to decimal

0.1011

$1 \times 2^{-1} = 0.5$

$0 \times 2^{-2} = 0$

$1 \times 2^{-3} = 0.125$

$1 \times 2^{-4} = 0.0625$

$(0.6875)_{10}$

$\boxed{y(1) = 0.6875}$

$\underline{n=2}$

$y'(2) = Q[0.95\, y(1)] + x(2)$

$= Q[0.95 \times 0.6875] + 0$

$= Q[0.6531]$

$= (0.6531)_{10}$

$\downarrow$

Convert to Binary

$\downarrow$

$(0.10100)_2$

$\downarrow$ Rounding 4 Bits

0.1010_2

$\downarrow$

convert to decimal.

$\downarrow$

$(0.625)_{10}$

$\boxed{y(2) = 0.625}$

0.6531×2

1.3062

0.3062×2

0.6124

0.6124×2

1.2248

0.2248×2

0.4496

0.4496×2

0.8992

0.1010

$1 \times 2^{-1} = 0.5$

$0 \times 2^{-2} = 0$

$1 \times 2^{-3} = 0.125$

$0 \times 2^{-4} = 0$

0.625

$n=3$

$$y'(3) = Q[0.95\,y(2)] + x(3)$$
$$= Q[0.95 \times 0.625] + 0$$
$$= Q[0.59375]$$
$$= 0.59375_{10}$$

↓

Convert to Binary

↓

0.10011_2.

↓ Rounding 4 bit

$(0.1010)_2$

↓

Convert to decimal

↓

$(0.625)_{10}$.

0.59375×2
1.18750
0.18750×2
0.3750
0.3750×2
0.7500
0.7500×2
1.50
0.500×2
1.0

$\boxed{0.1001}1$

$$\begin{array}{r} 0.1001 \\ 1 \\ \hline 0.1010 \end{array}$$

0.101
$1 \times 2^{-1} = 0.5$
$0 \times 2^{-2} = 0$
$1 \times 2^{-3} = 0.125$
0.625

$n=0 = 0.75$
$n=1 = 0.6875$
$n=2$, $n=3$ } 0.625

n	Binary	Decimal
0		
1		
2	$(0.1010)_2$	$(0.625)_{10}$
3	$(0.1010)_2$	$(0.625)_{10}$.

$$\text{Dead Band} = \pm \frac{2^{-B}}{2(1-|a|)}$$

$\Rightarrow 4$ Bit

$$= \pm \frac{2^{-4}}{2(1-|0.95|)}$$

$0.95 \approx y(n-1)$

$$= \pm \frac{2^{-4}}{2(0.05)}$$

$$\begin{array}{r} 1.00 \\ -\,0.95 \\ \hline 0.05 \end{array}$$

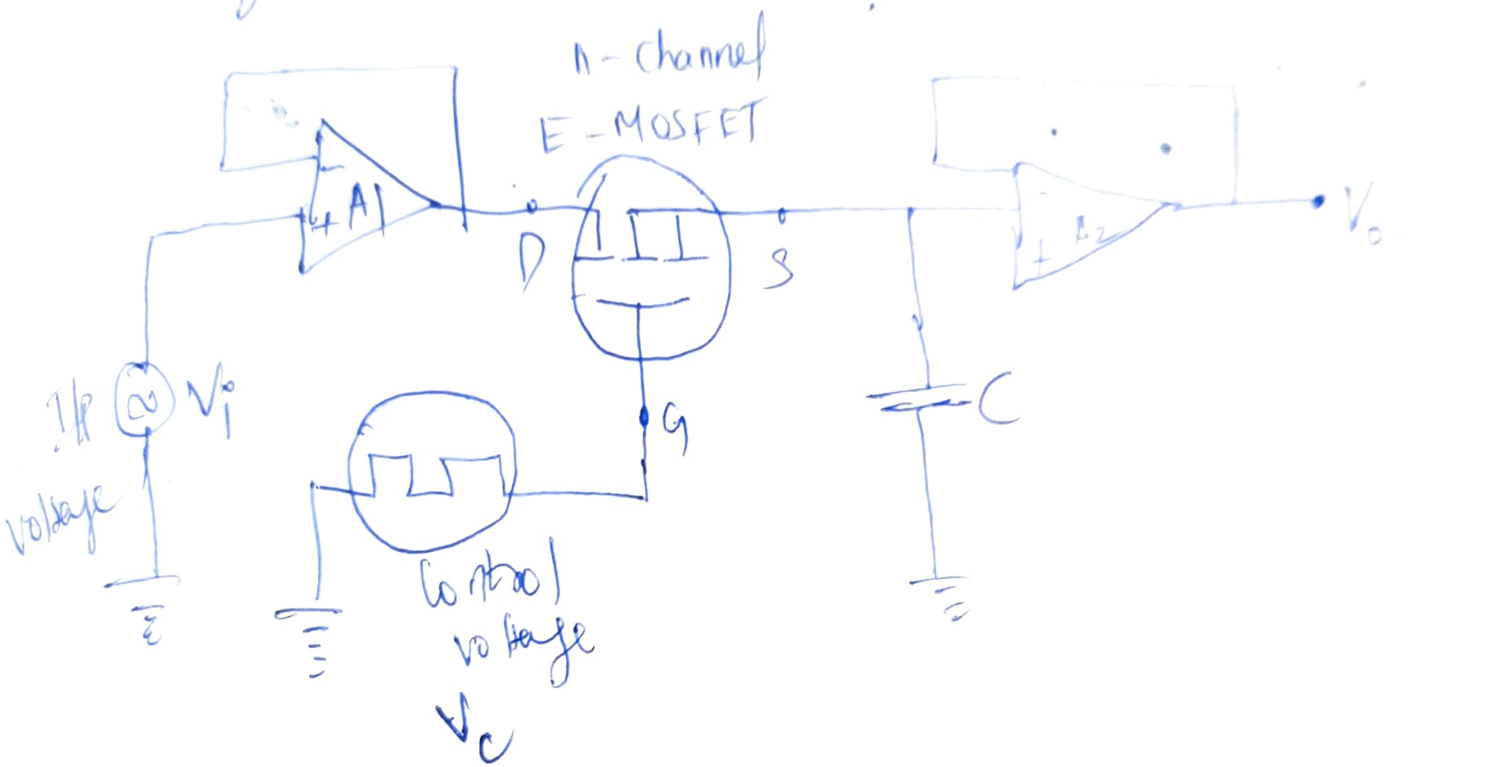
$$= \pm 0.625$$

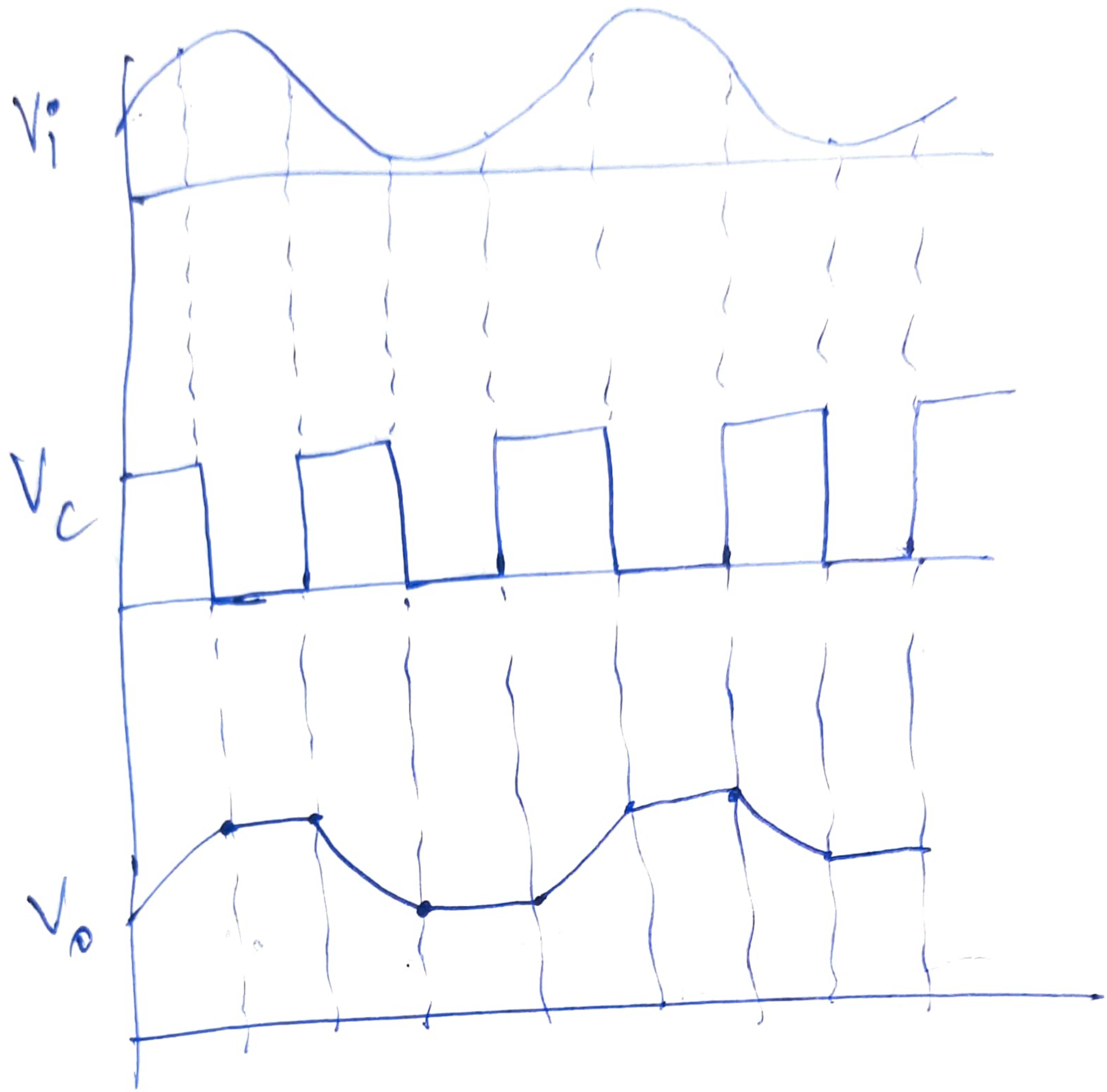
+ve −ve

$+625 \longrightarrow -0.625$ } Limit cycle oscillations

Sample & Hold circuit

- samples and holds the input signal.
- used in digital interfacing, analog to digital systems & pulse code modulation system





Vi
Vc
Vo

Random process :

"A Random process is a collection of time functions (or) signals corresponding to various outcomes of a random experiment".

Collection of time functions or signals.

denotes as $X(t, S)$

t – time

$S \rightarrow$ Sample point of the Random experiment.

Statistical properties,

- Mean
- Variance
- CDF
- PDF
- Autocorrelation
- Cross correlation..

Mean

Expected value of Random process

$$\mu_{X(t)} = E[x(t)]$$

$$= \int_{-\infty}^{\infty} x \, f_x(x;t)\, dx$$

Variance

$$\mathrm{Var}\, X(t) = E\left[[x(t) - \mu]^2\right]$$

$$\boxed{\mathrm{Var}\, X(t) = E[x^2] - E^2[X]}$$

$$E\left[(x(t) - \mu)^2\right]$$

$$= E\left[(x - \mu)^2\right]$$

$$= E\left[x^2 + \mu^2 - 2x\mu\right]$$

$$= E[x^2] + \mu^2 - 2E[x]\mu$$

$$= E[x^2] + \mu^2 - 2\mu^2$$

$$= E[x^2] - \mu^2$$

$$= E[x^2] - E^2[x]$$

$$= E[x^2]$$

Cumulative Distribution function (CDF)

$$F_x\, x(t) = P[x(t) \le x]$$

$$F\{x_1, x_2 \ldots x_n, t_1, t_2 \ldots t_n\}$$

$$= P\{x(t) \le x_1, x(t) \le x_2, x(t) \le x_3 \ldots x(t) \le x_n\}$$

Probability Density function (pdf)

$$f(x,t) = \frac{\partial^n F(x_1, x_2, x_3 \ldots x_n, t_1, t_2, t_3 \ldots t_n)}{\partial t_1 \partial t_2 \ldots \partial t_n}$$

Autocorrelation

Relation of a function with its shifted version.

$$R_{xx}(t_1, t_2) = E[X(t_1)\, X(t_2)]$$

Properties

$R_x(\tau)$ – for a stationary process $X(t)$

$R_x(\tau)$ is an Even function

$$\boxed{R_x(\tau) = R_x(-\tau)}$$

* Maximum Absolute value of $R_x(\tau)$ is at $\tau=0$

$$\boxed{R_x(\tau) \leq R_x(0)}$$

* For some T_0 $R_x(T_0) = R_x(0)$
then for all K

$$\boxed{R_x(KT_0) = R_x(0)}$$

Cross Correlation

$$R_{xy}(t_1, t_2) = E[x(t_1)y(t_2)]$$

$$R_{xy}(t_1, t_2) = R_{yx}(t_2, t_1)$$

Random process

Stationary or Non Stationary

Nature of signal.

→ Continuous time Continuous Amplitude

→ Continuous time discrete Amplitude

→ Discrete time Continuous Amplitude

→ Discrete time Discrete Amplitude

Four Classification based on Nature of signal.