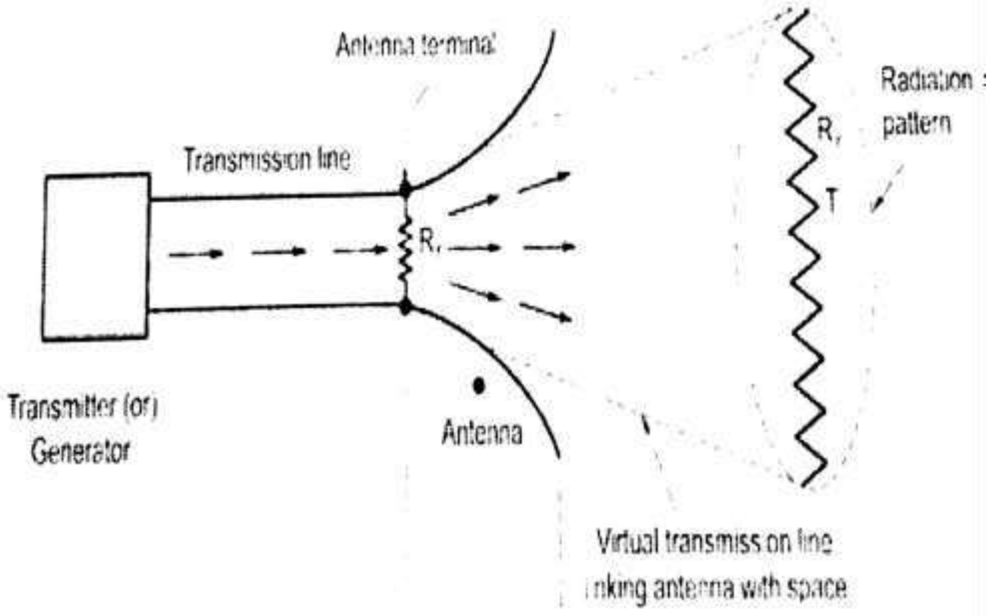
U18PCEC503-Antennas & Propagation

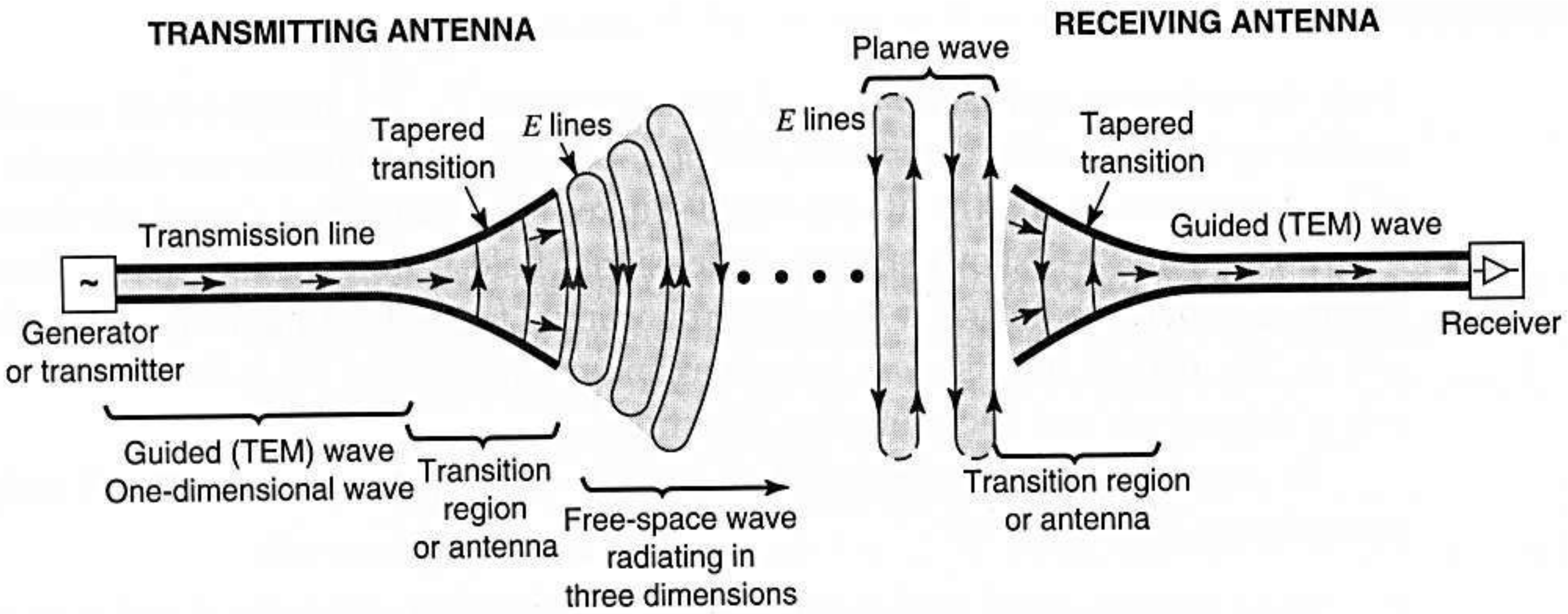
Unit – I

BASIC ANTENNA CONCEPTS

Introduction

An antenna is an important basic component in the communication system. Basically antennas are the metallic structures designed for radiating and receiving the ***Electromagnetic (EM) energy*** in an effective manner which is used for conveying the information.




TRANSMITTING ANTENNA
RECEIVING ANTENNA
Plane wave
E lines
Tapered transition
E lines
Tapered transition
Transmission line
Guided (TEM) wave
Generator or transmitter
Receiver
Guided (TEM) wave
One-dimensional wave
Transition region or antenna
Free-space wave radiating in three dimensions
Transition region or antenna

Antenna Characteristics

Irrespective of antenna type and applications, all the antennas possess certain basic properties (characteristics) and some of the important properties are:

(1) Radiation Pattern
   (a) Field Radiation Pattern
   (b) Power Radiation Pattern
(2) Beam Solid Angle (Beam Width)
(3) Radiation intensity
(4) Directive gain and Directivity
(5) Power gain
(6) Input impedance
(7) Polarization
(8) Bandwidth
(9) Effective Aperture and Effective Length
(10) Antenna temperature

Radiation Pattern

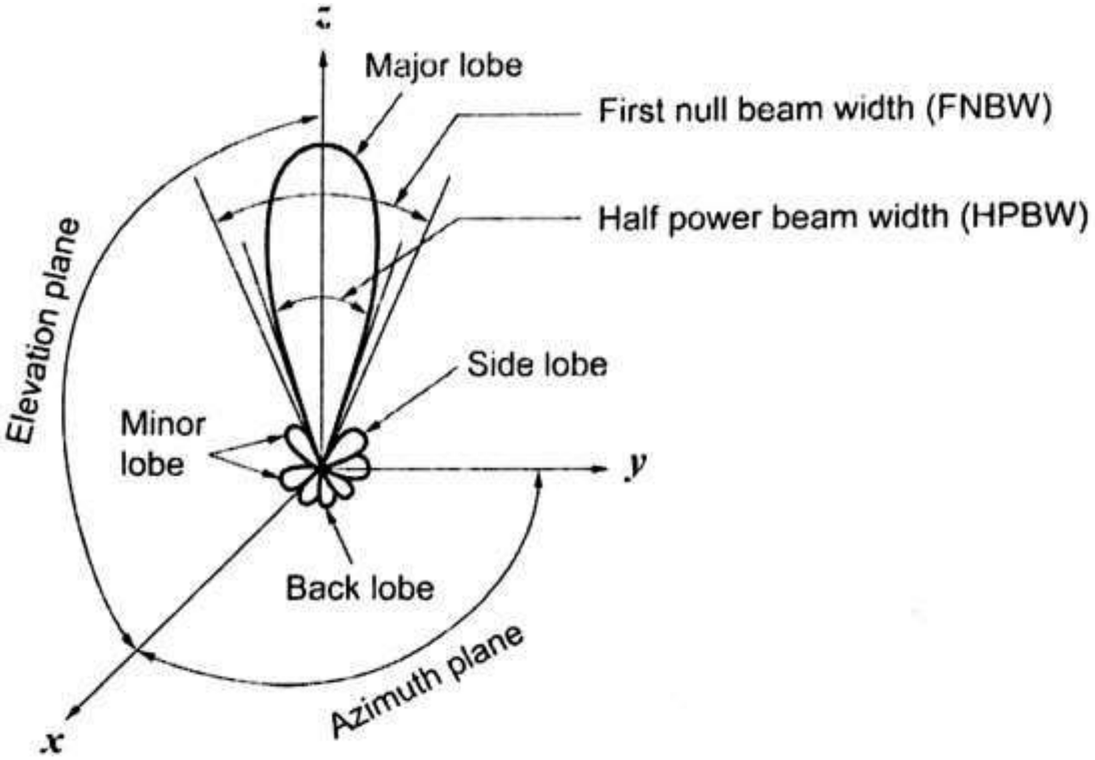
Any antenna is characterized by its radiation pattern which is a ***mathematical*** *or* ***graphical representation*** *of the radiations from an antenna as a function of space coordinates in a desired direction.*

Simply it is a graph which shows the variation in the actual field strength of electromagnetic field at all points, which are at equal distance from the antenna.

The radiation patterns are different for different antennas and are affected by the location of antenna with respect to the ground. There are two basic types radiation pattern:

If the radiation of the antenna is expressed in terms of the field strength (E) in V/m , then the graphical representation is called ***field strength pattern or field radiation pattern.***

Similarly, if the radiation of the antenna is expressed in terms of the *power per unit solid angle*, then the graphical representation is called ***power radiation pattern*** or simply ***power pattern.***



Radiation Pattern Lobes

A radiation lobe is a three dimensional portion of strong fields surrounded by a weak field. It is the portion of significant field strength in a particular direction. Depends on the field strength, the radiation lobes of an antenna may be classified into four types:

(i) Major Lobe

(ii) Minor Lobe

(iii) Side Lobe, and

(iv) Back Lobe

(a) Major Lobe

This is the radiation lobe containing the ***maximum radiation*** *in a desired direction,* which is also referred as ***main lobe*** or ***main beam***.

(b) Minor Lobe

All the lobes ***other than main lobe*** *are called the* ***minor lobes***. It represents the radiation in the undesired directions. The level of minor lobe is usually expressed as, *"a ratio of power density in that lobe to that of the major lobe"*.

In most wireless systems, minor lobes are undesired. Hence, a good antenna design should minimize the minor lobes.

(c) Side Lobe

These are the ***minor lobes adjacent to the main lobe*** and are separated by the various nulls (zero power). The side lobes are the largest among the minor lobes.

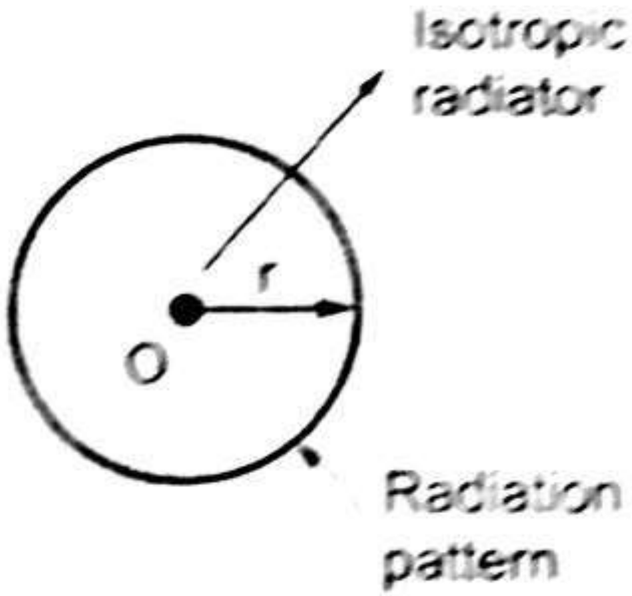
(d) Back Lobe

This is the minor lobe diametrically ***opposite to the main lobe***. It is an radiation lobe whose axis makes an angle of approximately 180° with the major lobe direction.

Isotropic Radiators

An isotropic radiator is a radiator which ***radiates uniformly in all the directions***.

It is also called *isotropic source or omni directional radiator or simply unipole*.



Basically, it is a lossless ideal *radiator or antenna*. Generally, all the practical antennas are compared with the characteristics of the isotropic radiator. So it is also called as ***reference antenna***.

Gain

The gain is an useful measure describes the performance of an antenna which act as the *figure of merit* for antenna. It is closely related to the directivity, which is a *measure* that takes into account the *antenna efficiency* as well as its *directional capabilities*.

The gain of the *transmitting antenna* is defined as, "*the ability of the antenna to concentrate the radiated power in a given direction*", where as for the *receiving antenna*, "*it is an ability of absorbing incident power effectively from the particular radiation direction*".

The relation between ***gain (G)*** and ***directivity (D)*** is expressed in terms of antenna efficiency (η).

$$G = \eta D \qquad 0 \le \eta \le 1$$

In most well-designed antennas, η may be close to the unity(100 %). In practice, G is always less than D (G < D) due to ***ohmic losses*** in the antenna.

When an antenna efficiency is 100% $(\eta = 1)$, the gain (G) and directivity (D) are used interchangeably. Gain of an antenna is expressed in decibel as,

$$G(dB) = 10\log_{10} G$$

1. In terms of Radiation Intensity

The ***radiation intensity*** in a given direction is defined as, "*the power radiated from an antenna per unit solid angle*".

$$\boxed{\text{Gain } (G) = \frac{\text{Maximum Radiation Intensity from Antenna Under Test (AUT)}}{\text{Maximum Radiation Intensity from a reference antenna with same input power}}}$$

2. In terms of Signal Power

In terms of signal power received by a receiver at a distant point in the direction of maximum radiation, then the gain of any antenna can be defined as,

$$\boxed{\text{Gain } (G) = \frac{\text{Maximum Power received from given Antenna } (P_1)}{\text{Maximum Power received from reference Antenna } (P_2)}}$$

3. In terms of Field Strength

In terms of field strength at a given distance from the given antenna, the gain is expressed as,

$$\boxed{Gain\ (G) = \frac{Maximum\ Field\ strength\ from\ given\ Antenna\ (E_1)}{Maximum\ Field\ strength\ from\ reference\ Antenna\ (E_2)}}$$

The field strength is calculated in the same distance as well as direction from both given antenna as well as the reference antenna.

DIRECTIVE GAIN (G_D)

Directive gain parameter is used by the practical antenna which concentrates on its radiated energy relative to that of the some standard antenna.

The directive gain is a function of angles $(\theta \, and \, \phi)$ which should be specified

$U(\theta,\phi)$ – Radiation Intensity in a particular direction,

U_{avg} – Average radiation intensity in that direction = $\frac{P_r}{4\pi}$

P_r – Radiated power in W/m^2

$$\boxed{Directive\ Gain\ (G_D) = \frac{Radiation\ Intensity\ in\ a\ particular\ direction}{Average\ radiated\ power}}$$

$$G_D(\theta,\phi) = \frac{U(\theta,\phi)}{U_{avg}} = \frac{U(\theta,\phi)}{\frac{P_r}{4\pi}}$$

$$G_D(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_r}$$

Directive gain is expressed in decibels, then

$$G_D(dB) = 10\log_{10} G_D(\theta,\phi)$$

$$G_D(dB) = 10\log_{10}\left\{\frac{4\pi U(\theta,\phi)}{P_r}\right\}$$

In term of Power Density

$$\boxed{G_D = \frac{\textit{Power density radiated in a particular direction by given antenna}}{\textit{Power density radiated in that partcicular direction by an isotropic antenna}}}$$

POWER GAIN (G_P)

The ***directive gain*** compares the radiated power densities of the actual and isotropic antennas when both are ***radiating the same total power***.

In terms of Power Density

But the ***power gain*** compares the radiated power density by both the actual antenna and an isotropic antenna and both have the ***same input power***.

$$\boxed{G_P = \frac{\textit{Power density radiated in a particular direction by the given antenna}}{\textit{Power density radiated in that direction by an isotropic antenna}}}$$

$$G_p = \eta\, G_D \qquad\qquad 0 \le \eta \le 1$$

In terms of Radiation Intensity

$$\boxed{G_P = \frac{\text{Radiation intensity in a given direction}}{\text{Average total input power}}}$$

$$G_p = \frac{U(\theta,\phi)}{P_t/4\pi}$$

where, total input power $P_i = P_r + P_l$, and P_l is the ***antenna ohmic power losses***.

In terms of Power Input

$$\boxed{G_P = \frac{\text{Power input supplied to the given antenna in the direction of maximum radiation}}{\text{Power input supplied to reference antenna}}}$$

DIRECTIVITY (D)

1. In terms of Radiation Intensity

The ***maximum directive gain in a particular direction*** is called the ***directivity*** of an antenna. It is defined as, "*the ratio of the maximum radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions*".

$$\text{Directivity}(D) = \frac{\text{Maximum Radiation Intensity of test antenna}}{\text{Average Radiation Intensity of test antenna}}$$

$$D = \frac{U(\theta,\phi)_{\max}}{U_{Avg}} = \frac{U(\theta,\phi)_{\max}}{P_r/4\pi} = \frac{4\pi \times U(\theta,\phi)_{\max}}{P_r}$$

$$D = \frac{4\pi \times (\text{Maximum Radiation Intensity})}{\text{Total Radiated Power}}$$

The directivity also defined as

$$\boxed{Directivity(D) = \frac{Maximum\ Radiation\ Intensity\ of\ the\ test\ antenna}{Radiation\ Intensity\ of\ an\ Isotropic\ antenna}}$$

$$D = \frac{U(\theta,\phi)_{Max}(test\ antenna)}{U_O(Isotropic\ antenna)}$$

2. In terms of Total Radiated Power

$$\boxed{Directivity(D) = \frac{Power\ radiated\ from\ a\ test\ antenna}{Power\ radiated\ from\ an\ Isotropic\ antenna}}$$

3. In terms of Power Density

The directivity (D) of an antenna is also defined as, "*the ratio of the maximum power density* $P(\theta,\phi)_{Max}\left(Watts/m^2\right)$ *to its average value over a sphere (isotropic pattern) as observed in the far field of an antenna*".

$$D = \frac{P(\theta,\phi)_{max}}{P(\theta,\phi)_{avg}}$$

The average power density over a sphere is given by,

$$P(\theta,\phi)_{avg} = \frac{1}{4\pi}\int_{\phi=0}^{\phi=2\pi}\int_{\theta=0}^{\theta=\pi} P(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{4\pi}\iint_{4\pi} P(\theta,\phi)\, d\Omega \quad \left(W\, sr^{-1}\right)$$

where, $d\Omega = \sin\theta\, d\theta\, d\phi$ = Elements of solid angle

$$D = \frac{P(\theta,\phi)_{\max}}{\frac{1}{4\pi}\iint\limits_{4\pi} P(\theta,\phi)\, d\Omega}$$

$$= \frac{1}{\frac{1}{4\pi}\iint\limits_{4\pi}[\, P(\theta,\phi)/\, P(\theta,\phi)_{\max}\,]\, d\Omega}$$

$P_n(\theta,\phi) = P(\theta,\phi)/\,P(\theta,\phi)_{\max}$ = Normalized power pattern

$$D = \frac{4\pi}{\iint\limits_{4\pi} P_n(\theta,\phi)\, d\Omega}$$

where, $\Omega_A = \iint\limits_{4\pi} P_n(\theta,\phi) d\Omega \;\; (sr) = Beam \;\; area$

4. In terms of Beam Area

The beam area (Ω_A) is *the solid angle through which all the power radiated by the antenna would flow*

$$D = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\phi)\, d\Omega} = \frac{4\pi}{\Omega_A}$$

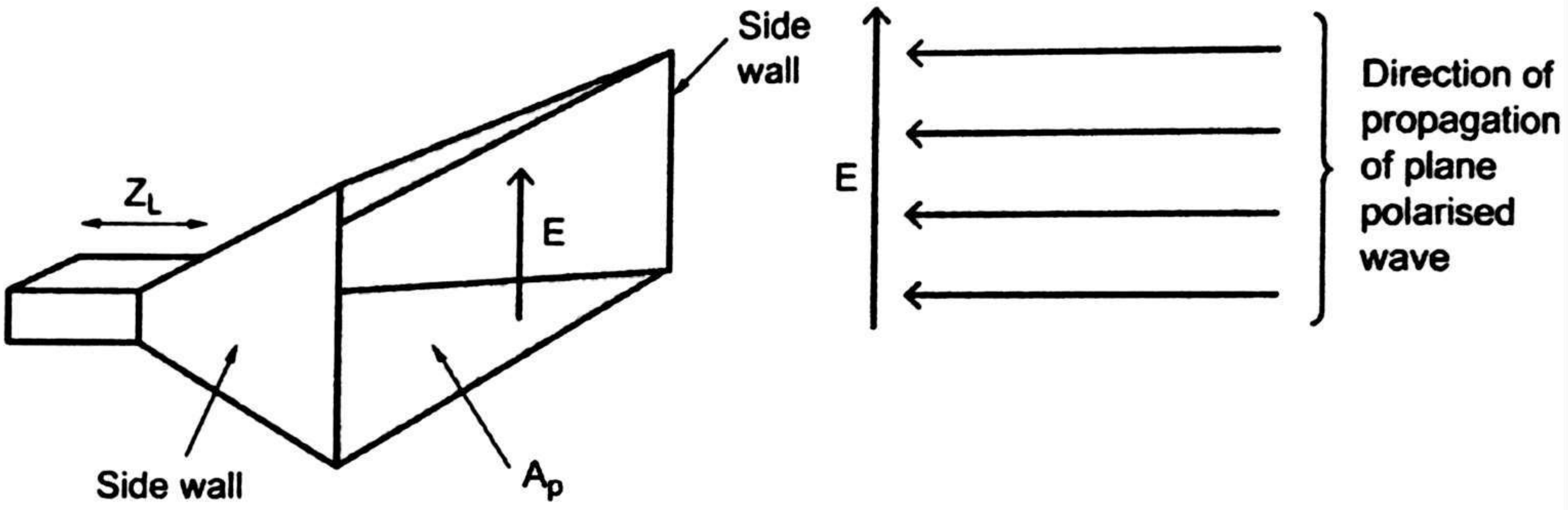
5. In terms of HPBW

If the half-power beam widths of an antenna are known, then the antenna directivity is expressed as,

$$D = \frac{41{,}253^o}{\theta^o_{HP}\phi^o_{HP}} \approx \frac{40{,}000}{\theta^o_{HP}\phi^o_{HP}} \Rightarrow \textit{Approximate directivity}$$

$41{,}253°$ = Number of square degrees in sphere

EFFECTIVE APERTURE [OR] EFFECTIVE AREA [OR] CAPTURE AREA [OR] ANTENNA APERTURE



Receiving Antenna

Effective aperture *(A_e) is an area over which an antenna extracts power from the incident radio waves.* It may be defined as, "*the ratio of power received at the antenna load terminal to the Poynting vector(power density) in W/m^2 of the incident wave.*

$$A_e = \frac{\textit{Power received by the antenna}}{\textit{Poynting vector of the incident wave}}$$

$$A_e = \frac{P_{ra}}{P_d} = \frac{P_d A_e}{P_d}$$

where,

P_{ra} - Power received in Watts

P_d - Power density [power flow per sq.metre] for the incident wave in W/ m^2, and

A_e - Effective area in m^2

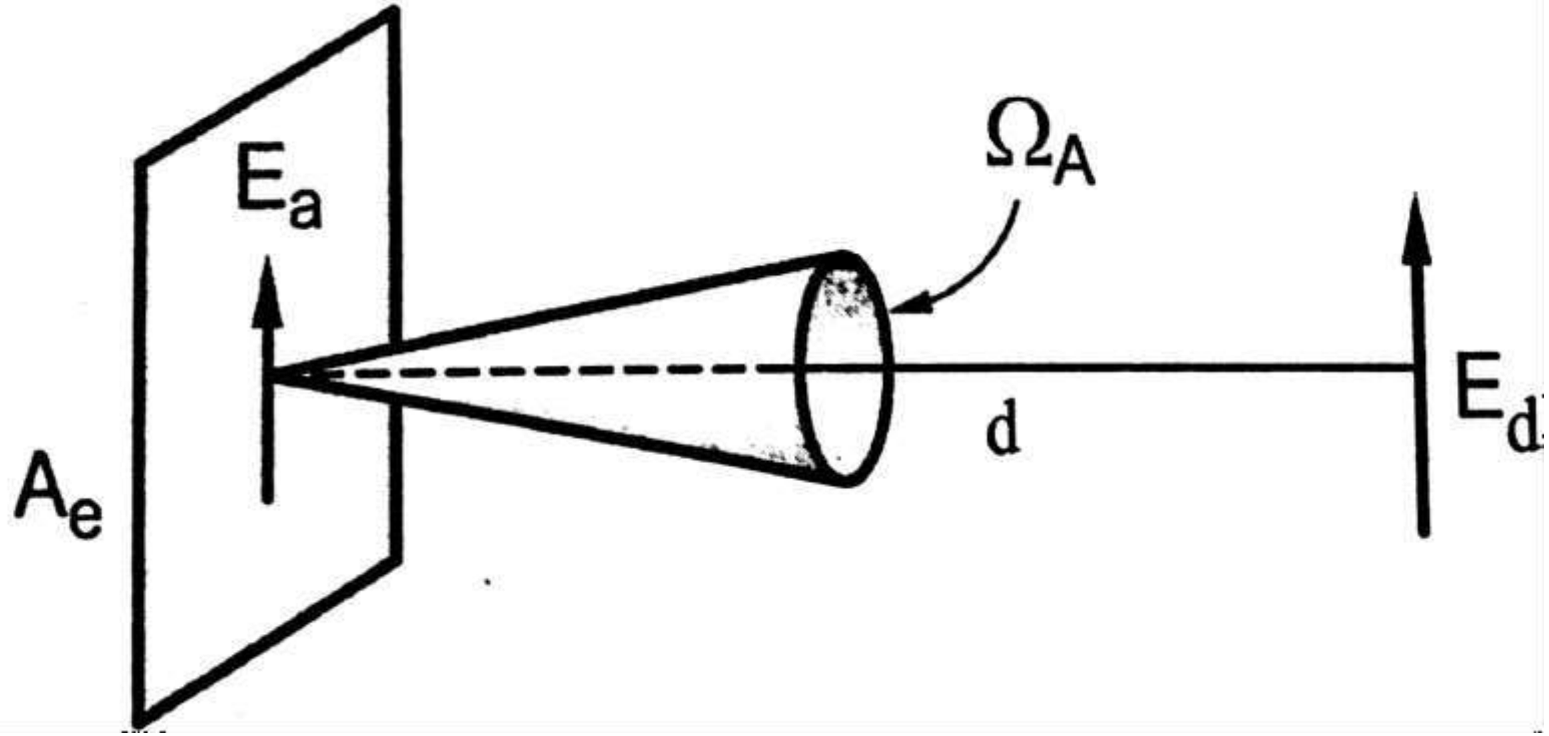
If an antenna extracts all the power from the incident wave over its entire physical aperture (A_P), then the total power (P_t) absorbed from the wave is expressed as,

$$P_t = \frac{E^2}{Z_L} A_p = P_d A_P \quad (\mathrm{W})$$

The incident field E (V/m) is not uniform across the aperture and it becomes zero at the side walls of the antenna. Thus, the effective aperture (A_e) of the antenna is less than the physical aperture (A_p) and hence an aperture efficiency (ε_{ant}) is expressed as

$$\varepsilon_{ant} = \frac{A_e}{A_p} \left(\text{dimensionless}\right)$$

For horn and parabolic reflector antenna, aperture efficiencies are commonly in the range of 50 to 80% $(0.5 \leq \varepsilon_{ant} \leq 0.8)$.



Radiation over beam area (Ω_A) form aperture (A_e)

total power radiated (P_r) $\quad P_r = \dfrac{E_a^2}{Z_0} A_e \quad$ (W)

Assume a uniform field 'E_d' in the far field at a distance 'd' from an antenna, then the power radiated becomes,

$$P_r = \frac{E_d^2}{Z_0} d^2 \Omega_A \quad \text{(W)}$$

By equating the equations $\frac{E_d^2}{Z_0} d^2 \Omega_A = \frac{E_a^2}{Z_0} A_e$

Replace, $E_d = \frac{E_a A_e}{d\lambda}$

$$\frac{E_a^2 A_e^2}{Z_0 d^2 \lambda^2} d^2 \Omega_A = \frac{E_a^2}{Z_0} A_e$$

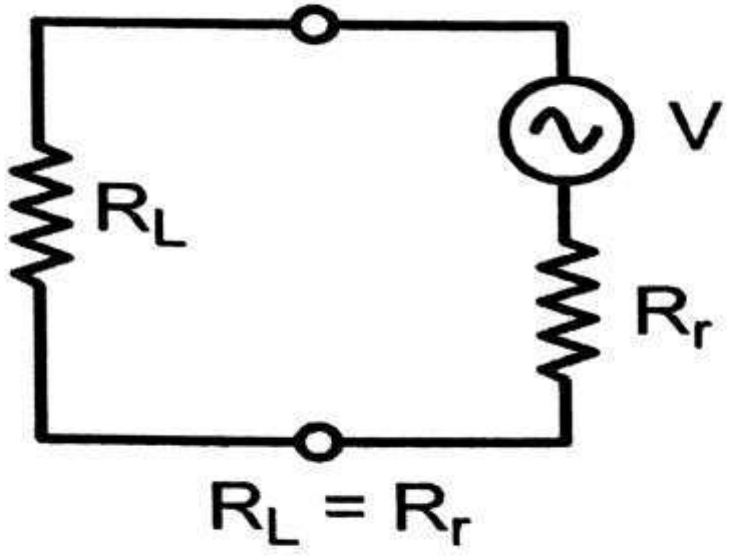
$$\lambda^2 = A_e \Omega_A \; (m^2)$$

We known that, the relation between directivity and beam area as,

$$D = \frac{4\pi}{\Omega_A}$$

Substitute equation, we get ***directivity from aperture***

$$D = 4\pi \frac{A_e}{\lambda^2} \qquad \because \Omega_A = \frac{\lambda^2}{A_e}$$



Thevenin's equivalent receiving antenna circuit

In a dipole antenna the load power is expressed as

$$P_{Load} = P_d\, A_e \quad (W)$$

Redirected power is nothing but the *power that are redirected through the aperture into the main beam.*

$$P_{rerad} = \frac{Power\ \ reradiated}{4\pi\ sr} = P_d A_r \quad (\mathrm{W})$$

where, A_r = Reradiating aperture = A_e, (m^2)

For a single dipole $\left(\lambda/2\ or\ \ shorter\right)$ antenna, the loaded power can be expressed as

$$P_{rerad} = P_{load}$$

Radiation Resistance

The radiation resistance is a fictitious resistance such that when it is connected in series with antenna and dissipates same power as the antenna actually radiates.

In general, an antenna is a radiating device, which radiates power into free space in the form of EM waves. Hence, the power dissipation from an antenna is given by

$$P_r = I^2 R_r$$

Assume that all the power dissipated in the form of EM waves, then the dissipation power can be divided by square of the current.

$$R_r = \frac{P_r}{I^2}$$

Total power loss = Radiation loss + Ohmic loss

$$P_{loss} = P_r + P_{ohm}$$

$$= I^2 R_r + I^2 R_l = I^2 (R_r + R_l)$$

$$P_{loss} = I^2 R \qquad \because R = R_r + R_l$$

where, R_l is the ohmic resistance of the antenna wire.

The value of the radiation resistance depends mainly on

(i) The configuration of antenna,

(ii) The point where radiation resistance is considered,

(iii) The location of antenna w.r.t. grounds and other objects, and

(iv) The ratio of length of diameter of the conductor used.

A dipole antenna has a radiation resistance of 72 Ω and a loss resistance of 18 Ω. Determine its efficiency.

Radiation resistance $R_r = 72\ \Omega$

Loss resistance $R_l = 18\ \Omega$

$$\text{Efficiency } (\eta) = \frac{P_{rad}}{P_{rad} + P_{loss}} = \frac{R_r}{R_r + R_l}$$

$$\eta\ \% = \frac{72}{72+18} \times 100 = 80\ \%$$

A dipole antenna has a radiation resistance of 75 Ω, a loss resistance of 20 Ω and a power gain of 12 dB. Determine the antenna efficiency and directivity.

Solution:

Given: Radiation resistance $R_r = 75\ \Omega$

Loss resistance $R_l = 20\ \Omega$

(i) Antenna efficiency (η) :

$$\eta = \frac{R_r}{R_r + R_l} = \frac{75}{75 + 20} = 0.79$$

(ii) Directivity (D):

The power gain is given by,

$$G_P\ (\text{dB}) = 12\ \text{dB}$$

$$G_P = \text{Antilog}\ (12\ \text{dB}) = 10^{(12/10)} = 15.84$$

The relation between power gain and directivity is given by

$$G_p = \eta D$$

$$D = \frac{G_P}{\eta} = \frac{15.84}{0.79} = 20.05$$

Hence the directivity in dB is given by

D (dB) = 10 $\log_{10}(20.05)$ = 13.02 dB

An antenna is operating at a wavelength of 2.5 m and has a directivity of 85. Determine its maximum effective aperture.

Given:

Wave length, $\lambda = 2.5$ m,

Directivity, $D = 85$

Effective aperture $A_e = \dfrac{\lambda^2}{4\pi} D$

$$= \frac{(2.5)^2}{4 \times 3.14} \times 85$$

$$= \frac{531.25}{12.56} = 42.3\, m^2$$

Determine the maximum effective aperture of an antenna, when the half power beam widths in the perpendicular planes intersecting in beam axis at 32° and 40°.

Given: $\theta_{HP}^{0} = 32^{0} \,\&\, \phi_{HP}^{0} = 40^{0}$

$$\text{Directivity, } D = \frac{41,253^{o}}{\theta_{HP}^{o}\phi_{HP}^{o}} = \frac{41,253}{32 \times 40} = \frac{41,253}{1280} = 32.23$$

The maximum effective aperture is given by

$$A_e = \frac{\lambda^2}{4\pi} D$$

$$= \frac{\lambda^2}{4 \times 3.14} \times 32.23 = \frac{32.23}{12.56} \lambda^2 = 2.57\,\lambda^2 \; m^2$$

Antenna Bandwidth

The bandwidth of an antenna is defined as, *"the width or range of frequency over which the antenna maintains certain required characteristics to the specified value like gain, pattern, polarization and impedance"*.

$$\text{Bandwidth (BW)} = \Delta\omega = \text{Upper limit} - \text{Lower limit} = \omega_U - \omega_L = \frac{\omega_r}{Q}$$

$$\Delta f = f_U - f_L = \frac{f_r}{Q} \qquad \because \Delta\omega = 2\pi\Delta f, \quad \omega_r = 2\pi f_r$$

f_r - centre frequency *or* design frequency *or* resonant frequency. For lower 'Q' antennas, the antenna bandwidth is very high and vice versa and for an antenna it is expressed as,

$$\boxed{Q = 2\pi \times \frac{\text{Total energy stored by Antenna}}{\text{Energy radiated per cycle}}}$$

Antenna Beam width

Basically antenna beam width is the ***measure of the directivity of an antenna*** and is defined as, "*the angular separation, that is, angular width in degrees between two identical points on the opposite side of the main radiation pattern*".

In an antenna pattern, there are a number of beam-widths possible, but two of the most widely used beam-widths are:

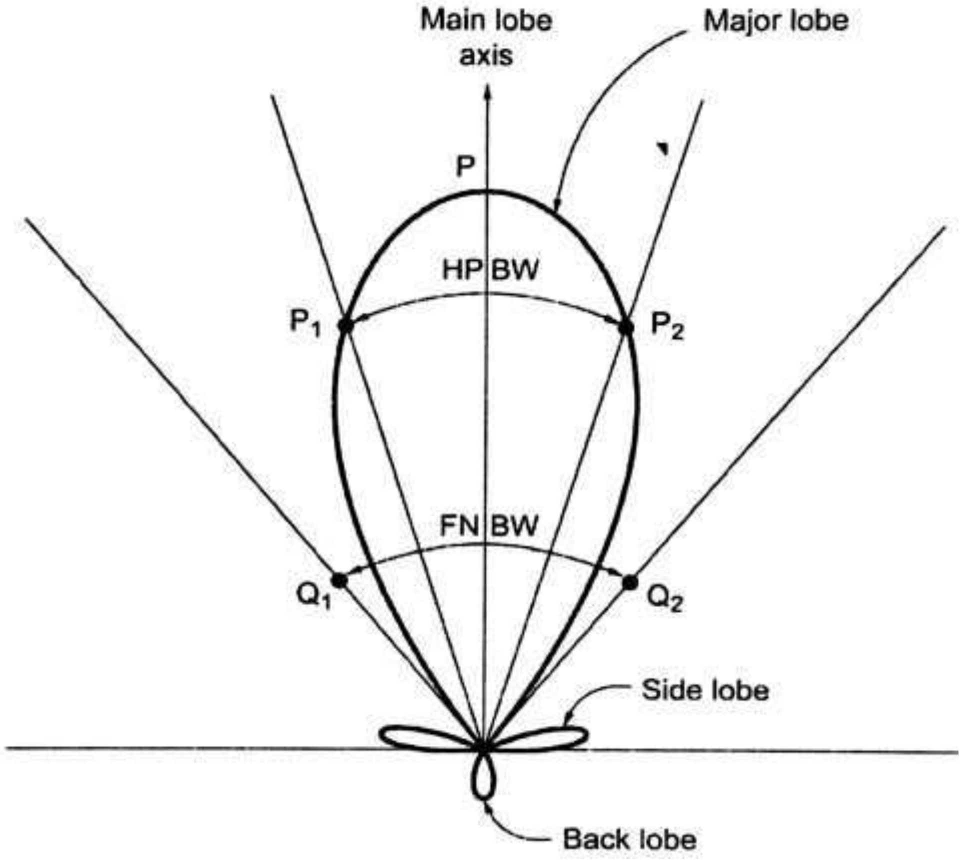
(i) Half – Power Beam Width (HPBW), and

(ii) First – Null Beam Width (FNBW).

HALF – POWER BEAM WIDTH (HPBW)

HPBW is an angular width in degrees, measured on the major lobe radiation pattern between points where the radiated power has fallen to half of its maximum value, which is called ***half power points***.

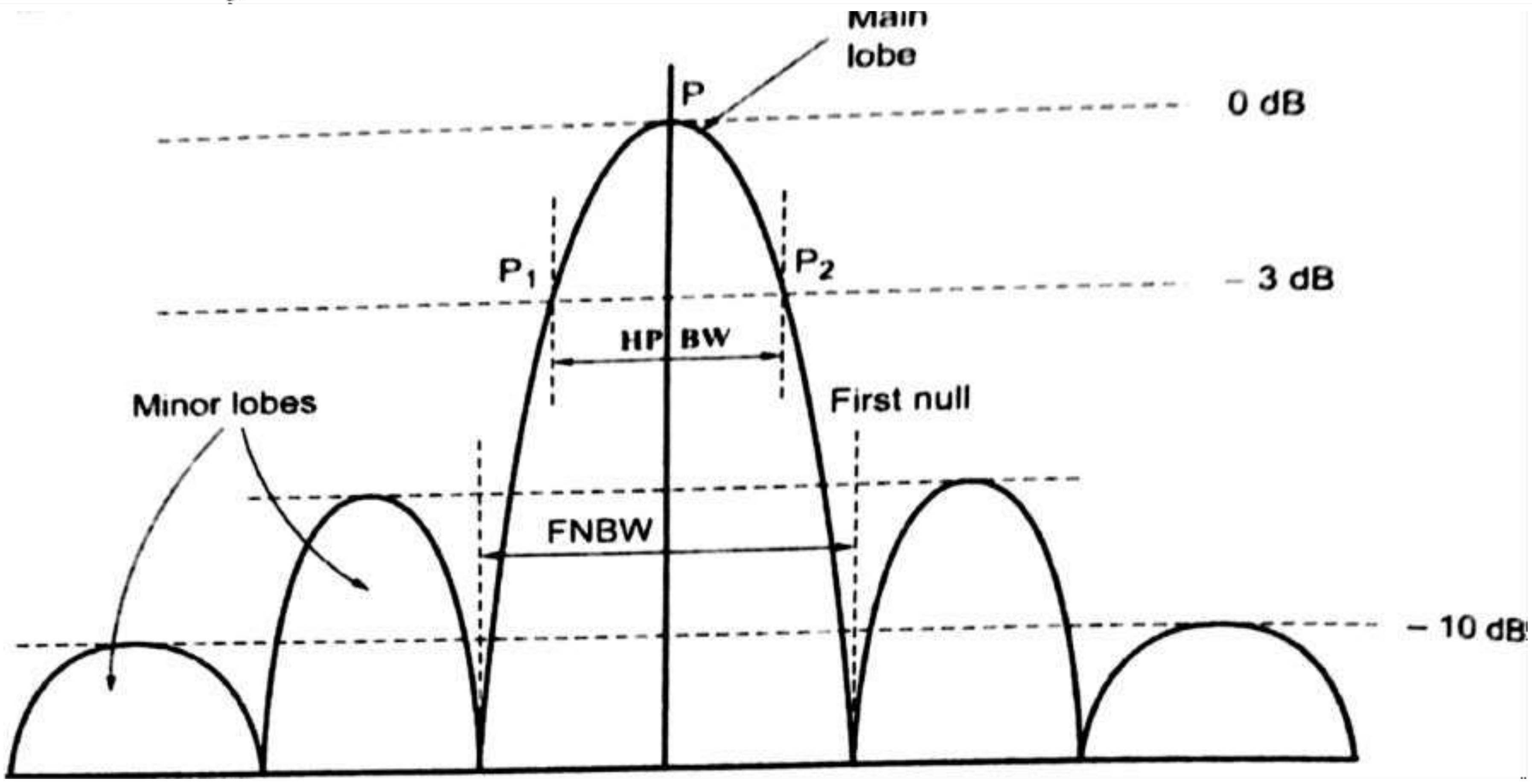
HPBW is also known as ***"3-dB beam width"*** because at half power points, the power is 3-dB down the maximum power value of the major lobe.



From the Fig.2.1, it is clear that the power is maximum at point P in main lobe, while it is half at half power points P_1 and P_2, where the power is ***3 dB down*** from the maximum power.

FIRST – NULL BEAM WIDTH (FNBW)

FNBW is defined as, *"the angular width(in degrees) between first nulls(zero power) or first side lobes, which has a beam width of* ***10 dB down*** *from the power maximum of the main lobe"*



Bandwidths on logarithmic scale (decibels)

FNBW is also known as ***10-dB beam width*** and is usually used to approximate the HPBW as,

$$\boxed{HPBW \approx \frac{FNBW}{2}}$$

The directivity (D) of the antenna is related with beam solid angle (Ω_A) or beam area (B) and it is given as,

$$\boxed{D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{B}}$$

where,

$$B \approx (HPBW)\ in\ horizontal\ \ plane \times (HPBW)\ in\ vertical\ \ plane\ (Sq.radians)$$

$$\approx (HPBW)\ in\ E\ plane \times (HPBW)\ in\ E\ plane$$

$$B \approx \theta_E \times \theta_H \ \ Sq.radians$$

H

$$\boxed{D = \frac{4\pi}{\theta_E \times \theta_H} \quad \text{if } \theta_E \text{ and } \theta_H \text{ are in radians}}$$

We can convert the angles expressed in radians into degrees by using the following relation,

$$1\ rad = \frac{180^0}{\pi} = 57.3^0$$

$$\boxed{D = \frac{4\pi \times (57.3)^2}{\theta_E^0 \times \theta_H^0} = \frac{41,257}{\theta_E^0 \times \theta_H^0} \quad \text{Sq.degrees}}$$

The error increases, if the beam widths of E and H planes increases. The factors affecting the beam width of an antenna are:

(i) The shape of radiation pattern,

(ii) The wavelength, and

(iii) The dimensions.

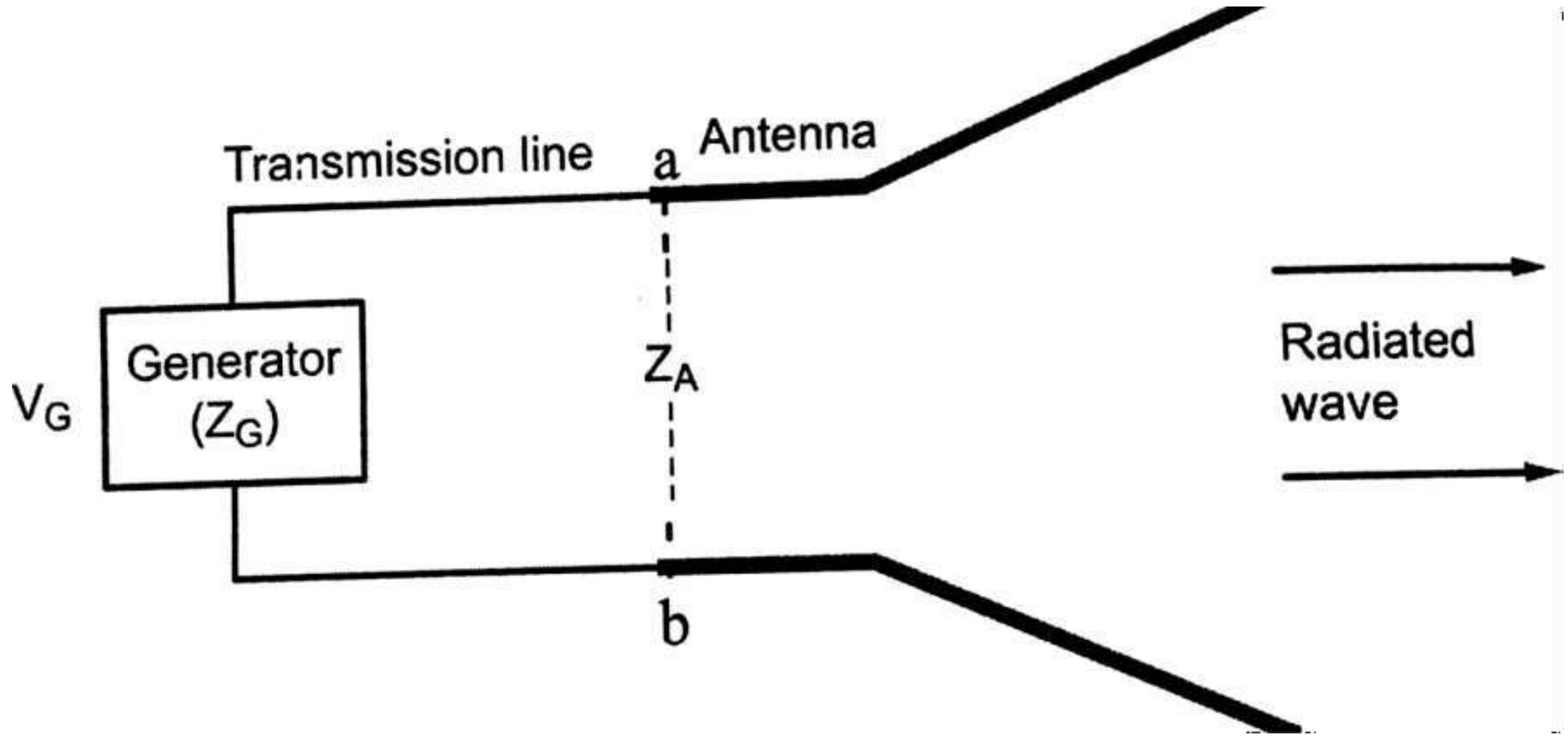
Input Impedance

Antenna input impedance is an impedance at the point where the transmission line carrying R.F power from the transmitter is connected to the antenna. It is also called ***feed point impedance or driving point impedance or terminal impedance.***

Input impedance of an antenna is simply defined as *"the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals or the ratio of the appropriate components of the electric to magnetic fields at a point".*

Antenna input impedance is an important one because it only decides the maximum available power from transmitter to the antenna or to extract maximum amount of received energy from the antenna.

Transmission mode:



Antenna in transmitting mode

In Figure 2.3, the antenna terminals are designated as a-b and there is no load attached, defines the antenna impedance as,

$$Z_A = R_A + jX_A$$

where,

Z_A – Antenna impedance at terminals a-b (ohms)

R_A – Antenna resistance at terminals a-b (ohms), and

X_A – Antenna reactance at terminals a-b (ohms).

The resistive part of the equation(1) consists of two components, that is,

$$R_A = R_r + R_l$$

where,

R_r – Radiation resistance of the antenna, and

R_l – Loss resistance of the antenna.

If we assume that the antenna is attached to a generator with ***internal impedance***

$$Z_G = R_G + jX_G$$

where,

R_G – Resistance of generator impedance (ohms), and

X_G – Reactance of generator impedance (ohms)

If the antenna input impedance Z_A is complex conjugate of the source impedance Z_G. $(Z_A^{*} \approx Z_G)$, then the condition for maximum power transfer can be given as

$$R_A = R_G \ and \ X_A = -X_G$$

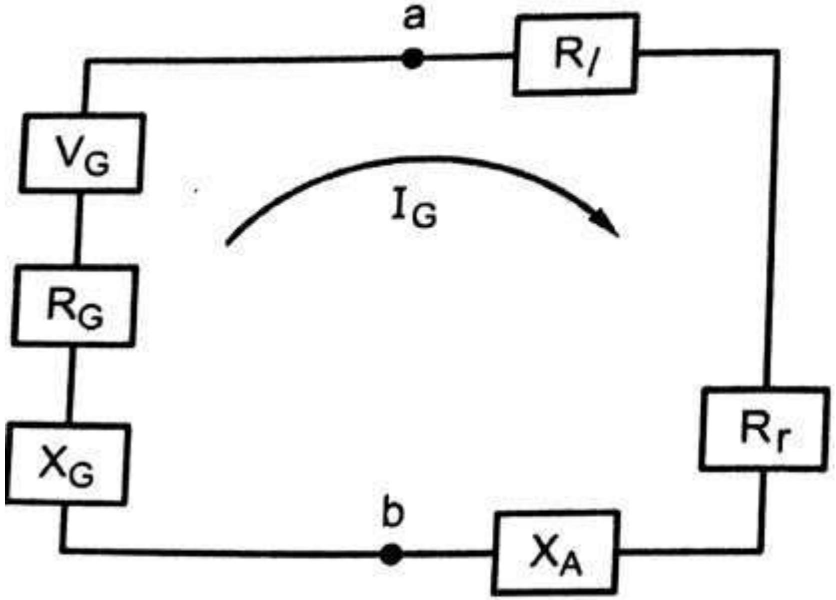
Under matching condition for maximum power transfer, the antenna current is given by,

$$I_G = \frac{V_G}{Z_T} = \frac{V_G}{Z_G + Z_A}$$

where,

Total impedance $= Z_T = Z_G + Z_A$, and

V_G = Peak generator voltage.



Thevenin equivalent for transmitting antenna circuit

For maximum power transfer condition, $R_G = R_A$

$$I_G = \frac{V_G}{R_A + R_A} = \frac{V_G}{2R_A}$$

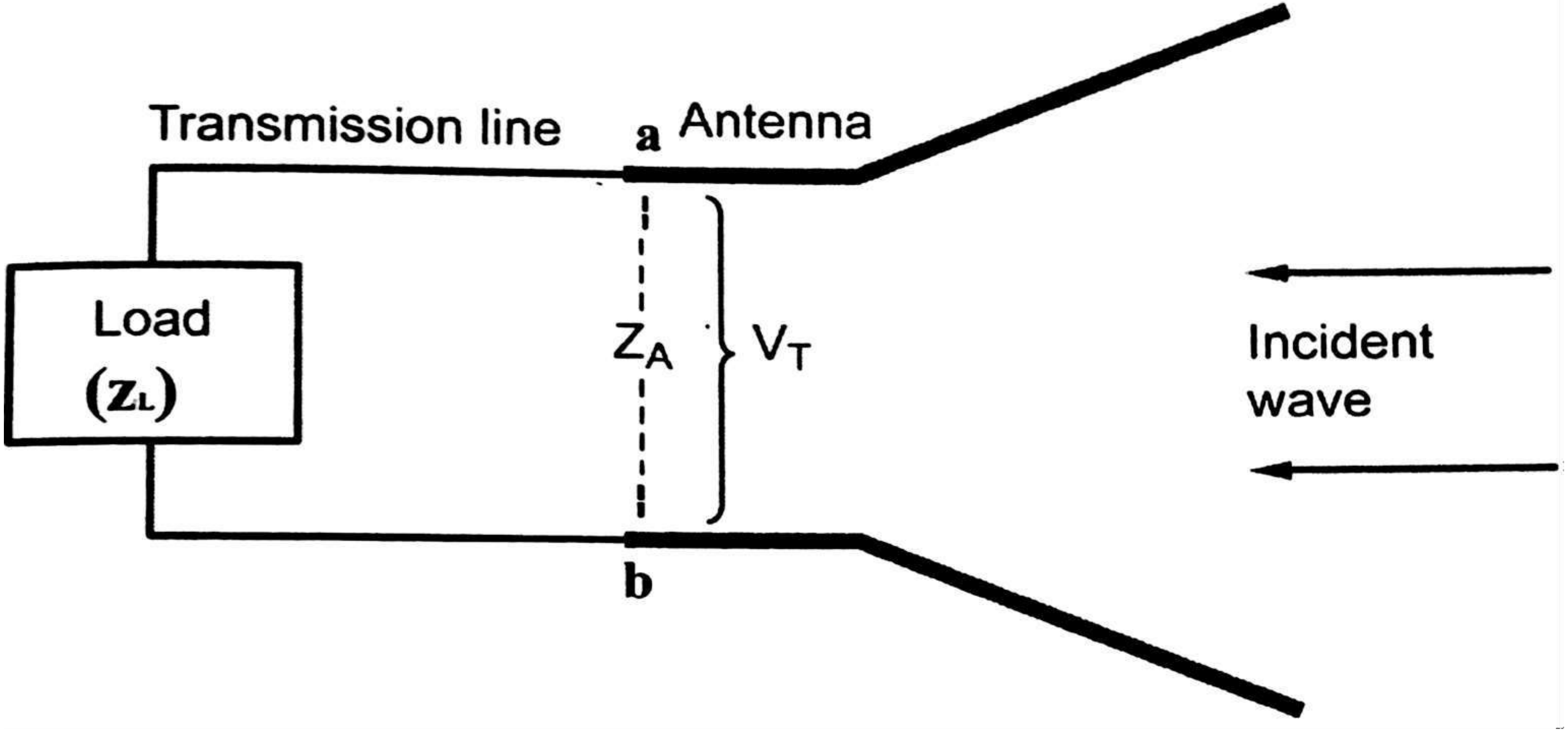
The real power supplied by the generator can be expressed as,

$$P_G = \frac{1}{2}\mathrm{Re}\{V_G I_G^*\}$$

$$= \frac{1}{2}\left[|V_G| \times \frac{|V_G|}{2R_A}\right]$$

$$\boxed{P_G = \frac{|V_G|^2}{4}\left[\frac{1}{R_A}\right] \; Watts = \frac{|V_G|^2}{4}\left[\frac{1}{R_r + R_l}\right] \; Watts}$$

Receiving mode



Antenna in receiving mode

In the receiving mode, the maximum power delivered to the antenna occurs under conjugate matching, when the total resistance is expressed as

$$R_T = R_A + R_L$$

where, R_A – Antenna resistance (ohm), and

R_L – Load resistance of the antenna (ohm)

The load impedance (Z_L) can be expressed as,

$$Z_L = R_L + jX_L$$

where,

X_L – Reactance of load impedance (ohms)

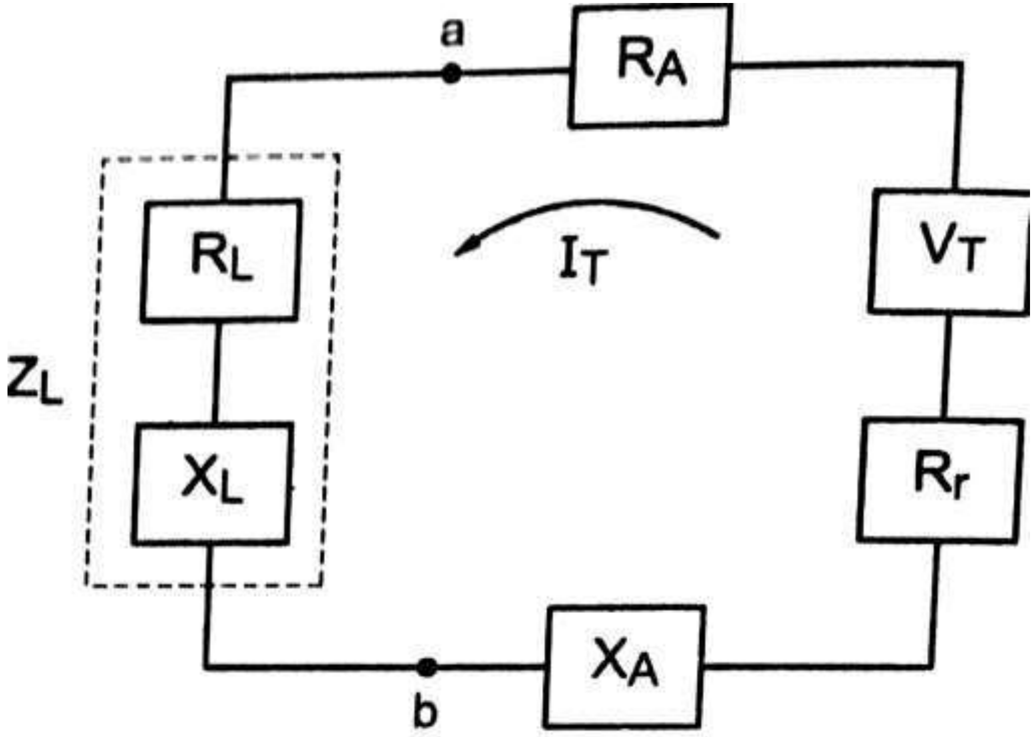
The conditions for maximum power transfer from incident power to antenna are:

(i) The antenna reactance is equal to the total reactance of the receiving mode.

$$X_A = -X_T$$

(ii) The antenna resistance is equivalent to load resistance.

$$R_A = R_L$$



The total antenna current for maximum power transfer is given by

$$I_T = \frac{V_T}{Z_T} = \frac{V_T}{Z_L + Z_A}$$

$$I_T = \frac{V_T}{R_L + R_A} = \frac{V_T}{R_A + R_A} = \frac{V_T}{2R_A} \qquad \because R_L = R_A$$

The induced (collected or captured) power in receiving antenna under complex matching is given as,

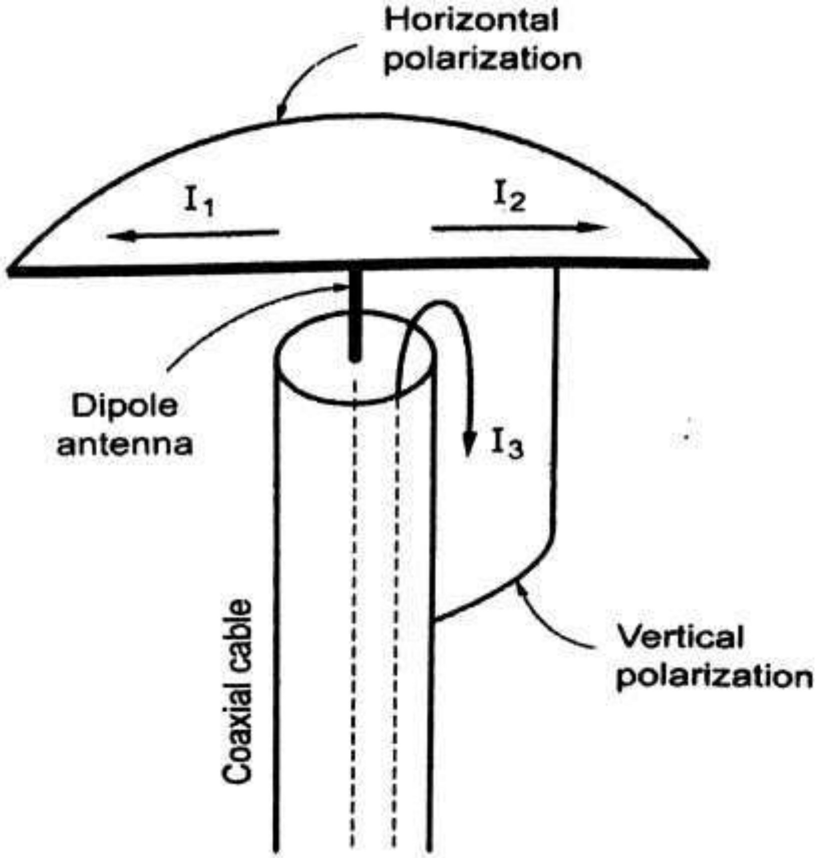
$$P_C = \frac{1}{2}\mathrm{Re}\{V_T I_T^{*}\}$$

Substitute equation $= \frac{1}{2}\left\{|V_T| \times \frac{|V_T|}{2R_A}\right\} = \frac{|V_T|^2}{4}\left[\frac{1}{R_A}\right]$

$$\underline{P_T = \frac{|V_T|^2}{4}\left[\frac{1}{R_r + R_l}\right] Watts}$$

Baluns

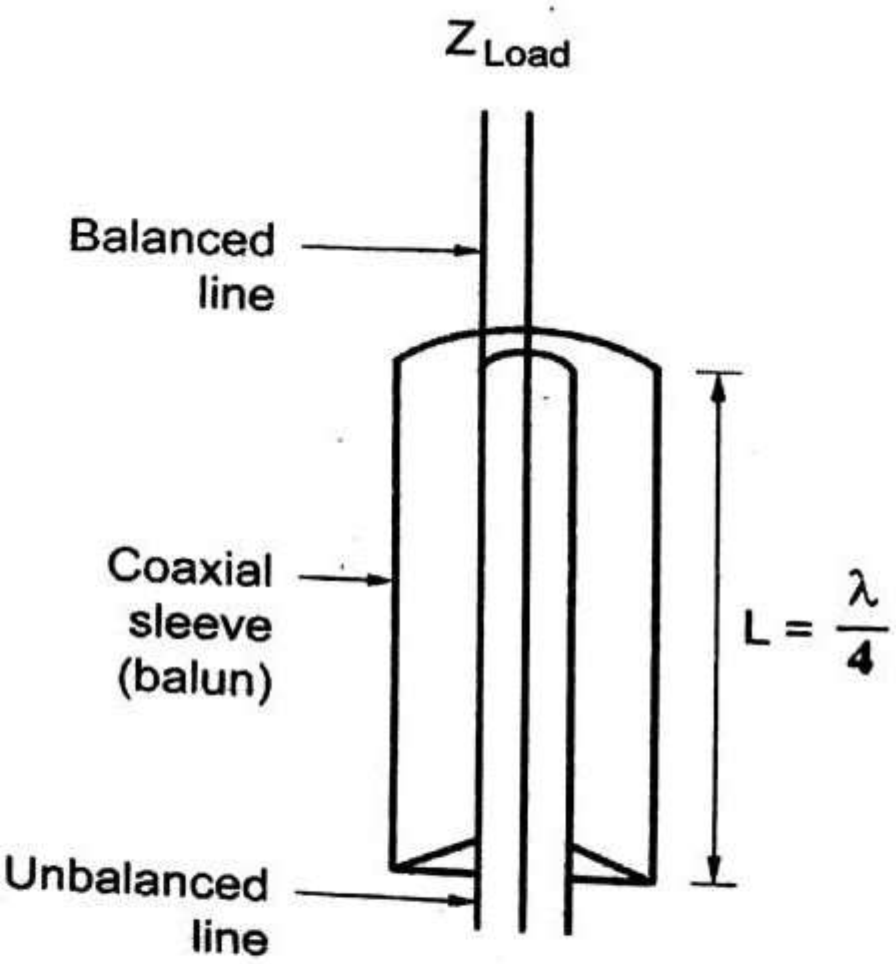
*The term **balun** is an abbreviation of the words **balance and unbalance**. It is a device used to couples (connects) a balanced two-conductor line to an unbalanced co-axial line, since baluns add complexity and expense to a system.*



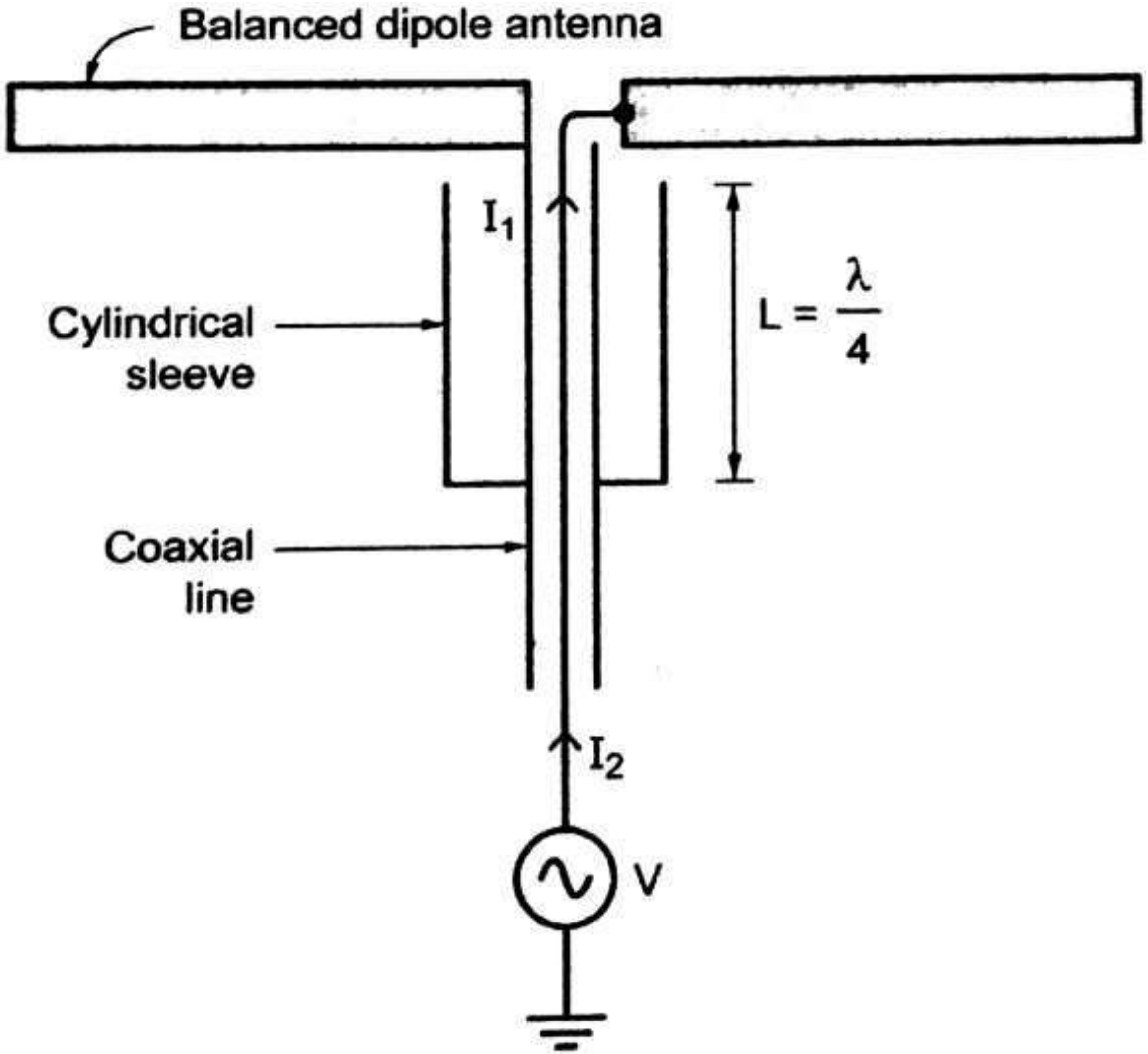
A horizontal dipole fed directly from a co-axial line

Types of Balun

Type I

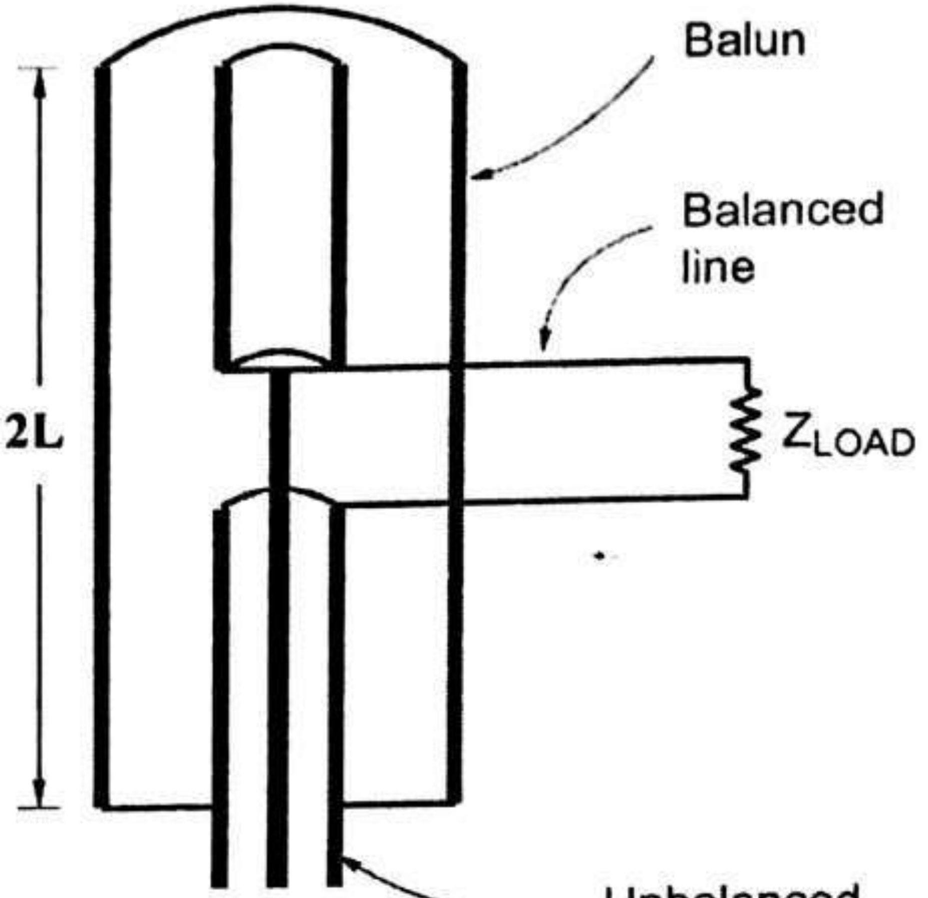


The quarter- wave choke balun (Type-I)

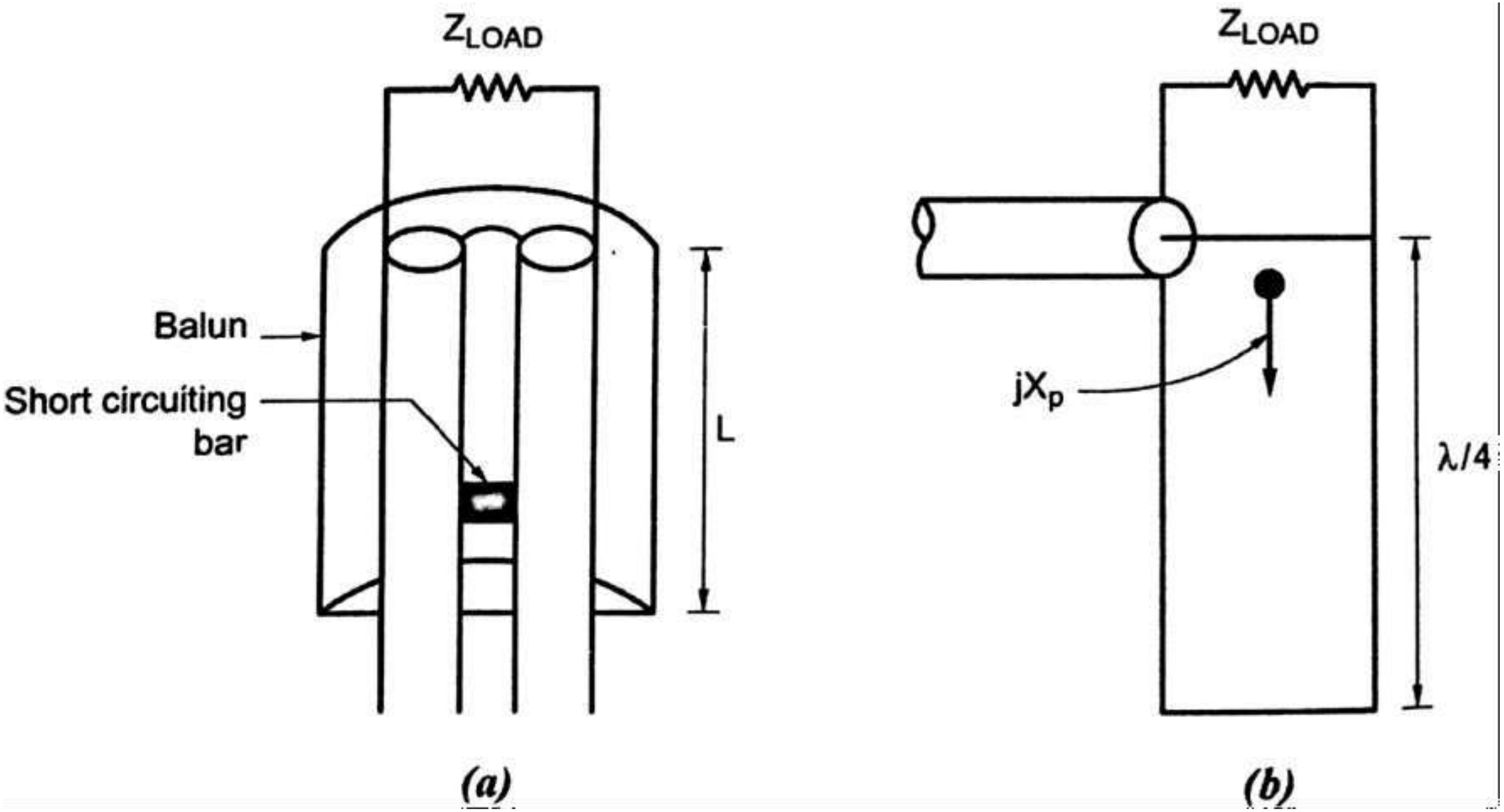


Type- I balun with dipole antenna

Type –II has two Type –I in series providing more bandwidth



Type-II balun

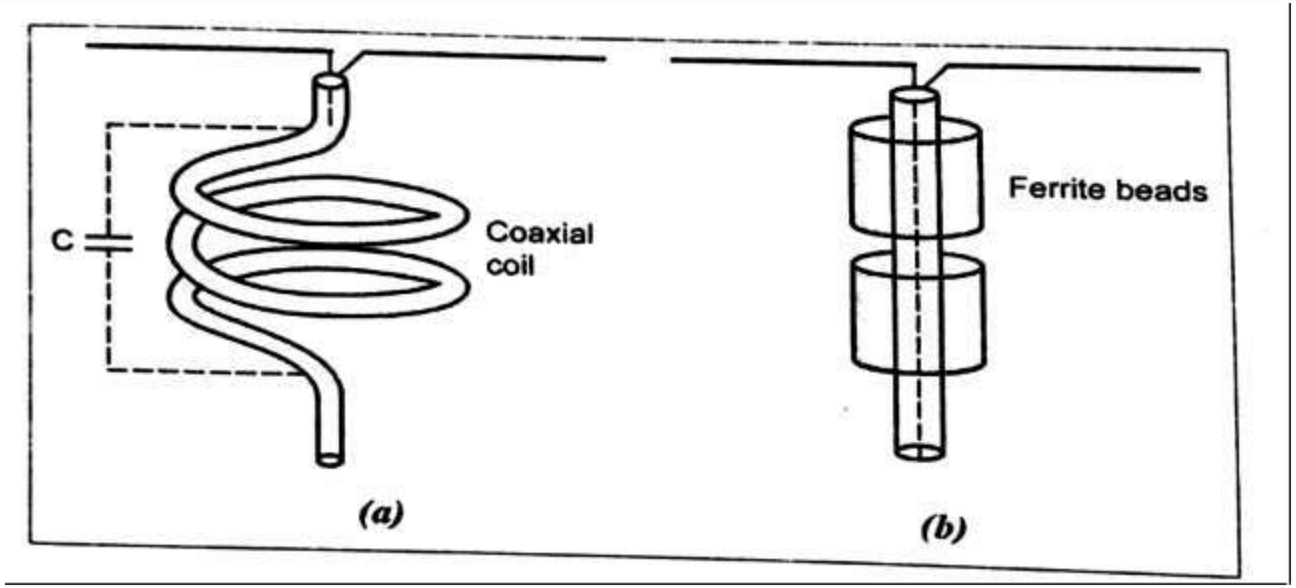


(a) Type –III balun (b) Equivalent circuit

Types of Choke Balun

Co axial coil balun: The coaxial cable wound into a coil producing a high impedance on the outside of the coil. The coil and its capacitance C form a parallel LC circuit that should resonate at the operating frequency.

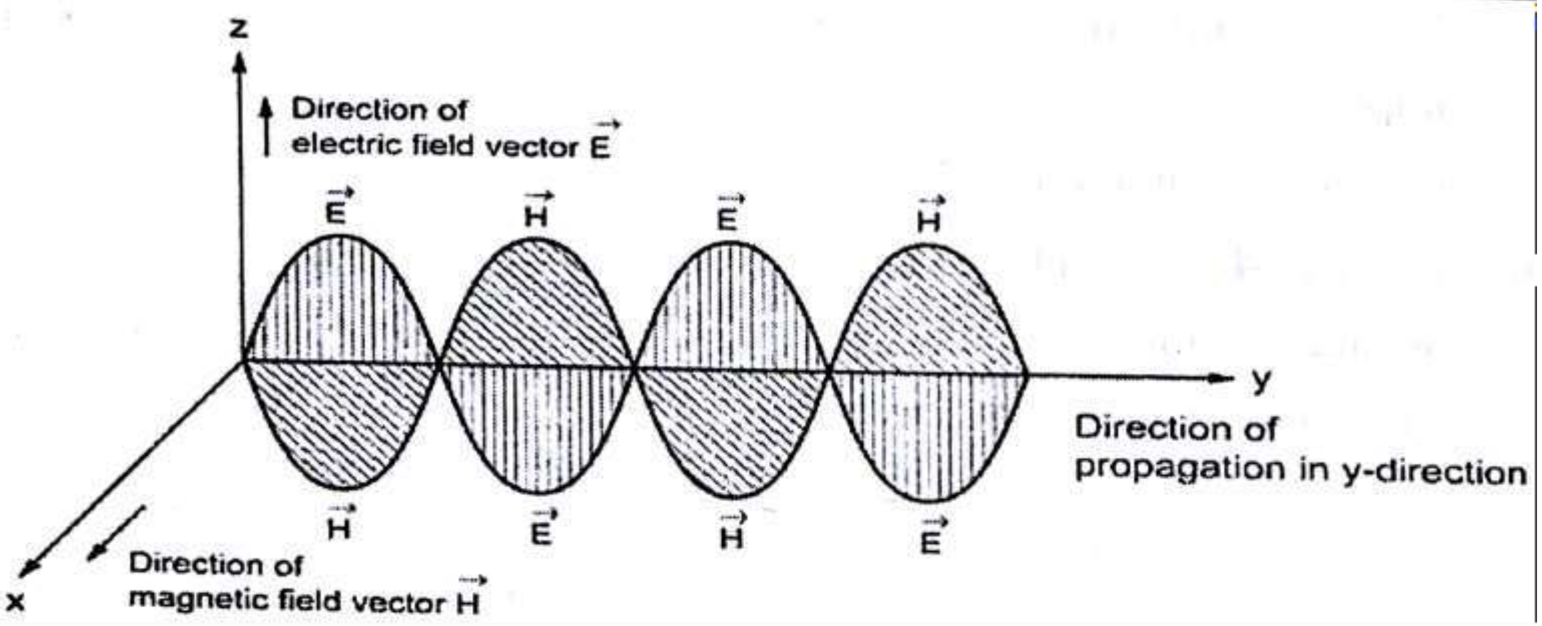
Ferrite –bead choke balun: The cylindrical ferrite beads placed on the outside of the co-axial cable. With a good quality ferrite beads large bandwidths may be obtained.



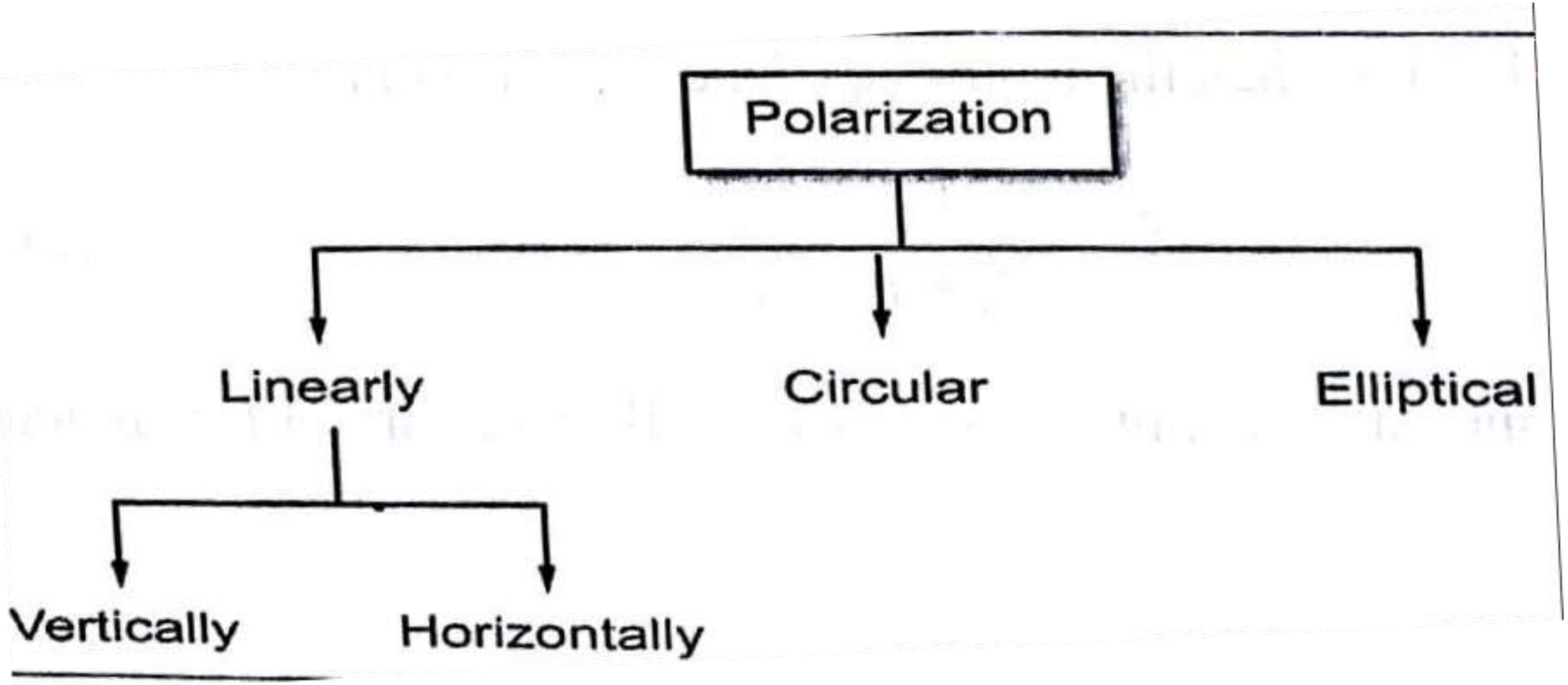
Polarization

The antenna polarization in a given direction refers to *the polarization of an electromagnetic wave radiated or transmitted by the antenna.*

Polarization of a radiated wave is defined as "*the property of an electromagnetic wave describing the time-varying direction and relative magnitude of the electric-field vector*".



Types of Polarization



Polarization Mismatch

In general, *the polarization of the receiving antenna will not be the same as the polarization of the incoming (incident) wave. This is commonly referred as "**polarization mismatch**" and the amount of power extracted by the antenna from the incoming signal will not be maximum because of the* **polarization loss.**

Assuming that the electric field of the incoming wave can be expressed as

$$E_i = \hat{\rho}_w E_i \qquad \qquad \text{........(1)}$$

where, $\hat{\rho}_w$ is the unit vector of the incident wave.

The polarization of electric field of the receiving antenna can be expressed as

$$E_a = \hat{\rho}_a E_a \qquad \qquad \text{........(2)}$$

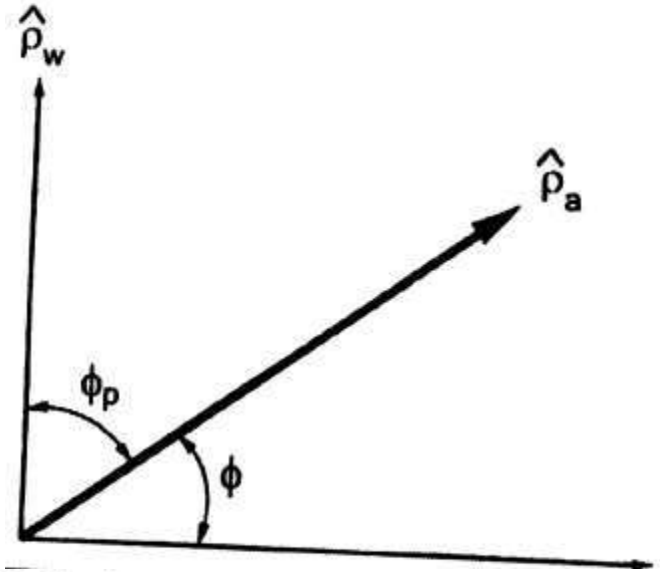
where, $\hat{\rho}_a$ is the unit vector of an antenna.

Polarization Loss Factor (PLF) *or* Polarization matching factor

PLF represents the polarization loss, based on the polarization of the transmitting antenna in its mode and expressed as

$$PLF(F) = \left|\hat{\rho}_w \, \hat{\rho}_a\right|^2 = \left|\cos\phi_p\right|^2 \qquad \text{.......(3)}$$

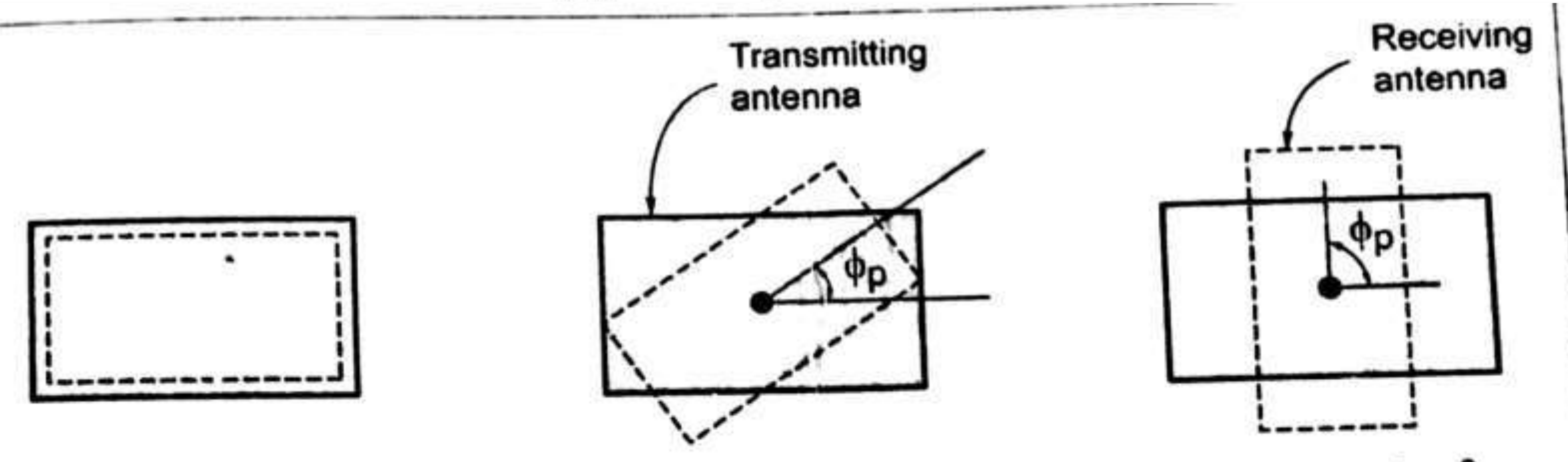
where, ϕ_p is the angle between the unit vectors of wave and antenna.



Polarization loss factor (PLF)

If the matching angle $\phi_p = 0^0$ and F = 1 , the antenna is matched to the wave, that is, when the polarization state of the wave is same as for the antenna then the response is maximized.

For a complete mismatch the matching angle $\phi_p = 90°$ and F = 0, when the antenna is mismatched to the wave then the response is zero which states that the wave and antenna have an opposite polarization.



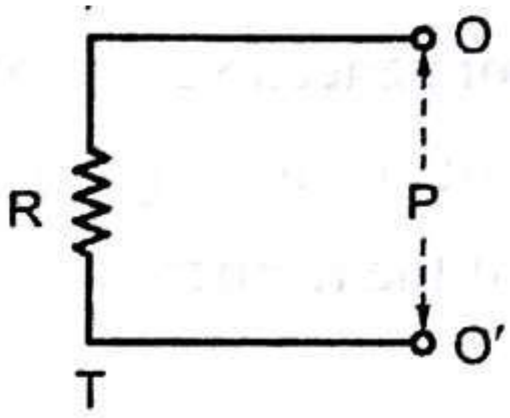
$PLF = \lvert \hat{\rho}_w \cdot \hat{\rho}_a \rvert^2 = 1$	$PLF = \lvert \hat{\rho}_w \cdot \hat{\rho}_a \rvert^2 = \cos^2 \phi_p$	$PLF = \lvert \hat{\rho}_w \cdot \hat{\rho}_a \rvert^2 = 0$
(aligned)	(rotated)	(orthogonal)

Antenna noise Temperature:

Every object with a physical temperature above the absolute zero value (i.e. above $0° K$ *or* $-273° C$) radiates heat energy. The antenna temperature has no relation with the physical temperature of the antenna.

The **antenna temperature** for a lossless antenna is defined as, "*the temperature of a far field region of space and near surroundings which are coupled to the antenna through radiation resistance*".

Both the antenna temperature (T_A) and radiation Resistance (R_r) are single valued scalar quantities.



Consider a simple resistor R at temperature T as shown in the Fig.3.1. According to the ***Nyquist relation***, the noise power per unit bandwidth available across the terminals (OO^1) is given by,

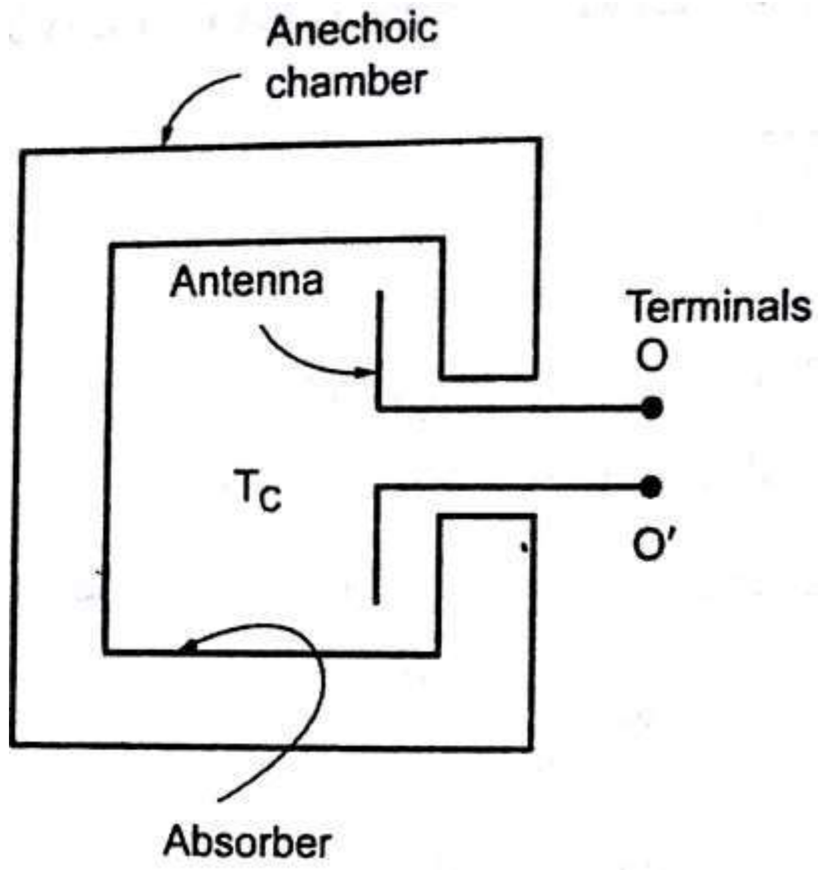
$$P = kT \text{ Watts/ Hz} \qquad \ldots\ldots (1)$$

where, P - Noise power per unit band-width in Watts/Hz

k - Boltzman's constant = 1.38×10^{-23} J/K, and

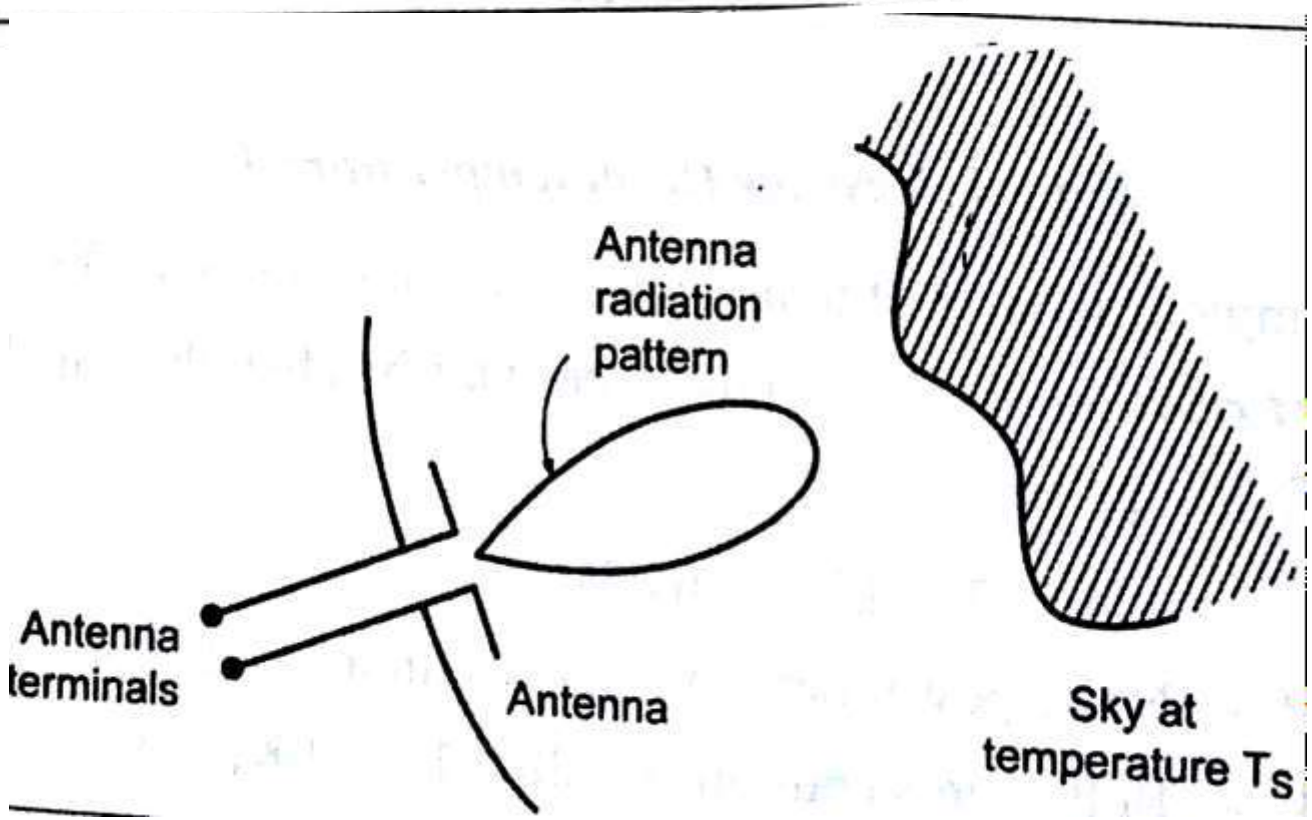
T - Absolute temperature of resistor in °K.

Now, if the resistor R is replaced by a lossless antenna of radiation resistance R_r in an anechoic (no echo) chamber at the temperature T_c , then under the condition $T = T_c$, the noise per unit bandwidth, available at the terminals is remain unchanged.



Antenna at anechoic chamber at temperature T_c

Finally, we remove antenna from anechoic chamber and the antenna pointed at a sky of temperature T_s as shown in the Fig.3.3, then the noise power per unit bandwidth remains unchanged if the temperatures T_c and T_s are same ($T=T_c=T_s$).



For a practical antenna used for the remote sensing, the noise per unit bandwidth is given as,

$$P = kT_A \quad W/Hz \qquad \text{.....(2)}$$

Here, T_A is the ***antenna temperature*** or ***antenna noise temperature*** which is equal to the temperature of the radiation resistance of the antenna.

If the power per unit bandwidth is ***independent of frequency***, then the total power is obtained by multiplying with the bandwidth (B) in Hz as follows

$$P = kT_A B \ \ Watts \qquad \text{......(3)}$$

Let the antenna has an effective aperture A_e and the power density per unit bandwidth is produced in the direction of radiation. This is called ***flux density*** and is denoted by S. The power received from the source is given by

$$\boxed{P = S A_e B \text{ Watts}} \qquad \text{......(4)}$$

where, S – Power density per unit bandwidth in $W/m^2\ Hz$, and

A_e – Effective-aperture in m^2, and

By equating equations (3) and (4), we get

$$kT_A B = S A_e B$$

$$\boxed{S = \frac{kT_A}{A_e} \text{ W/m}^2\text{ Hz}} \qquad \text{......(5)}$$

Then, the antenna temperature due to the source in degree K can be expressed as

$$\boxed{\therefore T_A = \frac{SA_e}{k} \, ^\circ\text{K}} \qquad \text{......(6)}$$

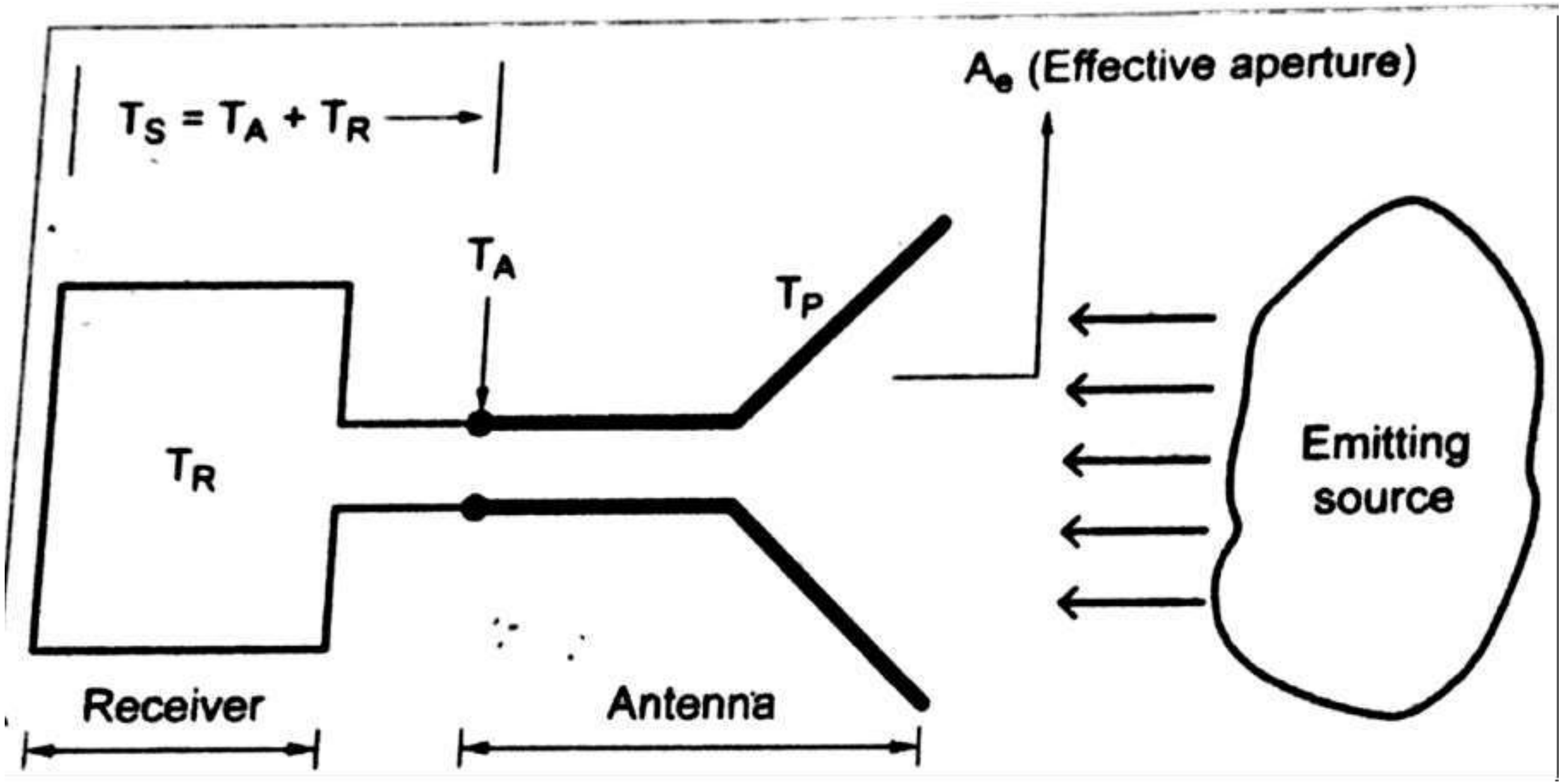
If the size of the source is small compared to the antenna beam solid angle 'Ω_A', then the antenna noise temperature is expressed as

$$\boxed{T_A = \frac{\Omega_S}{\Omega_A} T_S} \qquad \ldots\ldots (7)$$

where, Ω_A - Antenna beam solid angle in Sr

Ω_S - Source solid angle in Sr, and

T_S - Source temperature in °K



Noise temperatures at receiver terminals

If the receiver itself has a certain noise temperature T_R due to thermal noise in the receiver components, then the system noise power at the receiver terminals is given as

$$P_s = k(T_A + T_R)B = kT_s B \text{ Watts} \quad \ldots\ldots (8)$$

where, T_R - Receiver noise temperature at receiver terminals

T_A - Antenna noise temperature at receiver terminals

P_S - System noise power at receiver terminals, and

$T_S = T_A + T_R$ = Effective system noise temperature at receiver terminals

Equivalent Noise Temperature of Antenna(Te)

The noise introduced by a network may also be expressed as effective noise temperature and it is defined as, *the fictional temperature at the input of the network which would account for the noise ΔN at the output. ΔN is the additional noise introduced by the network itself.*

The noise figure (F) related with effective noise temperature is

$$F = 1 + \frac{T_e}{T_0}$$

$$F - 1 = \frac{T_e}{T_0}$$

$$\boxed{T_e = T_0 (F - 1)}$$

where, T_0 = Room temperature $(273^0 + 17^0) = 290°K$

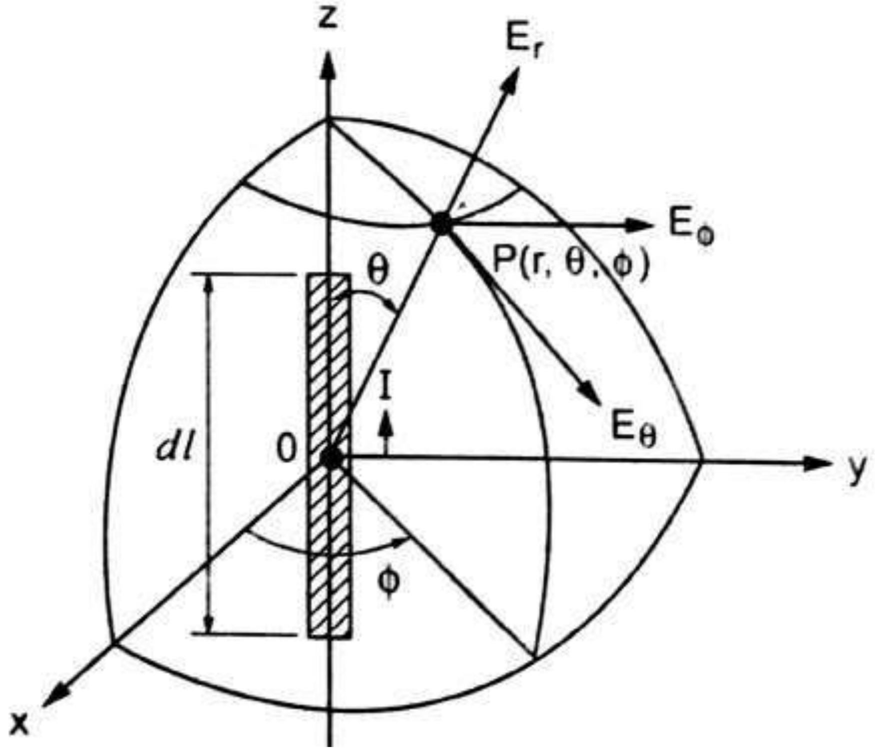
The noise figure 'F' in decibel is expressed as,

$$\boxed{F \text{ in dB} = 10 \log_{10} F}$$

Radiation from an oscillating dipole

Hertzian Dipole:

Hertzian dipole is an infinitesimal current element I dl which does not exist in real life. But the theory developed for this can be extended to practical antennas of specified current distribution. Thus the Hertzian dipole is the most basic antenna



Current element at the origin of spherical co-ordinate

Magnetic Field Components:

The vector potential 'A' is acting along 'Z' direction and so it will have only Z component. i.e., Az retarded in time by (r/c) seconds

Let us assume that the current element be excited by the

$$\text{Current } I = I_m \cos \omega t$$

The general expression for magnetic vector potential is given by

$$[A] = \frac{\mu}{4\pi} \int_v \frac{J\left(t - \frac{r}{c}\right)}{r} dv$$

$$= \frac{\mu}{4\pi} \int_V \frac{J\left(t - \frac{r}{c}\right)}{r} ds \; dl \qquad (\because dv = ds\, dl)$$

$$\text{But} \quad J\,ds = I$$

$$\therefore [A] = \frac{\mu}{4\pi} \int \frac{I\left(t - \frac{r}{c}\right)}{r}\, dl$$

Substituting the equation

$$\therefore [A_z] = \frac{\mu}{4\pi} \int \frac{I_m \cos \omega \left(t - \frac{r}{c}\right)}{r}\, dl$$

where A_z = Magnetic vector potential acting along 'z' direction

$$A_z = \frac{\mu}{4\pi r} I_m\, dl \cos \omega \left(t - \frac{r}{c}\right)$$

$$\left[\because \int dl = l = dl\right]$$

Now the magnetic field intensity 'H' is obtained from the magnetic vector potential by using the relation.

$$B = \nabla \times A = \mu H$$

In spherical co-ordinate system, $\nabla \times A$ given by

$$\nabla \times A = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \vec{a}_r & r\vec{a}_\theta & r\sin\theta\,\vec{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\,A_\phi \end{vmatrix}$$

$$(\nabla \times A)_r = \mu H_r \cancel{\vec{a}_r} = \frac{1}{r^2 \sin\theta} \cancel{\vec{a}_r} \left[\frac{\partial}{\partial \theta}(r\sin\theta\, A_\phi) - \frac{\partial}{\partial \phi}(r\, A_\theta) \right]$$

$$\mu H_r = \frac{\cancel{r}}{\cancel{r^2} \sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial}{\partial\phi} A_\theta\right]$$

$$\mu H_r = \frac{1}{r \sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial}{\partial\phi} A_\theta\right]$$

Due to spherical symmetry in x-y plane

$$\frac{\partial}{\partial\phi} = 0 \quad \text{and} \quad A_\phi = 0.$$

$$\therefore \quad \mu H_r = 0 \Rightarrow \boxed{H_r = 0}$$

$$(\nabla \times A)_\theta = \mu H_\theta \cancel{\vec{a}_\theta} = \frac{1}{r^2 \sin\theta} \cancel{\vec{a}_\theta}\, r\left[\frac{\partial}{\partial r}(r A_\phi \sin\theta) - \frac{\partial}{\partial\phi}(A_r)\right]$$

$$= \frac{1}{r \sin\theta}\left[\frac{\partial}{\partial r}(r A_\phi \sin\theta) - \frac{\partial}{\partial\phi}(A_r)\right] \quad \therefore \mu H_\theta = 0 \Rightarrow \boxed{H_\theta = 0}$$

Similarly for H_ϕ,

$$(\nabla \times A)_\phi = \mu H_\phi \cancel{\vec{a}_\phi} = \frac{1}{r^2 \sin\theta}\left[\cancel{\vec{a}_\phi}\, r \sin\theta\left[\frac{\partial}{\partial r} r A_\theta - \frac{\partial}{\partial \theta} A_r\right]\right]$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial}{\partial \theta}(A_r)\right]$$

Now we have to find A_r and A_θ

$$A_r = A \cdot \vec{a}_r$$

$$= (A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z) \cdot \vec{a}_r$$

	a_r	a_θ	a_φ
a_x	Sin θ cos φ	Cos θ cos φ	-sin φ
a_y	Sin θ sin φ	Cos θ cos φ	-cos φ
a_z	Cos θ	- Sin θ	0

$$A_x = A_y = 0$$

(since vector potential is acting alone in the 'z' direction.)

$$\therefore \quad A_r = A_z(\vec{a}_z \cdot \vec{a}_r)$$

$$\boxed{A_r = A_z \cos\theta}$$

Similarly,
$$A_\theta = (A_x \vec{a_x} + A_y \vec{a_y} + A_z \vec{a_z}) \cdot \vec{a_\theta}$$
$$A_\theta = A_z \vec{a_z} \cdot \vec{a_\theta}$$
$$= A_z(-\sin\theta) = -A_z \sin\theta$$
$$\boxed{A_\theta = -A_z \sin\theta}$$

Substituting equations
$$\mu H_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r} r(-A_z \sin\theta) - \frac{\partial}{\partial\theta}(A_z \cos\theta)\right]$$

Substituting for A_z from equations
$$\mu H_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(-r\frac{\mu}{4\pi r} I_m dl \cos\omega\left(t - \frac{r}{c}\right)\sin\theta\right) - \frac{\partial}{\partial\theta}\left(\frac{\mu}{4\pi r} I_m dl \cos\omega\left(t - \frac{r}{c}\right)\cos\theta\right)\right]$$

$$= \frac{1}{r}\left[\frac{-\mu}{4\pi}\, I_m\, dl\, \sin\theta\,(-\sin\omega\left(t-\frac{r}{c}\right)(-\frac{\omega}{c}) - \frac{\mu\, I_m\, dl}{4\pi r}\cos\omega(t-\frac{r}{c})(-\sin\theta)\right]$$

$$\mu\, H_\phi = \frac{1}{r}\left\{\frac{\mu\, I_m\, dl\, \sin\theta}{4\pi}\left(-\sin\omega\, t_1 \cdot \frac{\omega}{c} + \frac{\cos\omega\, t_1}{r}\right)\right\}$$

$$H_\phi = \frac{1}{\mu r}\,\frac{\mu\, I_m\, dl\, \sin\theta}{4\pi}\left(-\sin\omega\, t_1 \cdot \frac{\omega}{c} + \frac{\cos\omega\, t_1}{r}\right)$$

$$\boxed{H_\phi = \frac{I_m\, dl\, \sin\theta}{4\pi}\left(-\frac{\omega\sin\omega\, t_1}{r\,c} + \frac{\cos\omega\, t_1}{r^2}\right)} \quad \text{where } t_1 = t - \frac{r}{c}$$

Electric field components

Now let us calculate the electric field components E_r, E_θ, E_ϕ through Maxwell's equations.

From Maxwell's equations, we know that

$$(\nabla \times H) = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} \quad \text{since } D = \varepsilon E.$$

$$(\nabla \times H) = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \vec{a}_r & r\,\vec{a}_\theta & r\sin\theta\,\vec{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & rH_\theta & r\sin\theta\,H_\phi \end{vmatrix}$$

Writing equation in its component form, $\varepsilon \frac{\partial E_r}{\partial t} = (\nabla \times H)_r$

$$= \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial\theta}(r \sin\theta\, H_\phi) - \frac{\partial}{\partial\phi}(rH_\theta)\right]$$

$$\varepsilon \frac{\partial E_r}{\partial t} = \frac{1}{r \sin\theta}\left[\frac{\partial}{\partial\theta}(H_\phi \sin\theta) - \frac{\partial}{\partial\phi}(H_\theta)\right]$$

Substituting H_θ and H_ϕ from equations

$$\varepsilon \frac{\partial E_r}{\partial t} = \frac{1}{r \sin\theta}\left\{\frac{\partial}{\partial\theta}\left[\frac{I_m\, dl \sin^2\theta}{4\pi}\left(-\frac{\omega \sin\omega t_1}{cr} + \frac{\cos\omega t_1}{r^2}\right)\right]\right\} (\because H_\theta = 0)$$

$$\varepsilon \frac{\partial E_r}{\partial t} = \frac{1}{r \sin\theta}\, \frac{I_m\, dl}{4\pi}\left(-\frac{\omega \sin\omega t_1}{cr} + \frac{\cos\omega t_1}{r^2}\right)\frac{\partial}{\partial\theta}(\sin^2\theta)$$

$$\frac{\partial E_r}{\partial t} = \frac{1}{\varepsilon\, r \sin\theta}\, \frac{I_m\, dl}{4\pi}\left(-\frac{\omega \sin\omega t_1}{cr} + \frac{\cos\omega t_1}{r^2}\right) 2 \sin\theta \cos\theta$$

$$= \frac{2\,I_m\,dl}{4\pi\varepsilon r}\cos\theta\left(-\frac{\omega\sin\omega t_1}{cr} + \frac{\cos\omega t_1}{r^2}\right)$$

$$E_r = \int \frac{2\,I_m\,dl\,\cos\theta}{4\pi\varepsilon r}\left(-\frac{\omega\sin\omega t_1}{cr} + \frac{\cos\omega t_1}{r^2}\right)dt$$

$$= \frac{2\,I_m\,dl\,\cos\theta}{4\pi\varepsilon r}\left(+\frac{\omega\cos\omega t_1}{\omega\,cr} + \frac{\sin\omega t_1}{\omega\,r^2}\right)$$

$$\boxed{E_r = \frac{2\,I_m\,dl\,\cos\theta}{4\pi\varepsilon}\left[\frac{\cos\omega t_1}{c\,r^2} + \frac{\sin\omega t_1}{\omega\,r^3}\right]}$$

Similarly,

$$\varepsilon\frac{\partial E_\theta}{\partial t} = (\nabla\times H)_\theta = \frac{1}{r^2\sin\theta}\,r\left[-\frac{\partial}{\partial r}\,(r\,H_\phi\sin\theta - \frac{\partial}{\partial\phi}(H_r))\right]$$

$$= -\frac{1}{r\sin\theta}\frac{\partial}{\partial r}(r\ H_\phi \sin\theta) - 0$$

Substituting for H_ϕ from equation

$$= -\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{I_m\, dl\ \sin\theta}{4\pi}\left(-\frac{\omega\sin\omega t_1}{rc} + \frac{\cos\omega t_1}{r^2}\right)\right]$$

where $t_1 = t - \frac{r}{c}$ (retarded time)

$$= \frac{1}{r}\left[\frac{I_m\, dl\ \sin\theta}{4\pi}\left(\frac{\omega\cos\omega t_1}{c}\left(\frac{-\omega}{c}\right) - \left(\frac{-\omega}{c}\right)\frac{r(-\sin\omega t_1) - \cos\omega t_1}{r^2}\right)\right]$$

$$\frac{\partial E_\theta}{\partial t} = \frac{I_m\, dl \sin\theta}{4\pi\varepsilon r}\left[-\frac{\omega^2}{c^2}\cos\omega t_1 - \frac{\omega}{cr}\sin\omega t_1 + \frac{\cos\omega t_1}{r^2}\right]$$

$$E_\theta = \int \frac{I_m\, dl \sin\theta}{4\pi\varepsilon r}\left[\frac{-\omega^2}{c^2}\cos\omega t_1 - \frac{\omega}{cr}\sin\omega t_1 + \frac{\cos\omega t_1}{r^2}\right] dt$$

$$= \frac{I_m\, dl \sin\theta}{4\pi\varepsilon r}\left[\frac{-\omega^2}{c^2}\,\frac{\sin\omega t_1}{\omega} - \frac{\omega}{c\,r}\left(\frac{-\cos\omega t_1}{\omega}\right) + \frac{\sin\omega t_1}{\omega\, r^2}\right]$$

$$= \frac{I_m\, dl \sin\theta}{4\pi\varepsilon \cancel{r}}\left[-\frac{\omega\sin\omega t_1}{c^2\, r} + \frac{\cos\omega t_1}{c\, r^2} + \frac{\sin\omega t_1}{\omega\, r^3}\right]$$

$$\boxed{E_\theta = \frac{I_m\, dl \sin\theta}{4\pi\varepsilon}\left[-\frac{\omega\sin\omega t_1}{c^2 r} + \frac{\cos\omega t_1}{c\, r^2} + \frac{\sin\omega t_1}{\omega\, r^3}\right]}$$

$$\varepsilon \frac{\partial E_\phi}{\partial t} = (\nabla \times H)_\phi$$

$$= \left[r \sin\theta \, a_\phi \frac{\partial}{\partial r} (r \, H_\theta) - \frac{\partial}{\partial \phi} (H_r) \right] \frac{1}{r^2 \sin\theta}$$

$$\therefore \boxed{E_\phi = 0} \quad \text{(Since } H_r = H_\theta = 0\text{)}$$

INDUCTION FIELD AND RADIATION FIELD

Rewriting the equation

$$H_\phi = \underbrace{\frac{I_m \, dl \sin\theta}{4\pi}}_{\text{Amplitude}} \Bigg[\underbrace{\frac{\cos \omega t_1}{r^2}}_{\substack{\text{Induction} \\ \text{field (First term)}}} - \underbrace{\frac{\omega \sin \omega t_1}{c \; r}}_{\substack{\text{Radiation field} \\ \text{(Second term)}}} \Bigg]$$

At a distance $r >> \lambda$, the induction field and electrostatic field becomes negligible since it varies inversely with r^2 and r^3.

$$\eta = \frac{E_\theta}{H_\phi} = \frac{\dfrac{I_m\, dl \sin\theta}{4\pi\varepsilon}\left(-\dfrac{\omega \sin \omega t_1}{c^2 r}\right)}{\dfrac{I_m\, dl \sin\theta}{4\pi}\left(-\dfrac{\omega \sin \omega t_1}{c\, r}\right)}$$

$$= \frac{1}{\varepsilon c} = \frac{1}{(36\pi \times 10^9) \times (3 \times 10^8)} = 120\pi$$

$$\boxed{\eta = 120\pi}$$

POWER RADIATED BY A CURRENT ELEMENT AND ITS RADIATION RESISTANCE

$$P_r = E_\theta H_\phi,$$

Substituting for E_θ and H_ϕ from equation

$$P_r = \frac{I_m\, dl \sin\theta}{4\pi\varepsilon}\left[\frac{-\omega\sin\omega t_1}{c^2 r} + \frac{\cos\omega t_1}{c r^2} + \frac{\sin\omega t_1}{\omega r^3}\right]\times$$

$$\frac{I_m\, dl \sin\theta}{4\pi\cancel{\varepsilon}}\left[\frac{-\omega\sin\omega t_1}{c r} + \frac{\cos\omega t_1}{r^2}\right]$$

$$= \frac{I_m^2\, dl^2 \sin^2\theta}{16\pi^2\varepsilon}\left[+\frac{\omega^2\sin^2\omega t_1}{c^3 r^2} - \frac{\omega\cos\omega t_1\sin\omega t_1}{c^2 r^3}\right.$$

$$\left. - \frac{\omega\sin^2\omega t_1}{c r^4 \omega} - \frac{\omega\cos\omega t_1\sin\omega t_1}{c^2 r^3} + \frac{\cos^2\omega t_1}{c r^4} + \frac{\sin\omega t_1\cos\omega t_1}{\omega r^5}\right]$$

$$= \frac{I_m^2 \, dl^2 \sin^2\theta}{16\,\pi^2\,\varepsilon}\left[\frac{\omega^2\,(1-\cos 2\,\omega\, t_1)}{2\,c^3\, r^2} + \frac{\cos 2\,\omega\, t_1}{c\, r^4} - \frac{2\,\omega \cos \omega\, t_1 \sin \omega\, t_1}{c^2\, r^3} + \frac{\sin \omega\, t_1 \cos \omega\, t_1}{\omega\, r^5}\right]$$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{I_m^2\, dl^2 \sin^2\theta}{16\,\pi^2\,\varepsilon}\left[\frac{\omega^2\,(1-\cos 2\,\omega\, t_1)}{2\,c^3\,r^2} + \frac{\cos 2\,\omega\, t_1}{c\, r^4} - \frac{\omega \sin 2\,\omega\, t_1}{c^2\, r^3} + \frac{\sin 2\,\omega\, t_1}{2\,\omega\, r^5}\right]$$

$$P_r = \frac{I_m^2\, dl^2 \sin^2\theta}{16\,\pi^2\,\varepsilon} \times \frac{\omega^2}{2\,c^3\,r^2} + \text{Terms containing } \cos 2\,\omega\, t_1 \text{ and } \sin 2\,\omega\, t_1$$

The average value of $\sin 2\,\omega\, t_1$ and $\cos 2\,\omega\, t_1$, over a complete cycle is zero

$$P_r(\text{average}) = \frac{(I_m\, dl \sin\theta)^2}{16\,\pi^2\,\varepsilon} \times \frac{4\,\pi^2 f^2}{2\,c^3\,r^2} = \frac{(I_m\, dl \sin\theta)^2 f^2}{8\,\varepsilon\, c\,(c^2\, r^2)}$$

$$\boxed{P_r(\text{average}) = \frac{\eta\,(I_m\, dl \sin\theta)^2}{8\,\lambda^2\, r^2}} \quad \text{w/m}^2$$

The point 'p' is independent of the Azimuthal angle ϕ and therefore the elemental area on the spherical shell is

$$ds = 2\pi r^2 \sin\theta \, d\theta \qquad [\because dS_r = 2\pi r^2 \sin\theta \, d\theta]$$

Let W $\rightarrow$ be the total power radiated, then

$$W = \underbrace{\oint_S}_{\text{Surface integral}} P_r \cdot ds$$

Substituting the average power radiated from equation

$$W = \int_0^{\pi} \frac{\eta (I_m \, dl \sin\theta)^2}{8\lambda^2 r^2} \cdot 2\pi r^2 \sin\theta \, d\theta$$

$$= I_m^2 \left(\frac{dl}{\lambda}\right)^2 \pi \ \frac{\eta}{4} \int_0^{\pi} \sin^3\theta \ d\theta$$

$$= \left(\frac{dl}{\lambda}\right)^2 \pi . \frac{120\,\pi}{4} (\sqrt{2}\ I_{rms})^2 \ 2 \int_0^{\frac{\pi}{2}} \sin^3\theta \ d\theta$$

$[\because I_m = \sqrt{2}\ \ I_{rms} \ \& \ \eta = 120\,\pi]$

$$= \left(\frac{dl}{\lambda}\right)^2 \frac{120\,\pi^2}{4} \ 2\ I_{rms}^2 \ \ 2 \int_0^{\frac{\pi}{2}} \sin^3\theta \ d\theta$$

$$\int_0^{\frac{\pi}{2}} \sin^n\theta \, d\theta = \frac{n-1}{n}$$

$$\int_0^{\frac{\pi}{2}} \sin^3\theta \, d\theta = \frac{3-1}{3} = \frac{2}{3}$$

$$= \left(\frac{dl}{\lambda}\right)^2 120\,\pi^2\, I_{rms}^2\, \frac{2}{3}$$

$$\boxed{W = 80\,\pi^2 \left(\frac{dl}{\lambda}\right)^2 I_{rms}^2 \ \text{w/m}^2}$$

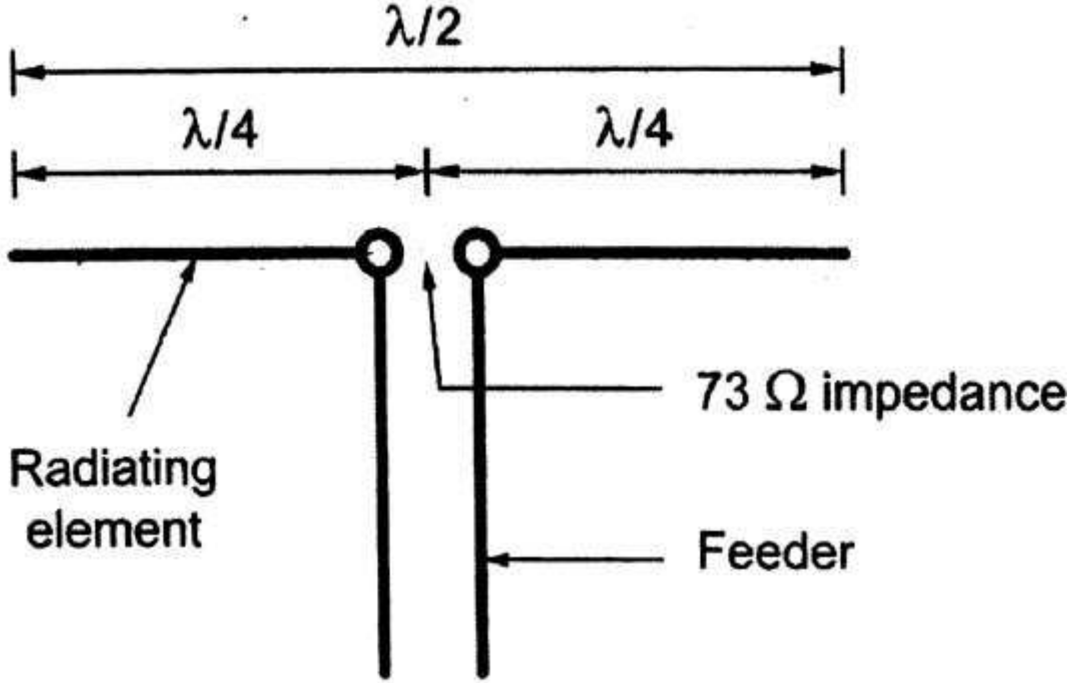
But $W = R_r\, I_{rms}^2$

where R_r = Radiation resistance

$$\boxed{R_r = 80\,\pi^2 \left(\frac{dl}{\lambda}\right)^2 \ \text{ohm}}$$

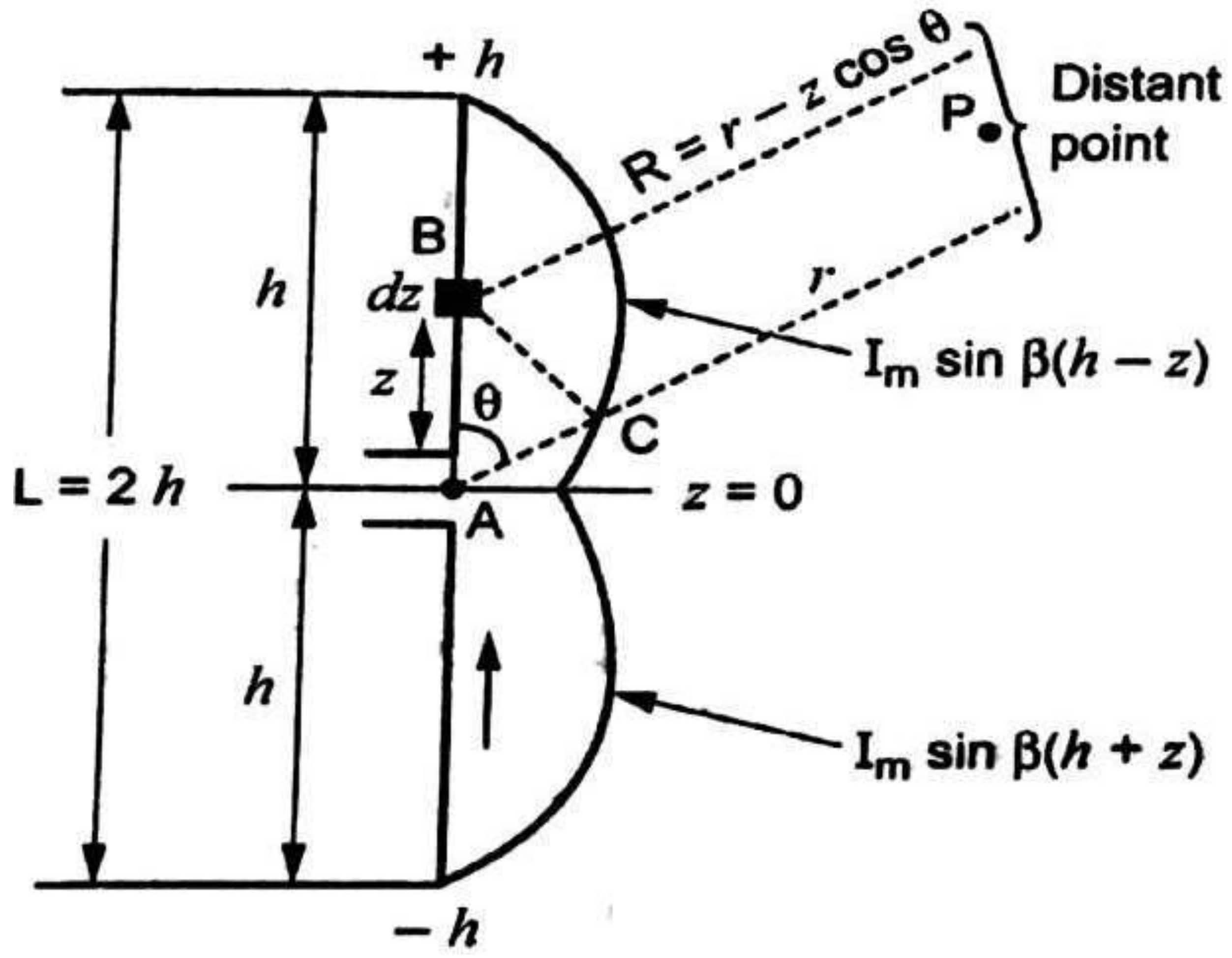
HALF WAVE DIPOLE (λ/2 ANTENNA)

The half wave dipole means half-wavelength dipole $\left(l = \lambda/2\right)$. It is one of the simplest antenna and is frequently employed as an element of the antenna array. It is also known as ***Hertz antenna*** or sometimes called ***half wave doublet***.



Half wave dipole antenna

POWER RADIATION FROM HALF WAVE DIPOLE



In ΔABC, AC = $r - R$

$$\frac{r - R}{z} = \cos\theta$$

$$r - R = z\cos\theta$$

$$\boxed{R = r - z\cos\theta}$$

Assumed sinusoidal current distribution in half wave dipole

current distribution

$$\left.\begin{aligned} I &= I_m \sin\beta\,(h-z) \quad \text{for } z>0 \\ I &= I_m \sin\beta\,(h+z) \quad \text{for } z<0 \end{aligned}\right\} \qquad \ldots (1)$$

where,

I_m = Current maximum at the current element Idz

$\beta = \dfrac{2\pi}{\lambda}$ = Phase constant, and

$I\,dz$ is the current element placed at a distance 'z' from $z = 0$ plane

Consider a point 'P' located at a far distance from the current element. Then the vector potential at point P due to the current element $I\,dz$ is given by

$$dA_z = \frac{\mu}{4\pi R}\, I\, dz\; e^{-j\beta R} \qquad \ldots\ldots(2)$$

where, R = Distance between $I\,dz$ to distant point P

The total vector potential

$$\int dA_z = \int_{-h}^{0} \frac{\mu\, I\, dz\, e^{-j\beta R}}{4\pi R} + \int_{0}^{+h} \frac{\mu\, I\, dz\, e^{-j\beta R}}{4\pi R}$$

$$A_z = \frac{\mu}{4\pi}\int_{-h}^{0} \frac{I_m \sin\beta(h+z)\; e^{-j\beta R}}{R}\, dz + \frac{\mu}{4\pi}\int_{0}^{+h} \frac{I_m \sin\beta(h-z)}{R}\, e^{-j\beta R}\, dz \quad \ldots (3)$$

Since the point 'P' is at a large distance, the lines to the point 'P' may be assumed to be parallel.

We know that $R = r - z\cos\theta$

$R \approx r$, when 'P' is at a large distance and replace R in the denominators of equation (3) only by r. But in numerator, 'R' represents the phase factor and therefore the difference between R and r is very important.

$$\therefore A_z = \frac{\mu}{4\pi}\int_{-h}^{0}\frac{I_m \sin\beta(h+z)}{r}\, e^{-j\beta(r - z\cos\theta)}\, dz + \frac{\mu}{4\pi}\int_{0}^{h}\frac{I_m \sin\beta(h-z)}{r}\, e^{-j\beta(r - z\cos\theta)}\, dz$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{-h}^{0}\sin\beta(h+z)\, e^{j\beta z\cos\theta}\, dz + \int_{0}^{h}\sin\beta(h-z)\, e^{j\beta z\cos\theta}\, dz\right]$$

For a $\lambda/2$ antenna, $L = 2h = \lambda/2$ & $z = h = \lambda/4 = \pi/2$ degrees

$$\sin\beta(h+z) = \sin\beta\left(\frac{\pi}{2}+z\right) = \cos\beta z \qquad \left[\because h = \frac{\pi}{2}\right]$$

$$\sin\beta(h-z) = \sin\beta\left(\frac{\pi}{2}-z\right) = \cos\beta z$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{-h}^{0} \cos\beta z\, e^{j\beta z\cos\theta}\, dz + \int_{0}^{h} \cos\beta z\, e^{+j\beta z\cos\theta}\, dz\right]$$

Now $\int_{-h}^{0} e^{+j\theta} d\theta = \int_{0}^{h} e^{-j\theta} d\theta$. Hence using this property, changing limits of integration of the first term.

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{h} \cos\beta z\, e^{-j\beta z\cos\theta}\, dz + \int_{0}^{h} \cos\beta z\, e^{+j\beta z\cos\theta}\, dz\right]$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_{0}^{h} \cos\beta z\, \{e^{-j\beta z\cos\theta} + e^{+j\beta z\cos\theta}\}\, dz \times \frac{2}{2}$$

We know that,

$$\frac{e^{-j\beta z\cos\theta} + e^{+j\beta z\cos\theta}}{2} = \cos(\beta z\cos\theta)$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^h [\cos\beta z\, 2\cos(\beta z\cos\theta)]\, dz \quad \ldots\ldots(4)$$

$$[\because 2\cos\alpha\cos\beta = \cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^h [\cos\{\beta z(1+\cos\theta)\} + \cos\{\beta z(1-\cos\theta)\}]\, dz \quad \ldots\ldots(5)$$

Integrating equation (5) and $h = z = \lambda/4$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{\sin(\beta z(1+\cos\theta))}{\beta(1+\cos\theta)} + \frac{\sin(\beta z(1-\cos\theta))}{\beta(1-\cos\theta)}\right]_0^{\lambda/4}$$

$$= \frac{\mu\, I_m\, e^{-j\beta r}}{4\pi\beta r}\left[\frac{(1-\cos\theta)\sin\beta z\,(1+\cos\theta)+(1+\cos\theta)\sin\beta z\,(1-\cos\theta)}{1-\cos^2\theta}\right]_0^{\lambda/4}$$

$$\left[\because \beta z = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{\pi}{2}\right]$$

$$= \frac{\mu\, I_m\, e^{-j\beta r}}{4\pi\beta r}\left[\frac{(1-\cos\theta)\sin(\pi/2+\pi/2\cos\theta)+(1+\cos\theta)\sin(\pi/2-\pi/2\cos\theta)}{1-\cos^2\theta}\right]$$

$$= \frac{\mu\, I_m\, e^{-j\beta r}}{4\pi\beta r}\left[\frac{(1-\cos\theta)\cos(\pi/2\cos\theta)+(1+\cos\theta)\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{\cos(\pi/2\cos\theta)\,[1-\cos\theta+1+\cos\theta]}{\sin^2\theta}\right]$$

$$\boxed{A_z = \frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]} \qquad \text{......(6)}$$

The ϕ components of H is given by,

$$H_\phi = \frac{1}{\mu}(\nabla\times A)\,\phi = \frac{1}{\mu}\times\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta)-\frac{\partial}{\partial\theta}(A_r)\right] \qquad \text{......(7)}$$

But now the current element is placed along z-axis, then $A_r = -A_z \sin\theta$ and $A_\theta = 0$ and substitute in equation (7), we get

$$H_\phi = \frac{1}{\mu} \times \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_z \cos\theta)\right] \qquad \ldots\ldots(8)$$

By substituting, A_z of equation (6) in equation (8), we get

$$= \frac{1}{\mu} \times \frac{1}{r}\left\{\frac{\partial}{\partial r}\left[-\frac{r\,\mu\,I_m e^{-j\beta r}}{2\pi\beta r}\left\{\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right\}\sin\theta\right]\right\}$$

$$H_\phi = \frac{-I_m}{2\pi\beta r}\left[\frac{\cos\left(\pi/2\,\cos\theta\right)}{\sin\theta}\right]\frac{\partial}{\partial r}\left[e^{-j\beta r}\right]$$

$$= \frac{-I_m}{2\pi\beta r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]\left[(e^{-j\beta r})(-j\beta)\right]$$

$$H_\phi = \frac{jI_m e^{-j\beta r}}{2\pi\cancel{\beta} r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

$$\boxed{|H_\phi| = \frac{I_m}{2\pi r}\left\{\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right\} \text{ A/m}} \quad \ldots\ldots (9)$$

The electric field expression for the radiation field can be achieved from the well known formula

$$\frac{E_\theta}{H_\phi} = \eta = 120\,\pi$$

$$|E_\theta| = 120\,\pi\,|H_\phi| \qquad \text{......(10)}$$

$$|E_\theta| = 120\,\pi \times \frac{I_m}{2\,\pi\,r}\left\{\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right\}$$

$$|E_\theta| = \frac{60\,I_m}{r}\left\{\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right\}\ \text{V/m} \qquad \text{...(11)}$$

POWER RADIATED BY A HALF WAVE DIPOLE AND ITS RADIATION RESISTANCE

$$P_{max} = |E_\theta||H_\phi|$$

$$= \left[\frac{60 I_m'}{r}\left\{\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right\}\right]\left[\frac{I_m}{2\pi r}\left\{\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right\}\right]$$

$$P_{\max} = \frac{30 I_m^2}{\pi r^2}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2 \qquad \ldots\ldots(12)$$

The *average value of the power is half of the maximum power* and it is expressed as

$$P_{avg} = \frac{E_\theta}{\sqrt{2}} \cdot \frac{H_\phi}{\sqrt{2}} = \frac{1}{2} E_\theta \cdot H_\phi = \frac{P_{max}}{2}$$

$$P_{avg} = \frac{15 I_m^2}{\pi r^2} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]^2 W/m^2 \qquad \text{......(13)}$$

The effective or R.M.S current is related to the maximum current by the relation and it is given by,

$$I_{r.m.s} = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = \sqrt{2}\, I_{r.m.s}$$

$$= \frac{15\,(\sqrt{2}\,I_{rms})^2}{\pi\, r^2} \left\{ \frac{\cos(\pi/2 \cos\theta)}{\sin\theta} \right\}^2$$

$$\boxed{P_{avg} = \frac{30\, I_{rms}^2}{\pi\, r^2} \left[\frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta} \right] \text{W/m}^2} \qquad \ldots\ldots(14)$$

Power radiated $(P_r) = \oint p_{avg} \cdot ds$ ......(15)

ds = elemental area of the spherical shell $= 2\,\pi\, r^2 \sin\theta\, d\theta$

By substituting equation (14) in equation (15), we get

$$= \int_0^{\pi} \frac{30\, I_{rms}^2}{\pi\, r^2} \left\{ \frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta} \right\} 2\,\pi\, r^2 \sin\theta\, d\theta$$

$$= 60\, I_{rms}^2 \int_0^{\pi} \frac{\cos^2 (\pi/2 \cos\theta)}{\sin\theta}\, d\theta$$

$$= 60\, I_{rms}^2 \int_0^{\pi} \frac{1}{2} \left\{ \frac{1 + \cos(\pi \cos\theta)}{\sin\theta} \right\} d\theta = 60\, I_{rms}^2 \cdot I$$

where, $$I = \frac{1}{2} \int_0^{\pi} \left\{ \frac{1 + \cos(\pi \cos\theta)}{\sin\theta} \right\} d\theta \qquad \text{......(16)}$$

The value of I after integration is

$$\boxed{I = 1.219}$$

$$\text{Power radiated} = 60\, I_{rms}^2 \times 1.219$$

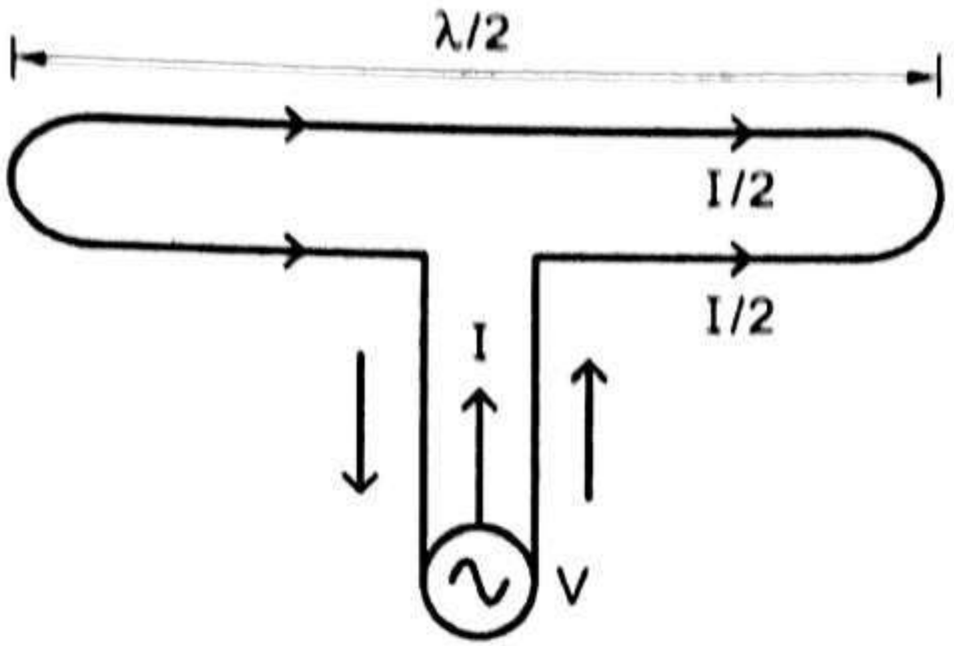
$$\boxed{P_r = 73.140\, I_{rms}^2}$$

Radiation Resistance:

$$W_r = R_r \cdot I_{rms}^2 \qquad \ldots\ldots (18)$$

$$\boxed{R_r = 73.14 \approx 73\ \Omega}$$

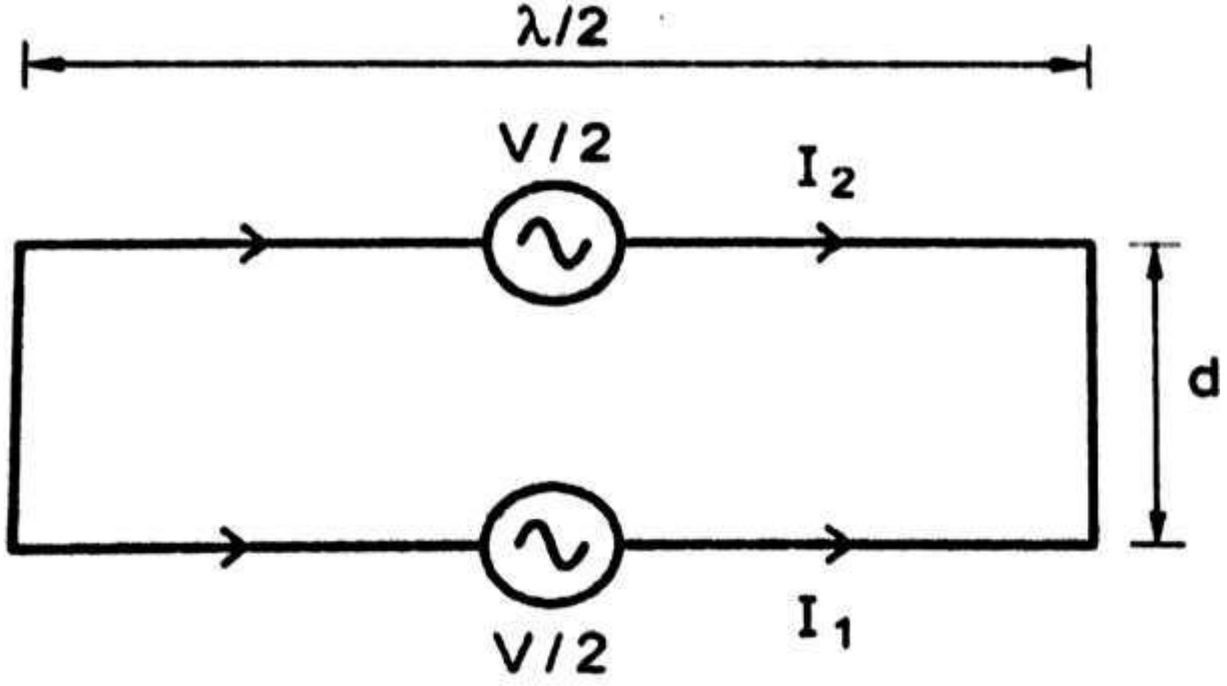
Folded Dipole



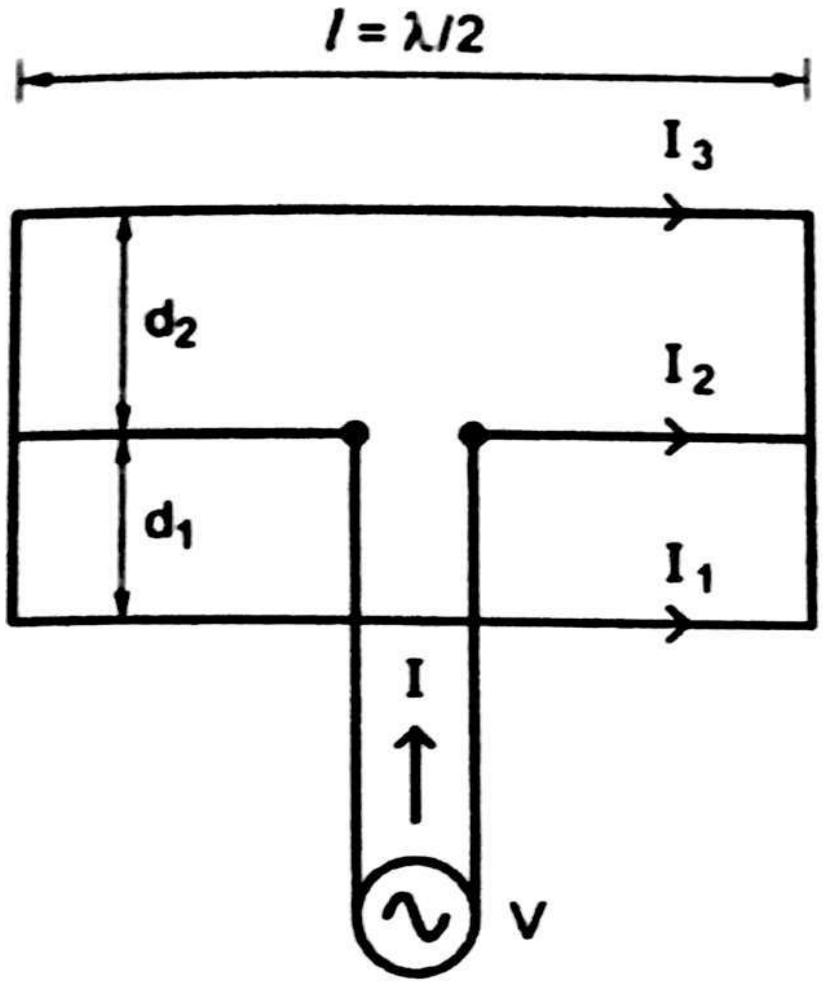
Folded antenna

DIFFERENT TYPES OF FOLDED DIPOLE ANTENNA

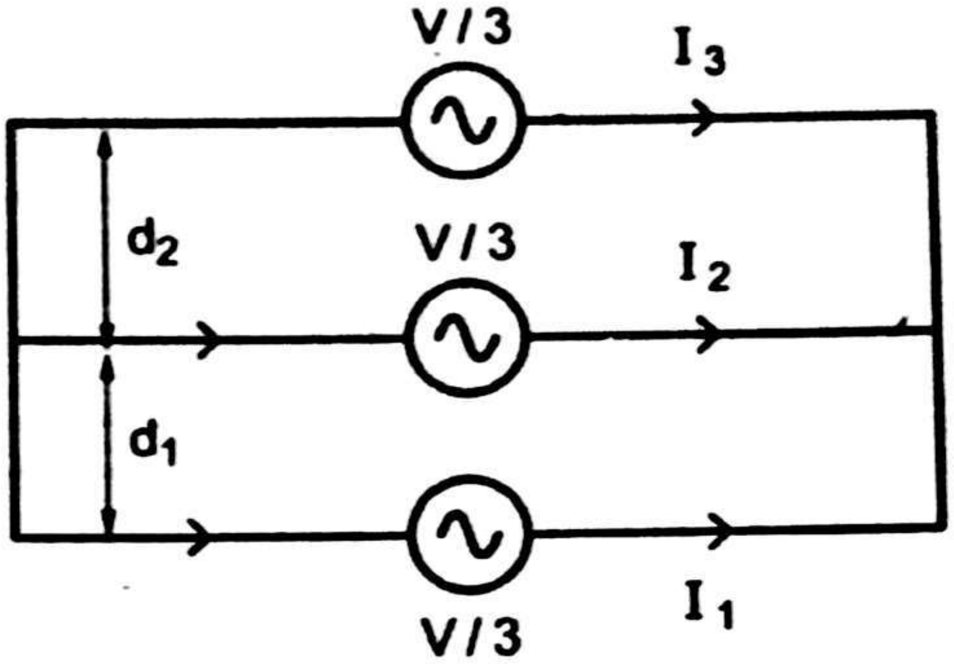
1. Equation of Input Impedance



Equivalent 2-wire folded dipole

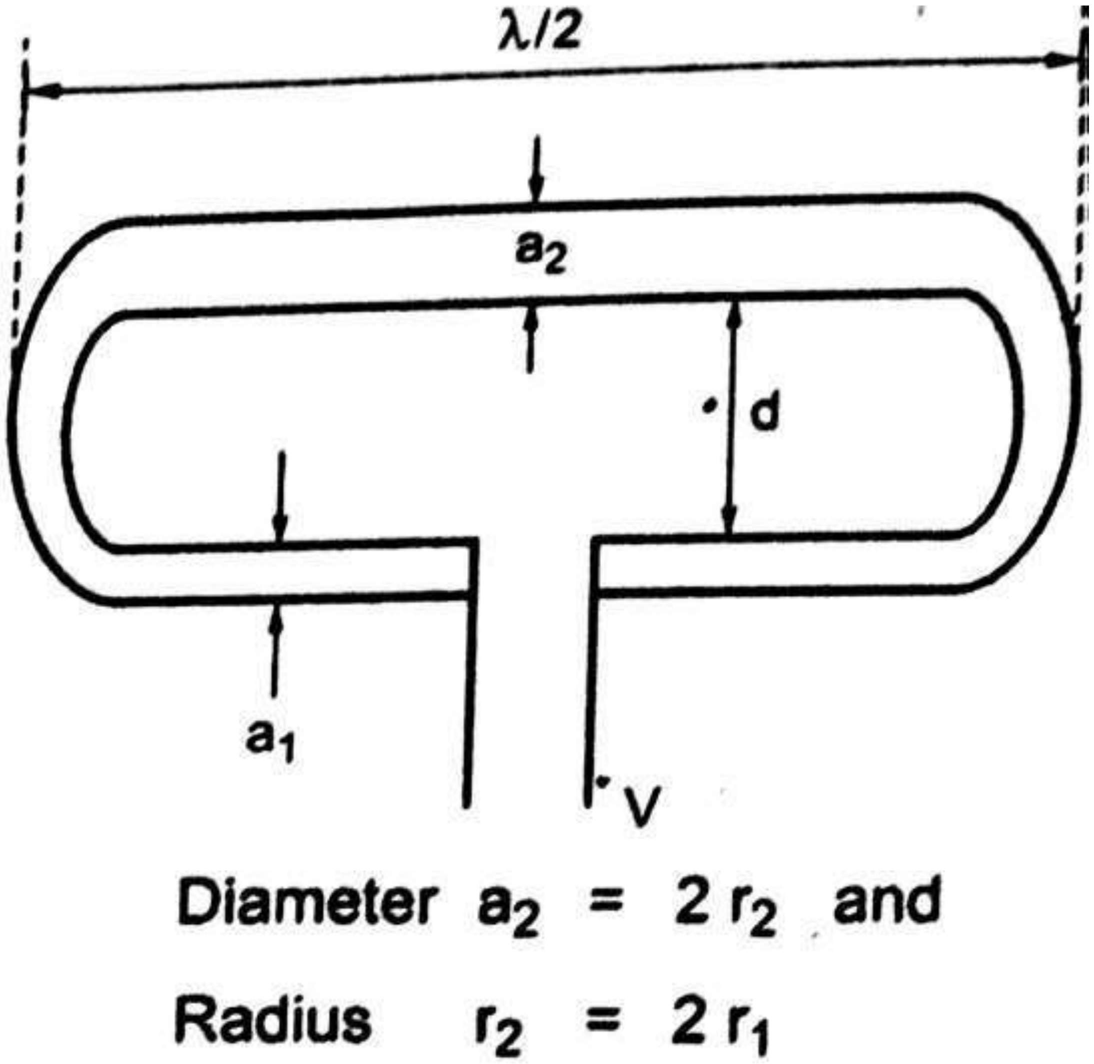


Three wire folded dipole



Equivalent circuit

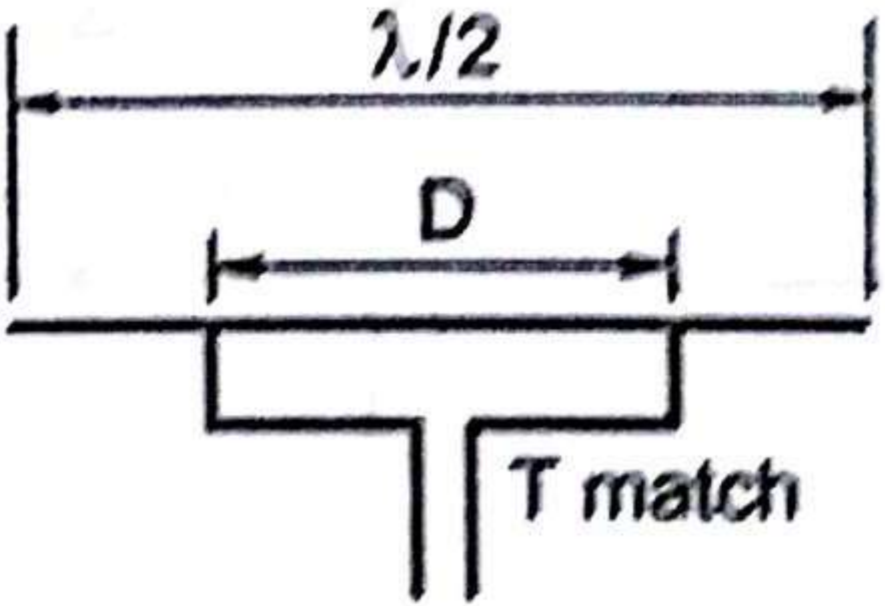
2. Unequal Radii



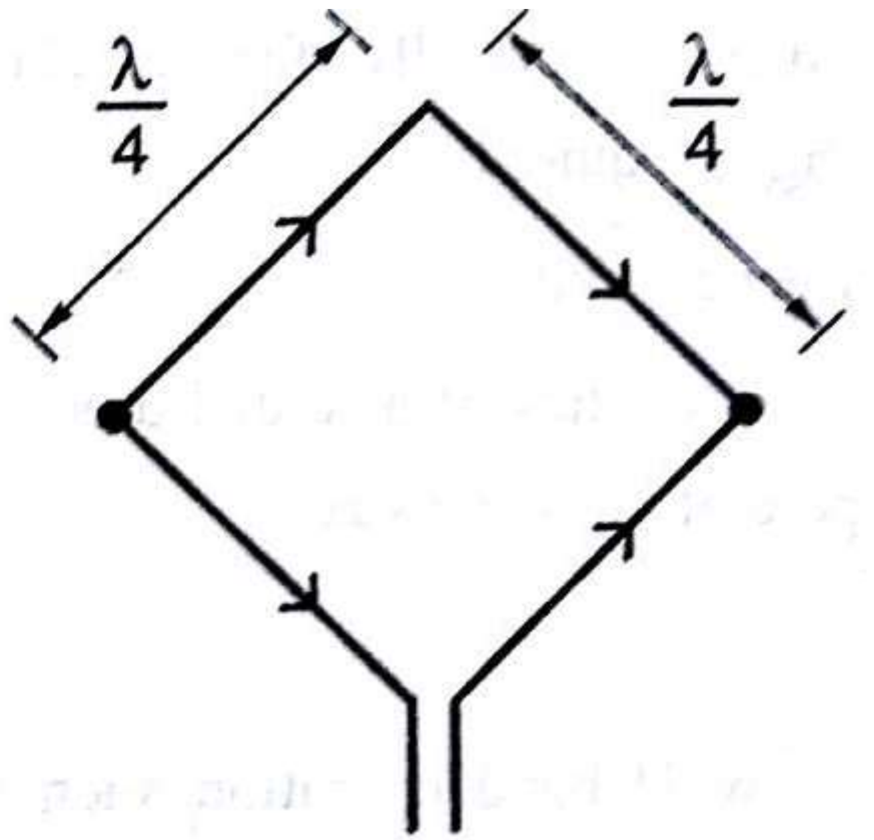
Folded antenna with different radii

MODIFICATIONS OF FOLDED DIPOLES

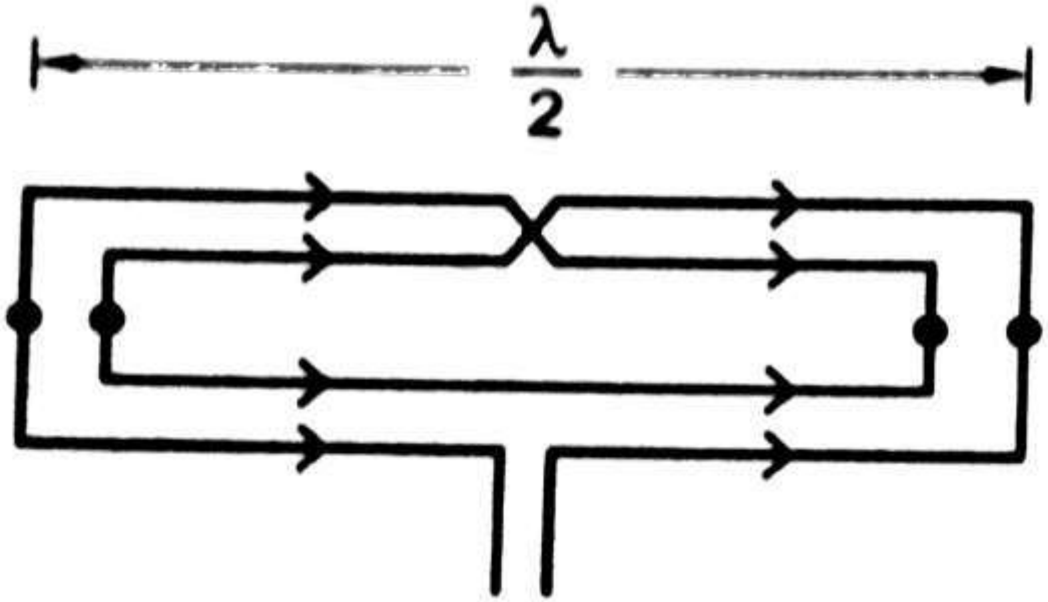
1. T-match antennas



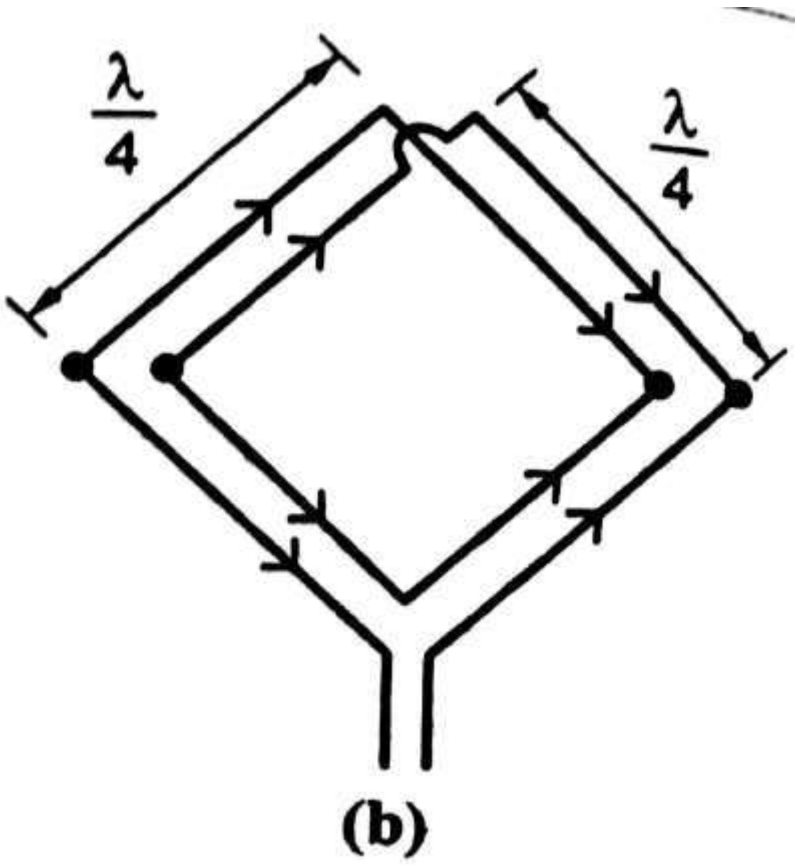
2. Single-turn loop antenna



3. Two-turn loop (or) quad antenna

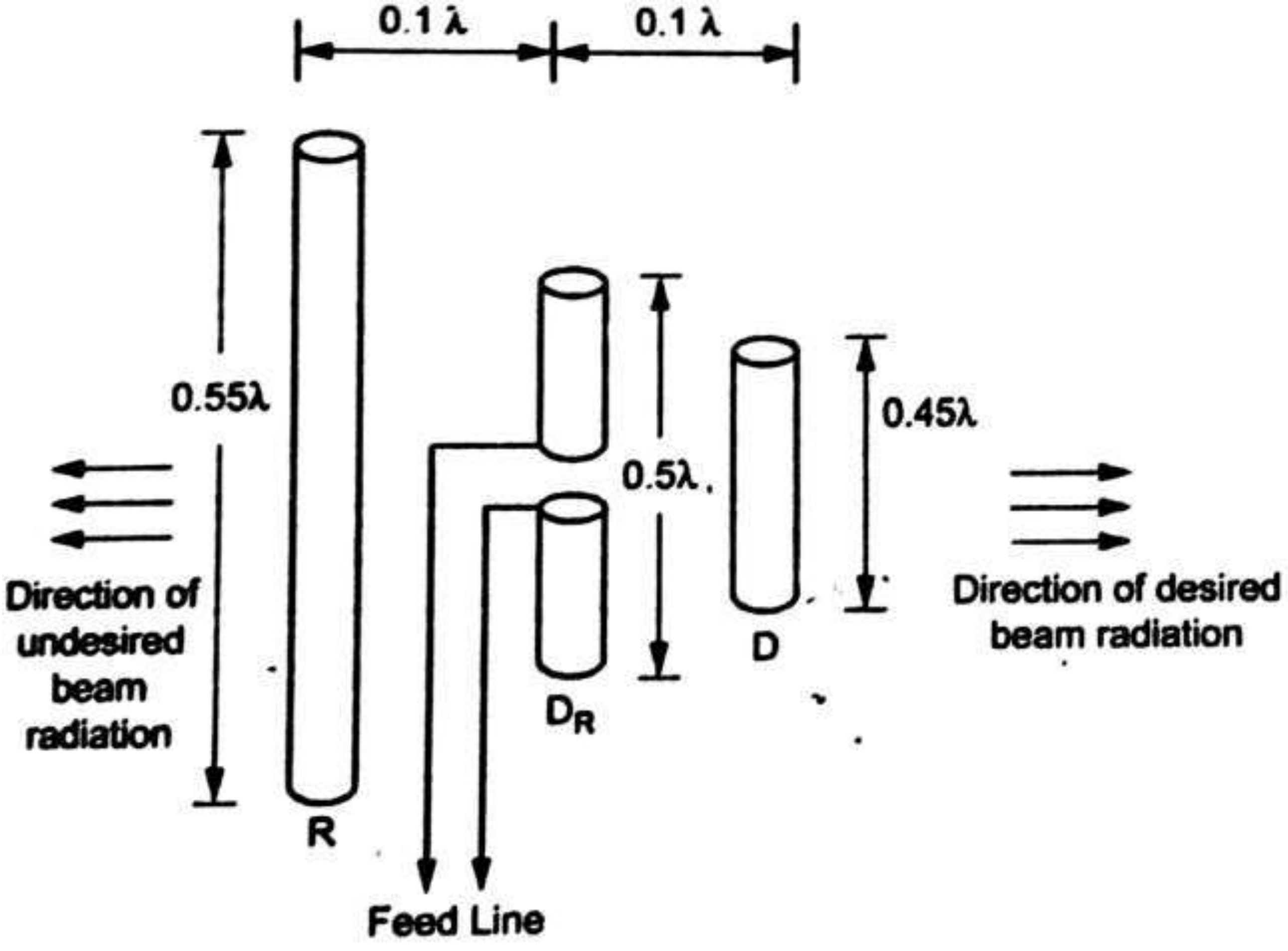


4-wire folded antenna

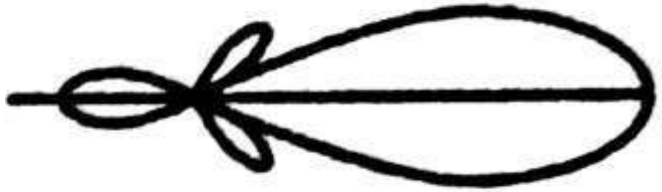


Two- turn loop antenna

YAGI-UDA Antenna



Yagi-Uda antenna



Radiation pattern

Construction:

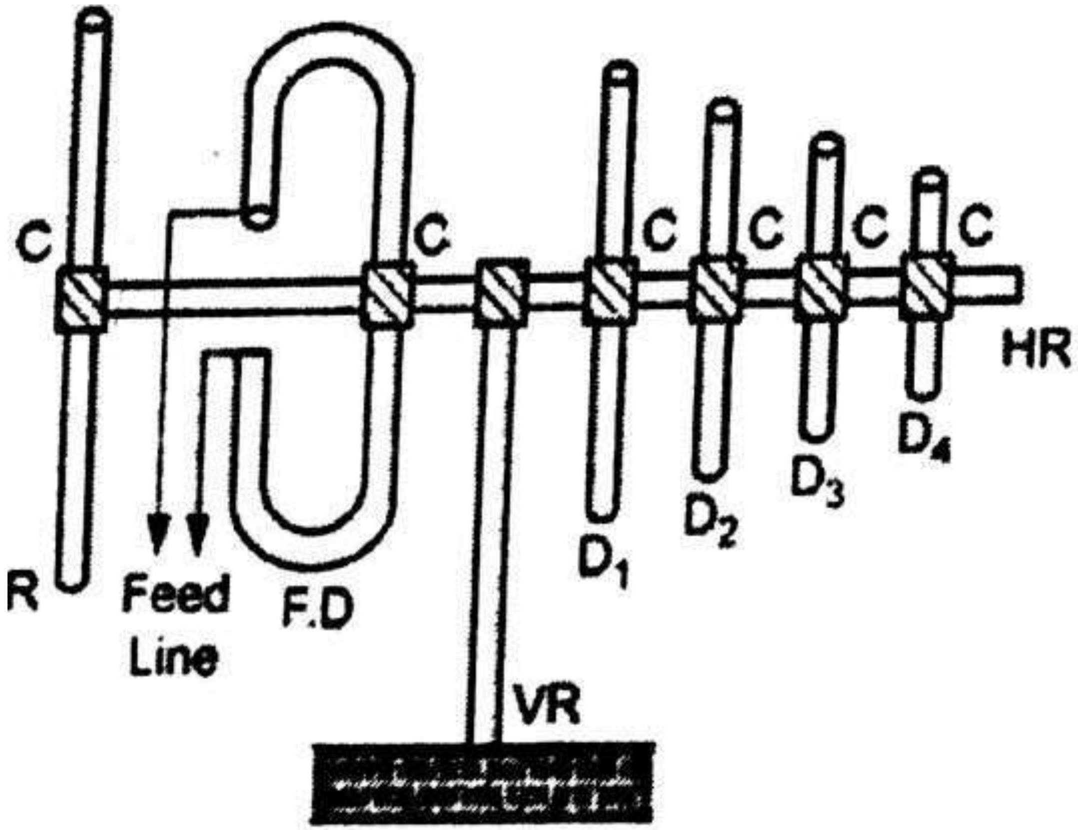
A basic Yagi-Uda antenna consists of

1. Driven Element
2. Reflector
3. Director

$$\text{Reflector (R) length} = \frac{500}{f(MHz)} \text{ feet (or) } \frac{152}{f(MHz)} \text{ meters}$$

$$\text{Driven } (D_R) \text{ element length} = \frac{475}{f(MHz)} \text{ feet (or) } \frac{143}{f(MHz)} \text{ meters}$$

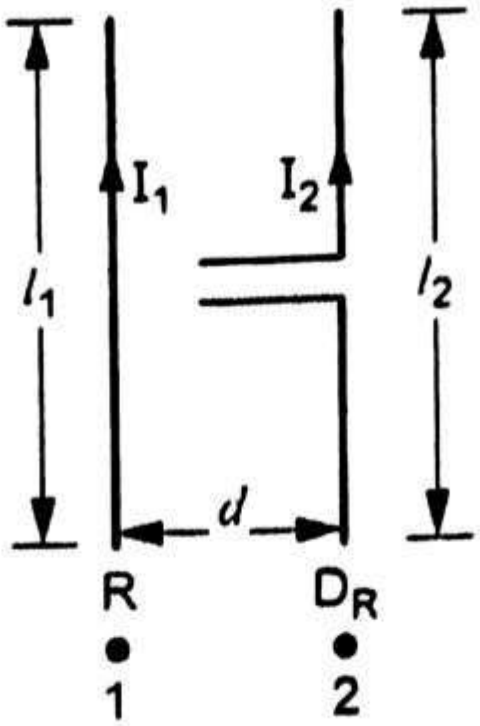
$$\text{Director (D) length} = \frac{455}{f(MHz)} \text{ feet (or) } \frac{137}{f(MHz)} \text{ meters}$$



Six-element yagi antenna with folded dipole

Voltage and Current Relations in Parasitic Antennas

Let us consider a yagi-uda antenna with 2 elements, one driven element and one reflector



Two element yagi-uda antenna

Let V_1 = Applied voltage in antenna number 1 *i.e.*, reflector

V_2 = Applied voltage in antenna number 2 *i.e.*, driven element

I_1 = Current through reflector

I_2 = Current through driven element

Z_{11}, Z_{22} = Self impedance of reflector and driven element

Z_{12}, Z_{22} = Mutual impedance between reflector and driven element

Now, the applied voltage in the reflector is given by

$$V_1 = I_1 Z_{11} + I_2 Z_{12}$$

Similarly applied voltage in the driven element is

$$V_2 = I_1 Z_{21} + I_2 Z_{22}$$

But if the individual antennas are not excited, the applied voltage becomes zero.

Here the reflector is not directly excited. Therefore $V_1 = 0$

$$I_1 Z_{11} + I_2 Z_{12} = 0$$

$$I_2 Z_{12} = -I_1 Z_{11}$$

$$I_1 = \frac{-I_2 Z_{12}}{Z_{11}}$$

$$V_2 = -I_2 \frac{Z_{12}}{Z_{11}} Z_{21} + I_2 Z_{22}$$

But $$Z_{12} = Z_{21}$$

$$V_2 = +I_2 \left[Z_{22} - \frac{Z_{12}^2}{Z_{11}} \right]$$

$$I_2 = \frac{V_2}{Z_{22} - \dfrac{Z_{12}^2}{Z_{11}}}$$

$$I_1 = -\frac{V_2}{Z_{22} - \dfrac{Z_{12}^2}{Z_{11}}} \; \frac{Z_{12}}{Z_{11}}$$

$$= -\frac{\dfrac{V_2}{Z_{22}\,Z_{11} - Z_{12}^2}}{Z_{11}} \cdot \frac{Z_{12}}{Z_{11}}$$

$$\boxed{I_1 = \frac{V_2\;Z_{12}}{Z_{12}^2 - Z_{22}\,Z_{11}}}$$

Taking Z_{12} to the denominator,

$$I_1 = \frac{V_2}{Z_{12} - \dfrac{Z_{11} Z_{22}}{Z_{12}}}$$

∴ The input impedance of reflector can be derived from the above equation

$$Z_1 = \frac{V_2}{I_1} = Z_{12} - \frac{Z_{11} Z_{22}}{Z_{12}}$$

$$\boxed{Z_1 = Z_{12} - \frac{Z_{11} Z_{22}}{Z_{12}}}$$

Similarly the input impedance of driven element can be derived from equation

$$\boxed{Z_2 = \frac{V_2}{I_2} = Z_{22} - \frac{Z_{12}^2}{Z_{11}}}$$

Unit II

Aperture Antennas

The term aperture refers to an opening in a closed surface.

The aperture antennas are most common at microwave frequency band. It must have an aperture length and width of atleast several wavelengths in order to have a high gain.

Typical antennas that fall in this category are the slot, horn, reflector, and lens antennas.

Radiation from Rectangular Aperture

A rectangular aperture which is an infinite conducting plane of dimensions '$2a$' along x-axis and '$2b$' along y-axis and located in the z plane (z = 0) . Let E_0 and H_0 be the tangential electric and magnetic field in the aperture.

In the region, z > 0 which is free-space, assume that the field in the aperture is uniform, to reduce the mathematical complexities and it is expressed as,

$$E_a = \begin{cases} E_0\, a_x & |x| \le a \;\&\; |y| \le b \\ 0 & Otherwise \end{cases} \qquad \ldots\ldots (1)$$

where, a_x – Unit vector in the x direction.

- Consider a function of variable x is p(x) and its Fourier transform is

$$P(k_x) = \int_{-\infty}^{+\infty} p(x) e^{j k_x x} dx \qquad \text{......} (2)$$

The Fourier transform of the aperture field is expressed as,

$$f_t = E_0 a_x \int_{-a}^{+a} \int_{-b}^{+b} e^{j k_x x + j k_y y} dy\, dx \qquad \text{......} (3)$$

where, k is the propagation constant which is represented as vector in a particular directions i.e. k_x, k_y, and k_z.

$$\int_{-a}^{a} e^{jk_x x} dx = \left[\frac{e^{jk_x x}}{jk_x} \right]_{-a}^{a} = \frac{e^{jk_x a} - e^{-jk_x a}}{jk_x} = \frac{2j \sin k_x a}{jk_x} = 2a \frac{\sin k_x a}{k_x a}$$

After integration, the equation (3) becomes

$$f_t = 4ab\, E_0 a_x \frac{\sin k_x a}{k_x a} \frac{\sin k_y b}{k_y b} \qquad \text{...... (4)}$$

The θ *and* ϕ are the spherical co-ordinate angles and the Fourier transform of the field with k_x and k_y are expressed as,

$$k_x = k_o \sin\theta \cos\phi \qquad \text{...... (5a)}$$

$$k_y = k_o \sin\theta \sin\phi \qquad \text{...... (5b)}$$

Here, k_z is the field at z direction and in the x-y plane z = 0. i.e. k_0 and substitute equations (5a) and (5b) in equation (4), we get

$$= 4ab\, E_0 a_x \frac{\sin(k_0 a \sin\theta \cos\phi)}{k_0 a \sin\theta \cos\phi} \frac{\sin(k_0 b \sin\theta \sin\phi)}{k_0 b \sin\theta \sin\phi}$$

$$f_t = 4ab\,E_0 a_x \frac{\sin u}{u}\,\frac{\sin v}{v} \qquad \ldots\ldots (6)$$

where,

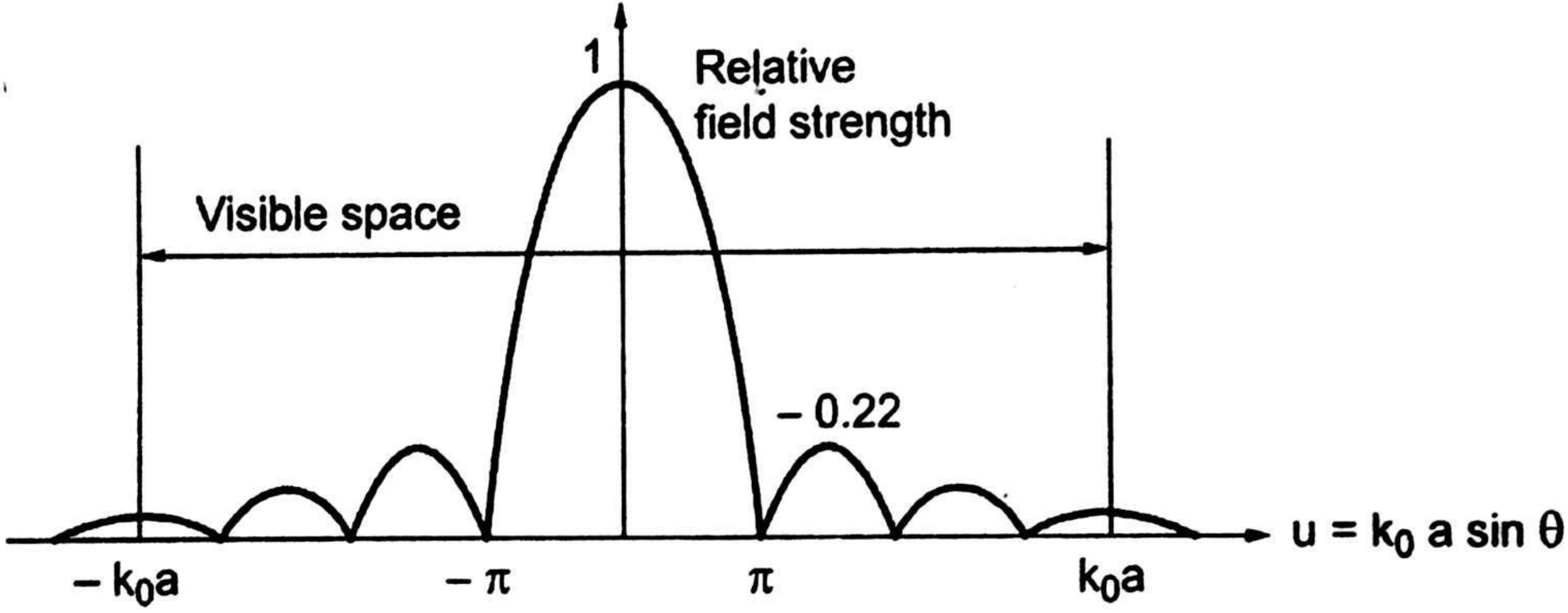
$$u = k_0 a \sin\theta \cos\phi\,,$$

$$v = k_0 b \sin\theta \sin\phi\,,$$ and

The radiated electric field is expressed in terms of f_t and it is given by,

$$\boxed{E(r) = \frac{jk_o 4abE_0}{2\pi r} e^{-jk_0 r} \frac{\sin u}{u}\frac{\sin v}{v}\left(a_\theta \cos\phi - a_\phi \sin\phi \cos\theta\right)} \qquad \ldots..(7)$$

where, a_θ *and* a_ϕ are the unit vector of spherical co-ordinate angle θ *and* ϕ



Radiation pattern of rectangular aperture with a uniform field

For the E-plane pattern, the maximum radiation is directed along z-axis $(\phi=0)$ and to get substitute $\phi=0$, $u=k_0 a\sin\theta\cos\phi$ and $v=k_0 b\sin\theta\sin\phi$ in equation (7),

$$E(r)=\frac{jk_o 4abE_0}{2\pi r}e^{-jk_0 r}\frac{\sin(k_0 a\sin\theta\cos\phi)}{k_0 a\sin\theta\cos\phi}\frac{\sin(k_0 b\sin\theta\sin\phi)}{k_0 a\sin\theta\sin\phi}(a_\theta\cos\phi-a_\phi\sin\phi\cos\theta)$$

$$\because \mid \cos 0=1 \ \& \ \sin 0=0$$

$$\boxed{E(r)=\frac{jk_o 4ab\,a_\theta E_0}{2\pi r}e^{-jk_0 r}\frac{\sin(k_0 a\sin\theta)}{k_0 a\sin\theta}} \qquad \ldots\ldots (8)$$

BeamWidth

From equation (8), the nulls (zeros) occurs, when

$$k_0 a \sin \theta \big|_{\theta=\theta_n} = n\pi , \quad \text{n} = 1,2,3,\ldots$$

At an angle, $$\theta_n = \sin^{-1}\left(\frac{n\pi}{k_o a}\right) \qquad \ldots\ldots (9)$$

Then, the total beam width between first nulls (n = 1) is given by,

$$BWFN = 2\theta_n = 2\sin^{-1}\left(\frac{n\pi}{k_0 a}\right) \qquad \ldots\ldots (9)$$

where, phase propagation constant $k_0 = \dfrac{2\pi}{\lambda_0}$ ….. (10)

By substituting the equation (10) in equation (9), we get

$$BWFN = 2\sin^{-1}\left(\frac{\pi \times \lambda_0}{2\pi a}\right)$$

$$= 2\sin^{-1}\left(\frac{\lambda_0}{2a}\right)$$

$$\boxed{BWFN \approx \frac{\lambda_0}{a} \text{ radian} \qquad \text{for } a >> \lambda_0}$$

Uniform Aperture Field with a Linear Phase Variation

Now we consider uniform aperture field with a linear phase variation, which will overcome the ***rectangular aperture problem***. Assume that the aperture field has a linear phase variation and it is expressed as,

$$E_a = E_0 a_x e^{-j\alpha x - j\beta y} \qquad |x| \leq a \quad |y| \leq b \qquad \text{.....(1)}$$

where, α – Phase variation in the x- axis direction, and

β – Phase variation in the y - axis direction.

The Fourier transform of the aperture field distribution and it is expressed as,

$$f_t = E_0 a_x \int_{-a}^{+a} \int_{-b}^{+b} e^{j(k_x-\alpha)x+j(k_y-\beta)y} \, dy \, dx \qquad \ldots\ldots (2)$$

By comparing with the rectangular field, the only modification in this field is achieved by replacing the k_x *by* $k_x - \alpha$ *and* k_y *by* $k_y - \beta$ and to get the radiated electric field consider $\alpha a = u_0$ *and* $\beta b = v_0$ for ***uniform aperture*** and the same is expressed as,

$$E(r) = \frac{jk_o 2abE_0}{\pi r} e^{-jk_0 r} \frac{\sin(u-u_0)}{u-u_0} \frac{\sin(v-v_0)}{v-v_0} \left(a_\theta \cos\phi - a_\phi \sin\phi \cos\theta\right) \qquad \ldots\ldots (3)$$

In the uv space the pattern is same as rectangular aperture except for a shift of the maximum from $u = v = 0$ to $u = u_0, v = v_0$ and the radiation lobe in specified in z-axis at the angles,

$$k_0 a \sin\theta \cos\phi = u_0 = \alpha a \qquad \text{...... (4a)}$$

$$k_0 b \sin\theta \sin\phi = v_0 = \beta b \qquad \text{...... (4b)}$$

From equations (4a) and (4b) ,we get

$$\frac{\beta b}{\alpha a} = \frac{k_0 b \sin\theta \sin\phi}{k_0 a \sin\theta \cos\phi}$$

$$\tan\phi = \frac{\beta}{\alpha}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{\beta}{\alpha}\right)} \qquad \text{...... (5)}$$

From equations (4a) and (4b) , we get

$$\alpha = k_0 \sin\theta \cos\phi \qquad \text{...... (6a)}$$

$$\beta = k_0 \sin\theta \sin\phi \qquad \text{...... (6b)}$$

By adding the above expressions,

$$\alpha^2 + \beta^2 = k_0^2 \sin^2\theta \left(\cos^2\phi + \sin^2\phi\right)$$

$$\sin^2\theta = \frac{\alpha^2 + \beta^2}{k_0^2}$$

$$\sin\theta = \frac{\sqrt{\alpha^2 + \beta^2}}{k_0}$$

$$\boxed{\theta = \sin^{-1}\left(\frac{\sqrt{\alpha^2+\beta^2}}{k_0}\right)} \qquad \text{...... (7)}$$

If the direction of maximum radiation is in $\phi = 0$ *or* xz plane at an angle $\theta = \theta_0$, then $\beta = 0$, based on equation (6b), then the radiation angle is given from equation (7) as,

$$\theta_0 = \sin^{-1}\left(\frac{\sqrt{\alpha^2+0}}{k_0}\right) = \sin^{-1}\left(\frac{\alpha}{k_0}\right) \qquad \text{...... (8)}$$

where, phase propagation constant $k_0 = \dfrac{2\pi}{\lambda_0}$ ...... (9)

By substituting the equation (9) in equation (8), we get

$$\boxed{\theta_0 = \sin^{-1}\left(\frac{\alpha\lambda_0}{2\pi}\right)} \qquad \text{.....(10)}$$

BEAMWIDTHS

The nulls (zeros) occurs where,

$$u - u_0 = \pm\pi$$

$$u = u_0 \pm \pi = k_0 a \sin\theta$$

Using Taylor series, the expansion of sin θ about θ_0 and approximated

$$\because f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \dots..$$

$$u = k_0 a \sin\theta_0 + k_0 a \cos\theta_0 (\theta - \theta_0) \qquad \dots\dots (11)$$

The first term $u_0 = k_0 a \sin\theta_0$ represent the maximum radiation position and the second term $k_0 a \cos\theta_0 (\theta - \theta_0)$ represent the first side lobe radiation position, that is, first null.

The nulls (zeros) occurs, when

$$k_0 a \cos\theta_0 (\theta - \theta_0) = n\pi \qquad n = 1,2,3,\ldots$$

At an angle,
$$(\theta - \theta_0) = \frac{n\pi}{k_0 a \cos\theta_0}$$

Then, the total beamwidth between first nulls (n = 1) from above expression is given by,

$$BWFN = 2(\theta - \theta_0) = \frac{2\pi}{k_0\, a \cos\theta_0} \qquad \ldots\ldots (12)$$

Phase propagation constant $k_0 = \frac{2\pi}{\lambda_0}$ .... (13)

By substituting equation (13) in equation (12), we get

$$BWFN = \frac{2\pi}{\frac{2\pi}{\lambda_o} a \cos\theta_0} = \frac{\lambda_0}{a \cos\theta_0} \qquad \text{......} (13)$$

Tapered Aperture:

In most of the antenna applications, it is desired to have very-low side band levels in order to reduce interference effects.

To illustrate the effect of a tapered aperture field, consider the rectangular aperture with a triangular aperture-field distribution and it is expressed as,

$$E = E_0 a_x \left(1 - \frac{|x|}{a}\right) \qquad |x| \le a \quad |y| \le b \qquad \ldots\ldots (1)$$

The Fourier transform of the aperture field is expressed as,

$$f_t = E_0 a_x \int_{-a}^{+a} \int_{-b}^{+b} \left(1 - \frac{|x|}{a}\right) e^{jk_x x + jk_y y} \, dy \, dx$$

$$= 4b\, E_0 a_x \frac{\sin k_y b}{k_y b} \int_0^a \left(1 - \frac{x}{a}\right) \cos k_x x dx \qquad \ldots\ldots(2)$$

From equation (2),

$$\int_0^a \left(1 - \frac{x}{a}\right) \cos k_x\, x\; dx$$

Consider, $\quad y = 1 - \dfrac{x}{a} \Rightarrow dy = -\dfrac{1}{a}$

$$dz = \cos k_x x \Rightarrow \int dz = z = \frac{\sin k_x x}{k_x} = \int z = -\frac{\cos k_x x}{k_x^2}$$

We know that, $\int y dz = yz - \int z dy$

$$= \left[\left(1-\frac{x}{a}\right)\frac{\sin k_x x}{k_x} - \frac{1}{a}\frac{\cos k_x x}{k_x^2}\right]_0^a$$

$$= 0 - \frac{1}{a}\frac{\cos k_x a}{k_x^2} - \left(-\frac{1}{ak_x^2}\right)$$

$$\int_0^a \left(1-\frac{x}{a}\right)\cos k_x x dx = \frac{1}{ak_x^2} - \frac{\cos k_x a}{ak_x^2} = \frac{1-\cos k_x a}{k_x^2 a} \quad \ldots\ldots (3)$$

By substituting equation (3) in equation (2), we get

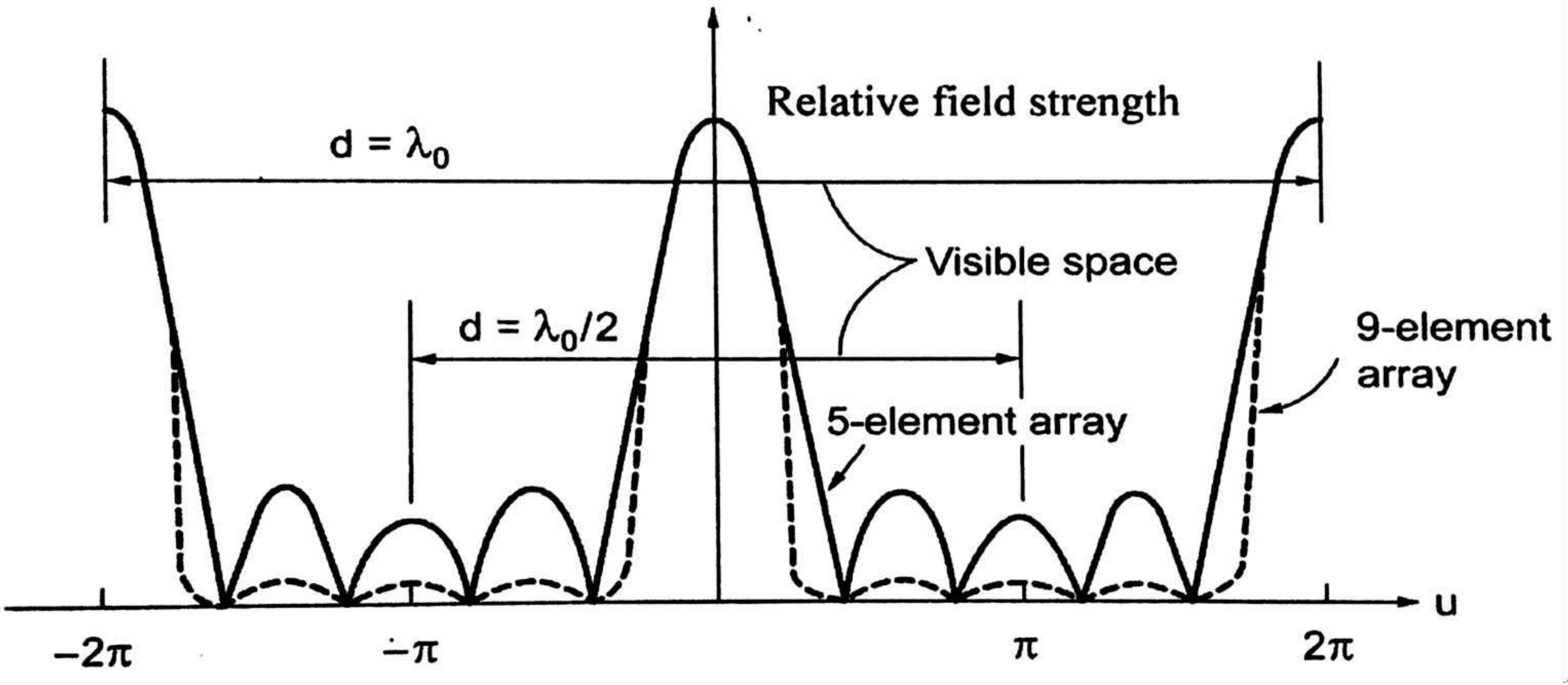
$$= 4ab\, E_0 a_x \frac{\sin k_y b}{k_y b}\frac{1-\cos k_x a}{k_x^2 a}$$

$$\because\ 1-\cos 2A = 2\sin^2 A \Rightarrow 1-\cos k_x a = 2\sin^2 k_x \left(a/2\right)$$

$$f_t = 2ab\, E_0 a_x \frac{\sin k_y b}{k_y b}\left[\frac{\sin k_x \left(a/2\right)}{k_x \left(a/2\right)}\right]^2 \qquad \ldots\ldots (4)$$

The radiated electric field is expressed as,

$$E(r) = \frac{jk_o abE_0}{\pi r} e^{-jk_0 r} \frac{\sin v}{v}\left(\frac{\sin u/2}{u/2}\right)^2 \left(a_\theta \cos\phi - a_\phi \sin\phi \cos\theta\right) \qquad \ldots\ldots (5)$$



Radiation pattern for tapered aperture field

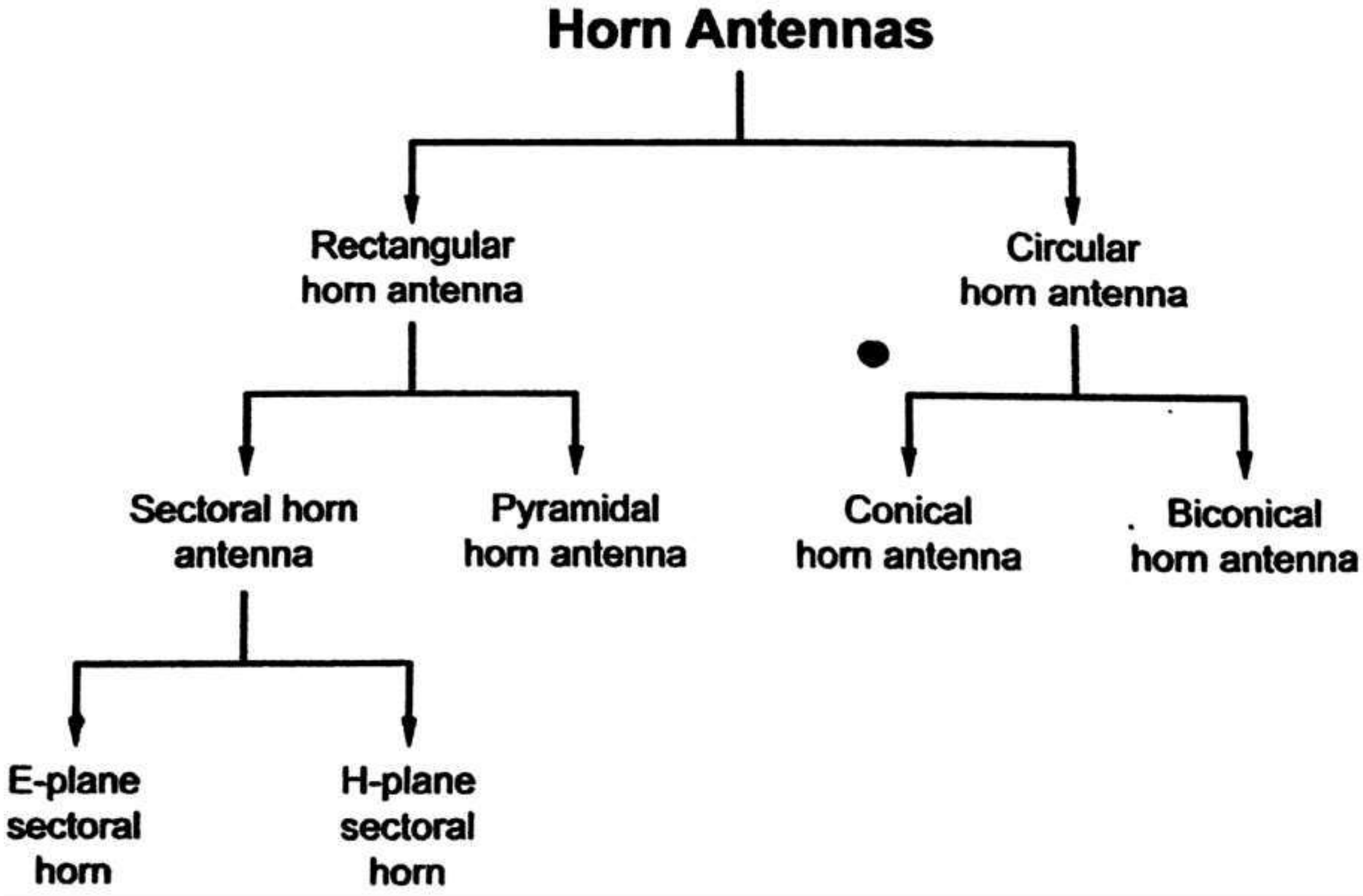
HORN ANTENNA

One of the simplest and probably the most widely used microwave antenna is the horn and may be considered as an aperture antenna.

A horn antenna may be regarded as a ***flared out*** or ***opened out waveguide***. When one end of the waveguide is excited and the other end is kept open, it radiates in open space in all directions.

Types:

1. Rectangular horn antenna
2. Circular horn antenna

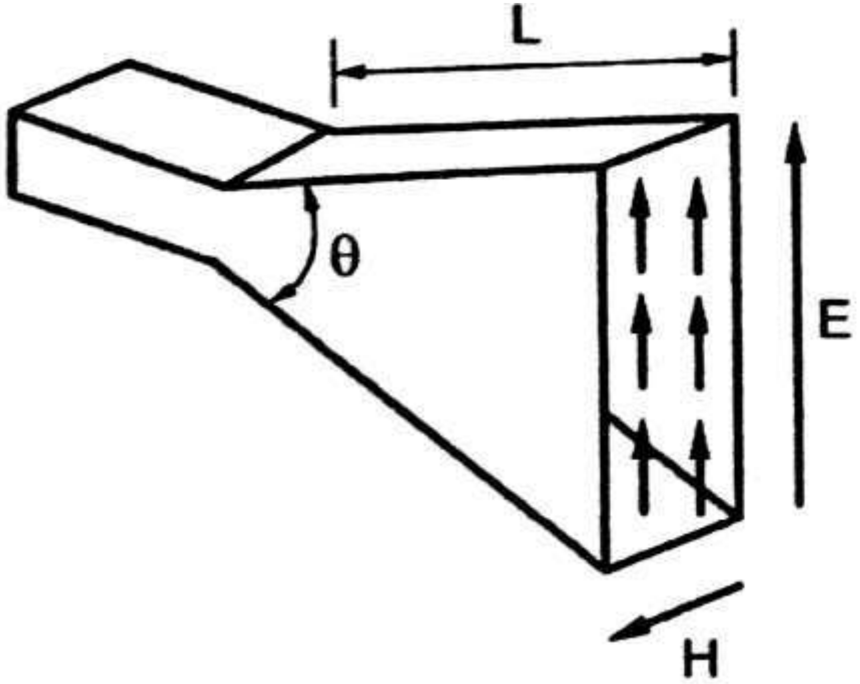

Horn Antennas
Rectangular horn antenna
Circular horn antenna
Sectoral horn antenna
Pyramidal horn antenna
Conical horn antenna
Biconical horn antenna
E-plane sectoral horn
H-plane sectoral horn

1. Rectangular horn antenna

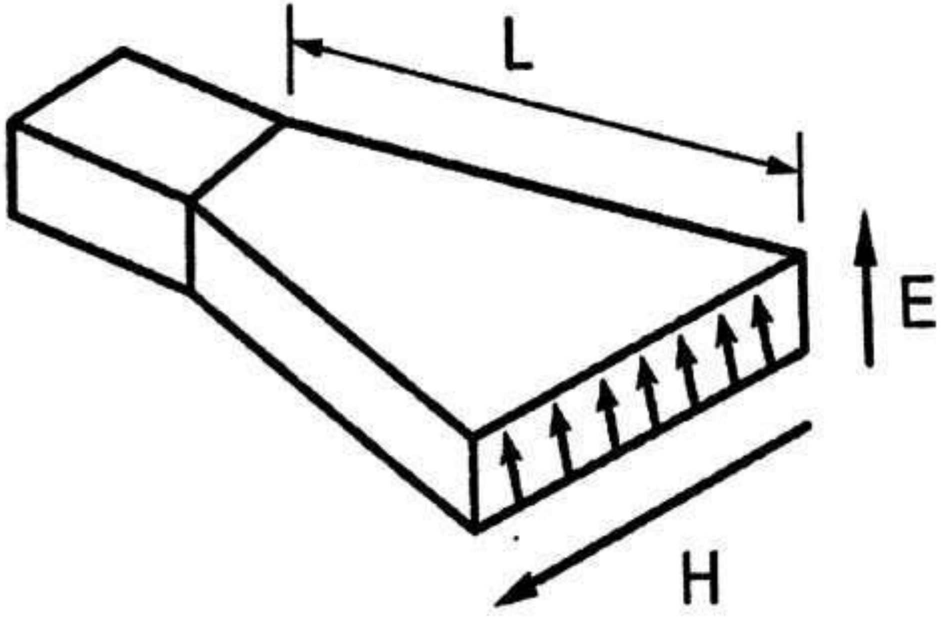
    1. Sectoral horn antenna

2. E – plane sectoral horn – Flaring is done in the direction of the electric field vector

3. H – plane sectoral horn - Flaring is done in the direction of the magnetic field vector

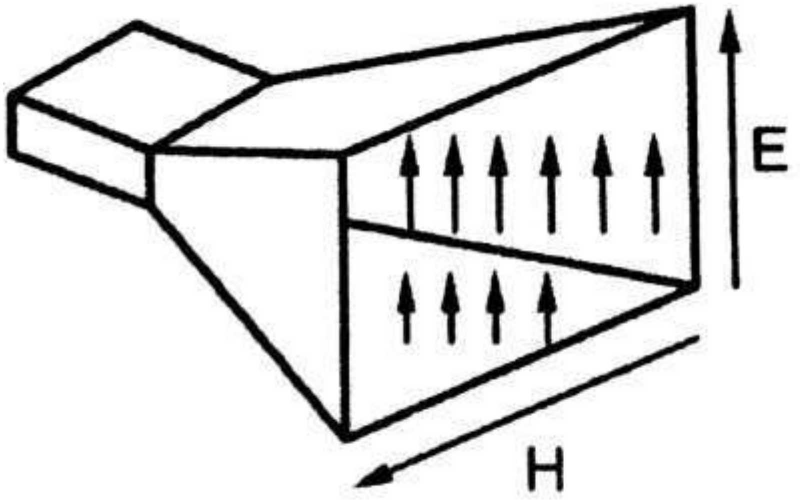


E-plane sectoral horn



H-plane sectoral horn

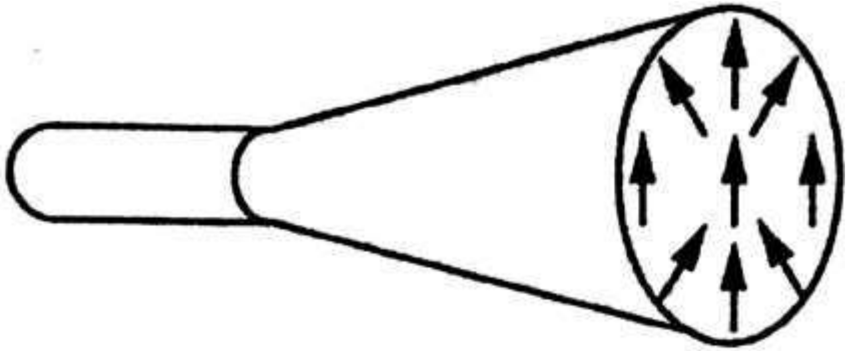
Pyramidal horn antenna - Flaring is done in the direction of both the electric field and magnetic field



Pyramidal horn antenna

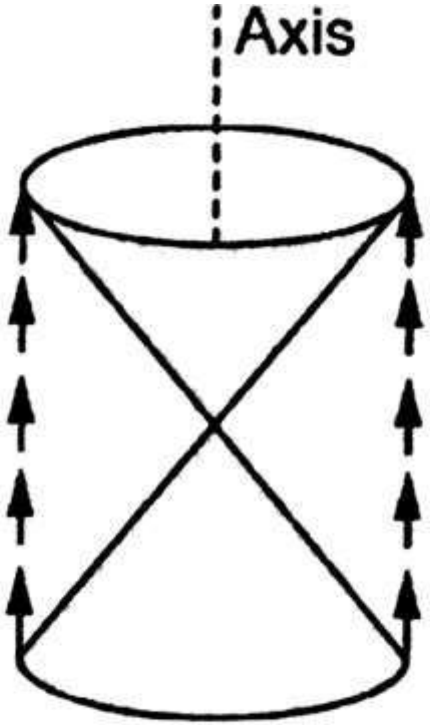
Circular horn antenna

1. Conical horn antenna



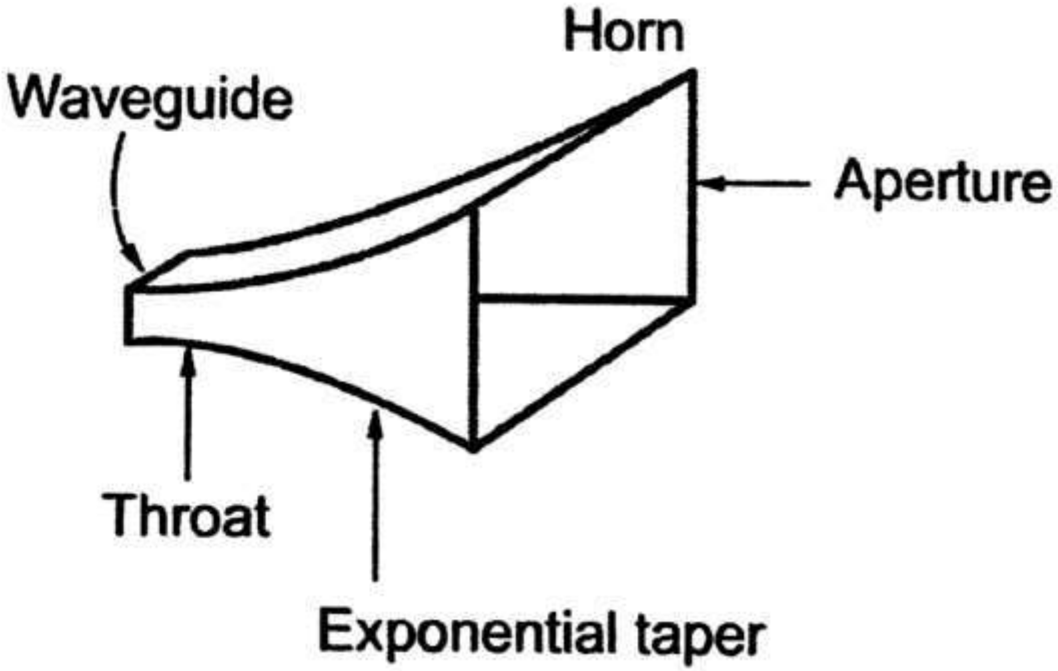
Conical horn

2. Biconical horn antenna

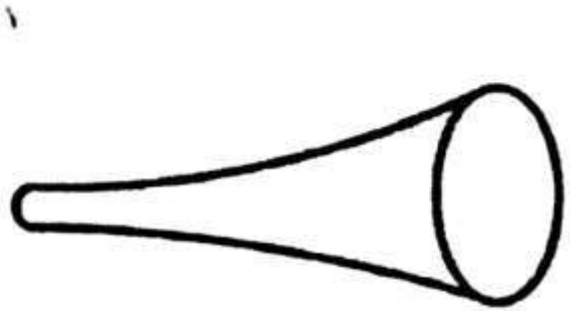


Biconical horn

Exponentially Tapered Horn Antenna



Exponentially tapered pyramidal



Exponentially tapered conical

Principle of Horn Antenna

***Huygene's** principle says that, each point on a primary wave front can be considered to be a new source of a secondary spherical wave and the secondary wave front can be constructed as the envelope of these secondary spherical waves.*

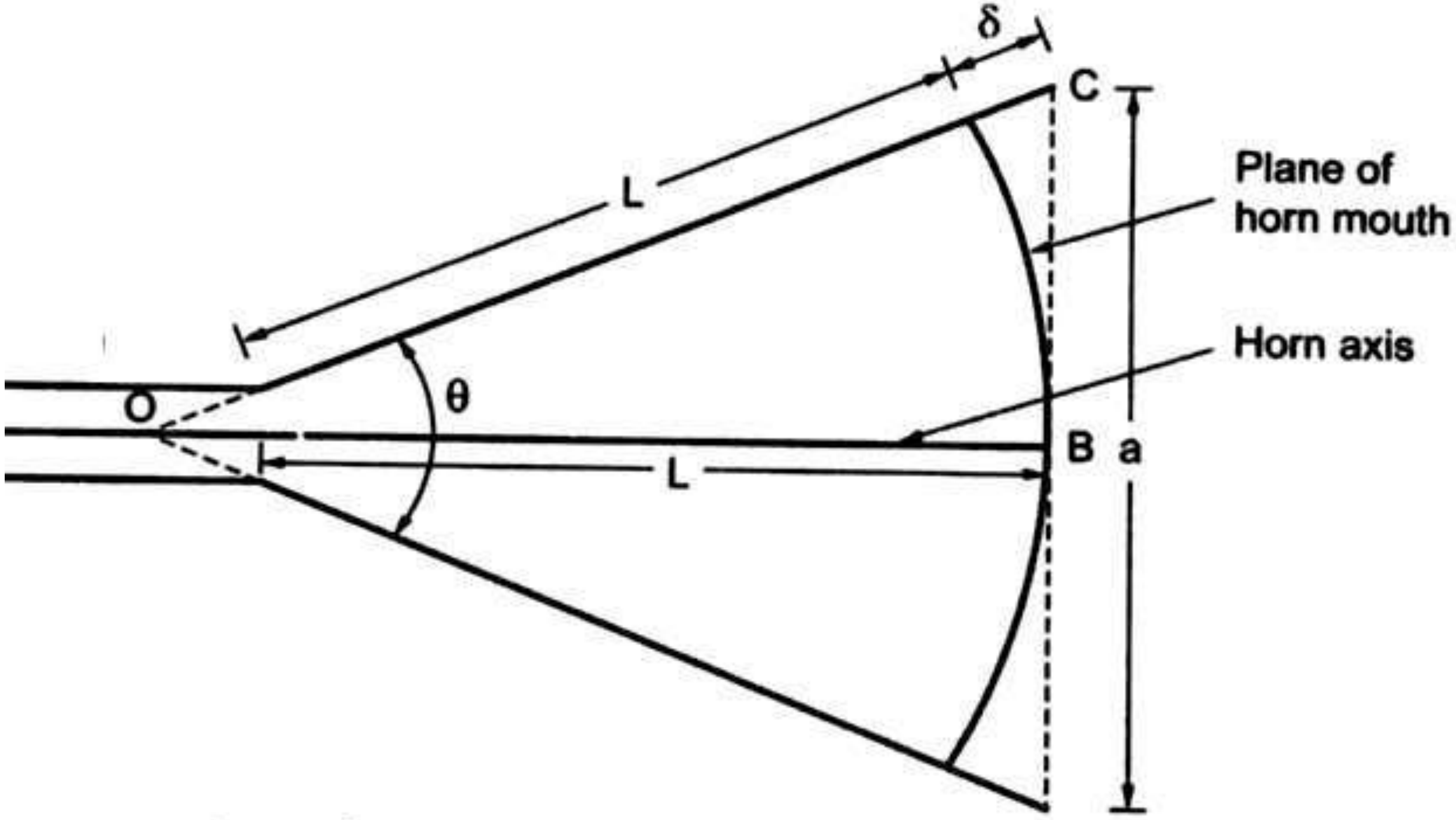
Design of Horn Antenna

Consider a pyramidal horn of length 'L' and aperture height 'a' with flaring along 'θ' as shown in Fig

From the geometry ΔOBC

$$\cos\frac{\theta}{2} = \frac{OB}{OC} = \frac{L}{L+\delta} \qquad \text{......(1)}$$

$$\tan\frac{\theta}{2} = \frac{BC}{OB} = \frac{a/2}{L} = \frac{a}{2L} \qquad \text{......(2)}$$



δ
C
L
Plane of
horn mouth
Horn axis
O
θ
B
a
L

where, θ – Flare angle (θ_E *for E plane*, θ_H *for H plane*) in degree

a – Aperture (a_E *for E plane*, a_H *for H plane*) in m,

L – Length of horn in m, and

δ – Path length difference in m.

The flare angle θ can be expressed from equations (1) and (2) as,

$$\boxed{\theta = 2\tan^{-1}\frac{a}{2L} = 2\cos^{-1}\left(\frac{L}{L+\delta}\right)} \quad \ldots\ldots (3)$$

In the E plane of the horn, δ is usually $0.25\,\lambda$ *or* less and in the H plane, it can be larger or about $0.4\,\lambda$

From triangle OBC,

$$(L+\delta)^2 = L^2 + \left(\frac{a}{2}\right)^2$$

$$L^2 + \delta^2 + 2\,L\,\delta = L^2 + \frac{a^2}{4}$$

If 'δ' is small, then δ^2 can be neglected.

$$\therefore \quad 2\,L\,\delta = \frac{a^2}{4}$$

$$\boxed{L = \frac{a^2}{8\delta}} \qquad \ldots\ldots (4)$$

Equations (3) and (4) are the ***design equations of the horn antenna.***

For an optimum flare horn, the ***half power beam width*** can be approximated as,

$$\theta_H = \frac{67^\circ \lambda}{a_H} = \frac{67\,\lambda}{w} \text{ degree} \qquad \ldots\ldots (5)$$

$$\theta_E = \frac{56^\circ \lambda}{a_E} = \frac{56\lambda}{a} \text{ degree} \qquad \text{......(6)}$$

Assume that there is no loss, the ***directivity*** is given in terms of the effective aperture of the horn as,

$$D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \varepsilon_{ap} A_p}{\lambda^2} \qquad \text{......(7)}$$

where, A_e = Effective aperture in m^2

A_p = Physical aperture in m^2 = Area of horn mouth opening, and

$\varepsilon_{ap} = \frac{A_e}{A_p}$ = Aperture efficiency

For a pyramidal rectangular horn,

$$A_p = a_E \cdot a_H = a \times w \qquad \text{........(8)}$$

where, $a = \textit{Height of the aperture} = a_E =$ E-plane aperture in m

$w = \textit{Height of the aperture} = a_H =$ H-plane aperture in m

Similarly for a conical horn,

$$A_p = \pi r^2 \qquad \text{.........(9)}$$

where, $r =$ Radius of aperture in metre

For example if $a_E = a_H = \lambda = 1$ m and $\varepsilon_{ap} \approx 0.6$, then the directivity of the rectangular horn is given by

$$\boxed{D = \frac{4\pi(0.6) A_p}{\lambda^2} \approx \frac{7.5 A_p}{\lambda^2}} \qquad \text{.........(10)}$$

$$\boxed{D(\text{dB}) \approx 10 \log_{10} \frac{7.5 A_p}{\lambda^2}} \qquad \text{......... (11)}$$

Reflector Antenna

Reflector type of antennas or reflectors are widely used to modify the radiation pattern of a radiating element

Reflector antenna means a reflector of suitable size and shape, which may produce a direct radiation(energy) in a desired direction

The antenna which is a radiating source in the reflector antenna is called ***primary antenna or feed***, while the reflector antenna is called the ***secondary antenna***. The most common feeds are dipole, horn and slot.

Types of Reflector antennas

(*i*) Plane reflector *or* flat sheet reflector,

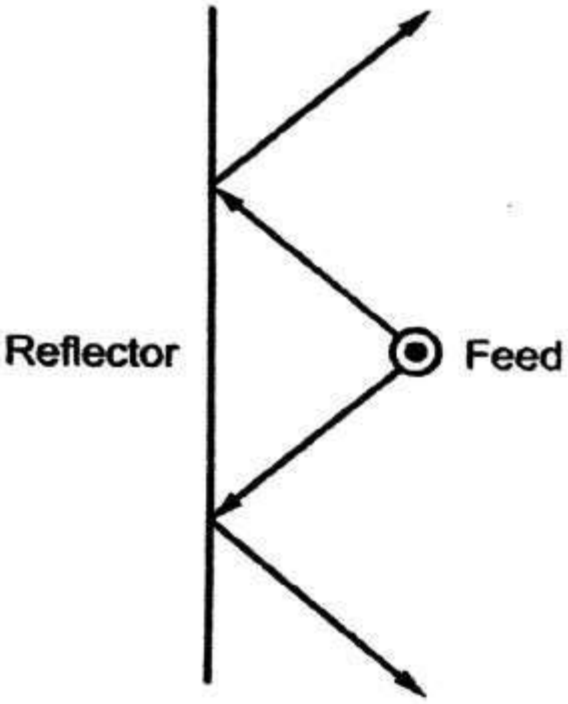
(*ii*) Corner reflector,

(*iii*) Parabolic reflector,

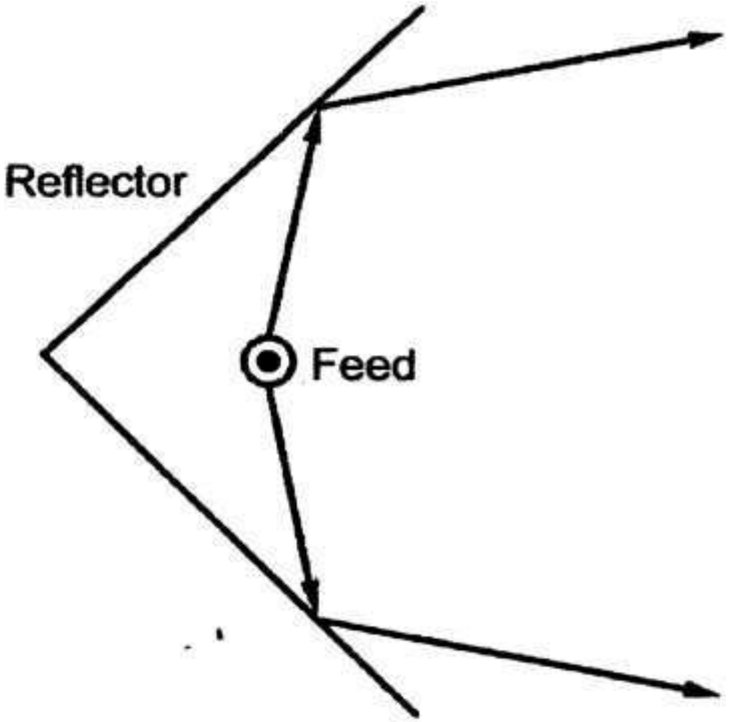
(*iv*) Hyperbolic reflector,

(*v*) Elliptical reflector, and

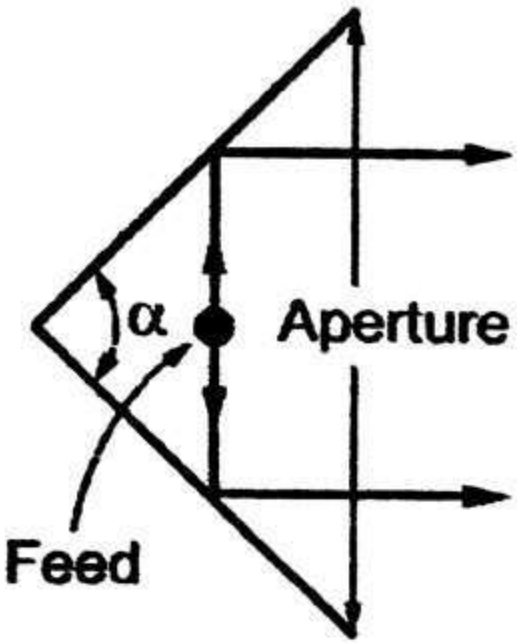
(*vi*) Circular reflector.



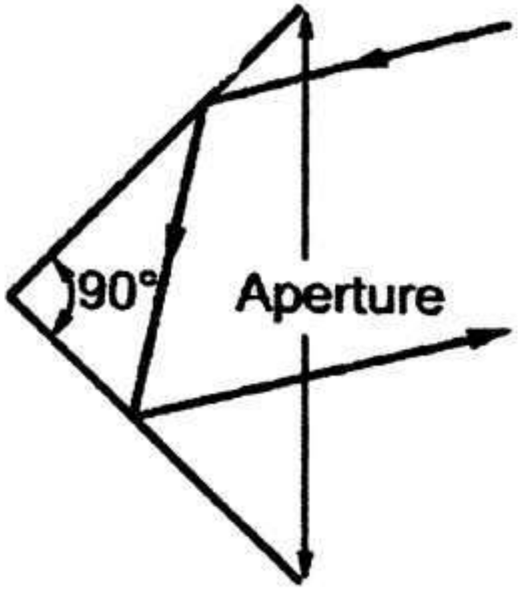
(a) Plane reflector



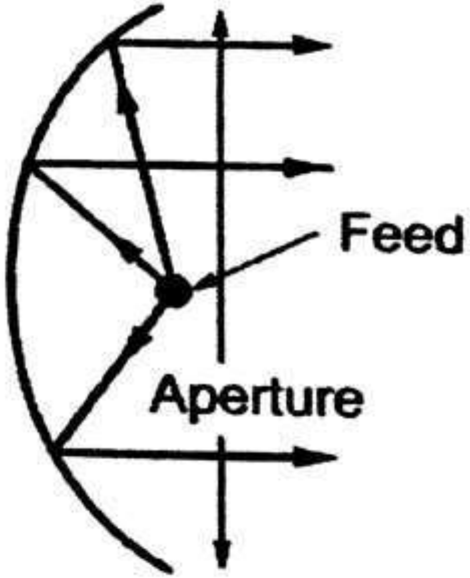
(b) Corner reflector



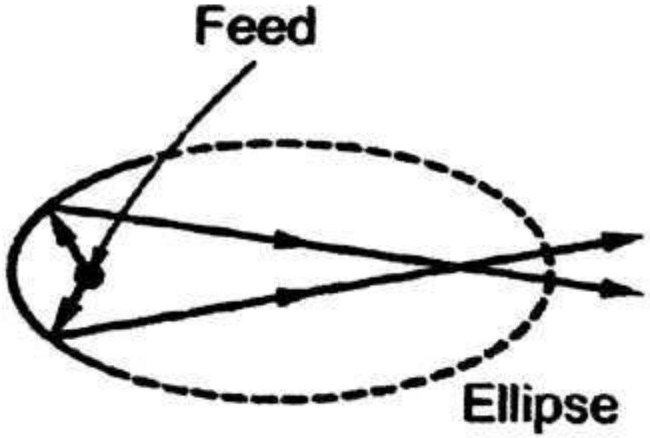
(C). Active corner reflector



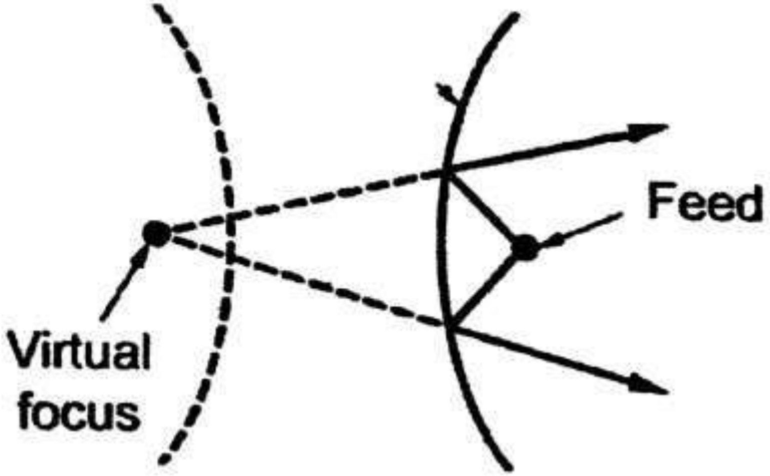
(d). Passive corner reflector



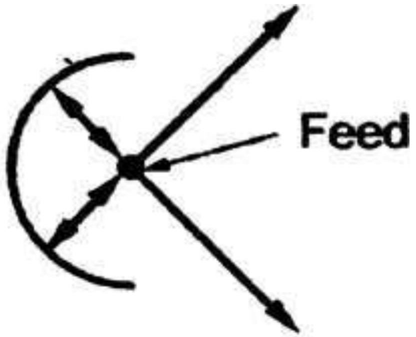
(d). Parabolic reflector



(e).Elliptical reflector



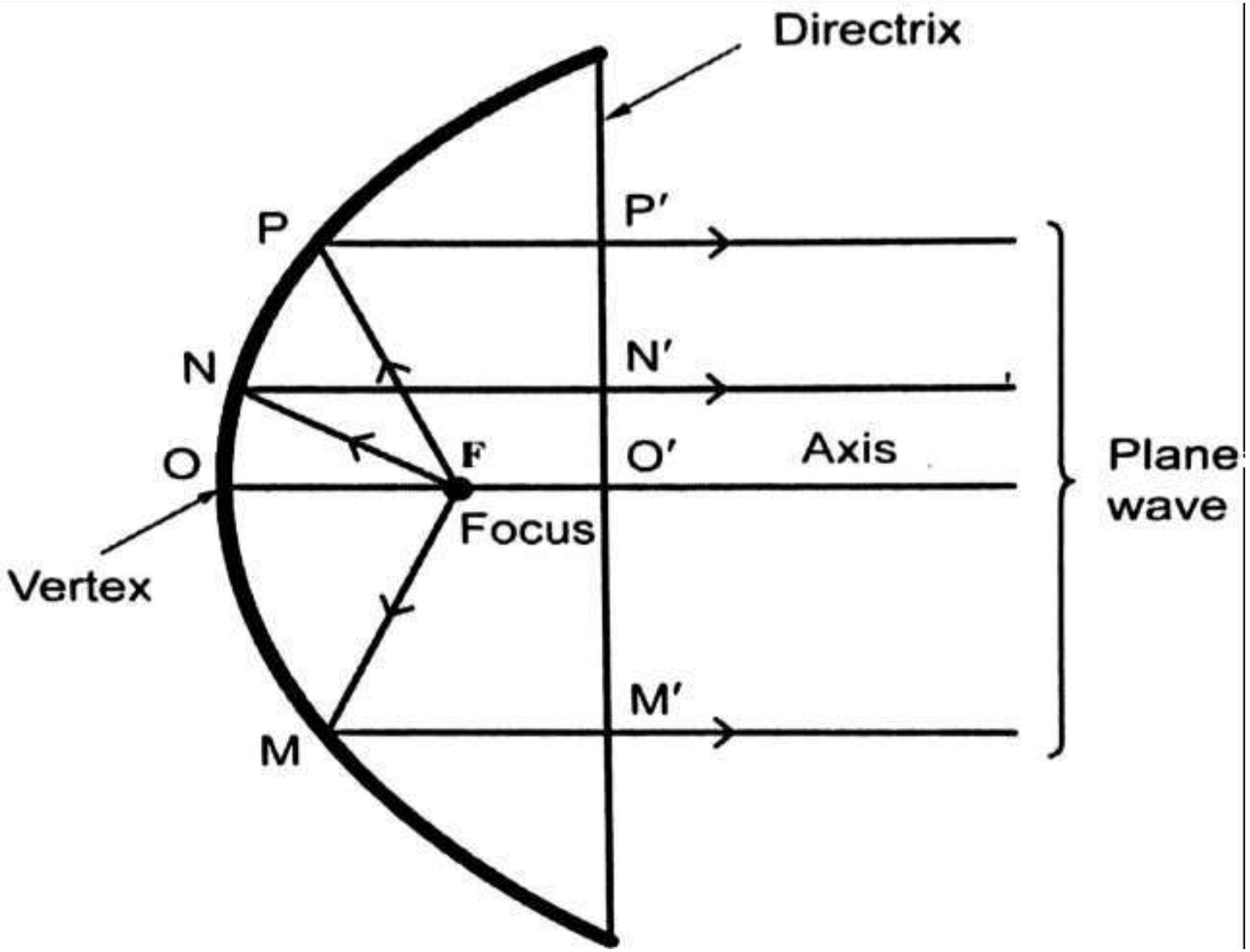
(f) Hyperbolic reflector



(g) Circular reflector

Parabolic Reflector

*The **parabolic structure** is used to improve the overall radiation characteristics such as antenna pattern, antenna efficiency, polarization etc of the reflector antenna.*



$$FN + NN' = FP + PP' = FM + MM'$$

The parabola is a two dimensional plane curve.

Where,

$$OF = \text{Focal length}$$

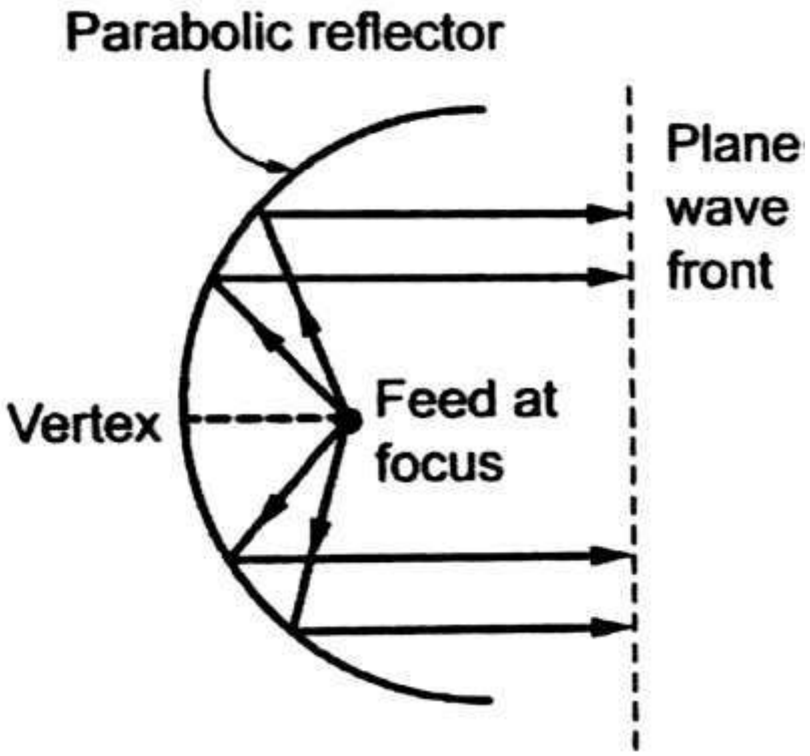
$$F = \text{Focus}$$

$$O = \text{Vertex}$$

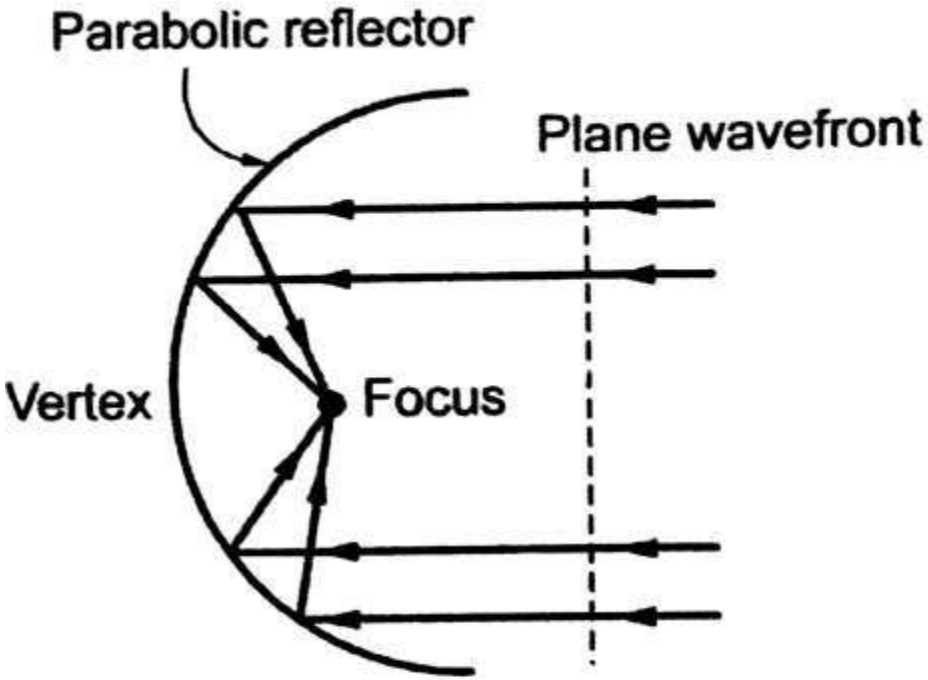
$$OO' = \text{Axis of parabola}$$

By the geometrical optics, when the point source is placed at the focal point, then the rays reflected by the parabolic reflector form a parallel wave front. This principle is normally used in the ***transmitting antenna****.*

Similarly at the receiving antenna, when the beam of parallel rays is incident on a parabolic reflector, then the radiations focus at a focal point.



(a) Parabolic reflector at transmitting end

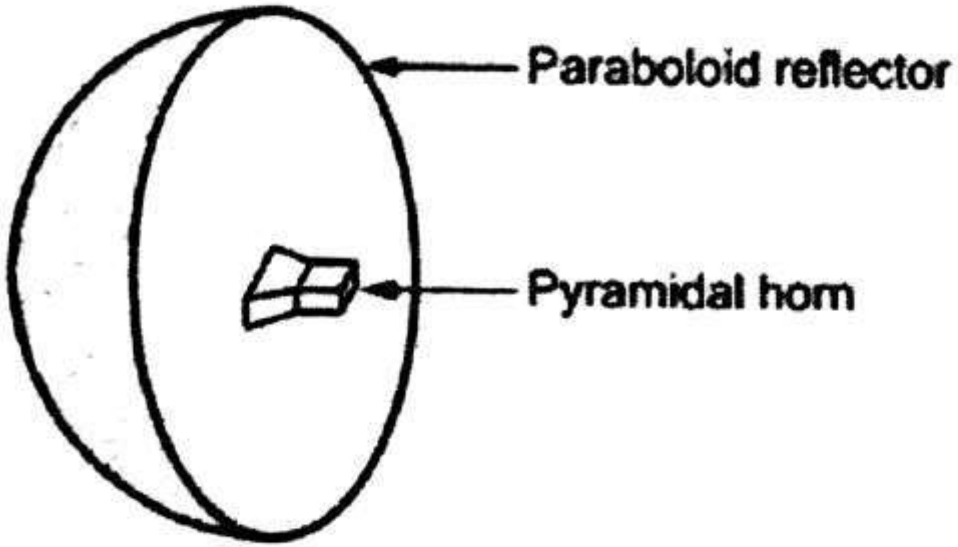


(b) Parabolic reflector at receiving end

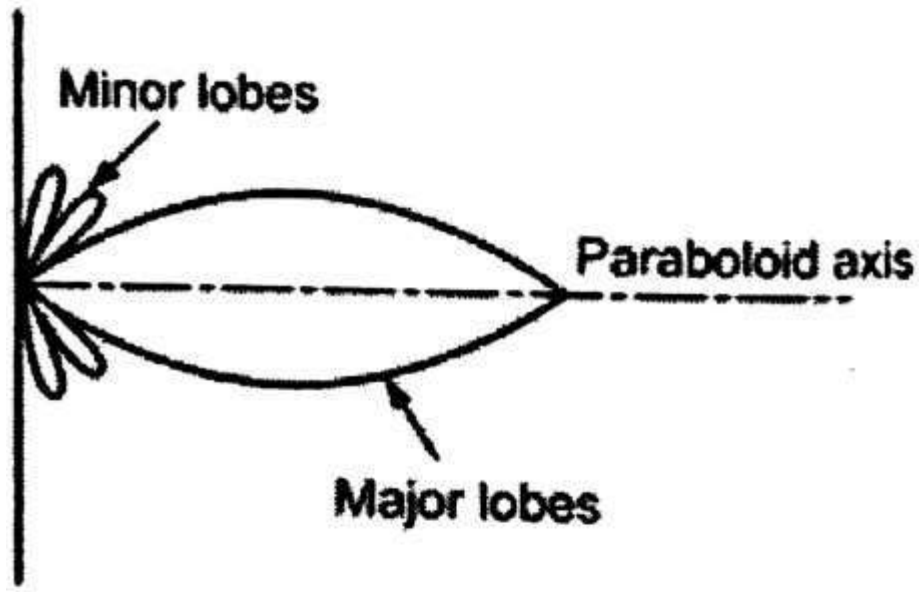
The open mouth (D) of the parabola is known as the aperture. The ratio of focal length to aperture (*i.e.*, *f*/D) is known as ***"f over D ratio"*** and it is an important characteristics of parabolic reflector (*f*/D varies from 0.25 to 0.50).

PARABOLOID (OR) PARABOLOIDAL REFLECTOR (OR) MICROWAVE DISH

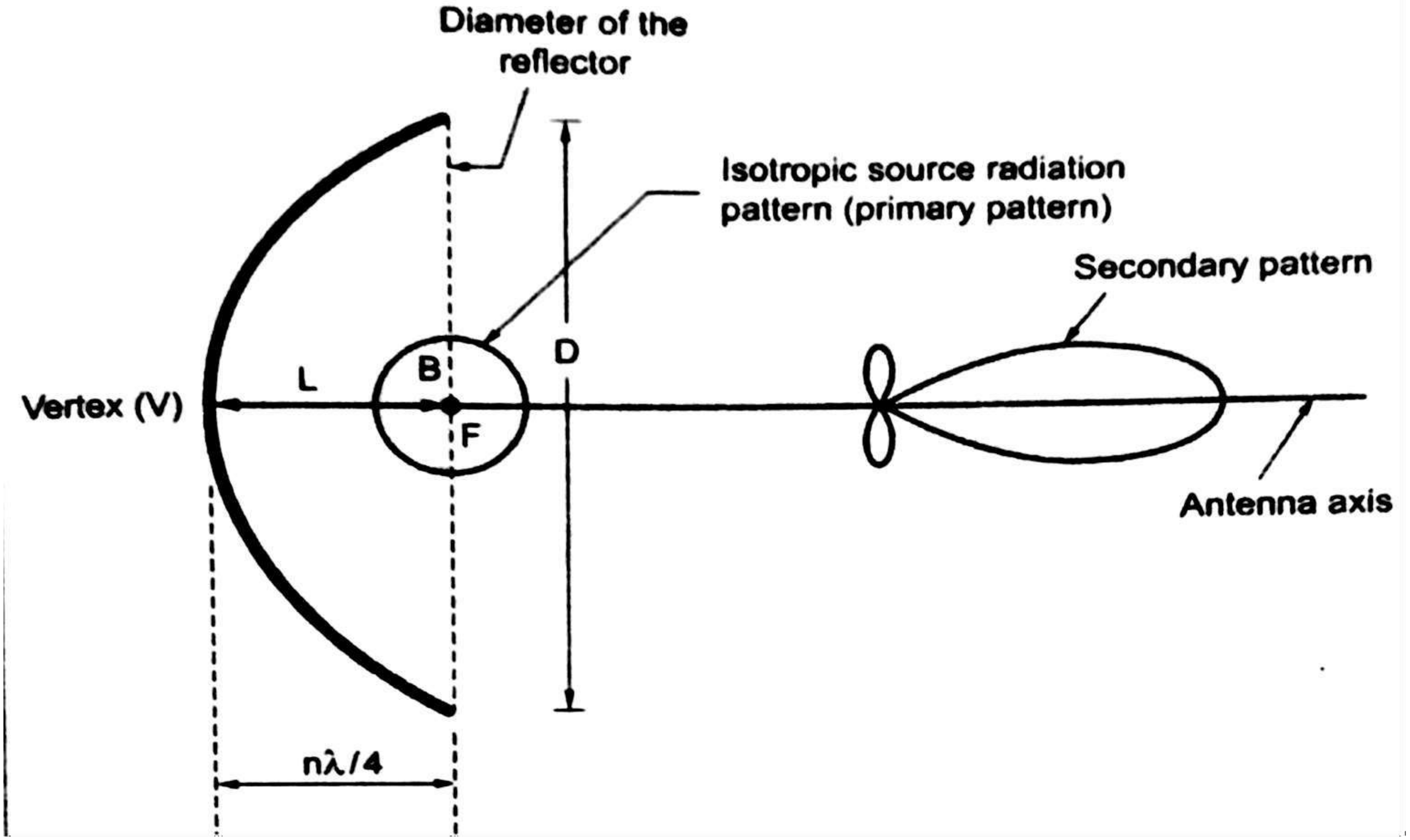
A parabola is a two dimensional plane curve. In practical applications, a three dimensional structure of the parabolic reflector is used.



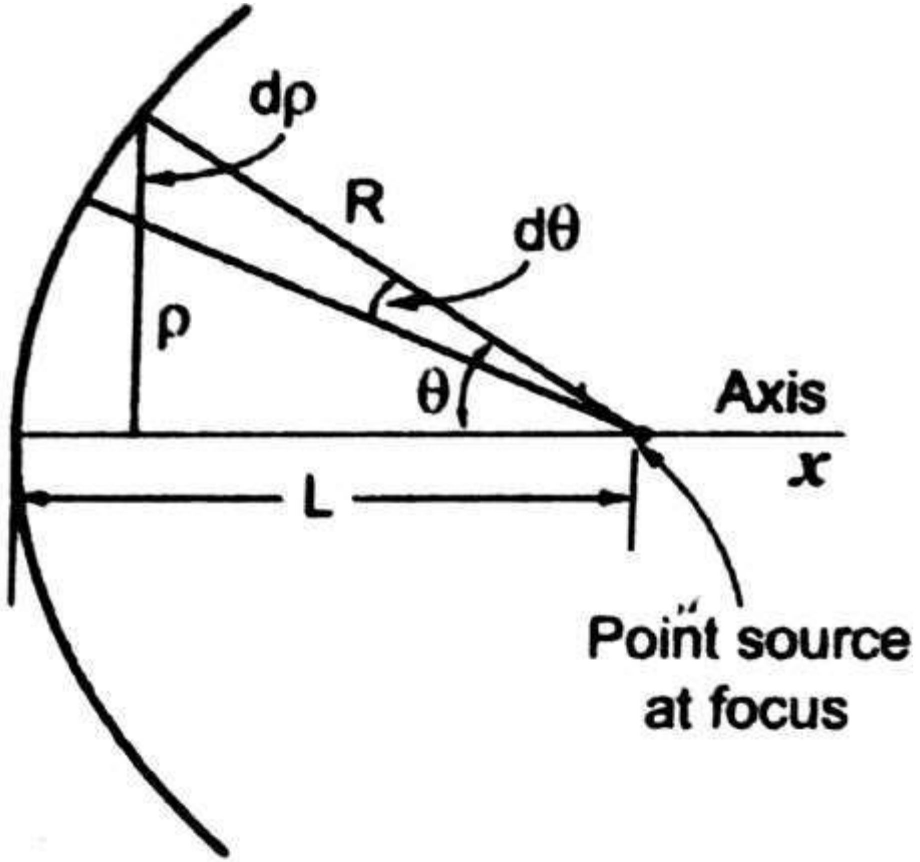
(a) Paraboloid



(b) Radiation pattern


Diameter of the reflector
Isotropic source radiation pattern (primary pattern)
Secondary pattern
D
B
L
Vertex (V)
F
Antenna axis
nλ/4

Field Distribution



Cross sections of paraboloid

Consider a paraboloid with an isotropic source used as a line source as given in Fig.5.21(b). The total power 'P' from distance of 'ρ' from the axis and strip of width '$\partial\rho$' is expressed as,

$$P = 2\pi\,\rho\,d\rho\,S_\rho$$

Where, S_ρ is the power density at a distance ρ from the axis, W/m^2

This power must be equal to the power radiated by the isotropic source over the solid angle $2\pi \sin\theta\, d\theta$.

$$P = 2\pi \sin\theta\, d\theta\, U$$

Where, U is the radiation intensity, W/sr

$$2\pi\rho\, d\rho\, S_\rho = 2\pi \sin\theta\, d\theta\, U$$

$$\frac{S_\rho}{U} = \frac{\sin\theta}{\rho(d\rho/d\theta)}$$

Where, $\rho = R\sin\theta = \dfrac{2L\sin\theta}{1+\cos\theta}$ $\quad \left|\therefore R = \dfrac{2L}{1+\cos\theta}\right.$

$$\boxed{S_\rho = \frac{(1+\cos\theta)^2}{4L^2}U}$$

The ratio of the power density $\dfrac{S_\theta}{S_0} = \dfrac{(1+\cos\theta)^2}{4}$

The field-intensity ratio $\dfrac{E_\theta}{E_0} = \dfrac{1+\cos\theta}{2}$

Feeding systems or structures:

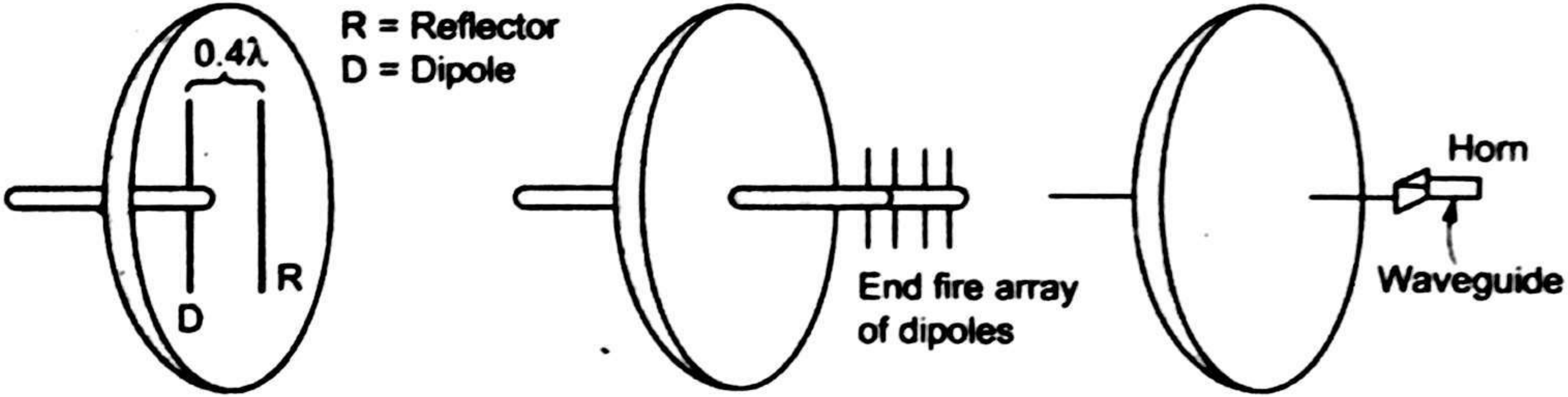
Parabolic reflector antenna consists of two basic parts

1. A source of radiation placed at the focus called primary radiator or feed

2.The reflector called secondary radiator

The various feed used in reflectors are

3. Dipole antenna
4. Horn antenna
5. End fire antenna
6. Cassegrain feed
7. Offset feed



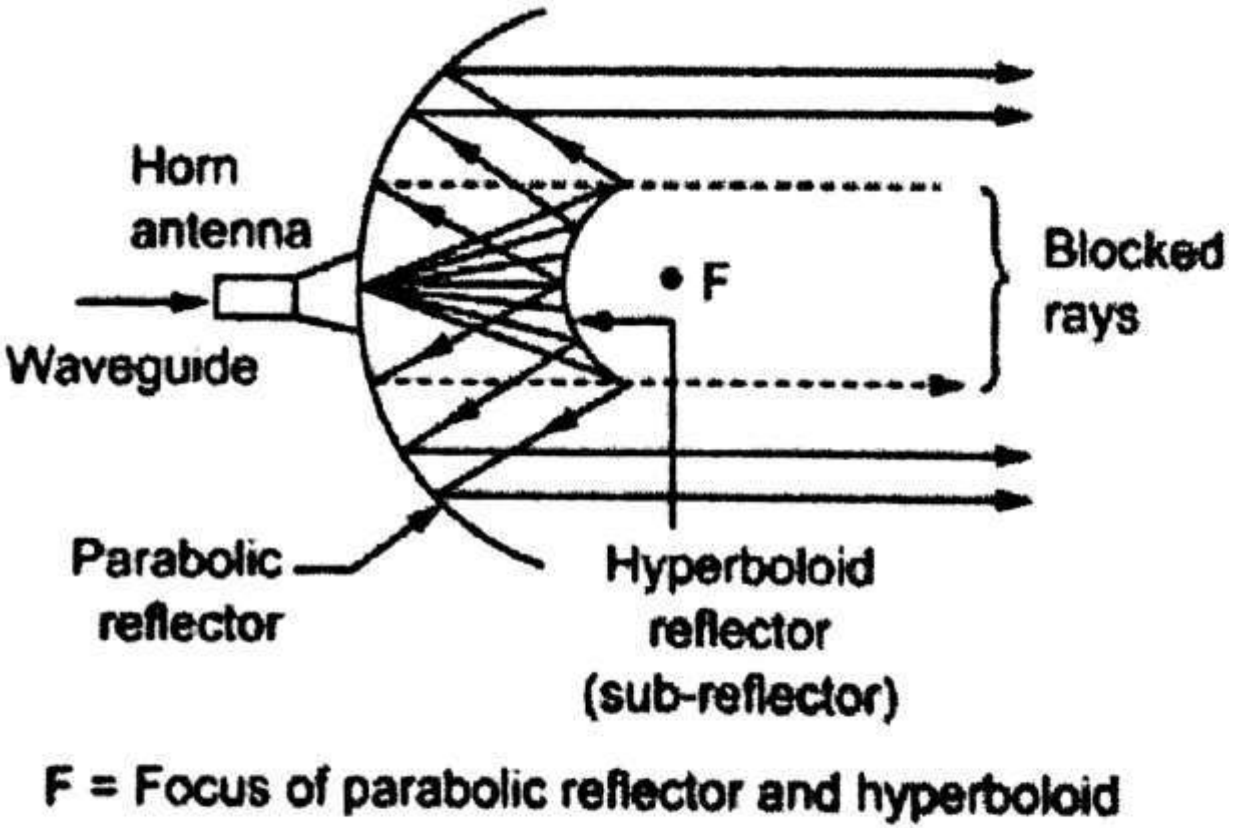
(a) Dipole with plane reflector

(b) End fire array of dipole

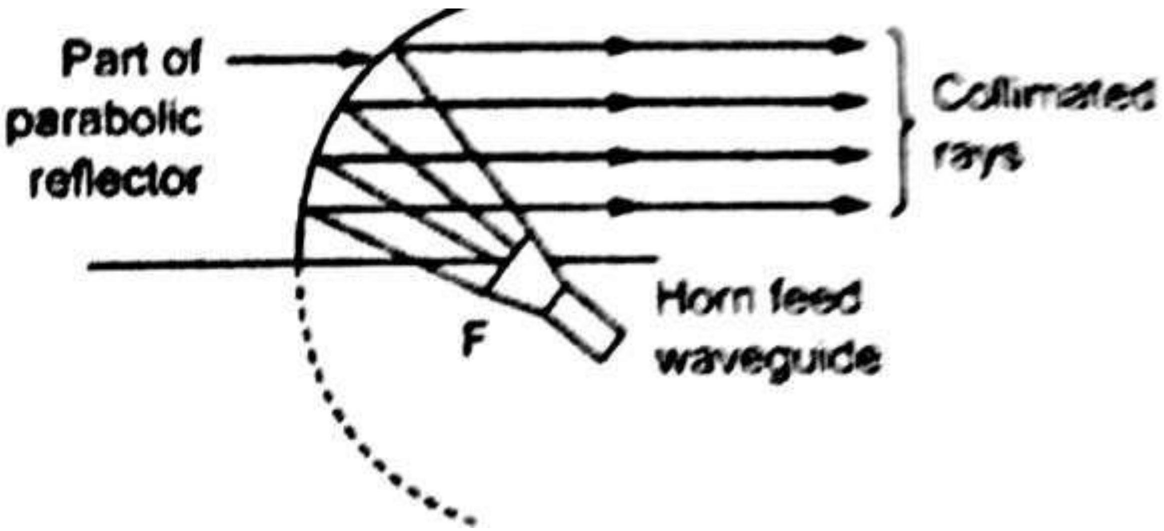
(c) Horn with waveguide

Different types of Feed system

Cassegrain feed

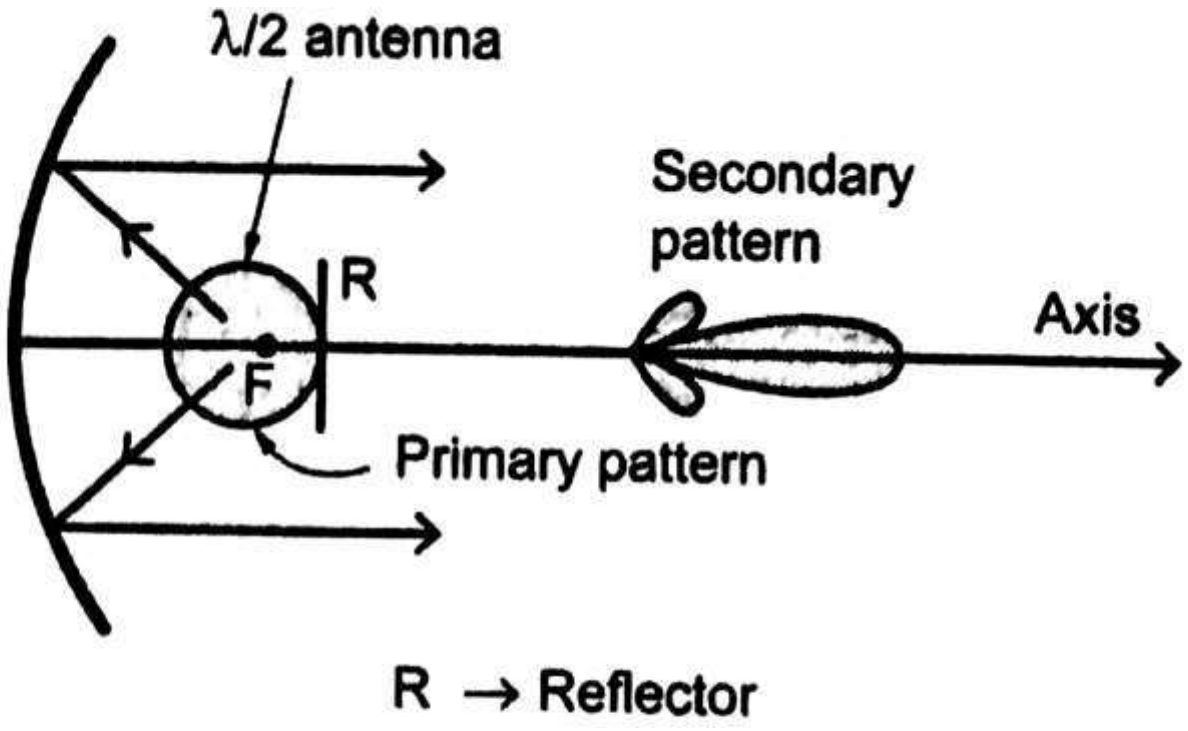


Cassegrain feed system



Offset feed system

Aperture Blockage



Full parabolic reflector using $\frac{\lambda}{2}$ antenna

Slot Antennas

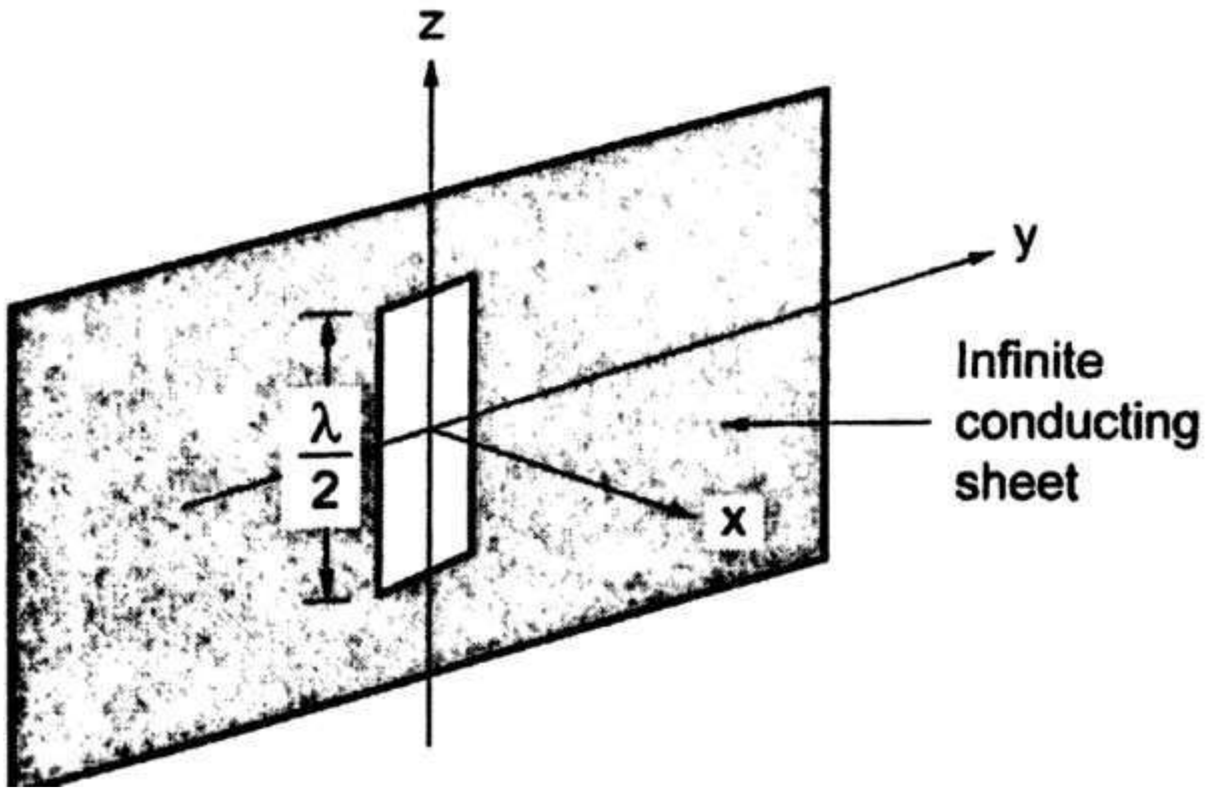
The slot antenna is an opening (slot) cut in a sheet of conductor which is energized through a co-axial cable or wave guide

It is the best suitable radiator at frequencies above 300MHz

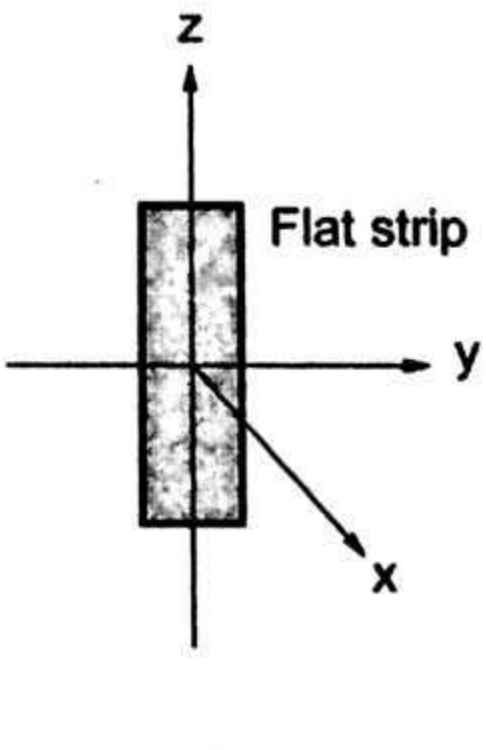
The shape, size and operating frequency of the slot determines the radiation pattern

Whenever a high frequency field exists across a very narrow slot in an infinite conducting sheet, the energy is radiated through that slot

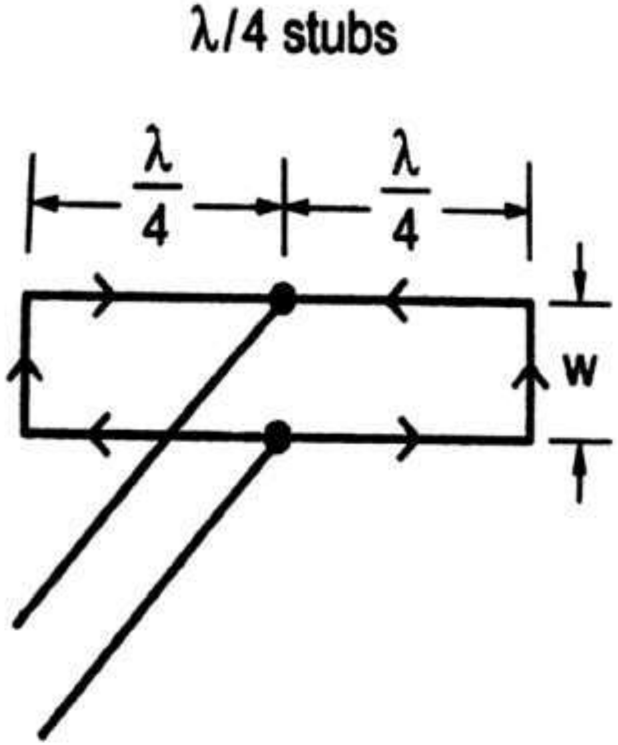
Construction



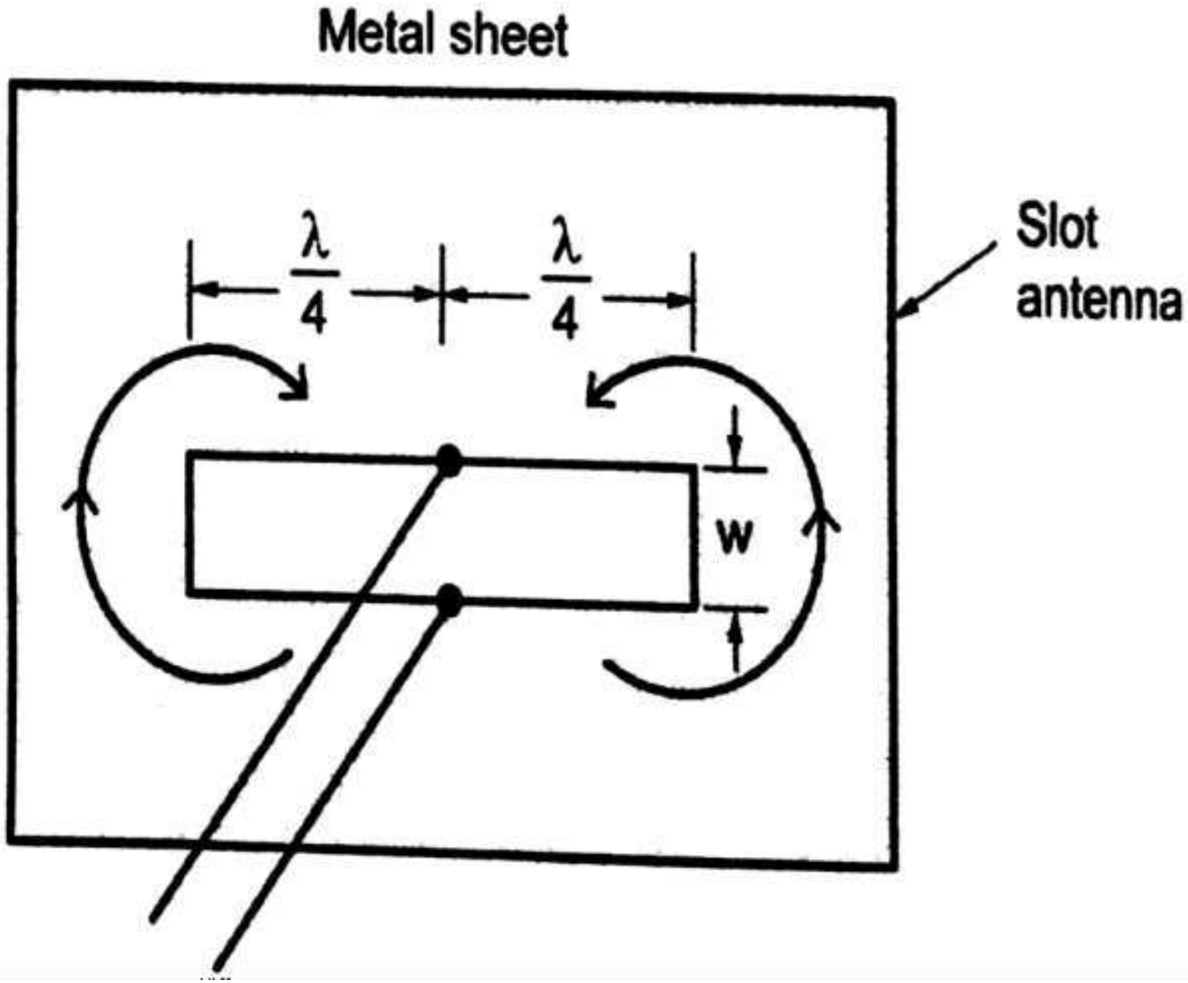
Metallic conducting sheet (slot antenna)



complementary flat strip

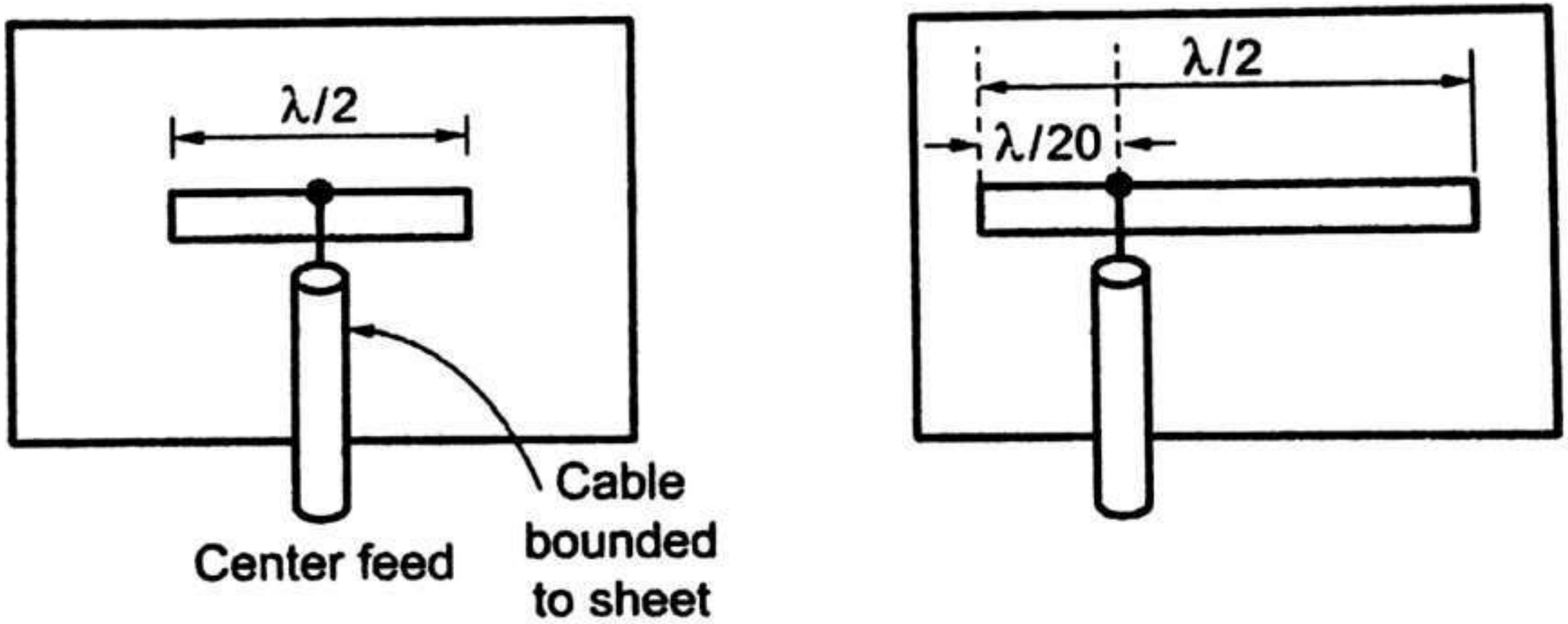


Radiator using stubs



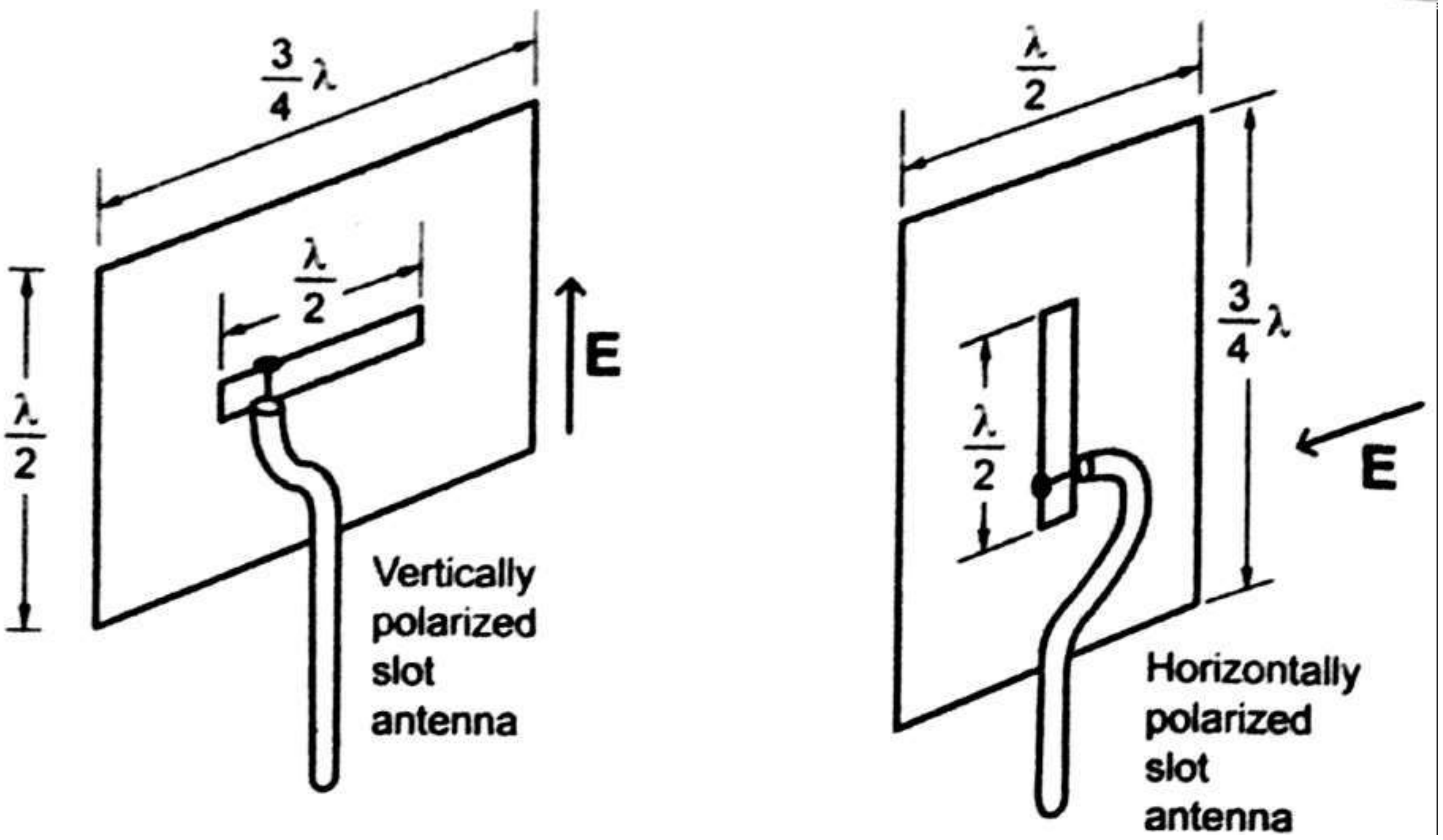
slot antenna

Method of feeding for Slot Antenna

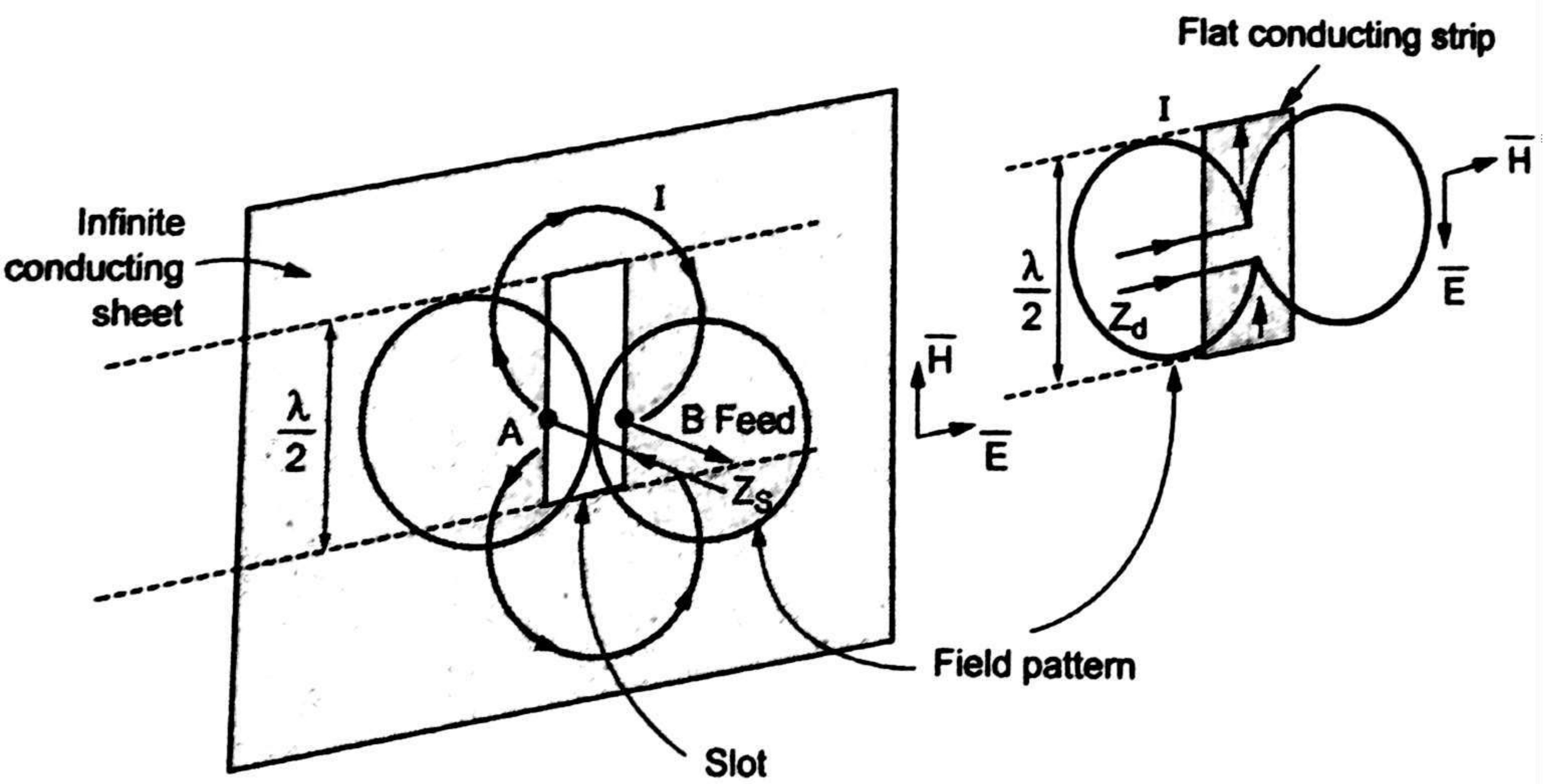


(a) Center feed *(b) Off-center feed*

Types of Slot Antenna



Working Principle: Pattern of the Slot Antenna



Slot and complementary dipole antenna

If, $Z_S \rightarrow$ Terminal impedance of the slot, and

$Z_d \rightarrow$ Terminal impedance of the dipole,

Then, Z_S and Z_d are related to each other in terms of intrinsic impedance of the free space η_0 and it is expressed as,

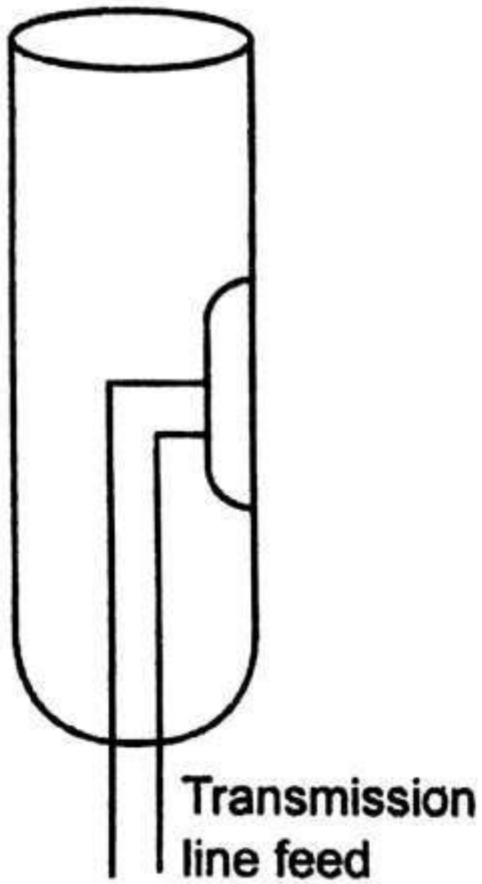
$$Z_d \cdot Z_S = \frac{\eta_0^2}{4} = \frac{(376.7)^2}{4} \approx 35,476 \qquad (\because \eta_0 = 120\,\pi \text{ ohms})$$

Hence the terminal impedance of the slot antenna is given as,

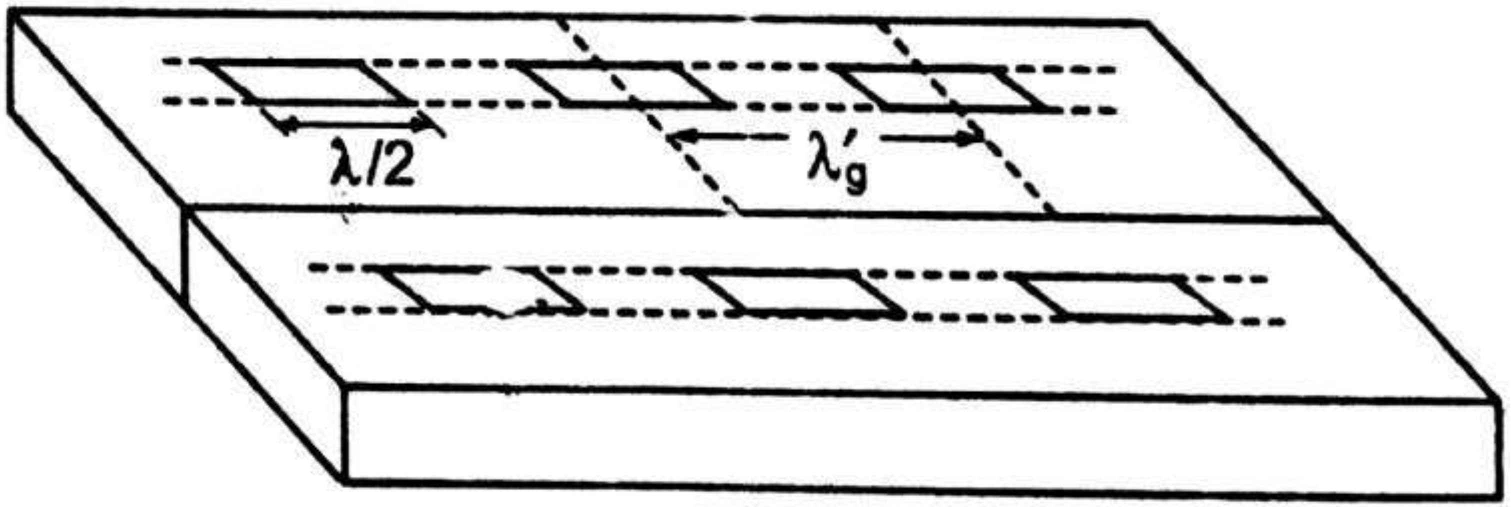
$$\boxed{Z_S = \frac{35,476}{Z_d}} \text{ or } \boxed{Z_S = 35,476\, Y_d} \qquad \text{.......(1)}$$

where, $Z_d = 73 + j\,42.5$ ohms

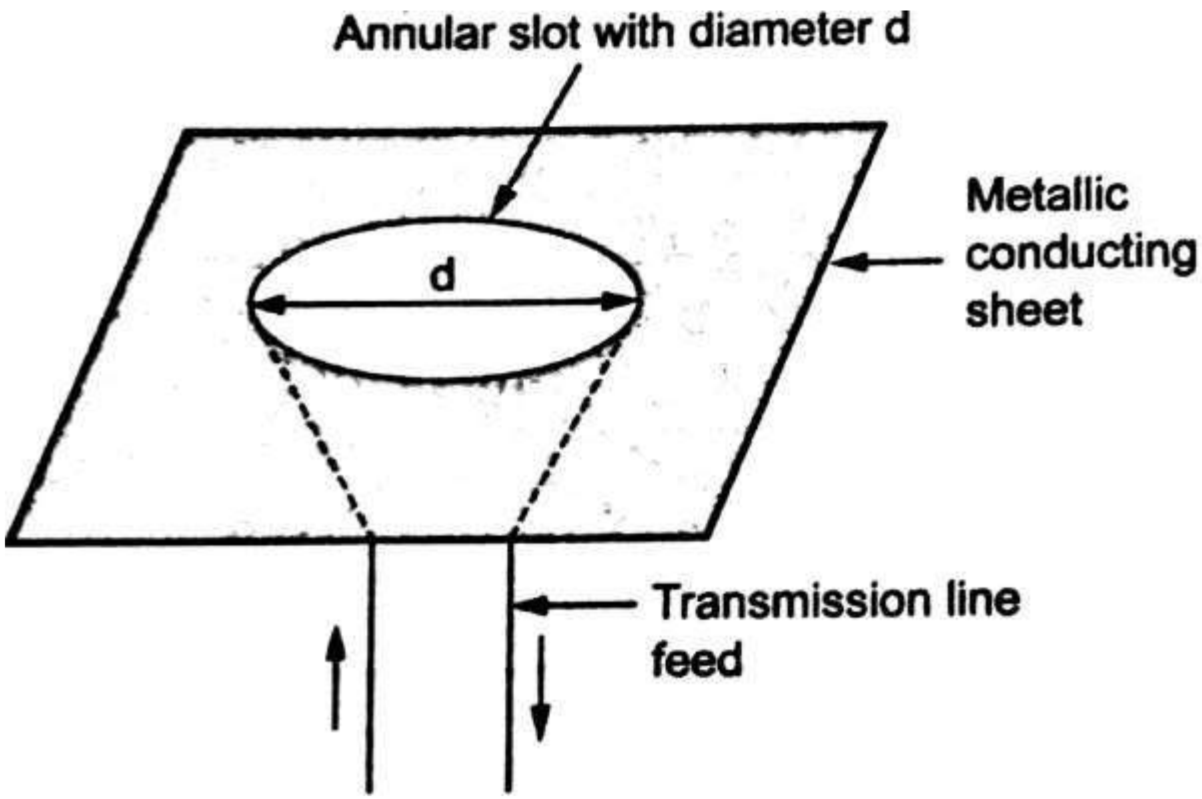
Various shapes of slot antenna



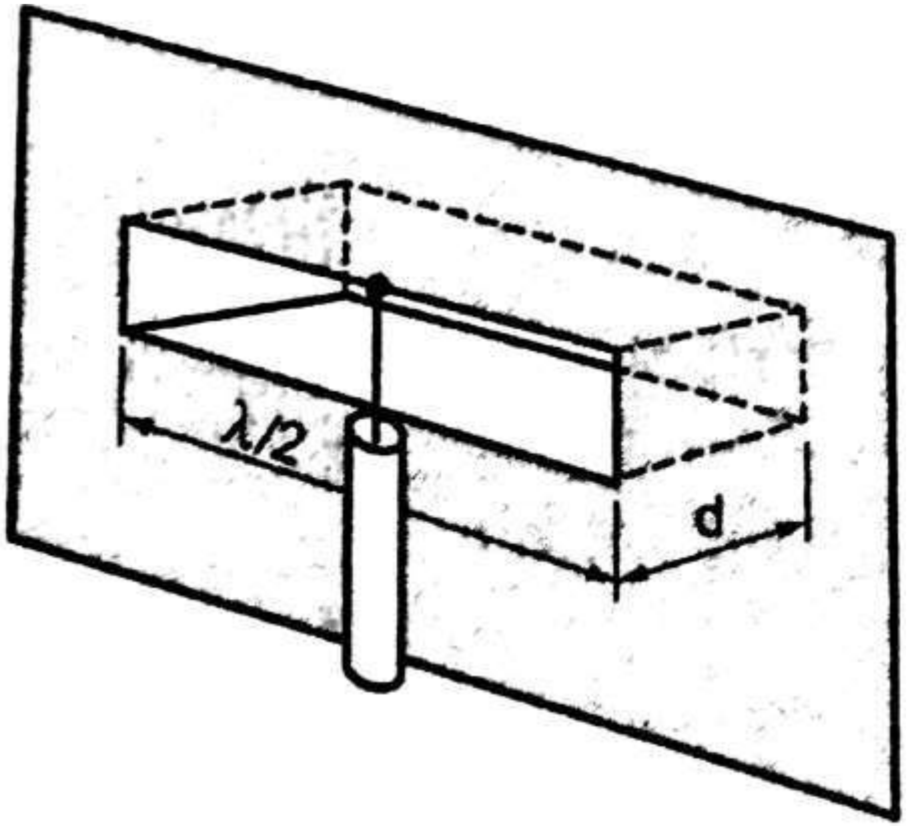
Slotted cylinder antenna



Planar array of slot antenna



Annular slot antenna



Boxed-in slot antenna

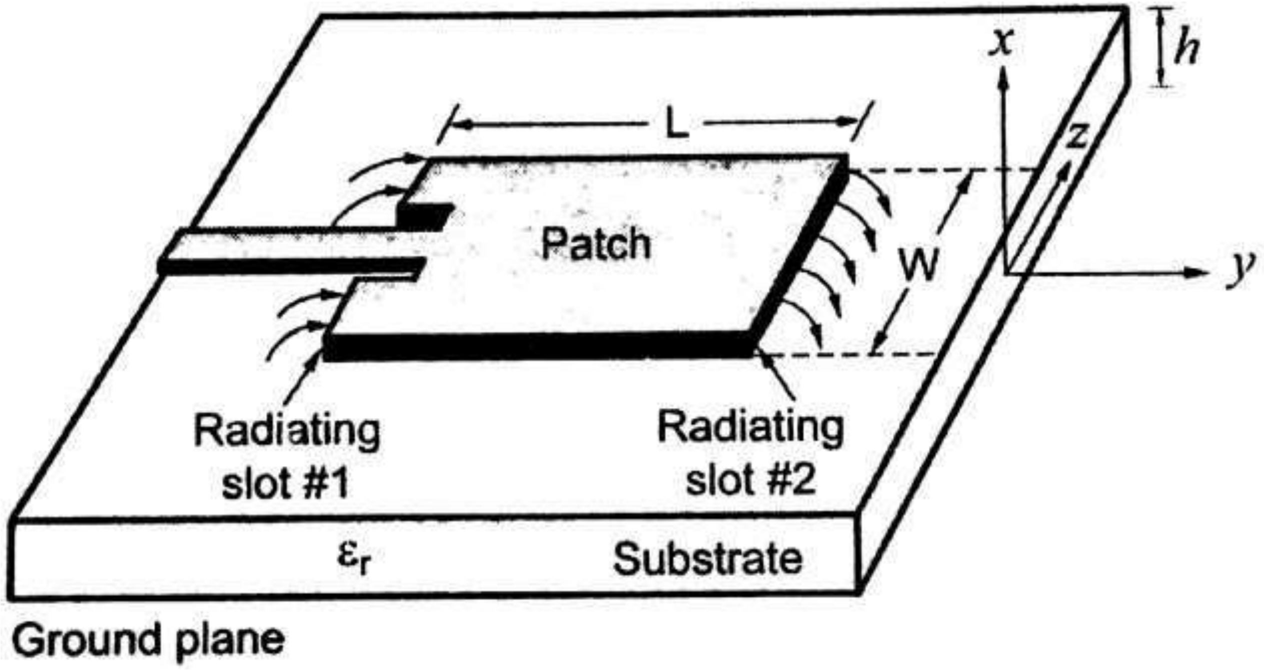
Microstrip Antennas (MAS) or Patch Antenna

The Antenna which is made up of metal patches placed on dielectric and fed by microstrip or coplanar transmission line is called ***microstrip antenna****. It is also called as* ***patch antenna*** *or* ***microstrip patch antenna****.*

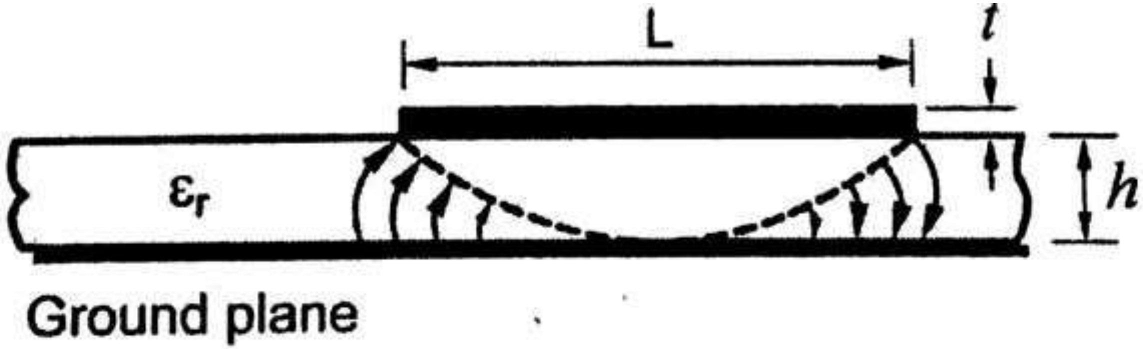
The simplest patch antenna uses a half-wavelength long patch with a larger ground plane to give better performance but at the cost of larger antenna size.

As the MSA are directly printed on to the circuit boards, so it is also called as ***printed antenna.*** The micro strip antenna is construcied on a thin dielectric sheet which uses a printed circuit board and etching techniques.

construction



Microstrip antenna



Side view

Types of patch in Microstrip Antenna

The following features are common for all MSA

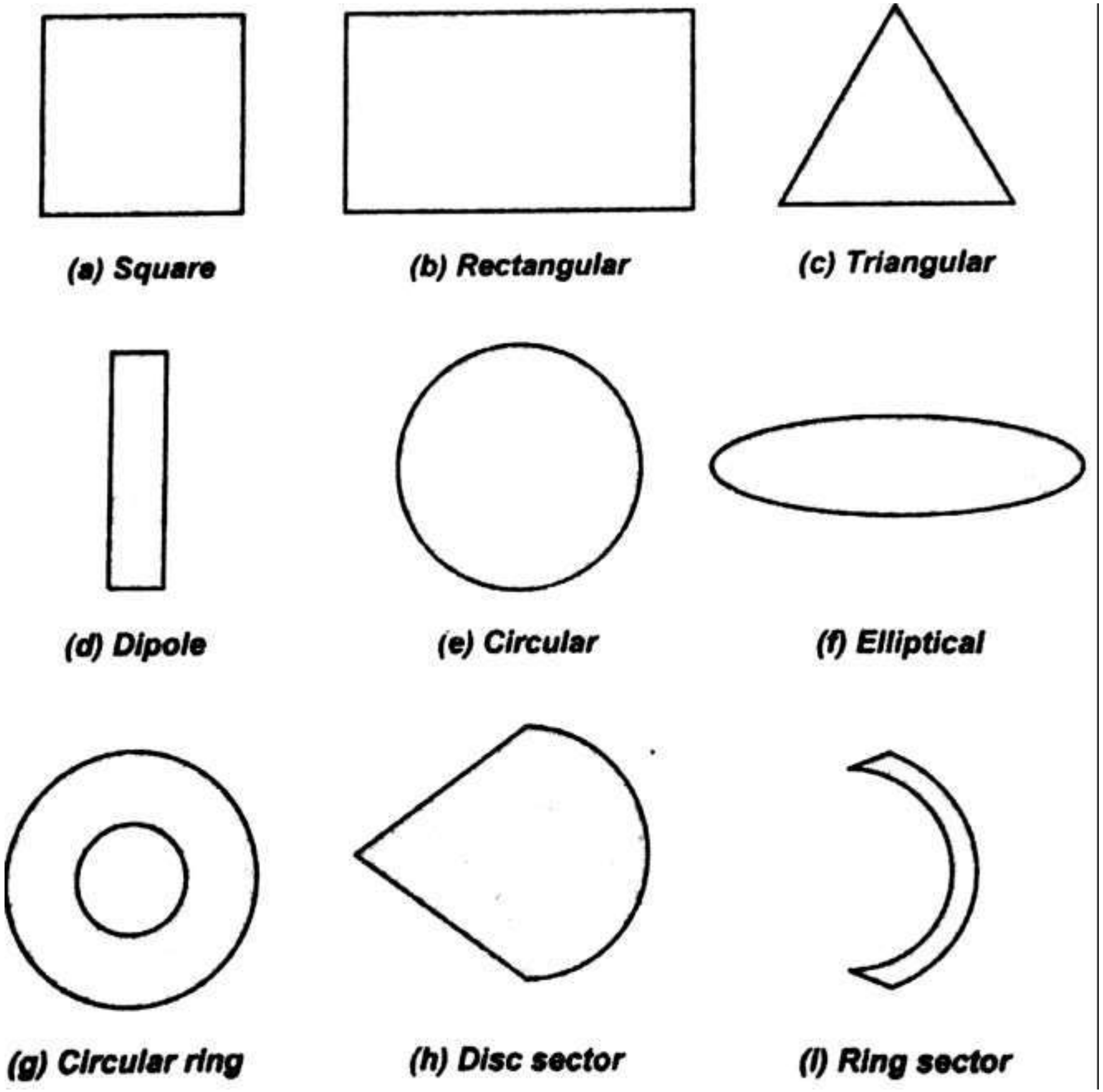
(i) A thin, flat metallic region which is commonly called patch

(ii) A dielectric substrate

(iii) A ground plane which is much larger than patch considering dimensions

(iv) A feed network which supplied power to antenna elements

In microstrip antenna, the radiating element and the feed lines are generally photo etched on the dielectric substrate



Different shapes of patch in microstrip antenna

Feed methods of Microstrip Antenna

1. Contacting feed
2. Non-contacting feed

(a) Microstrip feed

(i) Center feed

(ii) Offset feed

(iii) Inset feed

(iv) Quarter wave line feed

(b) Co-axial feed

(c) Aperture coupled feed

(d) Proximit coupled feed

Applications

(i) Mobile and satellite communication application

(ii) Radio frequency identification

(iii) Worldwide interoperability for Microwave access (WiMax)

(iv) Radar application

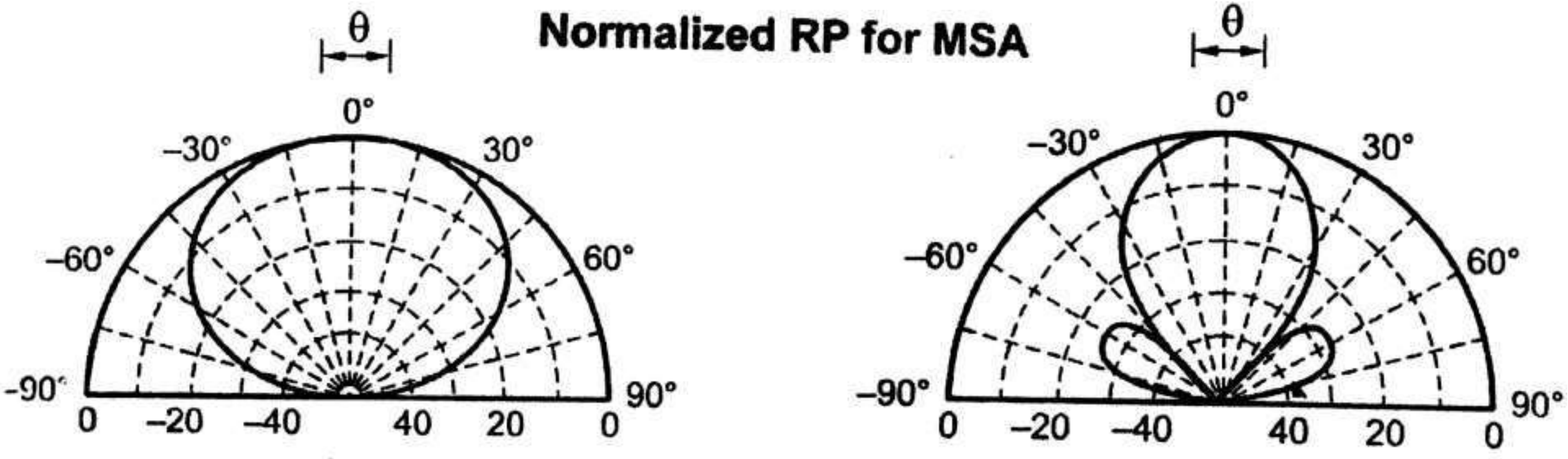
(v) Telemedicine application

(vi) Medicinal applications of patch

(vii) Military applications

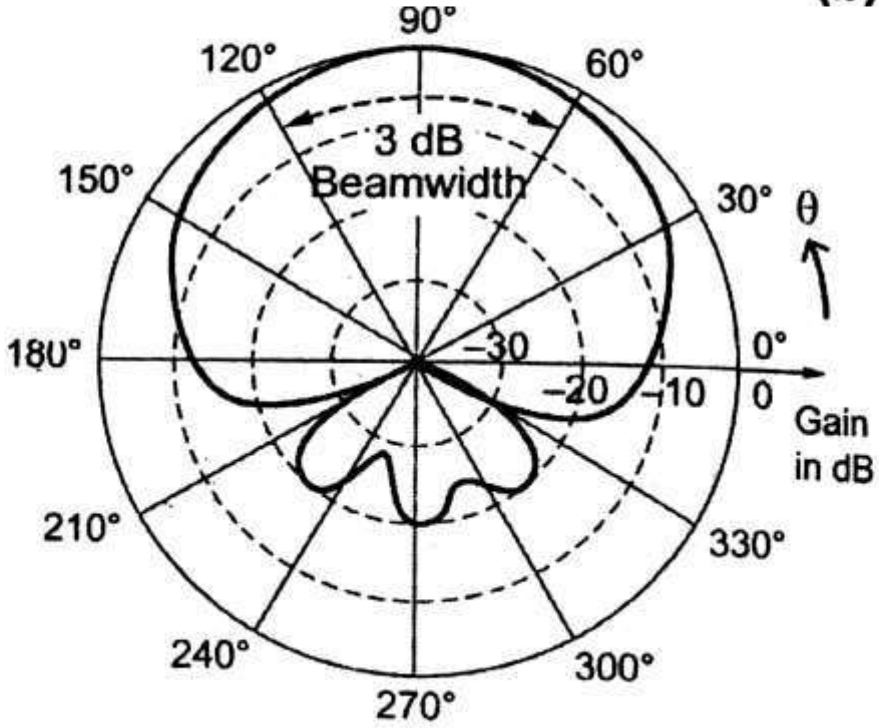
(viii) Space applications

Normalized RP for MSA



(a) $\phi = 0°$

(b) $\phi = 90°$



RP for linearly polarized MSA

Numerical tool for Antenna Synthesis

Computer Aided design (CAD) software

The main advantages of CAD tool are:

(i) CAD relations are independent of specific feeding mechanism with the exception of input resistance.

(ii) It requires less computational time.

(iii) Implementation is easy.

(iv) It does not require rigorous mathematical steps

(v) Accuracy is more

(vi) Results are closer to the experimental results.

Two of the commercially available CAD packages are listed as:

PCAAD 3.0

ENSEMBLE 2.0

CYLINDRICAL+

UNIT III

Antenna Arrays

Several antennas of similar type are arranged in a system to radiate more in desired direction with high gain

This can be achieved by combining the individual antenna radiations in desired direction and canceling the radiation in undesired direction

Such system is called an antenna array

An antenna array is a system of similar antennas oriented similarly to get greater directivity in a desired direction

The antenna array is said to be linear if the elements of the antenna array are equally spaced along a straight line

The linear antenna array is said to be uniform linear array if all the elements are fed with a current of equal magnitude with progressive uniform phase shift along the line

Various forms of Antenna Arrays

Practically various forms of the antenna array are used as radiating systems. Some of the practically used forms are as follows

(i) Broadside array

(ii) End fire array

(iii) Collinear array

(iv) Parasitic array

Array of 2 Point Sources

Point source is nothing but an isotropic radiator occupying zero volume

A number of similar point source is arranged in the form of array

The simplest condition of number of point sources in the array is two

The array of 2 point sources can be analyzed in 3 different ways

(i) Two point sources of equal magnitude and same phase

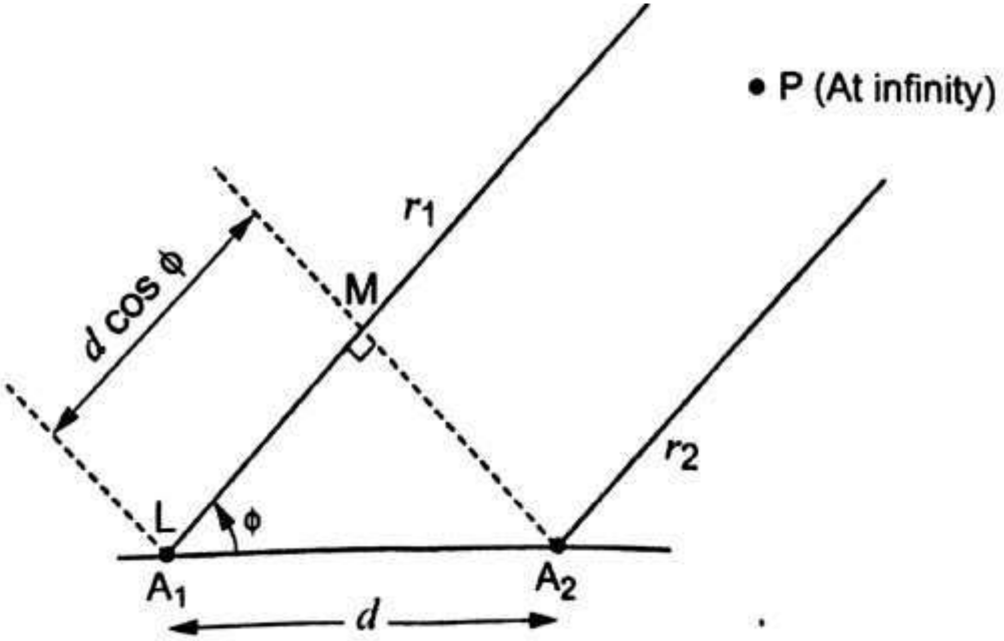
(ii) Two point sources with currents of equal magnitude and opposite phase

(iii) Two point sources with currents of unequal magnitudes and any phase

Two point sources with currents equal in magnitude and phase

Consider two point sources A_1 and A_2 separated by distance 'd' as shown in Fig.2.21. Let both the point sources are supplied with currents equal in magnitude and phase.

Consider a distant point 'p' far away from the array. Let the distance between point sources A_1 and A_2 and point 'p' be r_1 and r_2 respectively. As these radial distances are extremely large as compared with 'd' (distance between 2 point sources). We can assume $r_1 = r_2 = r$.



$$\text{the path difference} = d\cos\phi$$

In terms of wavelength,

$$\text{Path difference} = \frac{d\cos\phi}{\lambda}$$

$$\text{Phase angle } \psi = 2\pi \times \text{path difference}$$

$$\text{Phase angle } \psi = 2\pi\left(\frac{d\cos\phi}{\lambda}\right)$$

$$\boxed{\psi = \beta\, d\cos\phi \quad \text{radian}} \qquad \left(\because \beta = \frac{2\pi}{\lambda}\right)$$

Let $E_1 \rightarrow$ Far field at a distant point 'p' due to point source A_1

$$E_1 = E_0\, e^{-j\psi/2}$$

Similarly $E_2 \rightarrow$ Far field at point 'p' due to point source A_2

$$E_2 = E_0\, e^{j\psi/2}$$

where $E_0 \rightarrow$ Amplitude of both the field components

The total field (E_T) at point '*p*' is given by

$$E_T = E_1 + E_2 = E_0\, e^{-j\psi/2} + E_0 \cdot e^{j\psi/2}$$

$$E_T = E_0\,(e^{j\psi/2} + e^{-j\psi/2})$$

$$E_T = 2\,E_0 \cos(\psi/2) \qquad \left[\because \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}\right]$$

Substituting the value of ψ from equation

$$\boxed{E_T = 2\,E_0 \cos\left(\frac{\beta\, d \cos\phi}{2}\right)}$$

Array Factor

It is ratio of the magnitude of the resultant field to the magnitude of the maximum field.

$$\text{Array factor} = \frac{|E_T|}{|E_{max}|}$$

But maximum field is $E_{max} = 2\,E_0$

$$\text{Array factor} = \frac{|E_T|}{|2\,E_0|} = \cos\left(\frac{\beta\, d \cos\phi}{2}\right)$$

Field Pattern

To draw the field pattern, the directions of maxima, minima and half power points must be known which can be càlculated from equation

$$E_T = 2\,E_0 \cos\left(\frac{\beta\, d \cos\phi}{2}\right)$$

Here the amplitude of the total field is $2\,E_0$ whose maximum value may be 1

$\therefore$ By putting $2\,E_0 = 1$ or $E_0 = \frac{1}{2}$, the pattern is said to be normalized

$$E = \cos\left(\beta d \frac{\cos\phi}{2}\right)$$

Let $d = \lambda/2$ and $\beta = \dfrac{2\pi}{\lambda}$

$$E = \cos\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2} \frac{\cos\phi}{2}\right)$$

$$\boxed{E = \cos\left(\frac{\pi}{2}\cos\phi\right)}$$

Maxima Direction

The direction through which maximum radiation occurs is called as maxima direction or maxima. It is obvious that the electric field is maximum at maxima direction.

The total field strength 'E' is maximum when $\cos\left(\frac{\pi}{2}\cos\phi\right)$ is maximum and its maximum value is ± 1.

$$E = \cos\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

$$\frac{\pi}{2}\cos\phi_{max} = \cos^{-1}(\pm 1) = \pm n\pi \qquad \text{where } n = 0, 1, 2\ldots\ldots$$

If $n = 0$, then

$$\frac{\pi}{2}\cos\phi_{max} = 0$$

$$\cos\phi_{max} = 0$$

$$\boxed{\phi_{max} = 90^\circ \text{ or } 270^\circ}$$

Minima Direction

The total field strength 'E' is minimum when $E = \cos\left(\frac{\pi}{2}\cos\theta\right)$ is minimum and its minimum value is zero.

$$\therefore \quad E = \cos\left(\frac{\pi}{2}\cos\phi\right) = 0$$

$$\frac{\pi}{2}\cos\phi_{min} = \cos^{-1}(0) = \pm(2n+1)\frac{\pi}{2} \qquad \text{where } n = 0, 1, 2\ldots$$

If $n = 0$, then $\frac{\pi}{2}\cos\phi_{min} = \pm\frac{\pi}{2}$

$$\cos\phi_{min} = \pm 1$$

$$\boxed{\phi_{min} = 0° \text{ or } 180°}$$

Half power point directions

At half power points, power is $\frac{1}{2}$ (or) voltage and current is $\frac{1}{\sqrt{2}}$ times the maximum value.

$\therefore$ At half power point direction, the electric field is $\pm\frac{1}{\sqrt{2}}$

i.e.,
$$E = \pm\frac{1}{\sqrt{2}}$$

$$\therefore \quad E = \cos\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\phi = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = \pm(2n+1)\frac{\pi}{4}, \qquad \text{where } n = 0, 1, 2\ldots$$

If $n = 0$, then

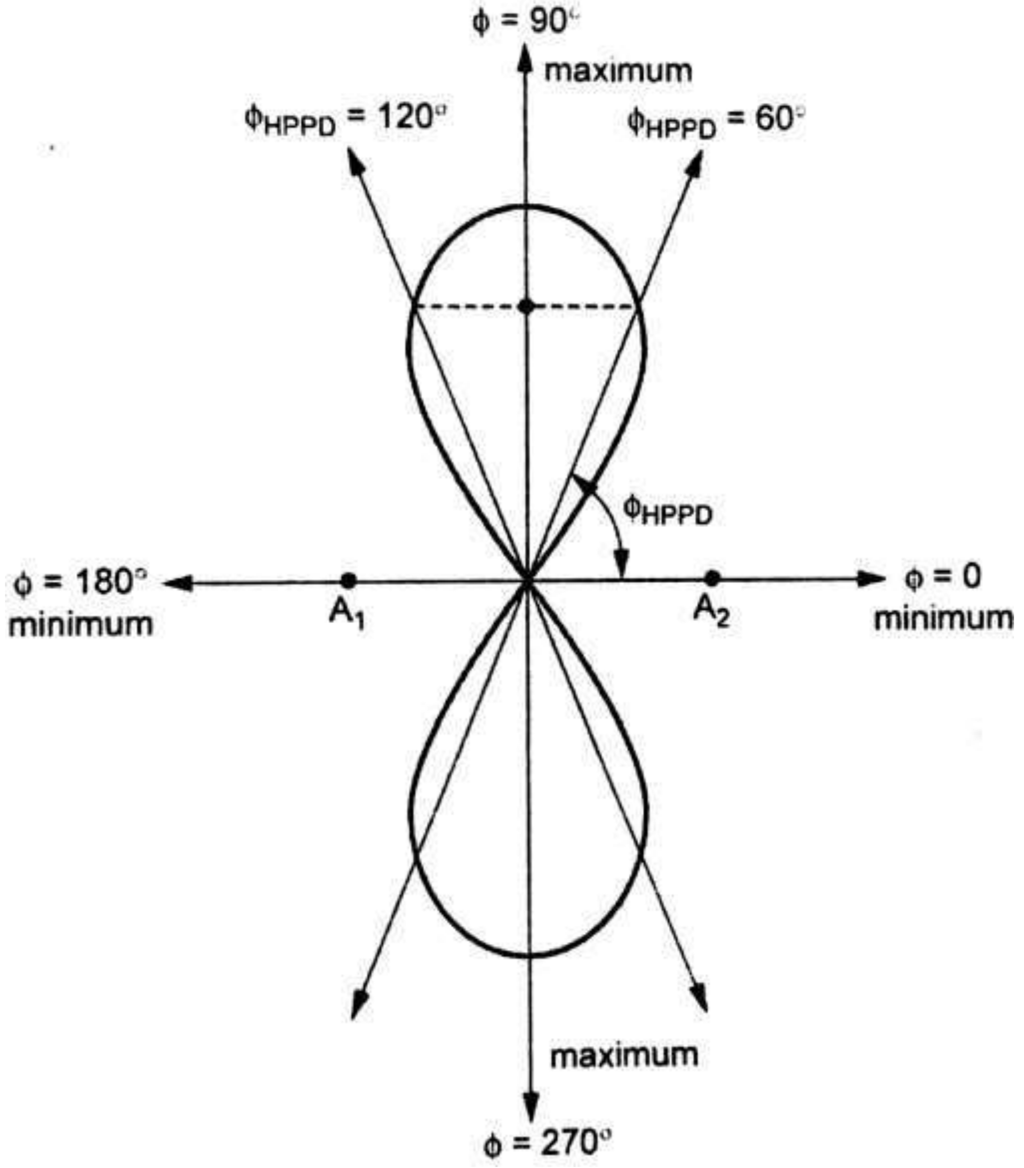
$$\frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$$

$$\cos\phi_{HPPD} = \pm\frac{1}{2}$$

$$\phi_{HPPD} = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

$$\boxed{\phi_{HPPD} = 60^\circ \text{ (or) } 120^\circ}$$

Now the field pattern with E against ϕ is drawn for $d = \lambda/2$



Two point sources with currents equal in magnitudes but opposite in phase

Consider 2 point sources separated by distance 'd' and supplied with currents ***equal in magnitude but opposite in phase***. It is similar to the previous case except that source A_1 has current out of phase (180°) (or) opposite phase to source A_2. *i.e.*, when there is maximum in source A_1 at one particular instant, then there is minimum in source A_2 at that instant and vice-versa.

Total far field at distant point 'p' is given by

$$E_T = -E_1 e^{-j\psi/2} + E_2 e^{j\psi/2}$$

$$\text{Let} \quad E_1 = E_2 = E_0$$

$$\therefore \quad E_T = E_0 \, e^{j\psi/2} - e^{-j\psi/2}$$

$$E_T = E_0 \cdot 2j \sin\frac{\psi}{2} \qquad \left[\because \frac{e^{j\theta/2} - e^{-j\theta/2}}{2j} = \sin\theta/2\right]$$

$$\underline{E_T = 2j\,E_0 \sin\left(\frac{\beta d \cos\phi}{2}\right)}$$

Field Pattern

To draw the field pattern, the directions of maxima, minima and half power points must be known which can be calculated from equation

$$E_T = 2j\,E_0 \sin\left(\frac{\beta\, d \cos\phi}{2}\right)$$

Here the amplitude of the total field is $2\,E_0$ whose maximum value may be 1

By putting $|2\,E_0| = 1$, the pattern is said to be normalized

$$\therefore \quad E = \sin\left(\frac{\beta\, d \cos\phi}{2}\right)$$

$$\text{Let} \quad d = \frac{\lambda}{2} \quad \text{and} \quad \beta = \frac{2\pi}{\lambda}$$

$$E = \sin\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2} \frac{\cos\phi}{2}\right)$$

$$\boxed{E = \sin\left(\frac{\pi}{2}\cos\phi\right)}$$

Maxima Directions

The direction through which maximum radiation occurs is called as maxima direction or maxima. It is obvious that the electric field is maximum at maxima direction.

$$E = \pm 1$$

$$E = \sin\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

$$\frac{\pi}{2}\cos\phi_{max} = \sin^{-1}(\pm 1) = \pm(2n+1)\frac{\pi}{2} \text{ where } n = 0, 1, 2\ldots\ldots$$

$$\text{If } n = 0, \text{ then } \frac{\pi}{2}\cos\phi_{max} = \pm\frac{\pi}{2}$$

$$\cos\phi_{max} = \pm 1$$

$$\boxed{\phi_{max} = 0° \text{ and } 180°}$$

Minima Direction

The total field strength 'E' is minimum when $E = \sin\left(\frac{\pi}{2}\cos\phi\right)$ is minimum *i.e.*, zero.

$$E = \sin\left(\frac{\pi}{2}\cos\phi\right) = 0$$

$$\frac{\pi}{2}\cos\phi_{min} = \sin^{-1}(0) = \pm n\pi \qquad \text{where } n = 0, 1, 2, \ldots\ldots$$

If $n = 0$, then

$$\frac{\pi}{2}\cos\phi_{min} = 0$$

$$\cos\phi_{min} = 0$$

$$\boxed{\phi_{min} = \pm 90^\circ}$$

Half Power Point Direction (HPPD)

At half power points, power is $\frac{1}{2}$ (or) voltage and current is $\frac{1}{\sqrt{2}}$ times the maximum value.

$\therefore$ At half power points direction, the electric field is $\pm\frac{1}{\sqrt{2}}$

$$E = \pm\frac{1}{\sqrt{2}}$$

$$E = \sin\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\phi = \sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$= \pm(2n+1)\frac{\pi}{4} \qquad \text{where } n = 0, 1, 2, \ldots\ldots$$

If $n = 0$, then

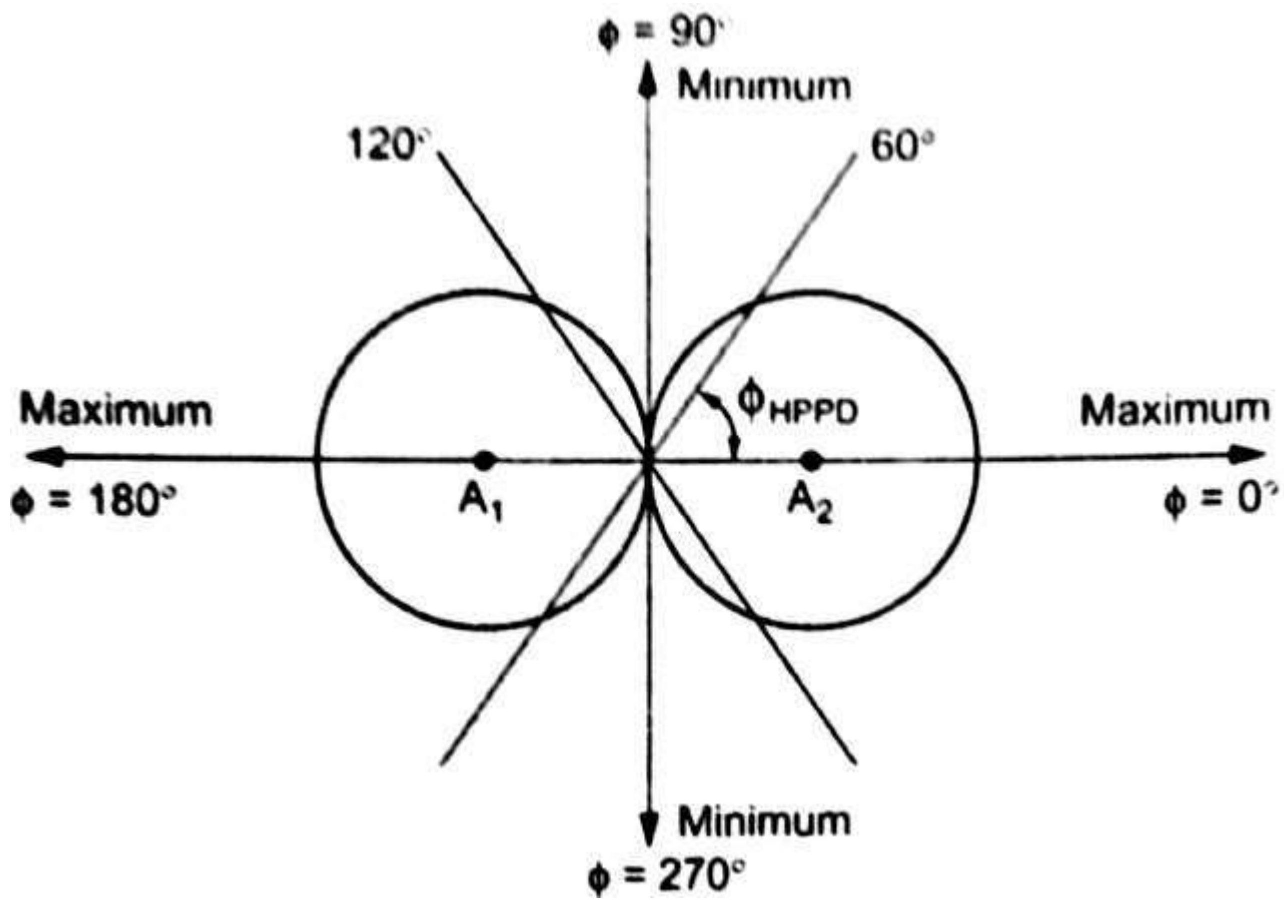
$$\frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$$

$$\cos\phi_{HPPD} = \pm\frac{1}{2}$$

$$\phi_{HPPD} = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

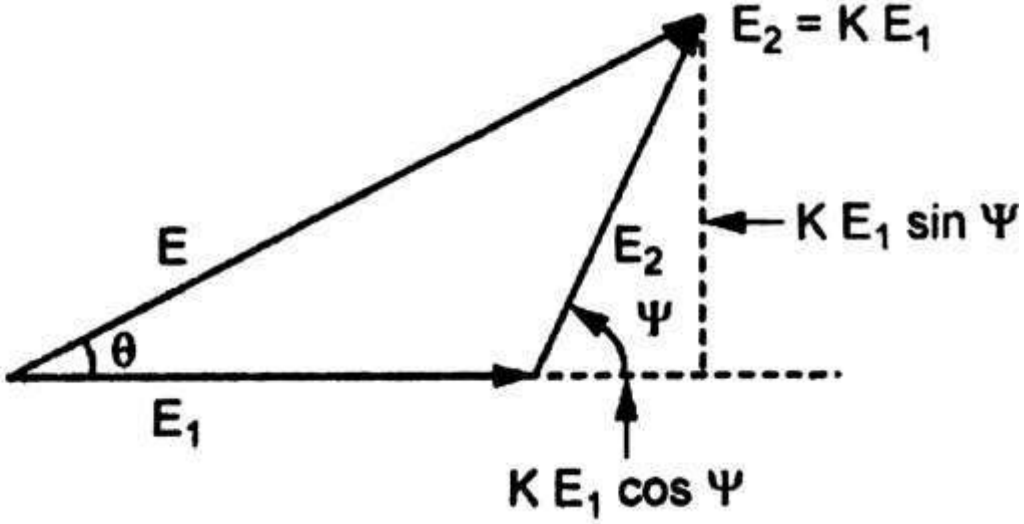
$$= 60^\circ \text{ and } 120^\circ$$

$$\boxed{\therefore \phi_{HPPD} = 60^\circ \text{ and } 120^\circ}$$



φ = 90°
Minimum
120°
60°
Maximum
φHPPD
Maximum
φ = 180°
A1
A2
φ = 0°
Minimum
φ = 270°

Two point sources with currents of unequal magnitudes and any phase



Vector diagram of fields E_1 and E_2

Now the total phase difference between the radiations by the 2 point sources at any far point '*p*' is given by

$$\boxed{\psi = \frac{2\pi}{\lambda} \cos \phi + \alpha}$$

Assume the value of 'α' as $0 < \alpha < 180°$, then the resultant field at point 'p' is given by

$$E_T = E_1 e^{j0} + E_2 e^{j\psi}$$

(Source 1 is assumed to be reference, hence phase angle is '0')

$$E_T = E_1 + E_2 e^{j\psi}$$

$$E_T = E_1 \left(1 + \frac{E_2}{E_1} e^{j\psi}\right)$$

Let
$$\frac{E_2}{E_1} = k$$

Since $E_1 > E_2$, the value of k is less than unity. $(0 \leq k \leq 1)$

$$E_T = E_1 (1 + k e^{j\psi})$$

$$\boxed{\therefore E_T = E_1 [1 + k (\cos\psi + j \sin\psi)}$$

∴ The magnitude of the resultant field at point 'p' is given by

$$|E_T| = \{E_1 [1 + k \cos \psi + j\, k \sin \psi]\}$$

$$\boxed{|E_T| = E_1 \sqrt{(1 + k \cos \psi)^2 + (k \sin \psi)^2}}$$

The phase angle between 2 fields at the far point 'p' is given by

$$\boxed{\theta = \tan^{-1} \frac{k \sin \psi}{1 + k \cos \psi}}$$

N Element uniform linear array

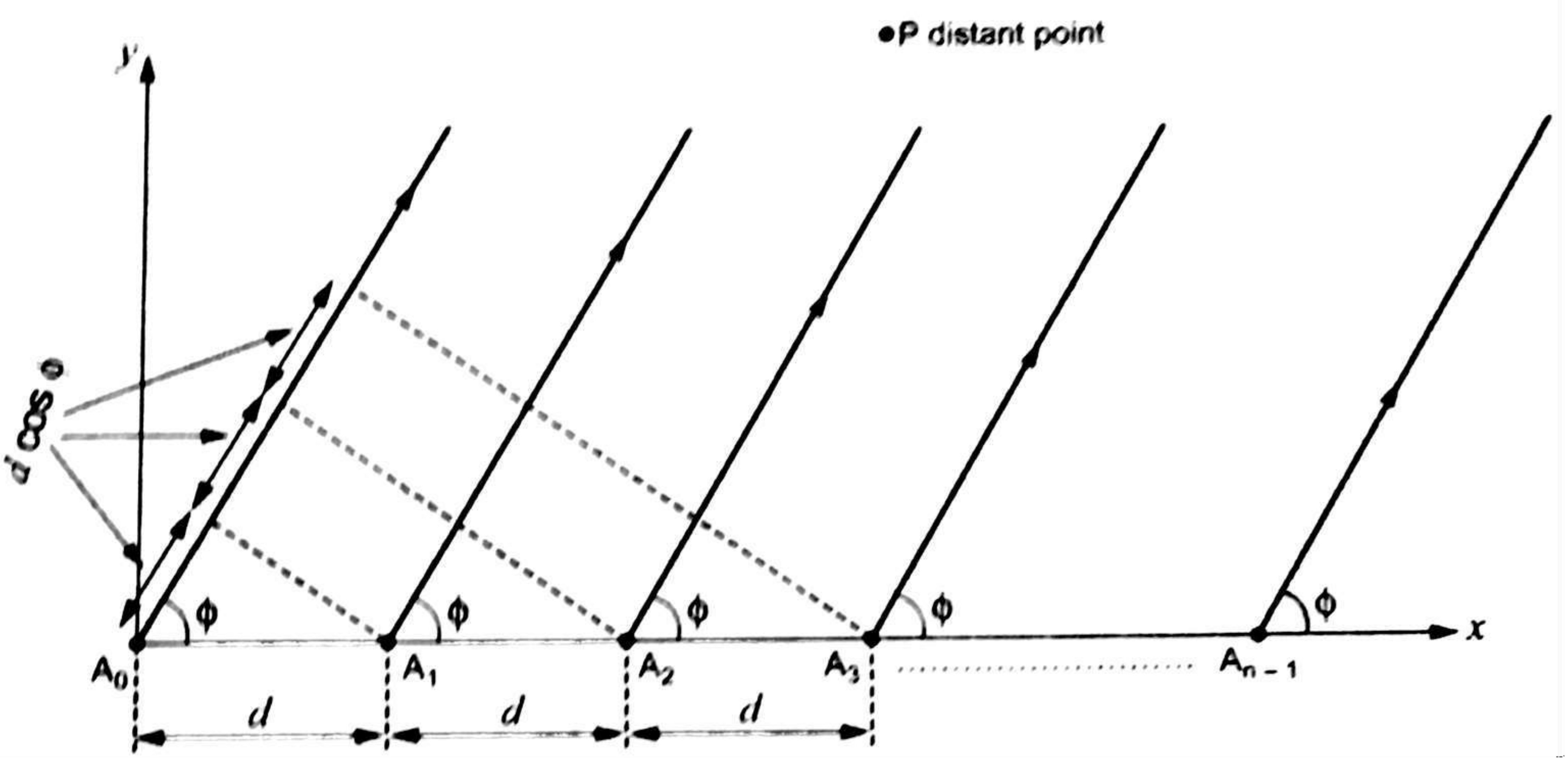
At higher frequencies, for point to point communications, it is necessary to have a pattern with single beam radiation. Such highly directive single beam pattern can be obtained by increasing the point sources in the array from 2 to '*n*' number of sources.

Linear Array

The antenna array is said to be linear if the elements of the antenna array are equally spaced along a straight line

Uniform Linear Array

The linear antenna array is said to be uniform linear array if all the elements are fed with a current of equal magnitude with progressive uniform phase shift along the line



Uniform linear array of 'n' elements

Consider a general 'n' element uniform linear array as shown in Fig

Here point sources are equally spaced and fed with a current of equal amplitude and phase shift is uniform progressive phase shift.

Total field at a distant point 'p' is obtained by adding the fields due to 'n' individual sources vectorically.

$$E_t = E_0 e^{0j\psi} + E_0 e^{j\psi} + E_0 e^{j2\psi} + E_0 e^{j3\psi} \ldots\ldots\ldots\ldots E_0 e^{j(n-1)\psi}$$

$$E_T = E_0 (1 + e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \ldots\ldots\ldots\ldots e^{j(n-1)\psi}) \quad \ldots\ldots (1)$$

ψ is the total phase difference of the fields at distant point 'P' from adjacent sources and it is expressed as,

$$\psi = \beta d \cos \theta + \alpha \quad radian \quad \ldots\ldots(2)$$

where,

α is the phase difference in adjacent point sources

$\beta\, d \cos\theta$ is the phase difference due to path difference, and

$$\text{Propagation constant } \beta = \frac{2\pi}{\lambda}$$

Multiplying equation (1) by $e^{j\Psi}$ becomes,

$$E_T e^{j\Psi} = E_0 (e^{j\Psi} + e^{j2\Psi} + e^{j3\Psi} + e^{j4\Psi} + \ldots\ldots\ldots\ldots e^{nj\Psi}) \qquad \ldots\ldots (3)$$

By subtracting equation (3) from equation (1), we get

$$E_T - E_T e^{j\psi} = E_O \left\{ \left[1 + e^{j\psi} + e^{j2\psi} + \ldots\ldots + e^{j(n-1)\psi} \right] - \left[e^{j\psi} + e^{j2\psi} + \ldots\ldots + e^{jn\psi} \right] \right\}$$

$$E_T (1 - e^{j\Psi}) = E_0 (1 - e^{jn\Psi})$$

$$\boxed{E_T = E_0 \left(\frac{1 - e^{jn\Psi}}{1 - e^{j\Psi}} \right)} \qquad \ldots\ldots (4)$$

Equation (4) may be written as

$$E_T = E_0 \frac{(1 - e^{jn\psi/2} \cdot e^{jn\psi/2})}{(1 - e^{j\psi/2} \cdot e^{j\psi/2})}$$

$$= E_0 \frac{(e^{jn\psi/2} \cdot e^{-jn\psi/2} - e^{jn\psi/2} \cdot e^{jn\psi/2})}{e^{j\psi/2} \cdot e^{-j\psi/2} - e^{j\psi/2} \cdot e^{j\psi/2}}$$

$$= E_O \left[\frac{e^{j\frac{n\psi}{2}}\left(e^{-j\frac{n\psi}{2}} - e^{j\frac{n\psi}{2}}\right)}{e^{j\frac{\psi}{2}}\left(e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}}\right)} \right]$$

According to trigonometric identity,

$$e^{-j\theta} - e^{j\theta} = -2j\sin\theta \qquad \text{.....(5)}$$

Using the equation (5), then the resultant field in equation (4) becomes,

$$E_T = E_O\left[\frac{\left(-2j\sin\frac{n\psi}{2}\right)e^{j\frac{n\psi}{2}}}{\left(-2j\sin\frac{n\psi}{2}\right)e^{j\frac{\psi}{2}}}\right]$$

$$\mathrm{E}_T = \mathrm{E}_0 \cdot e^{j\left(\frac{n-1}{2}\right)\psi}\,\frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}} \qquad \text{......(6)}$$

The phase angle of the resultant field at point P is given as

$$\phi = \frac{(n-1)}{2}\psi = \left(\frac{n-1}{2}\right)\beta\, d\cos\theta + \alpha \text{ (from equation 2)} \qquad \text{.....(7)}$$

Then, the equation (6) becomes,

$$E_T = E_0\left[\frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}\right]e^{j\phi} = E_0\left[\frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}\right](\cos\phi + j\sin\phi)$$

$$E_T = E_0\left[\frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}\right]\angle\phi \qquad \text{......(8)}$$

This equation (8) indicates the resultant field due to 'n' element linear array at distant point P. The magnitude of the resultant field is given as

$$E_T = E_0 \left[\frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \right] \qquad \text{......(9)}$$

maximum value of E_T is 'n' times the field from a single source

$$E_{T(Max)} = E_0 n$$

$$E_{Nor} = \frac{E_T}{E_{T(Max)}} = \frac{E_0 \dfrac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}}}{E_0 n}$$

$$\boxed{E_{Nor} = \frac{\sin \frac{n\psi}{2}}{n \sin \frac{\psi}{2}} = (Array\ Factor)_n}$$

Pattern Multiplication

"The total field pattern of an array of non-isotropic but similar sources is the multiplication of the individual source patterns and the pattern of array of isotropic point sources each located at the phase center of individual source and having the relative amplitude and phase, whereas the total phase pattern is the addition of the phase pattern of the ~~phase pattern of the~~ individual sources and that of the array of isotropic point sources"

The total field pattern of an array of non-isotropic but similar sources may be expressed as

$$Total\ Field\,(E_T) = (Multiplication\ of\ field\ pattern) \times (Addition\ of\ phase\ pattern)$$

$$E_T = \{E_i(\theta, \phi) \times E_a(\theta, \phi)\} \times \{E_{pi}(\theta, \phi) + E_{pa}(\theta, \phi)\}$$

where,

$E_i(\theta, \phi)$ = Field pattern of individual source,

$E_a(\theta, \phi)$ = Field pattern of array of isotropic point sources,

$E_{pi}(\theta, \phi)$ = Phase pattern of individual source,

$E_{pa}(\theta, \phi)$ = Phase pattern of array of isotropic point sources,

θ – Polar angles, and

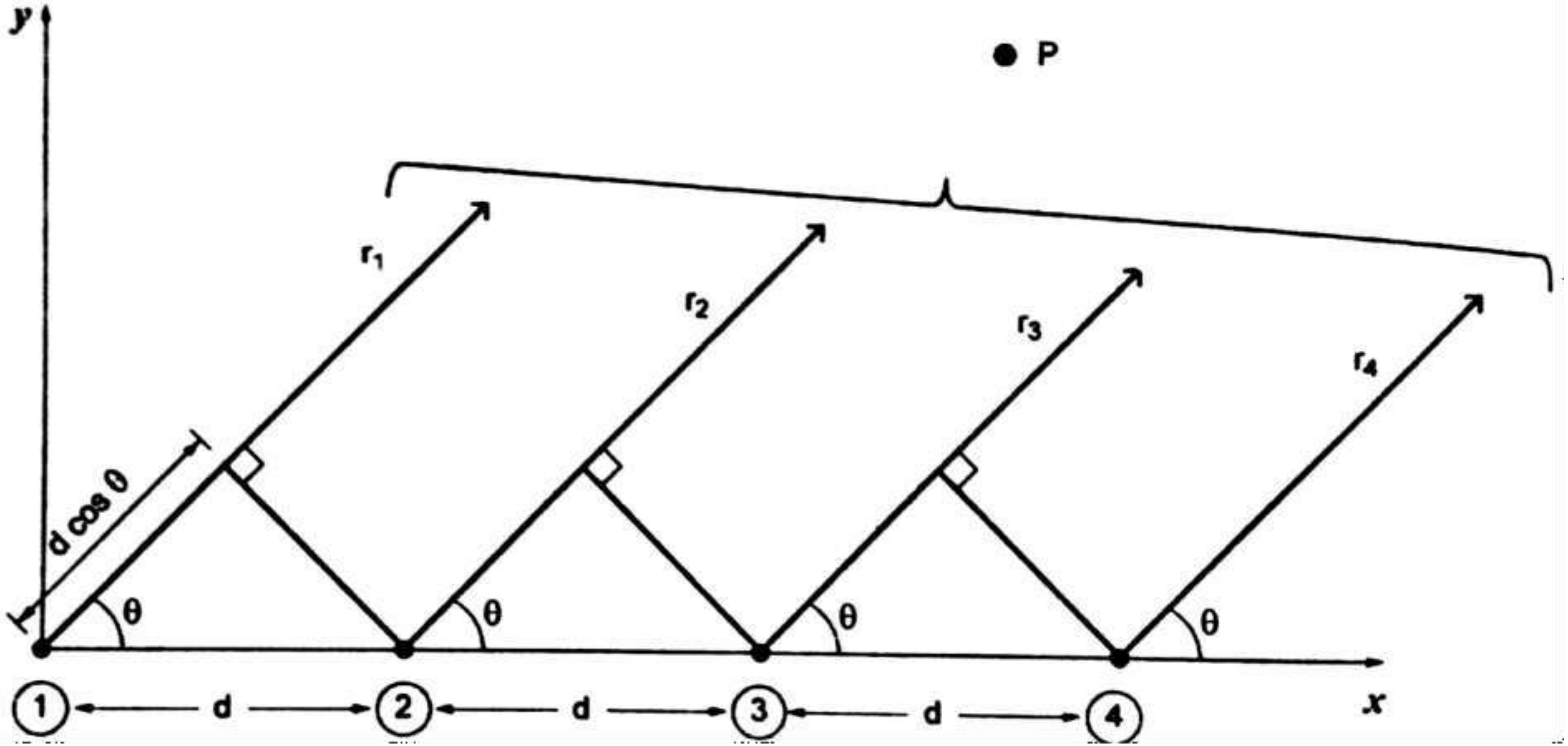
ϕ – Azimuth angles.

Advantages of pattern multiplication

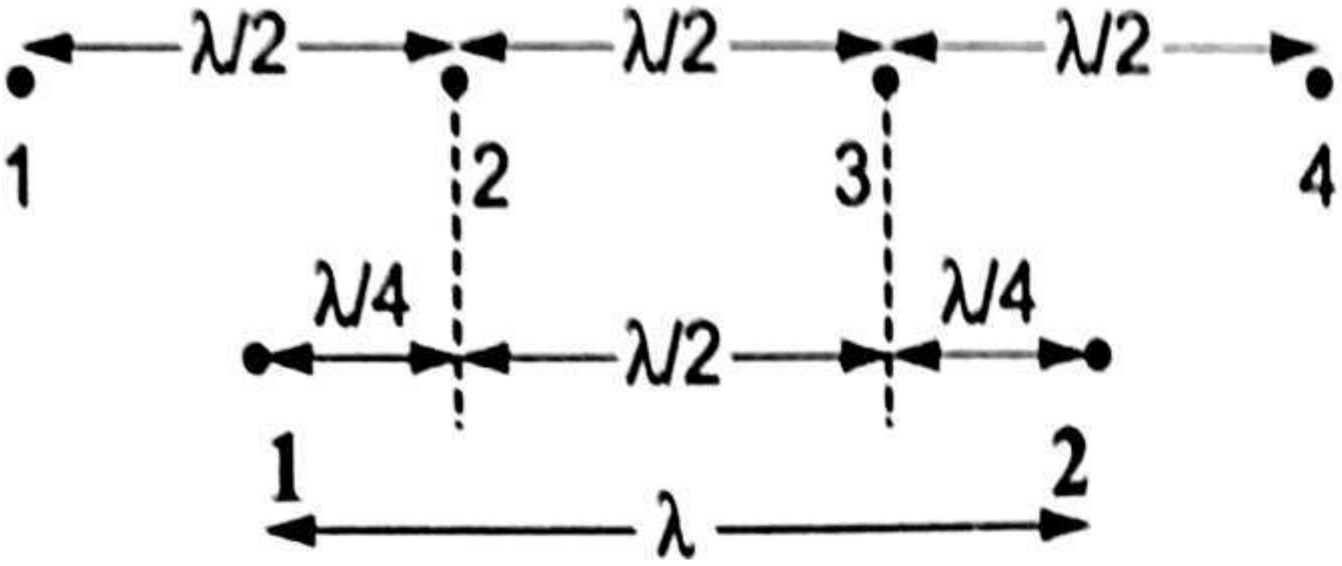
(*i*) It is a speedy method for sketching the pattern of complicated arrays just by inspection, and

(*ii*) It is a useful tool in the design of antenna arrays.

RADIATION PATTERN OF 4-ISOTROPIC ELEMENTS FED IN PHASE, SPACED $\frac{\lambda}{2}$ APART



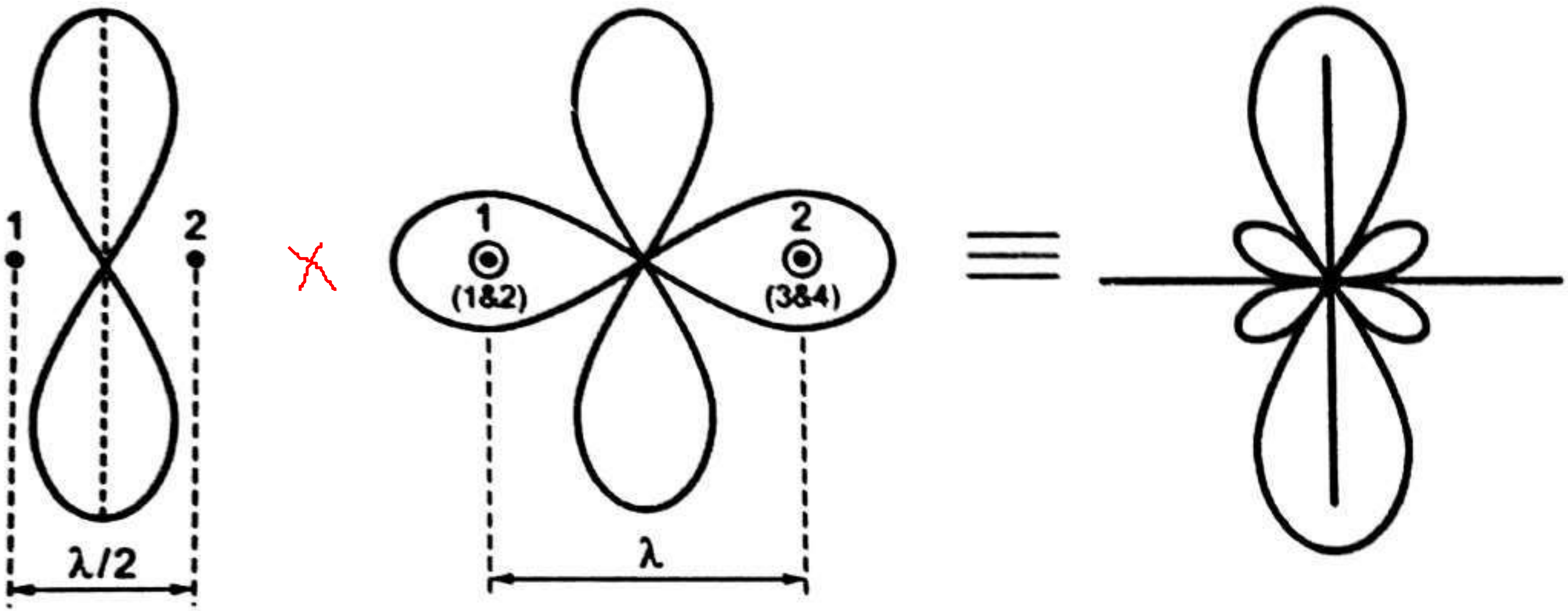
Linear array of 4 isotropic elements spaced $\frac{\lambda}{2}$ apart, fed in phase



Two units array spaced at λ

Two isotropic point source spaced λ/2 apart fed in phase provides a ***bidirectional pattern***. According to pattern multiplication, the radiation pattern of 4 elements is obtained as,

$$\left\{\begin{matrix}\therefore \text{ Resultant radiation} \\ \text{pattern of 4 elements}\end{matrix}\right\} = \left\{\begin{matrix}\text{Radiation pattern} \\ \text{of individual} \\ \text{elements}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Array of} \\ \text{two units} \\ \text{spaced '}\lambda\text{'}\end{matrix}\right\}$$



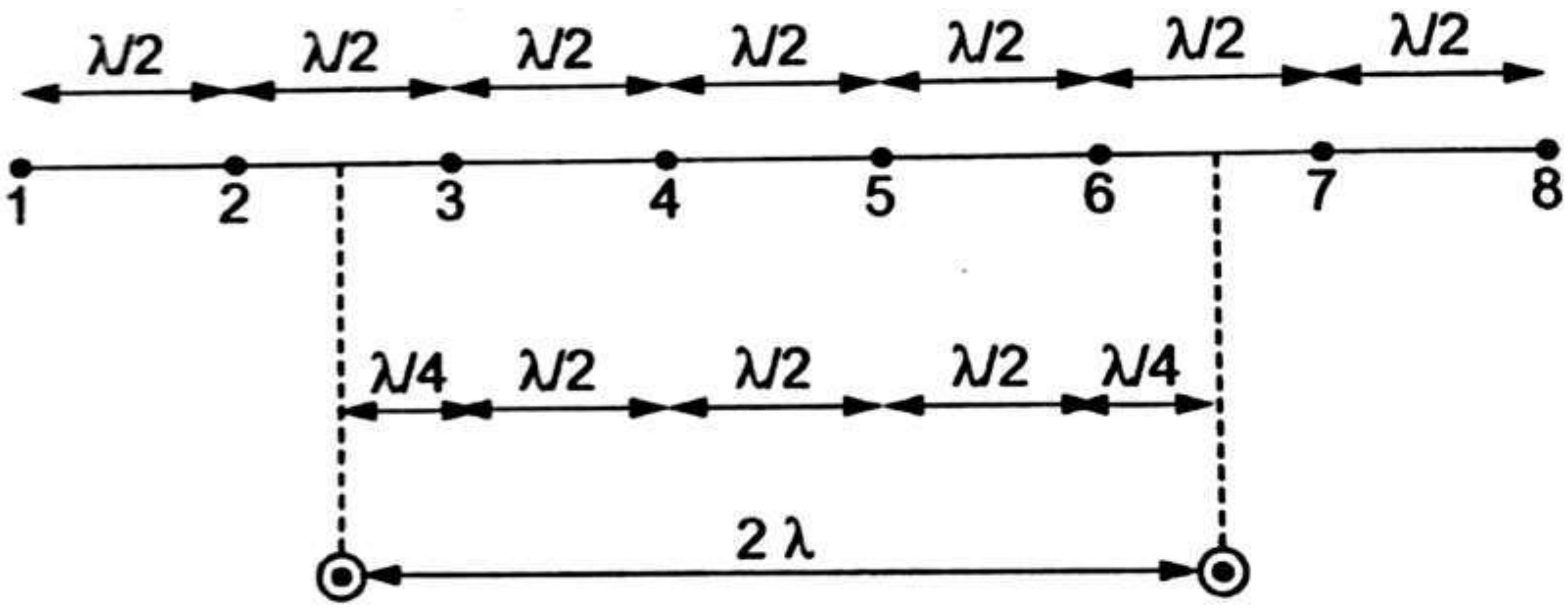
Individual (unit pattern) pattern due to 2 individual elements

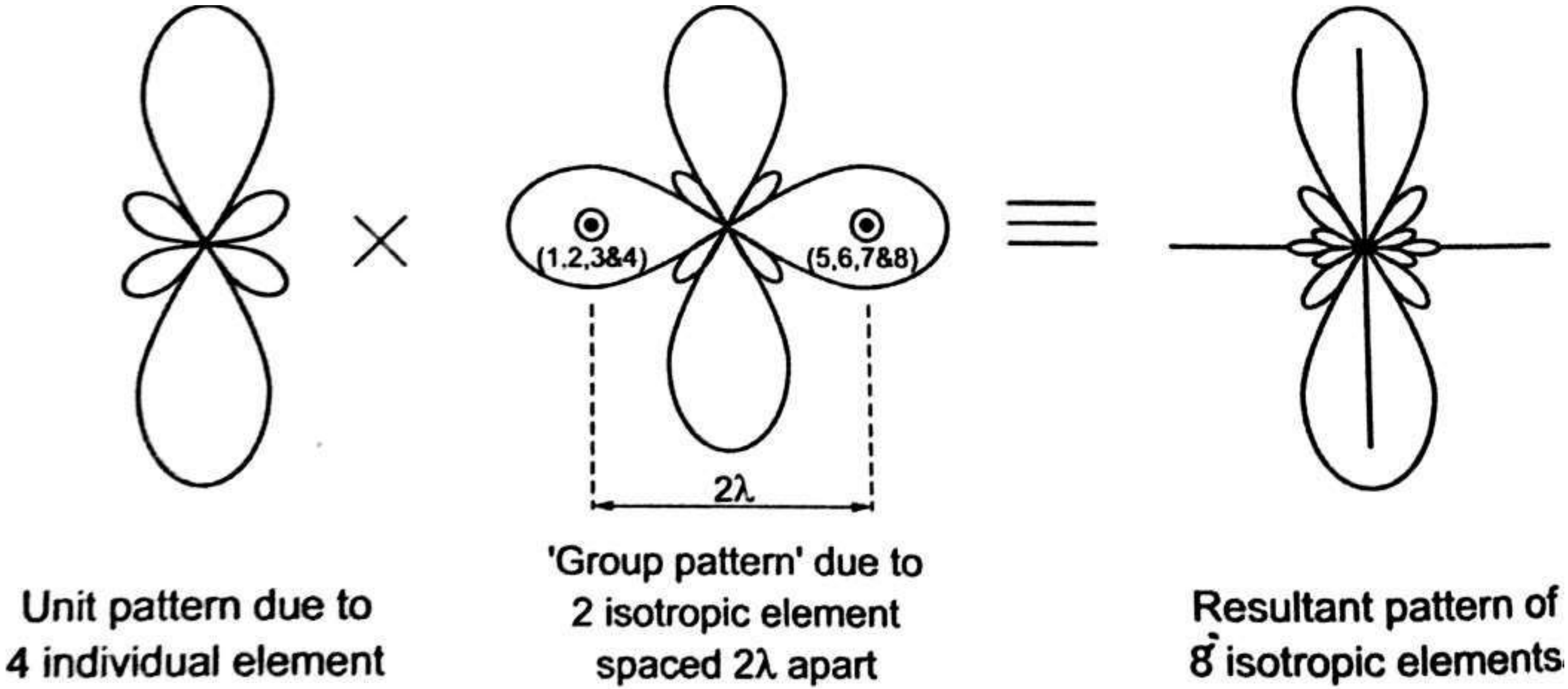
Group pattern due to array of two isotropic separated by λ

Resultant pattern of 4 isotropic elements

Resultant radiation pattern of 4 isotropic elements by pattern multiplication

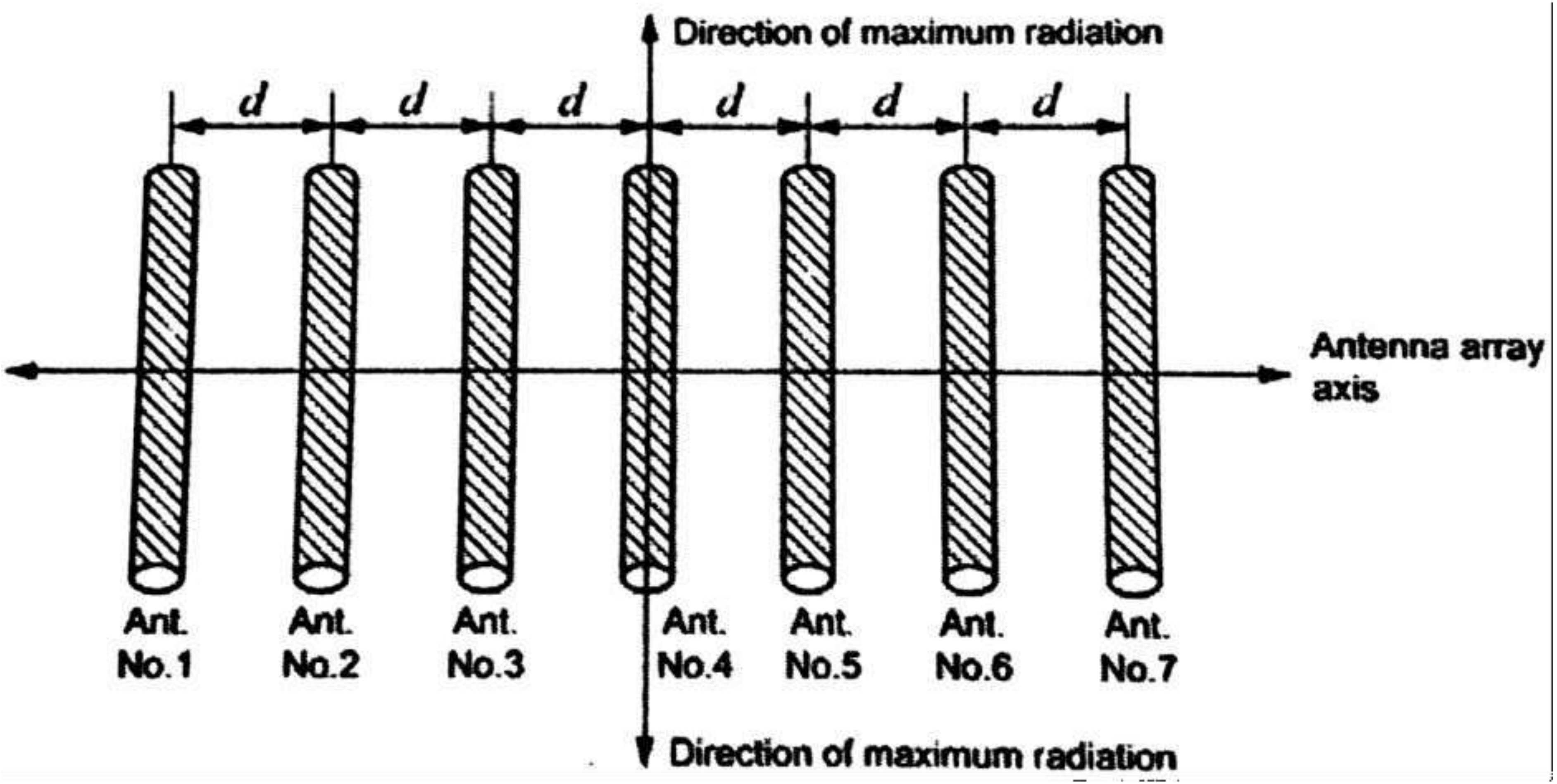
RADIATION PATTERN OF 8-ISOTROPIC ELEMENTS FED IN PHASE, AND $\frac{\lambda}{2}$ APART





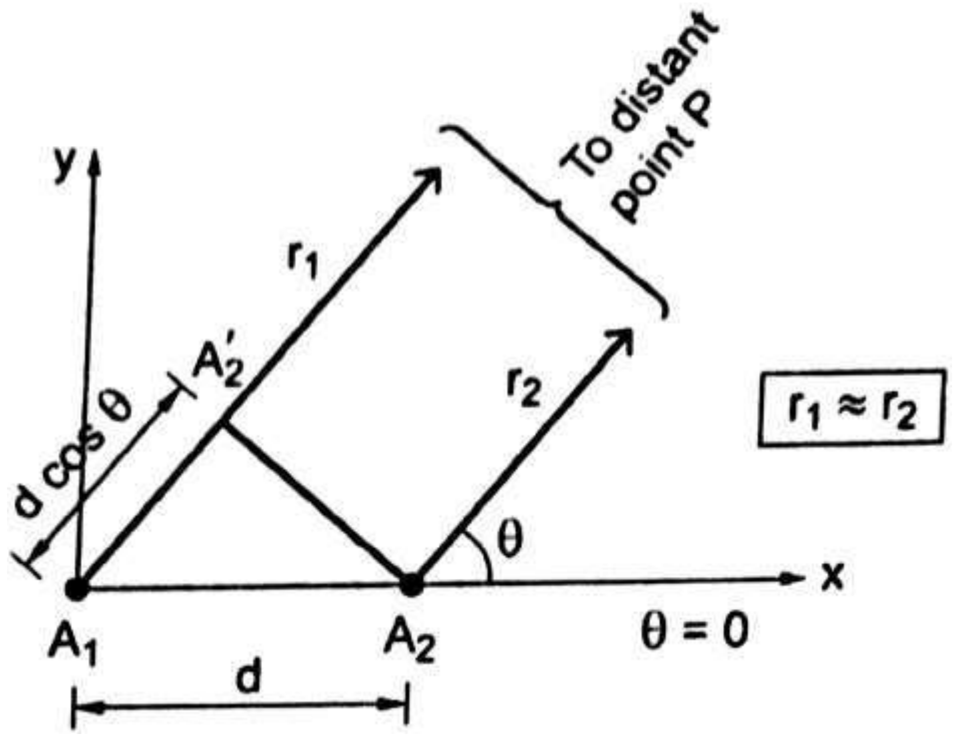
Resultant radiation pattern of 8-isotropic elements by pattern multiplication

Broadside Array



Broadside array of antennas

Array of 'n' isotropic sources of equal amplitude and spacing - Broadside Array



Two sources of equal amplitude and phase, separated by a distance 'd'

$$\text{path difference } (A_1 A_2^1) = d\cos\theta \text{ meter}$$

In terms of wavelength,

$$\text{Path difference} = \frac{d\cos\theta}{\lambda} \quad \text{......(1)}$$

$$\text{Phase angle } \psi = 2\pi \times \text{path difference}$$

$$= 2\pi\left(\frac{d\cos\theta}{\lambda}\right)$$

$$\psi = \frac{2\pi}{\lambda} d\cos\theta \text{ radians} \quad \text{......(2)}$$

$$\boxed{\psi = \beta d\cos\theta \text{ radians}} \quad \text{......(3)}$$

$$\psi = \beta d\cos\theta + \alpha \quad \text{......(4)}$$

1. Maxima Direction for Major Lobe

An array is said to be broadside array, if the phase angle makes maximum radiation perpendicular to the line of array. i.e. ***90° and 270°.*** In the broad side array, all sources are in phase. i.e. $\alpha = 0$ *and* $\psi = 0$.

$$\psi = \beta d \cos\theta + \alpha = 0 \qquad \ldots\ldots(5)$$

$$\beta d \cos\theta_{Max} = 0$$

$$\cos\theta_{Max} = 0$$

$$\boxed{\theta_{Max} = 90° \text{ or } 270°}$$

The major lobes maxima occurs in these directions

2. Maxima Direction for Minor Lobes

The minor lobe maxima occurs between first nulls and higher order nulls. The ***nulls*** are the directions through which an array ***radiate zero power***.

The total far field strength for array of '*n*' isotropic point sources of equal amplitude and spacing is expressed as,

$$E_T = E_0\left[\frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}\right] \qquad \ldots\ldots(6)$$

In the above expression, E_T is maximum, when numerator is maximum. *i.e.*

$\sin\frac{n\psi}{2}$ is maximum provided $\sin\frac{\psi}{2} \neq 0$.

$$\therefore \qquad \sin\frac{n\,\psi}{2} = 1$$

$$\frac{n\,\psi}{2} = \pm(2\,N+1)\frac{\pi}{2} \qquad \text{where, } N = 1, 2, 3, 4\ldots\ldots\ldots$$

N is a constant and N = 0 corresponds to major lobe maxima where, 'n' indicates the number of isotropic elements.

$$\frac{\psi}{2} = \pm(2\,N+1)\frac{\pi}{2\,n}$$

$$\psi = \pm(2\,N+1)\frac{\pi}{n} \qquad \ldots\ldots(7)$$

Equating equation (5) and equation (7), we get

$$\beta\,d\cos(\theta_{Max})_{minor} + \alpha = \pm(2\,N+1)\frac{\pi}{n}$$

$$\beta\,d\cos(\theta_{Max})_{minor} = \pm(2\,N+1)\frac{\pi}{n} - \alpha$$

$$\cos(\theta_{Max})_{minor} = \pm \frac{(2N+1)\frac{\pi}{n} - \alpha}{\beta d}$$

$$(\theta_{Max})_{minor} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm \frac{(2N+1)\pi}{n} - \alpha\right]\right\} \qquad \ldots\ldots(8)$$

For a broadside array $\alpha = 0$, then equation (8) becomes

$$(\theta_{Max})_{minor} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm \frac{(2N+1)\pi}{n}\right]\right\}$$

By substituting the propagation constant $\beta = \frac{2\pi}{\lambda}$ in the above expression, we get

$$\boxed{(\theta_{Max})_{minor} = \cos^{-1}\left\{\pm \frac{(2N+1)\lambda}{2nd}\right\}} \qquad \ldots\ldots(9)$$

where, $(\theta_{Max})_{minor}$ = Maxima direction of minor lobes

Consider n = 4, d = λ/2, N = 1 then equation (9) becomes

$$(\theta_{Max})_{minor} = \cos^{-1}\left\{\pm\frac{(2+1)}{2\times 4\times\frac{\lambda}{2}}\cdot\lambda\right\}$$

$$= \cos^{-1}\left(\pm\frac{3}{4}\right)$$

$$\boxed{(\theta_{Max})_{minor} = \pm\mathbf{41.4^\circ\ or\ \pm 138.6^\circ}}$$

∴ Thus $+41.4°$, $+138.6°$, $-41.4°$ and $+138.6°$ are the 4 minor lobe maxima of the array of 4 isotropic sources fed in phase and spaced $\frac{\lambda}{2}$ apart. No other maxima exist for $N \geq 2$, because for $N = 2$, $\cos(\theta_{max})_{minor} = \pm 5/4$ which is >> 1, whereas cosine value is always << 1.

3. Minima Directions for Minor Lobes

Minima is the direction through which an array radiate zero power. It is otherwise called as ***null direction*** and the electric field intensity is zero along the null direction.

The direction of minima of minor lobes is the array of 'n' isotropic sources of equal amplitude and phase is given as,

$$E_T = E_0 \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} = 0$$

Minima occurs, when $\sin \frac{n\psi}{2} = 0$

$$\frac{n\,\psi}{2} = \pm N\pi \qquad \text{where, } N = 1, 2, 3 \ldots\ldots\ldots$$

$$\psi = \pm \frac{2N\pi}{n} \qquad \ldots\ldots(10)$$

But, $\psi = \beta\, d \cos\phi + \alpha$, in equation (10), we get

$$\beta\, d\,(\cos\theta_{Min})_{minor} + \alpha = \pm \frac{2N\pi}{n}$$

For broad side array, $\alpha = 0$, then

$$\beta\, d\,(\cos\theta_{Min})_{minor} = \pm \frac{2N\pi}{n}$$

$$\cos(\theta_{Min})_{minor} = \frac{1}{\beta\, d}\left\{\pm \frac{2N\pi}{n}\right\}$$

$$(\theta_{Min})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n}\right\}\right] \qquad \ldots\ldots(11)$$

By substituting the propagation constant, $\beta = \frac{2\pi}{\lambda}$ in equation (11), we get

$$= \cos^{-1}\pm\left[\frac{1}{\frac{2\pi}{\lambda}d}\left\{\pm\frac{2N\pi}{n}\right\}\right]$$

$$\boxed{(\theta_{Min})_{minor} = \cos^{-1}\left[\pm\frac{N\lambda}{nd}\right]} \qquad \ldots\ldots(12)$$

where, $(\theta_{Min})_{\min or}$ = Direction of minor lobe minima

For example,

(i) If $N = 1, n = 4$ and $d = \lambda/2$

$$(\theta_{Min})_{minor} = \cos^{-1} \pm \frac{1 \cdot \lambda}{4 \cdot \frac{\lambda}{2}} = \cos^{-1}\left[\pm \frac{1}{2}\right]$$

$$(\theta_{Min})_{minor} = \pm 60, \pm 120°$$

(ii) If $N = 2$,

$$(\theta_{Min})_{minor} = \cos^{-1}\left[\pm \frac{2 \cdot \lambda}{4 \cdot \frac{\lambda}{2}}\right] = \cos^{-1}[\pm 1]$$

$$= \pm 0°, \pm 180° = 0°, 180°$$

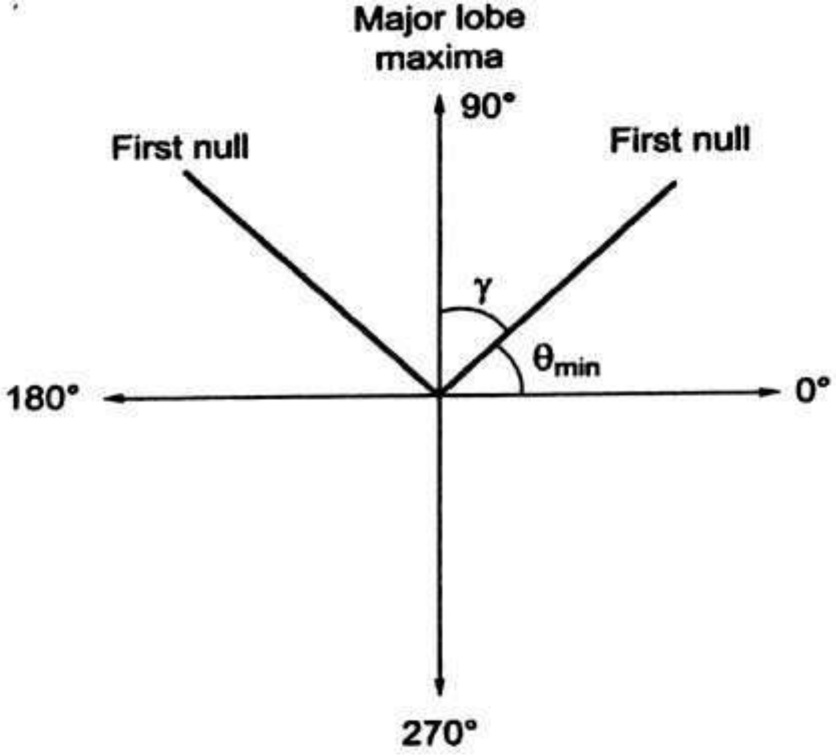
Thus 0°, 60°, 120°, 180°, – 60°, – 120° are the six minor lobe minima of the array of 4 isotropic sources spaced $\frac{\lambda}{2}$ apart. No other minima exist for which cosine functions becomes more than one which is not possible.

4. Beam Width of Major Lobe

(i) Beam width between First Null (BWFN)

BWFN is defined as,

The angle between first nulls (2γ) or double the angle between first null and major lobe in the maxima directions.



From Fig. $\gamma = 90 - \theta_{min} \Rightarrow \theta_{min} = 90° - \gamma$ ...... (13)

$$\text{Beam width (BW)} = 2 \times \left\{ \begin{matrix} \text{Angle between first null and} \\ \text{maximum of major lobe} \end{matrix} \right\}$$

$$\boxed{BW = 2 \times \gamma} \qquad \text{...... (14)}$$

By substituting equation (13) in equation (12), we get

$$90° - \gamma = \cos^{-1}\left\{ \pm \frac{N\lambda}{n\,d} \right\}$$

$$\cos(90° - \gamma) = \pm \frac{N\lambda}{n\,d}$$

$$\sin\gamma = \pm \frac{N\lambda}{n\,d} \qquad [\sin\gamma = \gamma \text{ when '}\gamma\text{' is very small}]$$

$$\boxed{\gamma = \pm \frac{N\lambda}{n\,d}} \qquad \text{......(15)}$$

First null occurs, when N = 1

$$\gamma_1 = \pm \frac{\lambda}{n\,d} \qquad \text{......(16)}$$

From equation (14), $\quad \text{BWFN} = 2 \times \gamma_1 = \dfrac{2\lambda}{n\,d}$ ......(17)

Let L = Total length of the array in meters.

$$L = (n-1)\,d \approx n\,d \qquad \text{(if } n \text{ is large)} \qquad \text{......(18)}$$

By substituting equation (18) in equation (16), we get

$$\therefore \qquad 2\gamma_1 = \frac{2\lambda}{L} = \frac{2}{L/\lambda} \text{ radian}$$

$$= \frac{2}{L/\lambda} \times 57.3 \text{ degree} = \frac{114.6^\circ}{L/\lambda}$$

$$\boxed{\text{BWFN} = \frac{114.6^\circ}{L/\lambda}} \qquad \text{......(19)}$$

(ii) Half Power Beam Width (HPBW)

$$\text{HPBW} = \frac{1}{2}\text{BWFN} \qquad \text{......(20)}$$

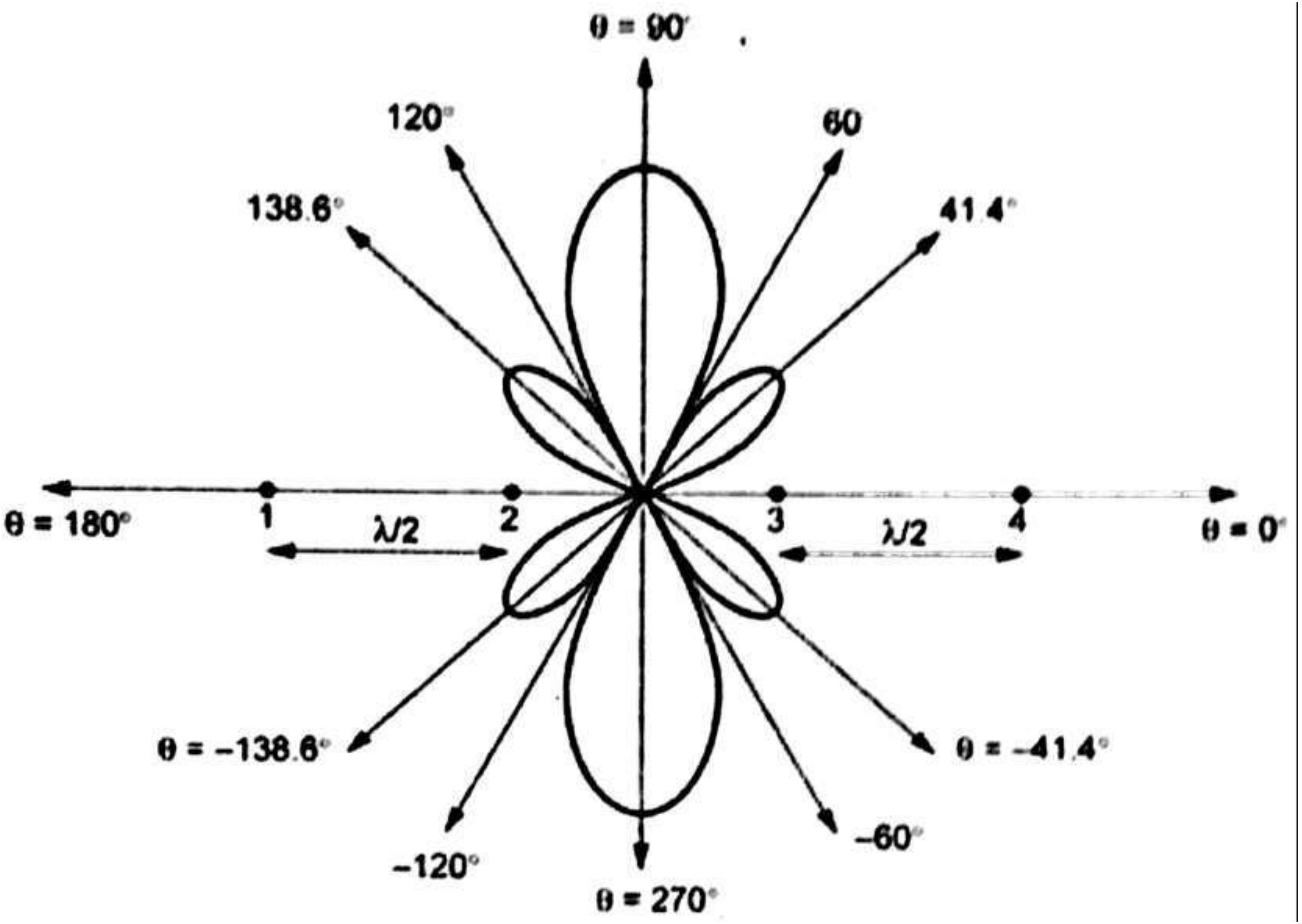
By substituting equation (19) in equation (20), we get

$$= \frac{57.3°}{L/\lambda}$$

$$\boxed{\text{HPBW} = \frac{57.3°}{L/\lambda}} \qquad \text{......(21)}$$

5. Directivity

$$D = 2n\left(\frac{d}{\lambda}\right) = 2\left(\frac{L}{\lambda}\right) \qquad \text{......(22)}$$

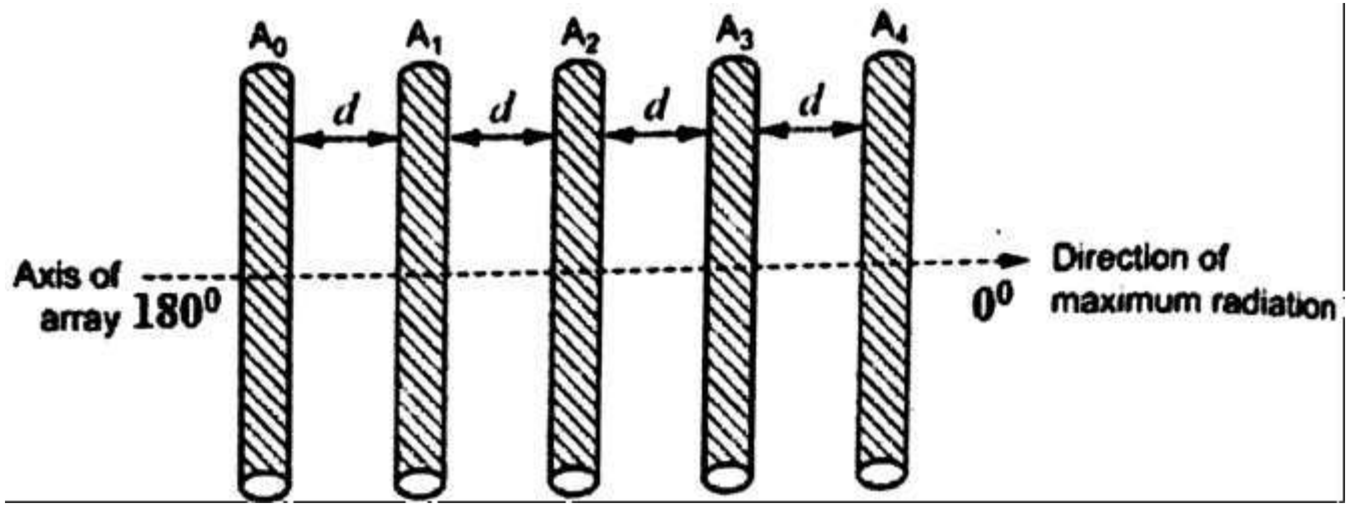


Field pattern of broadside array consisting of four isotropic sources of equal amplitude and in phase

End Fire Array

An array is said to be end fire, if the direction of maximum radiation coincides with the array axis to get unidirectional radiation.

In the end fire array, number of identical antennas are spaced equally along a line. All the antennas are fed individually with currents of equal magnitudes but their phases vary progressively along the line to make the entire arrangement to get unidirectional radiation along the axis of the array.



End fire array

For an array to be end fire, the phase angle is such that it makes the maximum radiation in the line of array. *i.e.,* $\theta = 0°$ or $180°$. The total phase difference is expressed as

$$\psi = \beta d \cos\theta + \alpha \qquad \ldots\ldots(1)$$

For end fire array $\psi = 0$ and $\theta = 0°$ (or) $180°$, then the equation (1) becomes,

$$\beta d \cos 0° = -\alpha$$

$$\boxed{\alpha = -\beta d = \frac{-2\pi}{\lambda} d} \qquad \ldots\ldots(2)$$

For an example, if spacing between 2 sources is $\frac{\lambda}{2}$ (or) $\frac{\lambda}{4}$, then the phase angle by which source 2 lags behind source 1 is

$$\frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \text{ (or) } \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2} \text{ radians}$$

1. Maxima Direction for Minor Lobe

The total field strength of 'n' element uniform linear array should be maximum for these directions.

$$E_T = E_0 \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}}$$

E_T will be maximum, when

$$\sin \frac{n\psi}{2} = 1 \quad \text{if } \sin \frac{\psi}{2} \neq 0$$

$$\frac{n\psi}{2} = \sin^{-1}(1) = \pm (2N+1)\frac{\pi}{2}$$

where, N = 1, 2, 3.........and N = 0 corresponds to major lobe maxima.

$$\psi = \pm \frac{(2N+1)\pi}{n} \qquad \text{......(3)}$$

By substituting equation (2) in equation (1),

$$\psi = \beta d \cos \theta - \beta d$$

$$\psi = \beta d (\cos \theta - 1) \qquad \text{......(4)}$$

Equating equation (3) and equation (4), we get

$$\beta d (\cos\theta - 1) = \pm \frac{(2N+1)\pi}{n}$$

$$\cos\theta - 1 = \pm \frac{(2N+1)\pi}{n \beta d}$$

$$\cos\theta = 1 \pm \frac{(2N+1)\pi}{n \beta d}$$

$$(\theta_{Max})_{minor} = \cos^{-1}\left[1 \pm \frac{(2N+1)\pi}{n \beta d}\right] \qquad \text{......(5)}$$

By substituting the propagation constant, $\beta = \frac{2\pi}{\lambda}$ in equation (5)

$$\boxed{(\theta_{Max})_{minor} = \cos^{-1}\left[1 \pm \frac{(2N+1)\lambda}{2\,n\cancel{\beta}\,d}\right]} \qquad \ldots\ldots(6)$$

For example, $n = 4$, and $d = \frac{\lambda}{2}$

$$(\theta_{Max})_{minor} = \cos^{-1}\left[1 \pm \frac{(2N+1)\lambda}{2.4.\frac{\lambda}{2}}\right]$$

$$\boxed{(\theta_{Max})_{minor} = \cos^{-1}\left(1 \pm \frac{(2N+1)}{4}\right)} \qquad \ldots\ldots(7)$$

If $N = 1$, $$(\theta_{Max})_{minor} = \cos^{-1}\left(1 \pm \frac{3}{4}\right)$$

$$= \cos^{-1}\left(\frac{1}{4}\right) \text{ [or] } \cos^{-1}\left(\frac{7}{4}\right) \Rightarrow \text{is invalid}$$

$$\boxed{(\theta_{Max})_{minor} = \cos^{-1}\left(\frac{1}{4}\right) = 75.5°}$$

If $N = 2$, $$(\theta_{Max})_{minor} = \cos^{-1}\left(1 \pm \frac{5}{4}\right)$$

$$= \cos^{-1}\left(\frac{-1}{4}\right) \text{ [or] } \cos^{-1}\left(\frac{9}{4}\right) \Rightarrow \text{is invalid}$$

$$= \cos^{-1}\left(\frac{-1}{4}\right)$$

$$\boxed{\therefore (\theta_{Max})_{minor} = -75.5°}$$

2. Minima Direction for Minor Lobe

Minima is the direction through which the array radiate zero power, which is also called as null direction. The electric field intensity is zero along the null direction.

$$E_T = E_0 \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} = 0$$

E_T is zero, when

$$\sin \frac{n\psi}{2} = 0$$

$$\frac{n\psi}{2} = \sin^{-1}(0) = \pm N\pi$$

Where N = 1, 2, 3........ and N = 0 corresponds to major lobe

$$\boxed{\psi = \pm \frac{2N\pi}{n}} \qquad \text{......(8)}$$

Equating the equations (4) and (8), we get

$$\beta\, d\, [\cos(\theta_{\text{Min}})_{\text{minor}} - 1] = \pm \frac{2\,N\,\pi}{n}$$

$$\cos(\theta_{\text{Min}})_{\text{minor}} - 1 = \pm \frac{2\,N\,\pi}{\beta\, n\, d}$$

$$= \pm \frac{2N\pi}{\dfrac{2\pi}{\lambda} \times d \times n}$$

$$\cos(\theta_{\text{Min}})_{\text{minor}} - 1 = \pm \frac{N\lambda}{nd} \qquad \text{......(9)}$$

$$1 - 2\sin^2 \frac{(\theta_{Min})_{\min or}}{2} - 1 = \pm \frac{N\,\lambda}{n\,d} \qquad \left[\because \cos\theta = 1 - 2\sin^2\frac{\theta}{2}\right]$$

$$-2\sin^2 \frac{(\theta_{Min})_{\min or}}{2} = \pm \frac{N\,\lambda}{n\,d}$$

$$2\sin^2\frac{(\theta_{Min})_{minor}}{2} = \pm\frac{N\lambda}{nd}$$

$$\sin\frac{(\theta_{Min})_{minor}}{2} = \pm\sqrt{\frac{N\lambda}{2nd}}$$

$$\frac{(\theta_{Min})_{minor}}{2} = \sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right)$$

$$(\theta_{Min})_{minor} = 2\sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right) \quad \text{......(10)}$$

For example; if $n = 4$ and $d = \lambda/2$

(i) N = 1, $$(\theta_{Min})_1 = 2\sin^{-1}\left(\pm\sqrt{\frac{1\times\lambda}{2\times4\times\frac{\lambda}{2}}}\right)$$

$$= 2\sin^{-1}\left(\pm\sqrt{\frac{1}{4}}\right) = 2\sin^{-1}\left(\pm\frac{1}{2}\right)$$

$$= 2 \times (\pm 30°)$$

$$(\theta_{Min})_1 = \pm 60°$$

(ii) N = 2,

$$(\theta_{Min})_2 = 2\sin^{-1}\left(\pm\sqrt{\frac{2\times\lambda}{2\times4\times\frac{\lambda}{2}}}\right)$$

$$= 2\sin^{-1}\left(\pm\sqrt{\frac{2}{4}}\right) = 2\sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$= 2 \times (\pm 45°)$$

$$(\theta_{Min})_2 = \pm 90°$$

(iii) $N = 3$, $\quad (\theta_{Min})_3 = 2\sin^{-1}\left(\pm\sqrt{\dfrac{3\times\lambda}{2\times 4\times\dfrac{\lambda}{2}}}\right)$

$$= 2\sin^{-1}\left(\pm\sqrt{\frac{3}{4}}\right) = 2\sin^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$$

$$= 2\times(\pm 60^\circ)$$

$$(\theta_{Min})_3 = \pm 120^\circ$$

(iv) $N = 4$, $\quad (\theta_{Min})_4 = 2\sin^{-1}\left(\pm\sqrt{\dfrac{4\times\lambda}{2\times 4\times\dfrac{\lambda}{2}}}\right) = 2\sin^{-1}\left(\pm\sqrt{\dfrac{4}{4}}\right) = 2\sin^{-1}(\pm 1)$

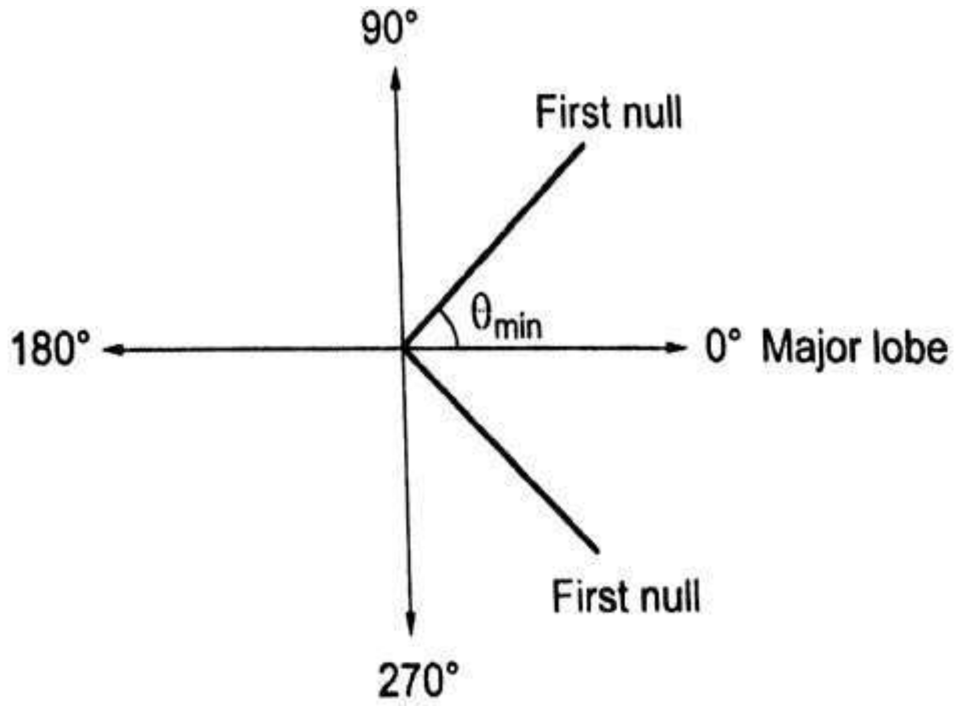
$$= 2\times(\pm 90^\circ)$$

$$(\theta_{Min})_4 = \pm 180^\circ$$

Therefore for an end fire array of 4 isotropic sources spaced $\lambda/2$ apart, there are six minor lobe maxima along the directions $\pm 60°$, $\pm 90°$, $\pm 120°$ and $\pm 180°$.

3. Beam Width for Major Lobe

(i) BWFN



Beam width of end fire array

$$\text{BWFN} = 2 \times \left\{ \begin{array}{c} \text{Angle between first nulls and the} \\ \text{maximum of major lobes} \end{array} \right\}$$

$$= 2 \times \theta_{min} \qquad \ldots\ldots(11)$$

From equation (10), $$\theta_{\min} = 2\sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right)$$

$$\sin\theta_{\min} = 2\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right) \qquad \ldots\ldots(12)$$

For small angles $\sin\theta_{\min} \approx \theta_{\min}$, then the equation (12) becomes

$$\theta_{\min} = \pm\sqrt{\frac{2N\lambda}{nd}} \qquad \ldots\ldots(13)$$

Let L = Total length of the array in meters.

$$L = (n-1)\,d \approx n\,d \text{ (if } n \text{ is large)} \qquad \ldots\ldots(14)$$

Therefore equation (13) becomes,

$$\theta_{\min} = \pm\sqrt{\frac{2Nd}{L}} \qquad \ldots\ldots(15)$$

By substitute equation (15) in equation (11), we get

$$\text{BWFN} = 2\times\theta_{\min} = 2\times\left(\pm\sqrt{\frac{2N\lambda}{L}}\right) \qquad \ldots\ldots.(16)$$

(a) If N = 1,

$$\text{BWFN} = \pm 2\sqrt{\frac{2}{\text{L}/\lambda}} = \pm\frac{2\sqrt{2}}{\sqrt{\frac{\text{L}}{\lambda}}} \text{ radians}$$

$$= \pm\frac{2\sqrt{2}}{\sqrt{\frac{L}{\lambda}}}\times 57.3 \text{ degrees}$$

$$\text{BWFN} = \pm 114.6\sqrt{\frac{2}{L/\lambda}}\ \text{degrees} \qquad \text{......(17)}$$

(ii) HPBW

$$\text{HPBW} = \frac{\text{BWFN}}{2}$$

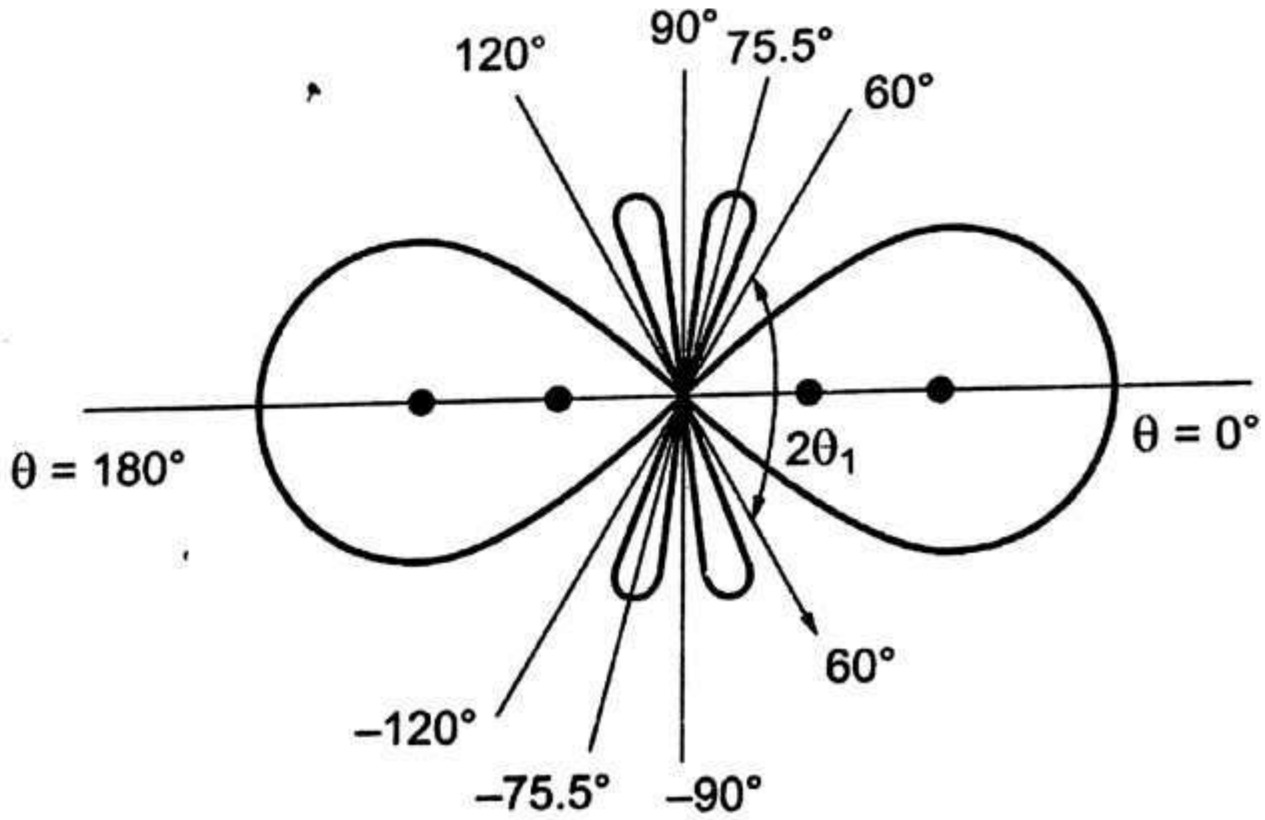
$$\text{HPBW} = \pm 57.3\sqrt{\frac{2}{L/\lambda}} \qquad \text{......(18)}$$

4. Directivity

$$D = 4n\left(\frac{d}{\lambda}\right) = 4\left(\frac{L}{\lambda}\right) \qquad \text{......(19)}$$

For an increased directivity,

$$D = 1.789\left[4n\left(\frac{d}{\lambda}\right)\right] = 1.789\left[4\left(\frac{L}{\lambda}\right)\right] \qquad \text{......(20)}$$



Field pattern of an end fire array

Phased Arrays

Phased array *means an array of many elements and the phase of each element being a variable that provides* ***control of the beam direction****, that is, maximum radiation in any desired direction and pattern shape including the side lobes.*

Some of the specialized phased arrays are:

(i) Frequency scanning array,

(ii) Retrodirective array, and

(iii) Adaptive array.

In the ***frequency scanning array*** *or* ***scanning array****, phase change is accomplished by varying the frequency. It is one of the simplest phased arrays since no phase control is required at each element.*

A ***retrodirective*** *or* ***self-focusing*** *array is an array that will receive a signal from any direction in space and automatically reflects the signal back toward its source, usually after suitable modulation and amplification.*

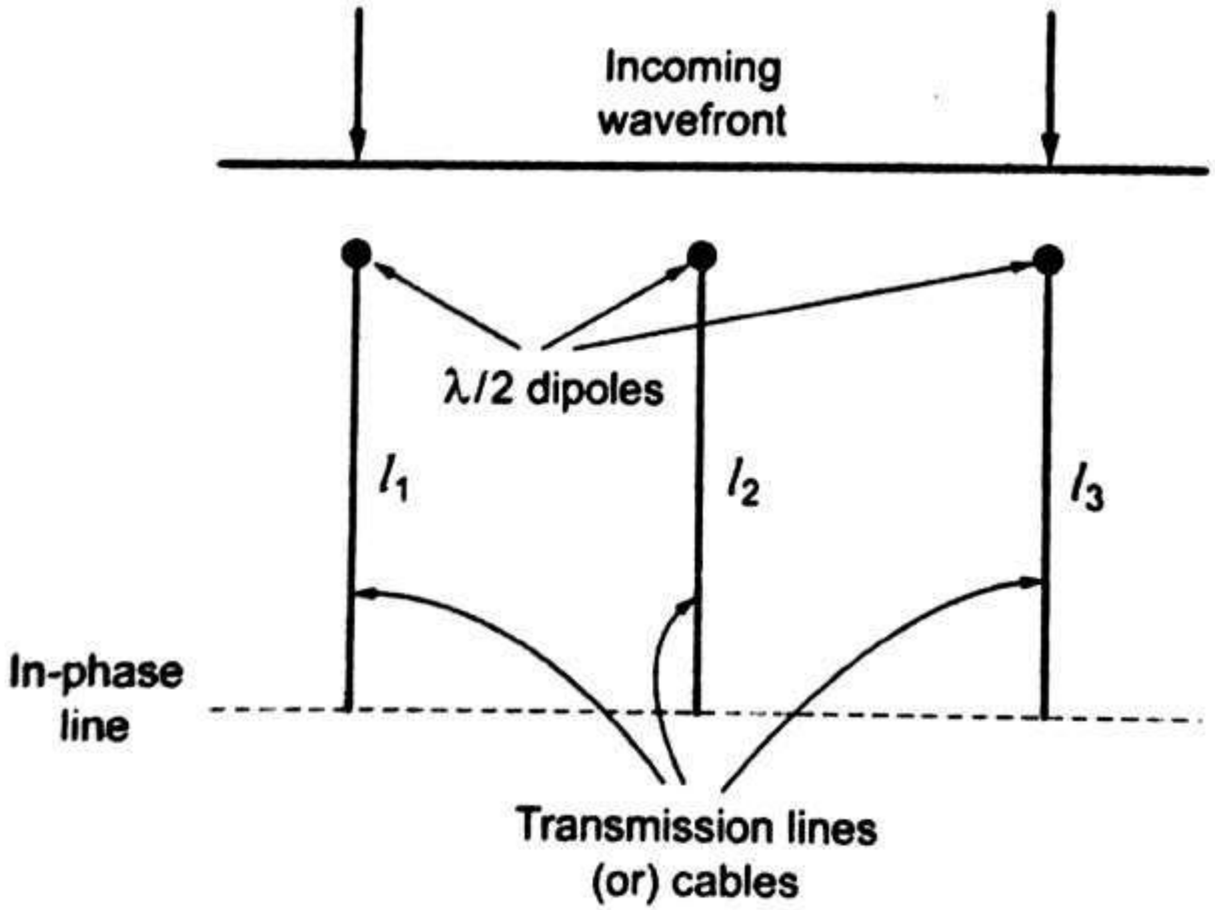
Phased array designs

The objectives of the phased array are:

(i) A phased array has to accomplish a beam steering without the mechanical and inertial problems of rotating the entire array, and

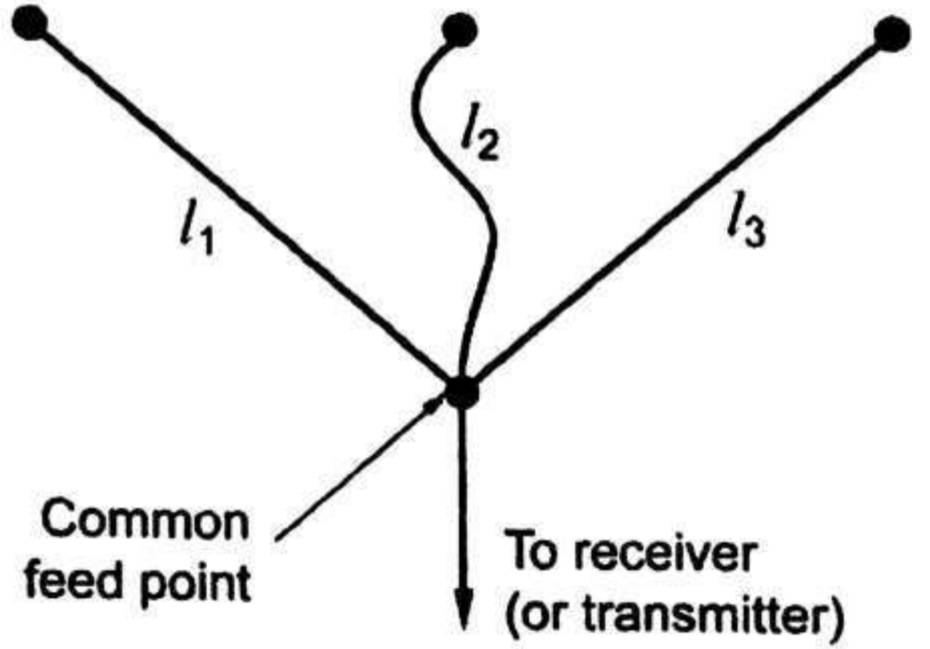
(ii) It has to provide beam control at a fixed frequency (or) at any number of frequencies within a certain bandwidth in a ***frequency-independent manner***.

In the simplest form of a phased array, beam steering can be done by mechanical switching. Consider a basic 3-elements array and each element be a $\frac{\lambda}{2}$ dipole antenna

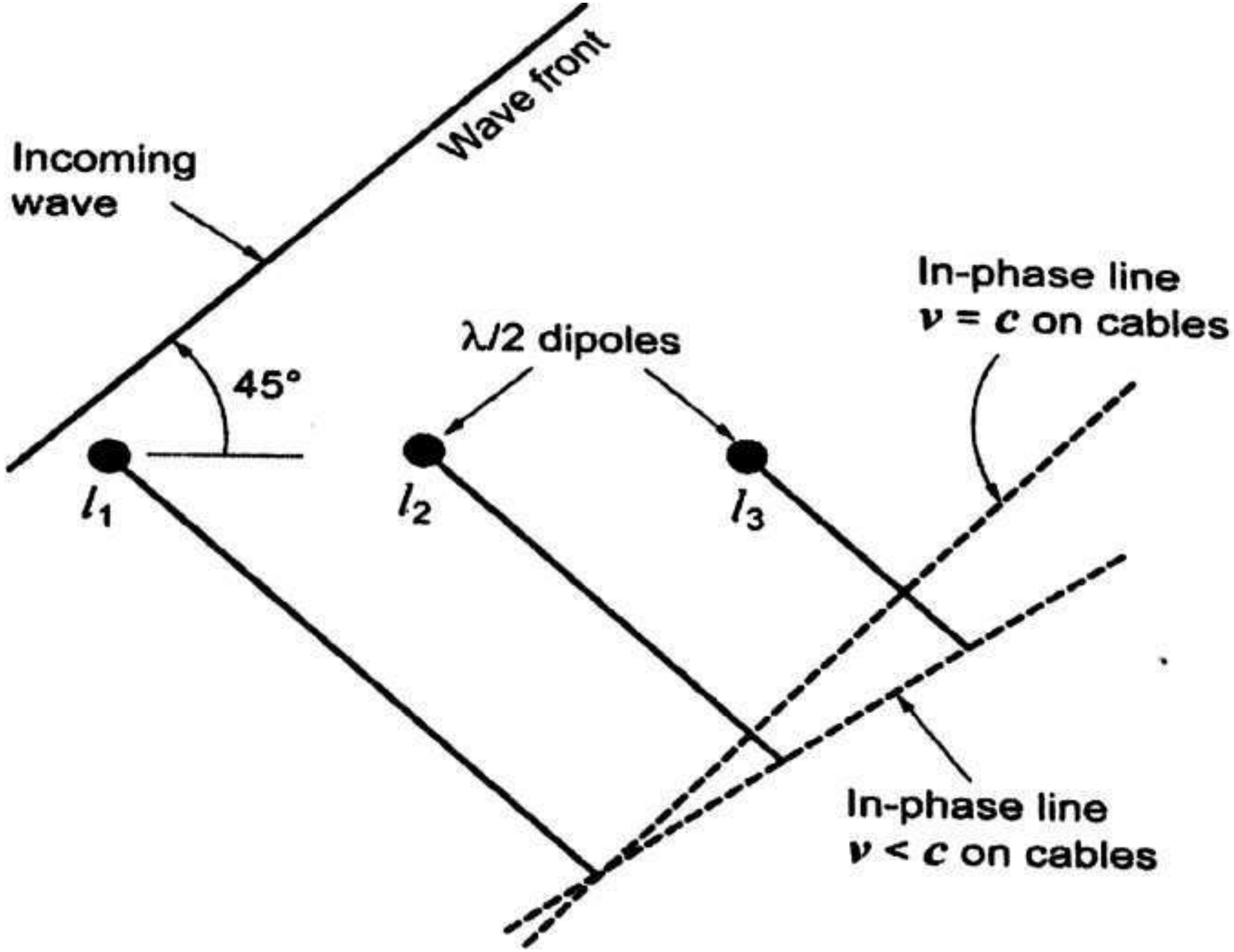


A simple phased array of three $\frac{\lambda}{2}$ dipoles

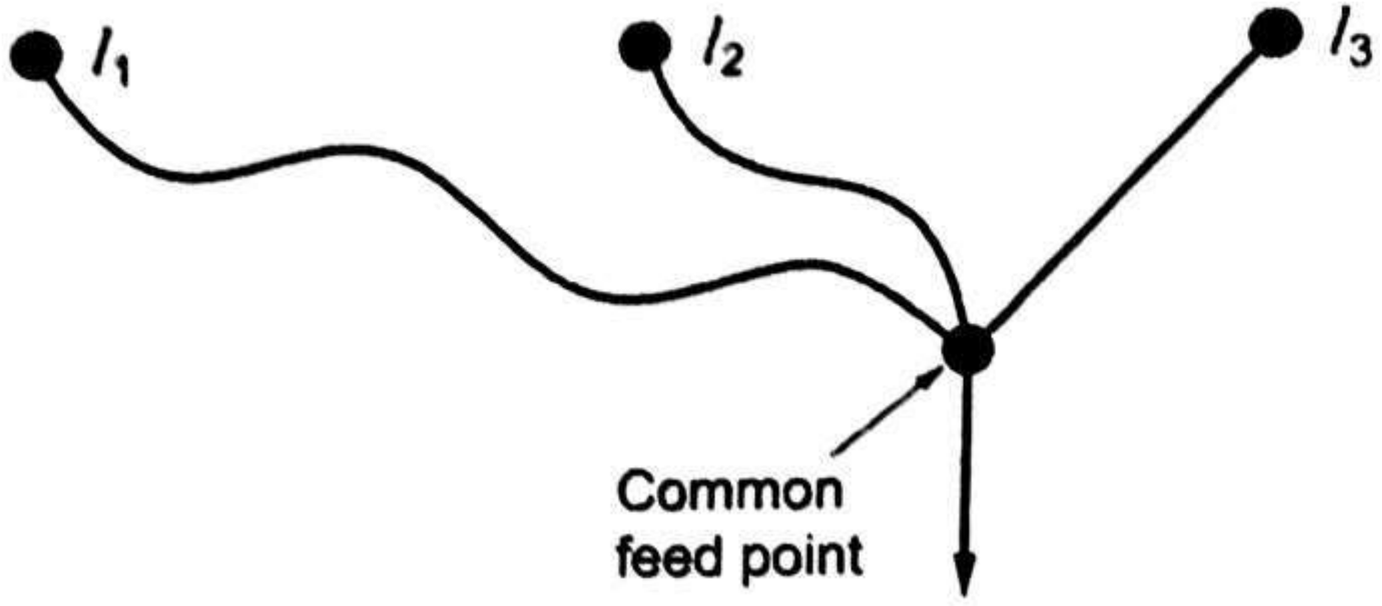
All three transmission cables are joined as a common point and this 3-element array will operate as a ***broadside array***.



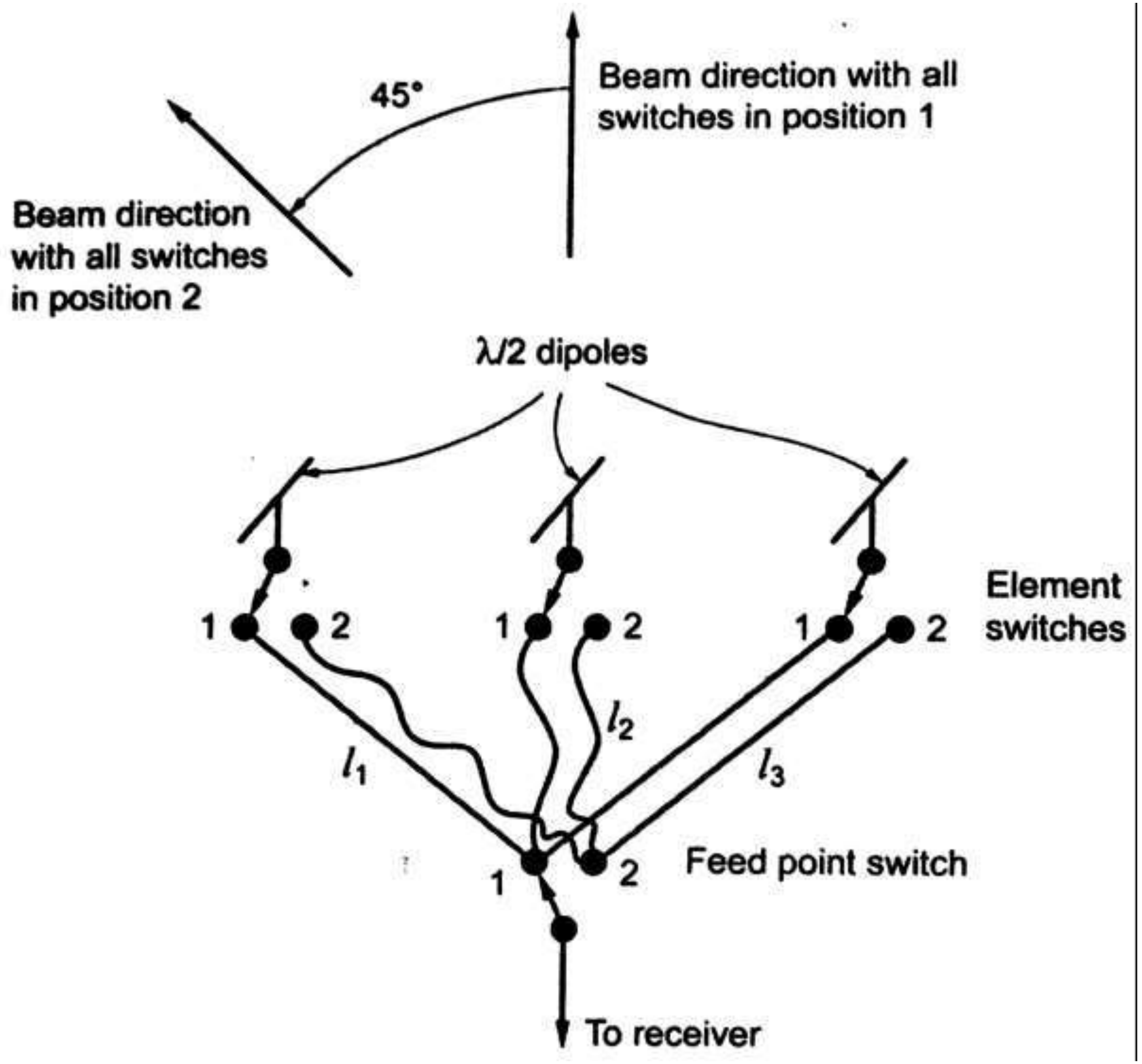
Equal length cables joined in common point



Incoming wave at 45° from broad side array



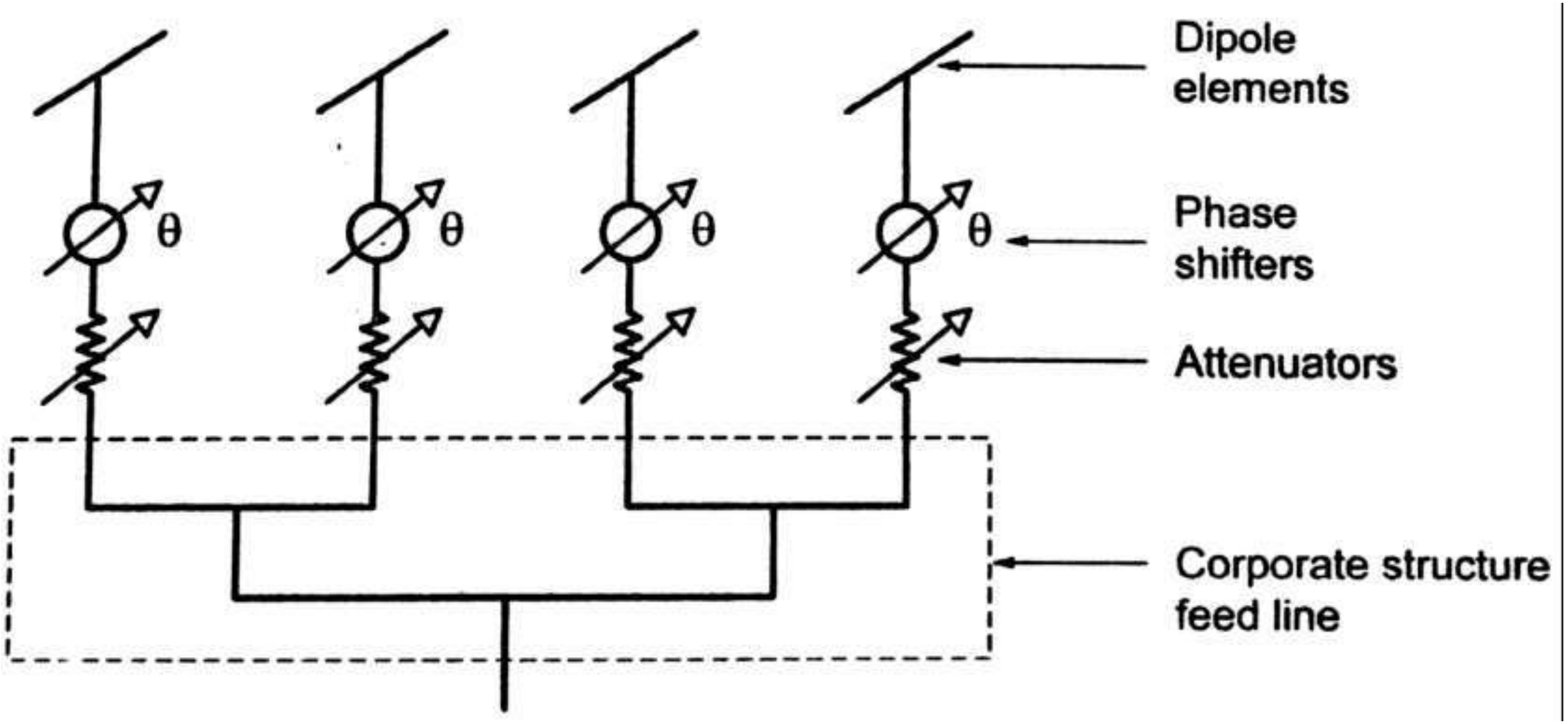
Joining on all cables in common point



Switches for shifting from broadside to 45° reception

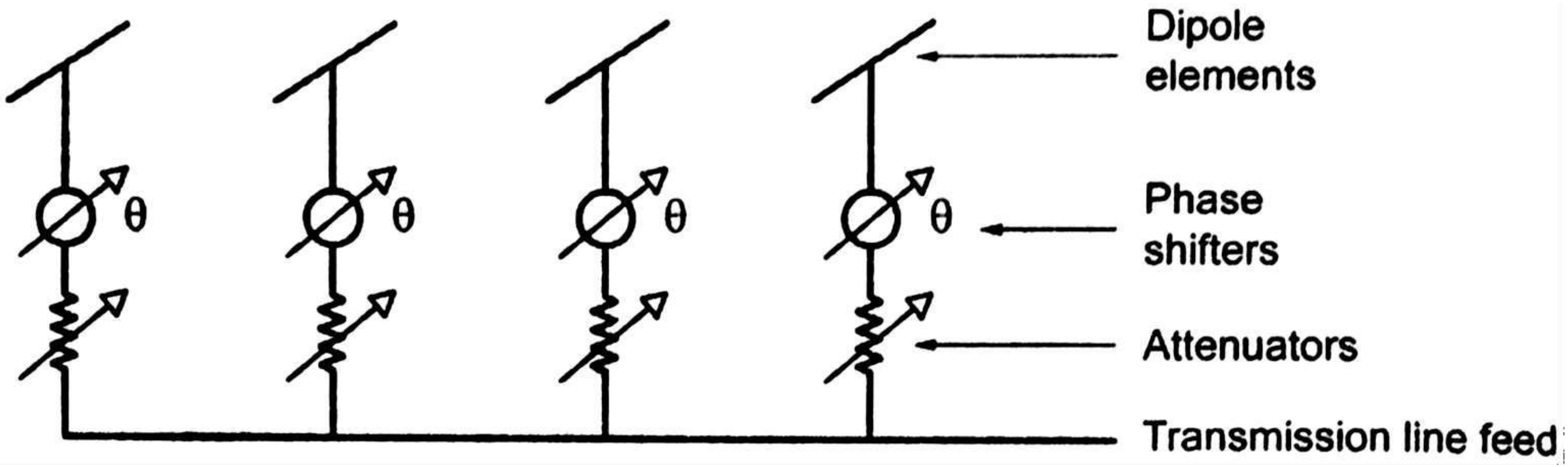
Different types of fed used in phased array

1. Corporate Structure:

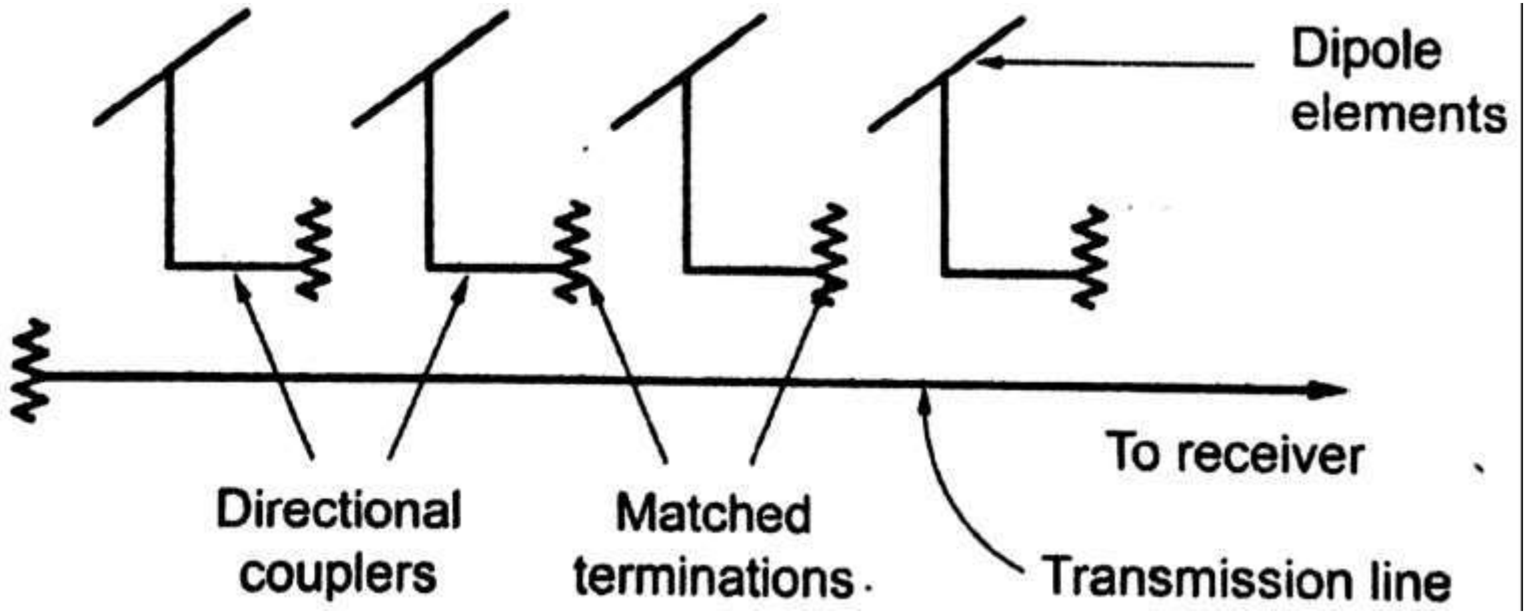


Schematic of phased array fed by corporate structure

2. End – Fed phased array



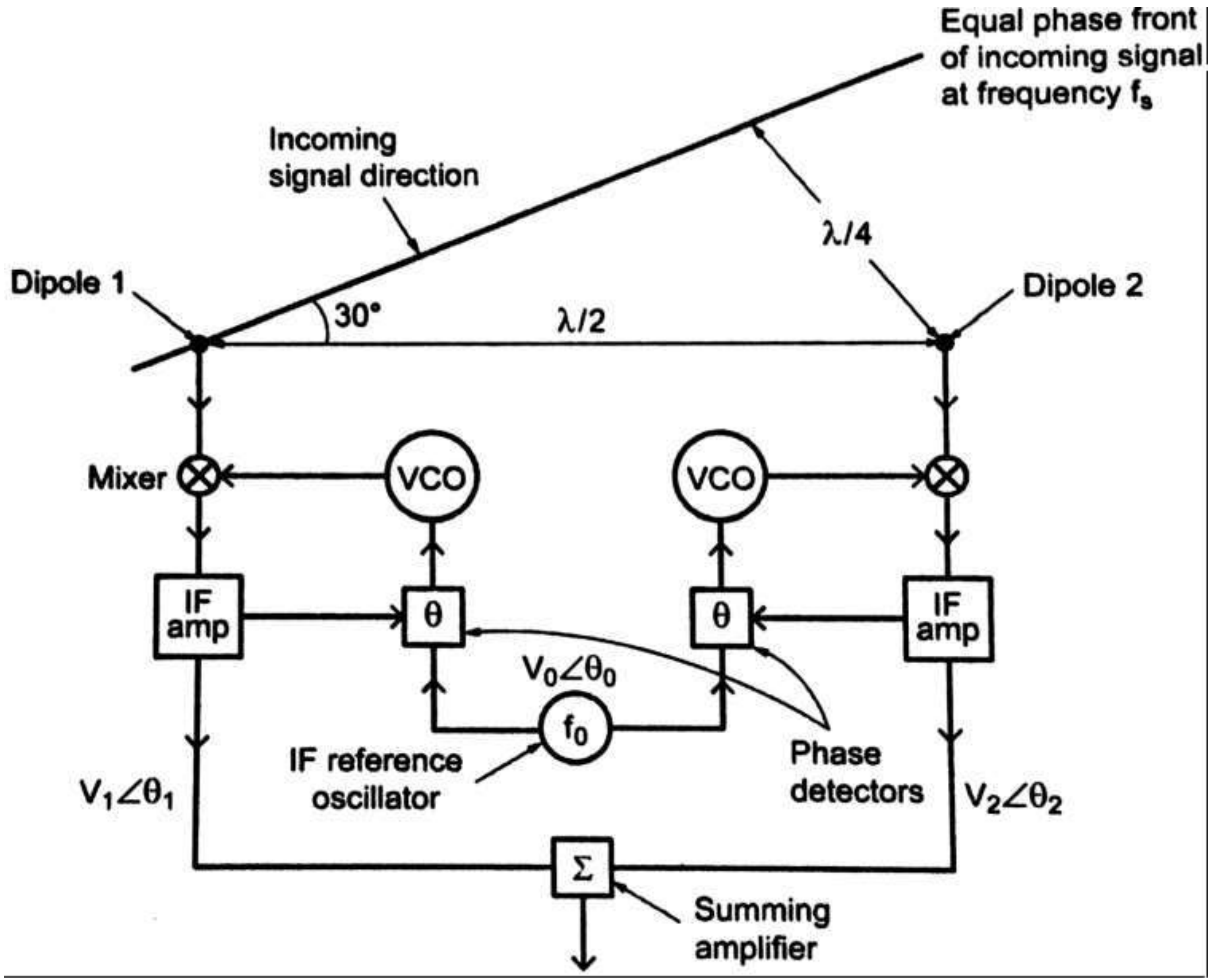
End-fed phase array with transmission line feed



End-fed phase array with directional coupler feed

Adaptive array

Adaptive arrays are arrays that can automatically self-adapt to various incoming signals conditions so as to maximize the signal from a particular source or to null out interfering signals.



Two- element adaptive array with signal-processing circuitry

Antenna Synthesis

Antenna analysis is the process of determining the radiation pattern for a given input distribution. Antenna synthesis is the inverse of antenna analysis.

Hence, antenna array synthesis is the process of determining input or source distribution for a specified radiation pattern.

In other words, antenna synthesis is the problem of determining the parameters of an antenna system that will produce a radiation pattern which accurately approximates some desired pattern.

The **various array synthesis techniques are as follows:**

(i) Fourier transformed method,

(ii) Dolph- Tchebyscheff method ,

(iii) Taylor's method,

(iv) Laplace transform method, and

(v) Binomial arrays

DOLPH-TCHEBYSCHEFF (D-T) OPTIMUM DISTRIBUTION [OR] CHEBYSHEV ARRAYS [OR] LINEAR ARRAY WITH NONUNIFORM AMPLITUDE DISTRIBUTIONS

While designing antenna arrays, it is necessary to determine the current ratios which results in smallest side lobe level for a specified beam width.

FUNDAMENTAL OF TCHEBYCHEFF POLYNOMIALS

The Tchebyscheff polynomial of *m*th degree with variable '*x*' is denoted by $T_m(x)$ and it is defined by

$$\left.\begin{aligned} T_m(x) &= \cos(m \cos^{-1} x), && -1 < x < +1 \\ &= \cos h(m \cos h^{-1} x), && |x| > 1 \end{aligned}\right\} \quad \ldots\ldots (1)$$

where '*m*' is an integer constant range from 0 to ∞

In general equation (1) can be written as,

$$T_m(x) = \cos\left(m\cos^{-1}x\right) = \cos(m\delta) = \cos\left(m\frac{\psi}{2}\right)$$

$$T_m(x) = \cos\left(m\frac{\psi}{2}\right) \qquad \text{......(2)}$$

where, $\delta = \cos^{-1}x \Rightarrow x = \cos\delta = \cos\dfrac{\psi}{2}$

Let us now obtain Tchebyscheff polynomials for different values of 'm'

If $m = 0$, $\quad T_0(x) = \cos(m\delta) = \cos(0.\delta)$

$$\boxed{T_0(x) = 1}$$

If $m = 1$, $T_1(x) = \cos(m\delta) = \cos(1.\delta) = x$

$$\boxed{T_1(x) = x}$$

If $m = 2$, $T_2(x) = \cos(m\delta)$

$\cos 2\theta = 2\cos^2\theta - 1$

$$= \cos(2\delta)$$

$$= 2\cos^2\delta - 1$$

$$\boxed{\therefore T_2(x) = 2x^2 - 1}$$

If $m = 3$, $T_3(x) = \cos 3\delta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$$= 4\cos^3\delta - 3\cos\delta$$

$$\boxed{T_3(x) = 4x^3 - 3x}$$

If $m = 4$, $\quad T_4(x) = \cos 4\delta$ $\qquad \cos 4\theta = 2\cos^2 2\theta - 1$

$$= \cos 2(2\delta) = 2\cos^2(2\delta) - 1$$

$$= 2\left[2\cos^2\delta - 1\right]^2 - 1$$

$$= 2\left[4\cos^4\delta - 4\cos^2\delta + 1\right] - 1$$

$$= 8\cos^4\delta - 8\cos^2\delta + 1$$

$$\boxed{T_4(x) = 8x^4 - 8x^2 + 1}$$

Further the polynomials with higher values of m can be obtained using recursive formula given by

$$T_{m+1}(x) = 2x\,T_m(x) - T_{m-1}(x)$$

For the particular values of 'm' the first ten Tchebyscheff polynomials are given as,

$m = 0$	$T_0(x) = 1$
$m = 1$	$T_1(x) = x$
$m = 2$	$T_2(x) = 2x^2 - 1$
$m = 3$	$T_3(x) = 4x^3 - 3x$
$m = 4$	$T_4(x) = 8x^4 - 8x^2 + 1$
$m = 5$	$T_5(x) = 16x^5 - 20x^3 + 5x$
$m = 6$	$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$
$m = 7$	$T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x$
$m = 8$	$T_8(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1$
$m = 9$	$T_9(x) = 256x^9 - 576x^7 - 432x^5 - 120x^3 + 9x$

Binomial Array

$$(a+b)^{n-1} = a^{n-1} + \frac{n-1}{1!} a^{n-2} \cdot b + \frac{(n-1)(n-2)}{2!} a^{n-3} b^2 + \frac{(n-1)(n-2)(n-3)}{3!} a^{n-4} b^3 \quad \ldots\ldots (1)$$

where, n - Number of radiating sources in the array

CONCEPTS OF BINOMIAL ARRAY

If the array is arranged in such a way that radiating sources are in the centre of the broad side array radiates more strongly than the radiating sources at the edges, minor lobes can be eliminated.

The secondary lobes can be eliminated entirely, when the following two conditions are satisfied:

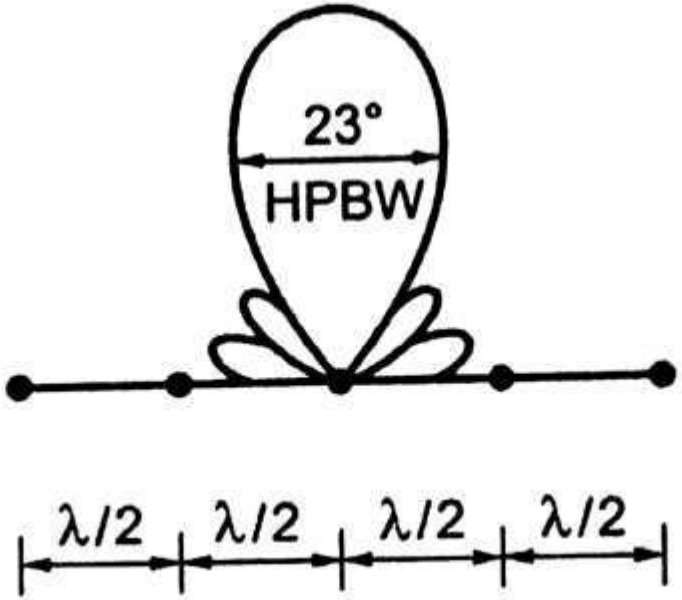
(i) The space between the 2 consecutive radiating sources does not exceed $\frac{\lambda}{2}$

(ii) The current amplitudes in radiating sources (from outer towards centre source) are proportional to the coefficients of the successive terms of the binomial series.

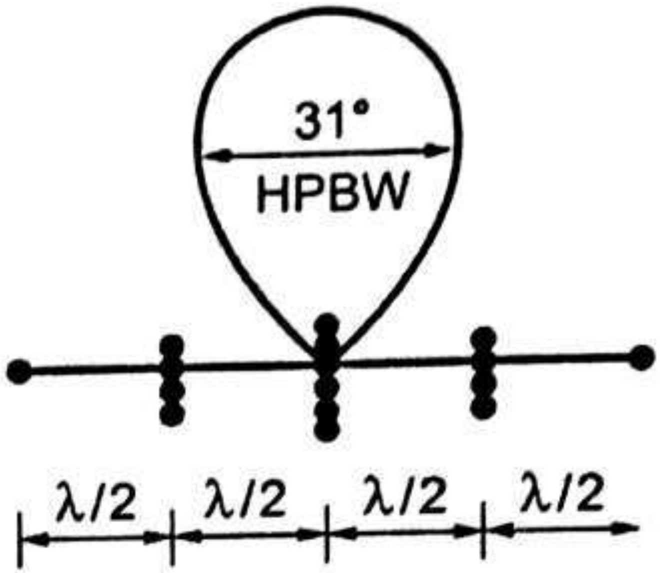
For example, the relative amplitudes for the arrays of 1 to 10 radiating sources are as follows:

Number of sources	Relative Amplitude
$n = 1$	1
$n = 2$	1, 1
$n = 3$	1, 2, 1
$n = 4$	1, 3, 3, 1
$n = 5$	1, 4, 6, 4, 1
$n = 6$	1, 5, 10, 10, 5, 1
$n = 7$	1, 6, 15, 20, 15, 6, 1
$n = 8$	1, 7, 21, 35, 35, 21, 7, 1
$n = 9$	1, 8, 28, 56, 70, 56, 28, 8, 1
$n = 10$	1, 9, 36, 84, 126, 126, 84, 36, 9, 1

consider $n = 5$, $d = \frac{\lambda}{2}$, HPBW of binomial array is 31° and HPBW of an uniform array is 23°



(a) Uniform array



(b) Binomial array with amplitude ratio 1 : 4 : 6 : 4 : 1

Disadvantages of Binomial Arrays

(i) HPBW increases and hence the directivity decreases.

(ii) For the design of a large array, the larger amplitude ratio of sources is required.

UNIT IV

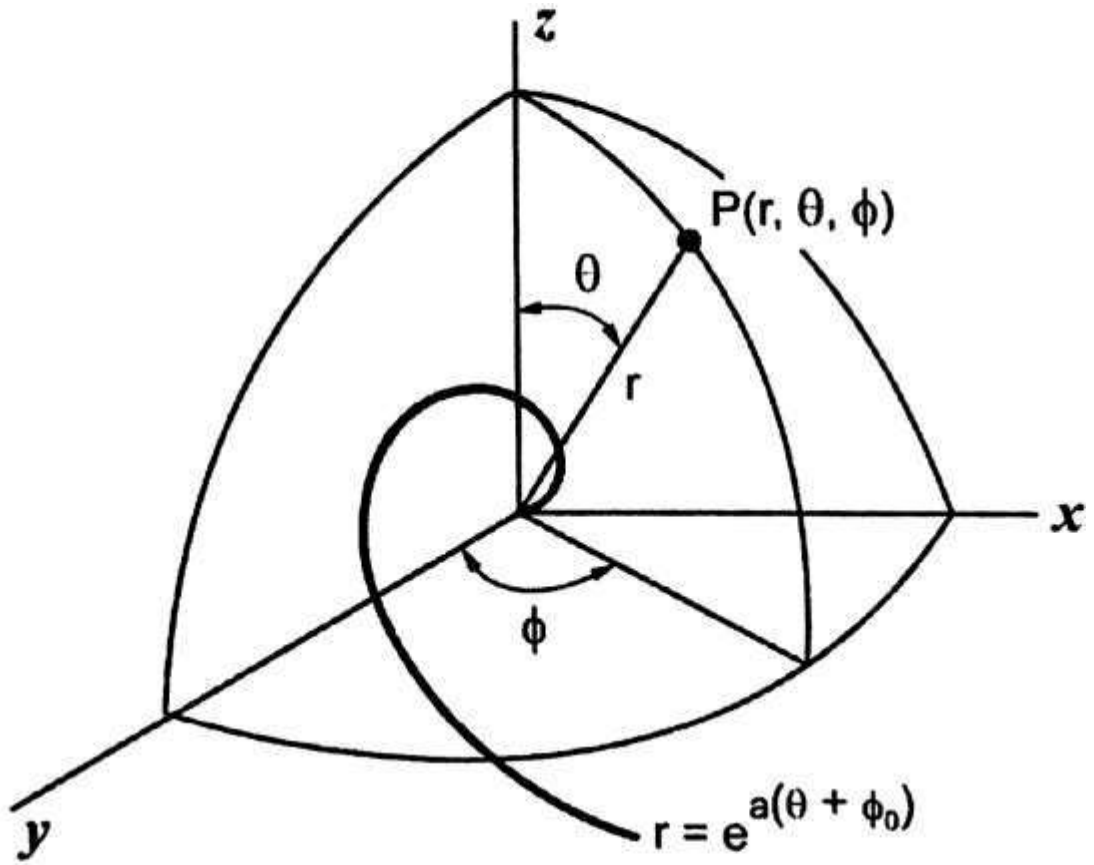
Principle of frequency independent antennas

A frequency independent antenna is physically fixed in size and operates on an over a wide bandwidth (entire frequency band) with relatively constant impedance, pattern, polarization and gain

These antennas are broadband antennas which are using 10 to 10,000 MHz

RUMSEY'S PRINCIPLE

"The performance that is, the impedance and pattern properties of a lossless antenna is independent of frequency if the dimensions of the antenna are specified in terms of angles such that they remain constant in terms of wavelength"



Spherical co-ordinate system for equiangular spiral antenna

This requirement can be fulfilled by any antenna if its equation in the spherical co-ordinates are in the form given as,

$$r = e^{a(\phi+\phi_0)} f(\theta) \qquad \ldots\ldots(1)$$

where, $f(\theta)$ is the function of θ

In case of planar antenna, the above equation reduces to,

$$r = e^{a(\phi+\phi_0)} \qquad \ldots\ldots(2)$$

Let us assume that, the antenna is perfectly conducting and it is surrounded by an isotropic homogeneous medium. The surface of this antenna is defined by,

$$r = F(\theta,\phi) \qquad \ldots\ldots(3)$$

where, r is the distance along the surface or the edge.

If the antenna is to be scaled to a new frequency that is K times lower than the original frequency, to maintain the same electrical properties

Physical surface of the antenna should be made K times greater

The new surface of the antenna is

$$r = KF(\theta,\phi) \qquad \ldots\ldots(4)$$

where, K is constant independent of θ and ϕ.

$$KF(\theta,\phi) = F(\theta,\phi + C) \qquad \ldots\ldots(5)$$

where, angle C depends on K but not on θ *and* ϕ.

To obtain the functional representation of $F(\theta,\phi)$, the both sides of equation (5) are differentiating with respect to C, we get

$$\frac{d}{dC}[K\,F(\theta,\phi)] = \frac{d}{dC}[F(\theta,\phi+C)]$$

$$\frac{\partial K}{\partial C}F(\theta,\phi) = \frac{\partial}{\partial(\phi+C)}[F(\theta,\phi+C)] \qquad \text{......(6)}$$

Now differentiating equation (5) with respect to ϕ, we get

$$\frac{\partial}{\partial\phi}[KF(\theta,\phi)] = \frac{\partial}{\partial(\phi+C)}[F(\theta,\phi+C)]$$

$$K\frac{\partial F(\theta,\phi)}{\partial\phi} = \frac{\partial}{\partial(\phi+C)}[F(\theta,\phi+C)] \qquad \text{......(7)}$$

By equating the equations(6) and (7) , we get

$$\frac{\partial K}{\partial C} F(\theta,\phi) = K \frac{\partial F(\theta,\phi)}{\partial \phi}$$

From the equation (3), that is, $r = F(\theta,\phi)$

$$\frac{\partial K}{\partial C}.r = K.\frac{\partial r}{\partial \phi}$$

$$\frac{1}{K}\frac{\partial K}{\partial C} = \frac{1}{r}\frac{\partial r}{\partial \phi} \qquad \ldots\ldots(8)$$

Since L.H.S. of the equation(8) is independent of θ and ϕ, the general solution for surface $r = F(\theta, \phi)$ of the antenna is given by,

$$r = F(\theta,\phi) = e^{a\phi} f(\theta) \quad \text{......(9a)}$$

where, parameter $a = \dfrac{1}{K}\dfrac{\partial K}{\partial C}$ ......(9b)

The above equation (9) is called ***Rumsey's Principle*** or ***general shape equation*** for the frequency independent antennas.

Spiral Antenna

Spiral antennas are a frequency independent antenna

(i) Planar spiral antenna

The shape of the equiangular plane spiral curve can be obtained from equation (9) by selecting derivation of f(θ) as,

$$f'(\theta) = \frac{df}{d\theta} = A\delta\left(\frac{\pi}{2} - \theta\right) \qquad \ldots\ldots(10)$$

where, A is a constant, and

δ is the Dirac delta function

Now using the equation (10), the equation (9) becomes,

$$r\big|_{\theta=\frac{\pi}{2}} = Ae^{a\phi}\ ,\ when\ \theta = \frac{\pi}{2} \qquad \text{......(11a)}$$

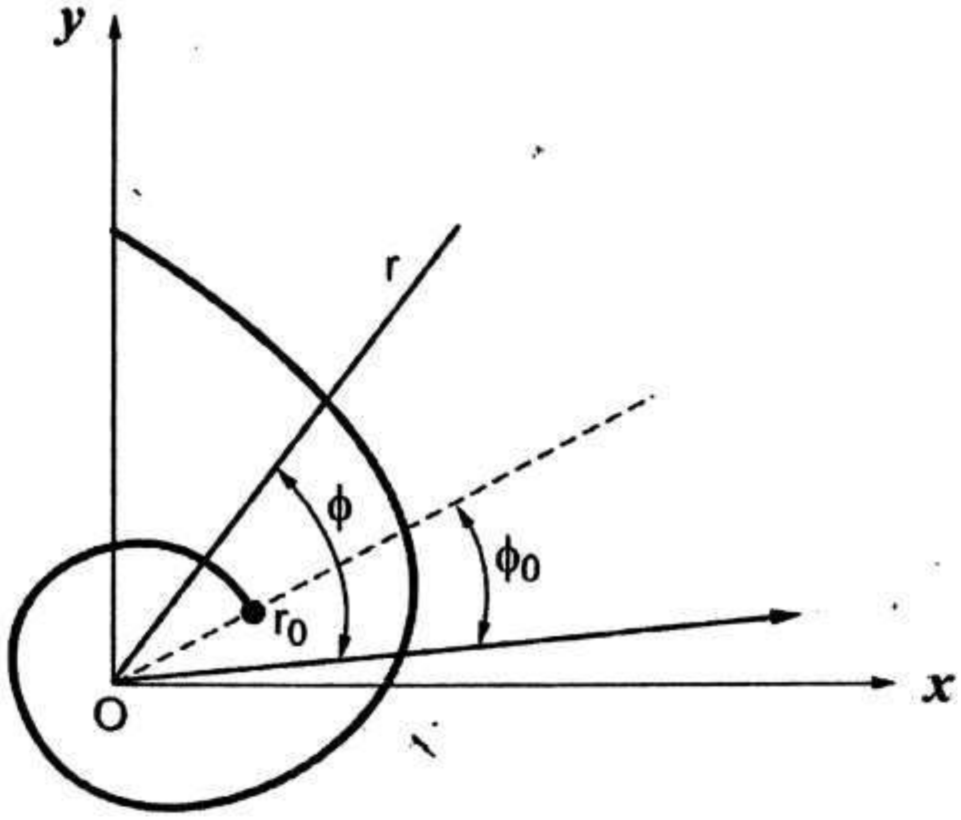
$$= 0, \qquad when\ \theta \neq \frac{\pi}{2} \qquad \text{......(11b)}$$

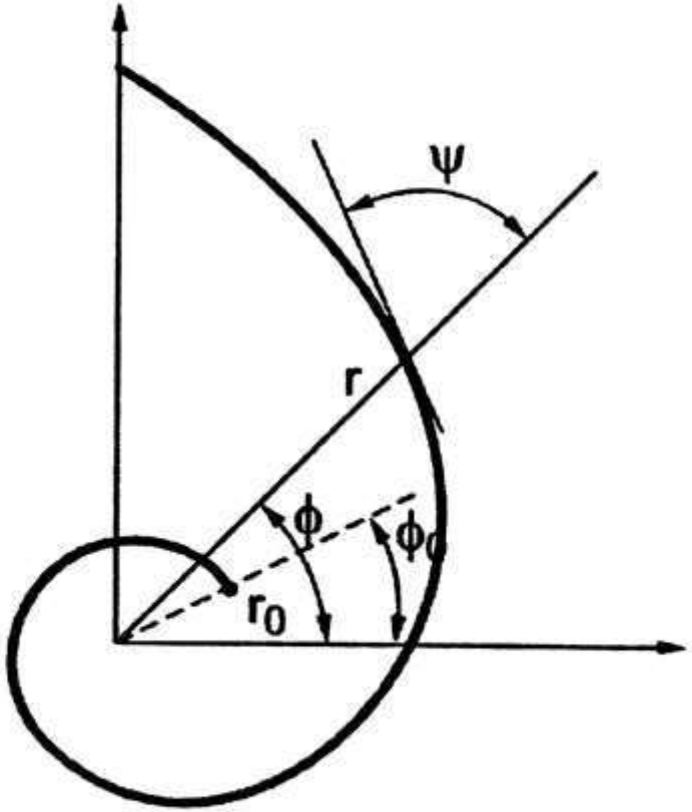
where, $A = r_0 e^{-a\phi_0}$ = Arbitrary positive constant

$$\left.\begin{aligned} \text{Then,}\quad r &= r_0 e^{-a\phi_0} e^{a\phi}\ ,when\ \ \theta = \frac{\pi}{2} \\ &= r_0 e^{a(\phi-\phi_0)} \end{aligned}\right\} \qquad \text{......(12a)}$$

$$= 0, \qquad when\ \theta \neq \frac{\pi}{2} \qquad \text{......(12b)}$$

This is the equation of an ***equiangular*** or ***logarithmic spiral*** where ***a*** is the rate of expansion and ϕ_0 is the orientation.





(a) Single spiral

(b) Two spiral ($\phi_0 = 0, \pi$)

(c) Multiple spiral ($\phi_0 = 0, \pi/2, \pi, 3\pi/2$)

(d) Multiple spiral ($\phi_0 = 0, \pi/2, \pi, 3\pi/2$)

Let, Z_1 = Input impedance of antenna for $\phi_1 = \alpha$

Z_2 = Input impedance of antenna for $\phi_1 = \pi - \alpha$

Two antennas form complementary screens and hence, we can write,

$$Z_1 Z_2 = \frac{\eta^2}{4} \qquad \ldots\ldots(13)$$

When $\phi_1 = \frac{\pi}{2}$, then $Z_1 = Z_2$, the equation (13) becomes

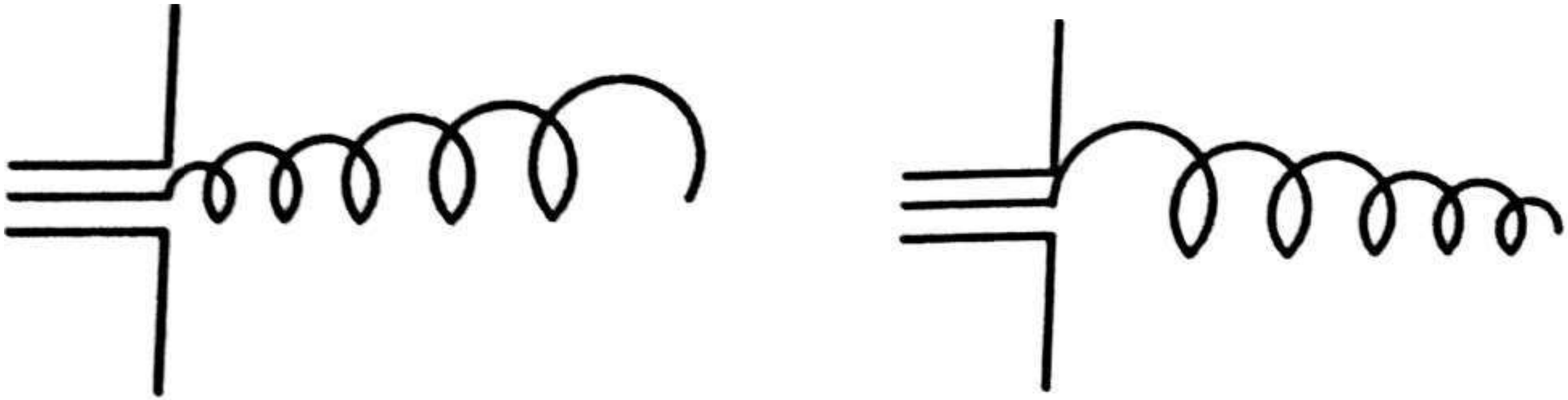
$$Z_1^2 = \frac{\eta^2}{4} = Z_2^2$$

$$Z_1 = Z_2 = \frac{\eta}{2} \qquad \ldots\ldots(14)$$

$$Z_1 = Z_2 = 60\pi = 188.4 \; ohms \qquad \because \eta = 120\pi$$

(ii) Conical – Spiral Antenna

A tapered helix is a conical-spiral antenna



(a) α-increasing (b) α-decreasing

Conical-spiral antenna

The shape of non-planar spiral antenna can be defined by selecting derivative of $f(\theta)$ as,

$$f'(\theta) = \frac{df}{d\theta} = A\delta(\beta-\theta) \qquad \text{......(15)}$$

Where, β = *Any angle in range* $0 \le \beta \le \pi$

The edges of such conical spiral surfaced can be given by, r_2'

$$r_2 = r_2' \ e^{(a\sin\theta_0)\phi} = r_2' \ e^{b\phi} \qquad \text{......(16a)}$$

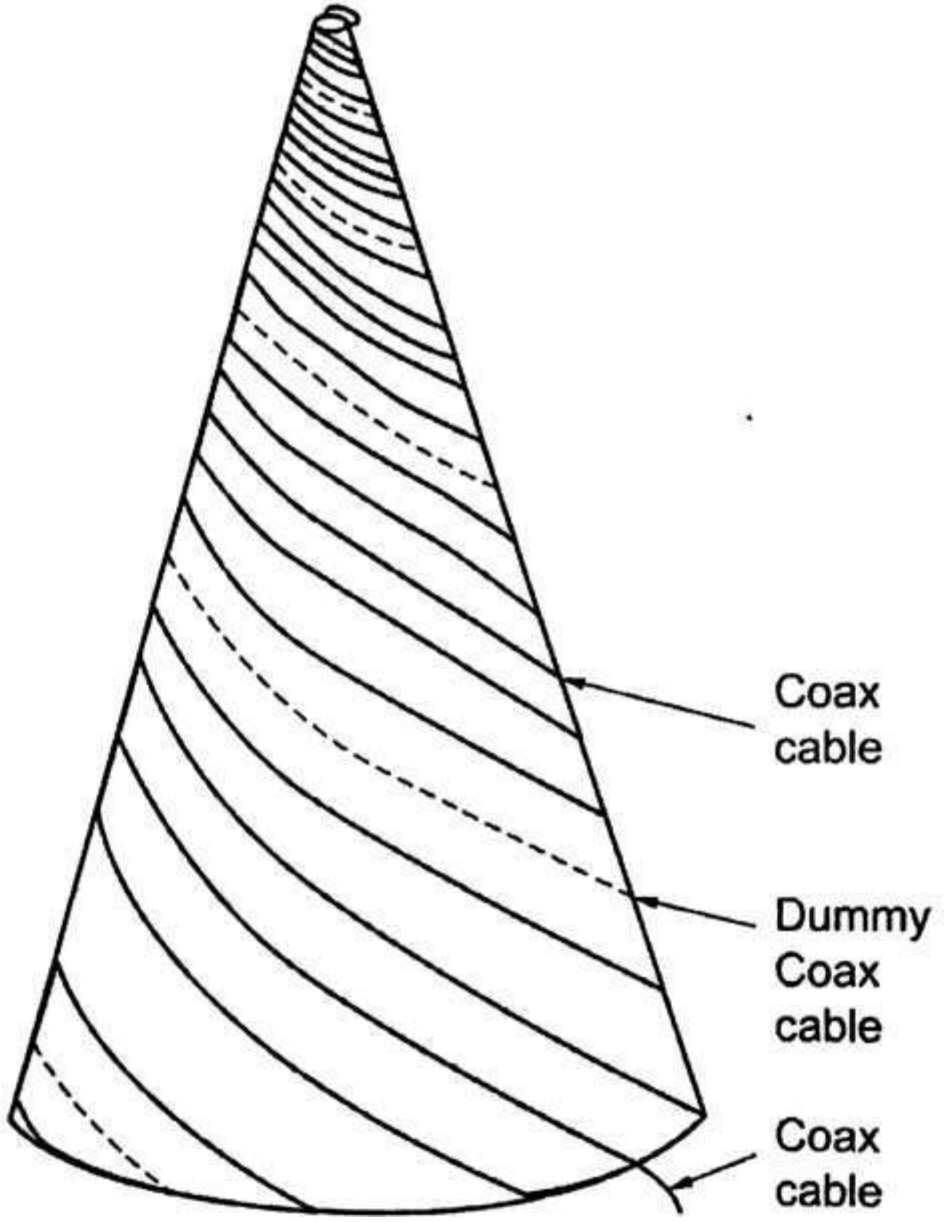
where, $b = a\sin\theta_0$

$$r_3 = r_3' \ e^{(a\sin\theta_0)\phi} = r_2' \ e^{-(a\sin\theta_0)\delta} . e^{(a\sin\theta)\phi}$$

$$= r_2' \ e^{a\sin\theta_0(\phi-\delta)} \qquad \text{......(16b)}$$

where, $r_3' = r_2' \ e^{-(a\sin\theta_0)\delta}$

θ_0 = half of the total included cone angle



Two arm balanced conical spiral antenna

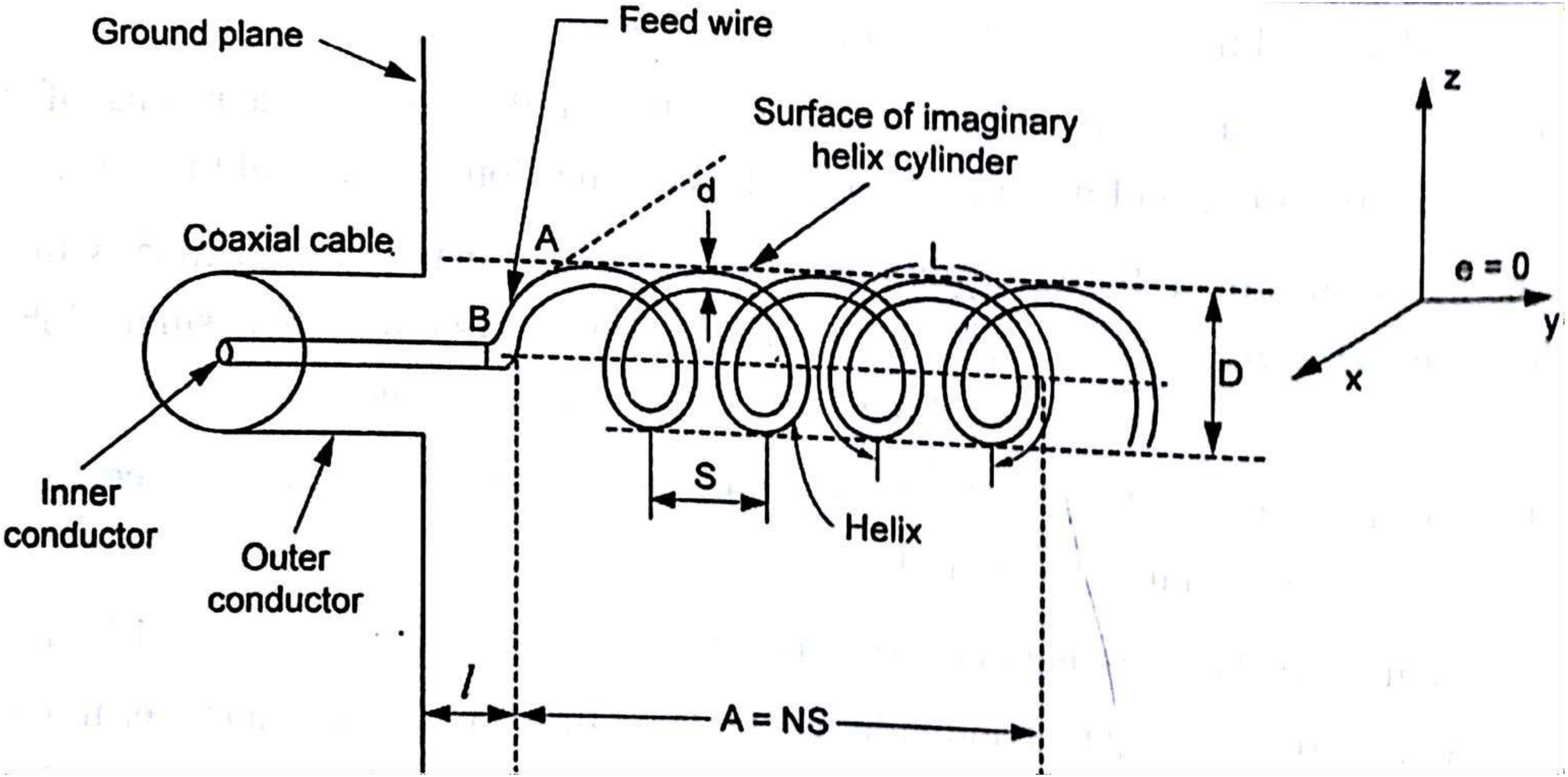
Helical Antenna

Helical antenna is a simplest type of antenna (radiator) which provides circularly polarized waves; it is used in extra terrestrial communications where satellite relays are involved.

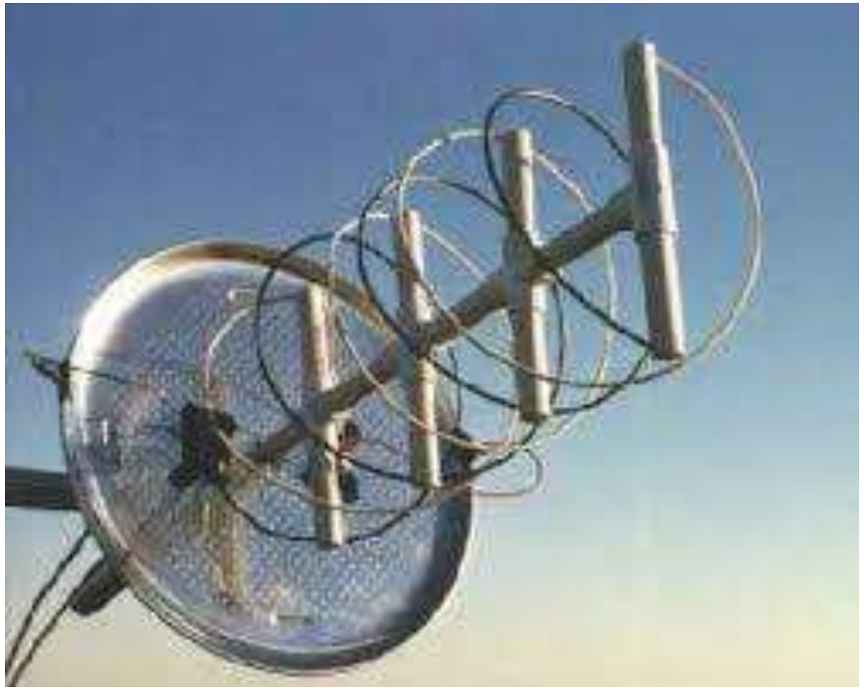
The helical antenna is a broadband VHF and UHF antenna to provide circular polarization characteristics.

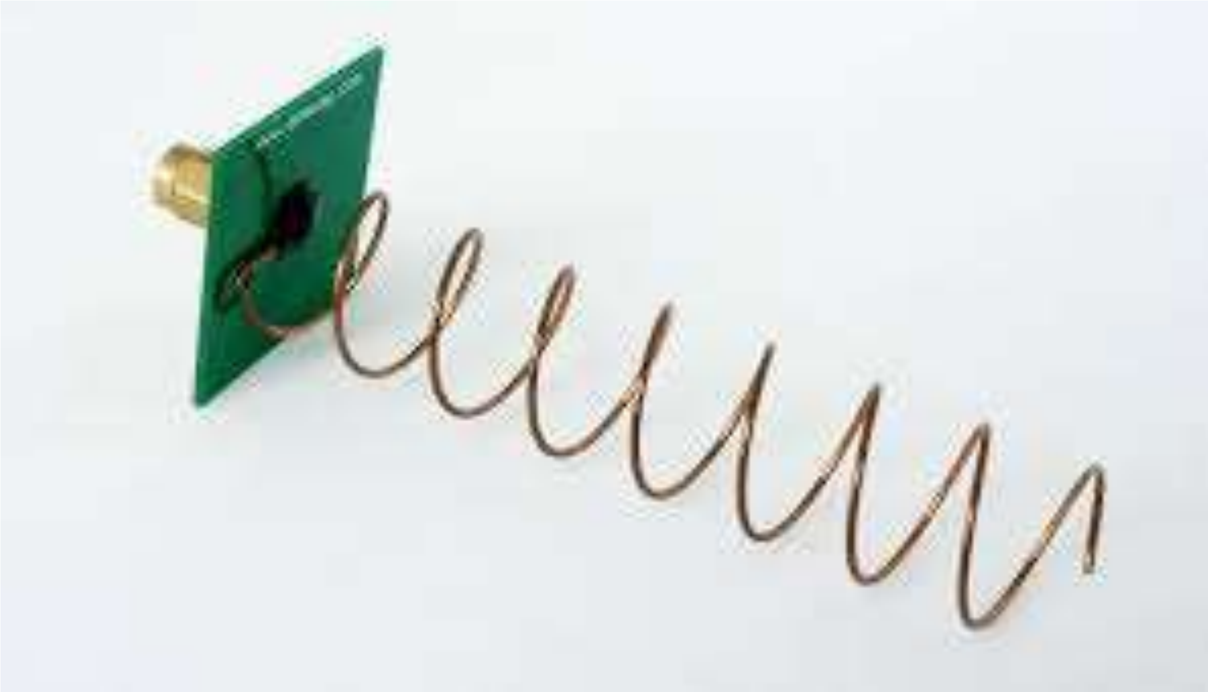
Construction

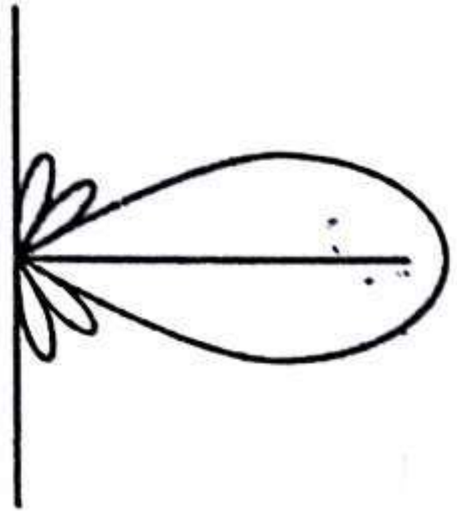
Helical antenna consists of a helix of thick copper wire or tubing wound in the shape of a screw thread and used with a flat metal called a ***ground plane*** or ***ground plate***



Helical Antenna







Radiation pattern of helical antenna (axial mode)

The following symbols are used to describe a helix

C = Circumference of helix = π D

d = Diameter of helix conductor

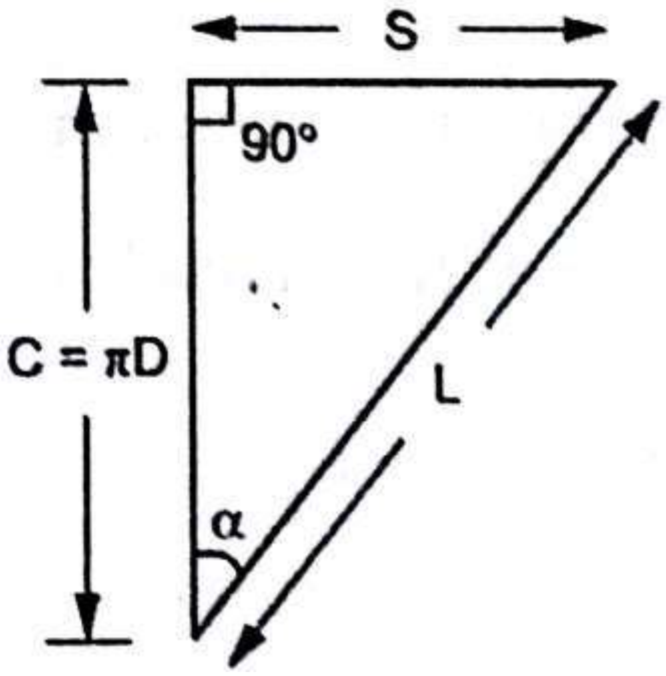
A = Axial length = NS

N = Number of turns

L = Length of one turn

l = Spacing of helix from ground plane

α = Pitch angle



Inter-relation between circumference, spacing, turn length and pitch angle

For N turn of helix, the total length of antenna is equal to NS

If one turn of helix is unrolled, then circumference (πD), spacing S, turn length "L" and pitch angle α are related by the triangle as shown in fig.

Then the length of one turn is expressed as

$$L = \sqrt{S^2 + C^2} = \sqrt{S^2 + (\pi D)^2} \qquad \text{......(1)}$$

Pitch angle (α) is the angle between a line tangent to the helix wire and the plane normal to the helix axis.

$$\tan \alpha = \frac{S}{C} = \frac{S}{\pi D}$$

$$\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right) \qquad \text{...... (2)}$$

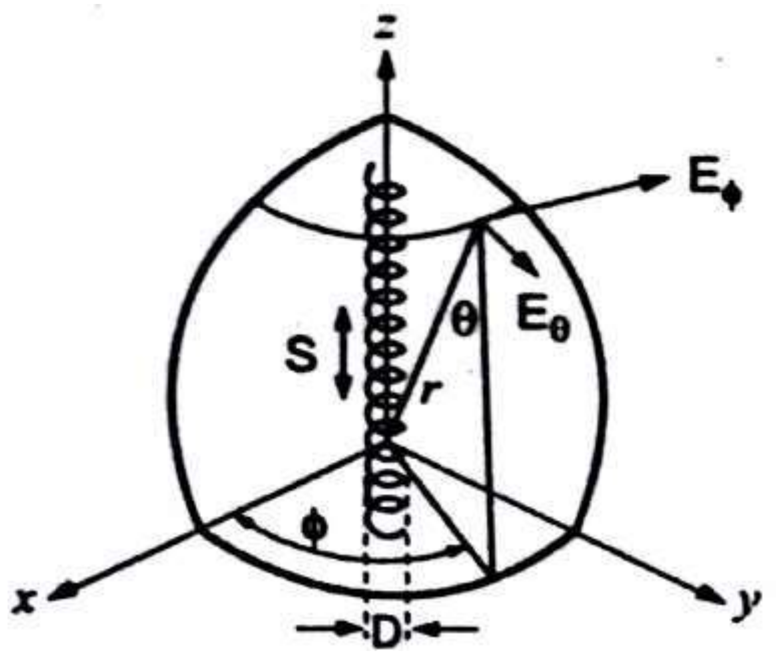
MODES OF RADIATION

In general, a helical antenna can radiate in many modes. But the most important modes of radiation are as follows:

(i) Normal mode or perpendicular mode.

(ii) Axial or End fire or Beam mode of radiation.

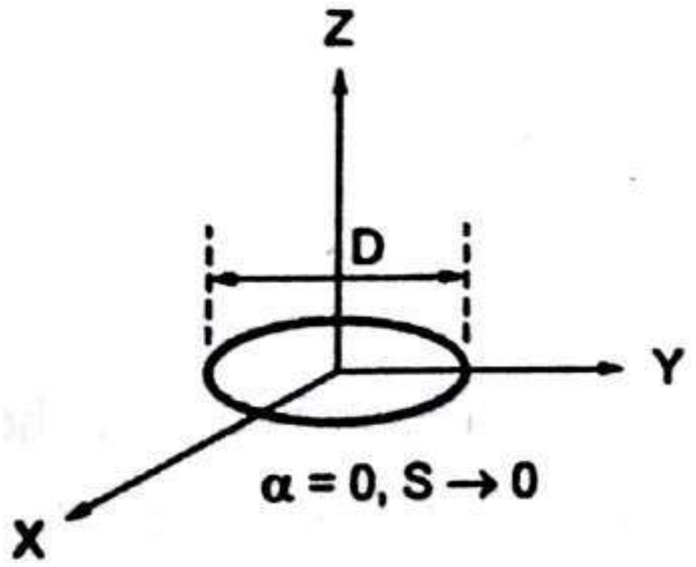
Normal Mode of Radiation



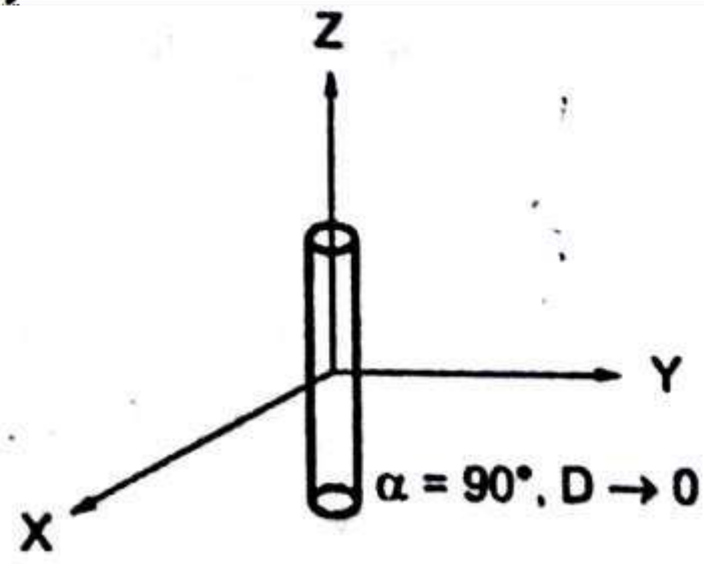
Helix in 3-dimensional spherical coordinate

When $\alpha = 0°$ helix corresponds to a loop and $\alpha = 90°$ the helix becomes a linear dipole as shown in Figure

If $S = 0$, helix collapse to a loop and if S = constant and $D = 0$, the helix straightens into a linear conductor (short dipole).



(a) Loop



(b) Short dipole

Limiting conditions on helix

Axial Ratio (AR)

The far field of the ***small loop*** is given by,

$$E_\phi = \frac{120\,\pi^2\,[I]\sin\theta}{r} \cdot \frac{A}{\lambda^2} \qquad \text{......(3)}$$

where, $[I]$ - Retarded current

r - Distance

A - Area of loop $= \dfrac{\pi D^2}{4}$

The far field of a ***short dipole*** is given by,

$$E_\theta = \frac{j\,60\,\pi\,[I]\sin\theta}{r} \cdot \frac{S}{\lambda} \qquad \text{......(4)}$$

where, $S = L =$ Length of dipole

The Equations (3) and (4) shows that there is 90^0 phase between them due to presence of 'j' operator. The Axial Ratio (AR) of Elliptical polarization is given by

$$AR = \frac{E_\theta}{E_\phi} = \frac{\left| \dfrac{j\, 60\, \pi\, [I] \sin\theta \cdot S}{\lambda r} \right|}{\left| \dfrac{120\, \pi^2\, [I] \sin\theta \cdot A}{r \lambda^2} \right|}$$

$$= \frac{S\lambda}{2\pi A} = \frac{2 S \lambda}{\pi^2 D^2} \qquad \text{where, } A = \frac{\pi D^2}{4}$$

$$AR = \frac{2 S \lambda}{\pi^2 D^2} = \text{Axial ratio}$$

For circular polarization, $AR = 1 = \dfrac{E_\theta}{E_\phi}$

$$|E_\theta| = |E_\phi|$$

$$\therefore \quad |2 S \lambda| = |\pi^2 D^2|$$

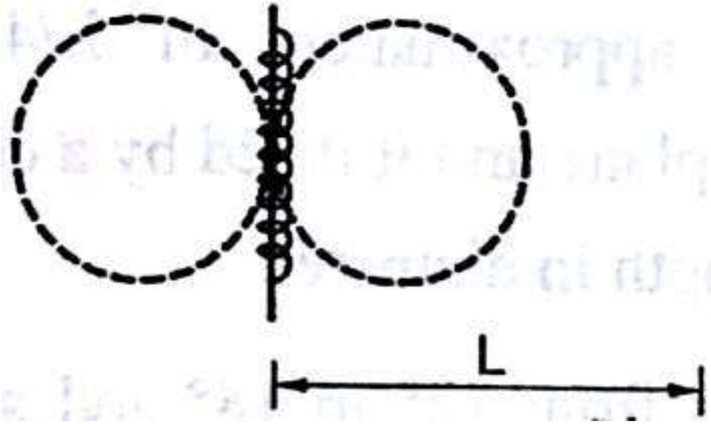
$$S = \frac{\pi^2 D^2}{2\lambda} = \frac{C^2}{2\lambda}, \text{ where } C = \pi D \qquad \ldots\ldots (6)$$

By substituting the equation (6) on equation (2), we get

$$\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right) = \tan^{-1}\frac{\frac{\pi^2 \cdot D^2}{2\lambda}}{\pi D}$$

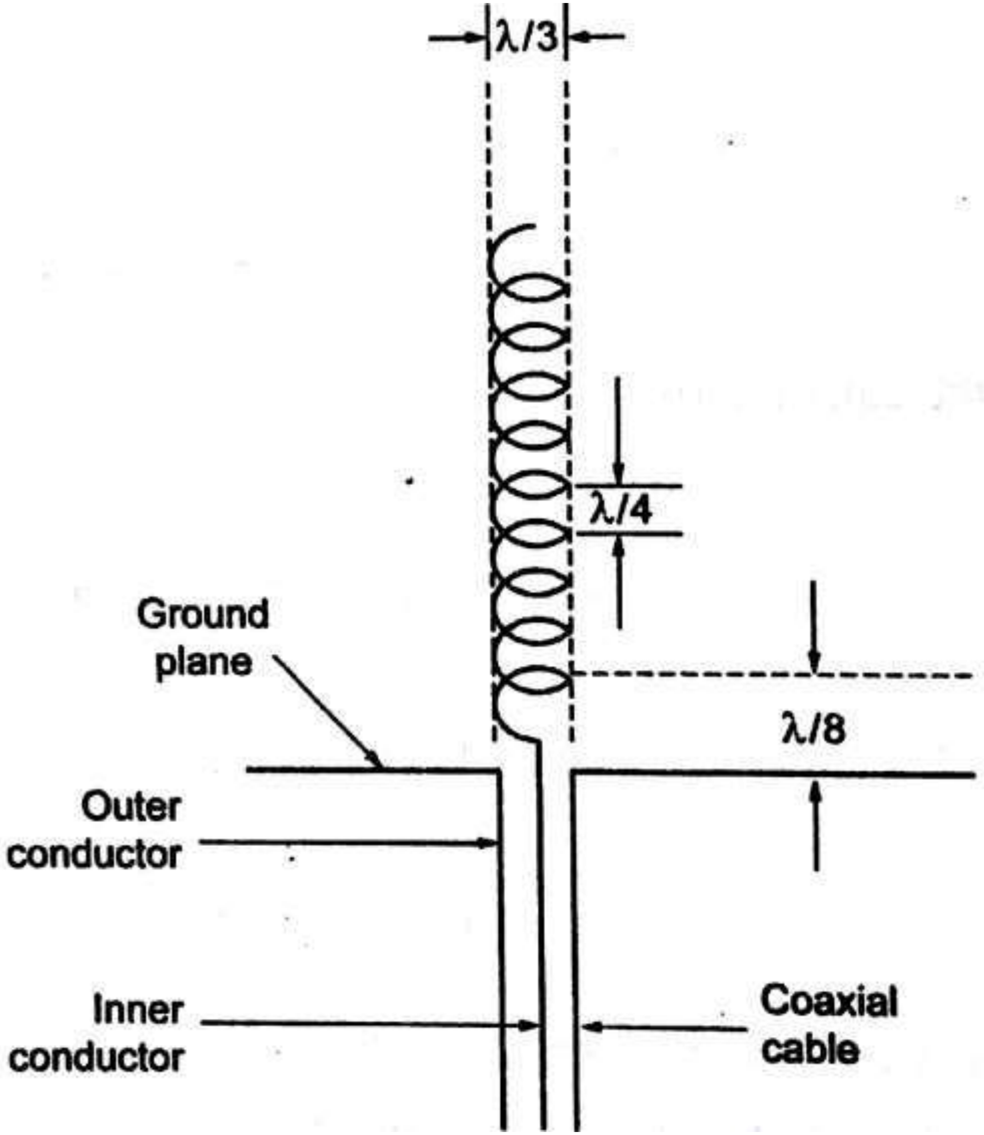
$$\alpha = \tan^{-1}\left(\frac{\pi D}{2\lambda}\right) = \tan^{-1}\left(\frac{C}{2\lambda}\right)$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{C}{2\lambda}\right)} \qquad \ldots\ldots (7)$$



Normal mode of radiation

Axial (OR) Beam Mode of Radiation



Arrangement for generating axial mode

In general, the terminal impedance of helical antenna lies between 100 Ω to 200 Ω pure resistive. Within 20% approximation, the ***terminal impedance*** is given by

$$\boxed{R = \frac{140\,C}{\lambda}\ \text{ohms}} \qquad \text{......(8)}$$

The HPBW (Beamwidth between half power points) is given by,

$$\text{HPBW} = \frac{52}{C}\sqrt{\frac{\lambda^3}{N\,S}}\ \text{degrees} \qquad \text{......(9)}$$

where, λ = free space wave length

S = Spacing

The beamwidth between first nulls is given by

$$\text{BWFN} = \frac{115}{C}\sqrt{\frac{\lambda^3}{NS}}\ \text{degree} \qquad \text{......(10)}$$

The maximum ***directive gain (directivity)*** for axial mode is given by

$$D = \frac{15\,N\,S\,C^2}{\lambda^3} \qquad \text{......(11)}$$

$$\text{Axial Ratio (AR)} = 1 + \frac{1}{2N} \qquad \text{......(12)}$$

The normalized far field pattern is given as,

$$E = \sin\left(\frac{\pi}{2N}\right)\cos\theta \cdot \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \qquad \text{......(13a)}$$

$$\psi = 2\pi\left[\frac{S}{\lambda}(1-\cos\theta) + \frac{1}{2N}\right] \qquad \text{......(13b)}$$

where, $\alpha = 12° \text{ to } 15°$, $N \geq 3$, $NS \leq 10$ and $C = \frac{3}{4}\lambda \text{ to } \frac{4}{3}\lambda$

Log Periodic Antenna

A log periodic antenna is a broadband narrow beam antenna. It is a frequency independent antenna.

This frequency independent concept can be obtained by adjusting the antenna structure (either expanded or contracted) in proportion to the wavelength. If it is not possible to adjust the antenna mechanically, then the size of ***active or radiating region*** should be proportional to the wavelength.

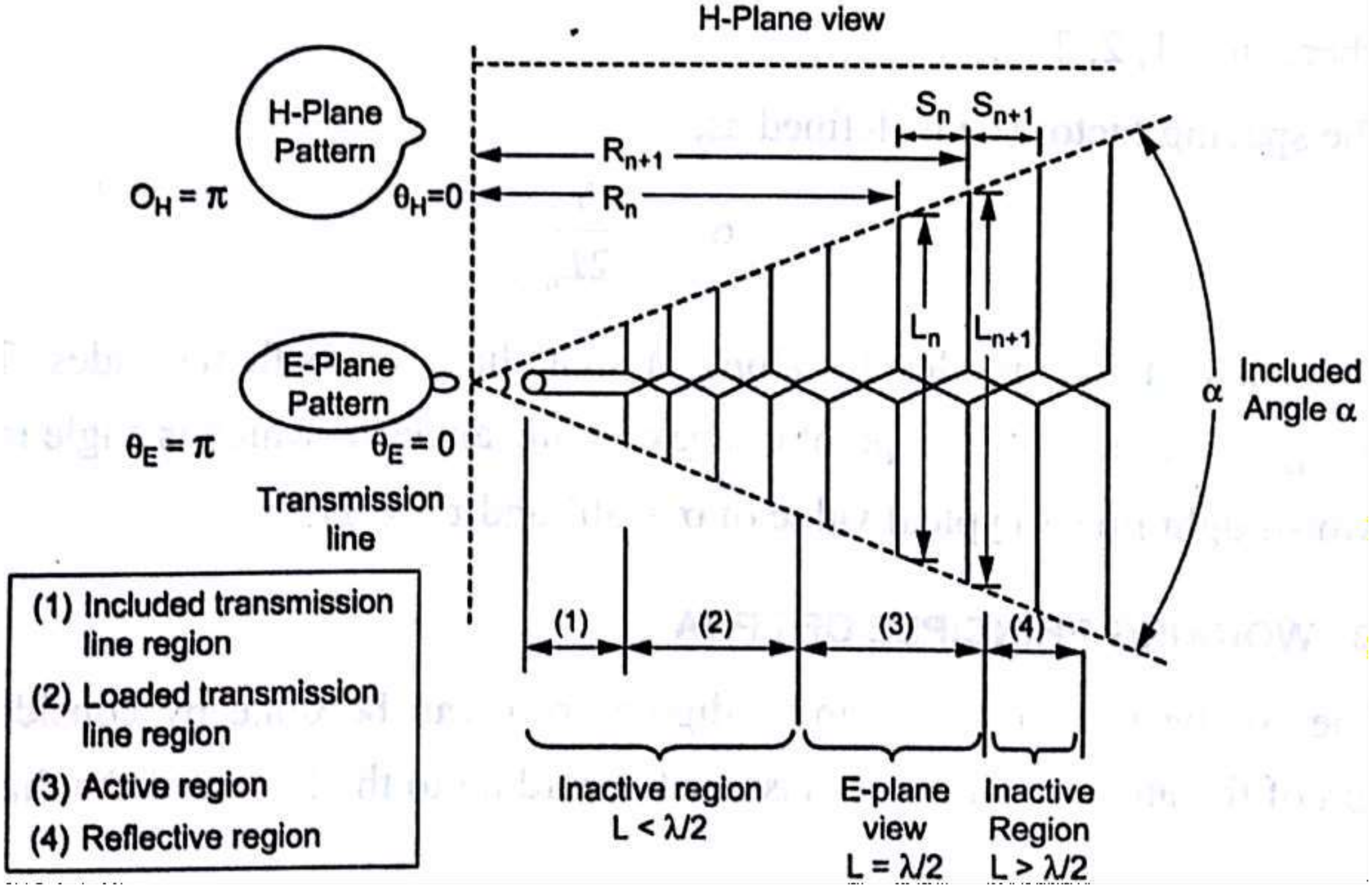
Log-Periodic Concept

Here, the geometry of the antenna structure is adjusted such that all the electrical properties of the antenna must repeat periodically with the logarithm of the frequency


BAZ


VHF Log Periodic Antenna

CONSTRUCTION OF LPDA



Radiation pattern of a LPDA in E-plane and H-plane

A log periodic dipole array with main region of operation

The relationship between spacings S and lengths L of adjacent elements are scaled as,

$$\frac{S_n}{S_{n+1}} = \frac{L_n}{L_{n+1}} = \tau \qquad \text{.....(1)}$$

τ is also called ***periodicity factor*** which is always less than 1. The above expression can be written in terms of constant k with the radii of the arm as

$$\frac{R_{n+1}}{R_n} = \frac{S_{n+1}}{S_n} = \frac{L_{n+1}}{L_n} = \frac{1}{\tau} = k; \quad k > 1 \qquad \text{......(2)}$$

where $n = 1, 2, 3 \ldots. n$

The spacing factor (σ) is defined as,

$$\sigma = \frac{S_n}{2L_n} \qquad \text{......(3)}$$

WORKING PRINCIPLE OF LPDA

(i) Inactive transmission - line region ($L < \lambda/2$)

(ii) Active region $L \approx \lambda/2$

(iii) Inactive reflective region ($L > \lambda/2$)

DESIGN OF LOG PERIODIC DIPOLE ARRAY

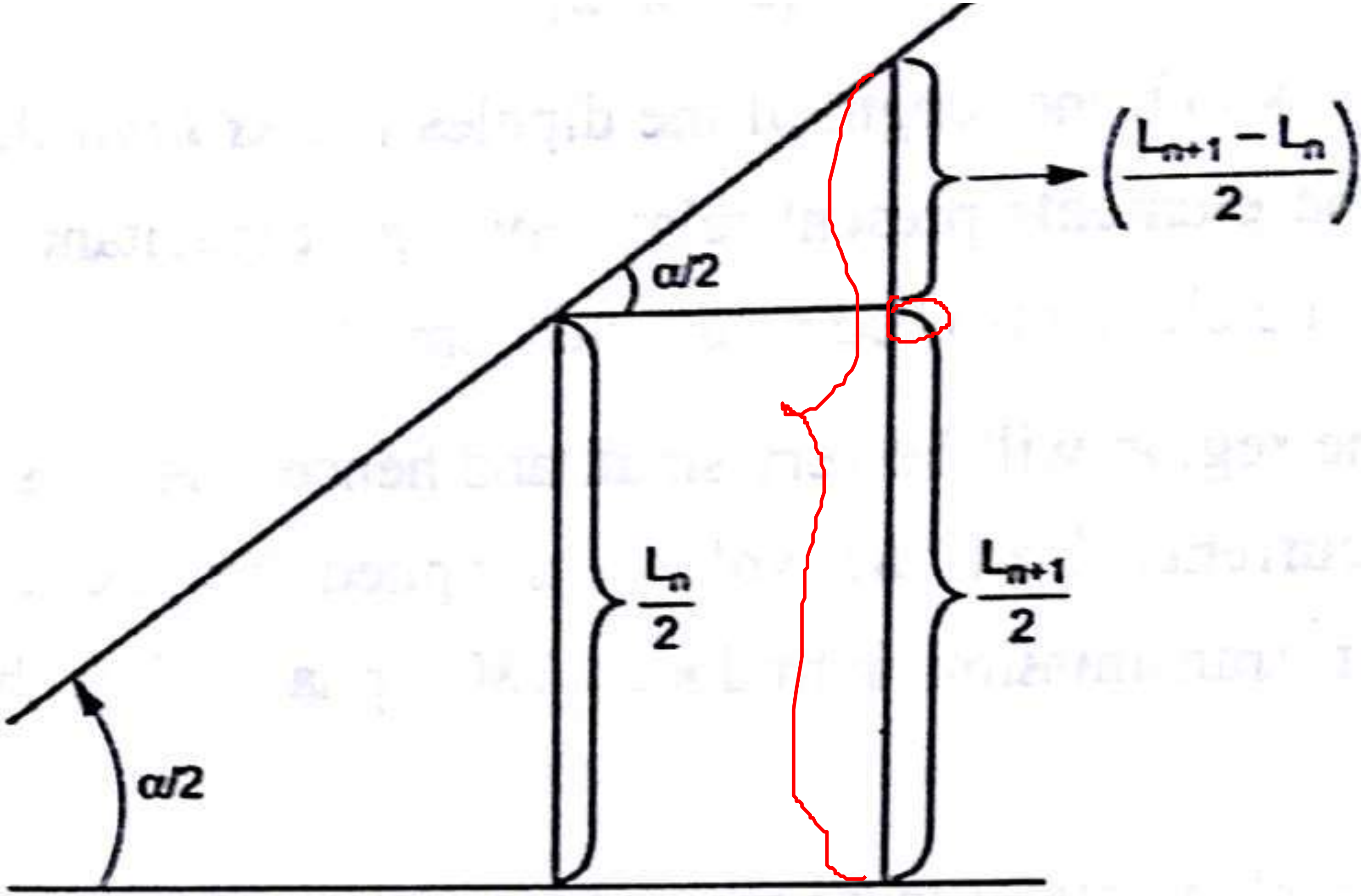
The performance of a log periodic dipole array depends on the following parameters.

(*i*) Apex angle (α)

(*ii*) Design ratio (τ)

(*iii*) Spacing factor (σ)

Consider a part of a log periodic array as shown in the Fig.



Geometry of log-periodic array

From Fig.

$$\tan(\alpha/2) = \frac{\frac{L_{n+1} - L_n}{2}}{S} \qquad \text{......(4)}$$

$$\tan(\alpha/2) = \frac{L_{n+1} - L_n}{2\,S}$$

$$= \frac{L_{n+1}\left[1 - \frac{L_n}{L_{n+1}}\right]}{2\,S} \qquad \text{......(5)}$$

But $$\frac{L_{n+1}}{L_n} = k$$

$$\frac{L_n}{L_{n+1}} = \frac{1}{k} \qquad \text{......(6)}$$

By substituting the equation (6) in equation (5), we get

$$\tan(\alpha/2) = \frac{\left(1 - \frac{1}{k}\right) L_{n+1}}{2\,S} \qquad \text{......(6)}$$

For active region $L_{n+1} = \lambda/2$ ......(7)

By substituting the equation (7) in equation (6), we get

$$\tan(\alpha/2) = \frac{\left(1 - \frac{1}{k}\right) \lambda/2}{2\,S} = \frac{\left(1 - \frac{1}{k}\right)}{4\left(\frac{S}{\lambda}\right)}$$

$$\tan(\alpha/2) = \frac{\left(1 - \frac{1}{k}\right)}{4\,\sigma} \qquad \text{......(8)}$$

where, $\sigma = \frac{S}{\lambda} =$ Spacing factor

α Apex angle

$k =$ Scale factor

But $\tau = \frac{1}{k}$

$$\boxed{\tan(\alpha/2) = \frac{1-\tau}{4\sigma}} \quad \ldots\ldots(9)$$

From equation (9), σ can be obtained as

$$\boxed{\sigma = \frac{1-\tau}{4\tan\alpha/2}} \quad \ldots\ldots(10)$$

$$\tan(\alpha/2) = \frac{1-\tau}{4\sigma}$$

$$\alpha/2 = \tan^{-1}\left(\frac{1-\tau}{4\sigma}\right)$$

$$\boxed{\alpha = 2\tan^{-1}\frac{1-\tau}{4\sigma}} \qquad \text{......(11)}$$

The number of elements in an array(n) can be obtained from the upper frequency (f_U) and lower frequency(f_L) and it is given as,

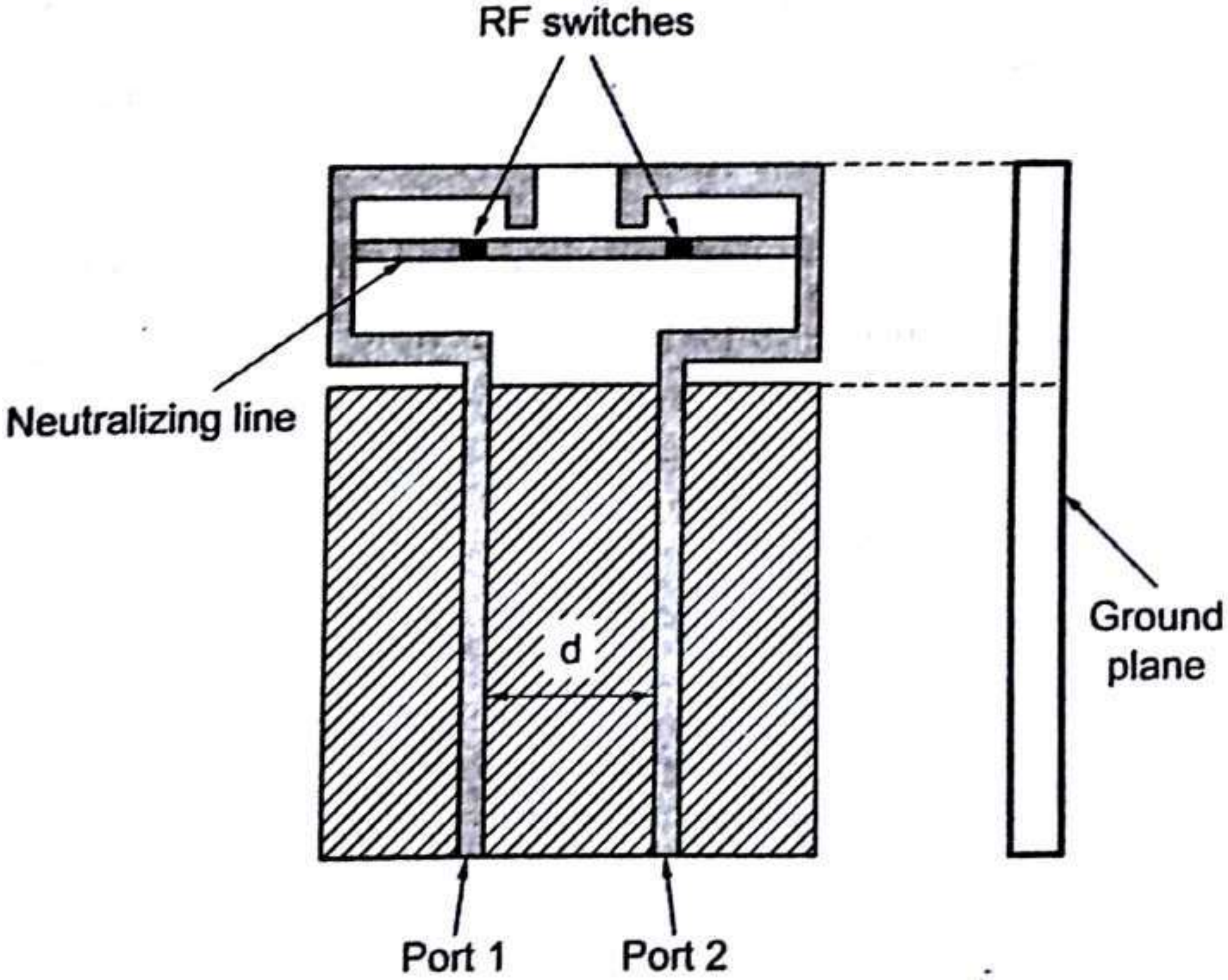
$$\log(f_U)-\log(f_L) = (n-1)\log\left(\frac{1}{\tau}\right) \qquad \text{......(12)}$$

Modern Antennas

Reconfigurable Antenna

*A **reconfigurable antenna** is an antenna capable of modifying dynamically its frequency and radiation properties in a controlled and reversible manner.*

Reconfigurable antennas, with the ability to radiate more than one pattern at different frequencies and polarizations, are necessary in modern telecommunication systems.



Reconfigurable antenna

Types of Antenna Reconfiguration

(i) Frequency reconfiguration

(ii) Radiation pattern reconfiguration

(iii) Polarization reconfiguration

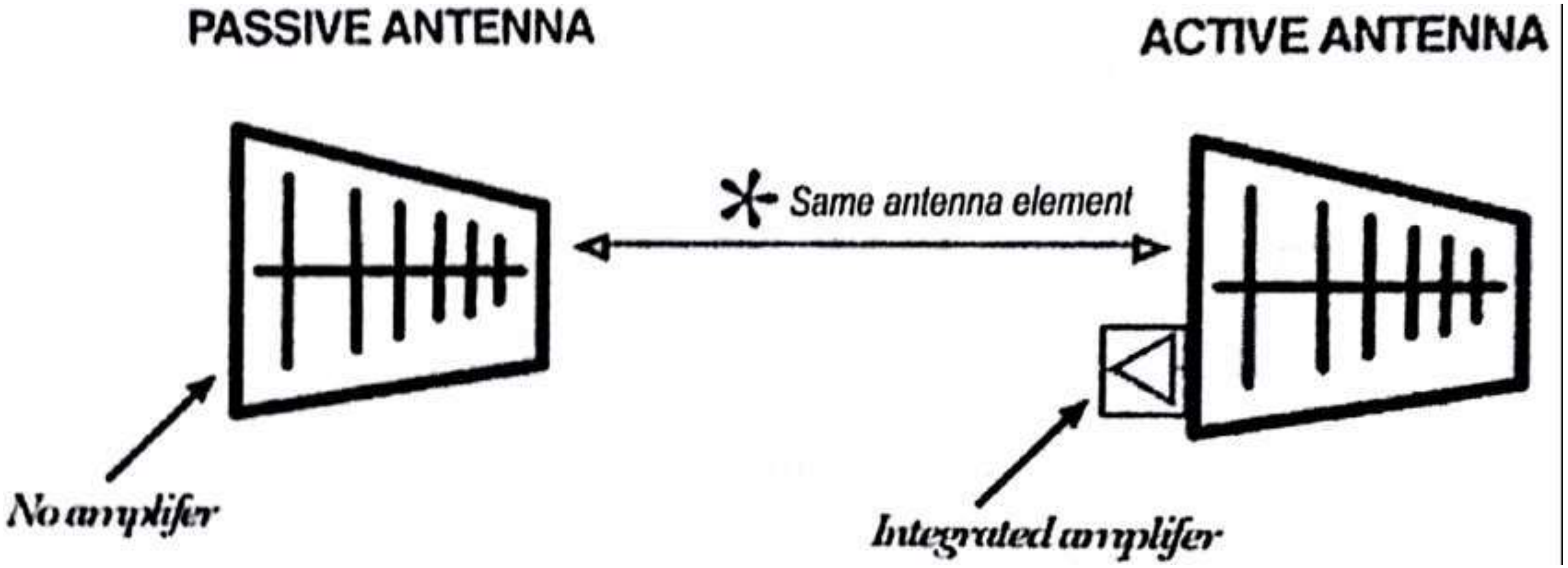
(iv) Compound reconfiguration

Active antenna

An active antenna is an antenna that contains active electronic components, as opposed to typical passive components

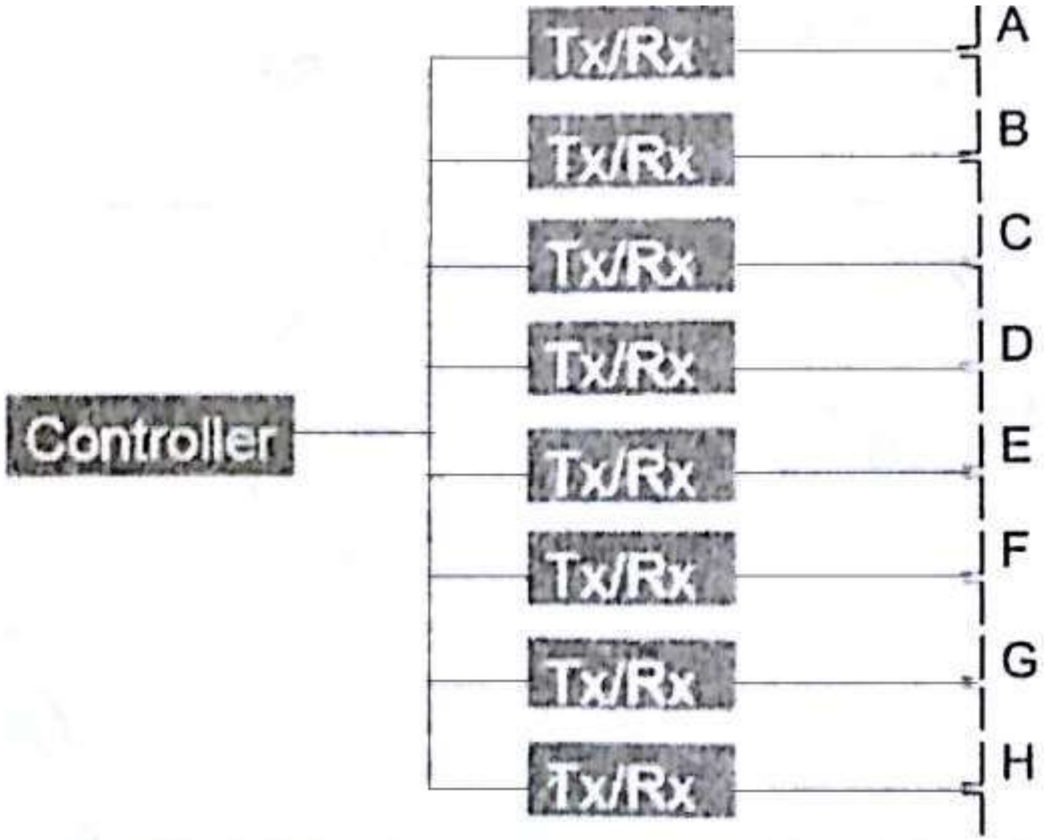
A passive antenna typically resembles a dipole antenna

A simple definition of active antennas are any antennas with integrated signal amplifiers built right into the unit



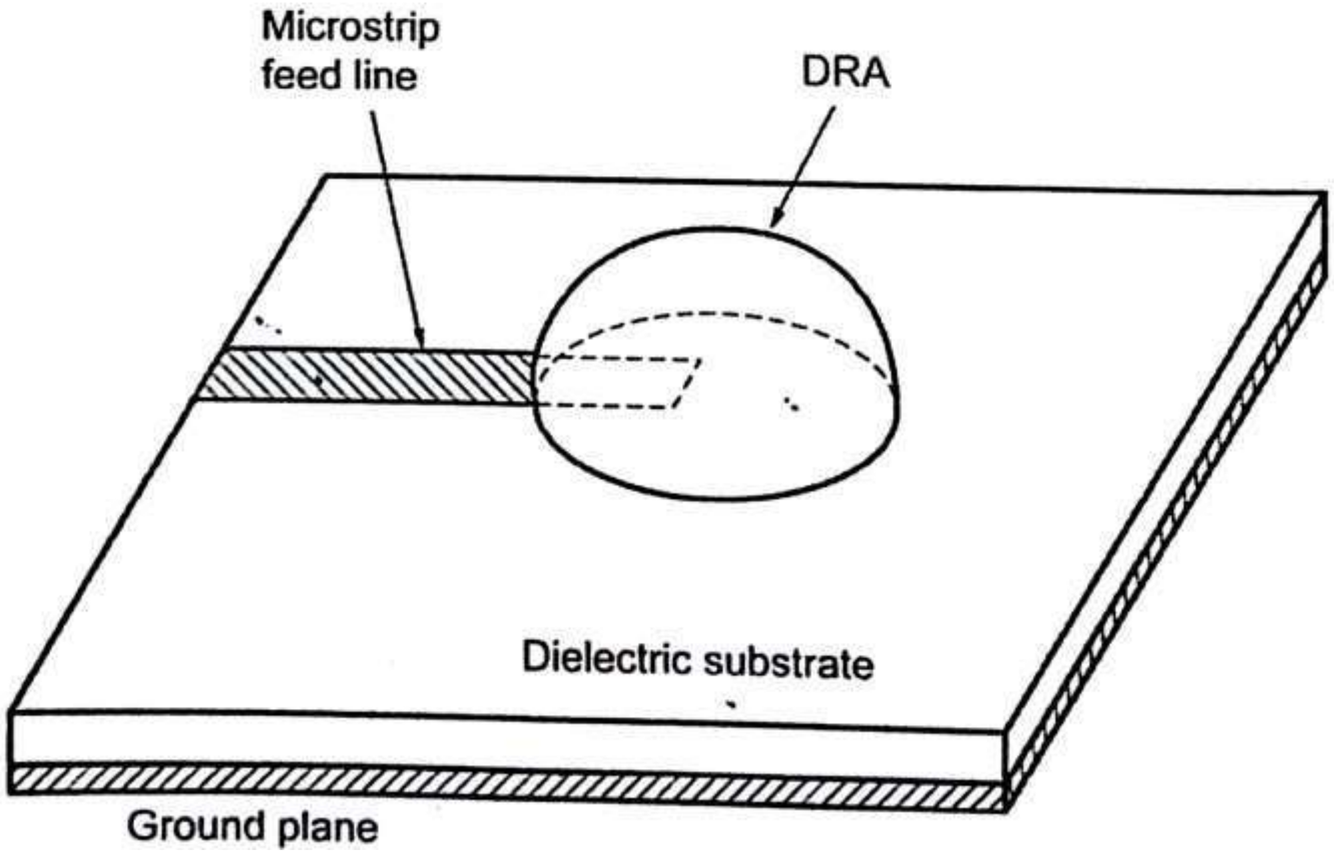
Integrated active antennas

Integrated active antennas are a single column antenna consists of multiple elements. If all the elements are fed in phase, a narrow horizontal beam is formed



Dielectric Resonator Antenna (DRA)

The DRA is an antenna that makes use of a radiating mode of a dielectric resonator (DR) and resonance frequency determined by the its dimensions and dielectric constant ε_r.



Antenna Measurements

Antenna Ranges

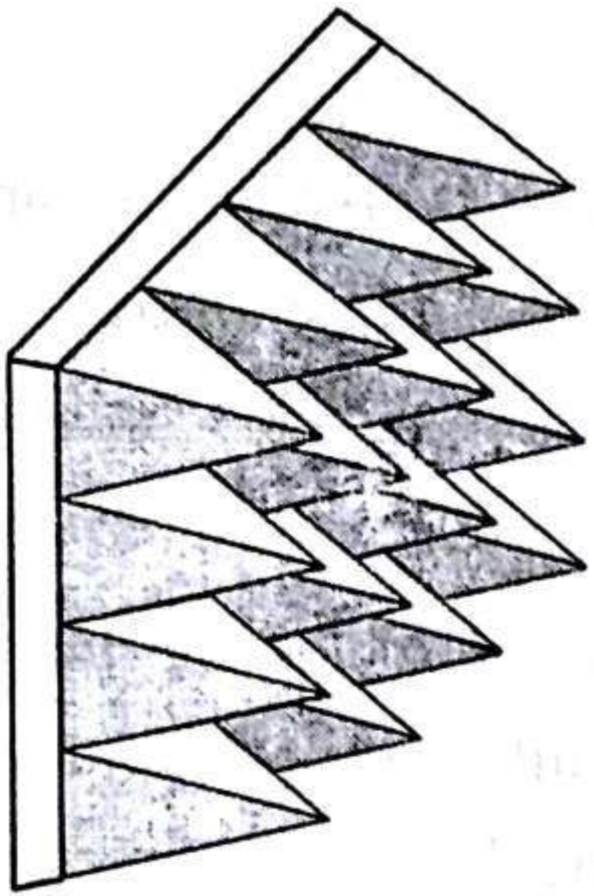
The antenna facilities are categorized as ***outdoor*** and ***indoor ranges***, and limitations are associated with both of them.

Outdoor ranges are not protected from environmental conditions whereas indoor facilities are limited by space restrictions.

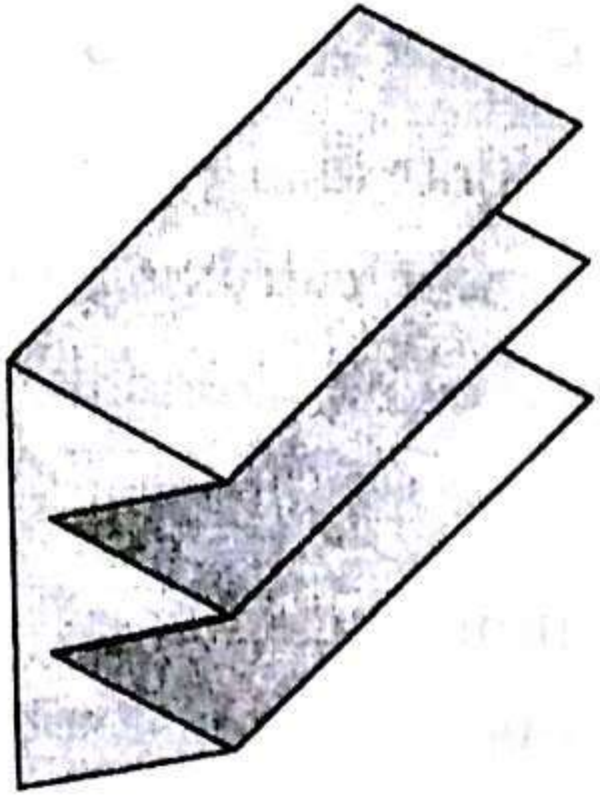
Anechoic Chambers

To provide a controlled environment and allows all-weather capability, security, and to minimize electromagnetic interference, indoor anechoic chambers have been developed as an alternative to outdoor testing which simulates reflectionless free space.

Absorbing Materials

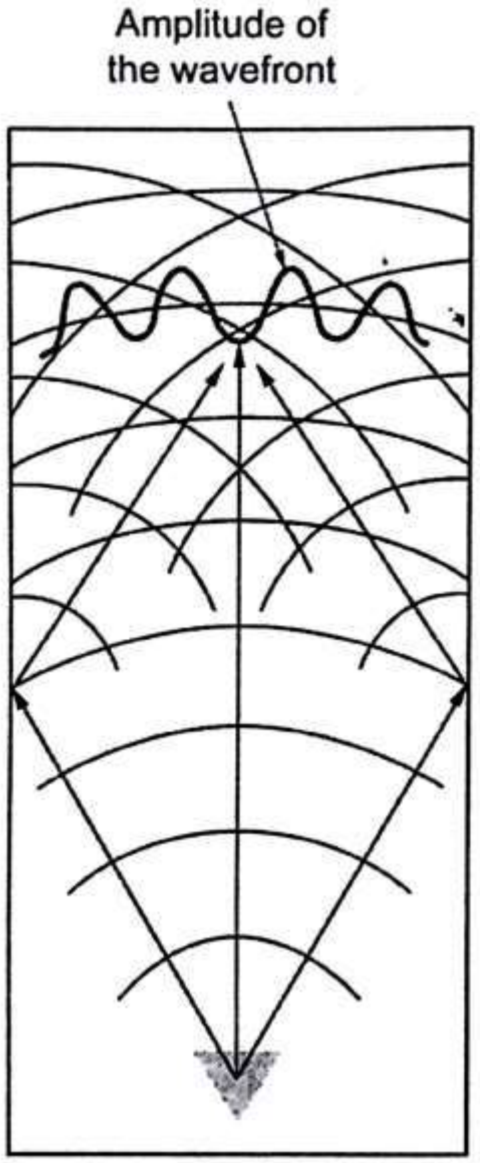


(a) Pyramids

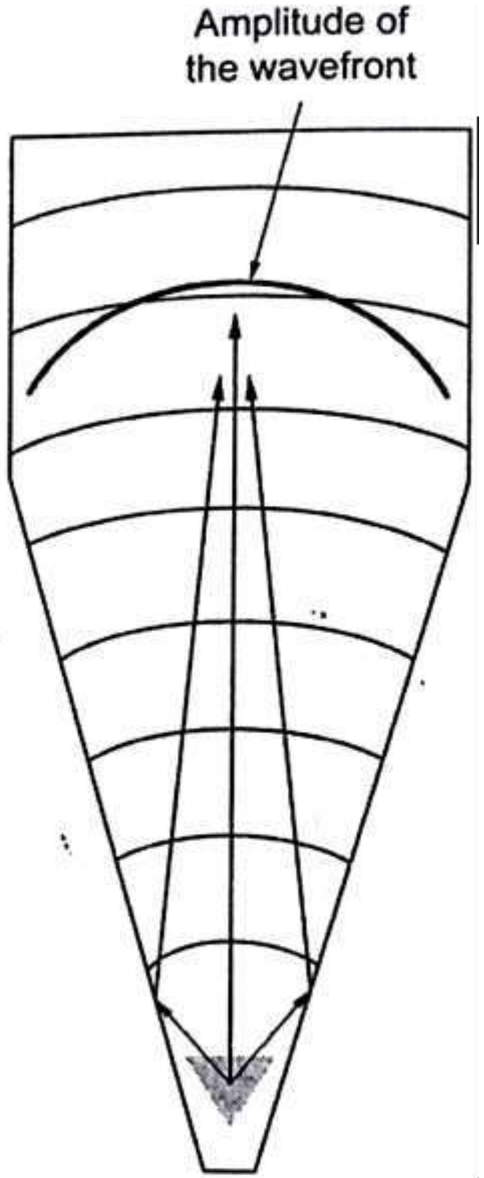


(b) Wedges

Anechoic Chamber Types



(a) Rectangular chamber



(b) Tapered chamber

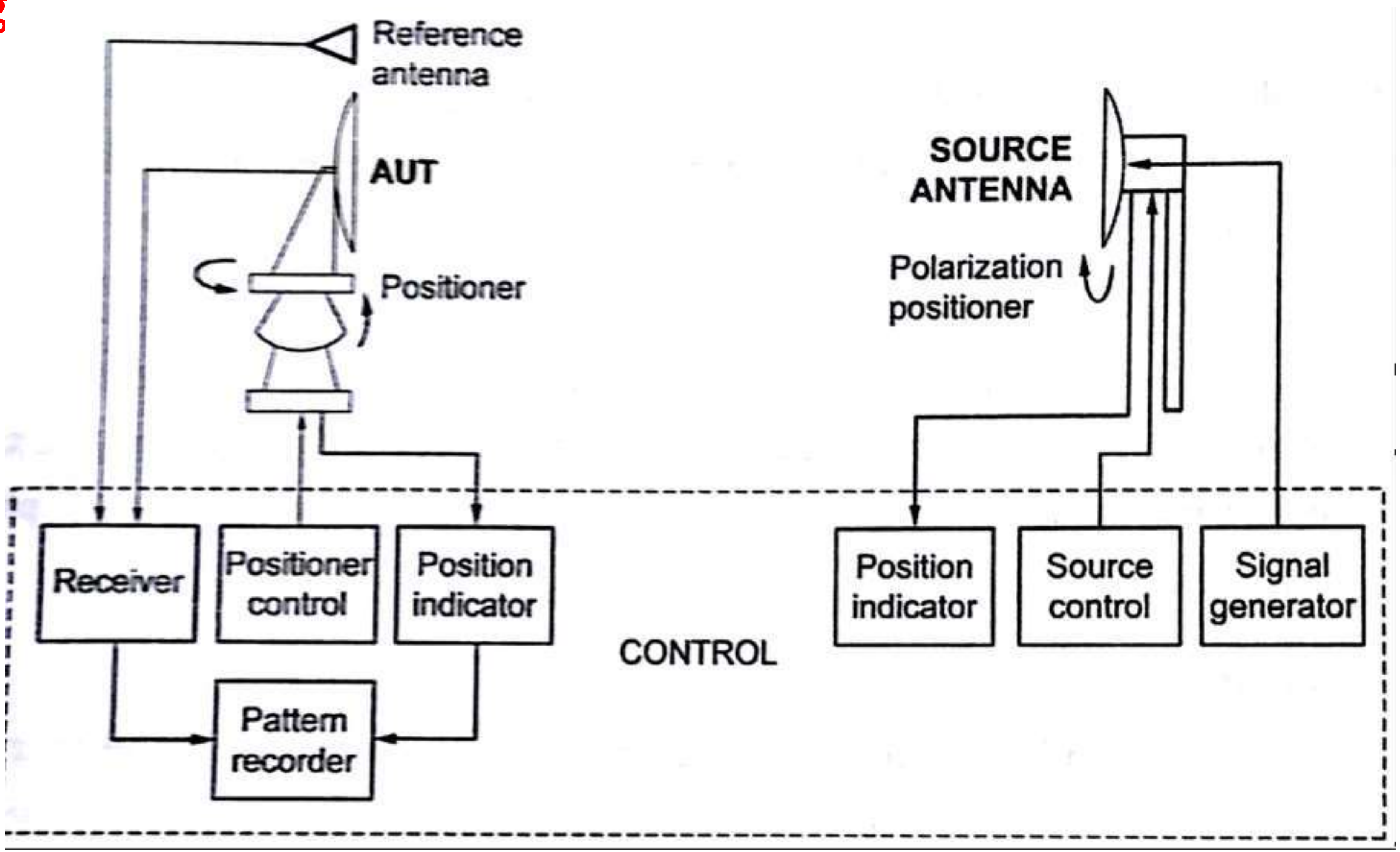
Range Instrumentation

Transmitting

Positioning

Receiving

Recording



Simplified block diagram of an antenna measurement range with AUT

Measurement of Gain

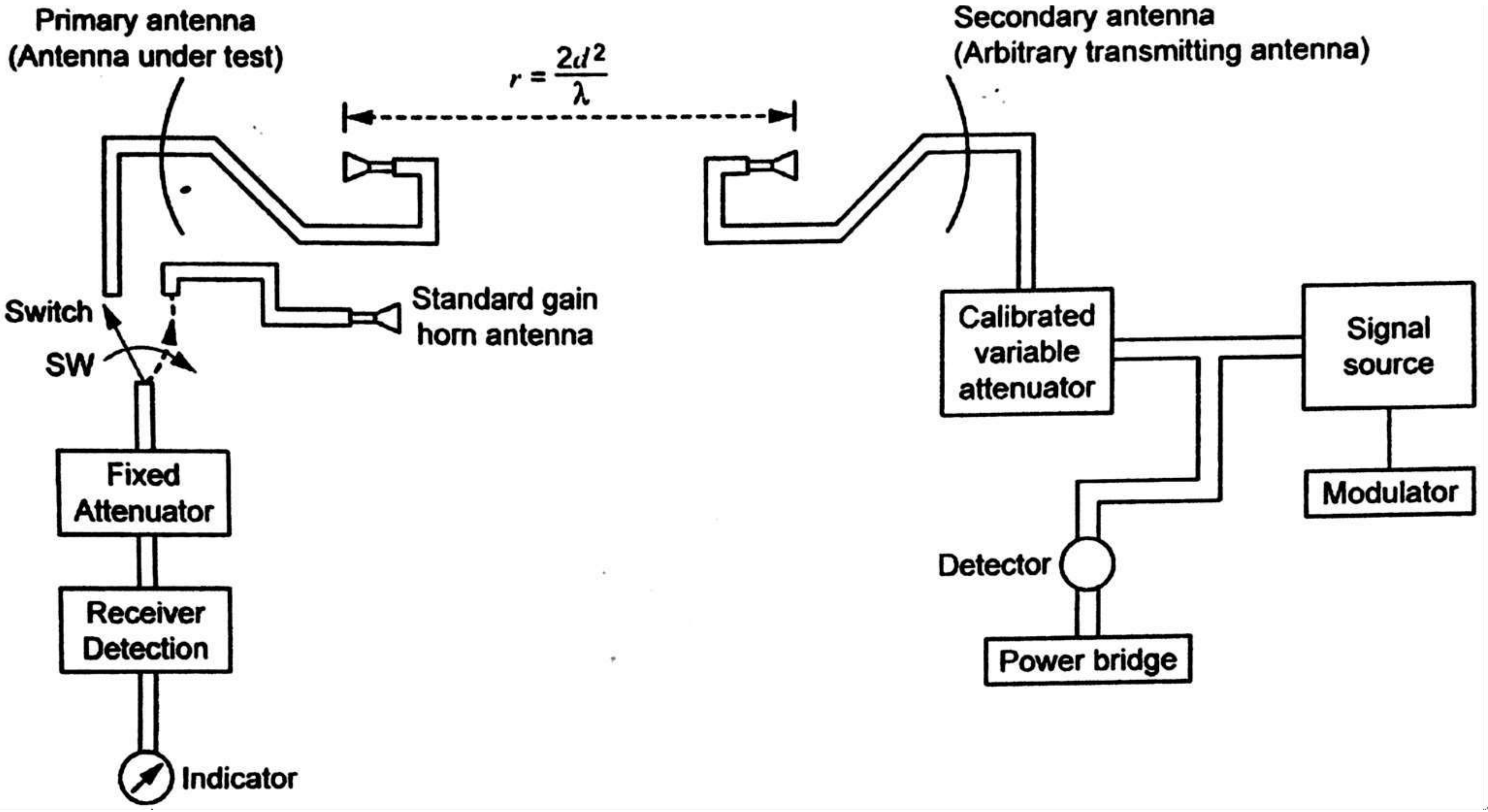
The performance of any antenna can be described in terms of ***figure of merit*** *i.e.*, ***gain*** of an antenna. Depending upon the frequency of operation various methods can be used for the measurement of gain of an antenna.

Basically there are two standard methods used for the measurement of gain of an antenna such as

(i) *Gain-transfer (or gain-comparison) method or direct comparison method*, and

(ii) *Absolute-gain method.*

Gain Measurement by Direct Comparison Method



Set up the gain measurement by gain comparison method

Now consider two different cases,

Case I: If $P_1 = P_2$, then no correction need to be applied and the gain of the subject antenna under test is given by,

Power gain $= G_P = \dfrac{A_2}{A_1}$, where A_1 and A_2 are relative power levels

By taking logarithms on both sides, we get,

$$\log_{10} G_P = \log_{10}\left(\frac{A_2}{A_1}\right) = \log_{10} A_2 - \log_{10} A_1$$

$$G_P(\text{dB}) = A_2(\text{dB}) - A_1(\text{dB}) \qquad \ldots\ldots(1)$$

Case II: If $P_1 \neq P_2$, then the correction need to be included

Let
$$\frac{P_1}{P_2} = P, \text{ then}$$

$$\log_{10}\frac{P_1}{P_2} = P(dB)$$

Hence, the power gain is given by,

$$G = G_P \times \frac{P_1}{P_2} = \frac{A_2}{A_1}\cdot\frac{P_1}{P_2}$$

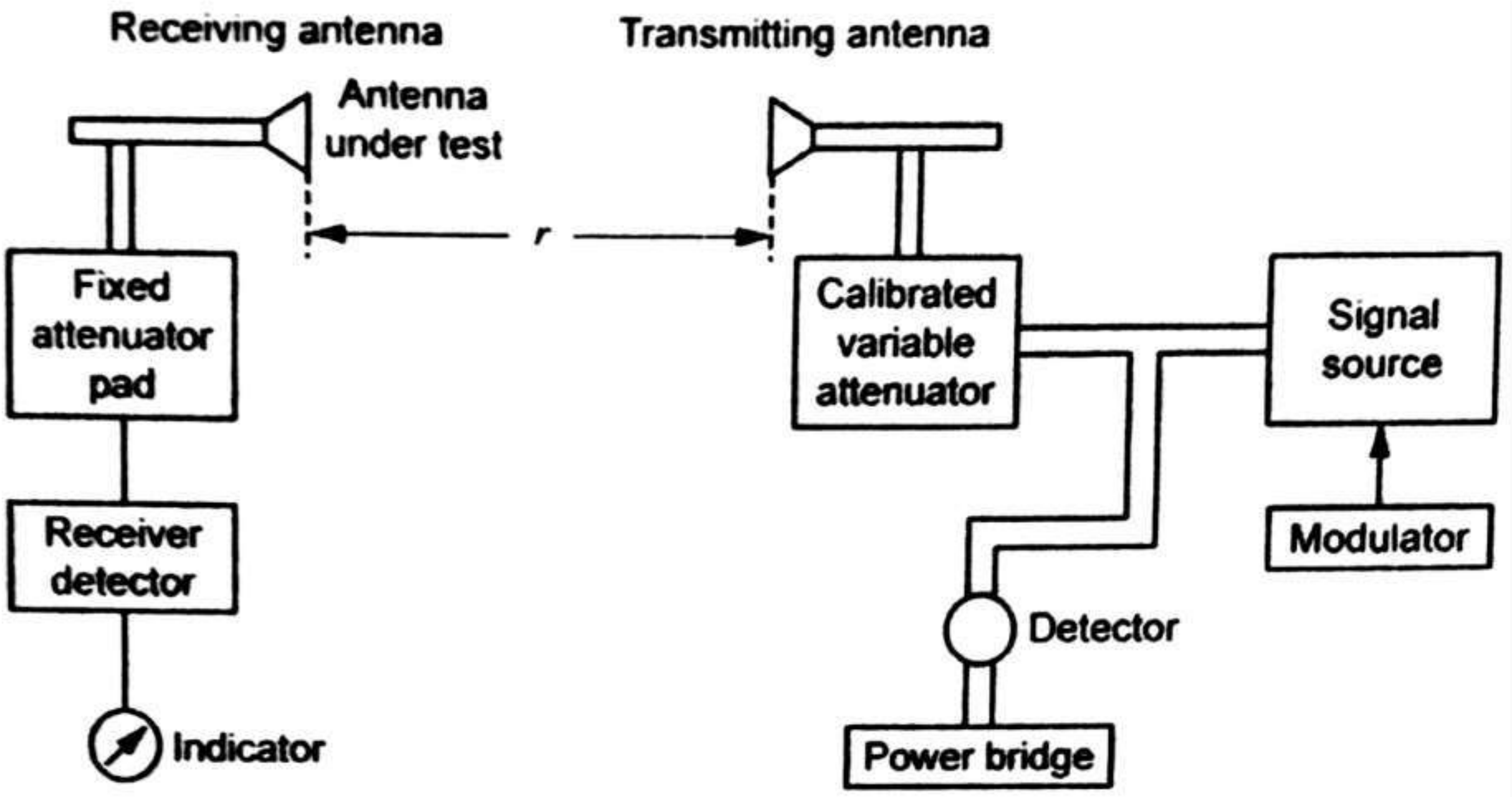
$$G = G_P \cdot P$$

By taking logarithms on both sides, we obtained

$$\log_{10} G = \log_{10} G_P + \log_{10} P$$

$$G(dB) = A_2(dB) - A_1(dB) + P(dB) \qquad \text{......(2)}$$

Measurement of Absolute Gain



Transmitting and receiving antennas for absolute gain measurement

$$A_{et} = A_{er} = \frac{G_D \lambda^2}{4\pi} \qquad \ldots\ldots(3)$$

From Friis's transmission equation, we can write,

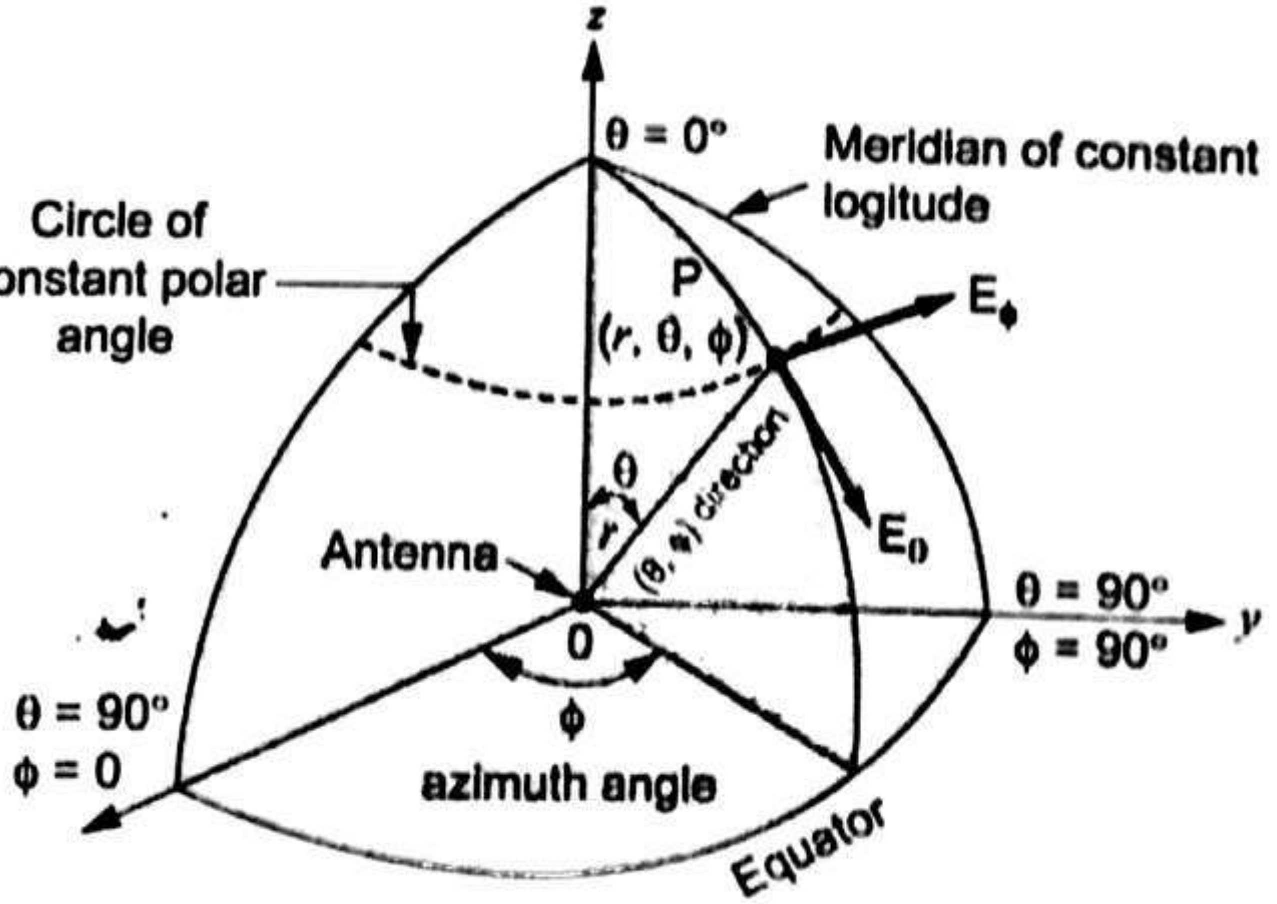
$$\frac{P_r}{P_t} = \frac{A_{er} \cdot A_{et}}{\lambda^2 \cdot r^2} = \left(\frac{G_D \lambda^2}{4\pi}\right)\left(\frac{G_D \lambda^2}{4\pi}\right)\frac{1}{\lambda^2 r^2}$$

$$\frac{P_r}{P_t} = \left(\frac{G_D \lambda}{4\pi r}\right)^2$$

$$\frac{G_D \lambda}{4\pi r} = \sqrt{\frac{P_r}{P_t}}$$

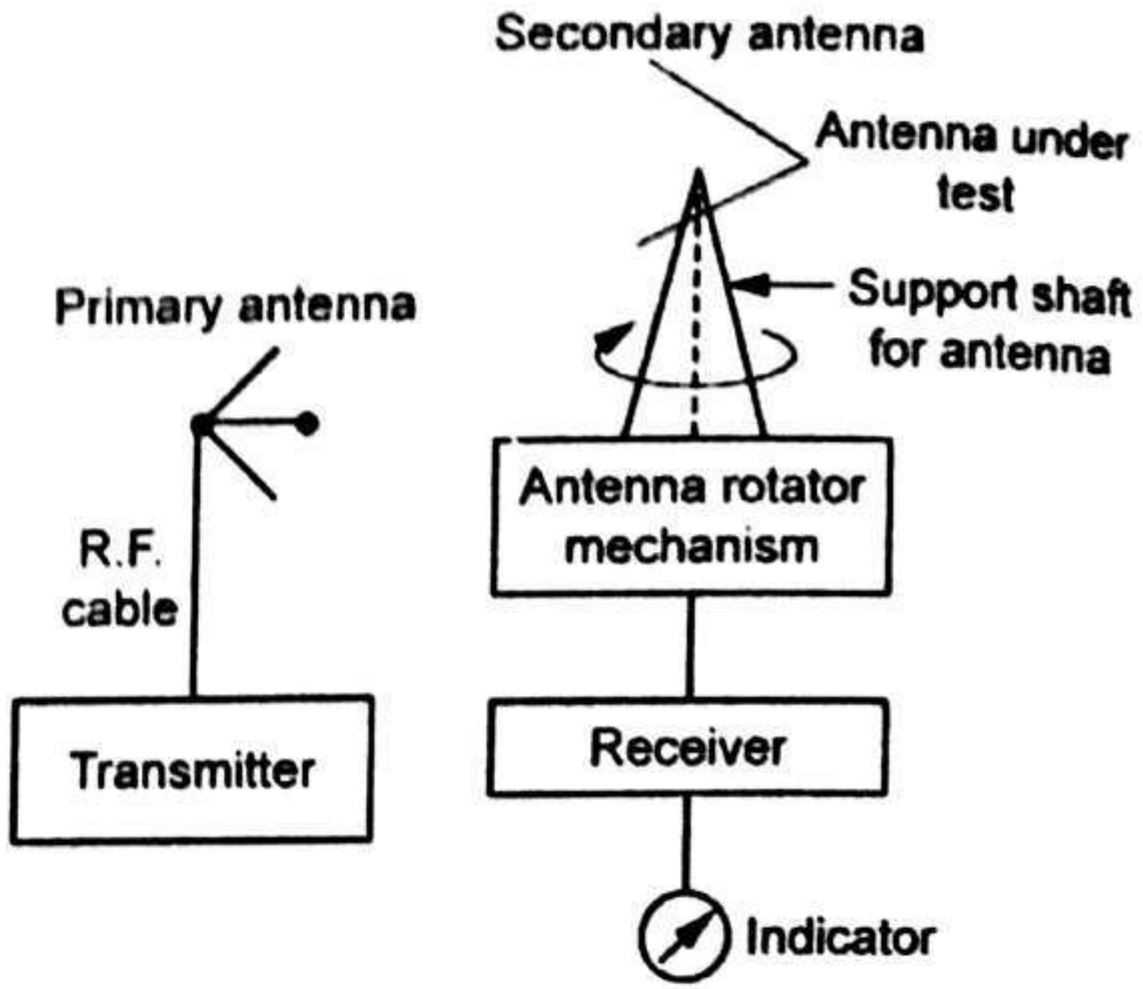
$$\boxed{G_D = \frac{4\pi r}{\lambda}\sqrt{\frac{P_r}{P_t}}} \qquad \ldots\ldots(4)$$

Measurement of Radiation Pattern



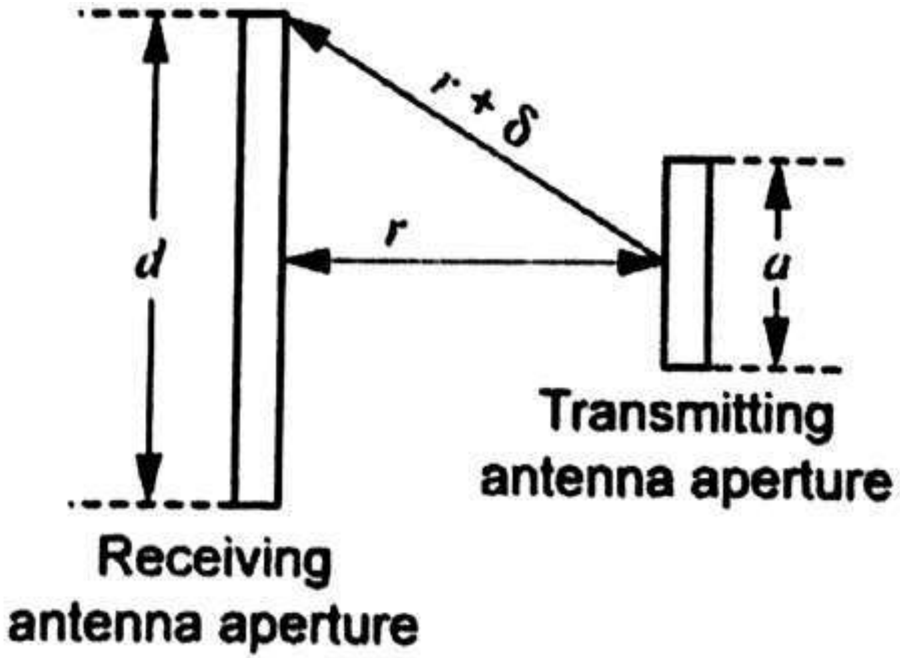
Spherical co-ordinate system representation for radiation pattern measurement

Setup for measurement of radiation pattern of an antenna



Measurement set up for radiation pattern

Uniform Distance Requirement



According to standard condition, the distance between the two antennas should be

$$r \geq \frac{2D^2}{\lambda} \quad \dots (1)$$

Where

r = Distance between transmitter and receiver

λ = Wavelength

D = Maximum dimension of either of the antenna

From Fig

$$(r+\delta)^2 = \left(\frac{D}{2}\right)^2 + r^2$$

where δ is phase difference error

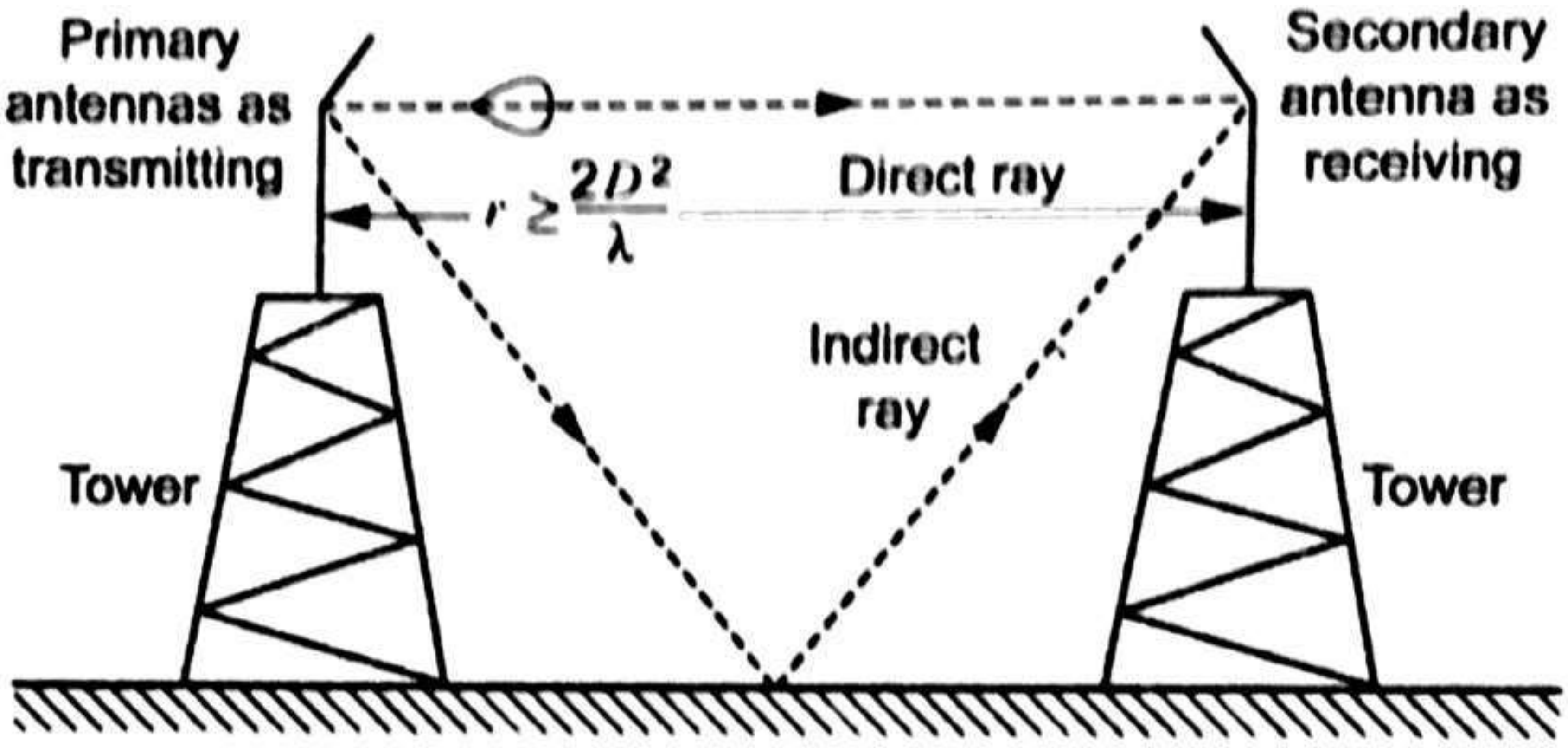
$$r^2 + 2r\delta + \delta^2 = \frac{D^2}{4} + r^2$$

When δ is very small, after neglecting δ^2, we can write

$$2r\delta = \frac{D^2}{4}$$

$$\boxed{r = \frac{D^2}{8\delta}} \qquad \dots (2)$$

Uniform Amplitude Requirement



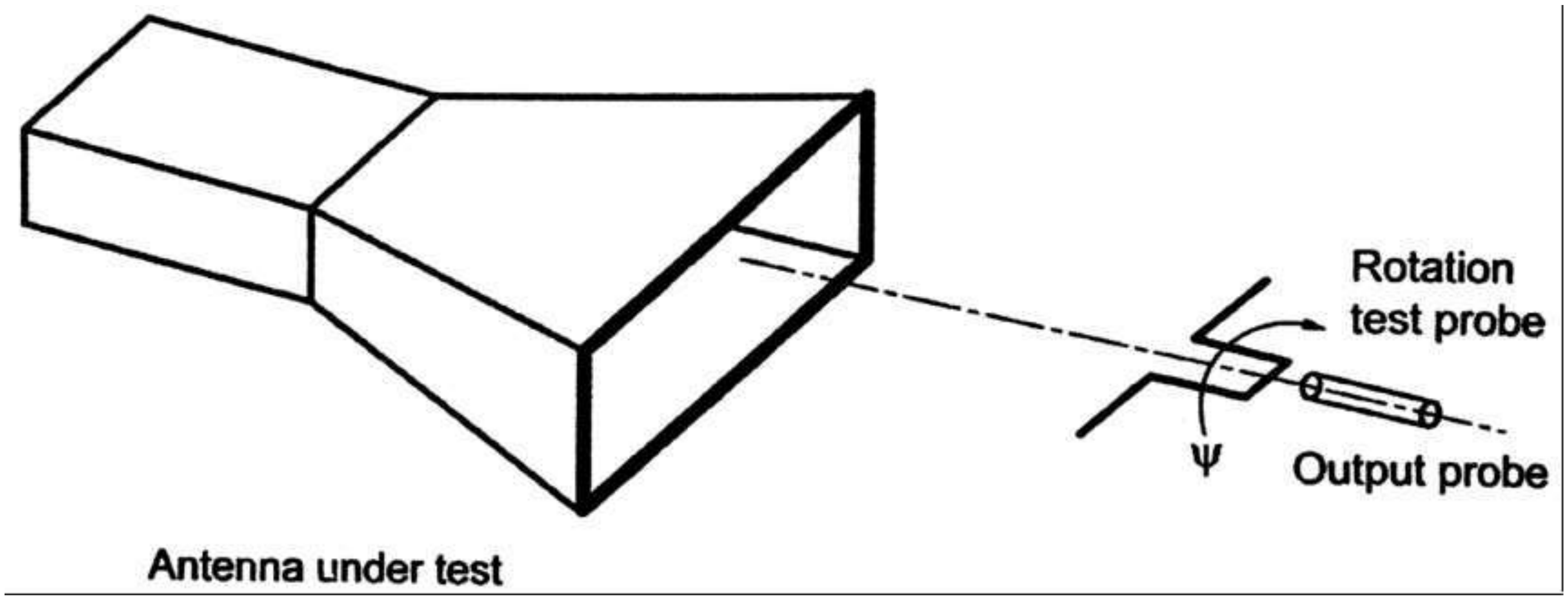
Uniform amplitude requirement

Measurement of Polarization

The polarization of a wave is defined as the curve traced by the instantaneous electric field in a plane perpendicular to the direction of wave travel, at a given frequency.

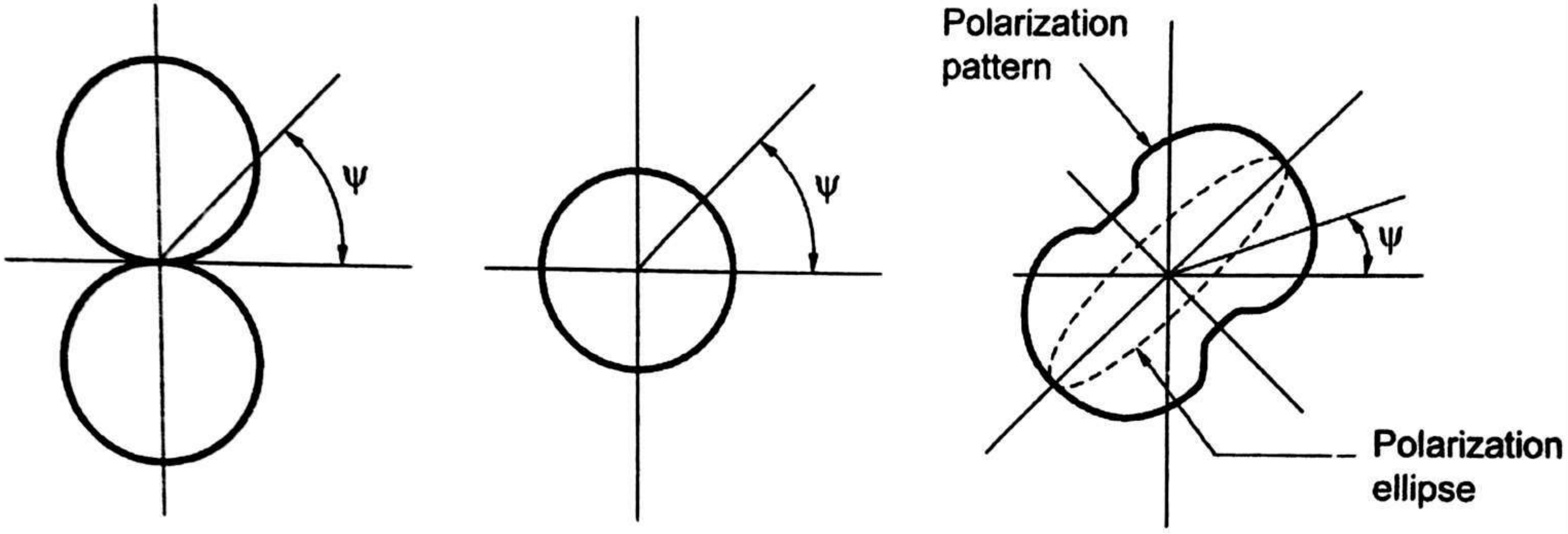
The polarization of the antenna is defined as the curve traced by the instantaneous electric field radiated by the antenna in a plane perpendicular to the radial direction.

1. Polarization Measurement Setup



2. Polarization Measuring Methods

(i) Polarization-Pattern method



(a) Linear *(b) Circular* *(c) Elliptical*

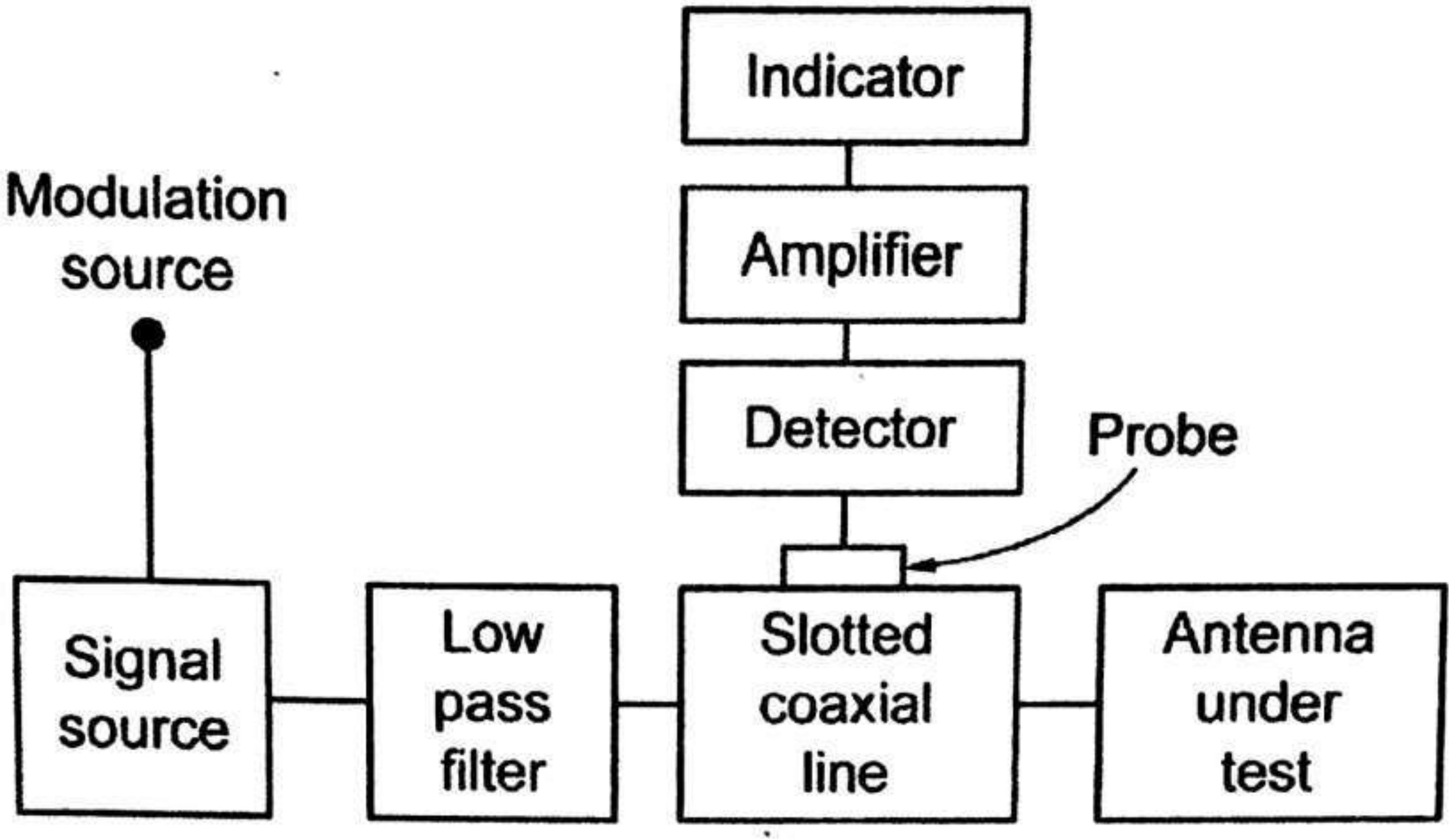
Typical polarization patterns for different polarization of the test antenna

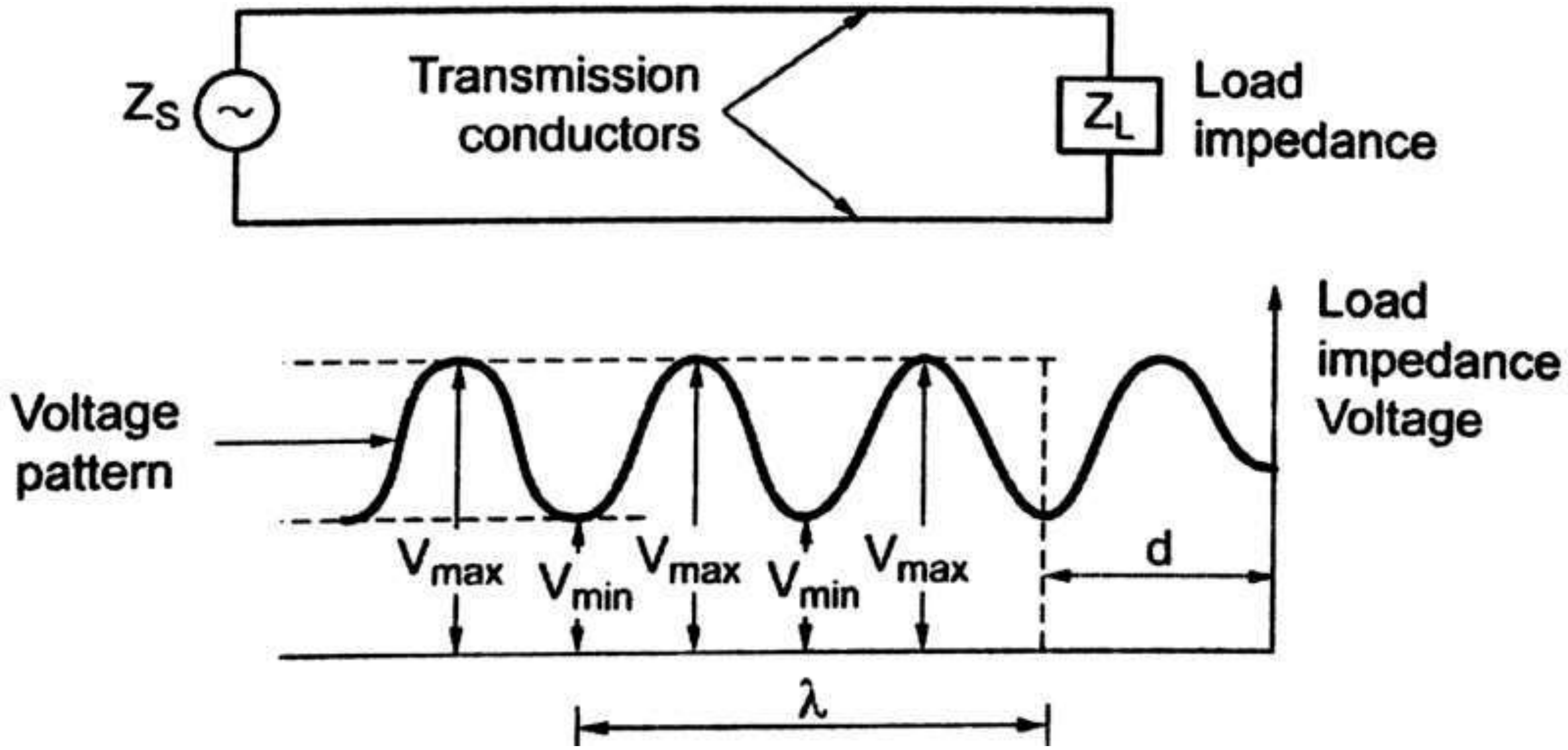
(ii) Rotating–Source Method

(iii) Linearly Component Method

(iv) Circular Component Method

STANDING WAVE RATIO (SWR) METHOD OR SLOTTED LINE METHOD FOR IMPEDANCE MEASUREMENT AT HIGH FREQUENCY





$$\text{Voltage standing wave ratio (S)} = \frac{V_{max}}{V_{min}} = \frac{1+\Gamma}{1-\Gamma}$$

The ratio of the electrical field strength of reflected and incident wave is called ***reflection coefficient*** and it is expressed in terms of impedance as,

$$\Gamma = \frac{E_R}{E_I} = \frac{Z - Z_0}{Z + Z_0}$$

where, Z is the impedance at a point and

Z_0 is characteristic impedance.

The reflection coefficient in terms of VSWR,

$$|\Gamma| = \frac{VSWR - 1}{VSWR + 1} = \frac{S - 1}{S + 1}$$

UNIT V

Introduction

Wave propagation is "the process of communication involves the transmission of information from one location to another".

Radio propagation is a term used to explain, "*how radio waves behave when they are transmitted or propagated from one point on the earth to another*".

Applications of Radio Wave Propagation

(i) Radio direction finding,

(ii) Radar application,

(iii) Satellite control and applications,

(iv) Control of machine from remote place, and

(v) Transmission of information over a required distance

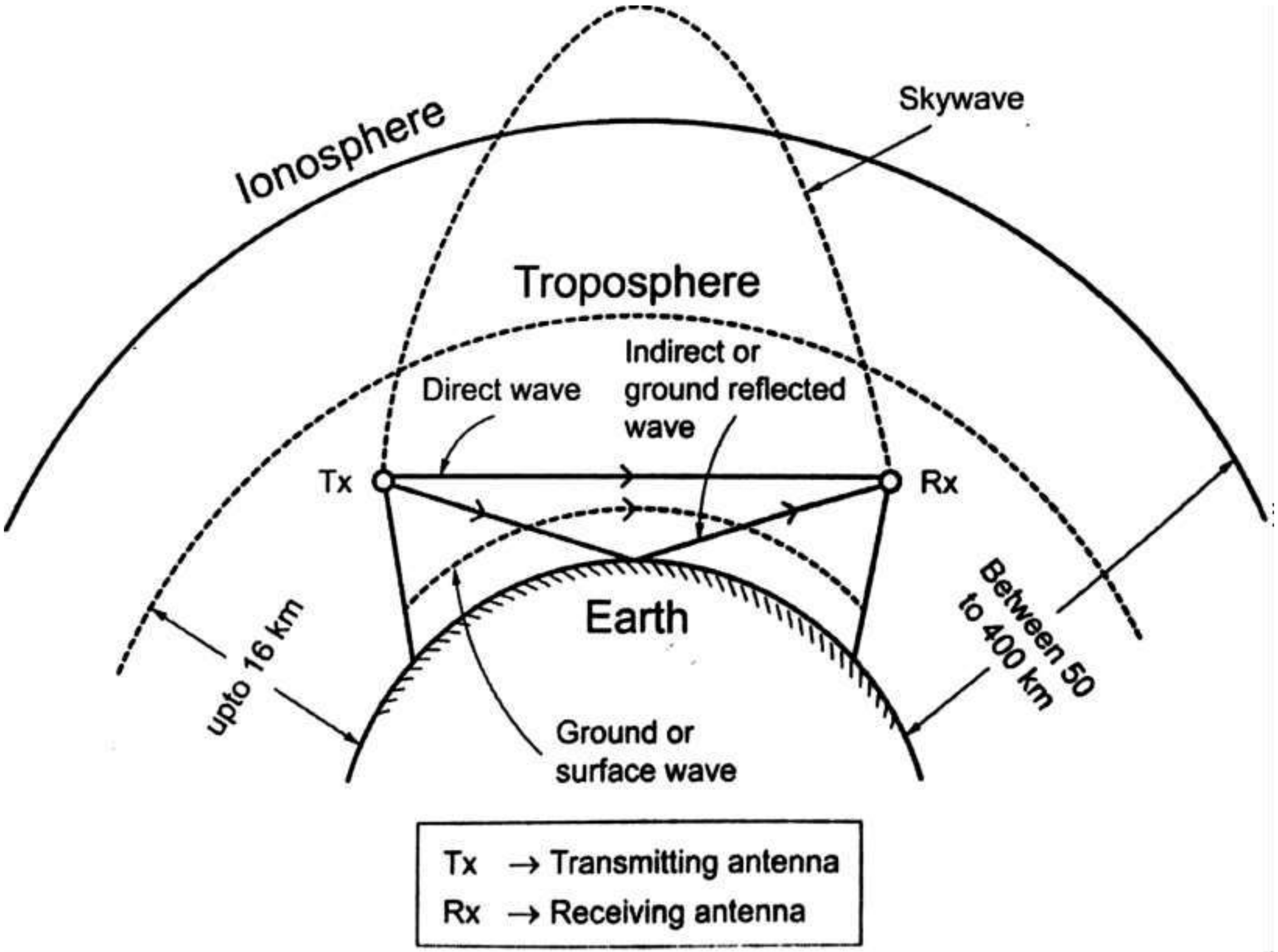
Modes of Propagation

(i) Ground wave (or) Surface wave propagation (Upto 2 MHz),

(ii) Sky wave or Ionospheric wave propagation (Between 2 to 30 MHz),

(iii) Space wave propagation (or) Tropospheric propagation (Above 30 MHz), and

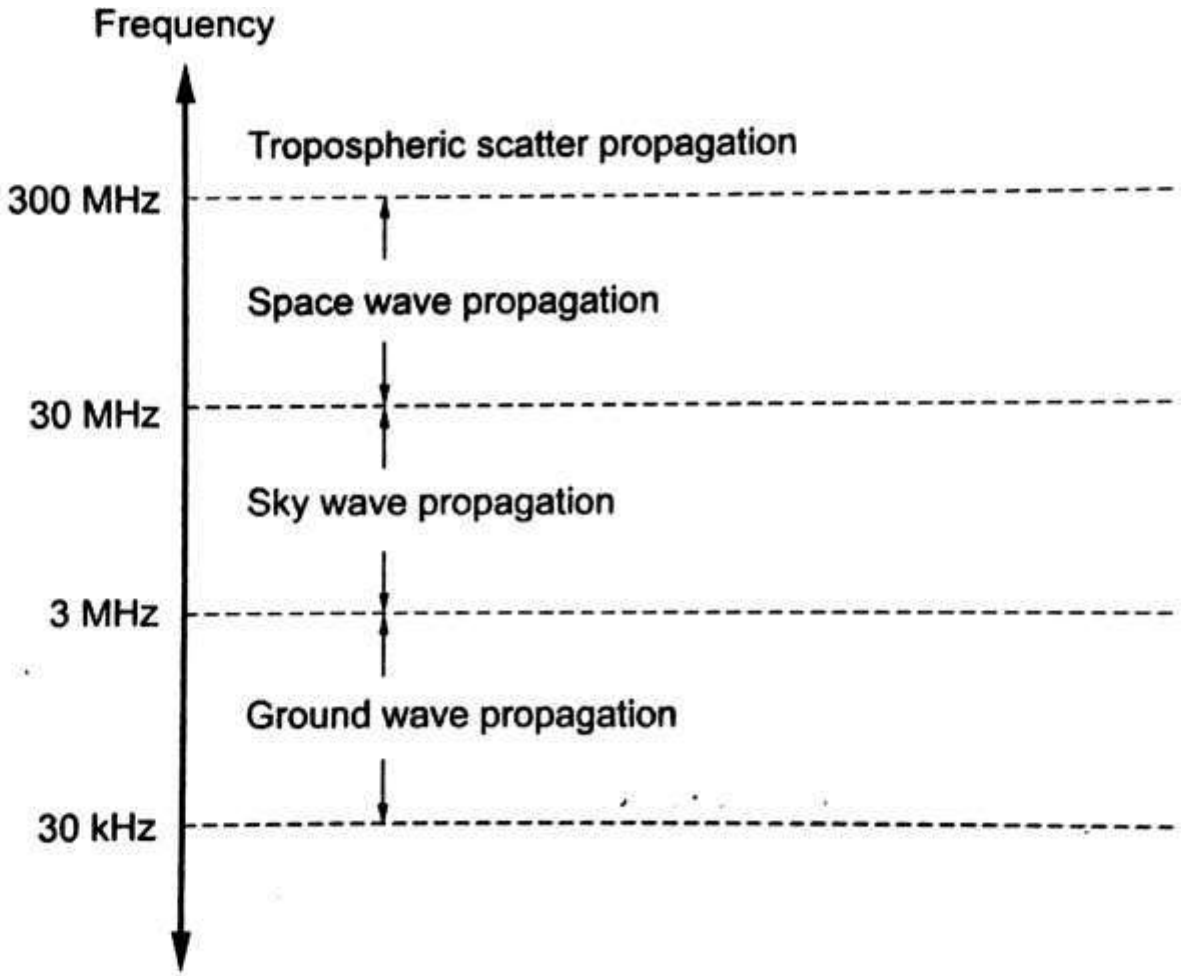
(v) Tropospheric Scatter Propagation (or) Forward Scatter Propagation (UHF and Microwaves above 300 MHz)



Different modes of propagations

1. Ground Wave (or) Surface Wave Propagation (Upto 2 MHz)

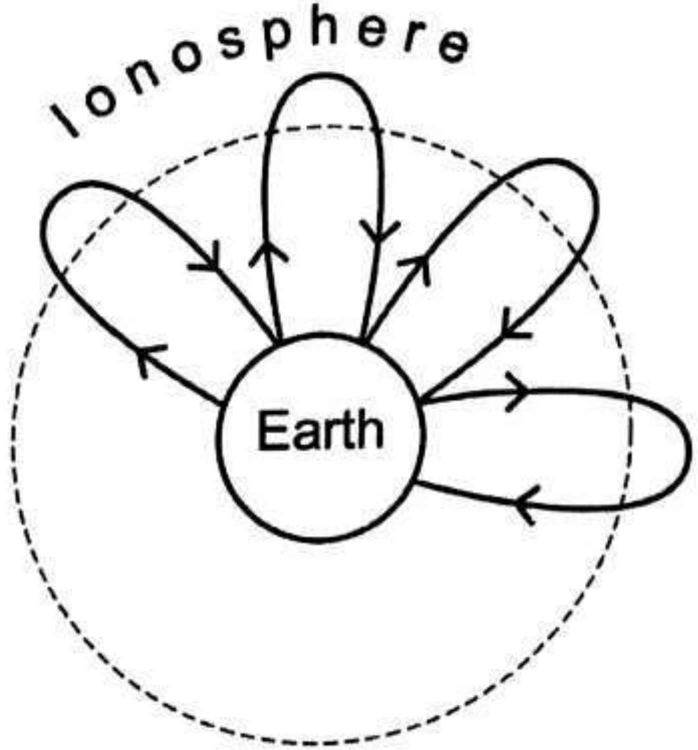
The propagation of electromagnetic waves near the surface of the earth is known as ***ground wave propagation***. Here, the transmitting and receiving antennas are close to the surface of the earth and are vertically polarized.



2. Sky Wave (or) Ionospheric Wave Propagation (Between 2 to 30 MHz)

The waves reaching the receiving antenna as a result of scattering and reflection from the ionosphere is called ***sky wave** (or)* ***ionospheric wave**.*

The *ionized region in the upper atmosphere located between 50 km to 400km above the earth surface is called* ***Ionosphere**.*



3. Space Wave Propagation (or) Tropospheric Propagation (Above 30 MHz)

The region in the atmosphere around 16 km from the earth surface is called ***troposphere****, which is the nearest portion to the surface of the carth.*

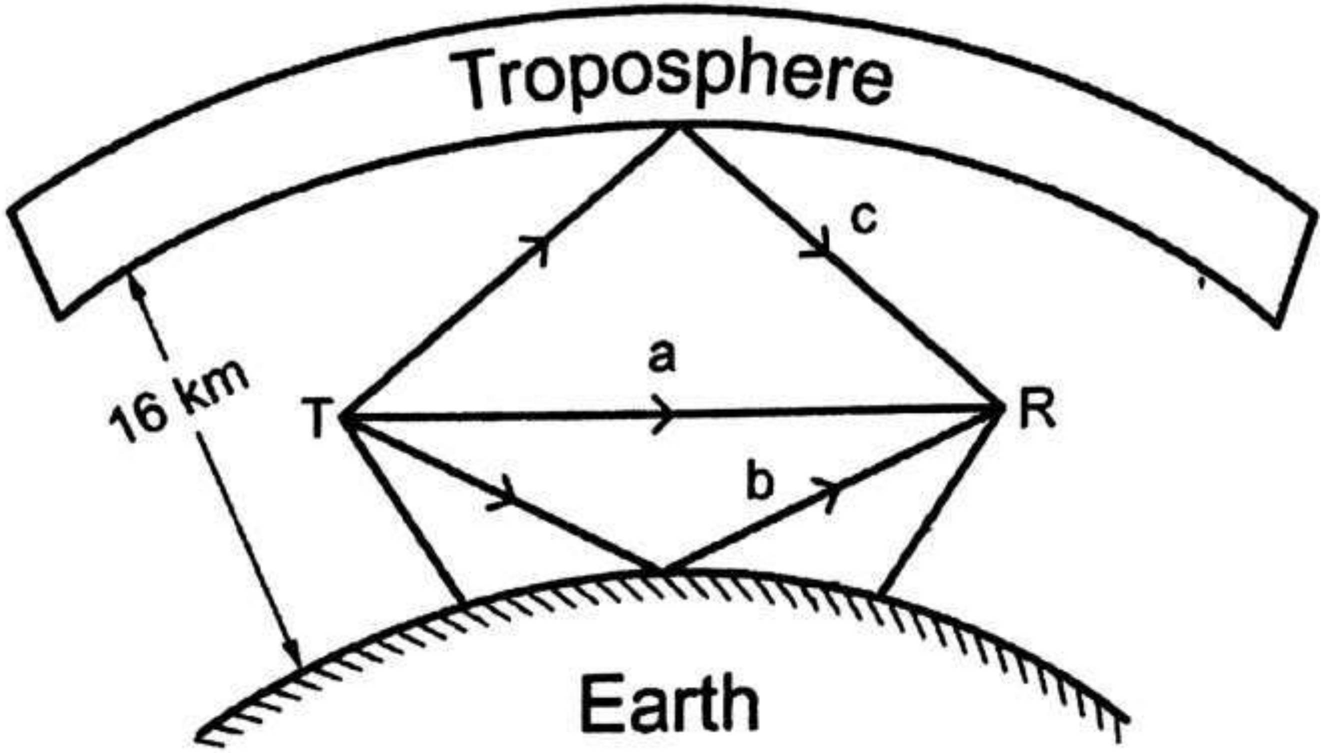
The radio waves which are reflected or scattered in the troposphere before reaching to the receiving antenna are called as ***tropospheric waves****.*

(i) ***Direct wave,*** and

(ii) ***Indirect wave*** (or) ***Ground reflected wave.***

The waves which travel from the transmitter to the receiver in a straight line (directly) without any reflection are called ***direct waves****.*

The transmitting wave arriving at the receiver through the reflections from the earth's surface are called ***ground reflected waves****.*



a → Direct wave
b → Ground reflected wave
c → Reflected wave from troposphere

Tropospheric propagation

4. Tropospheric Scatter Propagation (or) Forward Scatter Propagation (UHF and Microwaves above 300 MHz)

The UHF and microwave signals are propagated beyond line of sight propagation through the forward scattering in the ***tropospheric irregularities****, which is known as* ***tropospheric scatter propagation.***

This type of scatter propagation also leads to the ionospheric scatter propagation for frequencies in the lower range.

Both ionospheric scatter and tropospheric scatter produce undesirable noise and fading which can be ***minimized diversity reception***.

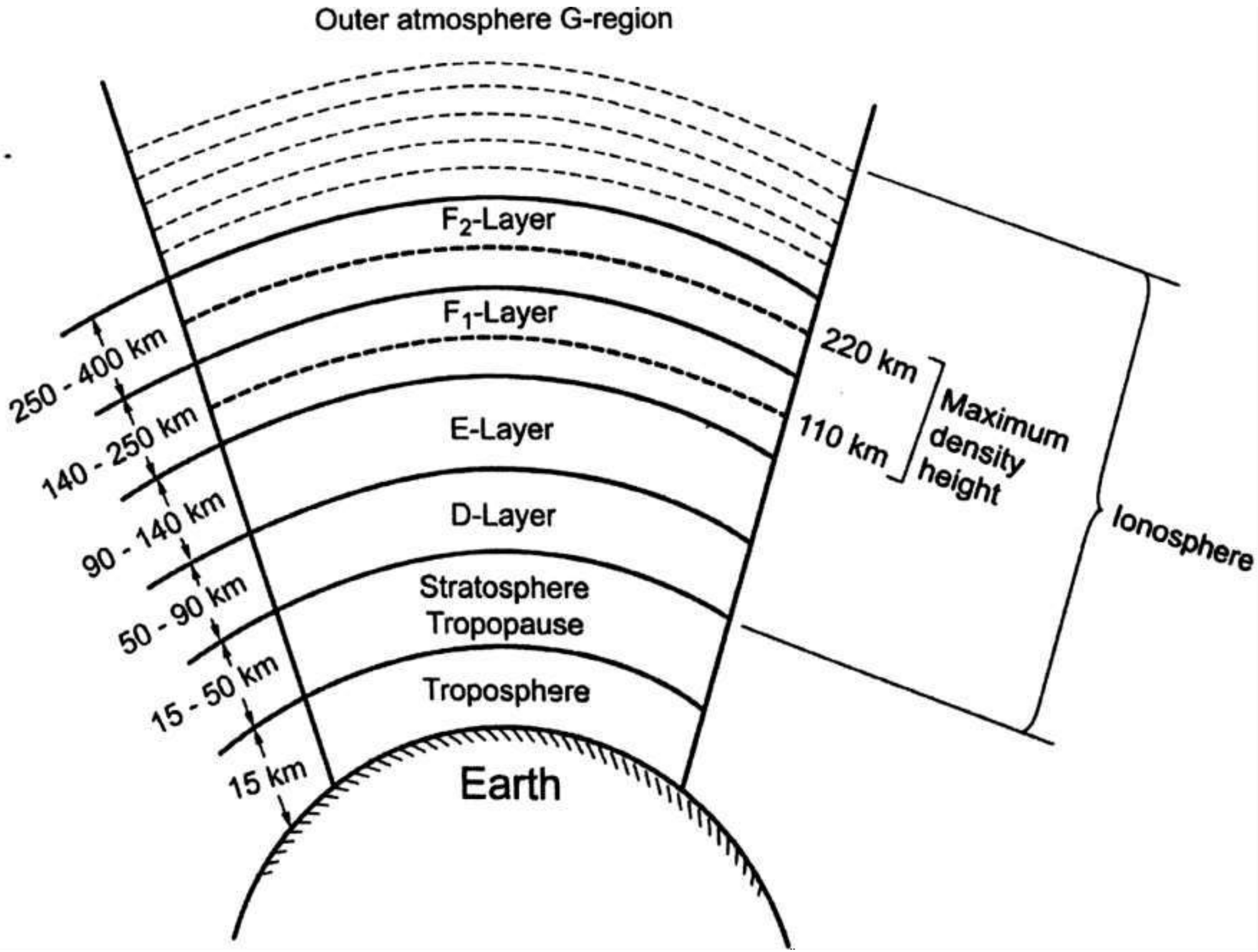
Structure of Atmosphere

The atmosphere consists of three major regions:

(i) Troposphere,

(ii) Ionosphere, and

(iii) Outer atmosphere.



Outer atmosphere G-region
F2-Layer
F1-Layer
E-Layer
D-Layer
Stratosphere
Tropopause
Troposphere
Earth
250 - 400 km
140 - 250 km
90 - 140 km
50 - 90 km
15 - 50 km
15 km
220 km
110 km
Maximum density height
Ionosphere

Structure of Troposphere

Lowest portion of the earth's atmosphere

Average height up to 16 Km from earth's surface

It contains approximately 75% OF atmosphere's mass and 99% of its water vapour

Height increases ➔ % of gas components constant

water vapour components decrease

temperature decreases ➔ minimum value -68 F

Tropopause – Located at the top of the troposphere

Temperature is constant

Separates troposphere from the stratosphere

Structure of Ionosphere

Earth’s upper atmosphere

Shell of electrons and electrically charged atoms and molecules that surround the earth from a height of 50 Km to 1000 Km

The upper part of the atmosphere where the ionization is possible is known as ionosphere. The upper part of the earth’s atmosphere absorb large quantities of radiant energy from the sun and produces ionizations which is the formation of negative and positive ions occurs.