Engineering Mechanics

GE 8292- ENGINEERING MECHANICS

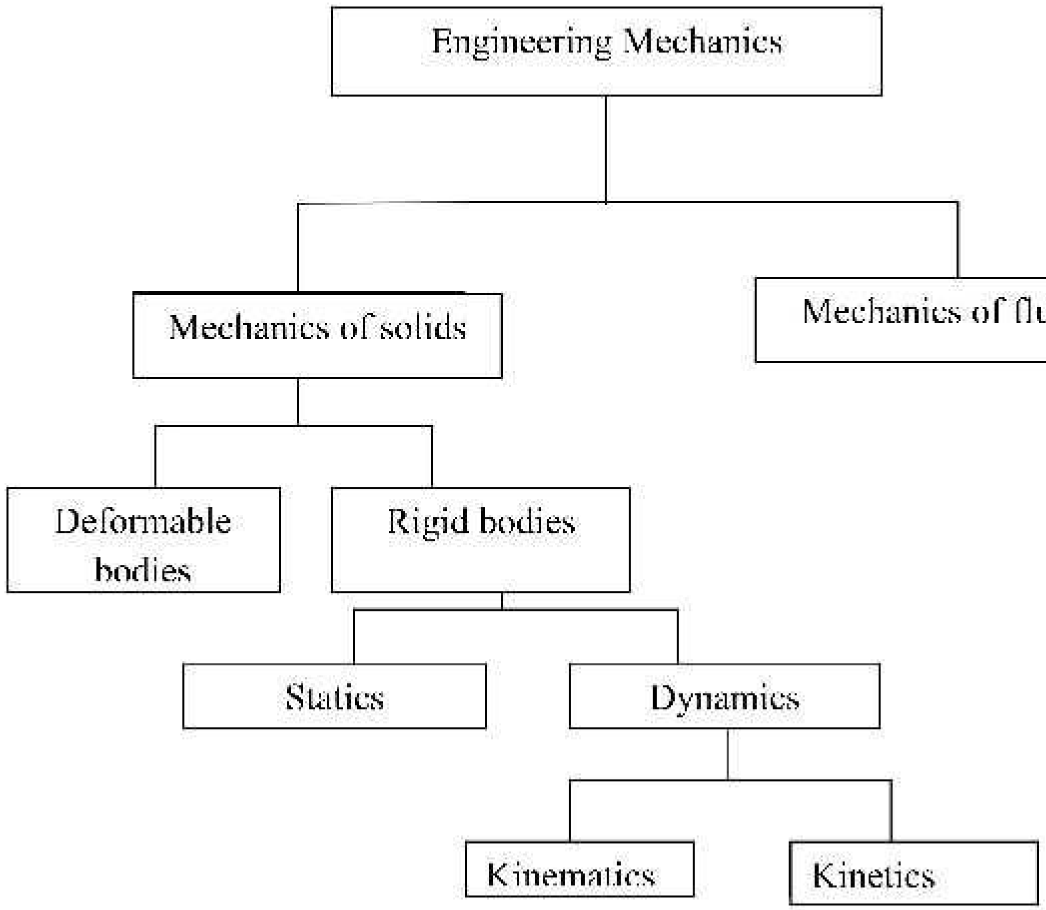
OBJECTIVES:

To develop capacity to predict the effect of force and motion in the course of carrying out the design function of Engineering.

UNIT-I BASIC & STATICS OF PARTICLES

Engineering Mechanics:

Engineering Mechanics may be defining as a branch of science which deals with the behavior of a body with the state of rest or motion, subjected to action of force.



Rigid Bodies:

When a body does not undergo any deformation under the application of forces then it is known as Rigid body.

Deformable body:

When a body undergoes a temporary or permanent change in its dimensions due to application of force it is known as deformable body.

Statics:

It is the branch of science, which deals with the study of a body at rest.

Dynamics:

It is the branch of science which deals with the study of a body in motion.

Kinematics:

It is the study of a body in motion without considering the forces, that cause the motion.

Kinetics:

It is the study of a body in motion, with considering the forces, that causes the motion.

Application of Mechanics:

Engineering mechanics has application in many areas of engineering projects. To cite of few examples, engineering mechanics is applied in the design of spacecrafts and rockets. Analysis of structural stability and machine strength, vibrations, robotics, electrical machines, flow and automatic controls.

Mass and weight

Mass:

The Quantity of matter contained in a body is called mass. The force with which a body is attracted towards the centre of the earth is called weight.

Weight = Mass of body × acceleration due to gravity

$W = mg$

$G = 9.81 \ m/s^2$.

Difference between Mass and Weight

	Mass	Weight
1.	It is a Quantity of matter contained in a body	It is the force with which the body is attracted towards the centre of the earth
2.	It is constant at all places	It is not constant at all places
3.	It resist motion in body	It provides motion in body
4.	It is a scalar quantity	It is a vector quantity
5.	It is never zero	It is zero at the centre of earth
6.	It is measured in kg both in MKS and SI units.	It is measured in Kgf in MKS units and Newton (N in SI units)

UNITS OF MEASUREMENTS

Measurement:

A physical Quantity can be measured by comparing the sample with a known standard amount.

Unit:

The known amount used in the measurement of physical quantity is called a unit.

1.Fundamental Units:

The units which are used for the measurement of basic or fundamental quantities (Mass, Length, Time) are known as fundamental units.

Eg. i) Mass ii) Length iii) Time.

2.Derived Units:

All units which are used for the measurement of physical quantities other than fundamental ones are called derived units.

Eg. Area, Volume, Speed, Velocity, etc….

System of Units:

1. Foot pound second system [FPS system]
2. Centimeter gram second system [CGS system]
3. Metre kilogram second system [MKS system]
4. System of International [SI system]

Six Fundamental Units in SI system:

1. The metre as the fundamental unit of length.
2. The kilogram as the fundamental unit of mass.
3. The second as the fundamental unit of time.
4. The ampere as the fundamental unit of electric current.
5. Kelvin as the fundamental unit of thermodynamics temperature.
6. The candela as the fundamental unit of luminous intensity.

LAW OF MECHANICS

Newton's First law:

It states that "A body continues in its states of rest or of uniform motion in astraight line, unless acted upon by some external force". This is known as First law of inertia.

Newton's second Law of motion:

It states "The rate of change of momentum is directly propositional to the impressed force and tasks places in same direction in which the force acts".

$$F = \text{Mass x Acceleration} = ma$$

Newton's third Law of motion:

It states "To every action there is always an equal and opposite reaction".

- A particle remains in its position (rest or motion) if the resultant force acting on the particle is zero.
- Acceleration of a particle will be proportional to the resultant force and in the same direction if the resultant force is not zero.
- Action and reaction b/w interacting bodies are in the same line of action equal in magnitude but act in the opposite direction.

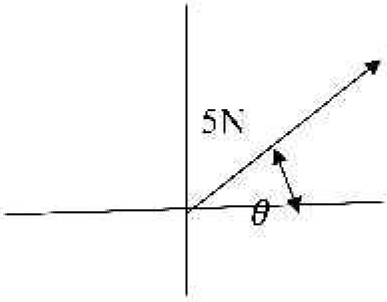
FORCE

Force is an agent which changes or tends to change the states (or) uniform motion of a body upon which it acts. Force is a vector quantity.

Characteristics of a Force:

1. Magnitude
2. Line of action
3. Direction

Graphical Representation of force:



Newton's second law of motion

Momentum = Mass $\times$ velocity

M= mass of the body

u = Initial velocity of body

v = final velocity of body

a = Constant acceleration

t = time required to change velocity from u to v

$\therefore$ change of momentum $= mv - mu$

Rate of change of momentum $= \frac{mv-mu}{t} = \frac{m(v-u)}{t} = ma$

By Newton's second Law

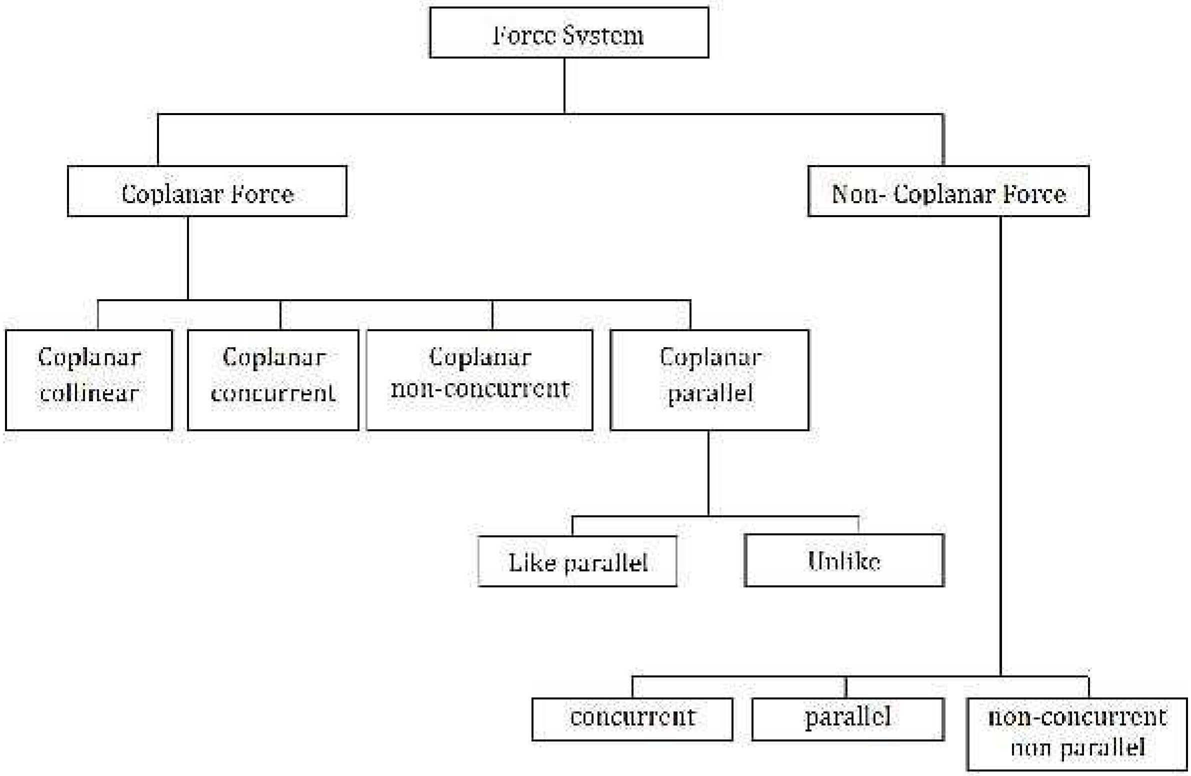
Force = Rate of change of momentum

$F = ma$

Unit of Force:

In SI system unit of force is (N) Newton. One Newton may be defined as the force while acting upon a mass of one kg. Produces an acceleration of 1 m/s^2 in the direction in which it acts.

$1\ N = 1\ kg \times 1 m/s^2 = 1 kgm/s^2$



Parallel

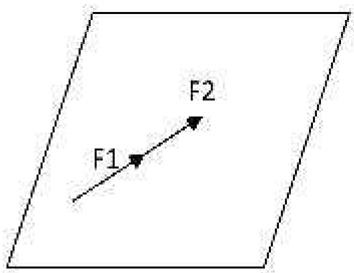
The forces do not line on same plane but their line of action is parallel to each other.

Non concurrent,Non parallel

The forces neither lie on same plane not their line of action meet at common point.

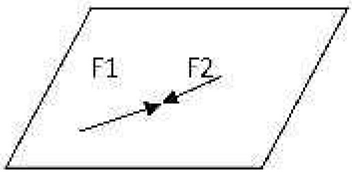
Like collinear coplanar forces

Forces acting in the same direction, lies on a common on line of action and acts in a single plane



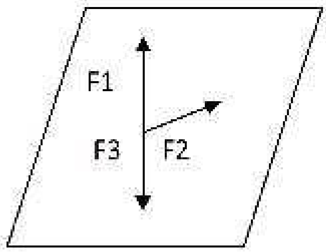
Unlike collinear coplanar forces:

Forces acting in the different lies on a common line of a action and act is a single plane.



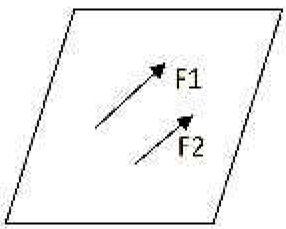
Coplanar concurrent forces:

Force intersects at a common point and lies in a single plane.



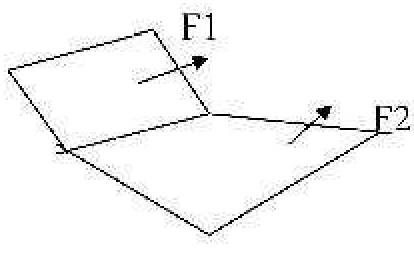
Coplanar Non concurrent flow:

Forces which do not intersects at a common point but acts in one plane. They may be parallel or non parallel



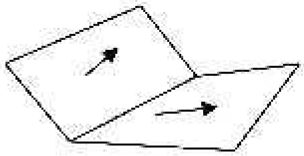
Non coplanar concurrent forces

Forces intersect at a common point but either line of action do not lie on same plane.



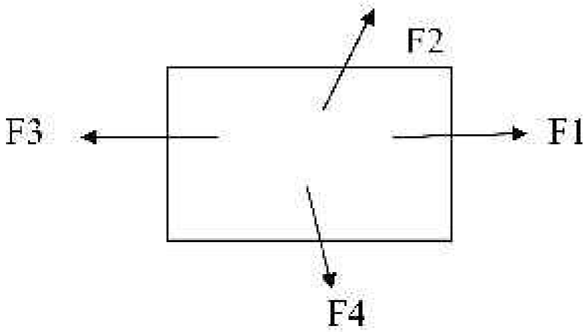
Non Coplanar Non concurrent force:

Forces do not intersect at one point and also their lines of action do not lie on same plane.



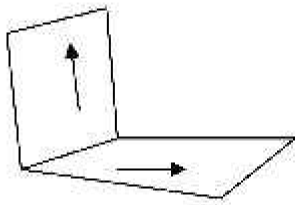
Coplanar forces:

The line of action of the forces lies on same plane.



Non coplanar forces:

The lines of action of the forces not lie on same plane.



Collinear:

The Line of action of the forces lie on same plane.

Like collinear:

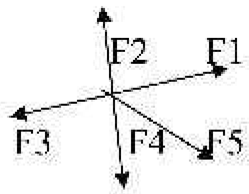
The line of action of forces lies on a same line and in same direction.

Unlike collinear

The lines of action of forces lie on same line but are in opposite direction.

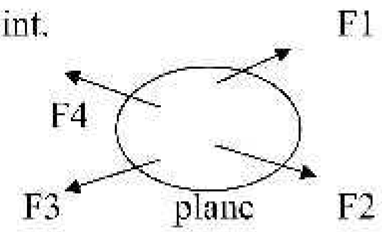
Concurrent forces:

The lines of action of all forces meet at a common point and lie in the same plane.



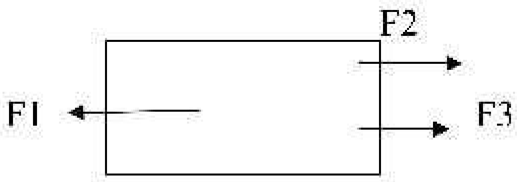
Non concurrent force system:

The forces will lie on same plane but their line of action will not intersect at a common point.



Parallel forces:

The forces lying on same plane whole line of action are parallel to each other.



Like parallel:

If the force acts in the same direction they are coplanar like parallel force system.

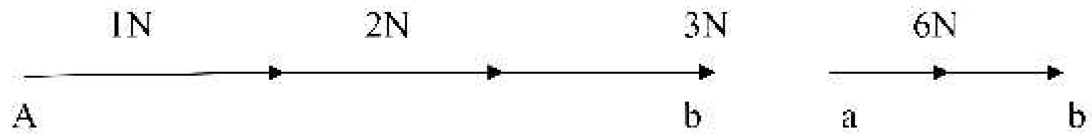
Unlike parallel: If the force acts in opposite direction, they are coplanar unlike parallel force system.

UNIT-1

Chapter-2 Statics of Particles in Two Dimensions- Resultant Force

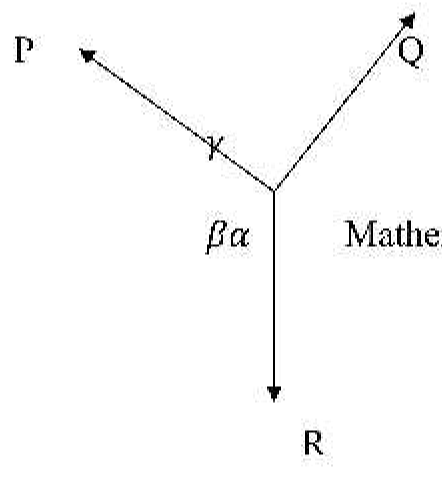
Resultant Force:

If a number of forces acting on a particle simultaneously are replaced by a single force which could produce the same effort as produced by the given forces, that single force is called resultant force.



Lami's Theorem:

It state that "if three coplanar forces acting at a point be in equilibrium, than each force is propositional to the sin of the angle between the other two forces.

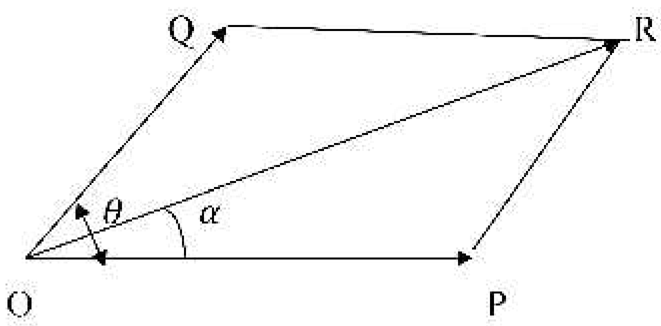


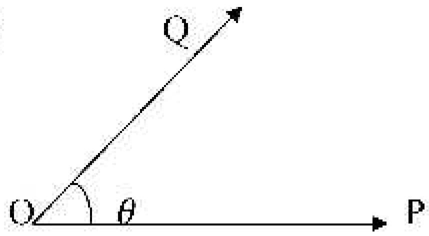
Mathematically

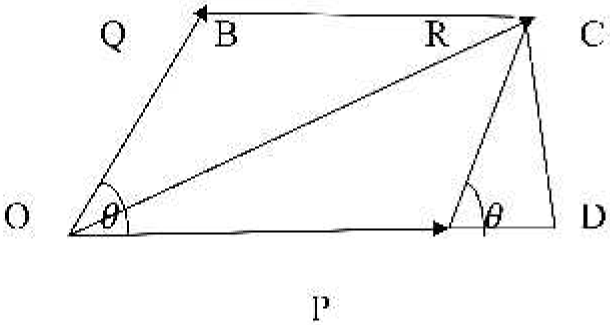
$$\frac{P}{sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma}$$

Parallelogram Law of forces:

It states that "if the two forces acting simultaneously at a point represented in magnitude and direction by the two adjacent sides of the parallelogram, then the resultant of these two forces is represented in magnitude and direction by the diagonal of the parallelogram originating from that point.







Let pand Q are two concurrent force acting on a point O at an angle of θ.

The forces P and Q are graphically represented by the lines OA and OB respectively.

The parallelogram θACB is completed by drawing the lines BC and AC parallel to OA and OB respectively.

In parallelogram OACB, the diagonal OC represents the resultant force of P and q. by II Law of forces.

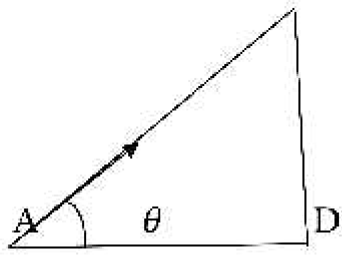
In order to prove the, parallelogram law of forces, extend the lines of action of force P, till its meet the perpendicular drawn from point C.

Let the point of intersection of these two lines be D. from the geometry of the parallelogram.

OB AC

OA=BC

In triangle ACD



$\cos\theta \ \frac{AD}{Q} \qquad \sin\theta \ \frac{CD}{Q}$

$AD = Q\cos\theta$ --------- (1) $\qquad$ $CD = Q\sin\theta$ ------------- (2)

Also = $AD^2 + CD^2 = AC^2$

$= AD^2 - CD^2 = Q^2$--------------- (3)

In triangle OCD

$OC^2 = OD^2 + CD^2$

$= (OA + AD)^2 + CD^2$

$= OA^2 + AD^2 + 2 \times OA \times AD + CD^2$

$= OA^2 \ (AD^2 + CD^2) + 2OA\ AD$

$= OA^2 + AC^2 + 2\ OA\ AD$

$R^2 = P^2 \quad Q^2 + 2 \times PQ\cos\theta$

$R = \sqrt{P^2 + Q^2 + 2 \times PQcos\theta}$

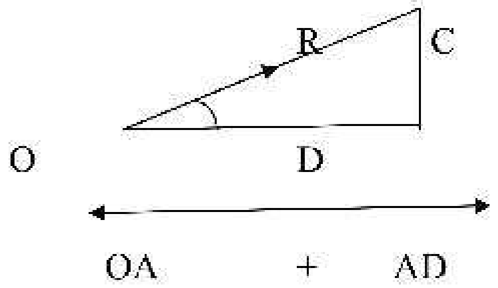
Inclination of the resultant force with the force P

Let the angle of inclinator of R with the line of action of the force P be $\propto$

In triangle OCD

$\tan\propto \frac{CD}{OD} \ \frac{CD}{OA+AD} - \frac{Qsin\theta}{P+Qcos\theta}$

$\therefore \tan\propto = \frac{Qsin\theta}{P+Qcos\theta}$

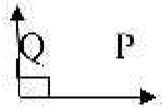


Important results:

1. If $\theta = 0°$ then the resultant forces pand Q will be like collinear, then,

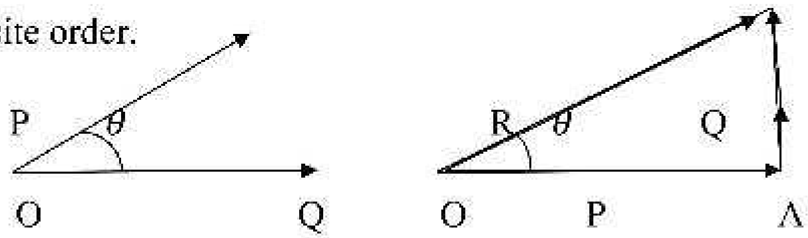
   $R = P + Q$ $\qquad$ $P \longrightarrow$ $\qquad$ $Q \longleftarrow$

2. If $\theta = 90°$, the forces P and Q are at right angles then R= $\sqrt{P^2 + Q^2}$
3. If $\theta = 180°$, then the forces P and Q will be unlike collinear forces, then R=P-Q.



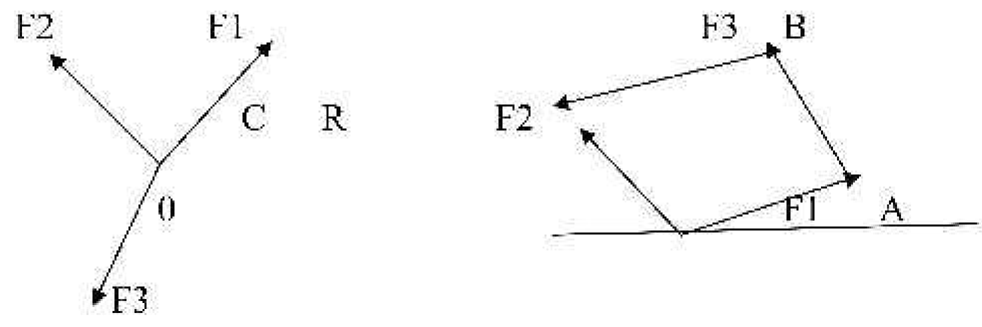
Triangle law of Forces:

If two forces acting at a point are represented by two sides of a triangle taken in order, then their resultant force is represented by the third side taken in opposite order.



Polygon Law of forces:

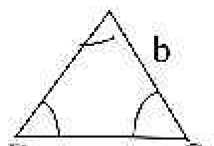
Polygon Law of forces states that, 'if a number of coplanar concurrent forces are represented in magnitude and direction by the sides of a polygon taken in an order then their resultant force is represented by the closing side of the polygon taken in the opposite order.



Sine Law:

The law of sines can be used when two angles and a side are known a technique known as triangulation.

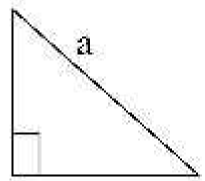
$$= \frac{a}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$$



Cosine Law:

It two side and the angle between the sides are known,

Then the third is given by



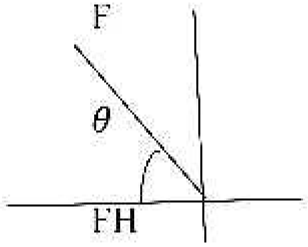
$$a^2 = b^2 + c^2 - 2bccos\alpha$$

$$b^2 = c^2 + a^2 - 2cacos\beta$$

$$c^2 = a^2 + b^2 - 2abcos\gamma$$

<u>Resolution of a force in to its horizontal and vertical part.</u>

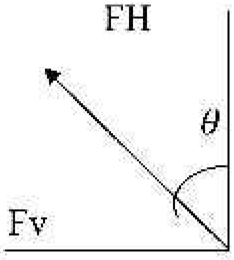
Method I



$$sin\theta = \frac{Fv}{F} => FV = Fsin\theta$$

$$cos\theta = \frac{FH}{F} => FH = Fcos\theta$$

Method II

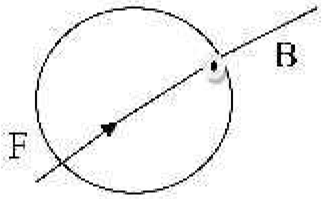


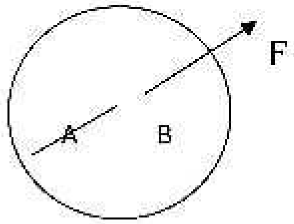
$$sin = \frac{FH}{F} => FH = Fsin\theta$$

$$cos\theta = \frac{Fv}{F} => Fv = Fcos\theta$$

Principle of transmissibility of Forces:

If a force act at any point of on a rigid body it may also be considered to act at any other point on its line of action.



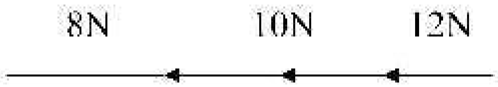


Resultant force of two concurrent forces:

1. Resultant force of two concurrent force
2. Resultant force of more than two concurrent force

Problem based on parallelogram & Resultant forces:

1. Find the resultant force of the collinear forces shown in fig.



Soln:

Resultant force R= 8+10+12=30N

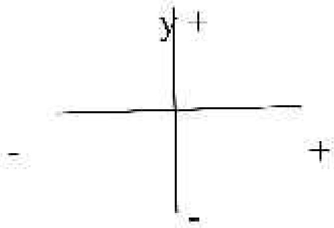
2. Find the resultant force of the collinear forces, shown in fig

Soln:

Magnitude of resultant force

2-3+6-11

R = -6N

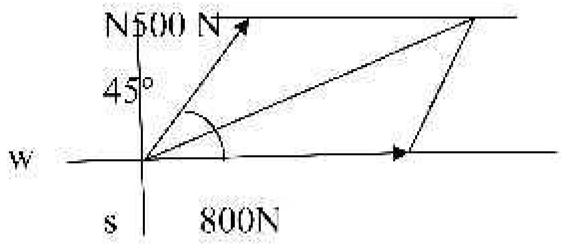


3. Find the resultant force an 800 N force acting towards eastern direction and a 500 n force acting towards north eastern direction.

by 1. Parallelogram Law

2. Triangle Law

Also find the direction



Given

P=800 N Q=500N θ=45°

To find

Resultant force & direction

Soln

1. Parallelogram Law

Resultant Force R= $\sqrt{P^2} + Q^2 + 2PQcos\theta$

$$R = \sqrt{800^2} + 500^2 + 2 \times 800 \times 500 \times cos45$$

R= 1206.52 N

Direction of magnitude

$$Q = tan^{-1}\left[\frac{Qsin\theta}{P+Q\ cos\theta}\right]$$

$$Q = tan^{-1}\left[\frac{500sin45}{800+500\ cos45}\right]$$

Q 17°04'

Summing of components:

R $= \sqrt{FH^2 + \sum FV^2}$

$\sum$FH = 800+500sin45=1153.55N

$\sum$FH = 500 sin 45 = 353.55 N

R = $\sqrt{1153.55^2 + 353.55^2}$

R = 1206.52 N

$\alpha = tan^{-1}\frac{|\sum FV|}{\sum FH}$

$= tan^{-1}\frac{|1153.55|}{353.55}$

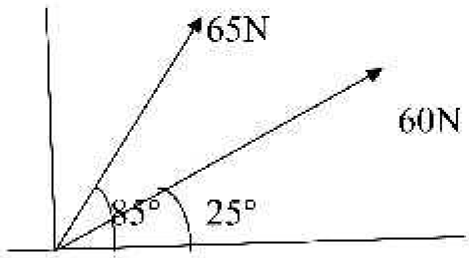
$\alpha = 17°04'$

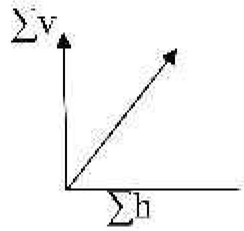
4. Two forces 60 N and 65 N act on a screw at an angle of 25° and 85° from the base. Determine the magnitude and direction of their resultant.

Given:

P_1=60 N1,θ_1=25°

P_2=65 N, θ_2=85°





To find:

Magnitude & direction of their resultant

Soln:

1. Magnitude of resultant force

$R = \sqrt{\sum FH^2 + \sum FV^2}$

$\sum FH = 60\ \cos\theta_1 + 65\ \cos\theta_2$

$= 60\ \cos 25 + 65\ \cos 85$

$\sum FH$ =60 N

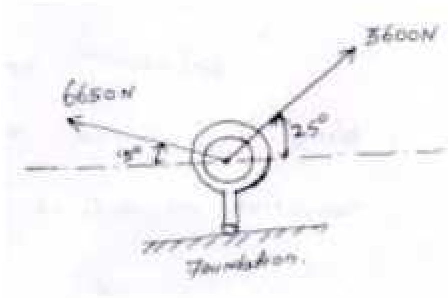
$\sum FV = 60\ \sin\theta_1 + 65\ \sin\theta_2 = 60\ \sin 25 + 65\ \sin 85$

$\sum FV$ = 90 N

$R = \sqrt{60^2 + 90^2}$

R= 108.17 N

5. Two wires are attached to a bolt in a foundation as shown in fig. below. Determine the pull exerted by the bolt on the foundation.



<u>Soln:</u>

Resultant force $R = \sqrt{\sum FH^2 + \sum FV^2}$

$\sum FH$ = 3600 cos 25-6650 cos 15

$\sum FH$ = -3160 N

$\sum FV$ = 3600 sin25+ 6650 sin 15

$\sum FV$ = 3242N

$R = \sqrt{-3160^2 + 3242^2}$

R = 4527 N

$\alpha = tan^{-1}\left[\frac{\sum FV}{\sum FH}\right]$

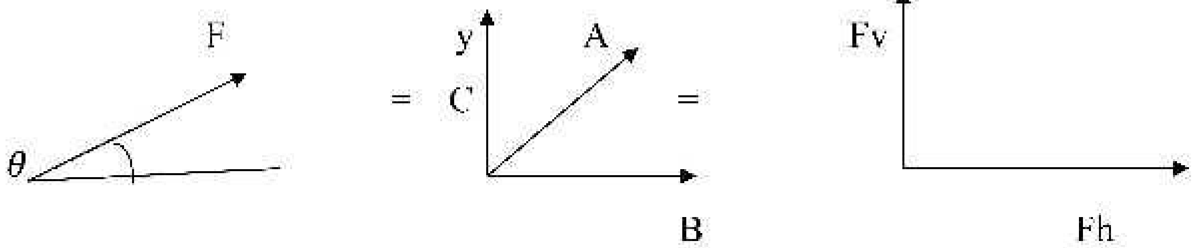
$\alpha = tan^{-1}\left[\frac{\sum 3242}{\sum 3160}\right]$

$\alpha = 45°73'$

⇒ **<u>Resultant force of more than Two concurrent Forces:</u>**

Resolution of Forces:

Splitting up a force into components along the fixed reference axis is called Resolution of a force.



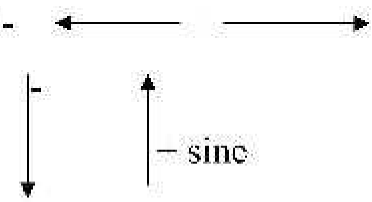
F_h = horizontal component = +F cosθ

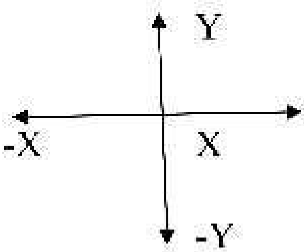
F_v = vertical component = +F sinθ

Sign conversion:

Horizontalcomponent -

Vertical component

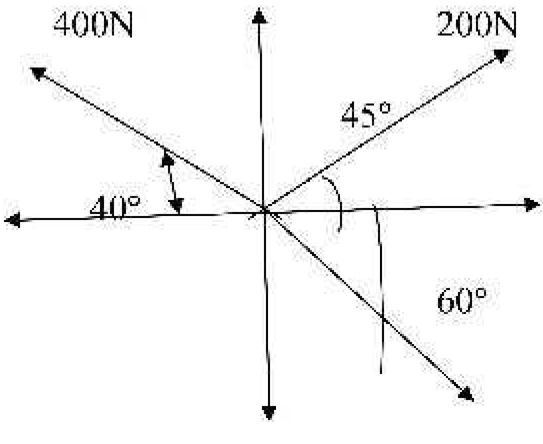




$$R = \sqrt{\sum FH^2 + \sum FV^2}$$

$$\alpha = tan^{-1}\left[\frac{\sum FV}{\sum FH}\right]$$

1. Three coplanar concurrent forces are acting at a point as shown in fig. Determine the resultant in magnitude of direction.



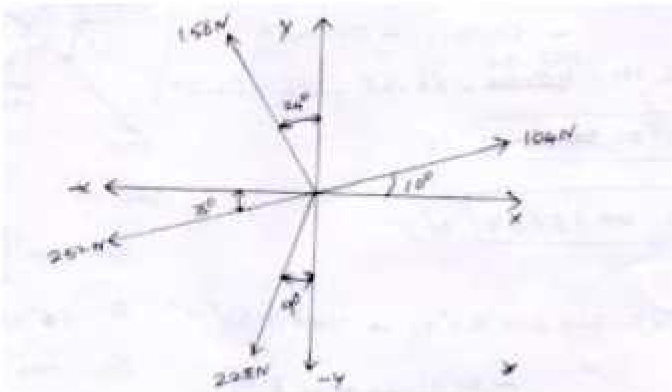
Soln:

Force & magnitude	θ	F$\cos\theta$	$F\sin\theta$
F_1=200	45°	200 cos45°=141.42	200 sin45°=141.42N
F_2=400	150°	400 cos150°=-326.41	400 sin150°=200
F_3=600	300°	600 cos300°=300 N	600 cos300°=-519.61
		$\sum FH$ = 95.01 N	$\sum FV$ = -178.19N

R $= \sqrt{\sum FH^2 + \sum FV^2} = \sqrt{95.01^2 + -178.19^2}$

R=201.95 N

2. The four coplanar forces acting at a point a as shown in fig. Determine the resultant in magnitude and direction.



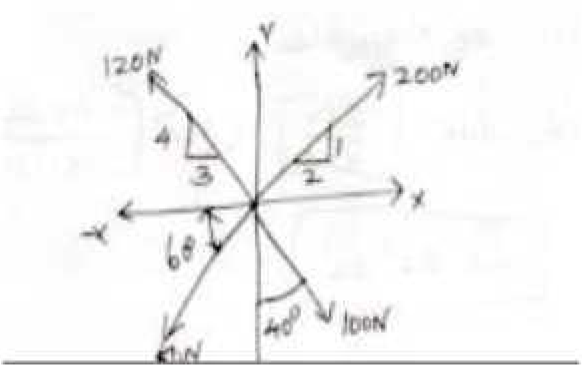
<u>Soln:2 nd method</u>

Note:θ_1 10° θ_2 90-24 66° θ_3 3° θ_4 90-9 81°

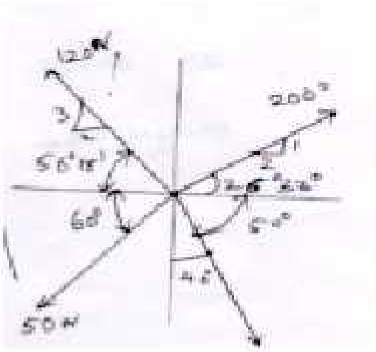
$\sum FH$ =-248.36 $\sum FV$ = -77.82

R=260.26N θ= 17.39°

3. A system of four forces acting on a body is shown in fig below. Determine the resultant force and direction.



Soln:



Resultant force R– $\sqrt{\sum FH^2 + \sum FV^2}$

$\sum FH$ $=200\cos 26°33' - 120\cos 56°18' - 50\cos 60 + 100cos50$

$\sum FH$ =178.90-66.58-25 64.27

$\sum FH$ =151.59

$\sum FV$ $= 200sin26°33' + 120sin56°18' - 50sin60 - 100sin50$

$\sum FV$ $= 89.39 + 99.83 - 43.30 - 76.60$

$\sum FV$ =69.32N

R $\sqrt{(151.59)^2 + (69.32)^2}$

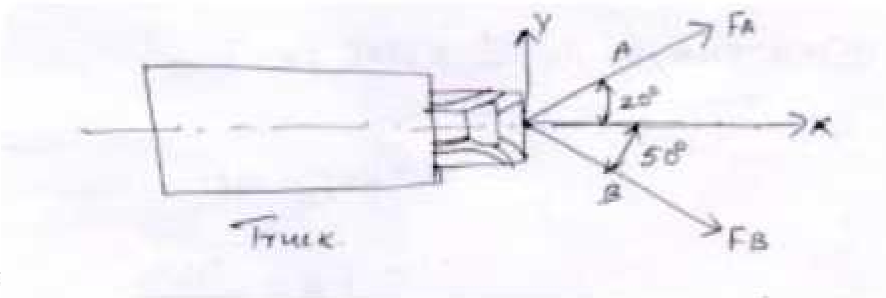
R = 166.68N

Direction of magnitude

$$\alpha = tan^{-1}\left[\frac{\sum FV}{\sum FH}\right] = tan^{-1}\left[\frac{69.32}{151.59}\right]$$

$$\alpha = 24°34'$$

4. The truck shown is to be toward using two ropes. Determine the magnitude of forces F_A& F_B acting on each rope in order to develop a resultant force of 950N directed along the positive X axis.



Resultant force R= 950 N in positive × $direction$

$\therefore$Hence $\sum FH$ = 950

$\sum FH$ = 0

Resolving forces horizontally

$\sum FH$=FAcos20°+FB cos 50°

$FAcos20 + FBcos50 = 950$--------- (1)

Resolving forces vertically

$\sum FV = FAsin20 - FBsin50 = 0$

$FAsin20 - FBsin50 = 0$------------------- (2)

Solving eq(1) & (2)

$FAcos20 + FBcos50 = 950$

$FAsin20 - FBsin50 = 0$

$0.939FA + 0.642FB = 950$---------------- (1)

$0.342FA + 0.766FB = 0$--------------- (2)

$0.939FA + 0.642FB = 950$--------------- (1)

(2)$\times$ 2.75 $\quad 0.342FA \pm 0.766FB = \quad 0$

2.748FB = 950

FB $=\frac{950}{2.748}$

FB = 345.64 N

FB value sub in eqn(1)

0.94$\times$ FA+0.642$\times$345.64=950

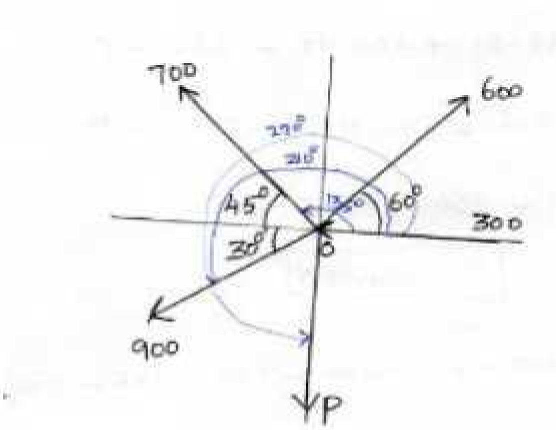
0.94 FA =950-221.9=728

FA=$\frac{728.09}{0.94}$

FA = 774.57 N

5. Five forces are acting on a particle. The magnitude of the forces are 300 N,600N,700N,900n and P and their respective angles with the horizontal are 0°,60°,135°,210°,270°. If the vertical component of all the force is -1000N, Find the value of P. Also calculate the magnitude and the direction of the resultant, assuming that the first force acts towards the point, while all the remaining forces act away from the point.

Given:



θ_1=0° θ_2=60° θ_3=180-135=45°

θ_4=180+[90-60]=210°=30°

θ_5=270=90°

F_1=300, F_2=600 F_3=700 F_4=900 F_5=P

$\sum FV$ = -1000N

Soln

Resultant force $R=\sqrt{(\sum FH)^2 + (\sum FV)^2}$

$\sum FV = -1000N$

To find the value of 'P'

Algebraic sum of vertical components

$\sum Fv$ = 600 sin60 +700 sin45-900 sin30-P

-1000 519.61−494.97-450-P

-1000=-519.61-494.97+450=-P

- 1564.58 = - P

P=1564.58N

$\sum$FH=-300+600 cos60-700 cos45-900 cos30

$\sum$FH=-300 +300-494.97-779.42

$\sum$FH=-1274.39 N

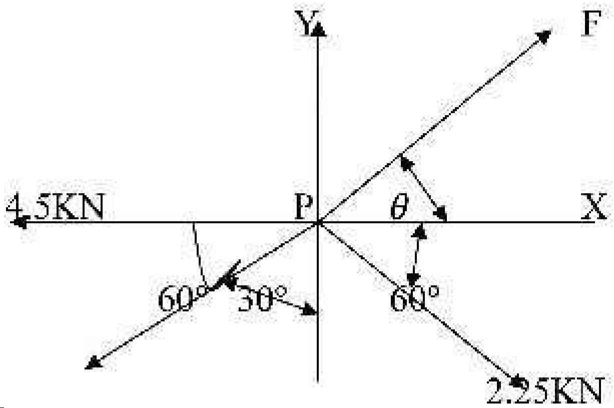
Resultant force R=$\sqrt{(-1274.39)^2 + (-1000)^2}$

R=1619.89 N

Direction $\alpha = tan^{-1}\left[\frac{\sum FV}{\sum FH}\right] = tan^{-1}\left[\frac{1000}{1274}\right]$

$\alpha = 38°7'$

6. Determine the magnitude and angle of f so that particle P shown in Fig



Soln:

$\sum$FH=F cosθ-4.5-7.5 cos60+2.25 cos60=0

Fcosθ-4.5-3.75+1.125=0---------------- (1)

$\sum$FV=F sinθ-7.5 sin60-2.25 sin 60=0

F sinθ-6.49-1.94=0-------------- (2)

Eqn (1) Rearrange

F cosθ-7.125=0

$F\cos\theta = 7.125$-----------------(3)

Eqn(2) Rearrange

$F\sin\theta - 8.43 = 0$

$F\sin\theta = 8.43$------------------ (4)

$\frac{Eqn(4)}{Eqn(3)} = \frac{F\sin\theta}{F\cos\theta} = \frac{8.43}{7.125}$

$\text{Tan}\theta = 1.183$

$\theta = \tan^{-1}[1.183]$

$\theta = 49°47'$

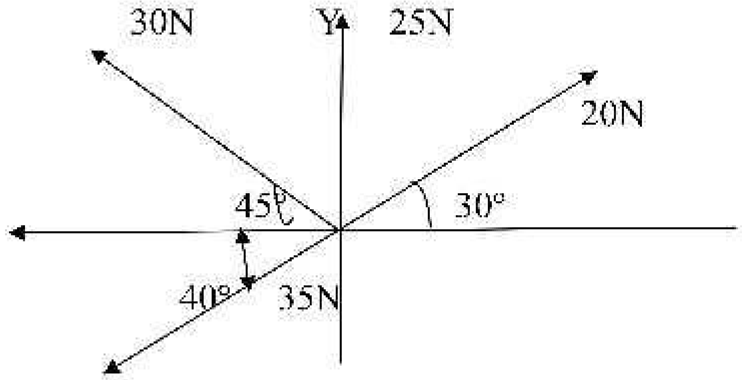
Substitute $\theta = 49°47'$ in eqn(3)

$F\cos\theta \; 7.125 \Rightarrow F\cos 49°47' \; 7.125$

$F = \frac{7.125}{\cos 49°47}$

$F - 11.03$ N

7. Particle 'O' is acted on by the following forces (HW)
   - (i) 20 N inclined 30° to north of east
   - (ii) 25 N towards the North
   - (iii) 30 N towards the north west
   - (iv) 35 N inclined 40° to south of west



Chapter -3

Equilibrium of Particles in Two Dimensions

Equilibrium:

A body is said to be in a state of equilibrium, if the body is either at rest or moving at a constant velocity.

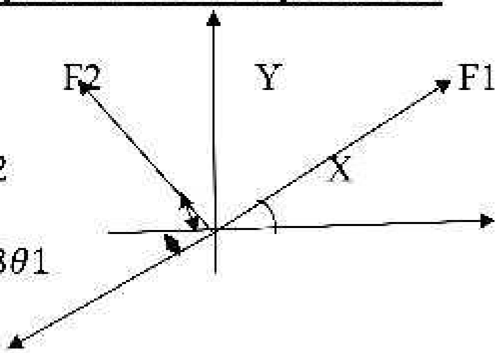
Equilibrium Force:

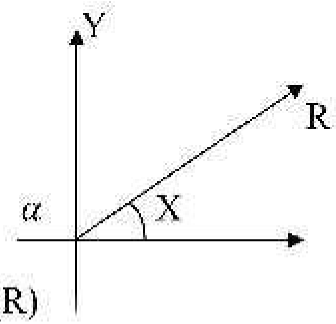
The set of forces where resultant is zero is called "Equilibrium Force".

Equilibrium and Equilibrant:

$\theta 2$

$\theta 3 \theta 1$

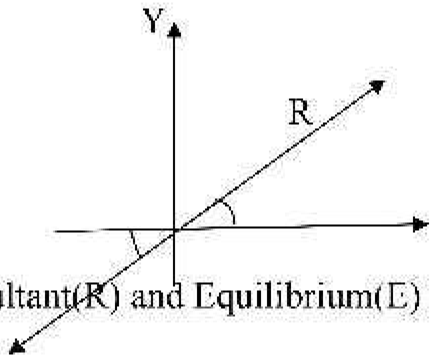




Resultant(R)

α X

α



Resultant(R) and Equilibrium(E)

Consider a particle subjected to three coplanar concurrent forces as shown in fig(1)

Let the resultant force of the force system R as shown in fig(2) with direction of α with horizontal. Due to this resultant force, the particle may starts moving in the direction of resultant force.

But if we apply an additional force of same magnitude and direction as that of resultant force, on the same line of action, but in opposite direction, then the movement of the particle will be arrested or the particle to said to be in Equilibrium.

The force E, which brings the particle (or set of force) to equilibrium, is called equilibrant.

Hence, Equilibrant (E) is Equal to the resultant force(R) in magnitude and direction, collinear but opposite in nature.

Conditions of Equilibrium:

For equilibrium condition of force system, the resultant is Zero.

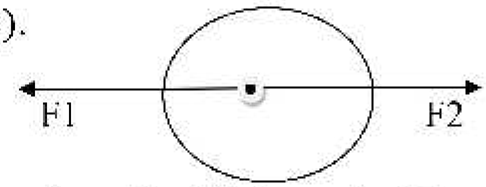
R=0

But R $= \sqrt{(\sum FH)^2 + (\sum FV)^2}$

$\sum FH=0$ $\sum FV=0$

Principle of Equilibrium:

Equilibrium principles are developed from the force Law of equilibrium ($\sum F = 0$).



1. Two force Equilibrium principle:

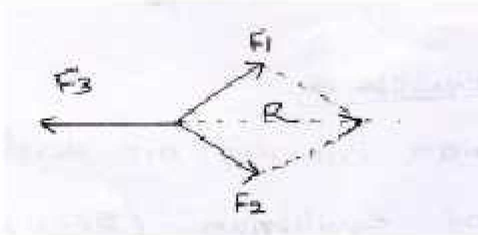
   If a body is subjected to two forces, then the body will be in equilibrium if the two forces are collinear, equal and opposite.

2. Three force equilibrium principle:

   If a body is subjected to three forces, then the body will be in equilibrium, if the resultant of any two forces is equal, opposite and collinear with the third force.

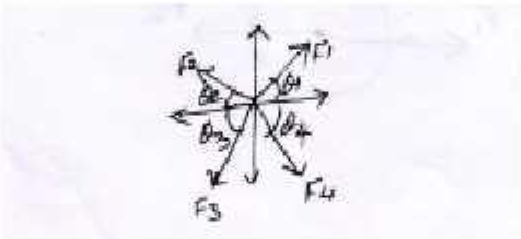
R is the resultant

F_1 and F_2 also $R=F_3$



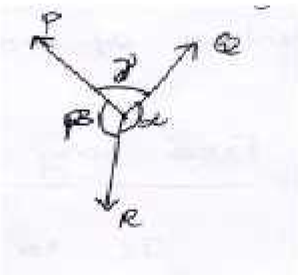
3. Four Force Equilibrium Principle:

   If a body is in equilibrium , acted upon by four forces, then the resultant of any two equal must be equal, opposite and collinear with the resultant of the other two.



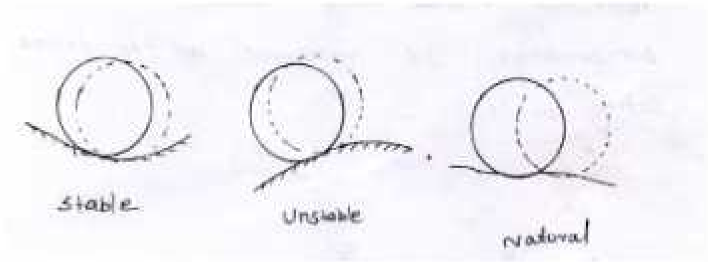
→Lami's Theorem:

If three coplanar forces acting at a point be in equilibrium, than each force is proportional to the sine of the angle b/w the other two.



$$\frac{P}{sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma}$$

Types of Equilibrium:



Stable Equilibrium:

A body is said to be in stable equilibrium, if it returns back to its original position after it is slightly displaced from its position.

Unstable Equilibrium:

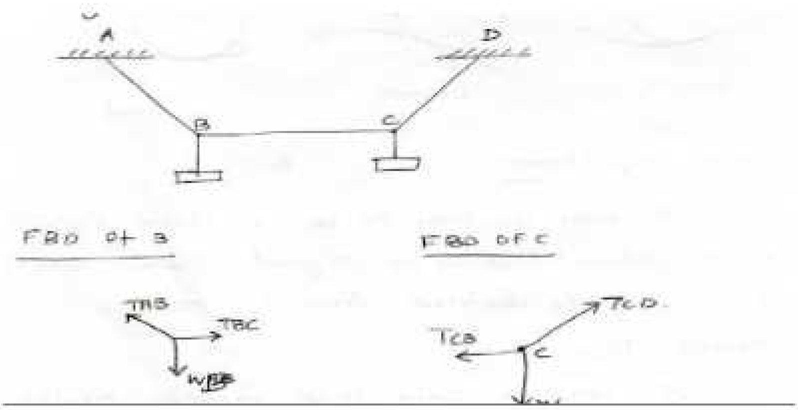
A body is said to be unstable equilibrium it does not return back to its original position and heals farther away after slightly displaced from its position of rest.

Natural Equilibrium:

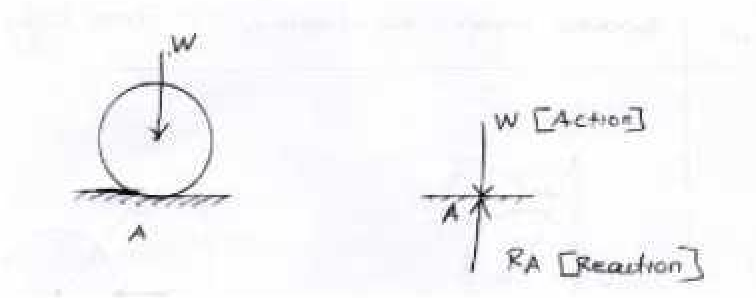
A body is said to be in natural equilibrium it in occupies a new position (also remain at rest) after slightly displaced from its position of rest.

⇒Free body Diagram:

It is a sketch of the particle which represents it as being isolated from its surroundings. It represents all the forces acting on it.



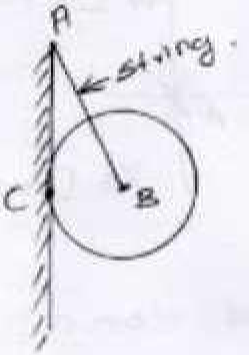
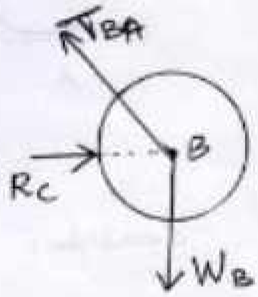
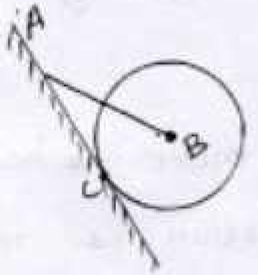
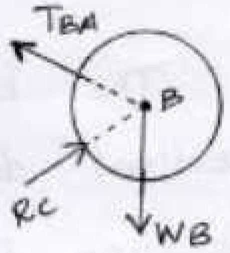
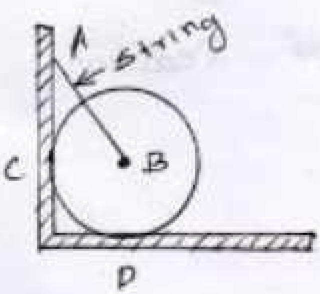
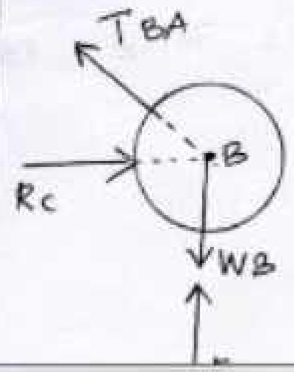
⇒Action and Reaction:



Consider a Ball placed on a horizontal surface shown in fig. The self weight of the ball (w) is acting vertically downwards through its centre of gravity. This force is called Action.

The ball can move horizontally, but its vertical downward motion is resisted due to resisting force developed at support (here, at the point of contact A) Acting vertically upwards. This force is called reaction.

⇒Free body diagram:

Sl NO	Bodies under equilibrium	Free body diagram.
1.		
2		
3		

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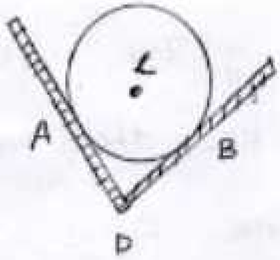

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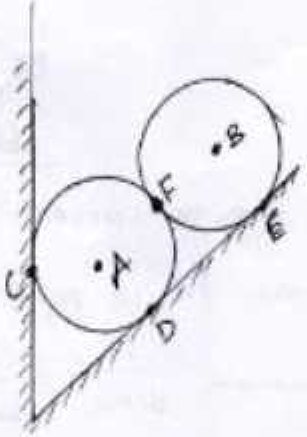
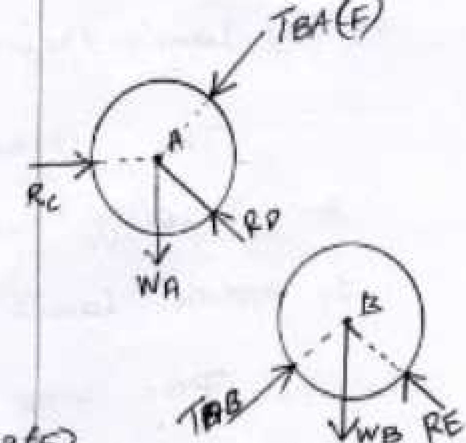
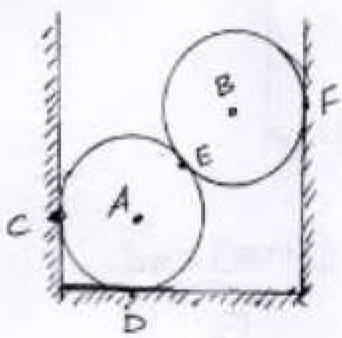
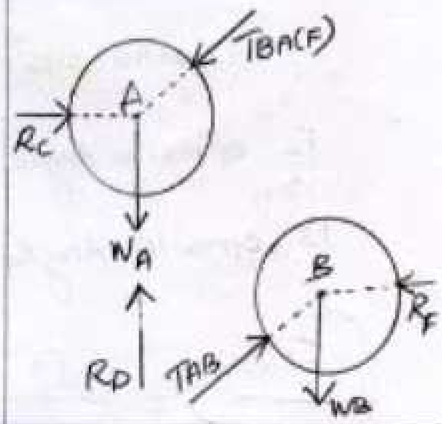




TBA(F)=TAB(F)

6





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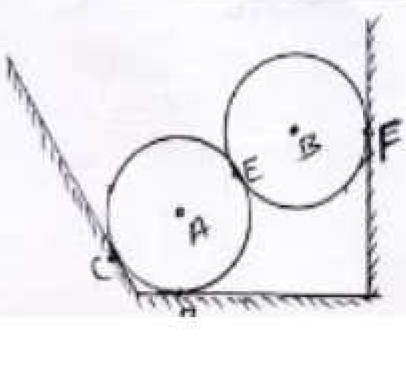
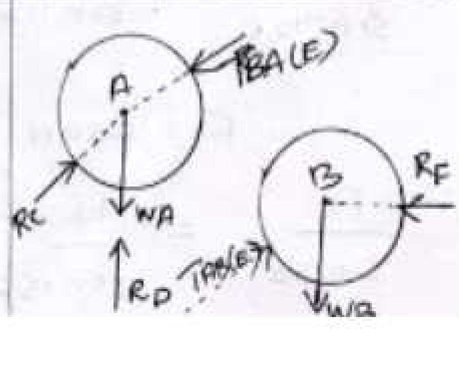
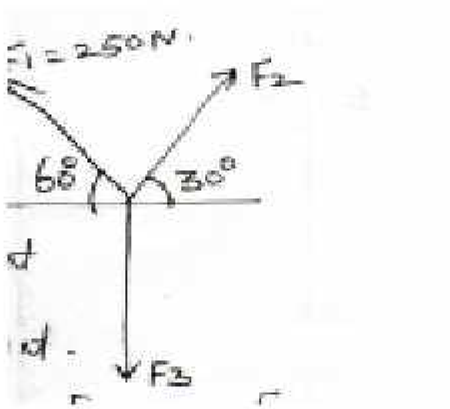




Problems:

1. The force shown in fig. is acting on a particle and keeps the particle in equilibrium. Then magnitude of force F1 is 250 N. Find the magnitude of forces F2 and F3.



Soln:

1. By Lami's theorem

The three concurrent force acting outwards from a point. So applied Lami's theorem. $\frac{F1}{sin\alpha} = \frac{F2}{sin\beta} = \frac{F3}{sin\gamma}$

First find the opposite angle of F1& F2&f3

F1 opposite angle

30+90=120°

F2 opposite angle (90+60)=150°

F3 opposite angle (180-[60+30]=90°

$\frac{F1}{sin120} = \frac{F2}{sin150} = \frac{F3}{sin90}$

F1=250 N

$\frac{F1}{sin120} = \frac{F2}{sin150}$

$$\frac{250}{sin120} = \frac{F2}{sin150}$$

F2=144.33 N

$$\frac{F1}{sin120} = \frac{F3}{sin90}$$

F3=288.67 N

2) By Equations of Equilibrium

$\sum$FH=0 $\sum$FV=0 F3 is No horizontal force

1) $\sum$FH= F2 cos30-F1 cos60=0

F2 cos30-250 cos60=0

$$F2=\frac{250cos60}{cos30}$$

F2=144.33N

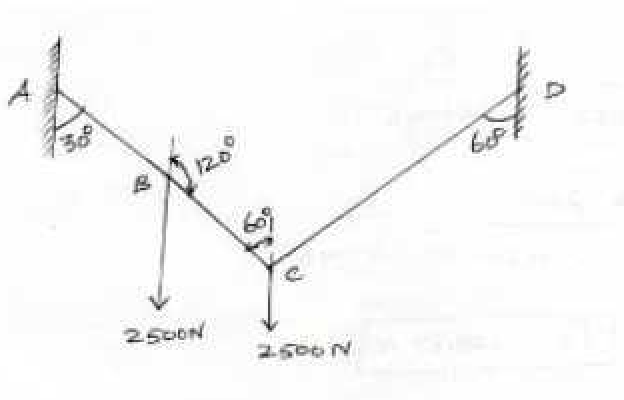
$\sum$FH= F1 sin 30+ F2 sin 60-F3=0

250sin30+144.33 sin 60-F3=0

250 sin30+144.33 sin 60 F3

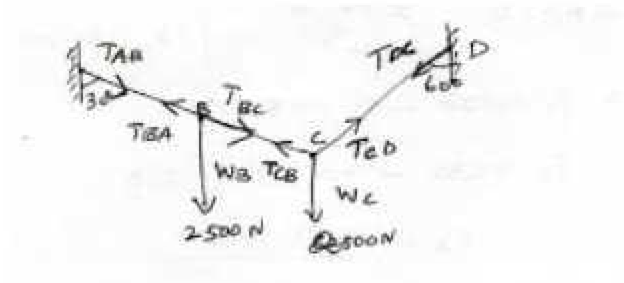
F3=228.67N

2. Two equal loads of 2500N are supported by a flexible string ABCD at points A &D. Find the tension in the portions AB,BC & CD of string.



Soln:

Free body diagram



$T_{AB} = T_{BA}$

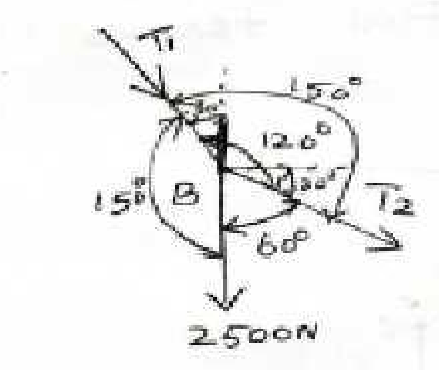
III ly$T_{BC} = T_{CB}$&$T_{CD} = T_{DC}$

⇒Let the tension in AB/BC &CD be T_1,T_2 & T_3 respectively

⇒ Let us split up the string

ABCD into two parts.

Consider A and B



By Lami's Theorem

$$\frac{TBA}{sin60} = \frac{TBC}{sin150} = \frac{Z500}{sin150}$$

$\frac{TBA}{sin60} = \frac{2500}{sin150}$

$T_{BA} = \frac{2500}{sin150} \times sin60$

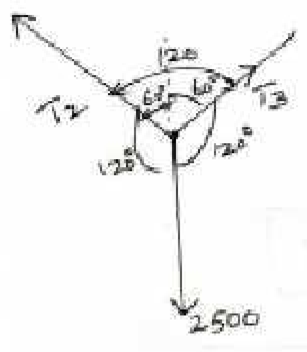
$T1 = 4330.13\ N$

$\frac{T_{CB}}{sin150} = \frac{2500}{sin150}$

$$T2 = \frac{2500}{sin150} \times sin150$$

$$T2 = 2500\ N$$

Consider a point c

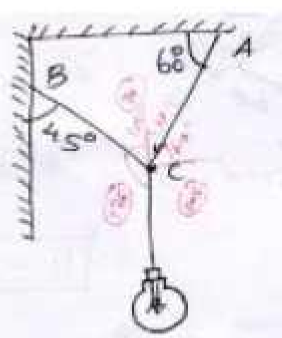


$$\frac{T2}{sin120} = \frac{T3}{sin120} = \frac{2500}{sin120}$$

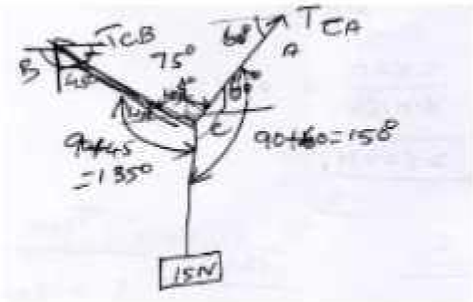
$$\frac{T3}{sin120} = \frac{2500}{sin120}$$

$$T3 = 2500\ N$$

3. An electric lamp weighting 15 N hangs from a point c, by S two strings AC and Bc as shown in fig. find tensions in string Ac & BC



Soln:



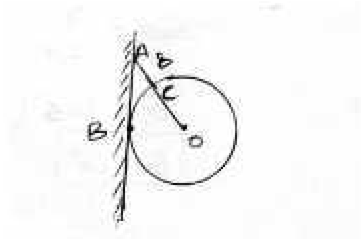
By Lami's theorem

$$\frac{TCB}{sin150} = \frac{TCA}{sin135} = \frac{15}{sin75}$$

$$\frac{TCB}{sin150} = \frac{15}{sin75} \; TCB = 7.76\,N$$

$$\frac{TCA}{sin135} = \frac{15}{sin75} = TCA = 7.76N$$

4. A smooth sphere w is supported by a string fastened to a point A on the smooth vertical wall, the other end is in contact with point b, on the wall as shown in fig. If the length of the string AC is equal to the radius of the sphere, find the tension in the string & reaction of the wall.



Given:

Radius of sphere OB=OC=radius length of string, AC= radius of sphere=r , weight of sphere =w

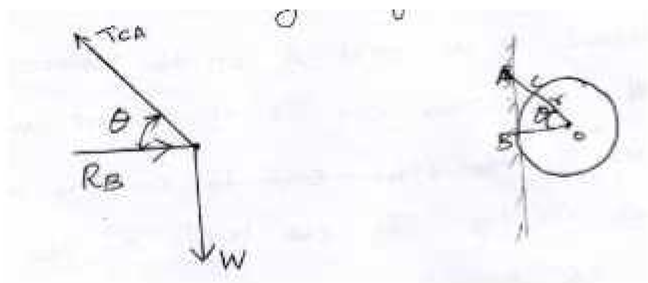
To find :

1. Tension in string

2. Reaction of the wall

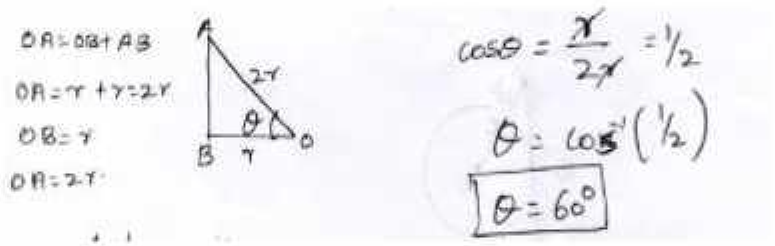
Soln:

Free body diagram



Find the angle b/w T_{CA}& R_B

From right angle triangle AOB



Apply $\sum FH=0$

R_B-T_{CA} cos 60 = 0-------------- (1)

$\sum FV=0$

T_{CA} sin 60- w=0-------------- (2)

T_{CA} sin60=w

$T_{CA}=\frac{w}{sin60}$

T_{CA}= 1.155w

T_{CA} value sub in Eqn (1)

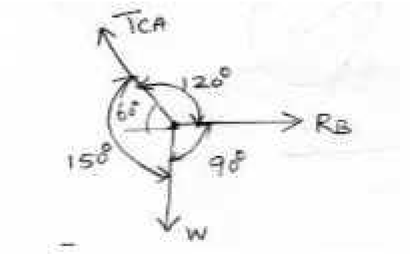
R_B-T_{CA} cos 60 = 0-------------- (1)

R_B-1.155w cos60=0

$R_B = 0.577w$

Other method

By Lami's theorem

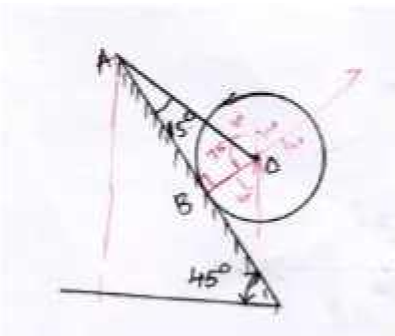


$$\frac{TCA}{sin90} = \frac{RB}{sin150} = \frac{w}{sin120}$$

$\frac{TCA}{sin90} = \frac{w}{sin120} \Rightarrow R_B = \frac{w}{sin120} \times sin150$

$R_B = 0.577\ W$

5. String AO holds a smooth sphere on an inclined plane ABC as shown in fig, the weight of the sphere is 1000 N and the plane is smooth. Calculate the tension in the string and the reaction at the point of contact B.



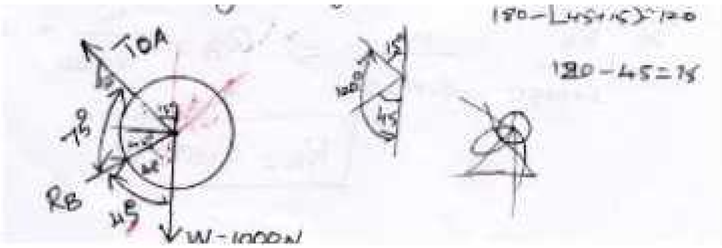
Given:

Weight of sphere W 1000N

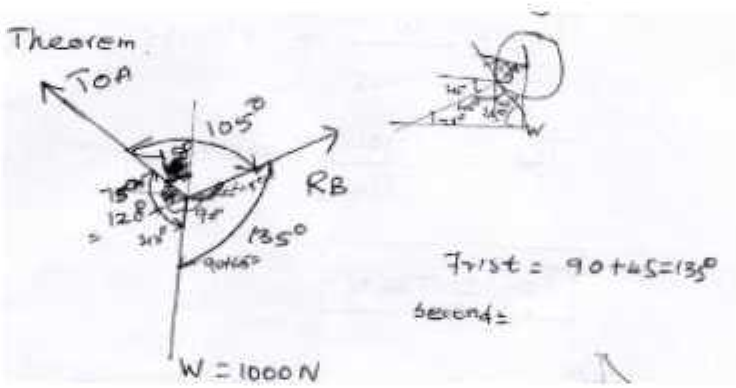
To find: Tension in string 7 Reaction

Soln:

Free body diagram



No of force is 3, so by using Lami's theorem



In right angled triangle OAB

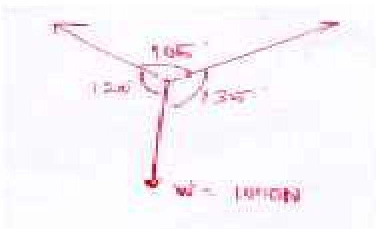
$\angle OAB + \angle ABO + \angle BOA = 180°$ $\quad\quad\quad$ $\angle OAB = 15°$

$15+90+\angle BOA = 180°$ $\quad\quad\quad$ $\angle ABO = 90°$

$BOA=180-(15+90)$

$\angle BOA = 75°$

Apply Lami's Eqn



$$\frac{TOA}{sin135} = \frac{RB}{sin120} = \frac{w}{sin105}$$

$$\therefore \frac{TOA}{sin135} = \frac{w}{sin105}$$

$T_{OA} = \frac{w}{sin105} \times sin135$

$T_{OA} = \frac{1000}{sin105} \times sin135$

$T_{OA} = 732N$

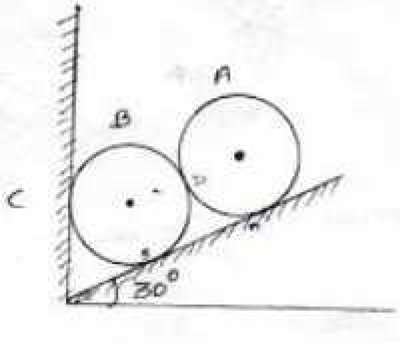
$III^{ly} \frac{R_B}{sin120} = \frac{w}{sin105}$

$$\frac{R_B}{sin120} = \frac{1000}{sin105}$$

$$R_B = \frac{1000}{sin105} \times sin120$$

$$R_B = 896.57N$$

6. Two identical rollers, each of weight 50 N, are supported by an inclined plane and a vertical wall as shown in fig. Find the reactions at the points of supports A,B and C. assume all the surface to be smooth.



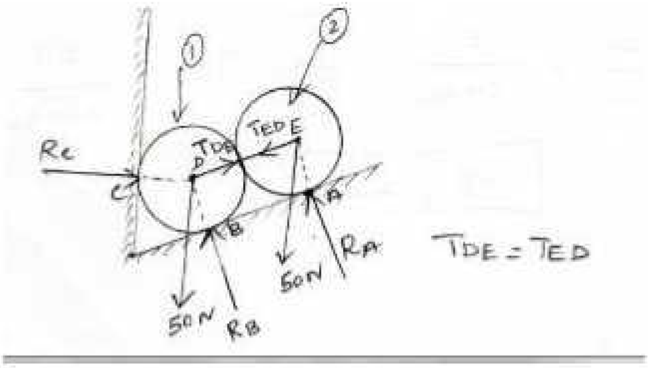
Given:

Weight of roller A & B = 50 N

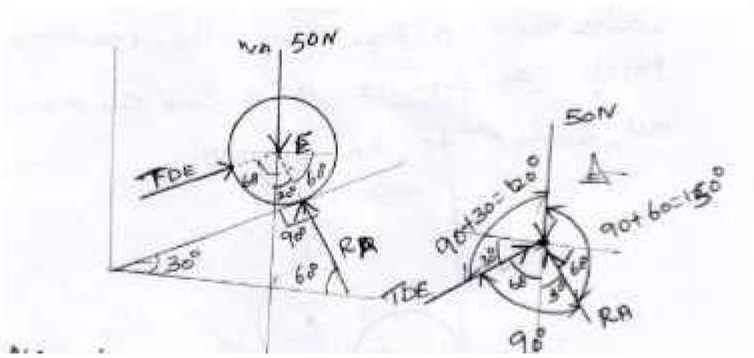
To find:

Reaction at the support A,B and C

Soln:



Free body diagram of roller 2



No of forces is three, apply Lami's Theorem

$$\frac{R_A}{sin120} = \frac{T_{DE}}{sin150} = \frac{50}{sin90}$$

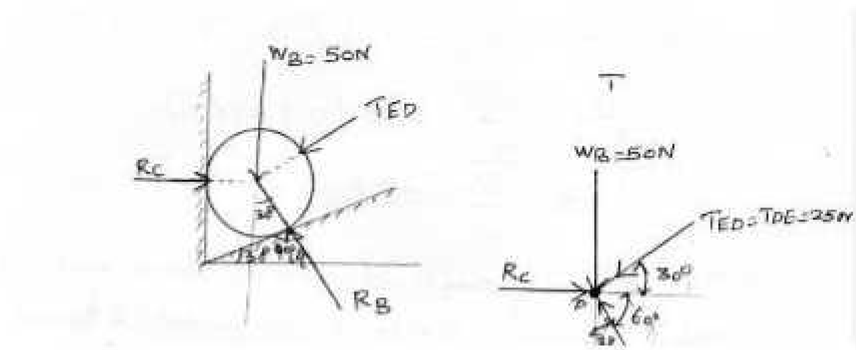
$$\frac{R_A}{sin120} = \frac{50}{sin90} \Rightarrow R_A = \frac{50}{sin90} \times sin120$$

$R_A = 43.3\ N$

$\frac{T_{DE}}{sin150} = \frac{50}{sin90} \rightarrow T_{DE} = \frac{50}{sin90} \times sin150$

$T_{DE} = 25N$

Free body diagram of roller 1



All forces acting at point D.

In equilibrium condition

$\sum FH=0\ \&\sum FH=0$

$\sum FH=0\ \rightarrow + \quad - \leftarrow$

R_C-TED cos 30-R_B cos 60 =0

R_C-25 cos 30-R_B cos 60 =0

R_C-21.65-0.5R_B =0-------- (1)

$\sum FH=0\ \downarrow - \ \uparrow +$

$R_B$sin60- TED sin 30-50 =0

R_B-sin60-25 sin30-50=0

R_B-sin 60 =25 sin 30+ 50=62.5

$R_B = \frac{62.5}{sin60}$

$R_B = 72.17$ N

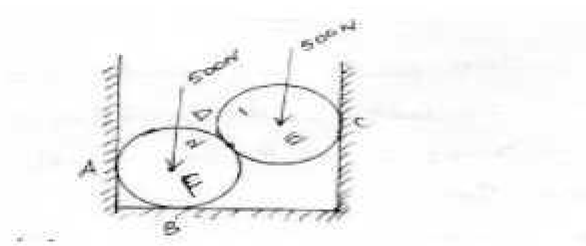
R_B value substitute Eqn(1)

$R_c - 21.65 - (0.5\times 72.17) = 0$

$R_c = 21.65 + (0.5\times 72.17)$

$R_C = 57.73\ N$

7. Two spheres each of weight 500 N and of radius 100mm rest in a horizontal channel of width of 360mm as shown in fig. find the reactions on the points of contact A ,B and C. Assume all the surface of contact are smooth.



Given data:

Weight of each Roller w = 500 N

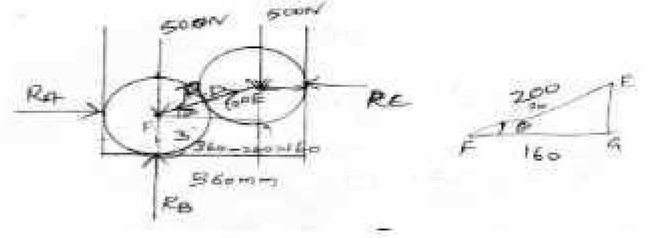
Width of channel = 360 mm

Radius of rollers r = 100mm

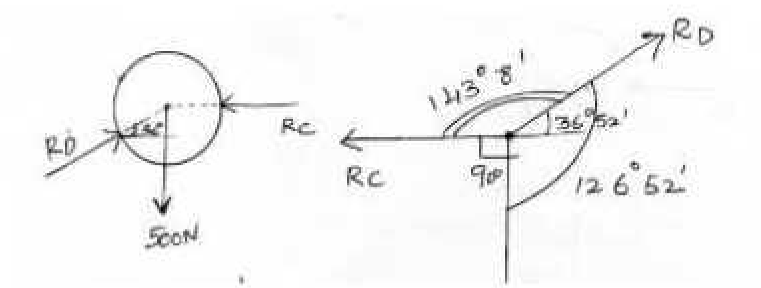
To find:

Reaction on the points of A,B & C.

Soln



Free body diagram of roller (1)



$\cos\theta = \frac{FG}{EF}$

$$\theta = cos^{-1}\left(\frac{FG}{EF}\right)$$

$$\theta = cos^{-1}\left(\frac{160}{200}\right)$$

$$\theta = 36°52'$$

By Lami's Theorem

$\frac{R_D}{sin90} = \frac{R_C}{sin126°52} = \frac{500}{sin143°8}$

$\frac{R_D}{sin90} = \frac{500}{sin143°8}$

$R_D = \frac{500}{sin143°8} \times sin90$

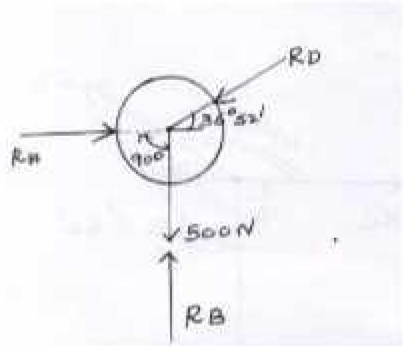
$R_D = 833.39\ N$

$\frac{R_C}{sin126°52} = \frac{500}{sin143°8}$

$R_C = \frac{500}{sin143°8} \times sin126°52'$

$R_C = 673.08N$

Free body diagram of roller(2)



$\sum FH =$ $R_A - R_D \cos 36°52'$=0

→− -← $R_A = R_D \cos 36°52'$

$R_A - 833.39 \times \cos 36°52'$

$R_A = 666.74\ N$

$\sum FV$ =0 ↓- ↑+

$R_B - R_D \sin 36°52' - 500$=0

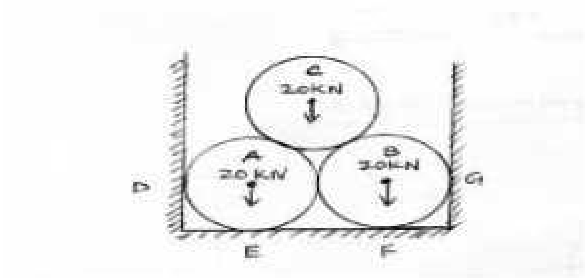
$R_B - 833.39 \sin 36°52'$-500=0

$R_B - 499.99 - 500$=0

$R_B = 999.99 N$

8. Three smooth pipes each weighting 20 KN and of diameter 60 cm are to be placed in a rectangular channel with horizontal base as shown in fig.

   Calculate the reactions at the points of contact b/w the pipes and b/w the channel and the pipes. Take width of the channel as 160 cm.



Given:

Weight of each pipe = $w_A = w_B = w_C = 20\ KN$

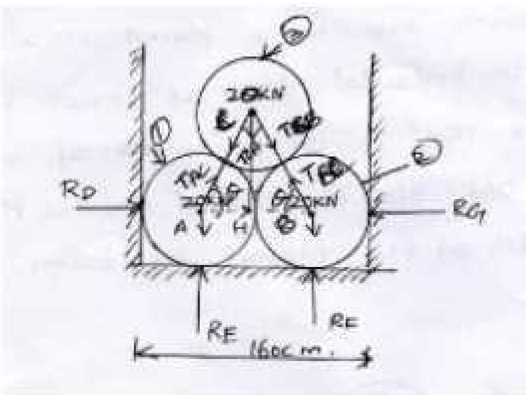
Diameter of each pipe = $D_A = D_B = D_C = 60cm$

Width of channel = 160 cm

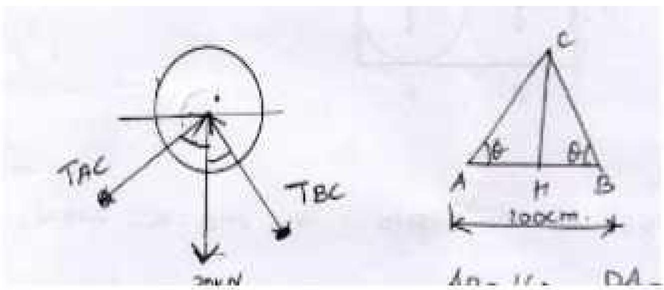
To find:

Reaction at the points : D,E,f,G

Soln:



Free body diagram of pipe 3



AB=160-DA-BG diameter

From triangle HAC AB=160-30-30 DA=Db=60cm

$\cos\theta = \frac{AH}{AC}$ AB 100cm

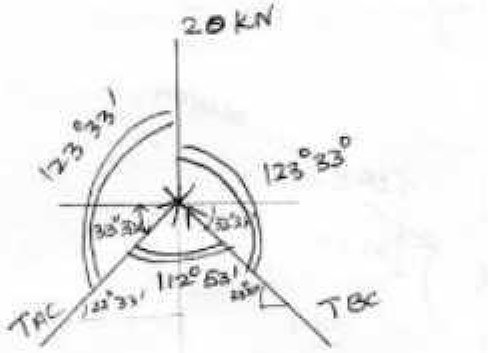
$\theta = cos^{-1}\left[\frac{AH}{AC}\right]$ AC BC BG DB/2

AC=BC=2×radius DA=BG=radius

$= cos^{-1}\left[\frac{50}{60}\right]$ AC=BC=2× 30

$\theta = 33°33'$ AC=BC=60 cm

AH=$\frac{AB}{2} = \frac{100}{2} = 50cm$



By Lami's theorem

$$\frac{T_{AC}}{sin123°33'} = \frac{T_{BC}}{sin123°33'} = \frac{20}{sin112°53'}$$

$$\frac{T_{AC}}{sin123°33'} = \frac{20}{sin112°53'}$$

$$T_{AC} = \frac{20}{sin112°53'} \times sin123°33'$$

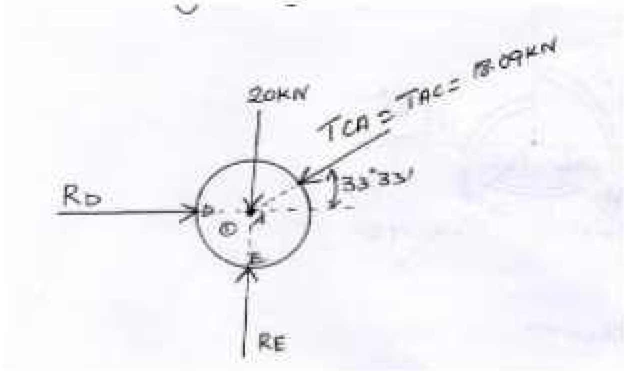
T_{AC} =18.09 KN

$$\frac{T_{BC}}{sin123°33'} = \frac{20}{sin112°53'}$$

$$T_{BC} = \frac{20}{sin112°53'} \times sin123°33'$$

$$T_{BC} = 18.09KN$$

Free body diagram of pipe(1)



$\sum FH = 0 \quad \rightarrow + \quad - \leftarrow$

$$R_D - T_{CA}\cos 33°33'=0$$

$$R_D = 18.09\cos 33°33' = 0$$

$$R_D - 15.07 = 0 \qquad T_{CA} = T_{Ac}$$

$$R_D = 15.07\ KN$$

$\sum FV = 0 \ \downarrow - \ \uparrow +$

$$R_E - 20 - T_{CA}\sin 33°33'=0$$

$$R_E - 20 - 18.09\sin 33°33'=0$$

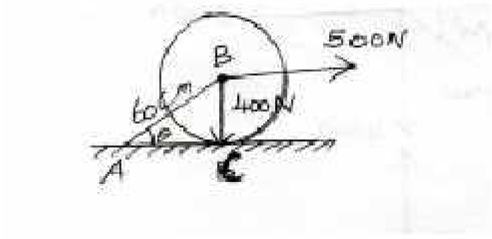
$$R_E - 29.99 = 0$$

$$R_E = 29.99KN$$

Similarly the free body diagram of pipe (2) is analyzed for pipe (1)

$$\therefore R_E = 29.99KN \ \ \& R_D = 15.07\ KN$$

9. A circular roller of radius 20 cm and of weight 400 N resets on a smooth horizontal surface and is held in position by an inclined bar AB of length 60 cm as shown in fig. a horizontal force of 500 n is acting at b. Find the Tension in bar AB and the reaction at C.



Given:

Radius of circular roller r=20 cm

Weight of roller w = 400 N

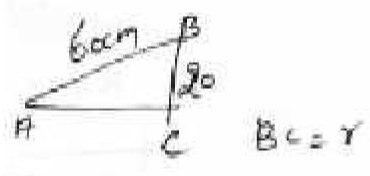
Horizontal force =500 N

To find:

1) Tension in AB
2) Reaction at C.

Soln:

From triangle ABC

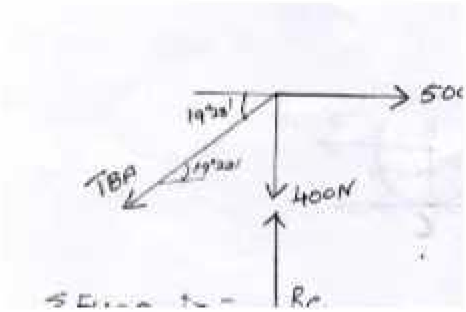


$\sin\theta = \frac{BC}{AB} \Rightarrow \theta = sin^{-1}\left(\frac{BC}{AB}\right)$

$\theta = sin^{-1}\left(\frac{20}{60}\right)$

$\theta = 19°28'$

Free body diagram



$\sum FH = 0 \quad \rightarrow + \quad - \leftarrow$

$\sum FH = -T_{BA} cos 19°28' + 500 = 0$

$-T_{BA} = cos 19°28' = -500$

$-T_{BA} = \frac{-500}{cos 19°28}$

$-T_{BA} = 530.31N$

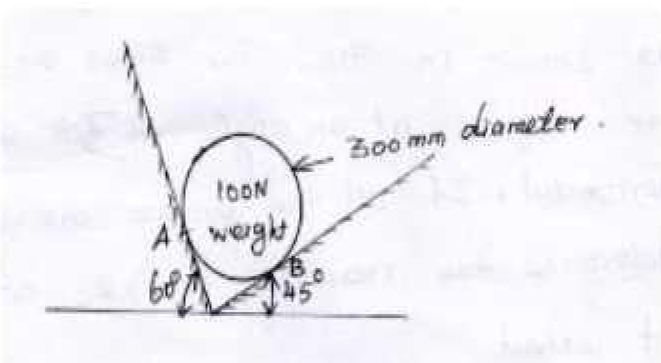
$\sum FV = 0 \ \downarrow - \ \uparrow +$

$-400 + R_c - T_{BA} \sin 19°28' = 0$

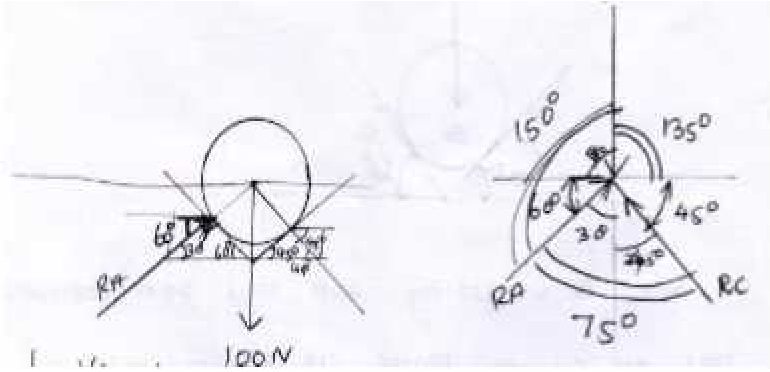
$R_c = T_{BA} sin 19°28' + 400 = 530.31 \times sin 19°28' + 400$

$R_C - 576.62N$

10.Determine the reaction at a and B



Soln:



By Lami's theorem

$$\frac{R_A}{sin135} = \frac{R_C}{sin150} = \frac{100}{sin75}$$

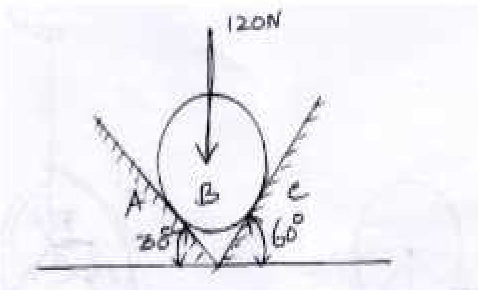
$$\frac{R_A}{sin135} = \frac{100}{sin75} \Rightarrow R_A = \frac{100}{sin75} \times sin135$$

$$R_A = 73.2N$$

$$\frac{R_C}{sin135} = \frac{100}{sin75} \Rightarrow R_C \frac{100}{sin75} \times sin150$$

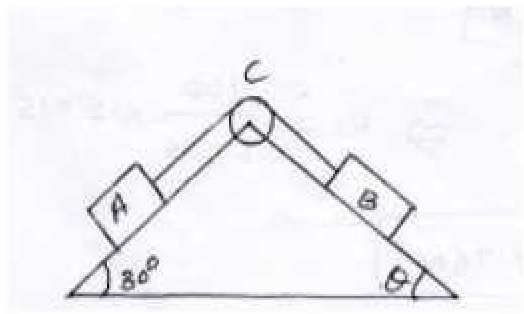
$$R_C = 51.76N$$

11.A Ball weighht120N in a right angle groove as shown in fig. The sides of the groove are inclined at an angle of $30°$ *and* $60°$to the horizontal. If all the surface are smooth,then determine the reaction RA&RC at the point of contact



12. A and B weighting 40 N and 30 N respectively rest on smooth planes as shown in fig. They are connected by a weight less chord passing over

a friction less pulley. Determine the angleθ &the tension in the chord for equilibrium. Also find the reaction of Block A&B



Given:

WA=40N

WB=30N

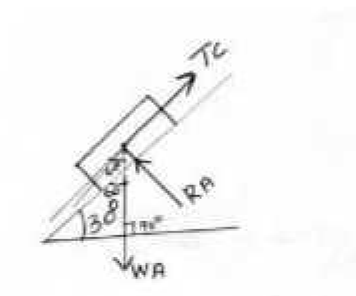
To find

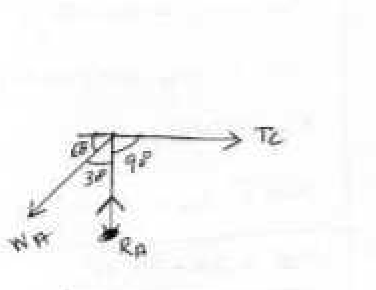
1.θ

2. Reaction of Block A&B

Soln

F B D of Block A





$\sum FH = 0$

TC-WA cos 60=0

TC = WA cos 60

TC = 40 cos 60

TC = 20N

$\sum FV = 0$

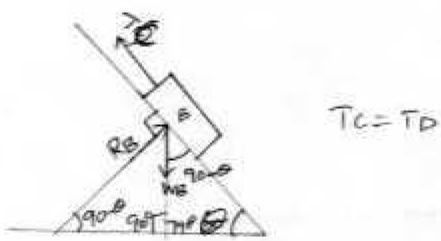
+RA - WA sin 60=0

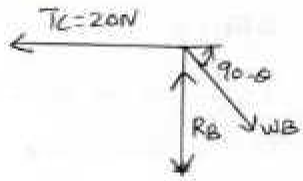
+RA = WA sin 60

RA = 40 sin 60

RA = 34.64N

FBD of Block B





$\sum FH = 0$

$-T_C + W_B \times \cos(90-\theta) = 0$

$-T_C = -W_B \cos(90-\theta)$

$\sum FV = 0$

$R_B - W_B \sin(90-\theta) = 0$

$R_B = W_B \sin(90-\theta)$

$T_C = W_B \sin\theta$

$20 = 30\sin\theta$

$\sin\theta = \frac{20}{30}$

$\theta = sin^{-1}\left(\frac{20}{30}\right)$

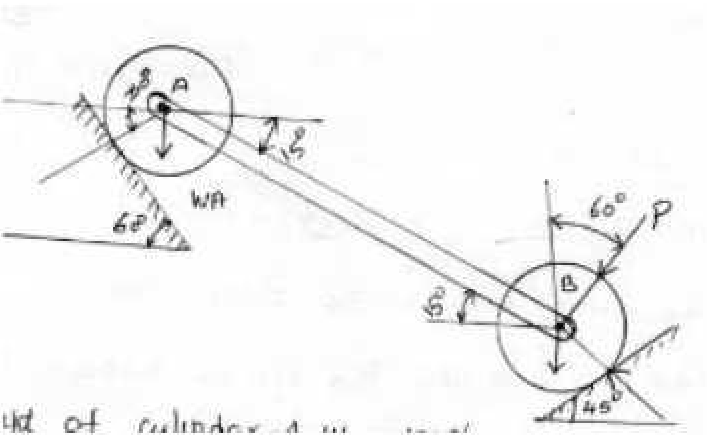
$\theta = 41°48'$

$R_B = W_B \cos\theta$

$R_B = 30 \times cos41°48'$

$R_B = 22.36N$

13. The following fig shows cylinders, A of weight 100 N and B weight 50 N resting on smooth inclined planes. They are connected by a bar of negligible weight hinged to each cylinder at their geometric centers by smooth pins. Find the force P, that can hold the system in the given position.



Given:

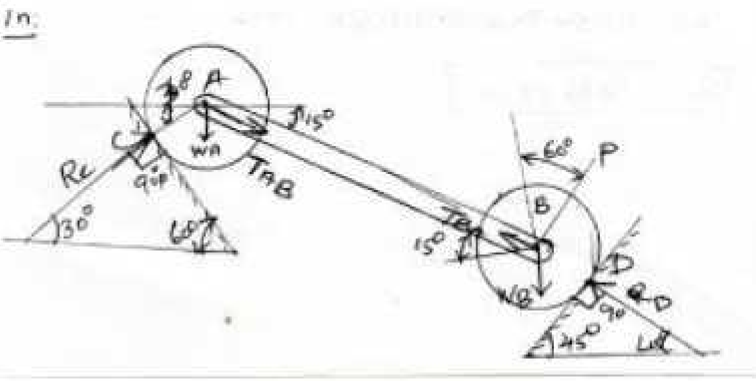
Weight of cylinder A $W_A = 100\ N$

Weight of cylinder B $W_B = 50\ N$

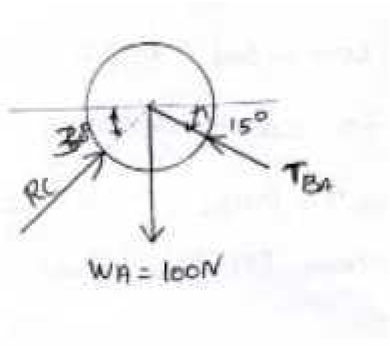
To find:

Force 'P'

Soln:



Free body diagram for cylinder A



$\sum$FH = 0

$\sum$FH $= R_C cos30 - -T_{BA} cos15$=0

$$R_C cos3 = T_{BA} cos15$$

$$R_C = \frac{T_{BA} cos15}{cos30}$$

$$R_C = 1.115 T_{BA} \text{----------------} (1)$$

$\sum$FV = 0

$\sum$FV - $W_A + T_{BA} sin15 + R_C sin30 = 0$

$T_{BA}sin15 + R_C sin30 = W_A = 0$

$T_{BA}sin15 + 1.115\, T_{BA}sin30 = 0$

$T_{BA}[sin15 + 1.115sin30] = 100$

$T_{BA} = \frac{100}{sin15+1.115sin30}$

$T_{BA} = 122.5\, N$

$R_C = 1.115 \times T_{BA} = 1.115 \times 122.5 R_C = 136.58N$

<u>Free body diagram of cylinder B</u>

$\sum FH = 0 \quad \rightarrow + \quad - \leftarrow$

$T_{AB}cos15 - Pcos30 - R_D 45 = 0$

$122.5\, cos15° - Pcos30 - R_D cos45 = 0$

$118.32 - 0.866P - 0.707R_D = 0$

$-0.866P - 0.707R_D = -118.32$-------- (1)

$\sum FV = 0 \downarrow - \uparrow +$

$-W_B\_Psin30 - T_{AB}sin15 + R_D sin45 = 0$

$-50 - P \times sin30 - 122.5sin15 + R_D sin45 = 0$

$-50 - 0.5P - 31.7 + 0.707\, R_D = 0$

$-81.7 - 0.5P + 0.707\, R_D = 0$

$0.707\, R_D - 0.5P = 81.7$

$-0.5P + 0.707R_D = 81.7$------------------ (2)

$(1) \Rightarrow -0.866P - 0.707R_D = -118.32$

$(2) \Rightarrow \quad -0.5P + 0.707R_D \quad = 81.7$

-0.366P = -36.62

$P = \frac{36.62}{0.366}$

P=100 N

Substitute in (1)

$-0.866P - 0.707R_D = -118.32$

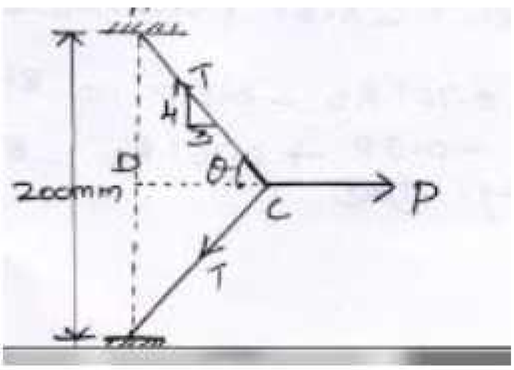
$-86.6 - 0.707 \times R_D = -118.32$

$-0.707R_D = -118.32 + 86.6 = -32$

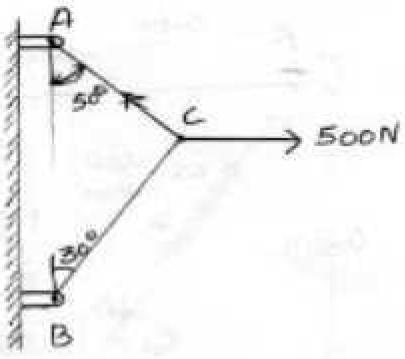
$R_D = \frac{-32}{-0.707}$

$R_D = 45.26N$

14. A rubber band has an unstretuched length of 200mm.It is pulled until its length is 250mm. as shown in fig. the horizontal force P is 1.75 n. what is the tension in the band (HW)



15. Two cables are tied together at c and are loaded as shown in fig below. Determine the tension in the cable AC and BC



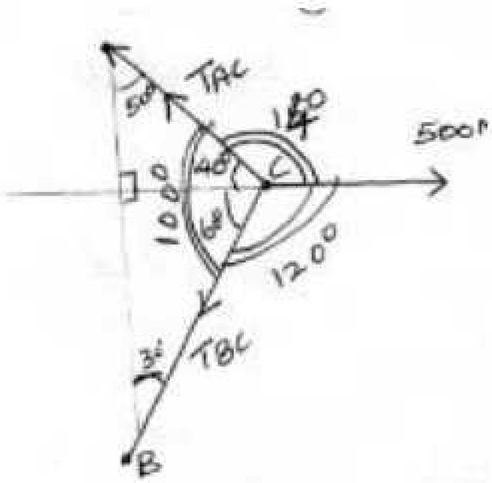
Given:

Force on C=500 N

To find: Tension of cable AC & Bc

Soln:

Free body Diagram



By using Lami's Theorem

$$\frac{T_{AC}}{sin120} = \frac{T_{BC}}{sin140} = \frac{500}{sin100}$$

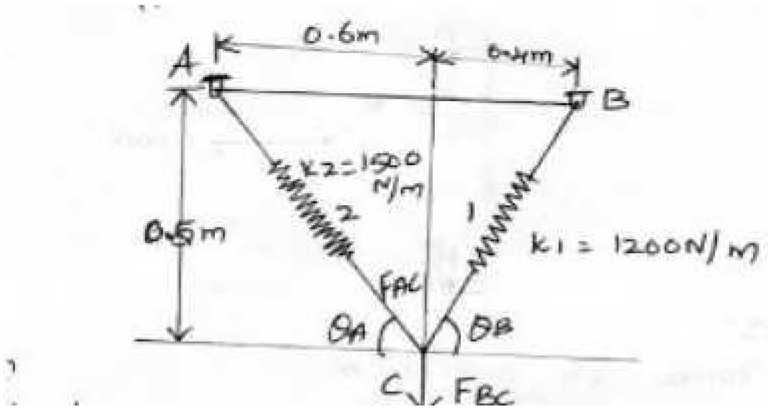
$$\frac{T_{AC}}{sin120} = \frac{500}{sin100} \Rightarrow T_{AC} = \frac{500}{sin100} \times sin120$$

$$T_{AC} = 439.69N$$

$$\frac{T_{BC}}{sin140} = \frac{500}{sin100} \Rightarrow T_{BC} \frac{500}{sin100} \times sin140$$

$$T_{BC} = 326.35N$$

16.A 30kg block is suspended by two spring having stiffness as shown. Determine the instructed length of each spring after the block is removed.

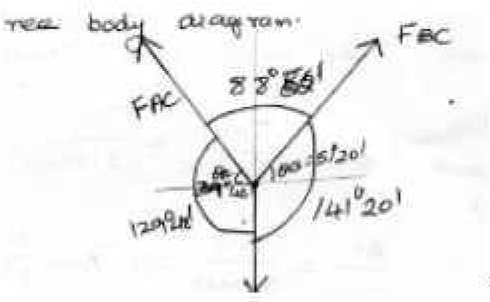


Unknown

Length of each spring L1&L2

Soln:

Free body diagram



$$= tan\theta_B = \frac{0.5}{0.4}$$

$$\theta_B = 51°20'$$

$$= tan\theta_A = \frac{0.5}{0.6}$$

$$\theta_A = 39°48'$$

F=30 kg=30× 9.81

=294.3N

By Lami's theorem $\frac{F_{AC}}{sin141°20'} = \frac{F_{BC}}{sin129°48'} = \frac{294.3}{sin88°52'}$

$$\frac{F_{AC}}{sin141°20'} = \frac{294.3}{sin88°52'} \Rightarrow F_{AC} = \frac{294.3}{sin88°521} \times sin141°20'$$

$$F_{AC} = 183.91N$$

$$\frac{F_{BC}}{sin129°48'} = \frac{294.3}{sin88°52'}$$

$$F_{BC} = \frac{294.3}{sin88°52'} \times sin129°48'$$

$$F_{BC} = 226.15N$$

Stiffness $= \frac{Force}{Deflection}$

$$k_2 = \frac{F_{AC}}{\delta_2}$$

$$1500 \frac{183.91}{\delta_2}$$

$$\delta_2 = 0.122m$$

$$k_1 = \frac{F_{BC}}{\delta_1}$$

$$1200 = \frac{226.15}{1}$$

$$\delta_2 = 0.188m$$

$$L_1 = \sqrt{(0.4)^2 + (0.5)^2} = 0.6403m$$

$$L_2 = \sqrt{(0.6)^2 + (0.5)^2} = 0.7810m$$

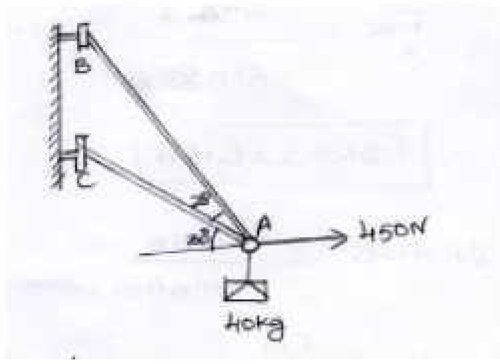
$$l_1 = L_1 - \delta_1 = 0.6403 - 0.188$$

$$l_1 = 0.452m$$

$$l_2 = L_2 - \delta_2 = 0.7810 - 0.122$$

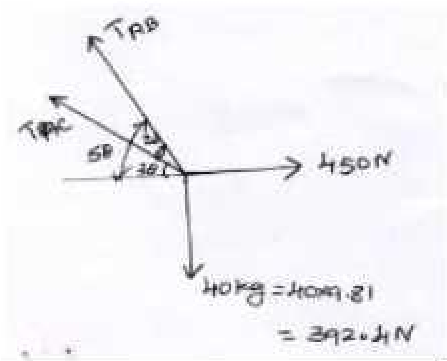
$l_2 = 0.728m$

17.Determine th3e tension in cables AB and AC required to hold the 40 kg crate shown in fig. below.



To find: T_{AB}&T_{AC}

Soln:



ΣFH = 0 $\quad$ >+ $\quad$ - <

450-$T_{AB}cos50 - T_{AC}cos30 = 0$

$T_{AB}cos50 + T_{AC}cos30 = 450$

$0.64T_{AB} + 0.86T_{AC} = 450$----------(1)

ΣFV =0 ↓- ↑+

$-392.4 + T_{AB}sin50 + \ T_{AC}sin30 = 0$

$T_{AB}sin50 + \ T_{AC}sin30 = 392.4$

$0.76T_{AB} + 0.5T_{AC} = 392.4$---------- (2)

Solving Eq(1)&(2)

(1) $\rightarrow 0.64T_{AB} + 0.86T_{AC} = 450$

(2)$\times$ 1.72 $\Rightarrow 0.76T_{AB} + 0.85T_{AC} = 392.4$

$-0.46T_{AB} \qquad = -224.92$

$$T_{AB} = \frac{224.92}{0.46}$$

$$T_{AB} = 488.95N$$

$T_{AB}\ sub\ in\ eqn(1)$

$0.64 \times 488.95 + 0.86T_{AC} = 450$

$$T_{AC} = 159.38N$$

Chapter-4

Forces in space –Resultant and Equilibrium of particles in Three Dimensions [Vector approach]

Quantities:

Physical Quantities are

i) Scalar quantity

ii) Vector quantity

Scalar Quantity:

Scalar quantity are those which are completely defined by their magnitude only.

Ex. 2kg of mass

25°C of temperature

10 m/s acceleration

Vector Quantity

The Quantity which are defined by their magnitude and direction is known as vector quantity.

Ex. 10 n force acting vertically downward direction

9.81 m/s^2acceleration directed towards is centre of the earth.

Types of Vectors

1. Free Vector
2. Fixed Vector
3. Sliding Vector

4. Unit Vector
5. Zero(or) Null Vector
6. Equal Vector
7. Like Vector

1. **Free Vector:**

If the vector may act at any point in space maintaining some magnitude and direction with no specific point of action is called Free vector.

2. **Fixed Vector:**

The vector whose point of action is same is called Fixed vector.

3. **Sliding vector:**

The vector may be applied at any point along its Line of action is called sliding vector.

4. **Unit vector:**

A vector whose magnitude is unity is called unit vector.

AB, n= $\frac{\overline{AB}}{|\vec{A}||\vec{B}|}$

5. **Zero (or) Null vector**

It is defind as the vector whose magnitude is zero.

6. **Equal vector**

Those vector which are similar to each other but have same magnitude and direction in same and equal is called equal vector.

7. **Like vector:**

These vector each are slimier to each other and have same direction and unequal magnitude is called like vector.

8. **Vector Addition:**

We Law of vector addition are

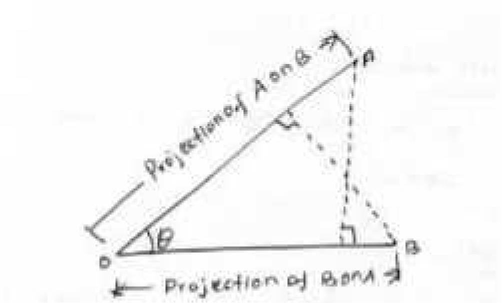
A+B=B+A [commutative Law]

A+ [B+C] =A+B+C [associative Law]

Vector Product

1. Scaler product(or) dot product
2. Vector product(or) cross product

1. **Scaler product (or) dot product:**



$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\,cos\theta$$

When the angle b/w two vector $\vec{A}\&\vec{B}$

$$cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}|.|\vec{B}|}$$

(i) When $\theta = 0°$

Then $\vec{A}.\vec{B} = |\vec{A}||\vec{B}|$

That is the two vector are in same direction.

(ii) when $\theta = 90° \vec{A}.\vec{B} = 0$

so the vectors are perpendicular

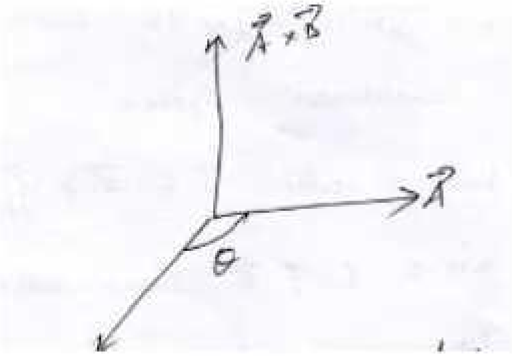
(iii) $\vec{A}.\vec{B} = |A|$

When the projection B or A

If the angle b/w the A & B given

$$\cos\theta = \frac{\vec{A}.\vec{B}}{AB}$$

2. <u>Vector Product (or) cross product</u>



In forms of Rectangular component $A = A_{xi} + A_{yi} + A_{zk}$

$B = B_{xi} + B_{yi} + B_{zk}$

$$A \times B = \begin{vmatrix} i & j & k \\ Ax & Aj & Az \\ Bx & By & Bz \end{vmatrix}$$

The Angle b/w the vector is given by

$$\sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}$$

Dot product of force and displacement given workdone.

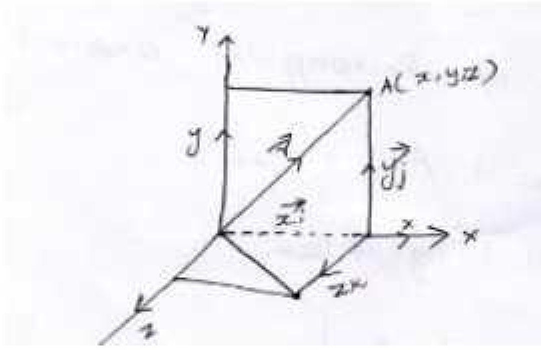
∴ Workdone=F_d

Position vector:

Position vector defines the position of points in any co-ordinate system.

Position vector $\vec{r} = \overline{x}\vec{\imath} + \overline{y}\vec{\jmath} + z\vec{k}$

Where $\vec{\imath}, \vec{\jmath}, \vec{k}$ – are unit vector



Magnitude $r = \vec{r} = \sqrt{x^2 + y^2 + z^2}$

Formula

Resultant vector $r = \vec{A} + \vec{B} + \vec{C}$

Unit vector to resultant vector $n = \frac{\vec{R}}{|\vec{R}|}$

$|R| = \sqrt{x^2 + y^2 + z^2}$

Magnitude $= \sqrt{x^2 + y^2 + z^2}$

Unit vector $n = \frac{\overrightarrow{AB}}{|AB|}$

$\overrightarrow{AB} = (x_2 - x_1)\vec{\imath} + (y_2 - y_1)\vec{\jmath} + (z_2 - z_1)\vec{k}$

$|AB| = \sqrt{x^2 + y^2 + z^2}$

Dot product vector $\vec{A}.\vec{B} = [\vec{\imath} + \vec{\jmath} + \vec{k}].[\vec{\imath} + \vec{\jmath} + \vec{k}]$

Angle b/w the vector $cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$

Cross Product vector $A \times B = \begin{vmatrix} i & j & k \\ x1 & y1 & z1 \\ x2 & y2 & z2 \end{vmatrix}$

Angle b/w the vector $sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}$

Problems:

1. Three vectors A, B, C are given as $A = 3\vec{\imath} + 2\vec{\jmath} + 4\vec{k}, B = 4\vec{\imath} - 2\vec{\jmath} + 6\vec{k}$

$C = 2\vec{\imath} - 3\vec{\jmath} - \vec{k}$, find

1. The resultant vector
2. A unit vector || er top resultant vector

Given:

$A = 3\vec{\imath} + 2\vec{\jmath} + 4\vec{k}$

$B = 4\vec{\imath} - 2\vec{\jmath} + 6\vec{k}$

$C = 2\vec{\imath} - 3\vec{\jmath} - \vec{k}$

To find:

1. The resultant vector
2. A unit vector || er top resultant vector

Solution:

1. The resultant vector

$R = \vec{A} + \vec{B} + \vec{C}$

$R = 3\vec{i} + 2\vec{j} + 4\vec{k} + 4\vec{i} - 2\vec{j} + 6\vec{k} + 2\vec{i} - 3\vec{j} - \vec{k}$

$R = 9\vec{i} - 3\vec{j} + 9\vec{k}$

2. A unit vector || er top resultant vector

Unit Vector $n = \frac{\vec{R}}{|\vec{R}|}$

$\vec{R} = 9\vec{i} - 3\vec{j} + 9\vec{k}$

$|R| = \sqrt{9^2 + (-3)^2 + 9^2} = \sqrt{81 + 9 + 81}$

$|R| = \sqrt{171}$

$R = 13.08$

$n = \frac{9\vec{i} - 3\vec{j} + 9\vec{k}}{13.08}$

$n = \frac{9}{13.08}\vec{i} - \frac{3}{13.08}\vec{j} + \frac{9}{13.08}\vec{k}$

Unit vector$n = 0.68i - 0.22j + 0.68k$

2.If $A = \vec{i} - \vec{j} - 2\vec{k}, B = 3\vec{i} + 2\vec{j} - 2\vec{k} C = 2\vec{i} + 3\vec{j} - 4\vec{k}$, find

$2A - 2B + 3C$ n terms of i, j, k and its magnitude.

Given :

$A = \vec{i} - \vec{j} - 2\vec{k}$

$B = 3\vec{i} + 2\vec{j} - 2\vec{k}$

$C = 2\vec{\imath} + 3\vec{\jmath} - 4\vec{k}$

To find:

$2A - 2B + 3C = ?$ magnitude

Solution:

$$2A - 2B + 3C = ?$$

$$2A = 2[\vec{\imath} - \vec{\jmath} - 2\vec{k}]$$

$$2A = 2\vec{\imath} - 2\vec{\jmath} - 4\vec{k}$$

$$2B = 2[3\vec{\imath} + 2\vec{\jmath} - 2\vec{k}]$$

$$2B = 6\vec{\imath} + 4\vec{\jmath} - 4\vec{k}$$

$$3C = 3[2\vec{\imath} + 3\vec{\jmath} - 4\vec{k}]$$

$$3C = 6\vec{\imath} + 9\vec{\jmath} - 12\vec{k}$$

$$2A - 2B + 3C = |2\vec{\imath} - 2\vec{\jmath} - 4\vec{k}| - |6\vec{\imath} + 4\vec{\jmath} - 4\vec{k}| + 6\vec{\imath} + 9\vec{\jmath} - 12\vec{k}$$

$$= 2\vec{\imath} - 2\vec{\jmath} - 4\vec{k} - 4\vec{\jmath} + 4\vec{k} + 9\vec{\jmath} - 12\vec{k}$$

$$2A - 2B + 3C = 2\vec{\imath} + 3\vec{\jmath} - 12\vec{k}$$

$$2A - 2B + 3C = \sqrt{2^2 + 3^2 + (-12)^2} = \sqrt{4 + 9 + 144}$$

$$|2A - 2B + 3C| = \sqrt{157} = 12.53$$

$$|2A - 2B + 3C| = 12.53$$

3. Find the unit vector along the line which ordinates at point (2,3,-2) and passes through the point(1,0,5)

Given:

At point (2,3,-2)=(x_1,y_1,z_1)(1,0,5) = (x_2,y_2,z_2)

To find:Unit vector 'n'=?

Soln:

Unit vector $n = \frac{AB}{|AB|}$

$AB = -1$

$A = x_1\vec{i} + y_1\vec{j} + z_1\vec{k}$

$B = x_2\vec{i} + y_2\vec{j} + z_2\vec{k}$

$$AB = (x_2 - x_1)\vec{i} + (y_2\text{-}y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

$$AB = (1 - 2)\vec{i} + (0\text{-}3)\vec{j} + (5 - (-2))\vec{k}$$

$$AB = -1\vec{i} - 3\vec{j} + 7\vec{k}$$

$$|AB| = \sqrt{(-1)^2 + (-3)^2 + (7)^2}$$

$$|AB| = \sqrt{1 + 9 + 49} = \sqrt{59}$$

$$|AB| = 7.68$$

$$n = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{-1\vec{i} - 3\vec{j} + 7\vec{k}}{7.68}$$

$$n = \frac{-1}{7.68}\vec{i} - \frac{3}{7.68}\vec{j} - \frac{7}{7.68}\vec{k}$$

$$\boldsymbol{ans}\ n = -0.13\vec{\imath} - 0.39\vec{\jmath} + 0.91\vec{k}$$

4. Find the dot product of two vector $A = 2\vec{\imath} - 6\vec{\jmath} - 3\vec{k}$,

   $B = 4\vec{\imath} + 3\vec{\jmath} - \vec{k}$ also find the angle b/w the angle b/w them.

   <u>Given Data:</u>

   $A = 2\vec{\imath} - 6\vec{\jmath} - 3\vec{k}$

   $b = 4\vec{\imath} + 3\vec{\jmath} - \vec{k}$

   <u>To find:</u>

   1. Dot product of Two vector
   2. Angle b/w the vector

<u>Soln:</u>

1. Dot product of two vector

   $A.B = \left[2\vec{\imath} - 6\vec{\jmath} - 3\vec{k}\right].\left[4\vec{\imath} + 3\vec{\jmath} - \vec{k}\right]$

   $= 2 \times 4 + [-6] \times 3 + [-3] \times [-1]$

   $= 8 - 18 + 3$

   $A.B = -7$

2. Angle b/w two vector

   $Cos\theta = \dfrac{A.B}{|A||B|}$

   $\left|\vec{A}\right| = \sqrt{(2)^2 + (-6)^2 + (-3)^2}$

   $\left|\vec{A}\right| = \sqrt{4 + 36 + 9}$

   $\left|\vec{A}\right| = 7$

   $\left|\vec{B}\right| = \sqrt{(4)^2 + (3)^2 + (-1)^2} = 16 + 9 + 1 = \sqrt{16 + 9 + 1}$

$|\vec{B}| = \sqrt{26}$

$$cos\theta = \frac{-7}{7\sqrt{26}} = \frac{-1}{\sqrt{26}}$$

$$\theta = cos^{-1}\left[\frac{-1}{\sqrt{26}}\right]$$

5. Find the cross product of vector $A = 2\vec{\imath} - 6\vec{\jmath} - 3\vec{k}, B = 4\vec{\imath} + 3\vec{\jmath} - \vec{k}$ and the angle b/w them.

Given:

$A = 2\vec{\imath} - 6\vec{\jmath} - 3\vec{k}$

$B = 4\vec{\imath} + 3\vec{\jmath} - \vec{k}$

To find:

1. Cross product of vector
2. Angle b/w hem two vector

Soln:

Cross product: $A \times B$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$$= \vec{\imath}[(-6 \times -1) - (-3 \times 3)] - \vec{\jmath}[(2 \times -1) - (4 \times -3)]$$

$$+\vec{k}[(2 \times 3) - (4 \times -6)$$

$$\vec{\imath}[6 + 9] - \vec{\jmath}[-2 + 12] + \vec{k}[6 + 24]$$

$$\vec{A} \times \vec{B} = 15\vec{\imath} - 10\vec{\jmath} + 30\vec{k}$$

$$sin\theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}$$

$$|\vec{A} \times \vec{B}| = \sqrt{15^2 + (-10)^2 + (30)^2}$$

$$\left|\vec{A} \times \vec{B}\right| = \sqrt{1225}$$

$$\left|\vec{A} \times \vec{B}\right| = 35$$

$$\left|\vec{A}\right| = \sqrt{2^2 + (-6)^2 + (-3)^2} = \sqrt{4 + 36 + 9}$$

$$\left|\vec{A}\right| = \sqrt{49}$$

$$\left|\vec{A}\right| = 7$$

$$\left|\vec{B}\right| = 4^2 + 3^2 + (-1)^2 = \sqrt{16 + 9 + 1} = \sqrt{26}$$

$$\left|\vec{B}\right| = \sqrt{26}$$

$$sin\theta = \frac{\left|\vec{A} \times \vec{B}\right|}{\left|\vec{A}\right|\left|\vec{B}\right|} = \frac{35}{7 \times \sqrt{26}}$$

$$sin\theta = \frac{5}{\sqrt{26}}$$

$$\theta = sin^{-1}\frac{5}{\sqrt{26}}$$

$$\theta = 78.69'$$

Formula used for three dimension force analysis

Force vector $\quad \vec{F} = \lambda \times F$

$$\lambda = \frac{\overrightarrow{OA}}{|OA|}$$

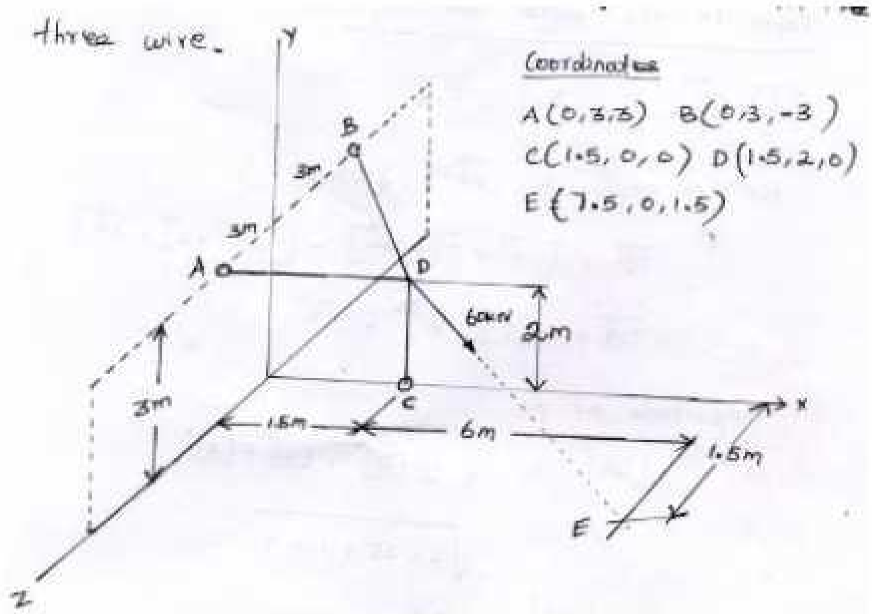
Magnitude $|OA| = \sqrt{(x)^2 + (y)^2 + (z)^2}$

$$\vec{R} = \overrightarrow{F_A} + \overrightarrow{F_B} + \overrightarrow{F_C}$$

$$R = \sqrt{{F_x}^2 + {F_y}^2 + {F_z}^2}$$

$$\theta = cos^{-1}\left(\frac{Rx}{R}\right)$$

1. In the figures shown, three wire jointed at D. The Two ends A and B are on the wall and the other end C is on the ground. The wire CD is vertical. A force of 60 KN is applied at 'D' and it passes through a point E on the ground as shown in fig. Find the forces in all the three wire.



Given:

$$F_{DE} = 60KN$$

To find:

$$F_{DA} = ?\ F_{DB} = ?\ F_{DC} = ?$$

Soln:

$OA = 0\vec{i} + 3\vec{j} + 3\vec{K}$

$OB = 0\vec{i} + 3\vec{j} - 3\vec{k}$

$OC = 1.5\vec{i} + 0\vec{j} + 0\vec{k}$

$OD = 1.5\vec{i} + 2\vec{j} + 0\vec{k}$

$OE = 7.5\vec{i} + 0\vec{j} + 1.5\vec{k}$

Force in the wire DA 'F_{DE}'

$\overrightarrow{F_{DA}} = \lambda_{DA} \times F_{DA}$

Position vector for $\overrightarrow{DA} = [\overrightarrow{OA} - \overrightarrow{OD}]$

$\overrightarrow{DA} = [0\vec{\imath} + 3\vec{\jmath} + 3\vec{k}] - [1.5\vec{\imath} + 2\vec{\jmath} + 0\vec{k}]$

$\overrightarrow{DA} = -1.5\vec{\imath} + \vec{\jmath} + 3\vec{k}$

Magnitude of DA

$|DA| = \sqrt{(-1.5)^2 + (1)^2 + (3)^2}$

$= \sqrt{2.25 + 1 + 9}$

$|DA| = 3.5$

$\lambda_{DA} = \frac{DA}{|DA|} = \frac{-1.5\vec{\imath} + \vec{\jmath} + 3\vec{k}}{3.5}$

$\lambda_{DA} = -0.428\vec{\imath} + 0.285\vec{\jmath} + 0.857\vec{k}$

$\overrightarrow{F_{DA}} = \lambda_{DA} \times F_{DA}$

$$\overrightarrow{F_{DA}} = -0.428\vec{\imath}F_{DA} + 0.285\vec{\jmath}F_{DA} + 0.857\vec{k}F_{DA}$$

$\overrightarrow{F_{DA}} = -0.428\vec{\imath}F_{DA} + 0.285\vec{\jmath}F_{DA} + 0.857\vec{k}F_{DA}$------- (1)

Force in the wire DB

Force from D to B coordinates

$\overrightarrow{F_{DB}} = \lambda_{DB} \times F_{DB}$

$$\lambda_{DB} = \frac{\overrightarrow{DB}}{|DB|}$$

$$\overrightarrow{DB} = [\overrightarrow{OB} - \overrightarrow{OD}]$$

$= [0\vec{\imath} + 3\vec{\jmath} - 3\vec{k}] - [1.5\vec{\imath} + 2\vec{\jmath} + 0\vec{k}]$

$\overrightarrow{DB} = -1.5\vec{\imath} + \vec{\jmath} - 3\vec{k}$

$$|\overrightarrow{DB}| = \sqrt{(1.5)^2 + (1)^2 + (-3)^2}$$

$$\left|\overrightarrow{DB}\right| = 3.5$$

$$\lambda_{DB} = \frac{DB}{|DB|} = \frac{-1.5\vec{i} + \vec{j} - 3\vec{k}}{3.5}$$

$\lambda_{DB} = 0.428\vec{i} + 0.285\vec{j} - 0.857\vec{k}$

$\overrightarrow{F_{DB}} = \lambda_{DB} \times F_{DB}$

$F_{DB} = -0.428F_{DB}\vec{i} + 0.285F_{DB}\vec{j} - 0.857F_{DB}\vec{k}$-------------- (2)

Force in the wire DC

Force from D to C coordinate

$\overrightarrow{F_{DC}} = \hat{\lambda}_{DC}.F_{DC}$

$\hat{\lambda}_{DB} = \frac{D\vec{C}}{|\overrightarrow{DC}|}$

$$\overrightarrow{DC} = [\overrightarrow{OC} - \overrightarrow{OD}]$$

$= \left[1.5\vec{i} + 0\vec{j} - 0\vec{k}\right] - [1.5\vec{i} + 2\vec{j} + 0\vec{k}]$

$\overrightarrow{DC} = 0\vec{i} - 2\vec{j} + 0\vec{k}$

$$\left|\overrightarrow{DC}\right| = \sqrt{(0)^2 + (-2)^2 + (0)^2} = \sqrt{4}$$

$$\left|\overrightarrow{DC}\right| = 2$$

$\lambda_{DC} = \frac{D\vec{B}}{|\overrightarrow{DB}|} = \frac{o\vec{i} - 2\vec{j} + 0\vec{k}}{2}$

$\hat{\lambda}_{DC} = \overrightarrow{-j}$

$$\overrightarrow{F_{DC}} = \lambda_{DC} \times F_{DC}$$

$\overrightarrow{F_{DC}} = -F_{DC}\vec{j}$------------------ (3)

Force in the wire DE

Force from D to E

$$\overrightarrow{F_{DE}} = \lambda_{DE} . F_{DE}$$

$$\lambda_{DE} = \frac{\overrightarrow{DE}}{|\overrightarrow{DE}|}$$

$$\overrightarrow{DE} = [\overrightarrow{OE} - \overrightarrow{OD}]$$

$$= [7.5\vec{\imath} + 0\vec{\jmath} - 1.5\vec{k}] - [1.5\vec{\imath} + 2\vec{\jmath} + 0\vec{k}]$$

$$\overrightarrow{DE} = 6\vec{\imath} - 2\vec{\jmath} + 1.5\vec{k}$$

$$|\overrightarrow{DE}| = \sqrt{(6)^2 + (-2)^2 + (1.5)^2} = \sqrt{36 + 4 + 2.25}$$

$$|\overrightarrow{DE}| = 6.5$$

$$\lambda_{DE} = 0.923\vec{\imath} + 0.307\vec{\jmath} + 0.23\vec{k}$$

$$\overrightarrow{F_{DE}} = \lambda_{DE} \times F_{DE}$$

$$F_{DE} = -0.923\vec{\imath} \times F_{DE} - 0.307\vec{\jmath} \times F_{DE} + 0.23\vec{k} \times F_{DE}$$

$$F_{DE} = -0.923 F_{DE}\vec{\imath} - 0.307 F_{DE}\vec{\jmath} + 0.23 F_{DE}\vec{k}$$

$$F_{DE} = 60KN$$

$$F_{DE} = -0.923 \times 60\vec{\imath} - 0.307 \times 60\vec{\jmath} + 0.23 \times 60\vec{k}$$

$$\overrightarrow{F_{DE}} = 55.38\vec{\imath} - 18.42\vec{\jmath} + 13.84\vec{k} \text{------------ (4)}$$

$$F_{DA} = -0.428\vec{\imath}F_{DA} + 0.285\vec{\jmath}F_{DA} + 0.857\vec{k}F_{DA}$$

$$F_{DB} = -0.428F_{DB}\vec{\imath} + 0.285F_{DB}\vec{\jmath} - 0.857F_{DB}\vec{k}$$

$$F_{DC} = -F_{DC}\vec{\jmath}$$

$$F_{DE} = 55.38\vec{\imath} - 18.42\vec{\jmath} + 13.84\vec{k}$$

Applying equilibrium condition

$\sum F_x = 0, F_{DA}.F_{DB}.F_{DC}.F_{DE} = 0$

$-0.428F_{DA} - 0.428F_{DB} + 55.38 = 0$

$$-0.428[F_{DA} + F_{DB}] + 55.38 = 0$$

$F_{DA} + F_{DB} = -55.38$

$F_{DA} + F_{DB} = \frac{-55.38}{-0.428}$

$F_{DA} + F_{DB} = 129.39$-------------- (5)

$\sum F_y = 0$

$-0.285F_{DA} + 0.285F_{DB} - F_{DC} - 18.42 = 0$

$-0.285F_{DA} + 0.285F_{DB} - F_{DC} = 18.42$

$\sum F_Z = 0$

$-0.857F_{DA} - 0.857F_{DB} + 18.42 = 0$

$0.857[F_{DA} - F_{DB}] + 13.84 = 0$

$F_{DA} - F_{DB} = \frac{-13.84}{0.857}$

$F_{DA} - F_{DB} = -16.15$------------- (7)

(5)⇒ $F_{DA} + F_{DB} = 129.39$

(7)⇒ $F_{DA} - F_{DB} = -16.15$

$2F_{DA} = 113.24$

$F_{DA} = \frac{113.24}{2}$

$F_{DA} = 56.62\ KN$

$(5) \Rightarrow F_{DA} + F_{DB} = 129.39$

$56.62 + F_{DB} = 129.39 \Rightarrow F_{DB} = 129.39 - 56.62$

$F_{DB} = 72.76\ KN$

$(6) \rightarrow -0.285 \times F_{DA} + 0.285 F_{DB} - F_{DC} = 18.42$

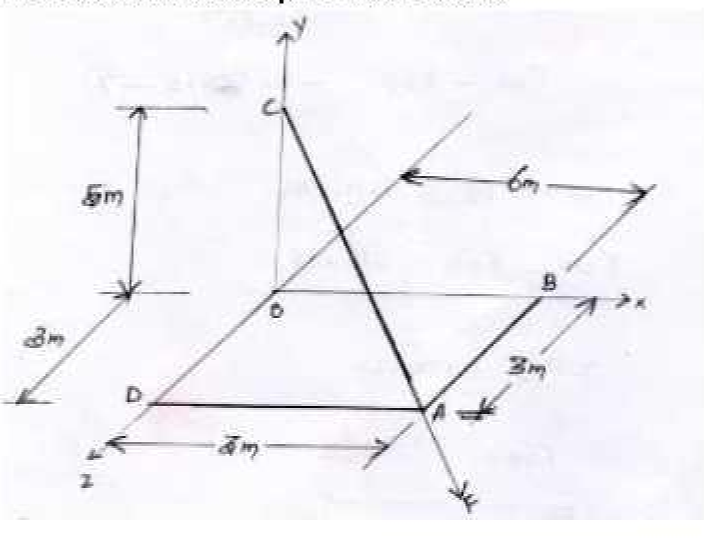
$$-0.285 \times 56.62 - 0.285 \times 72.76 - F_{DC} = 18.42$$

$$16.138 + 20.71 - F_{DA} = 18.42$$

$$F_{DA} = 16.138 + 20.71 - 18.42$$

$$F_{DA} = 18.428\ N$$

2. Fig shows three cables AB, AC,& AD that are used to support the end of a sign which exerts a force of $F = (250\vec{i} + 450\vec{j} - 150\vec{k})N$ at A. Determine the force develop in each cable.



Given:

$F = (250\vec{i} + 450\vec{j} - 150\vec{k})N$ at A

To find:

Force in AB, AC &AD

Soln:

$A = (3,0,3)$

$$B = (6,0,0)$$
$$C = (0,5,0)$$
$$D = (0,0,3)$$

$$OA = 3\vec{i} + 0\vec{j} + 3\vec{K}$$

$$OB = 6\vec{i} + 0\vec{j} + 3\vec{k}$$

$$OC = 0\vec{i} + 5\vec{j} + 0\vec{k}$$

$$OD = 0\vec{i} + 0\vec{j} + 3\vec{k}$$

Force of AB

$$F_{AB} = \lambda_{AB} \times F_{AB}$$
$$\lambda_{AB} = \frac{AB}{|AB|}$$
Position vector for AB

$$AB = OB - OA = [6\vec{i} + 0\vec{j} + 0\vec{k}] - [3\vec{i} + 0\vec{j} + 3\vec{k}]$$

$$|\overrightarrow{AB}| = 3\vec{i} + 0\vec{j} - 3\vec{k}$$

$$|\overrightarrow{AB}| = \sqrt{3^2 + 0^2 + [-3]^2} = \sqrt{9 + 0 + 9}$$

$$|\overrightarrow{AB}| = \sqrt{18}$$

$$|\overrightarrow{AB}| = 4.2$$

$$\lambda_{AB} = \frac{3\vec{i} + 0\vec{j} - 3\vec{k}}{4.2}$$

$$\lambda_{AB} = 0.714\vec{i} + 0\vec{j} - 0.714\vec{k}$$

$$\overrightarrow{F_{AB}} = \lambda_{AB} \times F_{AB}$$

$$\overrightarrow{F_{AB}} = 0.714F_{AB}\vec{i} + 0\vec{j} - 0.714F_{AB}\vec{k} \text{---------------------} (1)$$

Force on AC

$$\overrightarrow{F_{AC}} = \lambda_{AC} \times F_{AC}$$

$$\lambda_{AC} = \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|}$$

Position vector of $AC = OC - OA$

$$[0\vec{i} + 5\vec{j} + 0\vec{k}] - [3\vec{i} + 0\vec{j} + 3\vec{k}]$$

$$AC = -3\vec{i} + 5\vec{j} - 3\vec{k}$$

$$|\overrightarrow{AC}| = \sqrt{(-3)^2 + (5)^2 + (-3)^2} = \sqrt{9 + 25 + 9} = \sqrt{43}$$

$$|\overrightarrow{AC}| = 6.5$$

$$\lambda_{AC} = \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|} = \frac{-3\vec{i} + 5\vec{j} - 3\vec{k}}{6.5}$$

$$\lambda_{AC} = -0.461\vec{i} + 0.769\vec{j} - 0.461\vec{k}$$

$$\overrightarrow{F_{AC}} = \lambda_{AC} \times F_{AC}$$

$$\overrightarrow{F_{AC}} = -0.461F_{AC}\vec{i} + 0.769F_{AC}\vec{j} - 0.461F_{AC}\vec{k} \text{ ------------ } (2)$$

Force on AD

$$\overrightarrow{F_{AD}} = \lambda_{AD} \times F_{AD}$$

$$\overrightarrow{AD} = \overrightarrow{OA} - \overrightarrow{OA} = [0\vec{i} + 0\vec{j} + 3\vec{k}] - [3\vec{i} + 0\vec{j} + 3\vec{k}]$$

$$\overrightarrow{AD} = -3\vec{i} + 0\vec{j} + 0\vec{k}$$

Magnitude of AD

$$|\overrightarrow{AD}| = \sqrt{(-3)^2} = \sqrt{9}$$

$$|\overrightarrow{AD}| = 3$$

$$\lambda_{AD} = \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = \frac{-3\vec{\imath} + 0\vec{\jmath} + 0\vec{k}}{3}$$

$\lambda_{AD} = -\vec{\imath}$

$\overrightarrow{F_{AD}} = -F_{AD}\vec{\imath}$--------------- (3)

$F = 250\vec{\imath} + 450\vec{\jmath} - 150\vec{k}$ $[is\ given]$

Applying the equilibrium Eqn

$\sum F_x = 0$

$0.714F_{AB} - 0.461F_{AC} - F_{AD} + 250 = 0$

$0.714F_{AB} - 0.461F_{AC} - F_{AD} = -250$-------------- (4)

$\sum F_y = 0$

$0F_{AB} + 0.769F_{AC} + 450 = 0$--------------- (5)

$0.769F_{AC} = -450$

$F_{AC} = \frac{-450}{0.769}$

$F_{AC} = -585.17\ N$

$\sum F_z = 0$

$-0.714F_{AB} - 0.461\ F_{AC} - 150 = 0$--------------- (6)

$-0.714F_{AB} - 0.461\ \times (-585.17) - 150 = 0$

$-0.714F_{AB} + 269.76 - 150 = 0$

$-0.714F_{AB} + 119.76 = 0$

$F_{AB} = \frac{-119.76}{-0.714}$

$F_{AB} = 167.73N$

(4)$\Rightarrow$ $0.714F_{AB} - 0.461F_{AC} - F_{AD} = -250$

$$0.714 \times 167.73 - 0.461 \times -585.17 - F_{AD} = -250$$

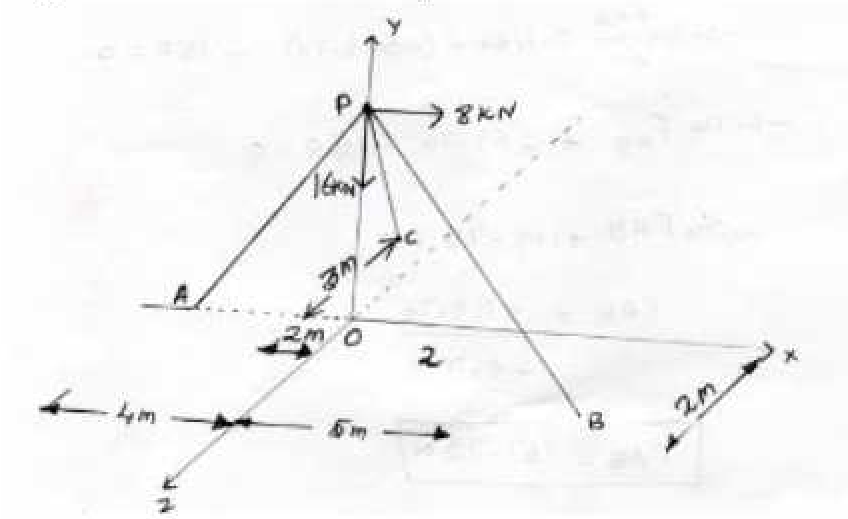
$$119.76 + 269.76 - F_{AD} = -250$$

$$389.52 - F_{AD} = -250$$

$$-F_{AD} = -250 - 389.52$$

$$-F_{AD} = -639.52 F_{AD} = 639.52\ N$$

3. Two force act upon the tripod at point P as shown in fig. The force 8 KN is parallel to X axis & the force 16 KN is parallel to Y axis. Determine the force acting at the legs of tripod if the rest on legs on ground at A, B, &C whose coordinates with respect to O are given the height of the P above the origin is 10m.



Given:

8 KN at point 'P' in horizontal
16 KN at point 'P' in vertical
Height of point P=10m from 0

To Find:

F_{PA}, F_{PB}, F_{PC}

Soln:

Coordinates

$A = (-4,0,0), B = (5,0,2), C = (-2,0,-3), P(0,10,0)$

$OA = -4\vec{i} + 0\vec{j} + 0\vec{k}, OB = 5\vec{i} + 0\vec{j} + 2\vec{k}, OC = -2\vec{i} + 0\vec{j} - 3\vec{k}$

$$OP = 0\vec{i} + 10\vec{j} + 0\vec{k}$$

Force on F_{PA}

$$\overrightarrow{F_{PA}} = \lambda_{PA} \times F_{PA} \lambda_{PA} = \frac{\vec{A}}{|PA|}$$

$$|\overrightarrow{PA}| = \overrightarrow{OA} - \overrightarrow{OP}$$

$$=-4\vec{i} - [10\vec{j}]$$

$$|\vec{PA}| = -4\vec{i} - 10\vec{j}$$

$$|\overrightarrow{PA}| = \sqrt{(-4)^2 + (-10)^2} = \sqrt{16 + 100} = \sqrt{116}$$

$$|\overrightarrow{PA}| = 10.77$$

$$\lambda_{PA} = \frac{PA}{|\overrightarrow{PA}|} = \frac{-4\vec{i} - 10\vec{j}}{10.77}$$

$$\lambda_{PA} = -0.371\vec{i} - 0.928\vec{j}$$

$$\overrightarrow{F_{PA}} = \lambda_{PA} \times F_{PA} = -0.371\vec{i} \times F_{PA} - 0.928\vec{j} \times F_{PA}$$

$$\overrightarrow{F_{PA}} = -0.371 F_{PA}\vec{i} - 0.928 F_{PA}\vec{j} \text{------------} (1)$$

Force of PB

$$\overrightarrow{F_{PB}} = \lambda_{PB} \times F_{PB} \lambda_{PB} = \frac{\overrightarrow{PB}}{|\overrightarrow{PB}|}$$

$$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP}$$

$$= [5\vec{i} + 2\vec{k}] - [10\vec{j}]$$

$$\overrightarrow{PB} = 5\vec{i} - 10\vec{j} + 2\vec{k}$$

$$|\overrightarrow{PB}| = 11.35$$

$$\lambda_{PB} = \frac{\vec{PB}}{|\overrightarrow{PB}|} = \frac{5\vec{i} - 10\vec{j} + 2\vec{k}}{11.35}$$

$$\lambda_{PB} = 0.44\vec{i} - 0.88\vec{j} + 0.176\vec{k}$$

$$\overrightarrow{F_{PB}} = \lambda_{PB} \times F_{PB}$$

$$\overrightarrow{F_{PB}} = 0.44F_{PB}\vec{\imath} - 0.88F_{PB}\vec{\jmath} + 0.176F_{PB}\vec{k}\text{--------- (2)}$$

Force on PC

$$\overrightarrow{F_{PC}} = \hat{\lambda}_{PC} \times F_{PC}\lambda_{PC} = \frac{\overrightarrow{PC}}{|\overrightarrow{PC}|}$$

$$\overrightarrow{PC} = \overrightarrow{OC} - \overrightarrow{OP}$$

$$= \left[2\vec{\imath} - 3\vec{k}\right] - 10\vec{\jmath}$$

$$\overrightarrow{PC} = -2\vec{\imath} - 10\vec{\jmath} - 3\vec{k}$$

$$\left|\overrightarrow{PC}\right| = \sqrt{(-2)^2 + (-10)^2 + (-3)^2} = \sqrt{4 + 100 + 9} = \sqrt{113}$$

$$\left|\overrightarrow{PC}\right| = 10.63$$

$$\lambda_{PC} = \frac{\overrightarrow{PC}}{|\overrightarrow{PC}|} = \frac{-2\vec{\imath} - 10\vec{\jmath} - 3\vec{k}}{10.63}$$

$$\lambda_{PC} = -0.188\vec{\imath} - 0.94\vec{\jmath} - 0.282\vec{k}$$

$$\overrightarrow{F_{PC}} = \lambda_{PC} \times F_{PC}$$

$$\overrightarrow{F_{PC}} = -0.188F_{PC}\vec{\imath} - 0.94F_{PC}\vec{\jmath} - 0.282\vec{k}$$

$$P = 0\vec{\imath} + 10\vec{\jmath} + 0\vec{k}\text{------------ (4)}$$

Apply Equilibrium condition

$$\sum F_x = 0$$

$$-0.371F_{PA} + 0.44F_{PB} - 0.188F_{PC} = 0\text{------------- (5)}$$

$$\sum F_y = 0$$

$$-0.928F_{PA} - 0.88F_{PB} - 0.94F_{PC} + 10 = 0$$

$$-0.928F_{PA} - 0.88F_{PB} - 0.94F_{PC} = -10$$

$$0.928F_{PA} - 0.88F_{PB} - 0.94F_{PC} = 10 \text{ -------- (6)}$$

$$\sum F_Z = 0$$

$$-0.282F_{PC} + 0.178F_{PB} = 0$$

$0.178F_{PB} - 0.282F_{PC} = 0$--------- (7)

Solve Eqn(5)&(6)

(5)× 0.928 $\quad - 0.344\, F_{PA} + 0.4F_{PB} - 0.174F_{PC} = 0$

(6) × 0.371 $\quad 0.344F_{PA} + 0.326F_{PB} + 0.348F_{PC} = 3.71$

$0.726F_{PB} - 0.174F_{PC} = 3.71$--------- (8)

Solve Eqn (7) &(8)

(7)⇒0.726⇒ $\quad 0.127F_{PB} - 0.2F_{PC} = 0$

(8)⇒0.176⇒ $\quad 0.127F_{PB} + 0.03F_{PC} = 0.652$

$$-0.23F_{PC} = -0.652$$

$$F_{PC} = \frac{-0.652}{-0.23}$$

Eqn(7) becomes $0.176F_{PB}$-$0.282F_{PC} = 0$

$$0.176 \times F_{PB} - 0.282 \times 2.834 = 0$$

$$F_{PB} = \frac{0.282 \times 2.834}{0.176}$$

$$F_{PB} = 4.539N$$

Eqn (5) becomes

$$-0.371F_{PA} + 0.44F_{PB} - 0.188F_{PC} = 0$$

$$-0.371F_{PA} + 0.44 \times 4.539 - 0.188 \times 2.834 = 0$$

$$-0.371 \times F_{PA} + 1.997 - 0.532 = 0$$

$$-0.371 \times F_{PA} + 1.465 = 0$$

$$F_{PA} = \frac{-1.465}{-0.371}$$

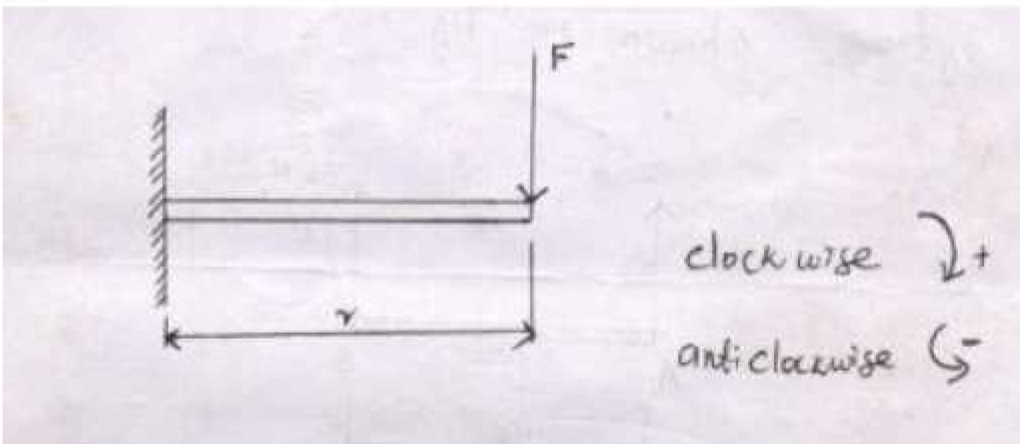
$$F_{PA} = 3.94N$$

UNIT II

Statics of Rigid bodies in Two Dimensional

Moment of force:

Moment of force is defined as the product of the force and perpendicular distance of the line of the force from the point.



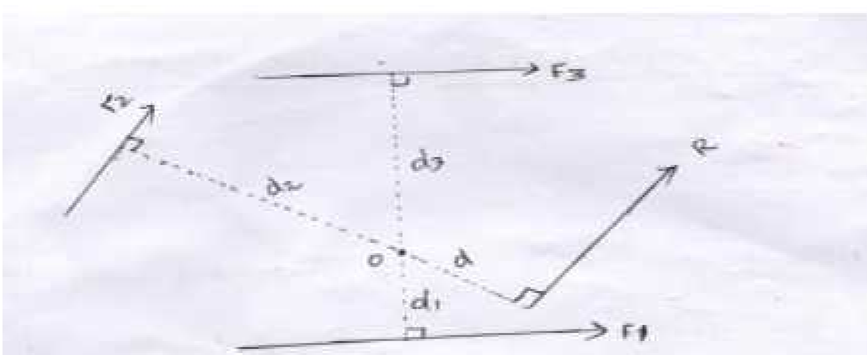
$Moment = Force \times perpendicular\ distance.$

$Mo = F \times d\ N.m$

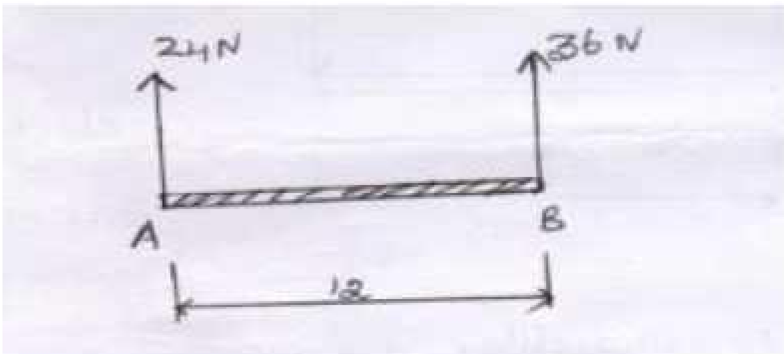
Varignon's Theorem:

The algebraic sum of the moment of any number of force about any point in their plane is equal to the moment of their resultant about the same point.

$F1 \times d1 + F2 \times d2 + F3 \times d3 = R \times d$



Find the resultant force for the parallel force System shown in fig.



Resultant force 'R'

$$R = 24 + 36$$

$$R = 60N$$

Location of resultant force:

Algebric sum of moment of all force about a

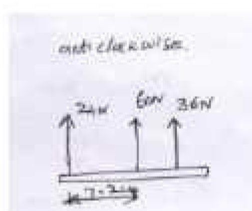
$\sum M_A = -36 \times 12$

$\sum M_A = -432\, N.m$

$\sum M_A = 432\, N.m(clockwise)$

By virginal theorem

$\sum M_A = R \times x$

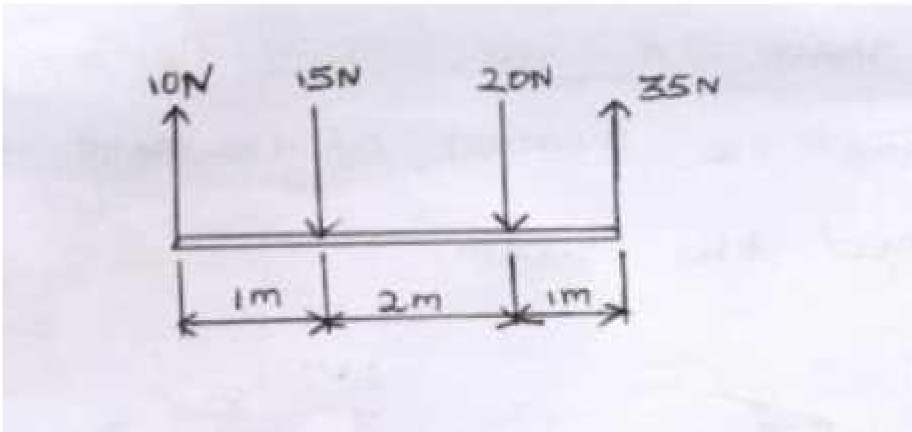


$+432 = +60 \times x$

$x = \frac{+432}{+60}$

$x = 7.2\, cm$

2. Four parallel forces of magnitude 10N, 50N, 20N and 35N as shown in fig. Determine the magnitude and direction of the resultant. Find the distance of the resultant from point A.



Solution:-

Magnitude of resultant:-

$R = 10 - 15 - 20 + 35$

$R = +10N$

Locating of the resultant

$\sum M_A = R \times x$

$\sum M_A = (15 \times 1) + (20 \times 3) + (-35 \times 4)$

$\sum M_A = -65\ N.m$

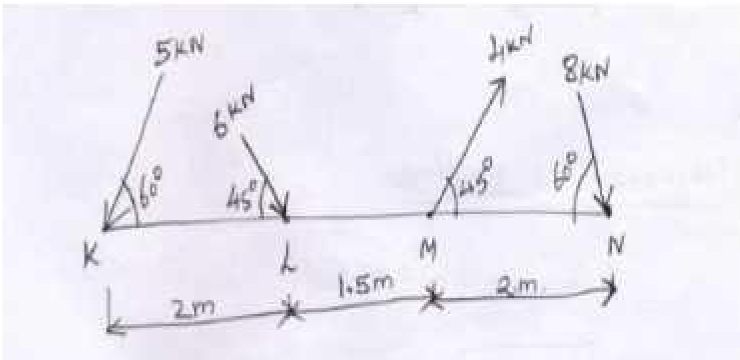
$\sum M_A = R \times x$

$+65 = 10 \times x$

$x = (+65)/(+10)$

$x = 6.5m$

1. A system of forces acts on a weightless beam as shown I fig. Find the magnitude of the resultant and the location of the point where the resultant met the beam.



Given:

$$Load\ at\ K = 5KN\ at\ 60°$$

$$K = 6KN\ at\ 45°$$

$$M = 4KN\ at\ 45°$$

$$N = 8KN\ at\ 60°$$

To find:

Resultant force & location

Soln:

Resultant R=$\sqrt{(\sum FH)^2 + (\sum FV)^2}$

$\sum F_H = 0 \overset{+}{\rightarrow}\overset{-}{\leftarrow}$

$= -5\ cos\,60\ + 6\ cos\,45 + 4\ cos\,45\ + 8\ cos\,60$

$\sum F_H = 8.57\ KN$

$\sum F_v = 0 \uparrow \quad \downarrow$

$= -5\ sin\ 60 + 6\ sin\ 45 + 4\ sin\ 45 + 8\ sin\ 60$

$\sum F_v = -12.67\ KN$

R $= \sqrt{(\sum FH)^2 + (\sum FV)^2}$

R $= \sqrt{(8.57)^2 + \sum(12.67)^2}$

$R = 15.3\ Kn$

Inclination of the resultant $\alpha = tan^{-1}\left(\frac{\sum_{F_V}}{\sum_{F_H}}\right)$

$\alpha = tan^{-1}(\frac{12.67}{8.57})$

$\alpha = 55.92^\circ$

To locate the resultant:

$\sum M_k = 0 \downarrow + \uparrow -$

$\sum M_k = 0 + |+sin45 \times 2| + |-4sin45 \times 3.5| + |+8sin\ 60 \times 5.5|$

$\sum M_k = +36.69\ KN.m\ (clockwise)$

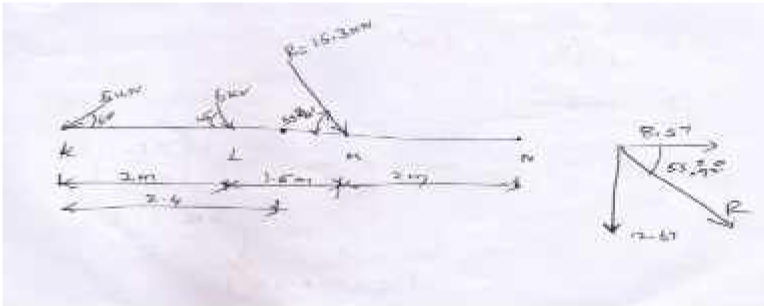
By varigon's Theorem

$\sum M_x = R \times x$

$+36.69 = 15.3 \times x$

$x = \frac{+36.69}{15.3}$

$x = 2.4m$

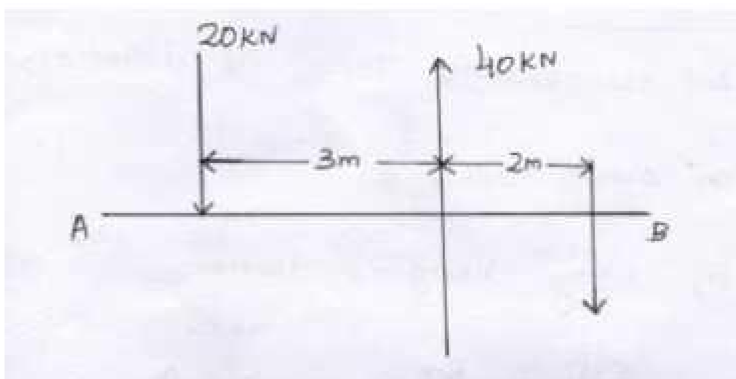


Problem:1

A coplanar parallel force system consisting of three forces acts on a rigid bar AB as shown fig. below

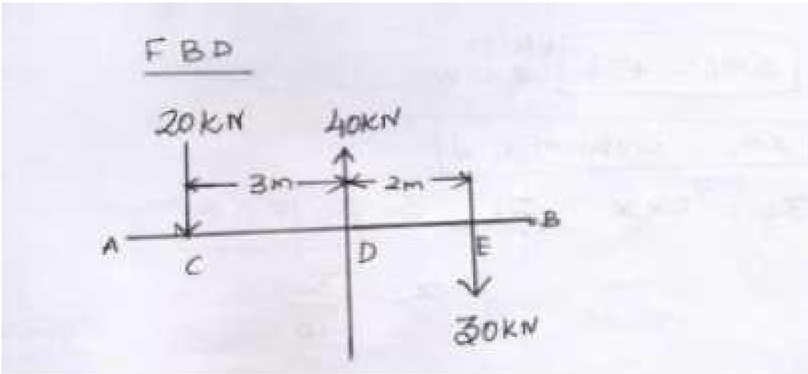
a) Determine the simplest equivalent action for the force system,

b. If an additional force of 10kn acts along the bar A to what be simplest equivalent action.



soln:

(a) simplest Equivalent force:



Sum of Horizontal force $\sum F_H = 0$

$$\sum FH = 0$$

Sum of vertical force $\sum FV = 0$

$\sum FV = -20 + 40 - 30 = -10KN$

Magnitude of Resulatant Force = R

$R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)^2}$

$= \sqrt{0^2 + (-10)^2}$

$R = \sqrt{100}$

$R \quad = 10N$

Line of Action:-

Let the resultant force at distance 'X' From the line of action 20KN

By using varigon's theorem

$\sum M_c - R \times x$

$\sum M_c = (-40 \times 3) + (30 \times 5) = 120 + 150$

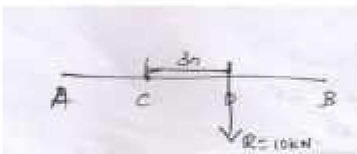
$\sum M_c = +30\ Nm$

$$\sum M_c = 30N.m\,c.w$$

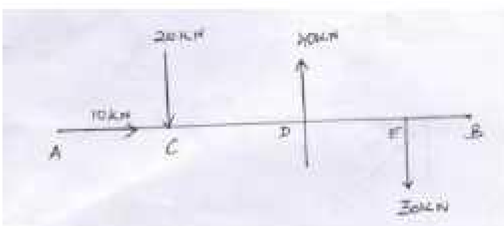
$30 = R \times x \quad \rightarrow \quad 30 = 10 \times x$

$$X = \frac{30}{10}$$

$x = 3m$



b) With additional force of 10KN from Ato B



Sum of Horizontal force $\sum F_H$=0

$$\sum F_H = 10KN$$

Sum of horizontal force $\sum F_v$=0

$\sum F_v = -20 + 20 - 30$

$$\sum F_v = -10KN$$

Resultant Force 'R' $R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)^2}$

$R = \sqrt{(10)^2 + [-10]^2}$

$R = \sqrt{100 + 100} = \sqrt{200}$

$R = 14.14\, KN$

Location

$\sum M_c = \sum F_V$

$\sum M_c = (-40 \times 3) + (30 \times 5) = -30kN.M$

$\sum M_c = 30\ KN.M$ clockwise

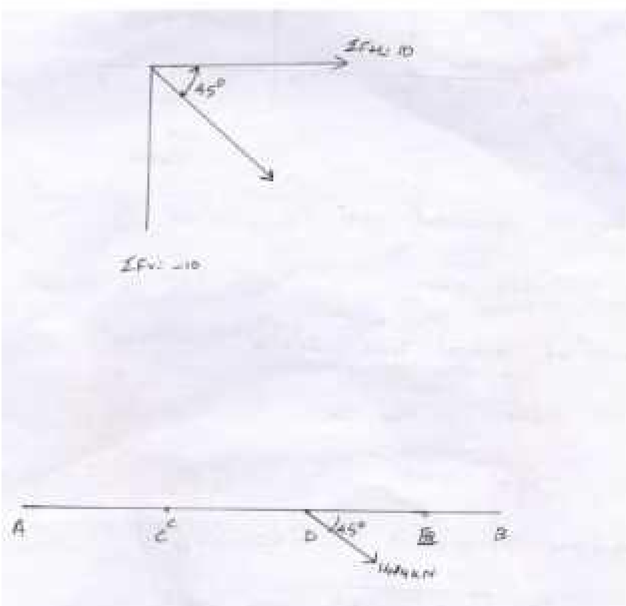
$30 = 10 \times x$

$x = 13m$

Location

$\theta = tan^{-1}\left(\frac{\sum_{F_V}}{\sum_{F_H}}\right) = tan^{-1}\left(\frac{10}{10}\right)$

$\theta = 45°$



$-80P = -4628.2$

$$P = \frac{4628.2}{80}$$

$P = 61.60N$

ii) Magnitude of the Resultant force:

Resultant $R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)}$

$$\sum F_H = -61.60 - 100\ cos60$$

$$\sum F_H = -111.60\ N$$

$$\sum F_v = 100 \sin 60$$

$$\sum F_v = 86.6N$$

$R = \sqrt{[-111.60]^2 + [86.6]^2}$

$R = 141.26N$

iii) Point of Application

By Varigon's theorem

$\Sigma M_o = R \times x$

$\Sigma M_o = 61.60 \times 40 + [-100\ sin\ 60 \times 80] = 0$

$\Sigma M_o\ 2464 - 6928$

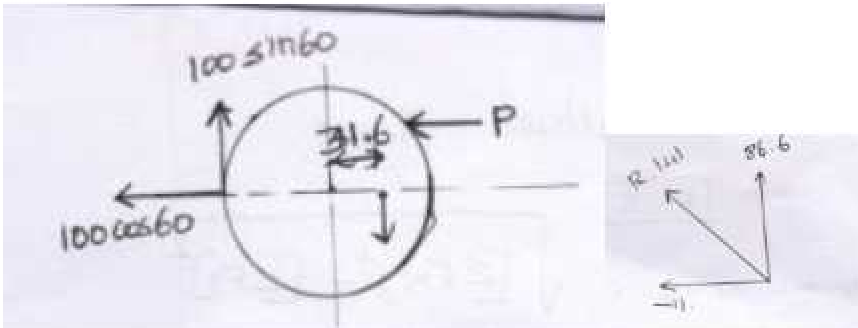
$\sum M_O = -4464.7$Counts clocks wise

$\sum M_O = 4464$ Clockwise

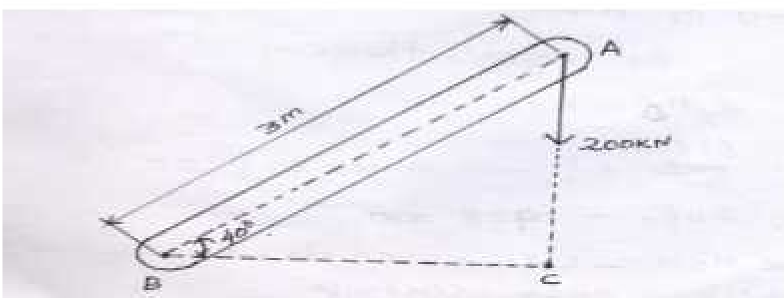
$\Sigma M_o = R \times x$

$4464.2 = 141.26 \times x$

$X = 31.60\,mm$



6. A 200KN vertical force is applied to the end of a lever which attached a shaft as B as shown in Fig Below. Determine the(i) magnitude of horizontal force (ii) The smallest force applied at which creates the same moment about B(iii) How far from the end B, at 400KN Vertical force must to create the same moment about B (iv) Replace the given system of force at B.



$Vertical\ load\ at\ point\ A = \ 200KN$

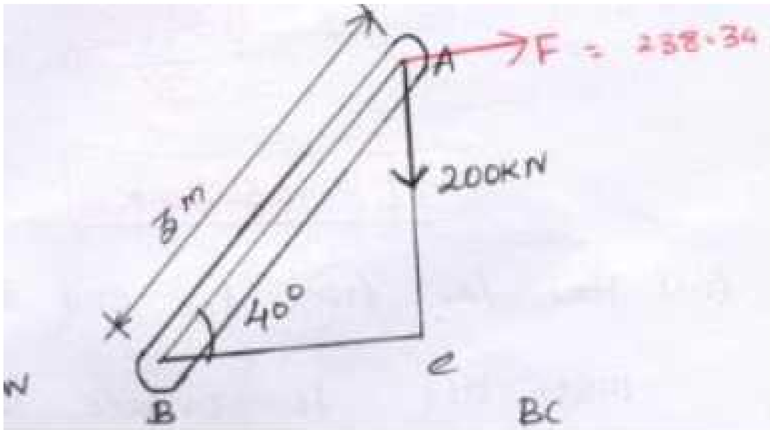
$Length\ of\ bar\ L = 3m$

$Angle\ \ = 40°$

Soln:

(i) The magnitude of horizontal force applied at 'A" which create same moment about 'B'

Take moment about 'B'



$M_o = +200 \times BC$ $\quad\quad$ $cos\theta = \frac{BC}{3}$

$M_O = +200 \times 2.29$ $\quad\quad$ $cos\theta = \frac{BC}{3}$

$M_O = +459.62\ KN.M$ $\quad\quad$ $BC = 2.29m$

→F

$MD = 459.62\ KN.m$ $\quad\quad$ $sin\,\theta = \frac{AC}{AB}$

Take moment About 'o' horizontal force $\quad\quad$ $sin\,\theta = \frac{AC}{AB}$

act forwards right $\quad\quad$ $\sin 40 = \frac{AC}{3}$

$M_D = F \times AC$ $\quad\quad$ $AC = 1.92m$

$459.62 = F \times 1.92$

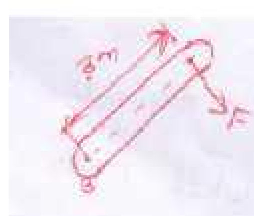
$F = \frac{459.62}{1.92}\frac{KN.m}{m}$

$F = 238.34\ KN$

ii) The smallest force applied at which create the same moment about 'B'

moment About $B = 459.62\ KN.m$

$M_B = F \times 3$



$459.62 = F \times 3$

$$F = \frac{459.62}{3}$$

$F = 153.20\ KN$

(iii) How far from the end B, a 400KN vertical force must act to create the same moment about B.

Let 400KN Vertical force act at a distance of 'x' A to have same moment −459.62 KN.m clockwise

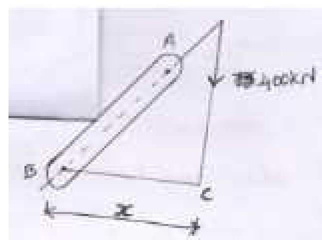
To have clockwise moment 400 N Vertical force on the right side of A

$Moment = -459.62\ KN.m$

$-400 \times x = -459.62$

$x = (-459.62)/(-400)$

$x = 1.149\ m$

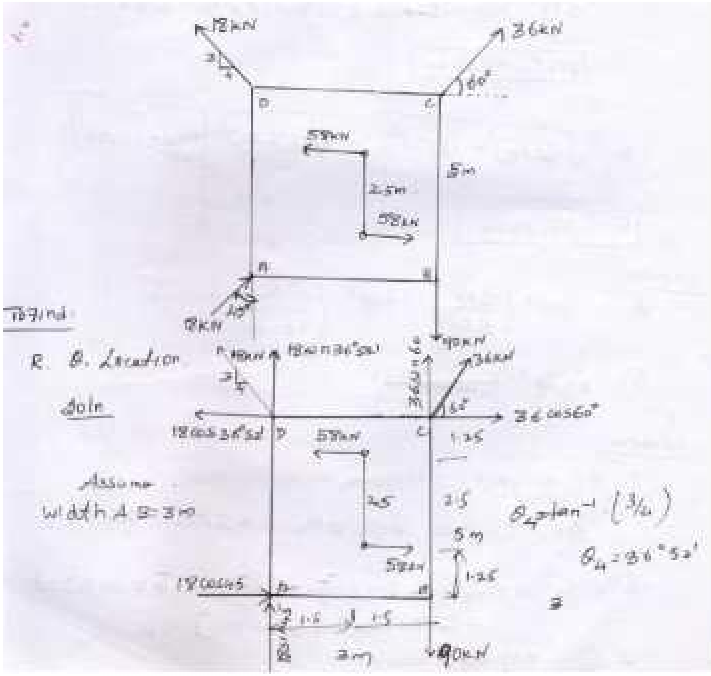


iv) Replace the given system of Force at B

$Downward\ load - 200KN$

$Moment\ at\ B = 459.63\ KN.m$

7. Determine the resultant of The calendar non concurrent force system shown in fig. below. Calculate its mangnitute and direction and locate its position with respect to the sides AB and AD



Resultant force

$R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)^2}$

$\sum F_H = 0+ \longrightarrow \quad \longleftarrow - F_H = F\ cos\theta$

$\Sigma F_H = 18cos\ 45\ + 36\ cos\ 60 - 18cos\ 36°\ 52' - 58 + 58$

$\Sigma F_H = 16.32\ KN$

$\sum F_V = 0 \quad +\uparrow \quad \downarrow$

$\Sigma F_v = 18\ sin\ 45 - 90 + \ 36\ sin60° + \ 18\ sin36°52' = 0$

$\sum F_V = -35.26\ KN$

$R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)^2} = \sqrt{(16.32)^2 + (-35.26)^2}$

$R = 30.88KN$

Direction:-

$$\theta - tan^{-1}\left(\frac{\Sigma_{FV}}{\Sigma_{FH}}\right) - tan^{-1}\left(\frac{35.26}{16.32}\right)$$

$$\theta = 65°9'$$

Location:

By varignon's theorem

$$\Sigma M_A - R \times x \text{ (or)} \Sigma M_A - \Sigma F_H \times y \text{ or } \Sigma F_V \times x$$

$$\Sigma M_A = (+90 \times 3) + (36 \cos 60 \times 5) + (-36 \sin 60 \times 3) + (-18 \cos 36°52' \times 5) + (+58 \times 1.25) + (-58 \times 3.75) = 0$$

$$\sum M_A = +49.46\ KN.M \text{ (clockwise)}$$

$$\sum M_A = 49.46 \text{ (clockwise)}$$

$$\Sigma M_A = \Sigma F_V \times x$$

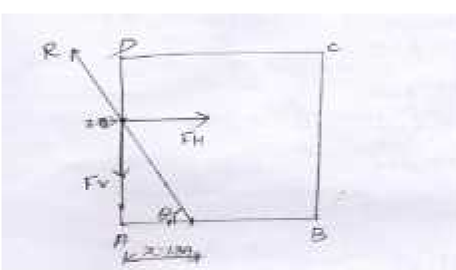
$$49.46 = 35.26 \times x$$

$$x = 1.39m$$

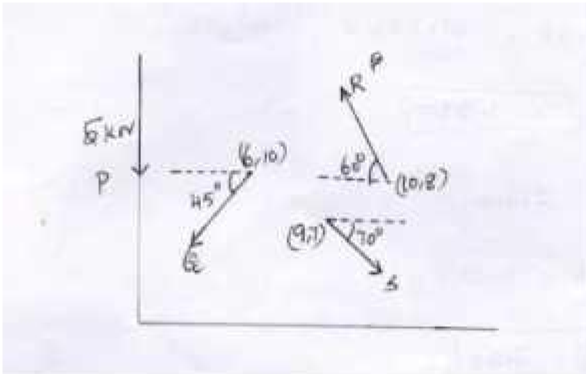
$$\sum M_A = \sum F_H \times y$$

$$49.46 = 16.32 \times y$$

$$y = 3.03$$



8. A system of four forces P, Q, R and s of magnitude 5KN, 8KN, 6KN And 4KN respectively acting on a body are shown in rectangular coordinates. As shown in fig find the moment of the forces about the origin O. also find the resultant moment of the forces about O. The distance are in meters.



Given:

$Load\ on\ P = 5KN$

$Load\ on\ Q = 3KN$

$Load\ on\ R = 6KN$

$Load\ on\ S = 4KN$

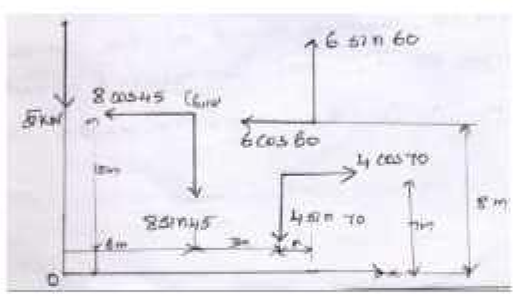
To Find:

1, moment of Forces

2, Resultant

Soln:-

Free body diagram



Moment of P

Moment of force 'P' about the orgin, M_P

$M_p = 5 \times 0$

$M_p - 0$

Moment of Q

Moment of force 'Q' about the orgin, M_Q

$M_Q = (8sin\,45 \times 6) + (-8cos\,45 \times 10)$

$M_Q = -22.64KN.m$

$M_Q = +2.64\,KN\ \ C.W$

Moment of R

Moment of force R about the orgin M_R

$M_R = -75.96\,KN.m$

$M_R = 75.96\,c.w$

Moment of S

Moment of force s about the Orgin 'M_S'

$M_s = (4\,cos \times 7) + (4sin\,70 \times 9)$

$M_s = 43.40\,KN.m$

Statics of Rigid bodies Force couple system

Moment of a Force:-

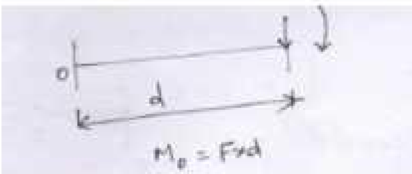
Moment of a Force about a point is defined as the product of the force and the perpendicular distance of the line of action of the force from the point

$M = F \times d$

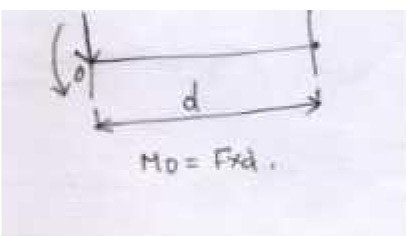
$F = Force$

$d = perpendicular\ distance$

The clockwise direction of moment is positive direction of moment



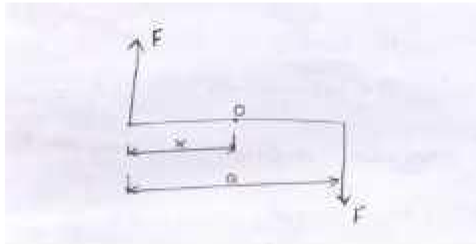
The Anticlockwise bending moment gives the negative direction of moment



Coupled force:

It is a turning effect produce in the body of object by applying two forces having same magnitude put in opposite Direction.

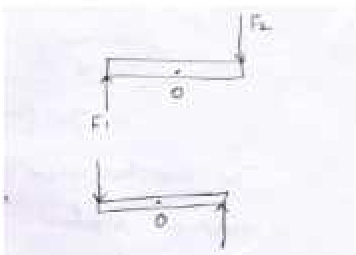
Two forces F and –F having the same magnitude, parallel lines of action and opposite sense are said to form a couple.



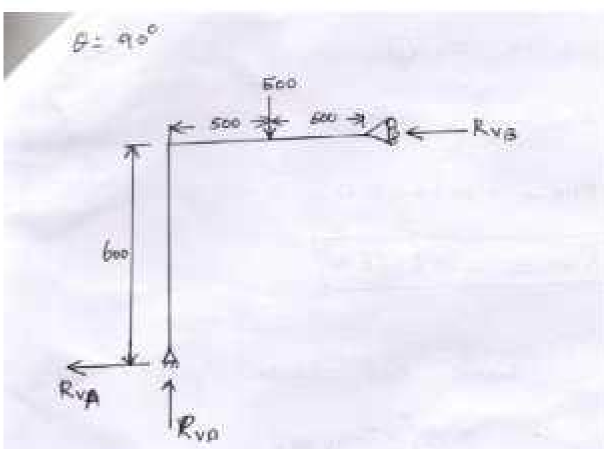
Types of couple:

1. clock wise couple

2, Anti- clockwise couple



$\theta = 90°$



$\Sigma F_H = 0$

$-RVA - RVB = 0$------------- (1)

$\sum FV = 0$

$RV_A - 500 = 0$

$RV_A = 500N$

$\sum M_A = 0$

$[500 \times 500] + [-RV_B \times 600] = 0$

$250 \times 103 = RV_B \times 600 = 0$

$-RV_B = -250 \times 103$

$$RV_B = \frac{-250 \times 10^3}{-600}$$

$RV_B = 416.66N$

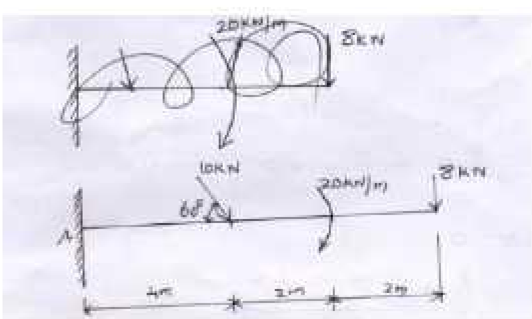
R_{VB} Sub in Eqn (1)

$-RH_A - RH_B = 0$

$RH_A = 416.66N$

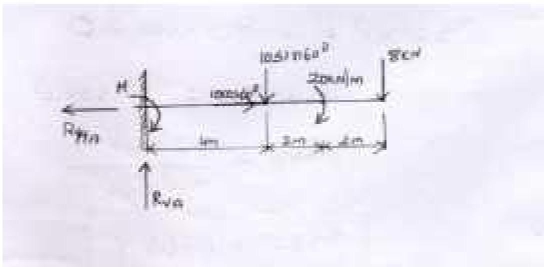
Problem:

Determine the support and reaction at A and B



Given

Free body diagram



$-1019.61\, RV_B = -250 \times 103$

$$RV_D = \frac{-250 \times 10^3}{1019.61}$$

$RV_B = 245.19\, N$

Sub in Eqn------ (1)

$-RH_A - RV_B \cos 30 = 0$

$-RH_A = -RV_B \cos 30 = -245.19 \cos 30$

$RH_A = 212\, N$

RV_A *Sub in* (2)

$RV_A + \sin 30 = 500$

$RV_A = -RV_B \sin 30 + 500$

$RV_A = -245.19 \times \sin 30 + 500$

$RV_A = -122.5 + 500$

$RV_A = 377.5 N$

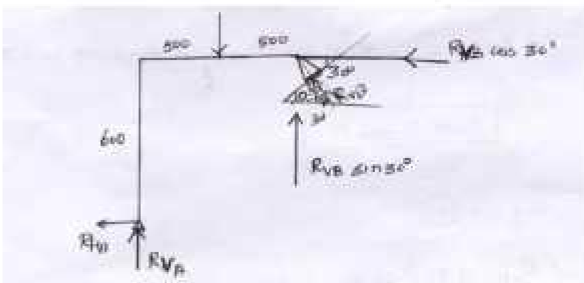
$$RV_B - \frac{-250 \times 10^3}{-1000}$$

$RV_B = 250\, N$

$RV_A + RV_B = 500 ---> RV_A = 500 - 250$

$RV_A = 250N$

ii) when $\theta = 60°$



$\sum F_H = 0$

$-RH_A - RV_B \cos 30 = 0$

$\sum F_v = 0$

$RV_A - 500 + Sin\,30 = 0$

$RV_A + RV_B\, Sin\,30° = 500$

$\sum M_A = 0$

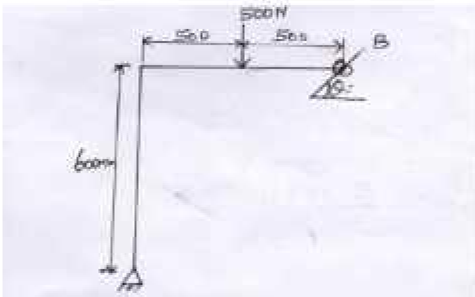
$\sum M_A = [500 \times 500] + RV_B \cos 30 \times 600] + [-RV_B \sin 30° \times 1000]$

$$\sum M_A = 250 \times 10^3 - RV_B\, 519.61 - RV_B\, 500 = 0$$

$250 \times 10^3 - 01019.61\, RV_B = 0$

Problem:

A Frame supported at A and B is subjected to force 500N as shown in fig compute the Reaction the support for the cases i) $\theta = 90° \theta = 60°$



Given $\theta = 0° \theta = 90° \theta = 60°$

To find

Reaction at the support

i)$\theta = 0^0$

$\sum F_H - 0$

$RH_A = 0$

$\sum F_V = 0$

$-500 + RV_B + RV_A = 0$

$RV_A + RV_B = 500$------> (1)

$\sum M_A = 0$

$[500 \times 500] + [RV_B \times 1000] = 100$

$250 \times 10^3 - 1000 RV_B = 0$

$-1000 RV_B = -250 \times 10^3$

To find reaction 'R'

$\sum F_H = 0$

$-RH_R - F_{PQ}\cos 25^0 = 0$

$-RH_R = F_{PQ}\cos 25^\circ$

$RH_R = -F_{PQ}\cos 25^\circ$

$RH_R = - \times 3 \times cos25$

$RH_R = 2.45\ N$

$\sum F_V = 0$

$RV_R - 4 - FP_Q sin\ 23 = 0$

$RV_R - 4 - 3 \times 25° = 0$

$RV_R = 4 + 3 sin\ 25^\circ$

$RV_R = 5.26\ N$

$\sqrt{[RH_R]^2 + [RV_R]^2}$

R= $\sqrt{(2.45)^2 (5.26)^2}$

$R = 5.80N$

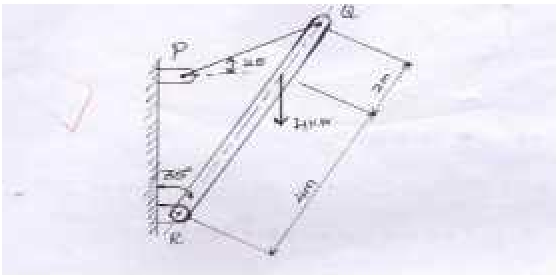
$\theta = tan^{-1}\left(\frac{\sum R_H}{\sum R_V}\right)$

$\theta = tan^{-1}\frac{5.26}{2.43}$

$\theta = 65^0$

Problem:

4000N load acts on the beam held by a cable PQ as shown in fig. The weight of the beam can be neglected. Draw the free body diagram of the beam and find tension in cable PQ. Also find the reaction force at R



To find:

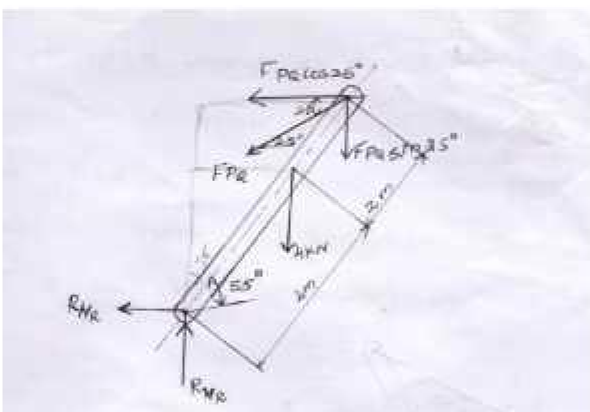
1. Free body diagram

2. Tension in cable PQ

3. Reaction on Force R

Soln:

1. Free body diagram:



2. Tension in cable 'PQ'

Moment at point 'R'

$$\sum M_R = [4 \times \sin 35^o] + [-F_{PQ}\cos 25^o \times 6\cos 35^o] + [F_{PQ}\sin 25^o \times 6\,sin35^o] = 0$$

$$\sum M_R = 9.177 - F_{PQ} \times 4.454 + 1.45F_{PQ} = 0$$

$-4.45F_{PQ} + 1.45F_{PQ} = -9.177$

$-3F_{PQ} = -9.177$

$F_{PQ} = \frac{-9.177}{-3}$

$F_{PQ} = 3N$

Procedure for finding out the resultant of non current coplander force system:

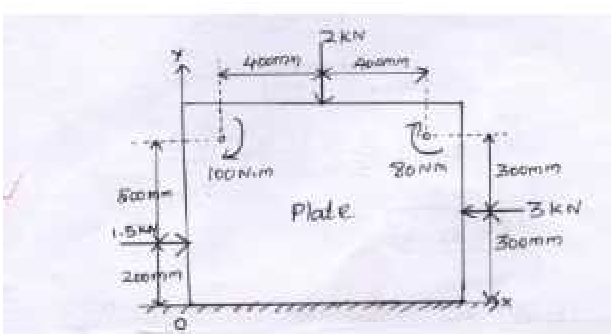
1. Resolve the given forces, if they are inclined to reference x and y Axis.
2. Find the sum of horizontal component of forces $\sum FH$
3. Find the sum of vertical component of forces$\sum FV$
4. 4.Calculate the resultant force R =$\sqrt{(\sum(F_H)^2 + (\sum(F_V)^2}$
5. Angle of inclination of resultant θ =tan^{-1}[$\frac{\sum FV}{\sum FH}$]
6. If the force moment system is converted into a single force, coordinate position is given by

$$\sum M_o = R \times x$$

$$\sum M_o = \sum F_v \times x$$

$$\sum M_o = \sum F_H \times y$$

A plate os acted upon by three force and two couple as shown in fig. determine the resultant of these force couple system and find co-ordinate x of the point on the x axis through which the resultant is passed



Given

Three force $1.5KN, 2KN, 3KN$

Two couple $100N.m$ $80\ N.m$

To find

Resultant force, location

Soln;

Resultant force R = $\sqrt{(\Sigma(F_H)^2 + (\Sigma(F_V)^2}$

Sum of horizontal

$$\sum F_H = 0$$

$\sum F_H = 1.5 - 3$

$\sum F_H = -1.5KN1$

Sum of vertical force $\sum Fv$ =0

$\sum F_V = -2\ KN$

Resultant $R = \sqrt{(\Sigma(F_H)^2 + (\Sigma(F_V)}$

$$R = \sqrt{[-1.5]^2[-2]^2}$$

$R = 2.5KN$

$$\theta = tan^{-1}\left[\frac{\sum F_V}{\sum F_V}\right] = tan^{-1}\left[\frac{-2}{-1.5}\right]$$

$\theta = 53.13^{\circ}$

To locate the resultant

By varigon's Thorem $\downarrow + \uparrow -$

$\sum M_o = R \times x$ and $\sum M_o = \sum F_y \times x$

$\sum M_o = [3 \times 0.3] + [-2 \times 0.5] + [-1.5 \times 0.2] + [-0.1] + [-0.08] = 0$

$\sum M_o = -0.58\ KN.M$

$\sum Mo = 0.58\ KN.M$ [clock wise]

The co-ordinate x of the point through which the resulted passes is given by

$\sum$Mo= $\sum FY \times x$ $x = \frac{0.58}{2}$

$0.58 = 2 \times x$

$x = 0.29m$

$x = 290mm$

we want to find the intersection

$\sum M_o = \sum F_H \times y$

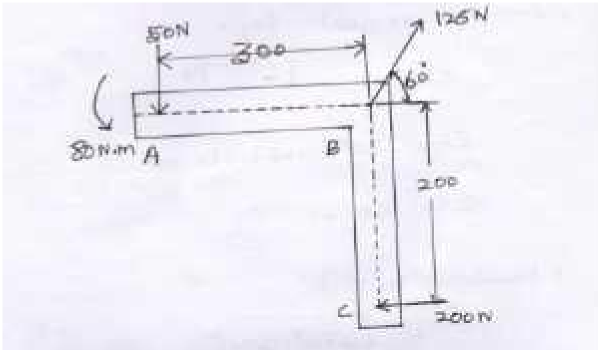
$0.581.5 \times y$

$y = 0.387\ m$

The three forces and a couple shown below are applied to an angel bracket

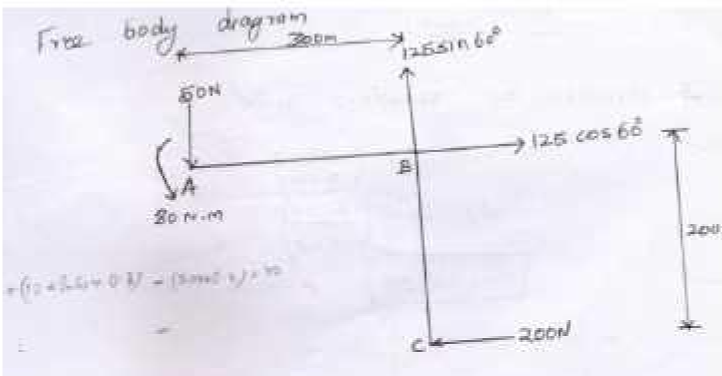
(i) Find he Resultant of this system of forces

(ii) Locate the points where the line o action of the resultant intersects line AB and the line BC



Soln

Free body diagram



1. Sum of Horizontal force

$$\sum F_H = 0 \qquad \overset{+}{\rightarrow}\underset{-}{\leftarrow}$$

$$\sum F_H = +125 \cos 60 - 200 = 0$$

$$\sum F_H = -137.5N$$

2. Sum of Vertical Force

$$\sum F_V = 0 \downarrow - \ \uparrow +$$

$$\sum F_V = -50 + 125 \sin 60 = 0$$

$$\sum F_V = 58.25$$

3. Resultant force' R'

$$R = \sqrt{(\Sigma(F_H)^2 + (\Sigma(F_V)^2}$$

$$R = \sqrt{[-137.5]^2 + [58.23]^2}$$

$$R = 149.32N$$

4. Direction of Resultant force α

$$\alpha = \tan^{-1}(\frac{\sum FV}{\sum FV})$$

$$\alpha = \tan^{-1}(\frac{58.25}{137.5})$$

$$\alpha = 22°57'$$

Location of Resultant Force:

By Varigon's Theorem

$$\sum M_A = \sum F_V \times x \; and \sum M_A = \sum F_H \times y$$

$$\sum M_A = (200 \times 0.2) + (-125 \sin 60 \times 0.3) - 80{:}0$$

$$\sum M_A = 40 \quad 32.47 \quad 80$$

$$\sum M_A = -7.5 \, N.m$$

$$\sum M_A = \sum F_V \times x$$

$7.5 = 58.25 \times x$

$x = 7.5/58.25 = 0.12\ m$

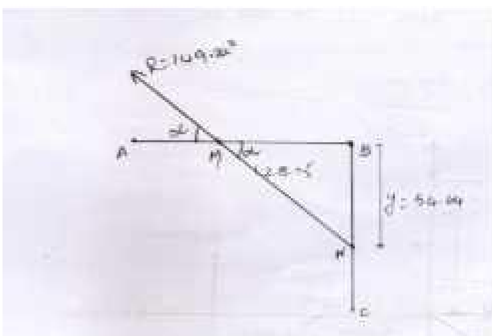
$x = 128.75mm$

$$\sum M_A = \sum F_v \times y$$

$7.5 = 137.25 \times y$

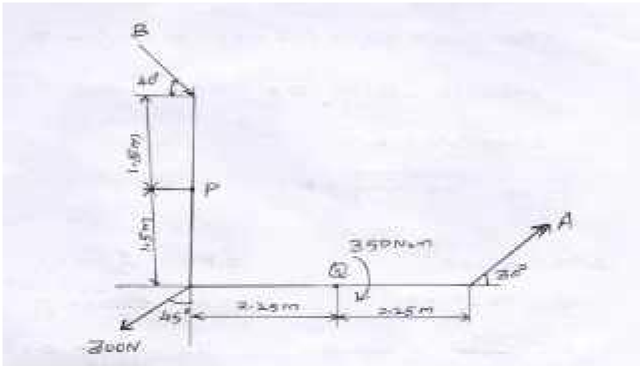
$y = 7.5/137.25 = 0.05m$

$y = 54.64\ mm$



Problem:

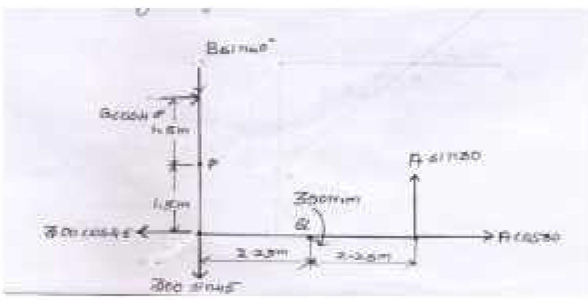
A system of forces acts as shown in fig.find the magnitude of A and B so that resultant of the force system passes through P and Q



To Find:

Forces acts on A and B

Soln: Free body diagram



The resultant forces passes through P and Q is moment About pis zero and also moment about Q 0

It only means that the algebraic sum of moment about P and Q is equal to zero

$\sum M_P = 0$ $\quad\quad\quad\quad \downarrow + \ \uparrow -$

$$\sum M_P = (+B\ \cos 40 \times 1.5) + (300 \cos 45 \times 1.5) + 350 + (-A \sin 30 \times 4.5) + (-A \cos 30 \times 1.5) = 0$$

$$\sum M_P = 1.149\,B + 318.19350 - 2.25A\ - 1.29$$

$$\Sigma M_P = 1.49B\ + 668.19 - 3.54A = 0$$

$-3.54A + 1.149B = -668.19$----------->(1)

$\sum M_Q = 0 \quad \downarrow + \uparrow -$

$\sum M_Q = (B\,cos40 \times 3) + (-B\,sin40 \times 2.25) + (-300sin\,45 \times 2.25) + 350 + (-A\,sin$
30×2.25)=0

$2.29\,B - 1.44B - 477 + 350 - 1.125A = 0$

$0.85B - 127 - 1.125A = 0$

$-1.25A + 0.85B = 127 ----------> (2)$

Solve 1&2

$-3.54A + 1.149B = -668.19$-----(1)

$-1.125A + 0.85B = 127$--------(2)

$(1) \times 1.25 \ \Rightarrow\!\!> -3.982A + 1.292B = -751.7$

$(2) \times 3.54 \ \Rightarrow\!\!> \ 3.982A \ | \ (\ ) \ 3.009B = < \ > 449.58$

$$-171B = -1201.29$$

$B = (-1201.29)/(-1.71)$

$B = 702.508N$

B Value substituting in Eqn (1)

$-3.54A + 1.149 \times 702.508 = -668.19$

$-3.54A + 807.182 = -668.19$

$-3.54A = -668.19 - 807.182$

$-3.54A = -1475.37$

$A = (-1475.37)/(-3.54)$

$A = 416.77\,N$

Result:-

$Force\ on\ A = 416.77N$

$Force\ on\ B\ = 702.508N$

Take moment about 'A'

$\sum M_A = 0$

$\sum M_A = (500 \times 11) + (-200 \times 7) + (1200 \times 5) + (-300 \times 2)$

$\sum M_A = 5500 = 1400 + 6000 - 600$

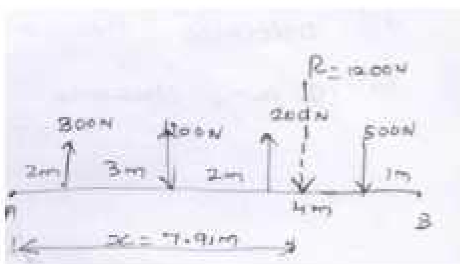
$\sum M_A = 9500\,N.m$

By varignon's theorem

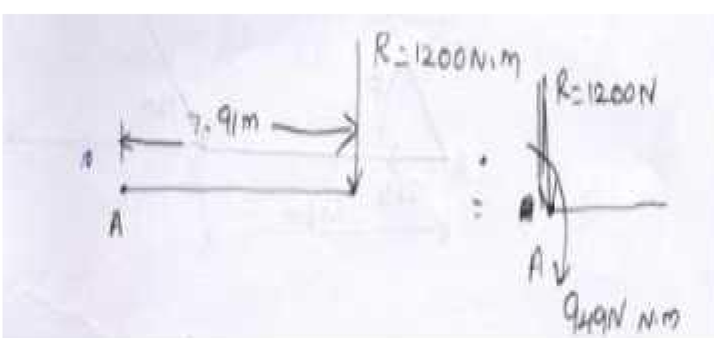
$\sum M_A = R \times x$

$9500 = 1200 \times x$

$x = 7.91m$



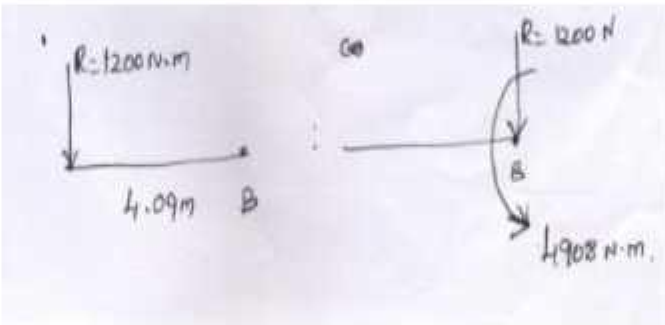
Force couple systemant 'A'



$Couple\ at\ A = 1200 \times 7.91$

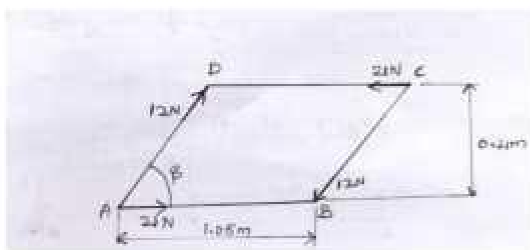
$A = 9492\ N.m$

Couple system at B



Problem:

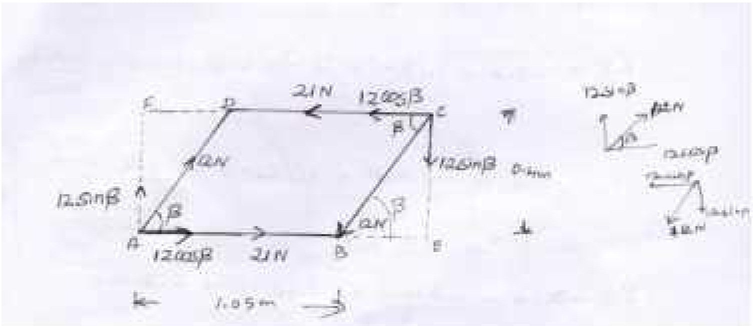
A plate ABCD in the shape of parallelogram is acted upon the two couples, as shown in the fig. Determine the angle B if the resultant couple is 1.8 N.m clockwise



Given:

Resultant couple =1.8N. m

Free body diagram



$Distance\ of\ AE = AB + BE$

$AB = 1.05\ m$

To find BE

$$tan\beta = \frac{CE}{BE} = \frac{0.4}{BE}$$

$BE = \ 0.4/tan\ \beta$

$AE = AB + BE$

$$AE = 1.05 + \frac{0.4}{tan\,\beta}$$

Given the resultant couple $\sum M_A = 1.8N.M$

Take moment about A

$\sum M_A = [-21 \times 0.4] + [-12\ cos\beta \times 0.4] + [12 sin\beta \times AE]$

$\sum M_A = 1.8\ N.M$

$$1.8 = -8.4\ - 4.8cos\beta + 12\ sin\beta \times [1.05 + \frac{0.4}{tan\beta}]$$

$$1.8 = -8.4\ - 4.8cos\beta + 12.6\ sin\beta + \frac{4.8}{\frac{sin\beta}{cos\beta}}\ sin\ \beta \qquad tan\theta = \frac{sin\,\theta}{cos\,\theta}$$

$$1.8 = -8.4 - 4.8\,cos\beta + 12.6\,sin\,\beta + 4.8\,cos\,\beta$$

$$1.8 + 8.4 = -4.8\,cos\beta + 12.6\,sin\beta + 4.8cos\,\beta$$

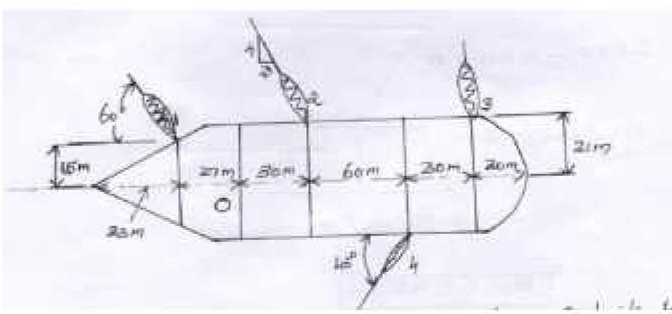
$$\mathbf{10.2} = \mathbf{12.6}sin\beta$$

$$Sin\beta = \frac{\mathbf{10.2}}{\mathbf{12.6}}$$

$$B = sin - \mathbf{1}\left(\frac{10.2}{12.6}\right) \qquad B = 54°$$

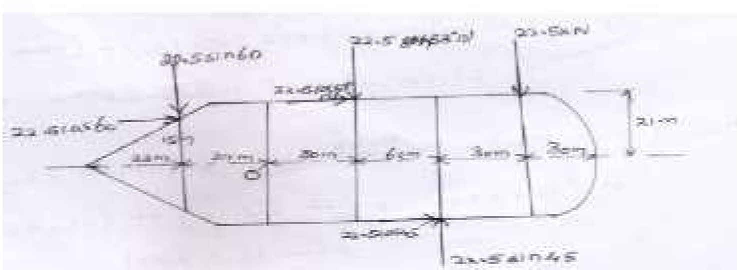
Problem

Four tugboats are used to bring an ocan large ship to us pier. Each tugboat exerts a 22.5KN force in direction as shown in fig (i) determine the equivalent force couple system at 'o'



(ii) Determine a single equivalent force and its location along the longitudinal axis of the ship

Soln: Free body diagram



$\Sigma F_H = 22.5\ cos\ 60 + 22.5\ cos + 53° + 22$

$\sum F_H$=40.65KN

$\sum F_v = -22.5\ sin\ 60 - 22.5 + \ sin\ 53° + 22.5\ sin\ 45 - 22.5$

$\sum F_v = -44.04N$

Resulant R= $\sqrt{(\sum(F_H)^2 + (\sum(F_V)^2}$

$R = \sqrt{[40.6.5]^2 + [-44.04]^2}$

$R = 59.95KN$

Direction $\theta = tan^{-1}\left(\frac{\sum F_V}{\sum F_H}\right) = tan^{-1}\left[\frac{44.04}{40.65}\right] = 47°3'$

To find location:

$\sum M_o = R \times x$

$\sum M_a = (22.5cos\ 60\ \times 15) + (-22.5sin\ 60 \times 27) + (22.5sin\ 53° \times 30) +$

$(22.5\ cos\ 53°\ 31 \times 21)\ |\ (22.5 \times 120)\ |\ (\ \ 22.5\ cos\ 45 \times 21)\ |\ (\ \ 22.5\ \ 45$
$\times 90)$

$\sum M_a = (11.25 \times 15) + (-19.48 \times 27) + (17.99 \times 30) + (13.5 \times 21) +$

$(22.5 \times 120) + (-15.9 \times 21) + (-15.9 \times 90)$

$\sum M_a = 1319.5\ KN.m$

Location

$\sum M_a = 1319.5$

$\sum M_a - R \times x$

$x = 1319/59.95$

$x = 22.01m$

Magnitude of couple

$M = R \times x$

$= 59.95 \times 22.01m$

$M = 1319.55\ KN.m$

<u>Equilibrium of Rigidbodies – support Reactions</u>

Beam:

A beam is horizontal structural member which carries a load transverse (perpendicular) to its axis and transfers the load through support reactions to supporting columns or walks

Frame:

A structure made up of up of several members riveted or welded together is known as frame.

Support Reactions of Beam:-

The force of resistance exerted by the support on the beam is called support reaction.

Types of support

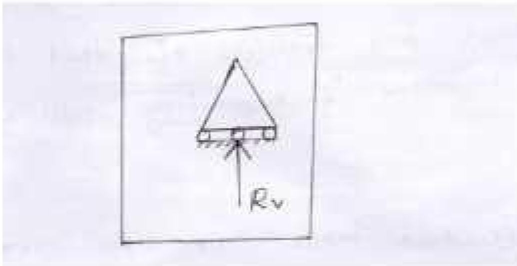
1. Roller support

2. Hinged support

3. Fixed support

1. Roller support:

It consist of the rollers as the bottom. It has only one vertical reaction.

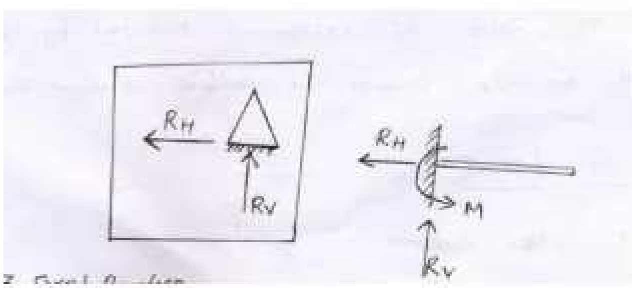


2. Hinged support:

It resists the horizontal and vertical moment. It has two Reaction

(i) Horizontal reaction

(ii) Vertical reaction



3. Fixed reaction:

It is the Stronged support. This support has following reaction

(i) Vertical reaction

(ii) Horizontal reaction

(iii) Rotational reaction (moment)

Types of load:

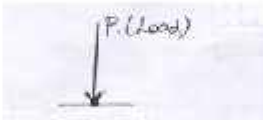
1. Point load

2. Uniformly distributed load (UDL)

3. Uniformly varying load (UVL)

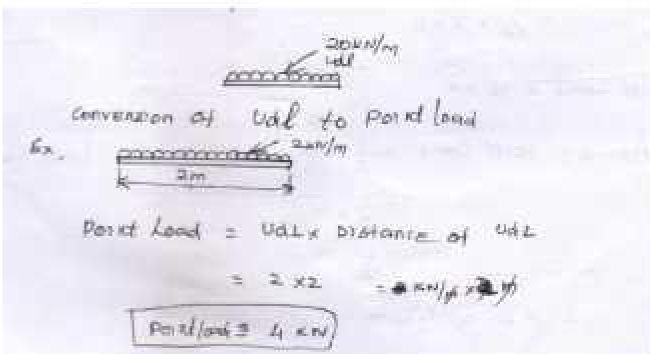
1. Point load:

Load which is acting at a particular point (i.e.) point load;



2. Uniformly distributed Load

The load which is spread over a beam in such a manner that each unit length at the beam carries same intensity of the load is called uniformly distributed load.



$$Point\ load\ = udl \times distance of\ udl$$

$$= 2 \times 2$$

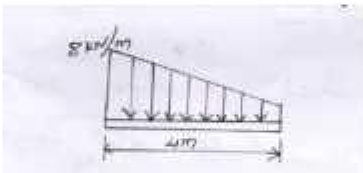
$$Point\ = 4KN$$

$$Location\ of\ point\ = \frac{uniformly\ disbuted\ load\ length}{2}$$

3. Uniformly varying load

A load which is varying from the particular point along particular length is called uniformly varying load

Conversion of uniformly varying load to point load



$Point\ load\ = 1/2 \times\ uniformly\ varying\ load\ \times\ \ length\ of\ uniformly\ load$

$= 1/2 \times 8 \times 4$

$Point\ load\ = \mathbf{16}KN$

$Location\ of\ point\ load\ = \frac{1}{3} \times\ uniformly\ varying\ load\ length$

$= \frac{1}{3} \times 4$

$= 4/3$

$L.P.L = \ 1.33m$

Procedure for solving the support reaction problem

1.Sum of all the horizontal force is zero $\sum F_H = 0$

To find R_H (R_{HB})

2.Sum of all the vertical force is zero $\sum FV=0$

To find R_{VA}–R_{VB}

3. Take moment of force about A ($\sum MA$=0 To R_{FV}=0 to find R_{VA}

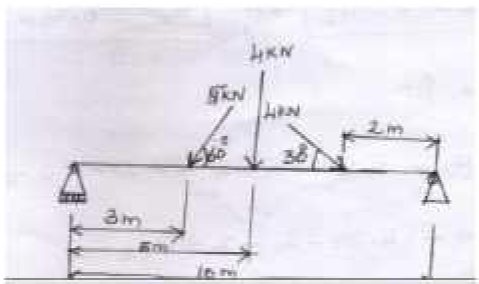
(OR)

4. Substitute R_{VB} in$\sum FV$=0 Eqn

To find R_{VA}

Problem-I

A beam is acted upon by a system of forces shown in fig. Find the support Reactions

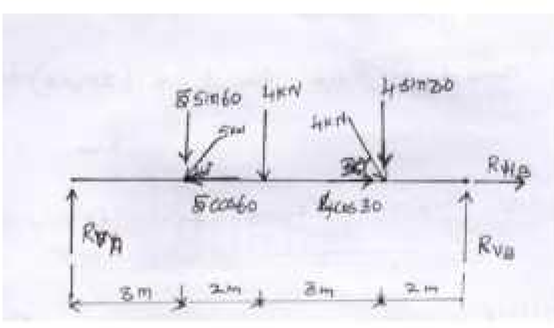


To find

Reaction at the support. R_V&R_{VB},R_{HB}

Soln:

Free body diagram



$\sum F_H$ =0 $\quad \overset{+}{\rightarrow}\overset{-}{\leftarrow}$

$\sum F_H = -5\cos 60 + 4cos30 + R_{HB}$

$\sum F_H = 0.96 + R_{HB}$

$R_{HB} = -0.96KN$

$R_{HB} = 0.96N(\rightarrow)$

$\sum F_V = 0$

$\sum F_V = -R_{VA} - 5sin\,60 - 4 - 4sin\,30 + R_{VB} = 0$

$RV_A - 4.33 - 4 - 2 + RV_B = 0$

$RV_A + RV_B - 10.33 = 0$

$RV_A + RV_B = 10.33$ ------> (1)

Take moment of force about A

$\sum M_A = 0$

$\sum M_A = (5sin\,60 \times 3) + (4 \times 2) + (4sin\,30 \times 8) + (Rvb \times 10) = 0$

$12.99 + 8 + 16 - 10\,R_{VB} = 0$

$36.99 - 10\,R_{VB} = 0$

$-10\,R_{VB} = (-36.99)/(-10)$

$R_{VB} = \dfrac{-36.99}{-10}$

$Ans: R_{VB} = 3.69\,N$

$R_{VB\ value}$ sub in Eqn ------> (1)

$R_{VA} + R_{VB} = 10.33$

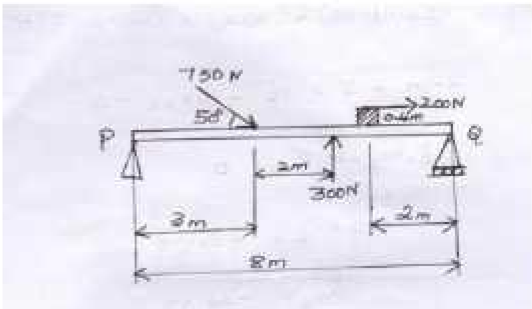
$R_{VA} + 3.69 = 10.33$

$R_{VA} = 10.33 - 3.69$

$R_{VA} = 6.64N$

Problem 2

A beam is loaded as shown in fig find the magnitude direction and the location of the resultant of the system of forces.



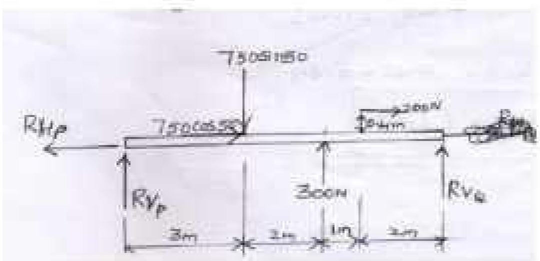
To find

1. Resultant force &direction

2. location of resultant force

Soln

Free body diagram



$\sum F_V = 0$ ↑+ ↓ -

$-R_{HP} + 750\, cos\, 50 + 200 = 0$

$-R_{HP} + 482 + 200 = 0$

$-R_{HP} + 682 = 0$

$-R_{HP} = -682N$

↑ ↓

$\sum F_V = 0 \quad + \quad -$

$R_{VP} - 750\ sin\ 50 + 300 + R_{VQ} = 0$

$R_{VP} + R_{VQ} - 574.53 + 300 = 0$

$R_{VP} + R_{VQ} = -300 + 574.53$

$R_{VP} + R_{VQ} = 274.53\ N$---------(1)

Take moment About 'p'

$\sum M_P = 0$

$(750\ sin\ 50 \times 3) + (200 \times 0.4)(-R_{VQ} \times 8) = 0$

$1753.59 - 1500 + 80 - 8\ R_{VQ} = 0$

$-8\ R_{VQ} = -1753.59 + 1500 - 80$

$-8\ R_{VQ} = 333.59$

$R_{VQ} = (-333.59)/(-8)$

$R_{VQ} = 41.9N$

$R_{VQ} = 41.69N$

R_{VQ} value sub in Eqn(1)

$R_{VP} + R_{VQ} = 274.53$

$R_{VP} + 41.69 - 274.53$

$R_{VP} = 274.53 + 41.69$

$R_{VP} = 232.83N$

Resultant $R = \sqrt{(\sum F_H)^2 + (\sum(F_V)^2}$

Resultant force consider $R = \sqrt{(\sum H_P)^2 + RV_P{}^2}$

Only hinged support $R = \sqrt{[682]^2 + [232.83]^2}$

$R = 720N$

Direction $\theta = tan^{-1}\left[\frac{\sum V_P}{\sum V_H}\right]$

$\theta = tan^{-1}\left(\frac{232.83}{682}\right)$

$\theta = 18°50'$

Location

By varignon's theorem

$\sum M_P = R \times x$

$\sum M_P = 0 \; +\downarrow \; \uparrow-$

$\sum M_P = (750 \times sin\,50 \times 3) + (300 \times 5) + (200 \times 0.4) + (-R_{VQ} \times 8)$

$\sum M_P = 1753.59 - 1500 - 800 - 333.52$

$\sum M_P = 0.07\,N.m$

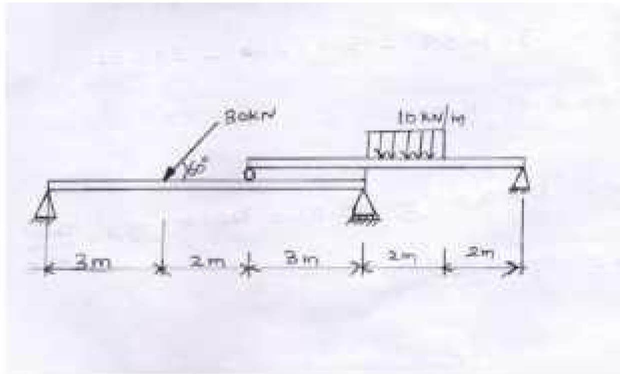
$\sum M_P - R \times x$ $\quad$ –> $0.07 - 720 \times x$ $\quad$ –> $\times - 0.07/720$

Problem: 3

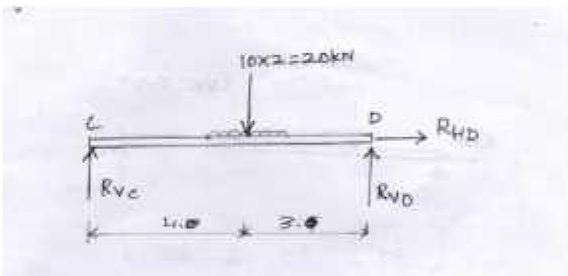
Two beams AB and CD are shown in fig Aand D are hinged supports Band C are rollers supports.

(i) Sketch the free body diagram of the beam AB and deter mine the reaction at the support A and B.

(ii) Sketch the free body diagram of the beam CD and determine the reactions at the supports C and D₀



Free diagram of beam CD



$\sum F_H = 0 \underset{+\ -}{\rightarrow\leftarrow}$

$R_{HD} = 0$

$R_{VC} - 20 + R_{VD} = 0$

$R_{VC} + R_{VD} = 20$---------- (2)

Take moment abut c=0

$\sum M_C = (20 \times 4)(-R_{VD} \times 7) = 0$

$80 - 7\,R_{VD} = 0$

$R_{VD} = \frac{-80}{-7}$

$R_{VD} = 11.42KN$

R_{VD} value sub in Eqn (1)

$R_{VC}\ + 11.42 = 20\ \ => R_{VC} = 20\ -\ 11.42$

Unit-III

PROPERTIES OF SURFACES AND SOLIDS

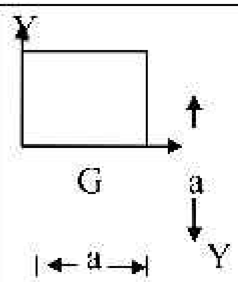
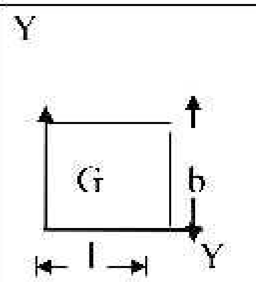
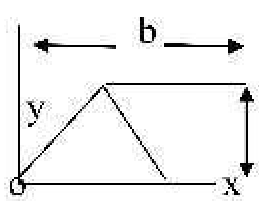
Centroid:

Centroid is defined as a point on a surface the whole area of the surface acts.

Centre of gravity:

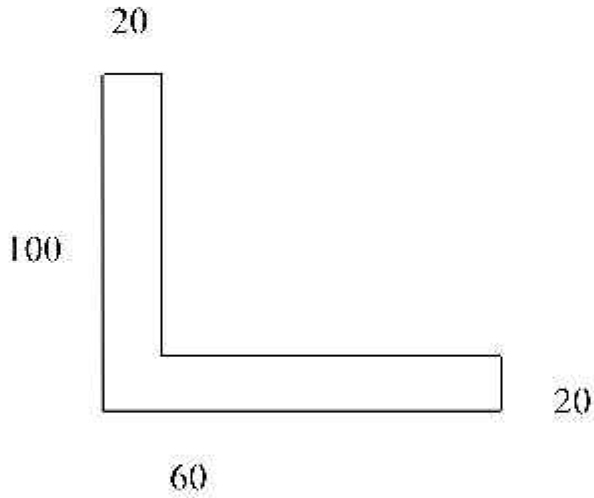
Centre of gravity is defined as the point through which the entire weight of the body acts.

Centroid of simple plane figure:

Sl.No	Name	Shape	X	Y	Area
1.	Square		$\frac{a}{2}$	$\frac{a}{2}$	a^2
2.	Rectangle		$\frac{l}{2}$	$\frac{l}{2}$	lb
3.	Triangle (Isosceles)		$\frac{b}{2}$	$\frac{h}{3}$	$\frac{1}{2}$bh

Problem 1:

Determine the Centroid of L section



Centroid

$$\bar{X} = \frac{a_1x_1+a_2x_2}{a_1+a_2}$$

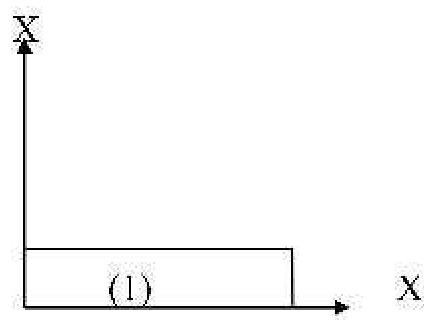
$$\bar{Y} = \frac{a_1y_1+a_2y_2}{a_1+a_2}$$

Section (1)

$$a_1 = 60 \times 20 = 1200mm^2$$

$$x_1 = \frac{60}{2} = 30mm$$

$$y_1 = \frac{20}{2} = 10mm$$

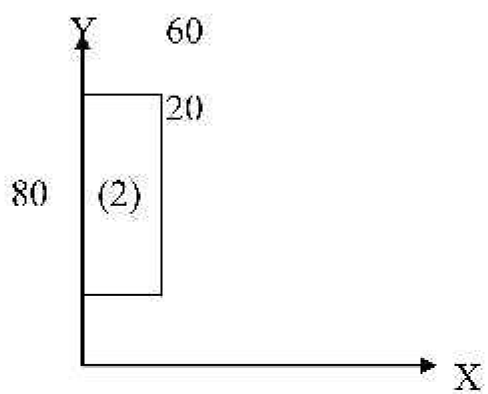


Section (2)

$$a_2 = 20 \times 80 = 160mm^2$$

$$x_2 = \frac{20}{2} = 10mm$$

$$y_2 = 20 + \frac{80}{2} = 60mm$$



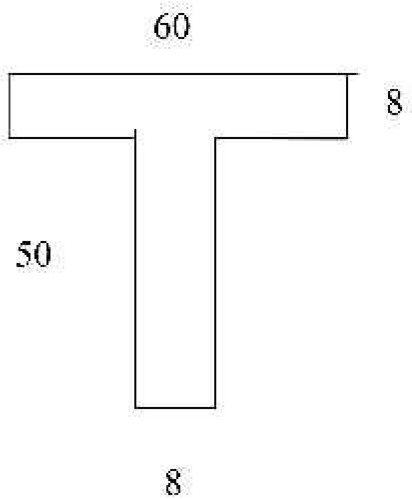
$$\bar{X} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{1200 \times 30 + 1600 \times 10}{1200 + 1600}$$

$$\bar{X} = 18.57mm$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{1200 \times 10 + 1600 \times 60}{1200 + 1600}$$

$$\bar{Y} = 38.57mm$$

2. Find the Centroid of T section



Section (1)

$$a_1 = 8 \times 50 = 400mm^2$$

$$x_1 = 26 + \frac{8}{2} = 30mm$$

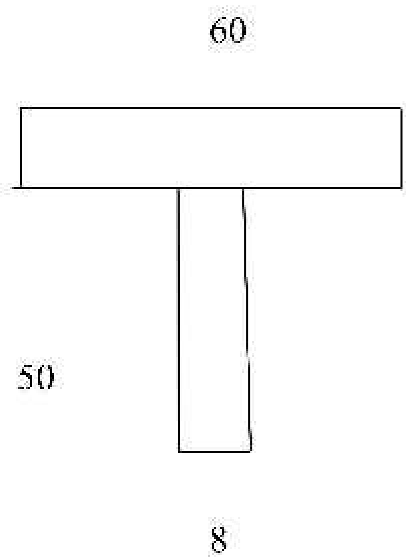
$$y_1 = \frac{50}{2} = 25mm$$

Section(2)

$$a_2 = 60 \times 8 = 480mm^2$$

$$x_2 = \frac{60}{2} = 30mm$$

$$y_2 = 50 + \frac{8}{2} = 54mm$$



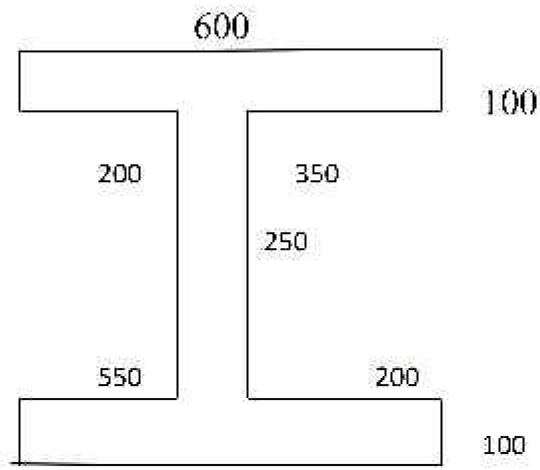
$$\bar{X} = \frac{a_1x_1+a_2x_2}{a_1+a_2} = \frac{400\times30+480\times300}{1200+480}$$

$$\bar{X} = 30mm$$

$$\bar{Y} = \frac{a_1y_1+a_2y_2}{a_1+a_2} = \frac{400\times25+480\times54}{400+480}$$

$$\bar{Y} = 40.81mm$$

3. Locate the Centroid of the I section shown.



Soln:

$$\bar{X} = \frac{a_1x_1+a_2x_2+a_3x_3}{a_1+a_2+a_3}$$

$$\bar{Y} = \frac{a_1y_1+a_2y_2+a_3y_3}{a_1+a_2+a_3}$$

Section (1)

$$a_1 = 800\times100 = 80000mm^2$$

$$x_1 = \frac{800}{2} = 400mm$$

$$y_1 = \frac{100}{2} = 50mm$$

Section(2)

$$a_2 = 250 \times 100 = 25 \times 10^3 mm^2$$

$$x_2 = 550 + \frac{100}{2} = 600mm$$

$$y_2 = 100 + \frac{250}{2} = 225mm$$

Section (3)

$$a_3 = 600 \times 100 = 60 \times 10^3 mm^2$$

$$x_3 = 350 + \frac{600}{2} = 650mm$$

$$y_3 = 100 + 250 + \frac{100}{2} = 400mm$$

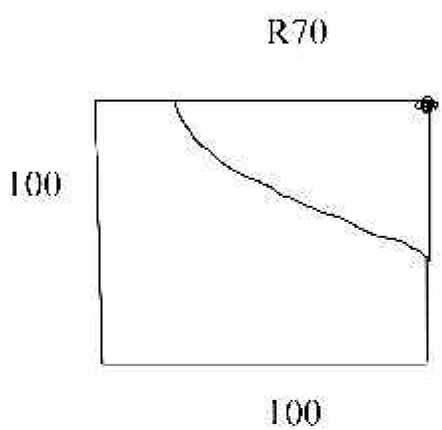
$$\bar{X} = \frac{(80 \times 10^3 \times 400) + (25 \times 10^3 \times 600) + (60 \times 10^3 \times 650)}{80 \times 10^3 + 25 \times 10^3 + 60 \times 10^3}$$

$$\bar{X} = 521.21mm$$

$$\bar{Y} = \frac{(80 \times 10^3 \times 50) + (25 \times 10^3 \times 225) + (60 \times 10^3 \times 400)}{80 \times 10^3 + 25 \times 10^3 + 60 \times 10^3}$$

$$\bar{Y} = 203.78mm$$

4. Locate the Centroid of the Area.

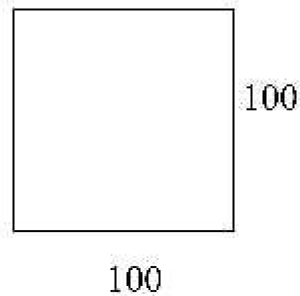


Soln:

$$\bar{X} = \frac{a_1 x_1 - a_2 x_2}{a_1 + a_2}$$

$$\bar{Y} = \frac{a_1 y_1 - a_2 y_2}{a_1 + a_2}$$

Section (1)



$a_1 = 100 \times 100 = 10 \times 10^3 mm^2$

$x_1 = \frac{100}{2} = 50mm$

$y_1 = \frac{100}{2} = 50mm$

Section (2)

$$a_2 = \frac{1}{4} \times \pi r^2 = \frac{1}{4} \times \pi \times 70^2 = 38.48mm^2$$

$$x_2 = \frac{4r}{3\pi} = \frac{4 \times 70}{3\pi} = 70.29mm$$

$$y_2 = \frac{4r}{3\pi} = \frac{4 \times 70}{3\pi} = 70.29mm$$

$$\bar{X} = \frac{a_1 x_1 - a_2 x_2}{a_1 + a_2}$$

$$\bar{X} = \frac{(10 \times 10^3 \times 50) - (3848 \times 70.29)}{10 \times 10^3 - 3848}$$

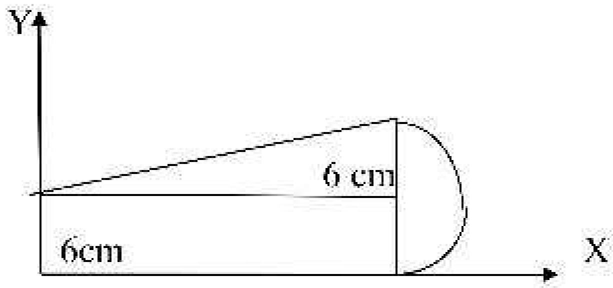
$$\bar{X} = 37.3mm$$

$$\bar{Y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

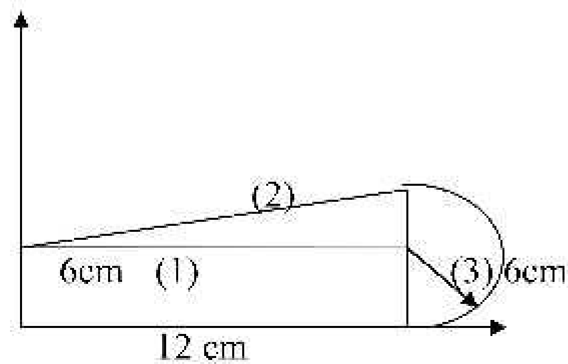
$$\bar{Y} = \frac{(10 \times 10^3 \times 50) - (3848 \times 70.29)}{10 \times 10^3 - 3848}$$

$$\bar{Y} = 373mm$$

5. Locate the Centroid of zero shown in fig.



Soln:



$\bar{X} = \frac{a_1x_1+a_2x_2+a_3x_3}{a_1+a_2+a_3}$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

Section (1)

$a_1 = 12 \times 6 = 72cm$

$$X_1 = \frac{12}{2} = 6cm$$

$$Y_1 = \frac{6}{2} = 3\ cm$$

Section (2) triangle

$$a_2 = \frac{1}{2}bh = \frac{1}{2} \times 12 \times 6$$

$a_2 = 36cm$

$$x_2 = \frac{b}{3} = \frac{12}{3} = 4cm$$

$$y_2 = 6 + \frac{h}{3} = 6 + \frac{6}{3} = 8cm$$

Section (3) Semi circle

$$a_3 = \frac{1}{2} \times \frac{\pi d^2}{4^2} = \frac{1}{2} \times \frac{\pi}{4} \times 12^2 = 56.24cm$$

$$x_3 = 12 + \frac{4r}{3\pi} = 12 + \frac{4 \times 6}{3\pi} = 14.5cm$$

$$y_3 = \frac{d}{2} = \frac{12}{2} = 6cm$$

$$\bar{X} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{(72 \times 6) + (36 \times 4) + (56.24 \times 14.5)}{72 + 36 + 56.24}$$

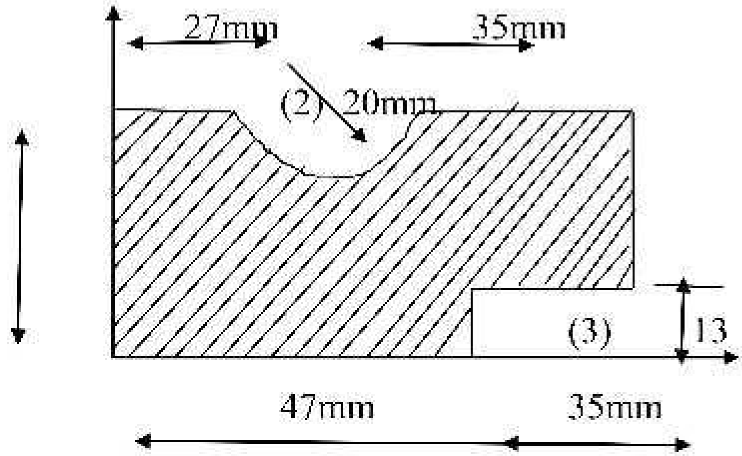
$$\bar{X} = 8.47cm$$

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(72 \times 3) + (36 \times 8) + (56.24 \times 6)}{72 + 36 + 56.24}$$

$$\bar{Y} = \frac{841.44}{164.24}$$

$$\bar{Y} = 5.12cm$$

5. Find the Centroid of the shaded area shown below



Soln:

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

Soln:

$$a_1 = 82 \times 40 = 3280mm^2$$

$$x_1 = \frac{82}{2} = 41mm$$

$$y_1 = \frac{40}{2} = 20mm$$

Section (2)

$$a_2 = \frac{1}{2} \times \frac{\pi}{4} \times d^2 = \frac{1}{2} \times \frac{\pi}{4} \times 40^2 = 628.31mm^2$$

$$x_2 = 27mm$$

$$y_2 = 40 - \frac{4r}{3\pi} = 40 - \frac{4 \times 20}{3\pi} = 31.51mm$$

Section (3)

$$a_3 = 35 \times 13 = 455mm^2$$

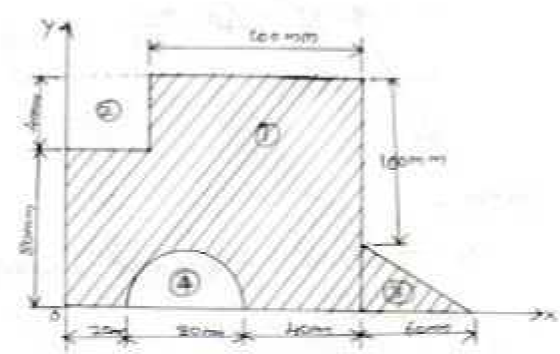
$$x_3 = 82 - \frac{35}{2} = 64.5mm$$

$$y_3 = \frac{13}{2} = 6.5\ mm$$

$$\bar{X} = \frac{(3280 \times 41) - (628.31 \times 27) - (455 \times 64.5)}{3280 - 628.31 - 455} = 40.14mm$$

$$\bar{Y} = \frac{(3280 \times 20) - (628.31 \times 31.51) - (455 \times 6.5)}{3280 - 628.31 - 455} = 19.50mm$$

7. Determine the centroid co-ordinates of the area shown in fig. below with respect to the shown x-y coordinate system.



Soln:

$$\bar{X} = \frac{a_1x_1 - a_2x_2 + a_3x_3 - a_4x_4}{a_1 - a_2 + a_3 - a_4}$$

$$\bar{Y} = \frac{a_1y_1 - a_2y_2 + a_3y_3 - a_4y_4}{a_1 - a_2 + a_3 - a_4}$$

Section (1)

$$a_1 = 140 \times 120 = 16800mm^2$$

$$x_1 = \frac{140}{2} = 70mm$$

$$y_1 = \frac{120}{2} = 60mm$$

Section (2)

$$a_2 = 40 \times 40 = 1600mm^2$$

$$x_2 = \frac{b}{2} = \frac{40}{2} = 20mm$$

$$y_2 = 80 + \frac{40}{2} = 20mm$$

Section (3)

$$a_3 = \frac{1}{2}bh = \frac{1}{2} \times 60 \times 20 = 600mm^2$$

$$x_3 = 140 + \frac{b}{3} = 140 + \frac{60}{3} = 160mm$$

$$y_3 = \frac{h}{3} = \frac{20}{3} = 6.66mm$$

Section (4)

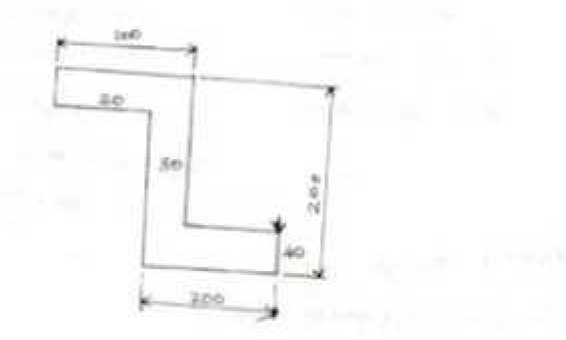
$$a_4 = \frac{1}{2} \times \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (80^2) \times \frac{1}{2} = a_4 = 2513mm^2$$

$$x_4 = 20 + \frac{d}{2} = 20 + \frac{80}{2} = 60mm$$

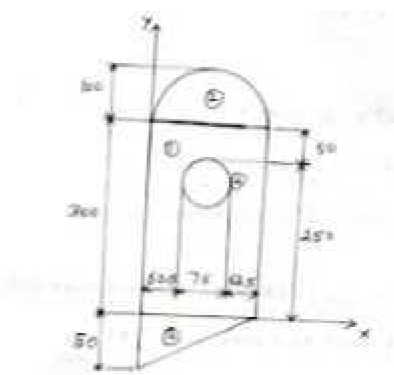
$$y_4 = \frac{4r}{3\pi} = \frac{4 \times 40}{3\pi} = 16.97mm$$

$\bar{X} = 81.97mm \qquad \bar{Y} = 60.97mm$

8. Determine the centroid of Z section



9. Locate the centroid of the plane area shown in fig:



Soln:

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3 - a_4x_4}{a_1 + a_2 + a_3 - a_4}$$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 - a_4y_4}{a_1 + a_2 + a_3 - a_4}$$

Section (1)

Rectangle

$$a_1 = 200 \times 400 = 600mm^2$$

$$x_1 = \frac{200}{2} = 100mm$$

$$y_1 = \frac{400}{2} = 200mm$$

Section (2)

$$a_2 = \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \pi \times 100^2$$

$$a_2 = 15707.96mm^2$$

$$x_2 = \frac{d}{2} = \frac{200}{2} = 100mm$$

$$y_2 = 300 + \frac{4r}{3\pi}$$

$$y_2 = 300 + \frac{4 \times 100}{3\pi} = 342.44mm$$

Section (3)

$$a_2 = \frac{1}{2}bh = \frac{1}{2} \times 200 \times 50 = 5000mm^2$$

$$x_3 = \frac{b}{3} = 66.66mm$$

$$y_3 = -\frac{50}{3} = -16.67mm$$

Section (4)

$$a_4 = \pi r^2 = \pi \times (37.5)^2 = 4417.86mm^2$$

$$x_4 = 62.5 + \frac{d}{2} = 62.5 + \frac{75}{2} = 100m$$

$$y_4 = 250 - \frac{d}{2} = 250 - \frac{75}{2} = 212.5\ mm$$

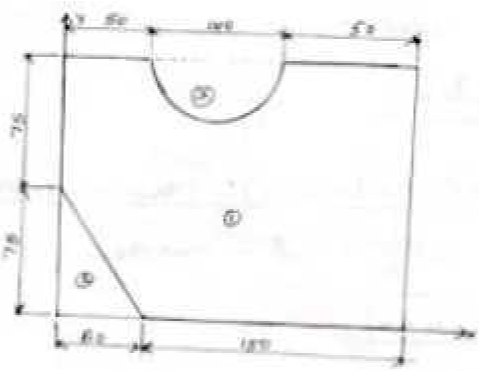
$$\bar{X} = \frac{(6000 \times 100) + (15707.96 \times 100) + (5000 \times 66.66) - (4417.86 \times 100)}{60000 + 15707.96 + 5000 - 4417.86}$$

$$\bar{X} = 97.812mm$$

$$\bar{Y} = \frac{(6000 \times 200) + (15707.96 \times 342.44) + \left(5000 \times (-16.67)\right) - (4417.86 \times 212.5)}{6000 + 15707.96 + 5000 - 4417.86}$$

$$\bar{Y} = 175.07mm.$$

10. Locate the centroid of the plane area shown in fig.



Soln:

$$\bar{X} = \frac{a_1x_1 - a_2x_2 - a_3x_3}{a_1 - a_2 - a_3}$$

$$\bar{Y} = \frac{a_1y_1 - a_2y_2 - a_3y_3}{a_1 - a_2 - a_3}$$

Section (1) Rectangle

$$a_1 = 200 \times 150 = 30 \times 10^3 mm^2$$

$$x_1 = \frac{200}{2} = 100mm$$

$$y_1 = \frac{150}{2} = 75mm$$

Section (2) Triangle

$$a_2 = \frac{1}{2}bh = \frac{1}{2} \times 50 \times 75 = 1875mm^2$$

$$x_2 = \frac{b}{3} = \frac{50}{3} = 16.66mm$$

$$y_3 = \frac{h}{3} = \frac{75}{3} = 25mm$$

Section (3)

$$a_3 = \frac{\pi r^2}{2} = \frac{\pi \times 50^2}{2} = 3926.99mm^2$$

$$y_3 = 150 - \frac{4r}{3\pi} = 150 - \frac{4 \times 50}{3\pi} = 128.77mm$$

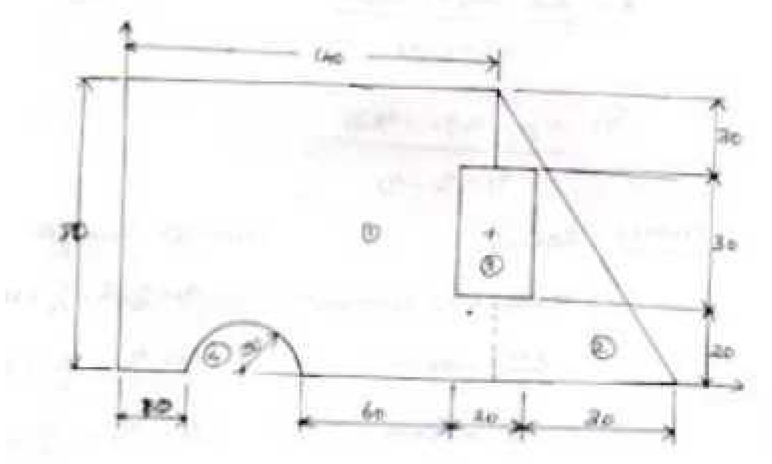
$$\bar{X} = \frac{(30 \times 10^3 \times 100) - (1875 \times 16.66) - (3926.99 \times 100)}{30 \times 10^3 - 1875 - 3926.99}$$

$$X = 105.45mm$$

$$\bar{Y} = \frac{(30 \times 10^3 \times 75) - (1875 \times 25) - (3926.99 \times 128.77)}{30 \times 10^3 - 1875 - 3926.99}$$

$$\bar{Y} = 70.14mm$$

11. Locate the centroid of the sectional area as shown in fig:



Soln:

$$\bar{X} = \frac{a_1x_1 + a_2x_2 - a_3x_3 - a_4x_4}{a_1 + a_2 - a_3 - a_4}$$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 - a_3y_3 - a_4y_4}{a_1 + a_2 - a_3 - a_4}$$

Section (1) Rectangle

$$a_1 = 140 \times 80 = 11200mm^2$$

$$x_1 = \frac{140}{2} = 70mm$$

$$y_1 = \frac{80}{2} = 40mm$$

Section (2) Triangle

$$a_2 = \frac{1}{2}bh = \frac{1}{2} \times 40 \times 80$$

$$a_2 = 1600mm^2$$

$$x_2 = 140 + \frac{40}{3} = 153.33mm$$

$$y_2 = \frac{h}{3} = \frac{80}{3} = 26.66mm$$

Section (3) Rectangle

$$a_3 = 20 \times 30 = 600mm^2$$

$$x_3 = 130 + \frac{20}{2} = 140mm$$

$$y_3 = 20 + \frac{30}{2} = 35mm$$

Section (4) Semicircle

$$a_4 = \frac{\pi r^2}{2} = \frac{\pi \times 30^2}{2} = 1413.71mm^2$$

$$x_4 = 10 + \frac{d}{2} = 10 + \frac{60}{2} = 40mm$$

$$y_4 = \frac{4r}{3\pi} = \frac{4 \times 30}{3\pi} = 12.73mm$$

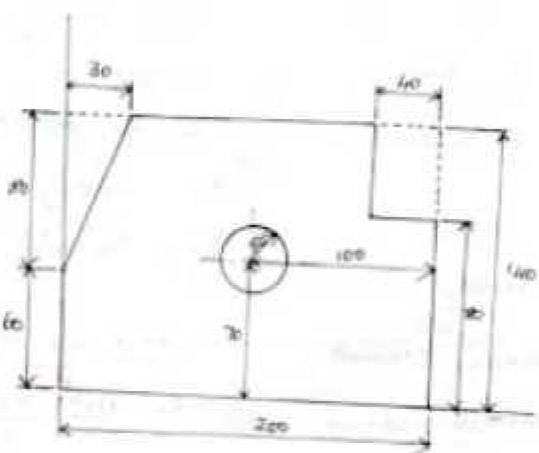
$$\bar{X} = \frac{(11200 \times 70) + (1600 \times 153.33) - (600 - 140) - (1413.71 \times 40)}{11200 + 1600 - 600 - 1413.71}$$

$$\bar{X} = 82.396\text{mm}$$

$$\bar{Y} = \frac{(11200 \times 40) + (1600 \times 26.68) + (600 \times 35) - (1413.71 \times 12.73)}{11200 + 1600 - 600 - 1413.71}$$

$$\bar{Y} = 41.875cm$$

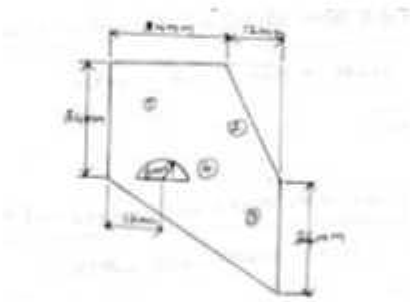
12. Find the centroid of the shaded area shown in fig.



Ans $X = 94.92mm$

$Y = 61.058mm$

13. Locate the centroid for the plane surface shown below.



$$\bar{X} = \frac{a_1x_1 - a_2x_2 - a_3x_3 - a_4x_4}{a_1 - a_2 - a_3 - a_4}$$

$$\bar{Y} = \frac{a_1y_1 - a_2y_2 - a_3y_3 - a_4y_4}{a_1 - a_2 - a_3 - a_4}$$

Section (1) rectangle

$$a_1 = 36 \times 48 = 1728mm^2$$

$$x_1 = \frac{36}{2} = 18mm$$

$$y_1 = \frac{48}{2} = 24mm$$

Section (2) Triangle

$$a_2 = \frac{1}{2}bh = \frac{1}{2} \times 12 \times 24$$

$$a_2 = 144mm^2$$

$$x_2 = 24 + \frac{12}{3} = 28mm$$

$$y_2 = 24 + \frac{24}{3} = 32mm$$

Section (3)

$$a_3 = \frac{1}{2}bh = \frac{1}{2} \times 36 \times 24$$

$$a_3 = 432mm^2$$

$$x_3 = \frac{b}{3} = \frac{36}{3} = 12mm$$

$$y_3 = \frac{h}{3} = \frac{24}{3} = 8mm$$

Section (4) semicircle

$$a_4 = \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \pi \times 6^2 = 56.54mm^2$$

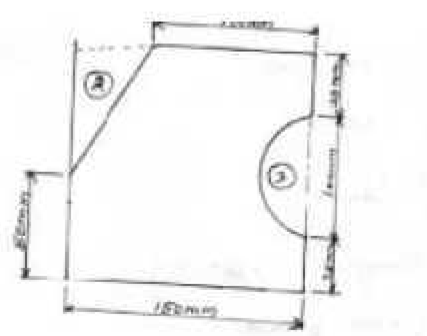
$$x_4 = 12mm$$

$$y_4 = 24 + \frac{4r}{3\pi} = 24 + \frac{4 \times 6}{3\pi} = 26.54mm$$

$$\bar{X} = \frac{(1728 \times 18) - (144 \times 28) - (432 \times 12) - (56.54 \times\times 12)}{1728 - 144 - 432 - 56.54} = 19.35mm$$

$$\bar{Y} = \frac{(1728 \times 24) - (144 \times 32) - (432 \times 8) - (56.54 \times 26.54)}{1728 - 144 - 432 - 56.54} = 29.12mm$$

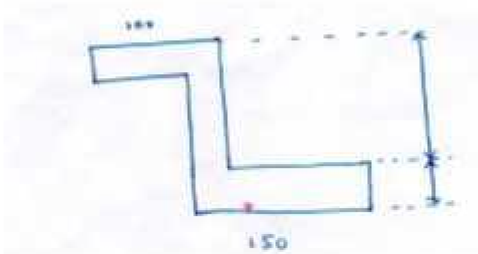
14. Locate the centroid of the plane area shown below.



Ans $X = 70.93mm$

$Y = 68.51mm$

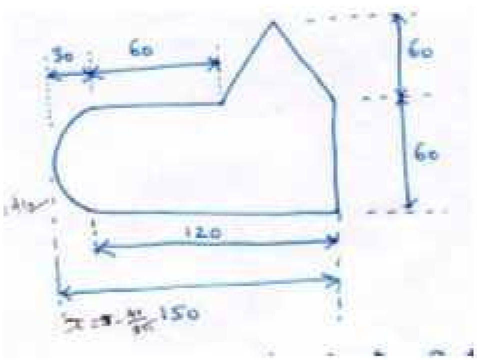
15.Find the centroid for the Z section shown in fig. All dimension are in mm.



$\bar{X} = 102.5mm$

$\bar{Y} = 77.5mm$

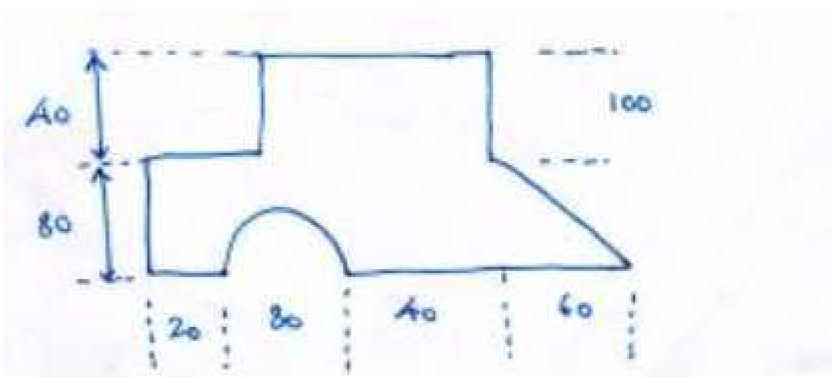
Determine the centroid of the plane uniform lamina as shown in fig. All dimensions are in mm



$\bar{X} = 85.31mm$

$\bar{Y} = 38.64mm$

16.Determine the centroidal coordinate of the given fig. All dimensions are in mm.



Moment of Inertia [polar]

State parallel Axis theorem

It state that, if the moment of inertia of a plane area about an axis through its centroid be denoted by IG the moment of inertia of the area about an axis AB parallel to the first and at a distance 'h' from the centroid is given by

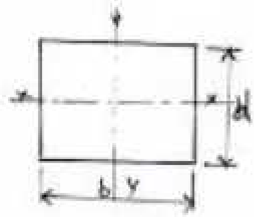
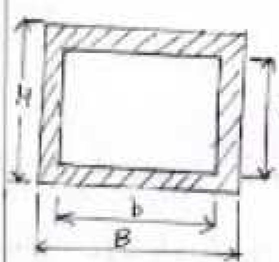
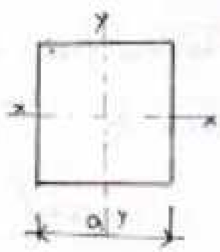
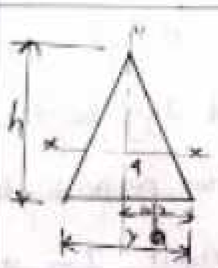
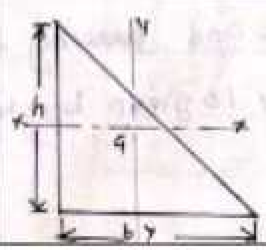
$$I_{AB} = IG + Ah^2$$

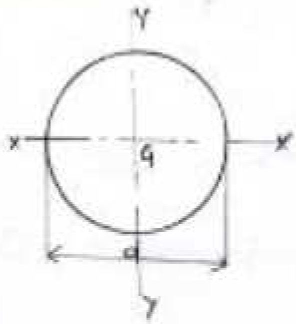
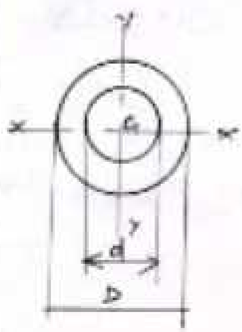
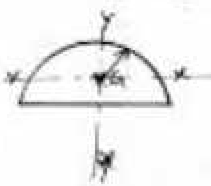
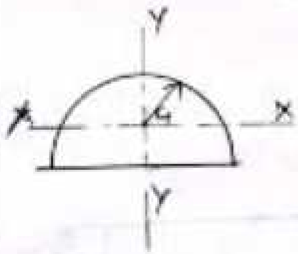
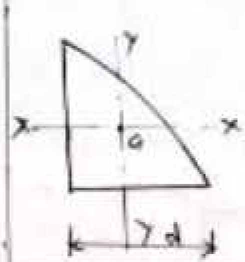
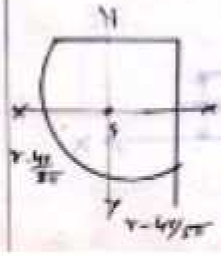
State perpendicular axis Theorem:

It states that 'If I_{xx} and I_{yy} be the moment of inertia of a plane section about two perpendicular axes meeting at 'o' the moment of inertia about I_{zz} about the axis z-z perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by the relation

$$I_{zz} = I_{xx} + I_{yy}$$

Moment of inertia.

Sl.No	Name	Figure	G ($\bar{x}$ and $\bar{y}$) about Area A $\bar{x}$ from left $\bar{y}$ from base	I_{xx}, I_{yy} self centroidal Axis
1	Rectangle		$\bar{x} = b/2$ $\bar{y} = d/2$ $A = b \times d$	$I_{xx} = \frac{bd^3}{12}$ $I_{yy} = \frac{db^3}{12}$ $I_{zz} = I_{xx} + I_{yy}$
2	Hollow Rectangle		$\bar{x} = B/2$ $\bar{y} = H/2$ $A = [Bb - Hh]$	$I_{xx} = \frac{1}{12}[BH^3 - bh^3]$ $I_{yy} = \frac{1}{12}[HB^3 - hb^3]$
3	square		$\bar{x} = a/2$ $\bar{y} = a/2$ $A = a^2$	$I_{xx} = \frac{a^4}{12}$ $I_{yy} = \frac{a^4}{12}$
4	Triangle		$\bar{x} = b/2$ $\bar{y} = h/3$ $A = \frac{1}{2} bh$	$I_{xx} = \frac{bh^3}{36}$ $I_{yy} = \frac{hb^3}{48}$
5	Rightangle Triangle		$\bar{x} = b/3$ $\bar{y} = h/3$ $A = \frac{1}{2} bh$	$I_{xx} = \frac{bh^3}{36}$ $I_{yy} = \frac{hb^3}{36}$ $I_{xx} + I_{yy} = I_{zz}$

6	circle		$A = \pi r^2$ or $\frac{\pi d^2}{4}$ $\bar{x} = d/2$ $\bar{y} = d/2$	$I_{xx} = \frac{\pi d^4}{64}$ $I_{yy} = \frac{\pi d^4}{64}$ ~~In~~ (or) $I_{xx} = \frac{\pi r^4}{4}$ $I_{yy} = \frac{\pi r^4}{4}$ $I_{zz} = \frac{\pi}{2} r^4$
7	hollow circle		$\bar{x} = D/2$ $\bar{y} = D/2$ $A = \frac{\pi}{4}[D^2 - d^2]$	$I_{xx} = \frac{\pi}{4}[R^2 - r^2]$ $I_{yy} = \frac{\pi}{4}[R^4 - r^2]$ $I_{zz} = \frac{\pi}{2}[R^2 - r^2]$
8	semi circle 		$\bar{x} = r$ $\bar{y} = \frac{4r}{3\pi}$ $A = \frac{\pi r^2}{2}$	$I_{xx} = 0.11 R^4$ $I_{yy} = \frac{\pi R^4}{8} = \frac{\pi D^4}{128}$
9	Quddrant		$\bar{x} = \frac{4r}{3\pi}$ $\bar{y} = \frac{4r}{3\pi}$ $A = \frac{\pi r^2}{4}$	$I_{xx} = 0.055 r^4$ $I_{yy} = 0.055 r^4$
10	Quadrant		~~Ixx~~ $\bar{x} = r - \frac{4r}{3\pi}$ $\bar{y} = r - \frac{4r}{3\pi}$	

Formula:

Moment of Inertia about the X axis

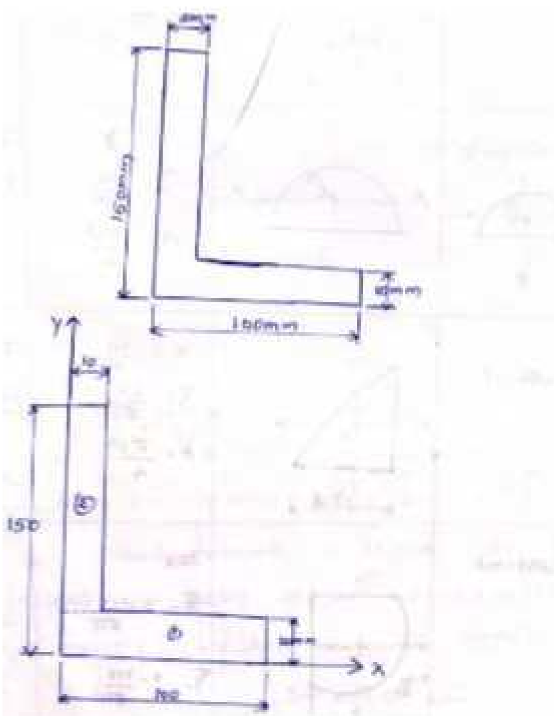
$$I_{xx} = I_{xx1} + A_1[\bar{y} - y_1]^2 + I_{xx2} + A_2[\bar{y} - y_2]^2 + I_{xx3} + A_3[\bar{y} - y_3]^2 + \ldots\ldots\ldots\ldots$$

Moment of Inertia about the Y axis

$$I_{yy} = I_{yy1} + A_1[\bar{x} - x_1]^2 + I_{yy2} + A_2[\bar{x} - x_2]^2 + I_{yy3} + A_3[\bar{x} - y_3]^2$$

Problem 1.

An area in the form of L section is shown in fig.



$$I_{xx} = I_{xx1} + A_1[\bar{y} - y_1]^2 + I_{xx2} + A_2[\bar{y} - y_2]^2$$

$$I_{yy} = I_{yy1} + A_1[\bar{x} - x_1]^2 + I_{yy2} + A_2[\bar{x} - x_2]^2$$

Section (1) Rectangle

$$a_1 = 100 \times 10 = 1000mm^2$$

$$x_1 = \frac{100}{2} = 50mm$$

$$y_1 = \frac{10}{2} = 5mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{100 \times 10^3}{12} = 8333.33mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{10 \times 100^3}{12} = 833.33 \times 10^3 mm^4$$

Section (2) rectangle

$$a_2 = 10 \times 140 = 1400mm^2$$

$$x_2 = \frac{10}{2} = 5mm$$

$$y_2 = 10 + \frac{140}{2} = 80mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{10 \times 140^3}{12} = 2.286 \times 10^6 mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{140 \times 10^3}{12} = 11.66 \times 10^3 mm^4$$

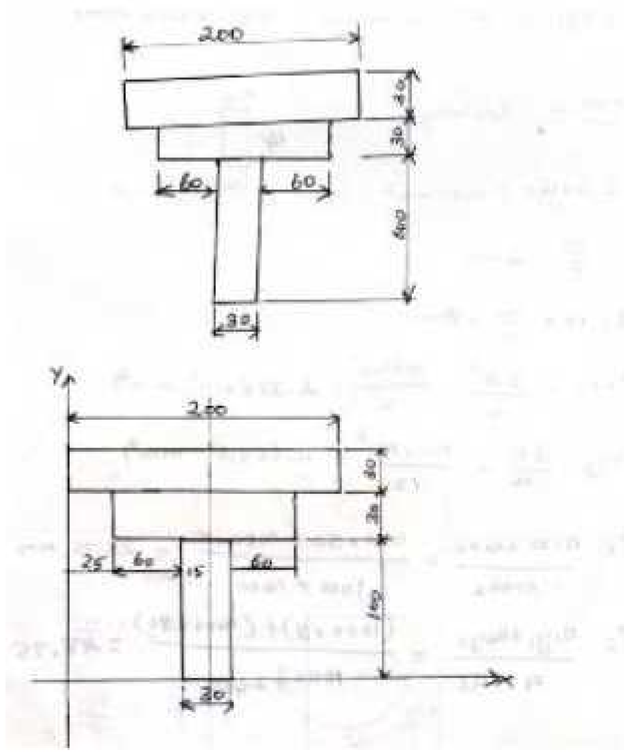
$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = \frac{1000 \times 50 + 1400 \times 5}{1000 + 1400} = 23.75mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{1000 \times 5 + 1400 \times 80}{1000 + 2400} = 48.75mm$$

$$I_{xx} = 8333.33 + 1000[48.75 - 5]^2 + 2.286 \times 10^6[1400(48.75 - 80)^2]$$
$$= 575 \times 10^6 mm^4$$

$$I_{yy} = 833.33 \times 10^3 + 1000[23.75 - 50]^2 + 11.66 \times 10^3[1400(23.75 - 5)^2]$$
$$= 2.02 \times 10^6 mm^4$$

2. Find the moment of inertia of the built up section shown in fig. about the axes passing through the centre of gravity parallel to the flange plate.



$$I_{xx} = I_{xx1} + A_1[\bar{y} - y_1]^2 + I_{xx2} + A_2[\bar{y} - y_2]^2 + I_{xx3} + A_3[\bar{y} - y_3]^2$$

$$I_{yy} = I_{yy1} + A_1[\bar{x} - x_1]^2 + I_{yy2} + A_2[\bar{x} - x_2]^2 + I_{yy3} + A_3[\bar{x} - y_3]^2$$

Section (1) Rectangle

$$a_1 = 30 \times 100 = 3000mm^2$$

$$x_1 = 25 + 60 + \frac{30}{2} = 100mm$$

$$y_1 = \frac{100}{2} = 50mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{30 \times 100^3}{12} = 2.5 \times 10^6 mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{100 \times 30^3}{12} = 2.25 \times 10^5 mm^4$$

Section (2) Rectangle

$$a_2 = 150 \times 30 = 4500mm^2$$

$$x_2 = 25 + \frac{150}{2} = 100mm$$

$$y_2 = 100 + \frac{30}{2} = 115mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{150 \times 30^3}{12} = 3.37 \times 10^5 mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{30 \times 150^3}{12} = 8.43 \times 10^5 mm^4$$

Section (3)

$$a_3 = 300 \times 30 = 9000mm^2$$

$$x_2 = \frac{200}{2} = 100mm$$

$$y_2 = 100 + 30 + \frac{30}{2} = 145mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{300 \times 30^3}{12} = 6.75 \times 10^5 mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{30 \times 300^3}{12} = 67.5 \times 10^5 mm^4$$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{(3000 \times 100) + (4500 \times 100) + (9000 \times 100)}{3000 + 4500 + 9000}$$

$$\bar{X} = 100mm$$

$$\bar{Y} = \frac{a_1x_1 + a_2x_2 + a_3y_3}{a_1 + a_2 + a_3} = \frac{(3000 \times 50) + (4500 \times 115) + (9000 \times 145)}{3000 + 4500 + 9000}$$

$$\bar{Y} = 119.54mm$$

$$I_{xx} = 2.5 \times 10^6 + 3000[119.54 - 50]^2 + 3.37 \times 10^5 + 4500[119.54 - 115]^2 + 6.75 \times 10^5 + 9000[119.54 - 145]^2$$

$$I_{xx} = 23.94 \times 10^6 mm^4$$

$$I_{yy} = 2.25 \times 10^5 + 3000[100 - 100]^2 + 8.43 \times 10^6 + 4500[100 - 100]^2 + 6.75 \times 10^5 + 9000[100 - 100]^2$$

$$I_{yy} = 76.15 \times 10^6 mm^4$$

Product of Inertia:

The moment of inertia of a plane fig. above a set of perpendicular axis is called product of inertia.

$$I_{xy} = \int_A xy \ da$$

$$= \sum axy$$

Problem: 1

Find the product of inertia and principal moment of inertia of the section about the centroidal axis

Product of inertia

$$I_{xy} = I_{x_1y_1} + I_{x2y2} + I_{x3y3}$$

$$I_{xy} = I_{xy} + a_1(\overline{x_1} - \bar{X})(\overline{y_1} - \bar{Y})$$

$I_{x_1y_1} = a_1 x_1^1 y_1^1$ $\quad X_1^1 = x_1 - \bar{X}$ $\quad Y_1^1 = y_1 - \bar{Y}$

$I_{x_2y_2} = a_2 x_2^1 y_2^1$ $\quad X_2^1 = x_2 - \bar{X}$ $\quad Y_2^1 = y_2 - \bar{Y}$

$I_{x_3y_3} = a_3 x_3^1 y_3^1$ $\quad X_3^1 = x_3 - \bar{X}$ $\quad Y_3^1 = y_3 - \bar{Y}$

Section (1)

$a_1 = 40 \times 8 = 320mm^2$

$x_1 = 25 + \frac{30}{2} = 52mm$

$y_1 = \frac{8}{2} = 4mm$

Section (2)

$a_3 = 40 \times 8 = 320mm^2$

$x_3 = \frac{40}{2} = 20mm$

$y_2 = 8 + \frac{44}{2} = 30mm$

Section (3)

$a_1 = 40 \times 8 = 320mm^2$

$x_3 = \frac{40}{2} = 20mm$

$y_3 = 8 + 44 + \frac{8}{2} = 56mm$

$$\bar{X} = \frac{a_1 x_1 - a_2 x_2 - a_3 x_3}{a_1 - a_2 - a_3} = \frac{(320 \times 52) + (352 \times 36) + (320 \times 20)}{320 + 352 + 320} = 36mm$$

$$\bar{Y} = \frac{a_1 y_1 - a_2 y_2 - a_3 y_3}{a_1 - a_2 - a_3} = \frac{(320 \times 4) + (352 \times 30) + (320 \times 56)}{320 + 352 + 320} = 30mm$$

$I_{x_1 y_1} = a_1 x_1^1 y_1^1 \qquad X_1^1 = x_1 - \bar{X} = 52 - 36 = 16$

$Y_1^1 = y_1 - \bar{Y} = 4 - 30 = -26$

$I_{x_1 y_1} = 320 \times 16 \times (-26)$

$I_{x_1y_1} = -133120mm^4$

$I_{x_2y_2} = a_2 x_2^1 y_2^1$ $\quad$ $X_2^1 = x_2 - \bar{X} = 36 - 36 = 0$

$Y_2^1 = y_2 - \bar{Y} = 30 - 30 = 0$

$I_{x_2y_2} = 0$

$I_{x_3y_3} = a_3 x_3^1 y_3^1$ $\quad$ $X_3^1 = x_3 - \bar{X} = 20 - 36 = -16$

$Y_3^1 = y_3 - \bar{Y} = 56 - 30 = 26mm$

$I_{x_3y_3} = -133120mm^4$

Product of Inertia:

$I_{xy} = I_{X1Y1} + I_{X2Y2} + I_{X3Y3}$

$= -133120 + 0 + (-133120)$

$I_{xy} = -266240mm^4$

Principal Moment of Inertia:

Maximum Principal moment of inertia:

$$I_{Max} = \frac{I_{XX} + I_{YY}}{2} \pm \sqrt{\left(\frac{I_{XX} + I_{YY}}{2}\right)^2 + I^2XY}$$

$I_{XX} = I_{XX1} + I_{XX2} + I_{XX3}$

$I_{YY} = I_{YY1} + I_{YY2} + I_{YY3}$

$$I_{XX1} = \frac{bd^3}{12} + a_1(Y_1 - \bar{Y_1})^2$$

$$I_{XX2} = \frac{bd^3}{12} + a_2(Y_1 - \bar{Y_1})^2$$

$$I_{XX3} = \frac{bd^3}{12} + a_3(Y_1 - \bar{Y_1})^2$$

$$I_{XX} = I_{XX1} + a_1(Y_1 - \overline{Y_1})^2 + I_{XX2} + a_2(\bar{Y} - Y_2)^2 + I_{XX3} + a_3(\bar{Y} - Y_3)^2$$

$$= 1706.66 + 320(30-4)^2 + 28394.66 + 352(30-30)^2 + 1706.66 + 320(30-56)^2$$

$$I_{XX} = 464.44 \times 10^3 mm^4$$

$$I_{yy} = I_{yy1} + a_1(X - X_1)^2 + I_{yy2} + a_2(X - X_2)^2 + I_{yy3} + a_3(X - X_3)^2$$

$$= 42666.66 + 320(36-52)^2 + 1877.33 + 352(36-36)^2 + 42666.66 + 320(36-20)^2$$

$$I_{yy} = 251.05 \times 10^3 mm^4$$

I_{Max}

$$I_{Max} = \frac{I_{xx} + I_{yy}}{2} \sqrt{+\left(\frac{I_{xx} + I_{yy}}{2}\right)^2 + I^2xy}$$

$$= \frac{464.44\times10^3 + 251.05\times10^3}{2} + \sqrt{\left(\frac{464.44\times10^3 - 251.05\times10^3}{2}\right)^2 + (-266240)^2}$$

$$I_{Max} = 357745 + \sqrt{1.138 \times 10^{10} + 7.083 \times 10^{10}}$$

$$I_{Max} = 357745 + 286.81 \times 10^3$$

$$I_{Max} = 644.55 \times 10^3 mm^4$$

$$I_{Min} = \frac{I_{xx} + I_{yy}}{2} \sqrt{+\left(\frac{I_{xx} + I_{yy}}{2}\right)^2 + I^2xy}$$

$$= \frac{464.44 \times 10^3 + 251.05 \times 10^3}{2} - \sqrt{\left(\frac{464.44 \times 10^3 + 251.05 \times 10^3}{2}\right) + (-266240)^2}$$

$$I_{Min} = 357745 - 268.81 \times 10^3$$

$I_{Min} = 88.935 \times 10^3 mm^4$

The position of Principal Axes is given by

$$tan2\theta = \left(\frac{I_{xy}}{\frac{I_{xx} - I_{yy}}{2}} \right) = \left(\frac{266240}{\frac{464.44 \times 10^3 - 251.05 \times 10^3}{2}} \right)$$

$$tan2\theta = \left(\frac{-2I_{xy}}{I_{xx} - I_{yy}} \right)$$

$$tan2\theta = 2.49$$

$$2\theta = tan^{-1}(2.49)$$

$$2\theta = 68.16 \ \ \theta = \frac{68.16}{2}$$

$$\theta = 34°4'$$

Principal Moment of inertia:

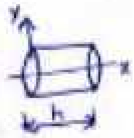
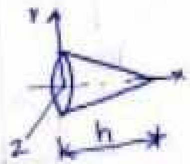
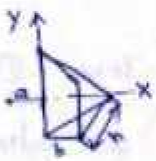
The perpendicular axis above which product of inertia is zero called principal axis and the moment of inertia with respect to these axis are called principal moment of inertia.

$$I_{max} \& I_{min} = \left(\frac{I_{xx} + I_{yy}}{2} \right) \pm \left(\frac{I_{xx} + I_{yy}}{2} \right)^2 + I_{xy}{}^2$$

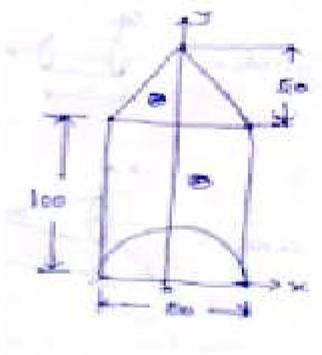
Location of principal Axes

$$tan2\theta = \frac{-2Ixy}{I_{xx} - I_{yy}}$$

Centre of gravity of common volume

shape	Fig	volume	centroid
	~~cylinder~~		
cylinder		$V = \pi r^2 h$	$\bar{x} = h/2$, $\bar{y}_1 =$ half
cone		$V = \frac{1}{3}\pi r^2 h$	$\bar{x} = h/4$
Pyramid		$V = \frac{1}{3}\, abh$	$\bar{x} = h/4$
Hemisphere		$V = \frac{2}{3}\pi r^2$	$\bar{x} = \frac{3r}{8}$
parabola		$V = \frac{1}{2}\pi a^2 h$	$\bar{x} = h/3$
semi ellipse		$V = \frac{2}{3}\pi a^2 h$	$\bar{x} = \frac{3h}{8}$

1. Find the position of the centroid of the solid combine shown in fig. consist of a solid cone of height 50 mm and base diameter 80 mm and a cylinder of height 100 mm and diameter 80mm with a semicircular cut as shown.



$$\bar{X} = \frac{v_1x_1 + v_2x_2 - v_3x_3}{v_1 + v_2 - v_3}$$

$$\bar{Y} = \frac{v_1y_1 + v_2y_2 - v_3y_3}{v_1 + v_2 - v_3}$$

$$v_1 = {}^1\!/_3\,\pi r^2 h = {}^1\!/_3 \times \pi \times 40^2 \times 50 = 83775.80mm^3$$

$$v_2 = \pi r^2 h = \pi \times 40^2 \times 100 = 502654.82mm^3$$

$$v_3 = \frac{\pi r^2 h}{2} = \frac{\pi \times 40^2 \times 100}{2} = 251327.41mm^3$$

$$X_1 = {}^b\!/_2 = {}^{80}\!/_2 = 40mm \qquad Y_1 = 100 + {}^{50}\!/_3 = 116.66mm$$

$$X_2 = {}^b\!/_2 = {}^{80}\!/_2 = 40mm \qquad Y_2 = {}^{100}\!/_2 = 50mm$$

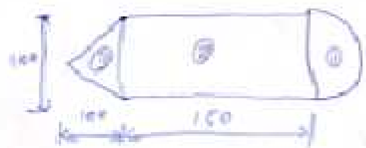
$$X_3 = {}^d\!/_2 = {}^{80}\!/_2 = 40mm \qquad Y_3 = \frac{4r}{3\pi} + \frac{4\times 40}{3\pi} = 16.97mm$$

$$\bar{X} = \frac{(83775.80 \times 40) + (502654.82 \times 40) - (251327.4 \times 40)}{83775.5 + 502654.82 - 251327.41}$$

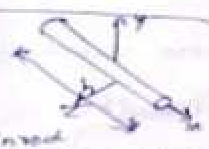
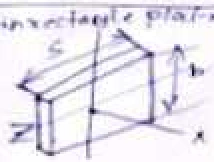
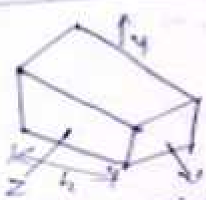
$$\bar{X} = 40mm$$

$$\bar{Y} = \frac{(83775.80 \times 416.66) + (502654.82 \times 50) - (251327.41 \times 16.97)}{83775.80 + 502654.82 - 251325.41}$$

2. A hemisphere of diameter 100 mm is fixed to cylinder is OA hemisphere diameter 100mm and cone is fixed another and of the cylinder its length is 100mm as shown fig. Locate the centroid of combine fig.



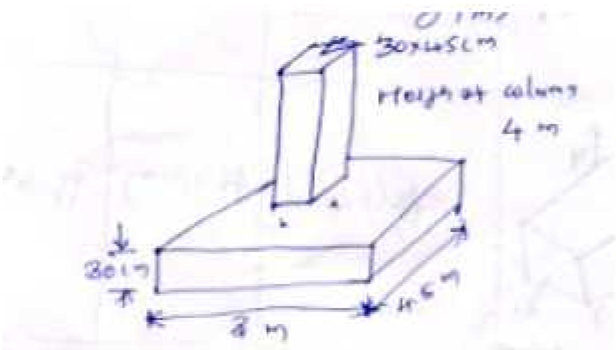
Mass Moment of irertia:

Figure	Moment of Inertia about xx	about yy	about zz	Mass
 Thin rod	$\frac{M}{12}L^2$	$\frac{M}{12}L^2$	$\frac{M}{12}L^2$	
Thin rectangle plate 	$\frac{M}{12}(b^2+c^2)$	$\frac{M}{12}L^2$	$\frac{M}{12}b^2$	
 Rectangular prism	$\frac{M}{12}(b^2+a^2)$	$\frac{M}{12}(a^2+b^2)$	$\frac{M}{12}(a^2+b^2)$	$\rho(abc)$
 cylinder	$\frac{M}{2}r^2$	$\frac{M}{12}(3r^2+h^2)$	$\frac{M}{12}(3r^2+h^2)$	$\rho(\pi r^2 h)$
sphere 	$\frac{2}{5}ma^2$	$\frac{2}{5}ma^2$	$\frac{2}{5}Ma^2$	$\rho\left(\frac{4}{3}\pi a^3\right)$

3. A rectangular RCC column is centrally cast over a concrete bed R.C.C. in Fig. column is of section 30× 45 cm and height 4m.the concrete bed is of size

3× 4.5m and thickness 30cm. find the mass moment of inertia of the column and bed combination about its vertical centroidal axis.

Mass density of concrete=2500 kg/m^3



Soln:

I_{yy} Combosite body $(I_{yy})column + (I_{yy})bed$

$$(I_{yy})_{column} = \frac{M}{12}(b^2 + d^2)$$

M=mass volume× mass density

M=$(0.3 \times 0.45 \times 4)m^3 \times 2500\, {}^{kg}/_{m^3}$

M=3500 kg

I_{yy} Column $= \frac{M}{12}(b^2 + d^2)$

$= \frac{1350}{12}(0.3^2 + 0.45^2)$

I_{yy} Column=32.91kg.m^2

I_{yy}bed $= \frac{M}{12}(b^2 + d^2)$ M=massvolume× $density$

$= \frac{10125}{12}(3^2 + 4.5^2)$ $(3 \times 0.3 \times 4.5) \times 2500m^3 \times {}^{kg}/_{m^3}$

M=10125kg

I_{yy} bed 24679.69kgm^2

I_{yy}Composite body $=$ Iyy column +I_{yy} bed

= 32.91+24679.69

I_{YY}= 24712.6kg.m^2

UNIT-IV

DYNAMIC OF PARTICLES

Newton's Law Of Motion

Newton"s Law

The rate of change of momentum is directly proportional to the resultant force.

The Resultant Force acting in the direction of equal to the product of mass and the acceleration in the direction of resultant Force.

$\sum F = ma$

m= mass

a= acceleration

D' Alembert' Principle:

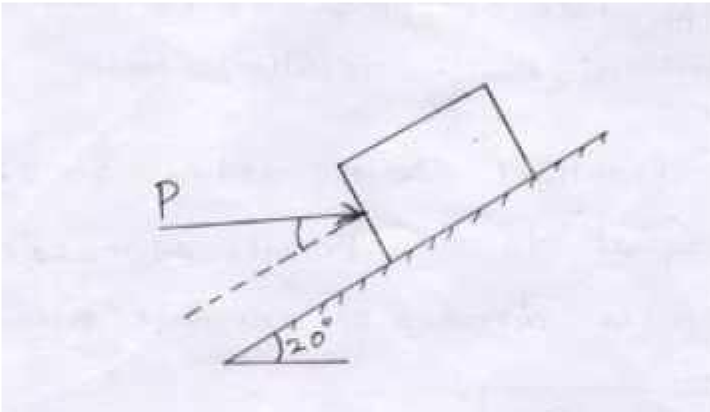
States that the inertia forces and couples, and the external forces and torques on a body together give statical equilibrium.

Inertia is a property of mater by virture of which a body resists ay change in velocity

$$F_I = \quad -mg$$

Problem:1

What horizontal force is needed to give the 50 kg block shown in fig. With an acceleration of $3^m/_{s^2}$ up the 20° plane. Assume the coefficient of friction b/w the block and plane is 0.25.



<u>Given:</u>

Weight of block $W = 50\ kg = 50 \times 9.81 = 490.5N$

Acceleration $a = 3\ {}^{m}/{}_{s^2}$

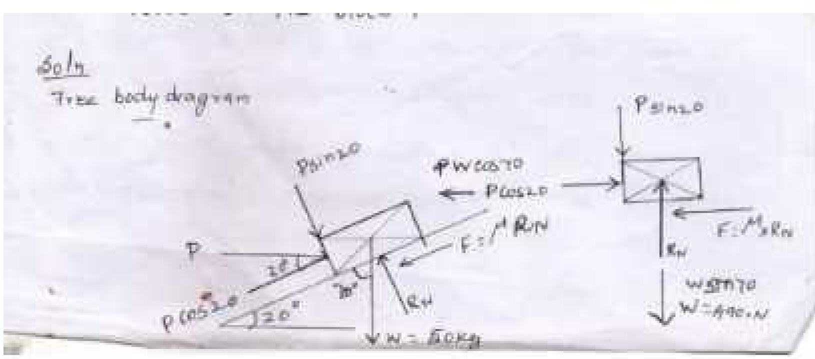
Coefficient of friction $= 0.25$

<u>To find:</u>

Force on the block P

<u>Soln:</u>

Free body diagram



$\sum FX = m\ a$

$\text{P} \cos 20 - M_{XRN} - \text{w} \cos 70 = 50 \times 3$

$$P\ \cos 20 - 0.25 \times R_N - 490.5 \times \cos 70 = 150 \quad ------> (1)$$

$\sum FY = 0$

$R_N - p \ sin\, 20 - w \ sin\, 70 = 0$

$R_N - p \ sin\, 20 - 490.5 \ sin\, 70 = 0$

$R\_N - psin20 - 490.5 \ sin\, 70 = 0$

$RN = 0.34P - 460.91$

R_N value in Eqn (1)

$P \ cos20 - 0.25 \ [0.34 \times p - 490.91] - 490.5 \times cos70 = 150$

$0.93p - 0.085p + 122.72 - 167.76 = 150$

$0.845P - 45.04 = 150$

$0.845P = 150 + 45.04$

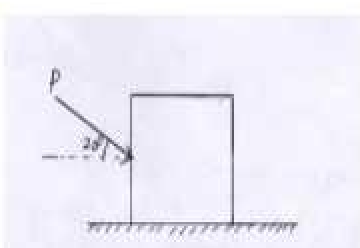
$0.845P = 195.04$

$$P = \frac{195.04}{0.845}$$

$P = 230.81N$

Problem:2

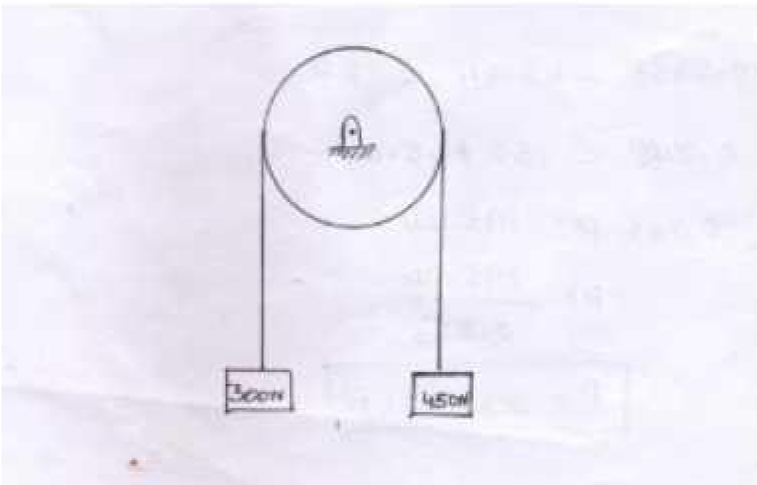
A block weighting 1KN, rest on a horizontal plane as shown in fig. Find the force **P** required to give an acceleration of 3 m/s^2 to right. Take the coefficient of friction M_K=0.25.



$P = 750.056N$

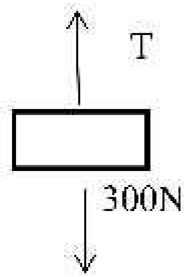
Problem:3

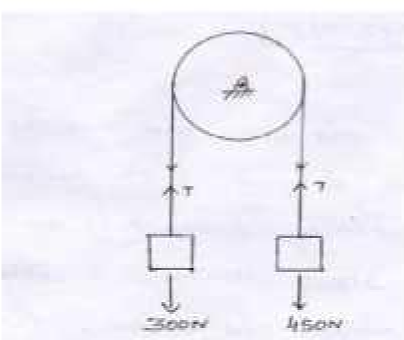
Two blocks weighting 300N and 450N are connected by a rope as shown fig. With what acceleration the heavier block comes down, and what is the tension of the rope, Pulley is frictionless and weight less.

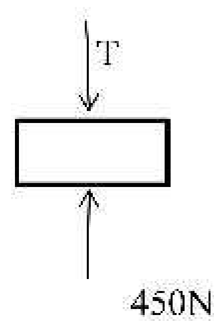


soln :

Free body diagram







$\sum F_X = ma$

$T - 300 = \frac{300}{9.81} \times a$

$T - 300 = 30.58 \times a$-----(1)

$\sum F_Y$= m a

$450 - T = \frac{450}{9.81} \times a$-----(1)

Solving Eqn (1) & (2)

$T - 300 = 30.58 \times a$

$\underline{450 - T = 45.87 \times a}$

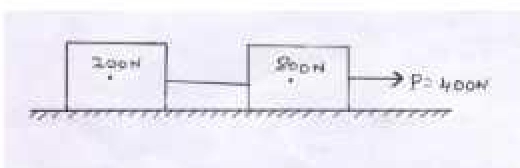
$150 \quad = 76.45 \times a$

$$a = \frac{150}{76.45}$$

$a = 1.962\ m/s2$

Problem 4:

Two weight 800N and 400N are connected by a thread and they move along a rough horizontal plane under the action of force P of 400N applied to 800N block, as shown in Fig. Find the acceleration of the weight and tension in the thread.



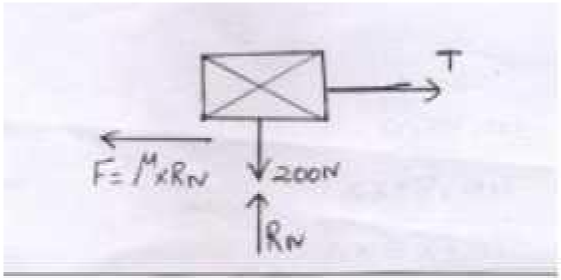
To find

Acceleration a

Tension T

Soln:

Consider block '200N'



$\sum F_Y$=0

$R_N - 200 = 0$

$R_N = 200N$

$\sum F_X$ = m a

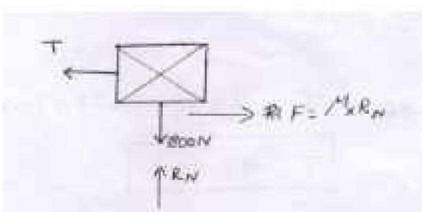
$T = F = m \times a$

$T - \mu \times RN = \frac{200}{9.81} \times a$ μ = 0.3 assume

$T - 0.3 \times 200 = 20.38 \times a$

$T - 60 = 20.38 \times a$--------->(1)

<u>Consider 800N block</u>



$\sum F_Y$=0

$R_N - 800 = 0$

$R_N = 800N$

$\sum F_X$ = m a

$-T + F_N = \frac{800}{9.81} \times$a

$-T + \mu \times R_N = 81.54 \times a$

$-T + 0.3 \times 800 = 81.54 \times a$

$-T + 240 = 81.54 \times a$

$-T + 240 = 81.54 \times a$-------->(2)

Solving Eqn 1&2

$T - 60 = 20.38 \times a$

$-T + 240 = 81.54 \times a$

$180 = 101.54 \times a$

$a = \frac{180}{101.92}$

$\boxed{a = 1.766 \ m/s^2}$

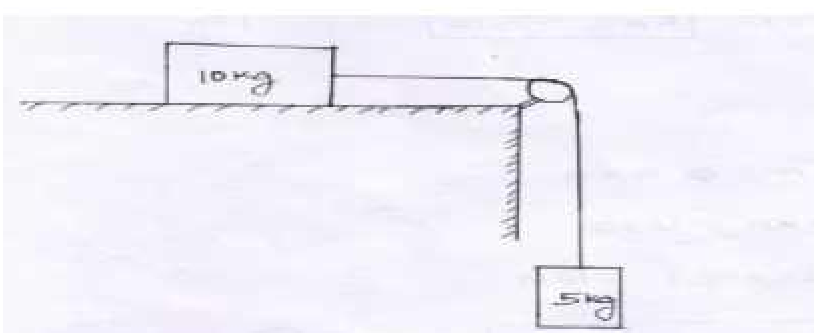
'a' Value sub in Eqn 1

$T - 60 = 20.38 \times 1.766$

$T = 95.99N$

Problem:5

Two blocks of mass 10kg and 5kg are connected as shown in fig. Assume $M_k = 0.25$. Find the acceleration and the tension in the string if pulley is weightless and frictionless.



Given:

$$Block\ A = 10kg$$

$$Block\ B = \ 5kg$$

$$M_k = 0.25$$

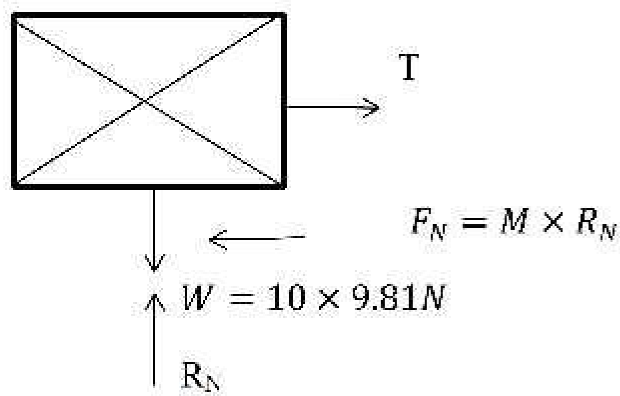
To Find:

1.Acceleration a

2.Tension T

Soln:

Consider block A (10kg)



$\sum F_Y = 0$

$$R_N - w = 0$$

$$R_N = w$$

$$R_N = 98.1\ N$$

$\sum F_X = ma$

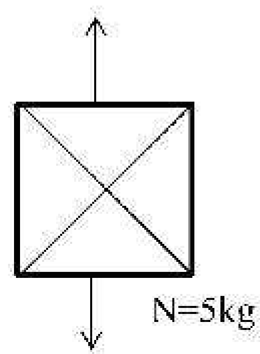
$$T - \ F_N = N10 \times a$$

$$T - M \times R_N = 10 \times a$$

$$T - 0.25 \times 98.1 = 10a$$

$$T - \ 24.52 = 10a \text{ ------> } (1)$$

Consider 5 kg block



$\sum F_Y =$ m a

$$T - \ w = m\ a$$

$T - 5 \times 9.81 = 5 \times a$

$W = 5 \times 9.81 = 49.05N$

$+T - 49.05 = 5a$ ----->(2)

Solving Eqn 1&2

$$T - 24.52 = 10\,a$$

$$T - 49.05 = 5\,a$$

$$24.52 = 5\,a$$

$$a = \frac{24.52}{5}$$

$$a = 4.905 \ {}^{m}/_{s}2$$

'a' Value sub in Eqn 1

$$T - 24.52 = 10\,a$$

$$T = 10\,a + 24.52$$

$$T = 10 \times 4.905 + 24.52$$

$$T = 73.57\,N$$

Problem 6

A block of 1200 N rest on a rough inclined plane at 12° to the horizontal. It is pulled up the plane by means of a light flexible rope running parallel to the plant and passing over a light frictionless pulley at the top of the plane. The portion of the rope beyond the pulley hangs vertically down and carries a weight of 800N at its end.

If Coefficient of friction = 0.2, find a) tension in the rope (b) acceleration with which the body moves up the plane (c) distance moved is after 3sec after starts from rest.

Given:

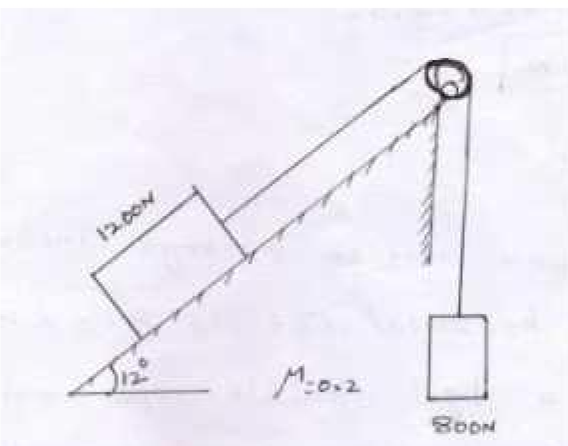
$$Coefficient\ of\ friction\ \mu\ = 0.2$$

$$Weight\ of\ block\ w = 1200\ N$$

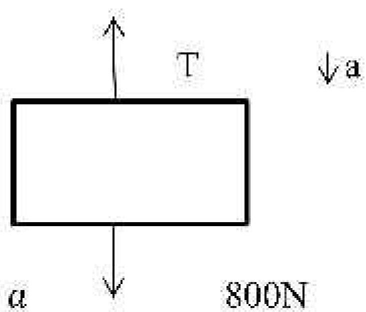
To Find:

1. Tension 'T'
2. Acceleration 'a'
3. Distance moved 3sec after starts from rest

Soln:



Consider 800 N block



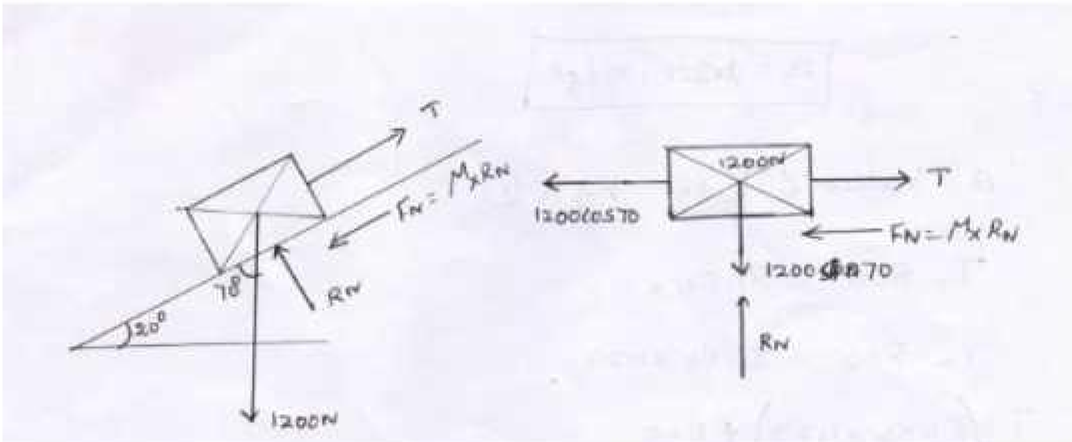
$\sum F_Y =$ m a

T−800= m a

$$T - 800 = \ 800/9.81 \times\ a$$

$$T - 800 = 81.54\ a \text{ -----> } (1)$$

Consider 1200 N block:



$\sum F_Y = 0$

$R_N - 1200 \;\; 70 = 0$

$R_N = 1127.63 = \;0$

$R_N = 1127.63\,N$

$\sum F_Y = \text{m a}$

$$T - \;1200\,cos\,70 - F_N \;=\; m\,a$$

$$T - 1200\,cos \;-\; M \times R_N = \frac{1200}{9.81} \times a$$

$$T - \;410.42 \;-\; 0.3 \times 1127.63 = 122.32 \times a$$

$$T - \;410.42 - 338.28 = 122.3 \times a$$

$T - \;748.70 \;=\; 122.3 \times a$ -------> (2)

Solve Eqn (1) & (2)

$$T \;- 800 \;=\; 81.54\,a$$

$$T - \;748 \;=\; 122.3\,a$$

$$-\,51.8 \;=\; -\,40.76\,a$$

$$a = \frac{-51.8}{-40.76}$$

$$a = 1.27\ m/s2$$

'a' Value sub in Eqn (1)

$$T - 800 = 81.54 \times a$$

$$T - 800 = 81.54 \times 1.27$$

$$T = (81.54 \times 1.27) + 800$$

$$T = 903.55\ N$$

Consider kinetic Eqn

To Find Distance

$$S = u\,t + \frac{1}{2}\,a\,t2$$

$$S = 0 \times 3 + 1/2(1.27) \times (3)2$$

$$S = 5.71\ m$$

Initial condition

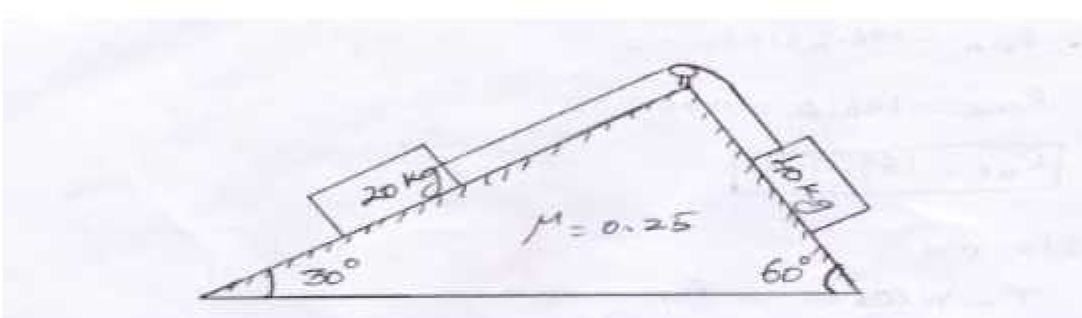
$u = 0$

$a = 1.27\ m/s2$

$t = 3sec$

Problem 7

Two blocks of mass 20 kg and 40 kg are connected by a rope passing over a frictionless pulley as shown in fig. (a) Assuming the coefficient of friction as 0.3 for all contact surfaces. Find the tension in the string and the aueleration of the system. Also compute the velocity of the system after 4 sec starting from rest.



Given:

$Mass\ of\ block\ A\ mA = 20\ kg$

$Mass\ of\ block\ B\ mB = 40\ kg$

$Coefficient\ of\ friction\ \mu\ = \ 0.3$

To Find:

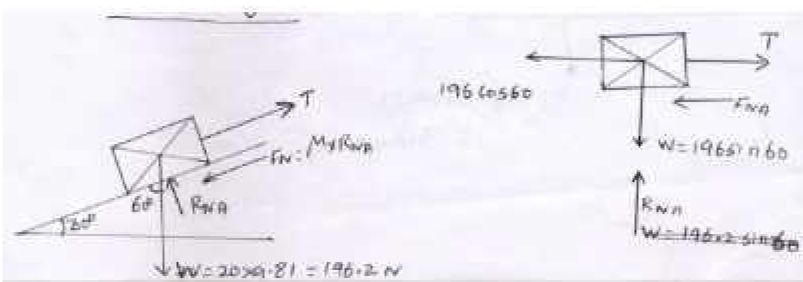
Tension in the string T

Acceleration 'a'

Velocity of the system after 4 sec

Solution:

Consider 20 kg block



$\sum F_Y = 0$

$R_{NA} - 196.2\ sin\ 60\ = \ 0$

$R_{NA} - \ 196.2\ sin\ 60$

$R_{NA} - \ 169.91\ N$

$\sum F_X = \text{m a}$

$T - \ w\ cos\ 60\ - \ F_{NA} = \ m\ a$

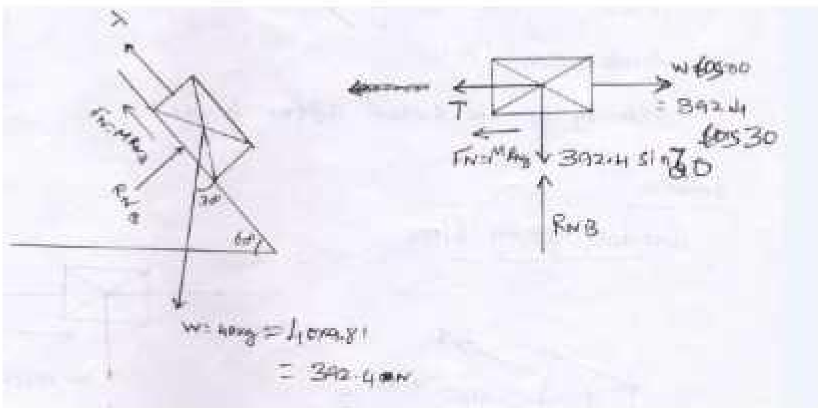
$T - \ 196.2\ cos\ 60\ - \ \mu \times R_{NA} = 20 \times a$

$$T - 196.2\,cos\,60 - 0.3 \times 169.91 = 20 \times a$$

$$T - 98.1 - 50.973 = 20\,a$$

$$T - 149.07 = 20\,a \text{ ----->} \quad (1)$$

Consider 40 kg block



$\sum F_Y$=0

$$R_{NB} = -392.4\,sin\,30 = 0$$

$$R_{NB} = 392.4 sin\,30$$

$$R_{NB} = 196.2N$$

$\sum F_x$ =ma

$$392.4\ cos\,30 - T = ma$$

$$339.82 - 0.3 \times R_{NB} - T = 40 \times a$$

$$339.82 - 0.3 \times 19.2 - T = 40a$$

$$280.96 - T = 40a \text{------->}(2)$$

solve Eqn 1 & 2

$$T - 149.07 = 20a$$

$$280.96 - T = 40a$$

$$131.89 = 60a$$

$$a = 131.89/60$$

$$a = 2.19\, m/s^2$$

'a' value sub in Eqn (1)

$$T - 149.07 = 20 \times 2.19$$

$$T = 193.03\, N$$

Using kinetic Eqn

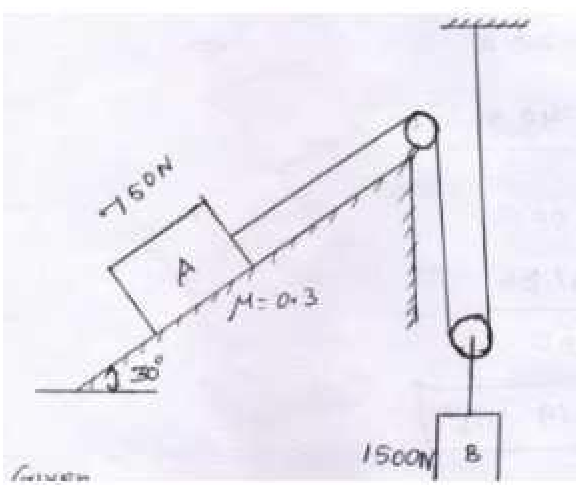
$V = u + at$ u=0 Initial stage

$V = 0 + 2.198 \times 4$ t=4sec

$$V = 8.79\, m/sec$$

Problem:

Two blocks of weight 750 N and 1500 N start from shown in fig. Find the acceleration of each block and the distance travelled by the 750 N block in 2 sec Also find the tension in the string.



Given:

$Weight\ of\ block\ A\ W_A = 750\ N$

$Weight\ of\ block\ B\ W_B = 1500\ N$

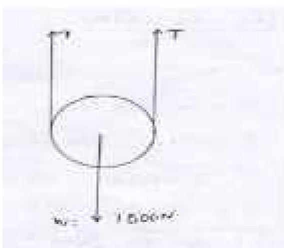
$Coefficient\ of\ friction\ \mu\ =\ 0.3$

<u>To Find:</u>

1. Acceleration
2. Tension
3. Distance travelled by the 750 N in 2 Sec.

<u>Soln</u>

<u>Consider1500N block</u>



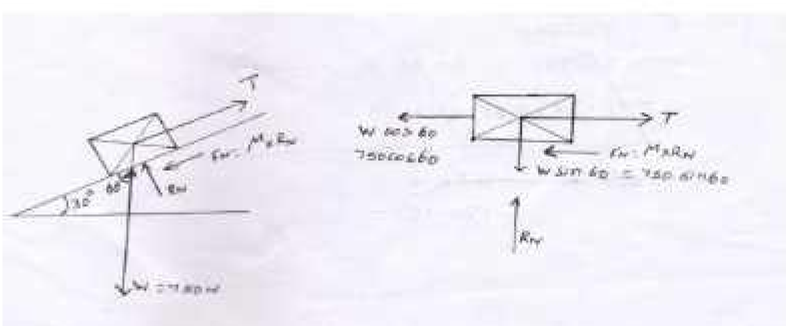
$\sum F_Y = ma$

$2\,T - 1500 = m \times a$

$2\,T - 1500 = \frac{1500}{9.81} \times a$

$2\,T - 1500 = 152.90 \times 2a$ --------> (1)

<u>Consider 750 N block</u>



$\sum F_Y = 0$

$$R_N - w\ sin\ 60\ =\ 0$$

$R_N = w\ sin\ 60$

$R_N = 750\ sin\ 60$

$R_N = 649.51N$

$\sum F_X = \text{ma}$

$$\text{T}-F_N - \text{w cos } 60 = ma$$

$$\text{T}-\mu R_N - 750 \cos 60 = \frac{750}{9.81} \times a$$

$$\text{T}-0.3 \times 649.51 - 750 \cos 60 = 76.45 \times a$$

$$\text{T}-569 - 85 = 76.45a\text{---------}(2)$$

Solve eqn (1) &(2)

$2T - 1500 = 152.90 \times 2\ a$-------------(1)

$2\ T - 1500 = 305.8\ a$

$\div 2 \qquad T - 750 = 152.9\ a$ --------> (1)

$$T - 750 \quad = \quad 152.90 \times a \text{ -------> } (1)$$

$$T - 569.85 = \quad 76.45 \times a \text{ ----------> } (1)$$

$$-\ 180.15 = 76.45\ a$$

$$a = \frac{-180.15}{76.45}$$

$$\text{a} = -2.35^{m}/_{s^2}$$

a value sub in Eqn (1)

$$T - 750 = 152.9 \times (-2.35)$$

$$T - 750 = 152.9 \times (-359.31)$$

$T = 750 = -359.31$

$T = -359.3 + 750$

$T = 390.68\ N$

Distance travelled

$s = ut + {}^{1}/_{2}\, at2$ u=0 t=2

$s = 0 \times 2 + {}^{1}/_{2} \times (-2.35) \times (2)2$

$s = -4.7\ m$

Impact of Elastic Bodies:

A collision between two bodies to be an impact, if the bodies are in contact for short interval of a time and exert very large force on a short period of time.

On impact bodies deform first and then recover due to elastic properties and start moving with different velocities.

Types of Impact:

- Line of impact
- Direct impact
- Oblique impact
- Central impact
- Eccentric impact

Perfectly Elastic impact: [e=1]

If both of bodies regain to their original shape and size after the impact. Both momentum and energy is conserved.

In elastic impact [e<1]

The collision do not return to their original shape and size completely after the collection. Only the momentum remains conserved, but there is a loss energy.

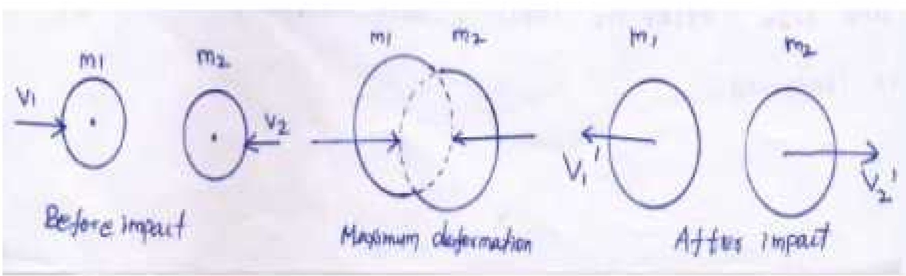
Period of collision:

During the collection, the bodies undergo a deformation for a small time interval and then recover the deformation in a further small interval.

Time elapse b/w initial contact and maximum deformation is called the period of deformation. And the instant of separation is called time of restitution or period of recovery.

Principal of collision:

Consider 2 bodies approach each other with the velocity v_1 and v_2 masses m_1 and m_2 are shown in fig.



Let 'F' be force entered due to collection at a small time. Apply conservation of momentum principal for both bodies

$$m_1 v_1 + m_2 v_2 = m_1 v_1{}^1 + m_2 v_2{}^1$$

Newton's impact Eqn:

Coefficient of restitution, e= $\frac{relative\ velocity\ of\ separation}{relative\ velocity\ of\ approach}$

$$e = \frac{v_2{}^1 v_1{}^1}{v_1 - v_2}$$

Total kinetic energy at before impact

$$=\frac{1}{2}\, m_1 v + \frac{1}{2} m_2 v_2^2$$

Total kinetic energy at after impact

$$=\frac{1}{2} m_1 v_1^{1\ 2} + \frac{1}{2} m_2 v_2^{1\ 2}$$

Loss of K.E=Intial K.E – Final K.E

<u>Oblique:</u>

$V_1 \sin \propto_1 = V_1{}^1 \sin \theta_1$

$V_2 \sin \propto_1 = V_2 \sin \theta_1$

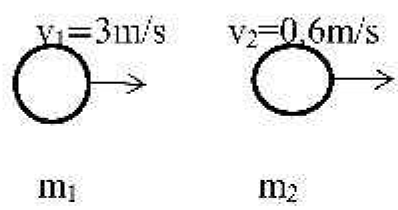
$m_1\, v_1 \cos \propto_1 + m_1 v_1{}^1 \cos \theta_1 + m_2 v_2{}^1 \cos \theta_2$ $\qquad\qquad$ $m_1 = m_2$

$$e = \frac{V_2{}^1 \cos \theta_2 - V_1{}^1 \cos \theta_1}{V_1 \cos \propto_1 - V_2 \cos \propto_2}$$

<u>Problem based on impact of elastic body:</u>

1. A sphere of 1 kg moving at 3 m/s, collides with another sphere of weight of 5 kg in the same Direction at 0.6 m/s. If the collision is perfectly elastic, find the velocity after impact.

<u>Given:</u>



$m_1 = 1$ kg

$m_2 = 5$ kg

$v_1 = 3$ m/s

$v_2 = 0.6$ m/s

Perfectly elastic impact e= 1

<u>To find:</u>

Velocity at after the impact V_1^1 & V_2^1

Soln:1

Law of conservation of momentum

$m_1v_1 + m_2v_2 = m_1v_1^1 + m_2v_2$

$1\times3+5\times0.6=1\ v_1^1+5\ v_2^1$

$V_1^1+5\ V_2^1 = 6$ -------> (1)

The coefficient of restitution, $e = \frac{V_2^1 - V_1^1}{V_1 - V_2}$

$e = 1$[perfectly Elastic Impact]

$1 = \frac{V_2^1 - V_1^1}{3 - 0.6}$

$V_2^1 - V_2^1 = 1\times[3-0.6]$

$V_2^1 - V_1^1 = 2.4$ ------> (2)

Solve Eqn (1) & (2)

$V_1^1+5\ V_2^1 = 6$

$V_2^1 - V_1^1 = 2.4$

$6\ V_2^1 = 8.4$

$V_2^1 = 8.4/6$

$V_2^1 = 1.4$ m/s

V_2^1 value sub in Eqn (1)

$V_1^1+5\ V_2^1 = 6$ --------> $V_1^1 = 6 - [5\times V_2^1]$

$V_1^1 += 6 - [5\times1.4]$

$V_1^1 = -1$ m/s

$V_1^1 = 1$ m/s

2. A car weighting 5 KN is moving east with a velocity of 54 k m p h and collide with a second car weighting 12 KN is moving west with a velocity of 72 k m p h If the impact is perfectly plastic, what will be the velocities of the cars. V_1=54km/h v_2=-72km/h

<u>Given:</u>

$W_1 = 5KN$ $W_2 = 12KN$

$M_1 = {}^{5}/_{9.81}$ $M_2 = {}^{12}/_{981}$

$W_1 = 5$ KN $= {}^{5}/_{9.81} = 0.509$ kg $= m_1$

$W_2 = 12$ KN $= {}^{12}/_{9.81} = 1.22$ kg $= m_2$

$V_1 = 54$ km/hr

$V_2 = -72$ km/hr

<u>To Find:</u>

Velocity of car

<u>Soln:</u>

Law of conservation momentum

$m_1 v_1 + m_2 v_2 = m_1 v_1^1 + m_2 v_2^1$

Perfectly plastic means e=0

$\therefore v_1^1 = v_2^2 = e$

$0.509 \times 54 + 1.22 \times [-72] = 0.509 \times v_1^1 + 1.22 \times v_2^1$

$27.486 - 87.84 = [0.509\, V c + 1.22]$

$-60.354 = Vc \times 1.729$

$$V_c = \frac{-60.354}{1.729}$$

$$Vc = -34.90\ km/hr$$

3. Direct central impact occurs between 300N body moving to right with a velocity of 6 m/s and 150N body moving to the left with a velocity of 10 m/s. Find the velocity of each body after the impact if the coefficient of restitution is 0.8.

Same as problem No:1

Ans: V_2^1=9.2 m/s

$V_1^1 = -3.6$ m/s

4. Two bodies, one of which 20N and velocity 10 m/s and the other of 100N with a velocity of m/s downward,each other and implinges centerlly. Find the velocity of each body of the impact if the coefficient of restitution is 0.6. Find also the loss in kinetic energy due to impact.

<u>Given data:</u>

W_1=20N $\quad m_1 = \frac{20}{9.81}$ $\quad m_1$=2.038 kg

V_1=10 m/s

W_2=100N $\quad m_1 = \frac{100}{9.81}$ $\quad m_2$=10.19 kg

$V_2 = -10$ m/s

Coefficient of restitution, e= 0.6

<u>To find:</u>

Final velocity After impact V_1^1 & V_2^1

Loss of kinetic Energy.

Soln:

Law of conservation of Energy

$m_1 v_1 + m_2 v_2 = m_1 v_1^1 + m_2 v_2^1$

$(2.038 \times 10) + (10.19 \times -10) = 2.038\ v_1^1 + 10.19 \times v_2^1$

$20.38 - 101.9 = 2.038 v_1^1 + 10.19\ v_2^1$

$-81.52 = 2.038\ v_1^1 + 10.19\ v_2^1$

$2.038\ v_1^1 + 10.19\ v_2^1 = -81.52$ --------> (1)

If coefficient of restitution Eg 'e' is given

$$e = \frac{V_2^1 - V_1^1}{V_1 - V_2}$$

$$0.6 = \frac{V_2^1 - V_1^1}{10 - (-10)} \qquad \frac{V_2^1 - V_1^1}{20}$$

$V_2^1 - V_1^1 = 20 \times 0.6$

$V_2^1 - V_1^1 = 12$ -------> (2)

Solve Eqn (1) & (2)

$2.038\ V_1^1 + 10.19\ V_2^1 = -81.52$ ------> (1)

Eqn (2) × 2.037 $\quad 2.038\ V_2^1 - 2.038\ V_1^1 = 24.456$

$12.228\ V_2^1 = -57.06$

$V_2^1 = \frac{-57.06}{12.228}$

$V_2^1 = -4.66$ m/s

V_2^1 value sub in Eqn (1)

$2.038\ V_1^1 + 10.19\ V_2^1 = -81.52$ -------> (1)

$2.038\ V_1^{1} + 10.19 \times (-4.66) = -81.52$

$2.038\ V_1^{1} + [-47.55] = -81.52$

$V_1^{1} = \frac{-81.52+47.55}{2.038}$

$V_1^{1} = \frac{-33.96}{2.038}$

$V_1^{1} = 16.66$ m/s

Loss of kinetic Energy:

= Initial kinetic Energy – Final kinetic Energy

[before Impact] [after impact]

Total kinetic Energy before impact

$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2}(10.19)\ (-10)^2$

$= \frac{1}{2}\ (20038)\ (10)^2 + \frac{1}{2}(10.19)\ (-10)^2$

Before K.E = 611.4 N.m

Total kinetic at after impact [find K.E]

$= \frac{1}{2} m_1\ v_1^{1\,2} + \frac{1}{2}\ m_2^{\,1\,2}$

$= \frac{1}{2} \times (2.038)(-16.66)^{\,2} + \frac{1}{2} \times (10.19)(-4.66)^2$

After K.E= 394.26 N.m

Loss of kinetic energy during impact

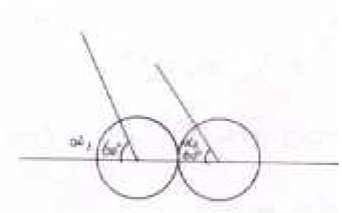
$=$ After K. E $=$ Before K. E

$=611.4 - 394.26$

Loss= 217.11 N.m

Problem 5

A ball of weight 500g moving with velocity of im/sec impings on a bar of mass 1kg moving with velocity 0.75m/s at the time of impact the velocity of the body are parallel and inclined at $60^0$to the line joining there centers. Determine the velocity direction of the ball after the impact where e= 0.6 also find the loss of kinetic energy due to impact.



$\propto_1 = \propto_2 = 60°$

$m_1 = 500\text{g} = \frac{500}{1000} = 0.5kg$

$m_2 = 1\text{kg}$

$v_1 = 1m/s$

$v_2 = 0.75$

Coefficient of restitution e= 0.6

To find:

1. final velocity $v1^1$&$v\,2^1$

2. Direction θ_1& θ_2

3. loss of kinetic energy

Soln:

Law of conservation of momentum

$m_1 v_1 \cos \propto_1 + m_2 v_2 \cos \propto_2 \;\; m_1 v1^1 cos\theta_1 + 1\,v2^1 cos\theta_2$

0.635= $0.5v1^1 cos\theta_1 + v2^1 cos\theta_2$

If coefficient of restitution is given

$$e = \frac{v2^1 cos\theta_2 - v1^1 cos\theta_1}{v_1 \cos\propto_1 - v_2 \cos\propto_2}$$

$$0.6 = \frac{v2^1 cos\theta_2 - v1^1 cos\theta_1}{1 \cos 60 - 0.75\ os\ 60}$$

$$0.6[\cos - 0.7 \times \cos 60] = v2^1 cos\theta_2 - v1^1 cos\theta_1 \text{------------}(2)$$

Solve Eqn (1) &(2)

$$0.625 = 0.5v1^1 cos\theta_1 + v2^1 cos\theta_2$$

$$\underline{0.075 = v2^1 \cos\theta_2 \mp v1^1 \cos\theta_1}$$

$$0.55 = 1.5\ v1^1 \cos\theta_1$$

$$v1^1 \cos\theta_1 - \frac{0.55}{1.5}$$

$$\therefore v1^1 \cos\theta_1 - 0.366$$

$$v1^1 \sin\theta_1 = v_1 \sin \propto 1$$

$$= 1 \sin 60$$

$$= 0.866$$

$$\frac{V_1^1 \sin\theta_1}{V_1^1 \cos\theta_1} = \frac{0.866}{0.366}$$

$$\tan\theta_1 = 2.366$$

$$\theta_1 = \tan^{-1}(2.366)$$

$$\theta = 67°$$

$$V_1^1 \cos\theta_1 = 0.366$$

$$V_1^1 \cos 67° = 0.366$$

$$V_1^1 = \frac{0.366}{\cos 67°}$$

$$V_1^1 = 0.94 m/s$$

------> (2) $0.075 = V_2^1 \cos\theta_2 - V_1^1 \cos\theta_1$

$0.075 = V_2^1 \cos\theta_2 - 0.94 \cos 67°$

$0.075 + 0.94 \cos 67° = V_2^1 \cos\theta_2$

$0.442 = V_2^1 \cos\theta_2$

$V_2^1 \sin\theta_2 = 0.6495$

$$\frac{V_1^1 \sin\theta_1}{V_1^1 \cos\theta_1} = \frac{0.6495}{0.442}$$

$\tan\theta_2 = 1.469$

$\theta_2 = 55°$

$V_2^1 \cos\theta_2 = 0.442$

$V_2^1 \cos 55° = 0.442$

$V_2^1 = \frac{0.442}{\cos 55°}$

$V_2^1 = 0.785\ m/s$

Loss of kinetic Energy = Before K.E − After K.E

Before K.E= $1/2\, m_1v_1^2 + 1/2\, m_2v_2^2$

$= 1/2 \times 0.5 \times 1^2 + 1/2 \times 1 \times (0.75)^2$

Before K.E=0.25+0.281

Before K.E=0.531 N.m

After kinetic Energy= $1/2\, m_1v_1^2 + 1/2\, m_2v_2^2$

$= 1/2 \times 0.5 \times (0.94)^2 + 1/2 \times 1 \times (0.785)^2$

$= 0.2209 + 0.308$

After K.E= 0.528 N.m

Loss of K.E=before K.E−After K.E

$=0.531-0.528$

Loss K.E=2.1×10^{-3}N.m

DYNAMIC OF PARTICLES

Dynamics

It is the branch of science which deals with the study of a body in motion.

Dynamic is further classified into two branches 1. Kinematics 2. Kinetics

Kinematics:

Kinematics is the study of motion of a moving body without considering the force.

Kinetics:

Kinetics is the study of motion of a moving body with considering external force.

Types of plane motion:

1. Rectilinear motion
2. Curvilinear motion

Rectilinear motion:

The motion of particle along a straight line.

Ex: A car moving straight road.

Ex: A stone vertically downward.

Curvilinear motion:

The motion of a particle along a curved path

Characteristic of Kinematics:

1. Displacement: 's'

   The displacement of a moving particle is the change in its position, during which the particle remains in motion. It is denoted by 's'
2. Speed:

   It is distance travelled by the particle (or) body along the path per unit time.

   $$\text{Speed} = \frac{\textbf{Distance dravelled}}{\textbf{time taken}}$$
3. Velocity 'v'

   It is the rate of change displacement.

   $$\text{Velocity} = \frac{\text{Distance travelled in a particular direction}}{\text{Time taken}} \quad m/s$$

4. Acceleration 'a'

   It is the rate of change of velocity acceleration

   $$a = \frac{\textbf{change of velocity}}{\textbf{time taken}}$$

   $$a = \frac{\textbf{final velocity} - \textbf{Intial velocity}}{\textbf{time taken}}$$

Negative acceleration is called retardation [When final velocity < Initial velocity]

5. Average velocity

   $$\text{Average velocity} = \frac{\textbf{Change in position}}{\textbf{Change in time}} = \frac{\Delta x}{\Delta t}$$

6. $$\frac{\text{Average speed}}{\text{Average speed}} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Mathematically Expression for Velocity and Acceleration:

Let s=Distance travelled by a particle in a straight line

t=time taken by the particle travelled this distance

Velocity= $\frac{ds}{dt}$

Acceleration= $\frac{dv}{dt} = \frac{d}{dt}(\frac{d}{dt})$ =d^2s/dt^2

Types of Rectilinear Motion:

1. Uniform acceleration
2. Variable acceleration

Rectilinear motion with uniform acceleration:

Eqn of motion in a straight line:

Consider the particle moving the uniform acceleration is a straight line.

Let u = Initial velocity (m/s)

v = final velocity (m/s)

s=Distance travelled (m)

t=time taken by the particle by the change from the u to v

a=acceleration of particle m/s^2

change o velocity=final velocity−Intial velocity

=v−u

Acceleration= $\frac{\text{change of velocity}}{\text{time taken}}$

a= $\frac{v-u}{t}$

a t = v−u

v=u+at ----------------> (1)

Average velocity= $\frac{\text{Initial velocity+final velocity}}{2}$

$$= \frac{u+v}{2}$$

Distance traveled by the particle in +sec

s=Average velocity $\times$time

s=($\frac{u+v}{2}$)t ----------------->(2)

velocity= $\frac{\text{distance}}{\text{time}}$

v= $\frac{s}{t}$

$$s = v\,t$$

$$s = \frac{u+v}{2} \times t$$

$$2\,s = u + v + t$$

$$2s/t \;= u + v$$

$u + v = \frac{2s}{t}$ v= u+at

$$u+u + at = \frac{2s}{t}$$

$$s = \frac{(2ut + at)t}{2}$$

$$s = \frac{2ut + at^2}{2}$$

$$s = \frac{2ut + at^2}{2}$$

$$s = \frac{2ut}{2} \neq \frac{at^2}{2}$$

$$s = ut + {}^{1}\!/_{2}\,at$$

s= $ut + {}^{1}/_{2}\, at^2$

from (1) v= $u + at$ t= $\frac{v-u}{a}$

$$s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}\, a \times \left(\frac{v-u}{a}\right)2$$

$$s = \frac{uv - u^2}{a} + \frac{1}{2}\, a\left(\frac{v-u}{a^2}\right)$$

$$s = \frac{uv}{a} - \frac{u^2}{a} + \frac{1}{2}\ \frac{v^2 + u^2 - 2vu}{a}$$

$$s = \frac{uv}{a} - \frac{u^2}{a} + \frac{v^2}{2a} + \frac{u^2}{2a} - \frac{2uv}{2a}$$

$$s = \frac{1}{2a}[uv \times 2 - u^2 \times 2 + v^2 + u^2 - 2uv]$$

s= $\frac{1}{2a}[2uv - 2u^2 + v^2 + u^2 - 2uv]$

$$s = \frac{1}{2a}[v^2 - u^2]$$

$2\,as = v^2 - u^2$

$v^2 = u^2 + 2a$

Problem I:

An automobile travels 600m in 40s when it is accelerated at a constant rate of $0.6m/s^2$. Determine the initial and final velocity and the distance travelled for the first 12s.

<u>Given:</u>

Total travels distance=600m

Total time=40s

Acceleration a=$0.6m/s^2$

To find

Intitial and final velocity u &v

Distance travelled for the first 12s

Soln

Now

Distance travelled at 60 m

$s = ut + \frac{1}{2} at2$

$600 = u \times 40 + \frac{1}{2} \times 0.6 \times (40)^2$

Intial velocity.u=$3 {}^{m}/_{s}$

Velocity v=u+at

$V = 3 + 0.6 \times 40$

Final velocity v=27 ${}^{m}/_{s}$

The distance travelled for the first 12s,1-2[1]

$a = 0.6\, m/m^2 \quad u = 3\, m/s$

s=ut+$\frac{1}{2}$at^2

s=3$\times$ $12 + \frac{1}{2} \times 0.6 \times (12)^2$

s=79.2 m

2. The motion of a p[article is defined by the relation $x = 3t^3 - 18t^2 + 26t + 8$

Where is the position expressed in metres and t is the time in seconds Determine (i)When the velocity is zero and (ii)The position and the total distance travelled when the acceleration becomes zero.

Given:

$x = 3t^3 - 18t^2 + 26t + 8$

x=position

t=seconds.

<u>Soln:</u>

Velocity=v= $\frac{dx}{dt}$

$$v = \frac{d}{dt}(3t^2 - 18t^2 + 26 \times 1 + 0$$

$$v = 9t^2 - 36t + 26$$

(ii)When velocity v= 0

0= $9t^2 - 36t + 26$

$9t^2 - 36t + 26$ =0

a=9   b= -36 c=26

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-36) \pm \sqrt{(-36)^2 - 4 \times 9 \times 26}}{2 \times 9}$$

$$t = \frac{-36 \pm 18.97}{18}$$

$t = \frac{36 \pm 18.97}{18}$ t= $3.094s$

$t = \frac{36 - 18.97}{18}$ t=0.946s

the velocity becomes zero t=0.946s and t=3.054s

Acceleration a= $\frac{dv}{dt}$

a= $\frac{d}{dt}[9t^2 - 36t + 26]$

$a = 9\times 2t - 36\times 1 + 0$

$a = 18t - 36$

Acceleration $a = 0$

$0 = 18t - 36$

$18t = 36$

$t = \frac{36}{18}$

$t = 2s$

Distance travelled from t=0 to t=2s

t= $2s$ $\quad x = 3t^3 - 18t^2 + 26 + 8$

$x = 3\times(2)^3 - 18\times(2)^2 + 26\times 2 + 8$

$x = 12\ m$

t= $0s$ $\quad x = 3\times(0)^3 - 18\times(0)^2 + 26\times 0 + 8$

$x = 8\ m$

When t= $0.946\ s$ 'v' becomes zero

$x = 3(0.946)^3 - 18\times(0.946)^2 + 26\times 0.946 + 8$

$x = 19\ m$

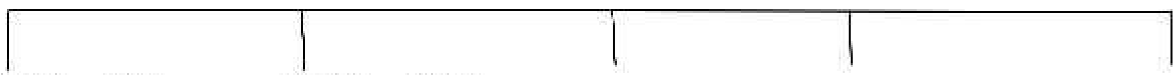
Total distance travelled= $(19 - 8) + (19 - 12)$

$= 18\ m$

3, A particle under constant deceleration is moving on a straight line and covers a distance of 25 m in the first 3s and 40 m in nest 6s. Calculate the distance it covers in subsequent 2s and the total distance covered before it come to rest.

Given:

A B C D E



S AB=25m S BC= 40 m

t= 3s t= 6s t= 2s

(A-B) (UDRM)

S= ut+$\frac{1}{2}at^2$

$S_{A\text{-}B}= u_a\ t_{A-B} + a\ t_{AB^2}$

$25 \pm u + 3 + \frac{1}{2}(a) \times (3)^2$

$25 = 3\ u + \frac{1}{2}\ a \times 9$

25=3 u+4.5 a

Both side÷by 3

$\frac{25}{3} = \frac{3u}{3} + \frac{4.5}{3}$ a

8.33=u+4.5 a

u+1.5 a = 8.33 -----------> (1)

A−C

  s= $ut + \frac{1}{2}at^2$

$65 \pm u \times 9 + \frac{1}{2}at^2$

s=65 t= 3 + 6 = 9

$65 = 9u + 40.s\ u$

÷9

7.22=u+4.5 a

u±4.5 a = 7.22

u= $7.22 - 4.5\,a$--------->(2)

sub (ii) in (i)

$7.22 - 4.5$ a $+ 1.5$ a=8.33

-3a= $8.33 - 7.22$

-3a=1.108

a=$1.108/-3$

a= $-0.369\ m/s^2$---------->(3)

sub (iii) in (2)

$u = 7.22 - 4.5 \times (0.369)$

u= $8.88\,m/s$

To find velocity at point c

v=u+at

$v_c = u_A + a t_{A-C}$

$= 8.88 + (-0.369)(9)$

$v_c = 5.56\ m/s$

For the motion from C to D (UDRM)

$v_c = 5.56 m/s$ $\quad t_{C-D} = 2s$ $\quad$ a= $-0.369\ m/s^2$

s= $ut + \frac{1}{2}\,at^2$

$S_{C-D} = u_c t_{C-D} + \frac{1}{2} a\, t_{CD^2}$

$= 5.56 \times 2 + \frac{1}{2} \times (-0.369) 2^2$

$S_{C-D} = 10.38$ m

Distance travelled in subsequent t=2s

s=10.38 m

For the motion from C-E (UDRM)

V_c=5.56 m/s a= $-0.369m/s$ V_E= 0

We have

$v^2-u^2=2as$

$v_E^2-v_C^2=2as$

$0^2-(5.56)^2=2\times(-0.369)\times s_{CE}$

s_{CE}=41.8 m

Total distance travelled before it comes to res

$=S_{AB}\ S_{BC}\ +S_{CE}$

=25+40+41.8

Total distance=106.9 m

4. The position of a particle which moves along a straight line is defined as $s = t^3 - 6t^2 - 15t + 40$ where s is expressed in m and + is in sec. Determine the (a) time at which the velocity will be zero. (b) the position and distance travelled by the particle at that time (c) acceleration of the particle at that time (d) the distance travelled by the particle when t–4 to t–6

Given:

$s = t^3 - 6t^2 - 15t + 40$

Soln:

a) t=? Velocity v=0

v= $\frac{ds}{dt}$

v= $\frac{d}{dt}(t^3 - 6t^2 - 15t + 4)$

v=$3t^2 - 6 \times 2t - 15 \times 1 + 0$

v=$3t^2 - 12t - 15$

v=0

$3t^2 - 12t - 15 = 0$

t= $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a= 3 b= -12 c-15

t= $\frac{-b \pm \sqrt{(-12)^2 - 4 \times 3(-15)}}{2 \times 3}$

t= $\frac{12 \pm \sqrt{144+130}}{6}$

t= $\frac{12 \pm \sqrt{324}}{6}$

t= $\frac{12 \pm 18}{6}$

t= $\frac{12+18}{6} = \frac{30}{6}$

T=5 Sec

&

t= $\frac{12-18}{6} = \frac{-6}{6}$

t= $-1\ sec$

t$\neq -1$

t= $5\ sec$

b) t=5 Sec & displacement s=?

s= $t^3 - 6t^2 - 15t + 40$

s= $5t^3 - 6(5)^2 - 15 \times 5 + 40$

s= $-60\ m$

t= 0

s= $0^3 - 6 \times 0^2 - 15 \times 0 + 40$

s= $40\ m$

Distance travelled= $[s_t = 5] - [s_t = 0]$

$$= -60 - 40 \qquad = -100\ m$$

Distance travelled = $100\ m$

3) when t=6 sec displacement 's'

s= $t^3 - 6t^2 - 15t + 40$

s= $6^3 - 6 \times 6^2 - 15 \times 6 + 40$

s= $4^3 - 6 \times 4^2 - 15 \times 4 + 40$

s= $-52\ m$

Distance travelled when t=4 to 5 sec

$= s_t = 5 - s_t = 4$

$= -60 - [-52]$

$= -60 + 52$

$= -8 = 8\ m$

Distance travelled when t=5 to 6

$= \mathrm{s_t} = 5 - \mathrm{s_t} = 5$

$= -50 - (-60)$

=10 m

Total distance travelled=8+10 $= 18\ m$

4) Acceleration a

a= $\frac{dv}{dt} = \frac{d}{dt}[3t^2 - 12t - 15]$

a=3$\times$ $2t - 12 \times 1$---(5)

a=6t−12

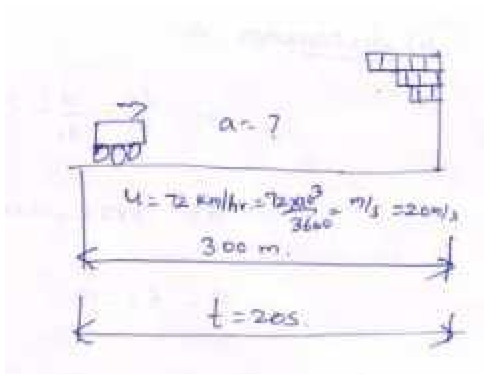
t=5 sec

a=6$\times$ $5 - 12$

a= $30 - 12$

a=18m/s^2

5) A driver of a car travelling at $72km/h$ Observes the traffic light 300 m ahead of him turning red. The traffic light is timed to remain red for 20 seconds before it turns without stopping to wait for its turn green, Determine (i) the required uniform acceleration of the car (ii) the speed with which the motorist crosses the traffic light.



Soln:

Displacement

s= $ut + \frac{1}{2}at^2$

300=20$\times$ $20 + \frac{1}{2} \times a \times 20^2$

a= $-0.5\ m/s^2$ (Retardation)

Final velocity

v=u+at

v= $20 + (-0.5) \times 20$

v= $10\, m/s$

v= $\frac{10\times3600}{1000} km/hr$

v= $36\ km/hr$

Problem:5

A particle starting from rest moves in a straight like and its acceleration is given by a=$50-36t^2 m/s^2$ Where t is in sec.Determine the velocity of the particle when it has travelled 52m.

Given

a= $50 - 36t^2$

s= $52m$

To find

Velocity

Soln

Acceleration a= $\frac{dv}{dt}$

dv=a$\times$ dt

dv=a$\times$ dt

dv=($50-36t^2$)dt

$\int dv\ \ \int(50 - 36t^2)dt$

$\int dv$=$\int(50 - 36t^2)dt$

v=$50t - 36 \times \frac{t^3}{3}$

v= $50t - 36 \times \frac{t^3}{3}$

v= $50t - 12t^3 + c_1$

when t= 0 $v = 0$ $c_1 = 0$

v= $50t - 12t^3$

ds= $v \times dt$

ds= $50t - 12t^3 \times dt$

ds= $50t - 12t^3 \times dt$

$\int ds = \int(50t - 12t^3)dt$

s= $\frac{50t^2}{2} - 12 \times \frac{t^4}{4} + c_2$

s= $25t^2 - 3t^4 + c_2$

when t=0 s= 0 $c_2 = 0$

s= $25t^2 - 3t^4$

Now s=52 m finding out t

52=25$\times t - 3t^4$

52=25$t - 3t^4$

Put $t^2 = t$

52=25 t$-3t^2$

$3t^2 - 25t + 52 = 0$

a=3

b=-25

c=52

$$t = \frac{-b \mp \sqrt{b^2-4ac}}{2a}$$

$$t= \frac{-(25) \mp \sqrt{(-25)^2 - 4 \times 3 \times 52}}{2 \times 3}$$

t=2.0816sec & t=2sec

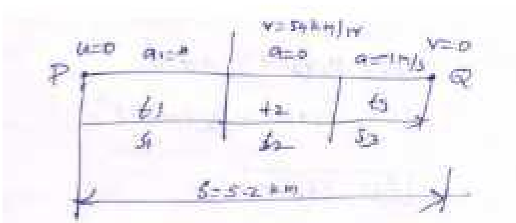
when t=2sec $v=50\times 2 - 12 \times 2^3$

v=2m/s

when t=200816 sec $v = 5050\times 2.0816 - 12 \times (2.0810)^2$

$v = -4.163/m/s$

6. Two stations pand Q are 5.2km apart. A train stars from rest at the station P and accelerates uniformly to attain a speed of 54 km/hr in 30 sec. The speed is maintained until the brakes are applied. The train comes to rest at the station Q with uniform retaration of 1m/s^2. Determine the total time required to cover the distance b/w these two station



<u>Consider Phase I</u>

U=0

t_1=30 sec

v_1=15m/s

v_1=u+$a_1 t_1$ $^{v=u+at}$

$15=0+a_1 \times 30$

$a_1=0.5\ m/s^2$

$s_1=ut_1 + \frac{1}{2}a_1 t1^2$

$s_1=0+\frac{1}{2} + a_1 t1^2$

$s_1=225m$

Consider Phase –III

$v_1=15m/s$

$a_3=-1m/s^2$

V=0

$V=u + at$

$0=15 - 1 \times t_3$

$0 = 15 - +3$

$t_3=15$ sec

$s_3=u_3 v_3 + {}^1/_2\, a_3 t3^2$

$=15\times 15 + {}^1/_2(-1)15^2$

$s_3 = 112.5m$

Consider Phase-II

$s_2 = s - [s_1 + s_3]$

$s_2=5200-[225 + 112.5]$

$s_2=4862.5m$

s_2 ut+ ${}^1/_2\, at^2$ $\qquad a = 0$

s_2=ut

4862.5=15$\times t$

t_2=$\frac{4862.5}{15}$

t_2=324.167sec

Total time=30+324.167+15

time=369.167 sec

multiply 2

120= $14\,u + 49\,a$

14 u+49 a=129=0-----(1)

÷=14

$u + 3.5a = 8.57$

u=8.57-3.5 a-----(2)

Sub Eqn (2) in (1)

$u + a$=10-------(1)

$3.57-3.5a + a = 10$

$8.57-2.5a = 10$

-2.5a= $10 - 8.57 = 1.43$

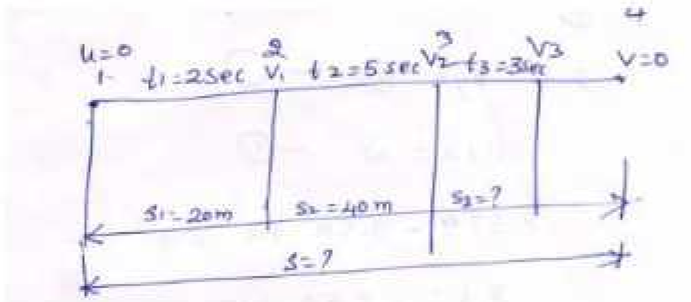
a=$\frac{1.43}{-2.5}$

a=$-0.572\ m/s^2$

u+$(-0.572) = 10$

u=10.572m/s

7. A particle under constant declaration is moving in a straight line and covers a distance of 20m in first 2seconds, and 40min the next 5sec. Calculate the distance it covers in the he subsequent 3sec and total distance travelled by the particle before it comes to rest.



Soln:

Phase (1)-2

The displacement s=$ut + {}^{1}/_{2}\, at^2$

t=2 sec s=20m

20=$u \times 2 + {}^{1}/_{2}\, at^2$

20=2u+2a

$\div$ 10 $u + a = 10$--------------(1)

Phase 1-3

s= $u + {}^{1}/_{2}\, at^2$

60=$u \times + {}^{1}/_{2} \times a \times 7^2$

60=$7u + {}^{1}/_{2} \times 49a$

S= $20 + 40 = 60$

t=2+5=7

Considered 3rd phase

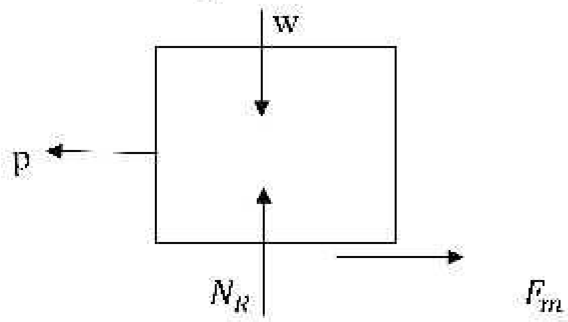
Unit –V

FRICTION

Friction:

When the two surface contact with each other and one surface tense to move with respect to another surface.

The tangent force developed in the contact6 surface and in the opposite reaction.



Types of Friction:

- Dry Friction or coulomb friction
- Fluid friction

Dry friction:

It is referred to the friction which is developed b/w two dry surface

Dry friction types

1. Static friction
2. Dynamic friction

Static friction:

It is the friction which experience by object when it is rest.

Dynamic friction:

It is the friction which experience by object in moving condition.

Dynamic friction further classified by two types

1. Sliding friction
2. Rolling friction

Coefficient of Friction:

It is the ratio b/w limiting friction and the normal reaction [normal friction] is called coefficient of friction.it is denoted by letter μ

$$\mu = \frac{F_N}{R}$$

Coulomb Law of Friction [Dry Law]

1. Law of static friction
2. Law of dynamic friction

Law of static friction:

* The frictional force is always act opposite direction to be movement of the object or body
* The frictional force does not depends upon the size and shape of the object
* The frictional force depends the degree of roughness of the contact surface b/w the two object
* The frictional force is equal to force applied to the body or object
* Limiting friction is directly propositional to the normal friction

$$F_N \alpha N_R$$

Coulomb Law of Dynamic Friction:

* The frictional force is always opposite to the movement of the object.
* The magnitude of the dynamic friction the contact ratio to normal reaction b/w two system
* The coefficient of kinetic friction less then coefficient of static friction

Angle of friction:

Angle of friction is Angle made by the resultant with the normal reaction and frictional force.

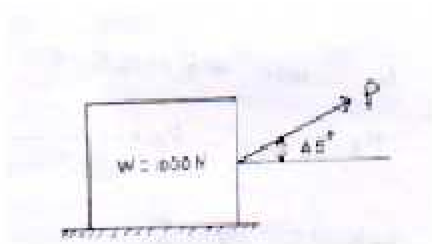
$$tan\emptyset = \frac{F_N}{N_R}$$

$$\emptyset = tan^{-1}\left[\frac{F_N}{N_R}\right]$$

Problem based on Friction:

1. A body weighting 1000N is lying on a horizontal plane. Determine the necessary force to move the body along the plane if the force is applied at angle of 45° to the horizontal with the coefficient of friction 0.24

Given:

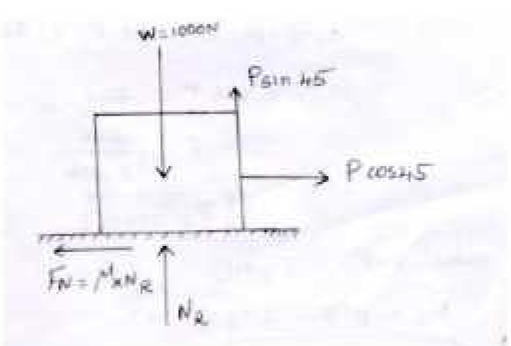


Coefficient of friction $\mu = 0.24$

Weight of body $w = 1000\ N$

To find

Soln



Sum of X Direction Force + - → - ←

$\sum F_x = 0$

$Pcos45 - F_N = 0$

$Pcos45 - \mu \times N_R = 0$

$Pcos45 - 0.24 \times N_R = 0$ ---------------- (1)

Sum of Y direction force ↑+ ↓-

$\sum F_y = 0$

$N_R - 1000 + Psin45 = 0$

$N_R = 1000 + psin45 = 0$

N_R value sub in Eqn(1)

$Pcos45 - 0.24 \times [1000 - Psin45] = 0$

$Pcos45 - 240 + [Psin45 \times 0.24] = 0$

$0.707P + 0.169\,P = 240$

$0.876P = 240$

$$P = \frac{240}{0.876}$$

$P = 273N$

P value sub in eqn (2)

$N_R = 1000 - 273 \times sin45$

$N_R = 806.95\ N$

Problem: 2

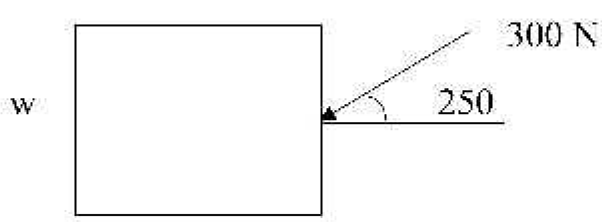
A pull of 250N inclined at 25° to the horizontal plane is required just to move a body kept on a rough horizontal plane. But the push required just to move the body is 300N. If the push is inclined at 25° to the horizontal. Find the weight of the body and the coefficient of friction b/w the body and the plane.

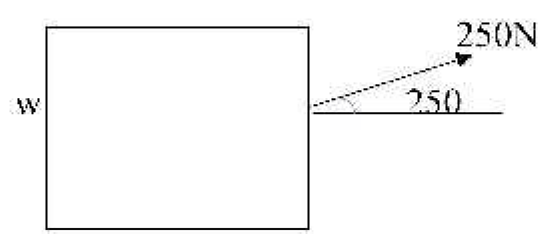
Given:

$P_1 = pull\ load = 250\ N\ at\ 25°$

$P_2 = push\ load = 300\ N\ at\ 25°$

To find



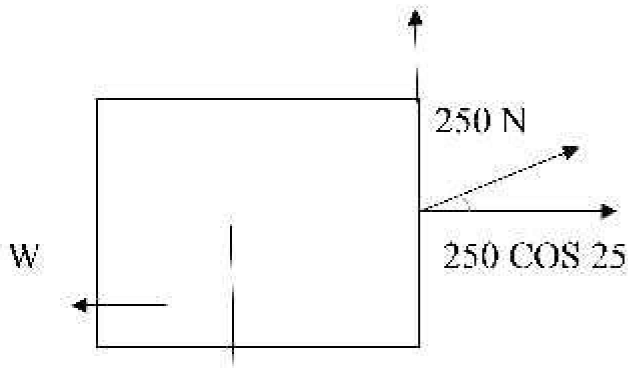


Weight and coefficient of friction

Soln

Case (i)

Free body diagram



Sum of x directional force + $\rightarrow$ $\leftarrow$ −

$\sum F_x = 0$

$250 \cos 25° - F_N = 0$

$250 \cos 25° - \mu \times N_{R1} = 0$

$226.57 - \mu \times N_{R1} = 0$

$-\mu \times N_{R1} = -226.57$

$\mu \times N_{R1} = 226.57$------------------- (1)

Sum of Y directional force $\downarrow$ - $\uparrow$ +

$\sum F_y = 0$

$N_{R1} - W + 250\, sin25 = 0$

$N_{R1} = W - 250 sin25$

$N_{R1} = W - 105.65$----------------- (2)

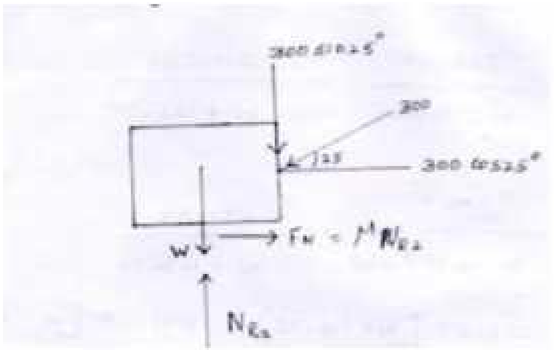
Sub eqn (1), from eqn (1)

$\mu N_{R1} = 226.57$

$$\mu = \frac{226.57}{N_{R1}}$$

$\mu = \frac{226.57}{W-105.65}$ -------------- (3)

Case (2) Free body diagram



Sum of X Directional force $\sum F_x = 0$

$F_N - 300cos25° = 0$

$\mu N_{R2} = 300cos25°°$

$\mu N_{R2} = 271.89N$------------ (4)

Sum of vertical force[Y direction] $\sum F_y = 0$

$N_{R2} - W - 300sin25° = 0$

$N_{R2} = W + 300sin25°$

$N_{R2} = W + 126.78$

N_{R2} value sub in Eqn (4)

$\mu N_{R2} = 271.89$

$$\mu = \frac{271.89}{N_{R2}}$$

$\mu = \frac{271.89}{W+126.78}$ ---------- (5)

Eqn (3) =Eqn (5)

$$\frac{226.57}{W - 105.65} = \frac{271.89}{W + 126.78}$$

$226.57[W + 126.78] = 271.89[W - 105.65]$

$226.57W + 28.72 \times 10^3 = 271.89W - 28.72 \times 10^3$

$226.57W - 271.89W = -28.72 \times 10^3 - 28.72 \times 10^3$

$-45.32W = -57.44 \times 10^3$

$W = \dfrac{57.44 \times 10^3}{45.32}$

$W = 1267.43N$

W value sub in eqn (3)

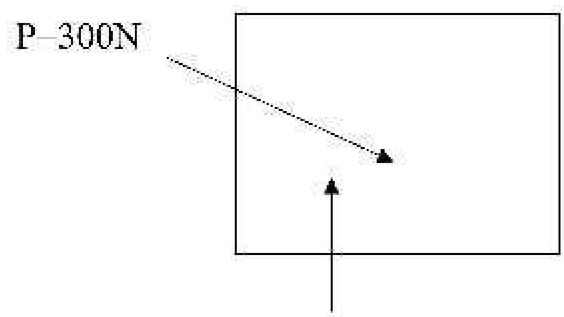
$\mu = \dfrac{226.57}{W - 105.65}$

$\mu = \dfrac{226.57}{1267.43 - 105.65}$

$\mu = 0.195$

Problem 3

Calculate the static coefficient of friction μ_sb/w the block shown in fig having a mass of 75kg and the surface. Also find the magnitude and direction of the friction force if the force P applied is inched at 45° to the horizontal and $\mu_s = 0.30$



Given:

Weight= $75\,kg = 75 \times 9.81 = 735.75N$

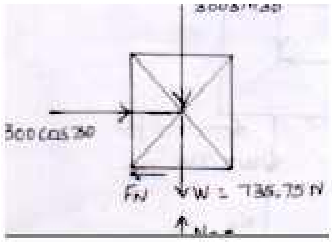
To find

Case (i) Coefficient of friction μ_s

Case (ii) Frictional force 'F_N', Direction $\emptyset$

Soln

Case (i) free body diagram



Sum of all the X direction force $\sum F_x = 0$

$300\cos 30 - F_N = 0$

$300\cos 30 - \mu N_R = 0$

$\mu N_R = -300cos30$

$\mu N_R = 259.8$---------------- (1)

Sum of all the Y direction force $\sum F_y = 0$

$-300\sin 30 - 735.75 + N_R = 0$

$N_R = 300sin30 + 735.75$

$N_R = 885.75\ N$

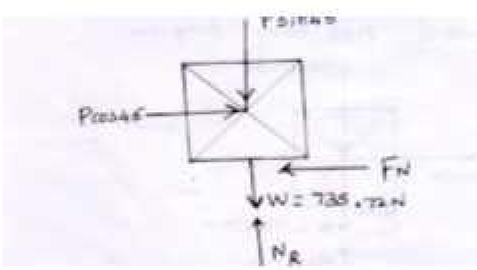
N_R value sub in eqn(i)

$\mu N_R = 259.8$

$$\mu = \frac{259.8}{N_R} = \frac{259.8}{885.75}$$

$\mu = 0.29$

Case (ii) free body diagram



Sum of all the horizontal force [X direction] $\sum F_x = 0$

$P\cos 45 - F_N = 0$

$$-F_N = -P\cos 45$$

$$F_N = P\cos 45$$

$$\mu N_R = p\ cos45$$

$$N_R = \frac{pcos45}{\mu} = \frac{pcos45}{0.3}$$

$$N_R = 2.35p$$

Sum of all the Direction force $\sum F_y = 0$

$$-p\ sin45 - 732.72 + N_R = 0$$

$$-p \times 0.7 - 735.72 + 2.35p = 0$$

$$-0.7p + 2.35p = 735.72$$

$$1.64p = 735.72$$

$$p = \frac{735.72}{1.64}$$

$$p = 447.81N$$

$$N_R = 2.35p$$

$$N_R = 2.35 \times 447.81$$

$$N_R = 1052.37N$$

$$F_N = \mu \times R = 0.3 \times 1052.37$$

F_N=315.71N

Direction ∅

$$\emptyset = tan^{-1}\left[\frac{F_N}{N_R}\right]$$

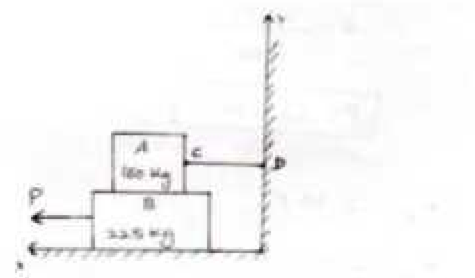
$$\emptyset = tan^{-1}\left[\frac{315.71}{1052.37}\right]$$

$$\emptyset = 16°41'$$

Problem 4:

Determine the smallest force P required to move the block B shown n fig below (i) block A is restrained by cable CD as shown in fig. (ii) Cable CD is removed. Take

$\mu_s = 0.30 \; and \; \mu_k = 0.25$



Given:

$W_A = 150kg = 150 \times 9.81 = 1471.5N$

$W_B = 225kg = 225 \times 9.81 = 2207.25N$

$\mu_s = 0.3$

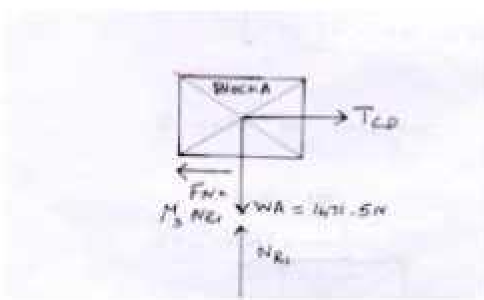
$\mu_k = 0.25$

To find

Force P

Soln

Block A is restrained by cable CD



Sum of X direction force $\sum F_x = 0$

$T_{CD} - F_N = 0$

$T_{CD} - \mu_s N_{R1} = 0$

$T_{CD} = \mu_s N_{R1}$

$T_{CD} = 0.3 N_{R1}$------------- (1)

Sum of vertical [Y direction force] $\sum F_y = 0$

$N_{R1} - W_A = 0$

$N_{R1} = W_A$

$N_{R1} = 1471.5N$

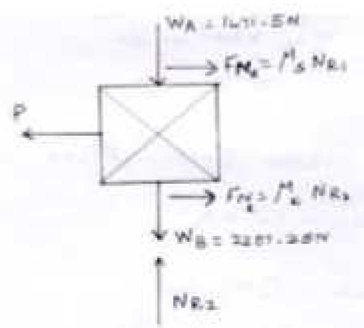
N_{R1} value sub in eqn(i)

$T_{CD} = 0.3\ R_1$

$T_{CD} = 0.3 \times 1471.3$

$T_{CD} = 441.45\ N$

Free body diagram of block B



Sum of X direction force

$F_{N_s} + F_{N_k} - p = 0$

$\mu_s N_{R1} + \mu_k N_{R2} - p = 0$

$p = \mu_s N_{R1} + \mu_k N_{R2}$-------- (1)

Sum of Y direction force

$-1471.5 - 2207.25 + N_{R2} = 0$

$N_{R2} = 1471.5 + 2207.25$

$N_{R2} = 3678.75N$

N_{R2} value sub in Eqn (1)

$p = \mu_s N_{R1} + \mu_k N_{R2}$

$p = 0.3 \times 1471.5 + 0.25 \times 3678.75$

$p = 1361.14\ N$

(i) Cable CD is removed

Both block is removed

Both the block will consider as a single body

$p = F_N$

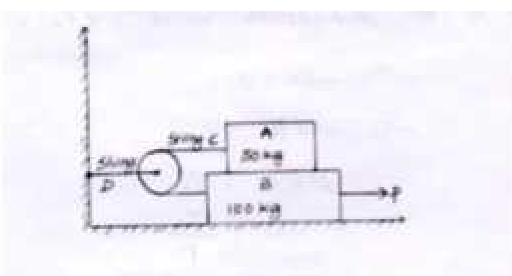
$p = \mu_k N_{R2} = 0.25 \times 3678.75$

$p = 919.68N$

Problem-6

Two blocks A and B of mass 5o kg and 10 kg respectively are connected by a string c which passes through a frictionless pulley connected with the fixed wall by another string D as shown in fig. Find the force P required to pull the block B. also find the tension in the string D.

Take coefficient of friction at all contact surface as 0.3°



Given:

Weight of block A $W_A = 50kg = 50 \times 9.81 = 490.5N$

Weight of block B $W_B = 100kg = 100 \times 9.81 = 981N$

Coefficient of friction$\mu = 0.3$

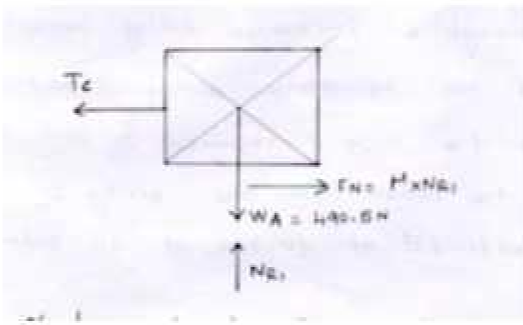
To find

(i) Force P

(ii) Tension in string T_D

Soln

Free body diagram of block A



Sum of x direction forces $\sum F_x = 0$

$-T_c$-$F_N = 0$

$-T_c + \mu N_{R1} = 0$

$\mu N_{R1} = T_c$

$N_{R1} = \dfrac{T_c}{\mu}$

$N_{R1} = \frac{T_c}{0.3}$ ---------- (1)

Sum of Y direction force $\sum F_Y = 0$

$-W_A + N_{R1} = 0$

$N_{R1} = W_A$

$N_{R1} = 490.5\ N$

N_{R1} value sub in Eqn (1)

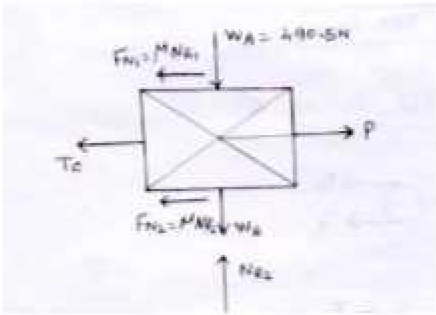
$N_{R1} = \dfrac{T_c}{0.3}$

$490.5 = \dfrac{T_c}{0.3}$

$T_c = 147.15\ N$

Consider block B

free body diagram



Sum of X direction force$\sum F_x = 0$

$p - T_c - F_{N1} - F_{N2} = 0$

$p - \mu N_{R1} + \mu N_{R2} = 0$

$p - 147.15 - 0.3 \times 490.15 - 0.3 \times N_{R2} = 0$

$p - 147.15 - 147.04 - 0.3 \times N_{R2} = 0$

$p - 294.19 - 0.3N_{R2} = 0$

$p = 294.19 - 0.3N_{R2} = 0$

$P = 294.19 + N_{R2}$------------- (2)

Sum of Y direction force $\sum F_Y = 0$

$N_{R2} - W_B - W_A = 0$

$N_{R2} - 981 - 490.5 = 0$

$N_{R2} = 1471.5N$

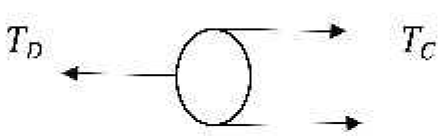
N_{R2} value sub in eqn(2)

$p = 294.19 + 0.3N_{R2}$

$p = 294.19 + 0.3 \times 1471.5$

$p = 735.64N$

Tension in the string D:



$\sum F_x = 0$

$T_c + T_c - T_D = 0$

$2T_c - T_D = 0$

$2 \times 147.15 - T_D = 0$

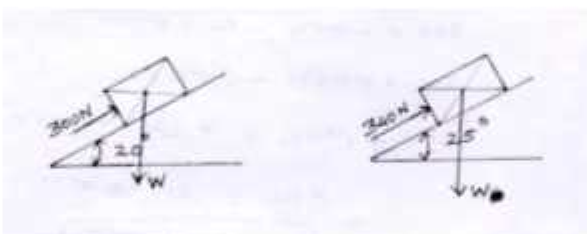
$294.3 - T_D = 0$

$-T_D = -294.3$

$T_D = 294.3N$

Problem -7

A force of 300 n is required just to move a block up a plane inclined at 20°.To the horizontal, the force being applied parallel to the plane shown in fig. if the inclination of the plane is increased to 25°, the force required just to move the block up is 340 N, [the force is acting parallel to the plane]. Determine the weight of the block and coefficient of friction.



Given:

Case (i)

Weight of body w=?

Force on body P= 300 N at 20°inclined on plane horizontal

Case (ii)

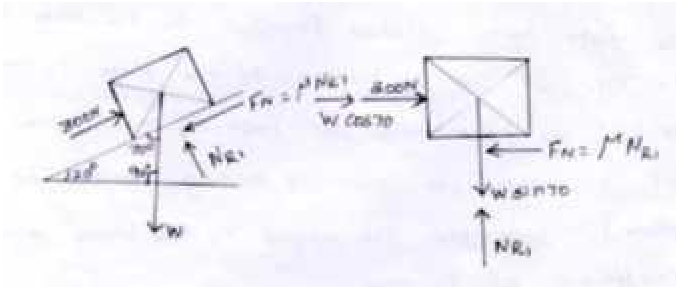
Force on body P= 340 N at 25°

To find:

Weight of body & coefficient of friction

Soln:

Case (i) free body diagram



Sum of X directional force $\sum F_X = 0$

$300 + wcos70 - F_N = 0$

$300 + wcos70 - \mu N_{R1} = 0$

$-N_{R1} = -[300 + wcos70]$

$\mu N_{R1} = 300 + w\cos 70$

$\mu = \frac{300+wcos70}{N_{R1}}$--------- (1)

Sum of all Y direction force $\sum F_y = 0$

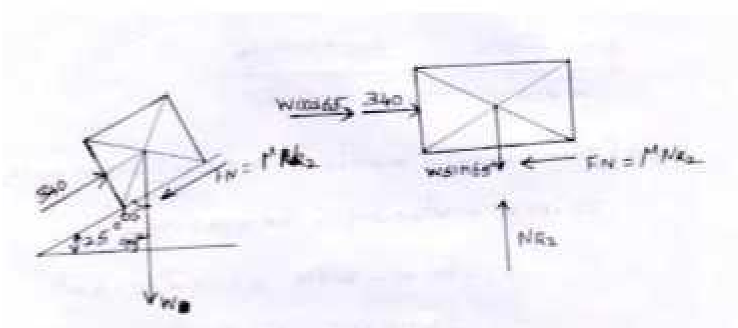
$N_{R1} - wsin70 = 0$

$N_{R1} = wsin70$------------ (2)

N_{R1} value sub in eqn (1)

$\mu = \frac{300+wcos70}{wsin70}$--------- (3)

Case (ii) consider block 2



Sum of all the X direction force $\sum F_X = 0$

$340 + w\cos 65 - F_N = 0$

$340 + wcos\,65 - \mu N_{R2} = 0$

$-\mu N_{R2} = -[340 + wcos65]$

$\mu = \frac{340+wcos65}{N_{R2}}$--------- (4)

Sum of all the Y direction force $\sum F_Y = 0$

$N_{R2} - wsin65 = 0$

$N_{R2} = wsin65$-------------- (5)

Eqn (5) sub in eqn (4)

$\mu = \frac{340+w\cos 65}{w\sin 65}$--------- (6)

Eqn(3) =eqn (6)

$$\frac{300 + wcos70}{wsin70} = \frac{340 + w\cos 65}{w\sin 65}$$

$[300 + wcos70] \times wsin65 = wsin70[340 + w\ cos65]$

$271.89w + w^2 \times 0.309 = 319w + 0.39w^2$

$271.86w - 319w = 0.39w^2 - 0.3w^2$

$-47.11\ w = 0.09w^2$

$-47.11\ w = 0.09w$

$$w = \frac{-47.11}{0.09}$$

ans $w = -523.47\ N$

w value sub in eqn(3)

$$\mu = \frac{300 + wcos70}{wsin70}$$

$$\mu = \frac{300 + (-523.47)cos70}{(-523.47)sin70}$$

Ans $\mu = -0.24$

Problem 8

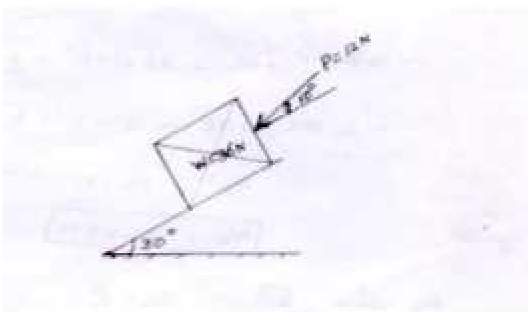
A block weighting 360 n is resting on a rough inclined plane having an inclination of 30°. A force of 12 N is applied at an angle of 10°up and the block is just on the point of moving down the plane. Determine the coefficient of friction

Given:

Block weight w=36 N

Inclination of the plane $\emptyset = 30°$

Force on block P–12 N at 10°

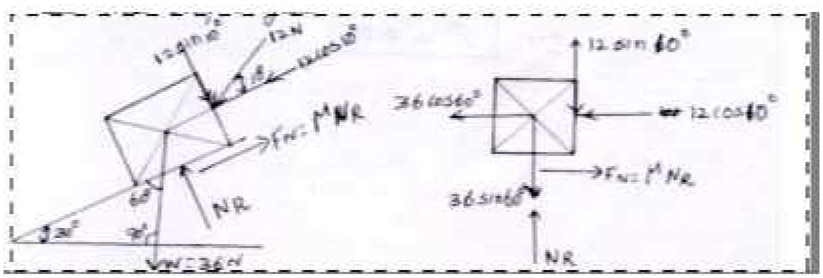


To find

Coefficient of frictionμ

Soln:

Free body diagram



Sum of X direction force $\sum F_X = 0$

$-12\ cos10 - 36cos60 + F_N$=0

$-11.87 - 18 + \mu N_R = 0$

$-29.87 + \mu N_R = 0$

$\mu N_R = 29.87$

$\mu = \frac{29.87}{N_R}$------ (1)

Sum of all the Y direction force $\sum F_y = 0$

$-12\ sin10° + N_R - 36\ sin60° = 0$

$-2.083 + N_R - 31.17 = 0$

$N_R - 33.25 = 0$

$N_R = 33.25\ N$

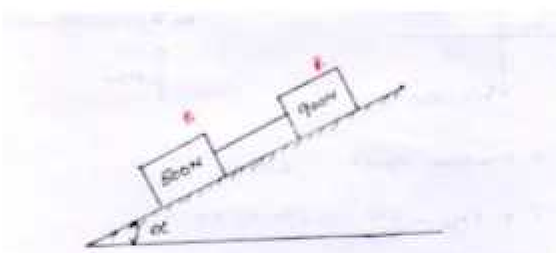
N_R value sub in Eqn (1)

$$\mu = \frac{29.87}{N_R} = \frac{29.87}{33.25}$$

$ans\ \mu = 0.89$

Problem 9

Two block of weight 500 N and 900 N connected by a rod are kept in an inclined plane is shown in fig. the rod is parallel to the plane. The coefficient of friction b/w 500 N block and the plane is 0.3 and that b/w 900 N block and the plane is 0.4 find the inclination of the plane with the horizontal and the rod when the motion down the plane is just about the start.



Given:

Weight of block A $W_A = 500\ N$

Weight of block B $W_B = 900\ N$

Coefficient of friction $\mu_A = 0.3$

Coefficient friction at block B $\mu_B = 0.4$

To find

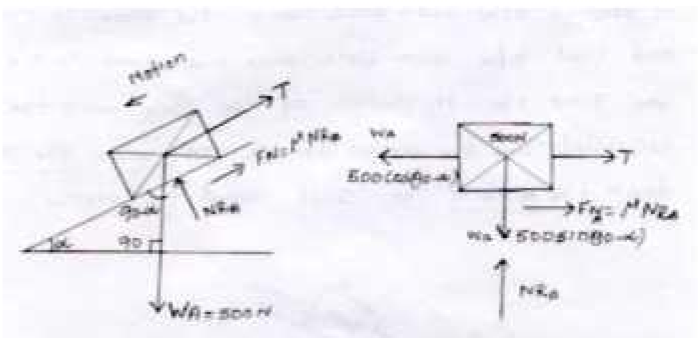
1. Inclination of the plane α

2. Tension

Soln:

Consider block 'A'

Free body diagram



Sum of X direction force $\Sigma F_X = 0$

$T + F_{N_A} - 500\cos(90-\alpha) = 0$

$T + \mu_A N_{RA} = 500\cos(90-\alpha)$

$T + \mu_A N_{RA} = 500 sin\alpha$---------- (1)

Sum of Y direction force $\Sigma F_Y = 0$

$N_{RA} - 500\sin(90-\alpha) = 0$

$N_{RA} = 500\sin(90-\alpha)$

$N_{RA} = 500\cos\alpha$--------------- (2)

$T + 0.3\ N_{RA} = 500\ sin\alpha$

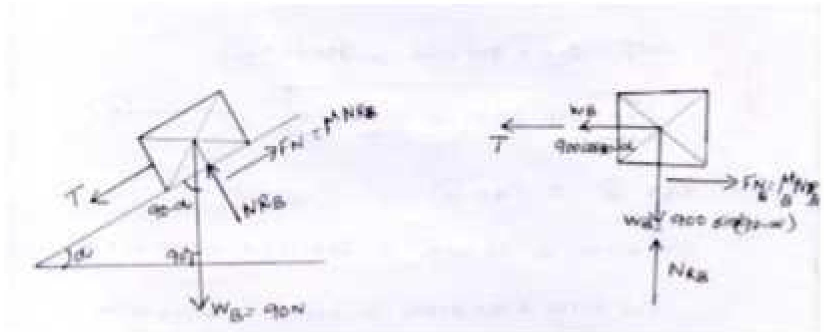
N_{RA} value sub in Eqn (1)

$T + 0.3\ \times 500\ cos\alpha = 500\ sin\alpha$

$T + 150 cos\alpha = 500\sin\alpha$

$T = 500\sin\alpha - 150\ cos\alpha$------------ (3)

Consider block B



Sum of X direction force $\Sigma F_X = 0$

$F_{NB} - T - 900\cos(90-\alpha) = 0$

$\mu_B N_{RB} - T - 900 sin\alpha = 0$

$0.4N_{RB} - T - 900\sin\alpha = 0$

$-T = -0.4N_{RB} + 900\ sin\alpha$

$-T = -[0.4N_{RB} - 900 sin\alpha]$

$T = [0.4N_{RB} - 900 sin\alpha]$------------- (4)

Sum of vertical [Y direction force] $\Sigma F_y = 0$

$N_{RB} - 900\sin(90-\alpha) = 0$

$N_{RB} = 900\sin(90-\alpha)$

$N_{RB} = 900\cos\alpha$--------- (5)

N_{RB} value sub in Eqn(4)

$T = 0.4 \times N_{RB} - 900\ sin\alpha$

$T = 0.4 \times 900\ cos\alpha - 900 sin\alpha$

$T = 360\ cos\alpha - 900\ sin\alpha$-------------- (6)

Eqn (3) = eqn(6)

$500\sin\alpha - 150\cos\alpha = 360\ \ cos\alpha - 900 sin\alpha$

$500\sin\alpha + 900\ sin\alpha = 360 cos\alpha + 150 cos\alpha$

$1400 \sin \alpha = 510 \cos \alpha$

$$\frac{sin\alpha}{cos\alpha} = \frac{510}{1400}$$

$$tan\alpha = \frac{510}{1400}$$

$$\alpha = tan^{-1}\left[\frac{510}{1400}\right]$$

Ans $\alpha = 20°$

α value sub in Eqn (3)

$T = 500\ sin\alpha - 150\ cos\alpha$

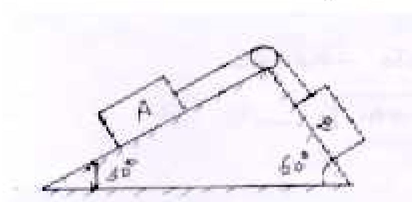
$T = 500 \sin 20 - 150\ cos20$

$T = 171 - 140$

$T = 31\ N$

Problem 10

Two block A and B are placed on inclined planes as shown in fig. the block a weight 1000N. Determine the minimum weight of the block b for maintaining the equilibrium of the system. Assume that the blocks are connected by an inextensible string passing over a frictionless pulley. Coefficient of friction μ_A b/w the block A and the plane is 0.25. Assume the same value for μ_B



Given:

Weight of block A $W_A = 1000N$

Coefficient of friction in block A & B $\mu_A = \mu_B = 0.25$

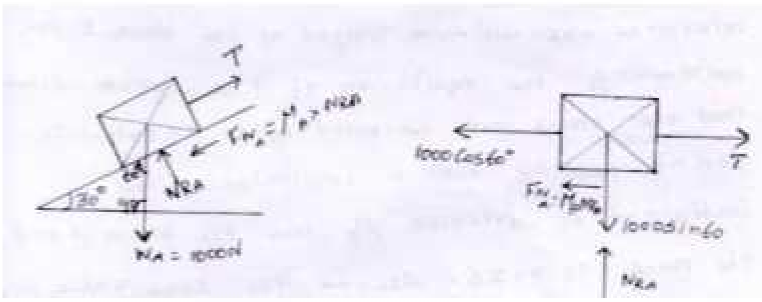
To find

Weight of block 'B' W_B

Solution:

Consider block 'A'

Free body diagram



Sum of X direction force Σ

$T - 1000\cos 60 - F_{NA} = 0$

$T - 1000\cos 60 \ - \mu_A N_{RA} = 0$

$T = 1000\ cos60 + 0.25\ N_{RA}$

$T = 500 + 0.25N_{RA}$-------------- (1)

Sum of all Y direction forces

$N_{RA} - W_A = 0$

$N_{RA} - 100\ sin60 = 0$

$N_{RA} = 1000\ sin60$

$N_{RA} = 866\ N$

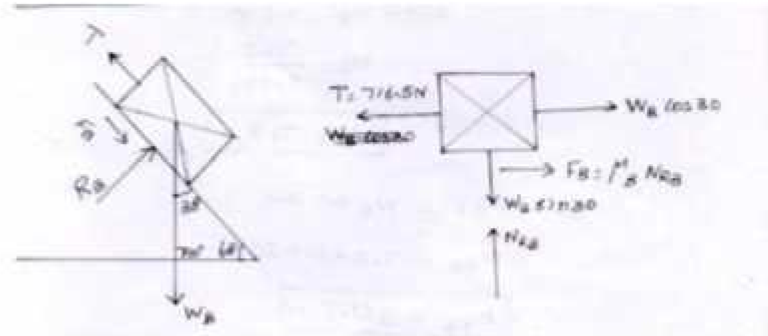
N_{RA} value sub in eqn (1)

$T = 500 + 0.25\ N_{RA}$

$T = 500 + 0.25\ \times 866$

$T = 716.5\ N$

Consider block B



Sum of X direction $\Sigma F_x = 0$

$-716.5 + W_B cos30 + \mu_B N_{RB} = 0$

$-716.5 + W_B cos30 + 0.25 \times N_{RB} = 0$

$W_B cos30 = 716.5 - 0.2 \times N_{RB}$---------- (2)

Sum of Y direction Force $\Sigma F_Y = 0$

$N_{RB} = W_B \sin 30 = 0$

$N_{RB} = W_B \sin 30$

N_{RB} value sub in Eqn (2)

$W_B cos30 = 716.5 - 0.2 \times N_{RB}$

$W_B cos30 = 716.5 - 0.2 \times [W_B \sin 30]$

$W_B cos30 = 716.5 - 0.125[W_B]$

$0.866W_B = 716.5 - 0.125W_B$

$0.866W_B + 0.125W_B = 716.5$

$0.991W_B = 716.5$

$$W_B = \frac{716.5}{0.991}$$

$$W_B = 723\ N$$

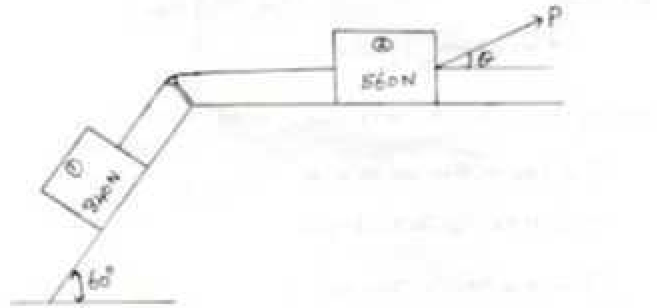
$N_{RB} = W_B \sin 30$

$N_{RB} = 723 \times \sin 30$

$N_{RB} = 361.5N$

Problem: 11

Determine the least value of P required to cause the motion impend the system shown in fig below. Assume coefficient of friction on all the contact surface as 0.2



Given:

Weight of block (1) W_1=840 N

Weight of block (2) $W_2 = 560\ N$

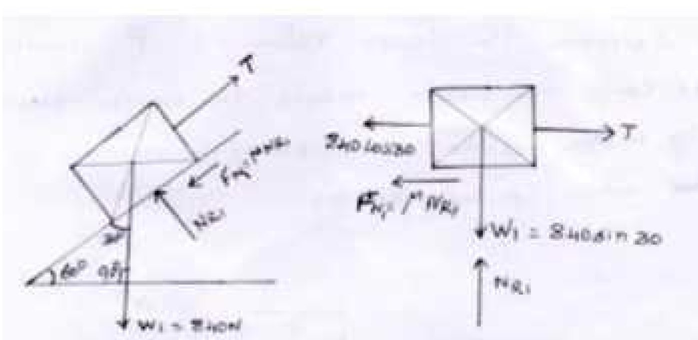
Coefficient of friction $\mu = 0.2$

To find

Load on block (2) P

Soln

Free body diagram of block



Sum of all the X direction forces

$T - F_{N1} - 840 \cos 30 = 0$

$T - \mu N_{R1} - 727.46 = 0$

$T - 0.2\ N_{R1} - 721.46 = 0$

$T = 0.2\ N_{R1} + 721.46$---------- (1)

Sum of All the Y direction force

$N_{R1} - w_1 \sin 30 = 0$

$N_{R1} - 840 \sin 30 = 0$

$N_{R1} - 420 = 0$

$N_{R1} = 420\ N$

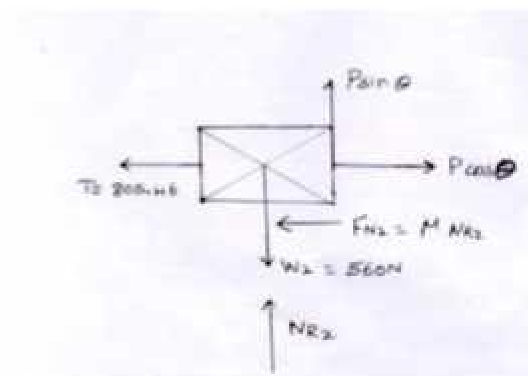
N_{R1} value sub in eqn (1)

$T = 0.2\ \times 420 + 721.46$

$T = 805.46\ N$

Consider block (2)

Free body diagram



Sum of X direction Force $\Sigma F_X = 0$

$-T - F_{N2} + p\ cos\theta = 0$

$-805.46 - \mu N_{R2} + Pcos\theta = 0$

Problem 12(H.W)

Determine whether the block shown in fig below having a mass of 40 kg is equilibrium and find the magnitude and direction of the friction force. Take $\mu_s =$ 0.40 and $\mu_k = 0.3$

Simple Contact Friction:

The friction force is the resisting force developed at the contact surface of two bodies due to their roughness and when the surface of one body moves over the surface of an another body.

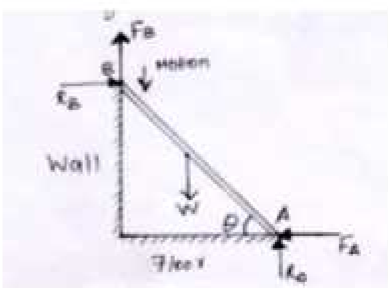
Some of important Engg application of simple contact friction are,

- Ladder friction
- Wedge friction
- Screw friction
- Belt friction

1. Ladder friction:

A ladder is a device used for climbing on roof or wall

Consider a ladder AB of length 'l' and weight 'w' resting on the ground at A and leaning against a rough wall at B as shown in fig.



Let the angle of ladder with horizontal be θ when the angle θ exceeds angle of friction (or) when the weight of the man on the ladder makes instability to the ladder, the ladder slips down.

During sliding the upper end of the ladder tends to slip downwards, hence the friction force at B, F_B will act upwards. If the coefficient of friction of wall surface B is μ_B

Then $\mu_B = \frac{F_B}{N_{RB}}$ (or) $F_B = \mu_B N_{RB}$

At the same time the lower end of the ladder A tends to slip away from the wall, hence the frictional force at A, F_A will act towards the wall if the coefficient of friction of floor surface A is μ_A

$\mu_A = \frac{F_A}{N_{RA}}$ (or) $F_A = \mu_A N_{RA}$

Condition:

$$\Sigma F_X = 0$$

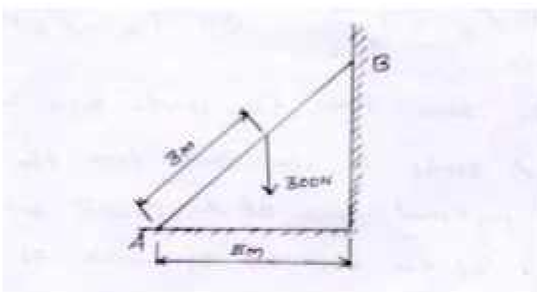
$$\Sigma F_Y = 0$$

$$\Sigma M = 0$$

$$F = \mu N_R$$

Problem 1:

A ladder is 8m long and weight 300N. The center of gravity of the ladder is 3m along the length of from the bottom end. The ladder rests against a vertical wall at B and on the horizontal floor at A as shown below. Determine the safe height upto which a man weighting 900N can climb without making the ladder slip. The coefficient of friction b/w ladder and floor is 0.4 and ladder top and wall is 0.3



Given:

Length of ladder l=8m

Weight of ladder w= 300N at 3m length from lower end

$\mu_A = 0.4$

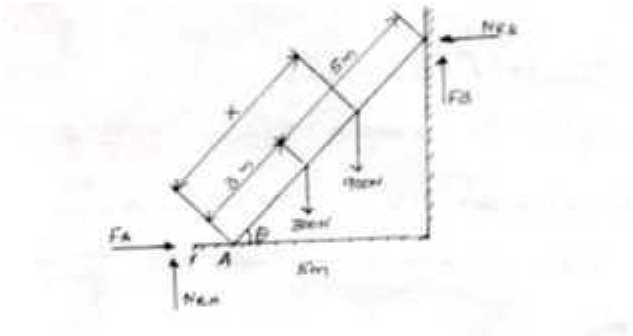
μ_B=0.3

To find

Safe height 'H'

Soln:

Free body diagram



Sum of X direction force $\Sigma F_X = 0$

$F_A - N_{RB} = 0$

$F_A - N_{RB}$-------------- (1)

Sum of Y direction force $\Sigma F_y = 0$

$N_{RA} + F_B - 300 - 900 = 0$

$N_{RA} + F_B = 300 + 900 = 0$

$N_{RA} + F_B = 1200$--------------- (2)

$\Sigma M_A = 0$ Moment about 'A'

Now find out the θ

$$\theta = cos^{-1}\left(\frac{5}{8}\right)$$

$$\theta = 51°19'$$

$\Sigma M_A = 0$ - +

$$(300 \times 3cos\theta) + (900 \times cos\theta) + [F_B \times 5] + \lfloor -N_{RB} \times 8sin\theta \rfloor = 0$$

$$[300 \times 3cos51°19'] + [900 \times cos51°19'] + [-F_B \times 5] + \lfloor -N_{RB} \times sin51°19' = 0$$

Sub $F_B = \mu N_{RB} = 0.3N_{RB}$

$$[300 \times 3cos51°19'] + [900 \times cos51°19'] + [-0.3NR_B \times 5] + [-N_{RB} \times 8sin51°19' = 0$$

$$562.51 + 562.51 \times -1.5N_{RB} - 6.24N_{RB} = 0$$

$562.51 + 562.21 \times -7.74 N_{RB} = 0$-------------- (3)

To find N_{RB}

From eqn (2) $\quad N_{RB} = F_A \quad F_A = \mu \times N_{RA}$

$N_{RA} + F_B = 1200$--------------- (2) $\quad F_A = 0.4 \times N_{RA}$

$\frac{F_A}{0.4} + 0.3 \times N_{RB} = 1200 \quad N_{RA} = \frac{F_A}{0.4}$

$$\frac{N_{RB}}{0.4} + 0.3 \times N_{RB} = 1200$$

$$2.5 N_{RB} + 0.3\, N_{RB} = 1200$$

$$2.8 N_{RB} = 1200$$

$$N_{RB} = \frac{1200}{2.8}$$

$$N_{RB} = 428.57N$$

N_{RB} value sub in Eqn(3)

$$562.51 + 562.51 \times -7.74 N_{RB} = 0$$

$$562.51 + 562.51 \times -7.74 \times 428.57 = 0$$

$$562.51 \times = [7.74 \times 428.57] - [562.51]$$

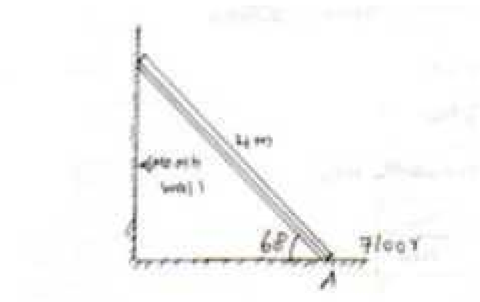
$$562.51 \times = 2754.63$$

$$X = \frac{2754.63}{562.51}$$

Ans $X = 4.89m$ from the floor of Ladder

Problem 2

A ladder of weight 1000 N and length 4m rest as shown in fig, if the 750 n weight is applied at distance of 3m from the top of ladder, it is at the point of sliding. Determine the coefficient of friction b/w ladder and the floor.



Given:

Length of Ladder l = 4m

Weight of ladder w = 1000N

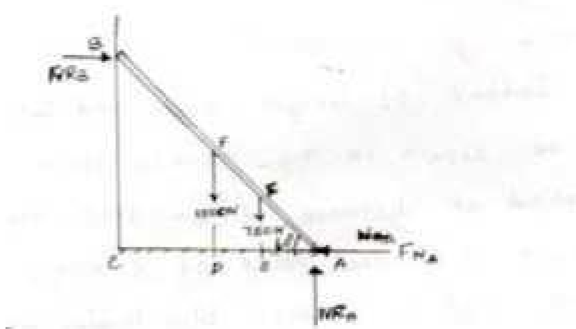
Weight of man = 750 N at 3m from toped

To find:

Coefficient of friction

Soln:

Free body diagram



Sum of X direction force $\Sigma F_X = 0$

$N_{RB} - F_{NA} = 0$

$N_{RB} - \mu_A N_{RA} = 0$

$-\mu_A N_{RA} = -N_{RB}$

$\mu_A = \frac{N_{RB}}{N_{RA}}$-------- (1)

Sum of all Y direction force

$N_{RA} - 1000 - 750 = 0$

$N_{RA} - 1750 = 0$

$N_{RA} = 1750N$

Take moment about 'A' $\downarrow$ + $\uparrow$ -

$\Sigma M_A = 0$

$\Sigma M_A = [N_{RB} \times BC] + [-1000 \times AD] + [-750 \times AE] = 0$

$BC = 4\ sin6$ $\quad$ $AD = 2\ cos60$ $\quad AE = -1\cos 60$

$[N_{RB} \times 4 \sin 60] + [-1000 \times 2\cos 60] + [-750 \times -1 cos60] = 0$

$3.46N_{RB} - 1000 - 375 = 0$

$3.46N_{RB} - 1375 = 0$

$$N_{RB} = \frac{1375}{3.46}$$

$N_{RB} = 397.39\ N$

N_{RA} & N_{RB} value sub in Eqn (1)

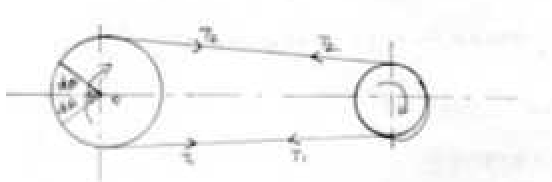
$$\mu = \frac{N_{RB}}{N_{RA}} = \frac{397.39}{1750}$$

$ans\ \mu = 0.22$

Belt friction:

Power is transmitted through a belt that is running round the two plugs.This is used in laths, diesel engine, and rise mills etc for power transmission.

The power is transmission is due to the friction existing b/w the belt and the pulley surface. The friction is called belt friction.



$\frac{T_2}{T_1} = e^{MQ}$ $\longrightarrow$ Tension Ratio

T_2= tension in fight side

T_1 = tension in slack side

θ = angle of contact

μ = coefficient of friction

$T_2 > T_1$

θ value sub in radian

θ in radian = $\pi/_{180} \times \theta \; value \; in \; degree$

Power $P = [T_2 - T_1] \times V$

$V = \frac{\pi d N}{60} \; m/s$

N– speed of drum

D = diameter of drum

V= belt speed (or) velocity of belt

Problem 1

A flat belt develops a tight side tension of 2000 N during power transmission the coefficient of friction b/w pulley and belt is 0.3, the angle of lap on smaller pulley is 165° and the belt speed is 18 m/s. determine the power that can be transmitted, if the belt is assumed to be perfectly elastic and without mass.

Given data:

Tension in tight side $T_1 = 2000N$

Coefficient of friction $\mu = 0.3$

Angle of contact $\theta = 165°$

Velocity of belt $V = 18\frac{m}{s}$

To find:

Power 'P'

Soln:

Power $P = [T_2 - T_1] \times V$

$T_2 = 2000, V = 18\, {}^{m}/_{s}\ given$

By Tension Ratio

$$\frac{T_2}{T_1} = e^{MQ}, \mu = 0.3$$

$\theta = 165^\circ$

$\theta = 165^\circ \times {}^{\pi}/_{180}$

$\theta = 2.87\ radian$

$$\frac{2000}{T_1} = e^{0.3\times 2.87}$$

$$T_1 = \frac{2000}{e^{0.3\times 2.87}} = \frac{2000}{e^{0.861}} = \frac{2000}{2.36}$$

$T_1 = 845.47\ N$

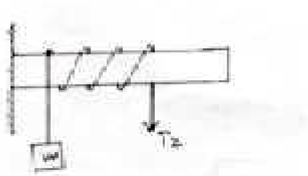
Power $P = [T_2 - T_1] \times V$

$P = [2000 - 845.47] \times 18$

$P = 20 \times 10^3 w$

Problem 2

A rope is wrapped three time around a rod shown in fig



Determine the force T required on the free end of the rope. To support a load of W= 20KN. Take μ *as* 0.3.

Given Data:

Weight w=20 KN $w = T_2$ $w = T_2$

$\mu = 0.3.$ $T_2 = T_1$ $T_1 = T$

To find:

Tension 'T'

Soln:

Tension Ratio

$$\frac{T_2}{T_1} = e^{MQ}$$

$$\theta = 3 \times {}^{\pi}/_{180} \times 360°$$

$$\theta = 18.84\ radian$$

$$\frac{T_2}{T_1} = e^{0.3 \times 18.84}$$

$$\frac{20}{\pi} = e^{0.3 \times 18.84}$$

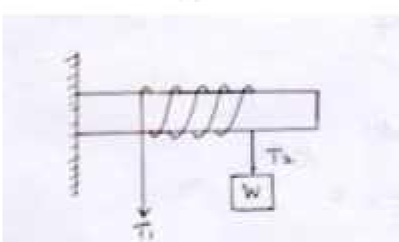
$$T_1 = \frac{20}{e^{0.3 \times 18.84}}$$

$$T_1 = 0.07\ KN$$

$$T = T_1 = 0.07\ KN$$

Problem 3

A wire rope is wrapped three and a half times around a cylinder as shown in below. Determine the force T_1 exerted on the free end of rope that is required to support a 1KN weight the coefficient of friction b/w the rope and the cylinder is 2.5



Given:

Coefficient of friction $\mu = 0.25$

No of turns=3.5

$T_2 = w = 1KN$

To find:

Force exerted T_1

Soln:

Tension ratio

Tension Ratio

$$\frac{T_2}{T_1} = e^{MQ}$$

$$\theta = 360 \times \left[{}^{\pi}/_{180}\right] \times 3.5$$

$$\theta = 21.99\ radian$$

$$\frac{T_2}{T_1} = e^{0.25\times 21.99}$$

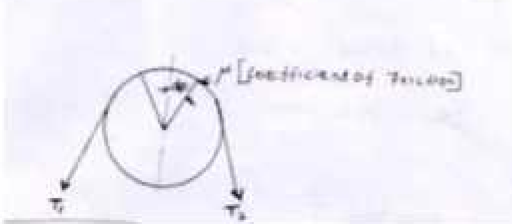
$$\frac{1}{T_1} = e^{0.25\times 21.99}$$

$$T_1 = \frac{1}{e^{0.25\times 21.99}}$$

$$T_1 = 4.09 \times 10^3\ KN$$

$$T = T_1 = 4.09\ KN$$

Derive the ratio b/w the two belt tension forces



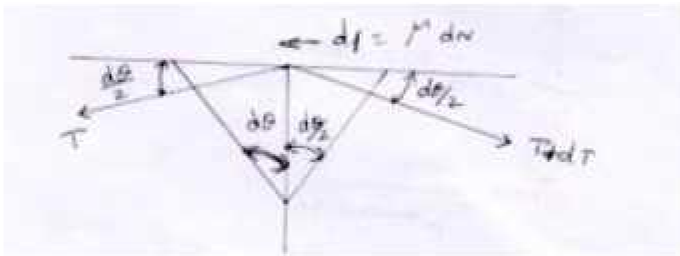
$$\frac{T_2}{T_1} = e^{M\theta}$$

T_1 = Tension in tight side

T_2 = Tension in slack side

θ = Angle of contact

μ = Coefficient of friction



$\Sigma F_X = 0$

$$-T\cos\frac{d\theta}{2} + (T + dT)cos\frac{d\theta}{2} - df = 0$$

$$-T\cos\frac{d\theta}{2} + T\cos\frac{d\theta}{2} + (dT)cos\frac{d\theta}{2} - df = 0$$

$df = \mu dN \quad \&\cos\frac{d\theta}{2} = 1$

$dT = -\mu dN = 0$

$dt = \mu dN$----------- (1)

$\Sigma F_Y = 0$

$$\mathrm{dN} + ([T + dT])sin\frac{d\theta}{2} - Tsin\frac{d\theta}{2} = 0$$

$$dN - T\sin\frac{d\theta}{2} - dT\sin\frac{d\theta}{2} - Tsin\frac{d\theta}{2} = 0$$

$$dN = 2T\frac{d\theta}{2} - dT\,do = 0$$

$dN - Td\theta = 0$

$dN - Td\theta$-------- (2)

dN value sub in Eqn (1)

$dT = \mu dN$--------- (1)

$$dT = \mu \times dN$$

$$\frac{dT}{T} = \mu d\theta$$

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\theta} \mu d\theta$$

$$[\log T]_{T_1}^{T_2} = [\mu]_0^{\theta}$$

$$logT_2 - logT_1 = \mu\theta = log\frac{T_2}{T_1} = \mu\theta$$

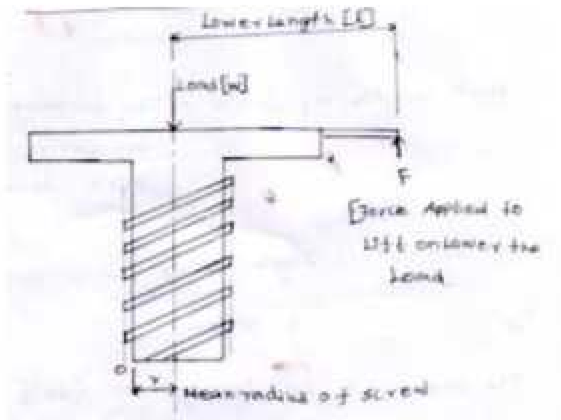
Flat for belt $\frac{T_1}{T_2} = e^{\mu\theta}$

$$\frac{T_2}{T_1} = e^{\mu\theta \, cosec\frac{B}{2}}$$

Problem 4:

A three turns of cables is used to hold the 100N weight with a tensile force of 50 N at the other end of shown in fig. find out the coefficient of friction b/w the cable and shaft surface

Screw friction



w=load active $w\ tan[\theta + \emptyset]$

It is used to raise and lower the load

Ex, screw jack

Taking moment about the axis of the screw

$W \tan(\emptyset + \theta) \times r - F \times l = 0$

$W \times r \times \tan(\emptyset + \theta) = Fl$

Force applied to raise (or) Lower the load F

$$F = \frac{wrtan(\emptyset + \theta)}{l}$$

θ =Lead angle

$\emptyset$ =Friction angle

$\tan \emptyset = \mu$

$\theta = tan^{-1}(\mu)$

Lead of screw the height lifted for one full from rotation

Pitch length of screw = The distance b/w the two connective thread head screw pitch length

If friction angle $\emptyset > lead\ angle\theta$

$$\emptyset > \theta$$

The screw is in locking or (self-locking) of the screw

$$\mu = \frac{tan\theta}{\tan(\emptyset + \theta)} \times 100$$

Mean radius $= r = \frac{pitch\ length}{2}$

$$r = \frac{dia\ meter}{2}$$

Problem: 1

A screw jack has a square thread of mean radius 5cm and pitch length of 1.5 cm. length of lever 50 cm. it is used to raise and lower the load of 25 KN. $\mu = 0.2$ find

(i) Force applied to raise the Load

(ii) Force applied to the lower the load

(iii) Threaded efficiency

Given

Load $w = 25KN = 25 \times 10^3 N$

Mean radius$r = 5\ cm$

Pith length$= 1.5cm$

Length of lever (l) –50cm $\mu = 0.2$

Soln

(i) Force applied to raise the road

$$F = \frac{Wrtan[\emptyset + \theta]}{l}$$

$tan\emptyset = \mu$

$\emptyset = tan^{-1}(\mu)$

$\emptyset = tan^{-1}(0.2)$

$\theta = Lead\ angle$

$$tan\theta = \frac{1.5}{2 \times \pi \times 5}$$

$$\theta = tan^{-1}\left(\frac{1.5}{2 \times \pi \times 5}\right)$$

$$\theta = 2°4'$$

$$F = \frac{25\times10^3\times5}{50}\tan[11°55' + 2°4']$$

$$F = 653.54\, N$$

(ii) Force applied to lower the load

$$F = \frac{wrtan(\emptyset - \theta)}{l}$$

$$F = \frac{25 \times 10^3 \times 5 \times \tan[11°55' - 2°4']}{50}$$

$$F = 0.104\, KN$$

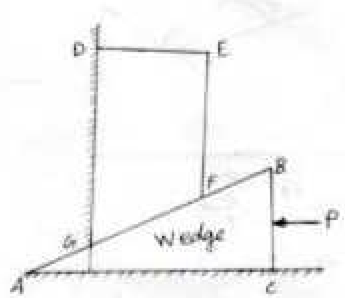
(iii) Efficiency: η

$$\eta = \frac{tan\theta}{\tan(\theta + \emptyset)} \times 100$$

$$\eta = \frac{\tan 2°4'}{\tan(2°4' + 11°55')}$$

$$\eta = 36$$

Wedge Friction

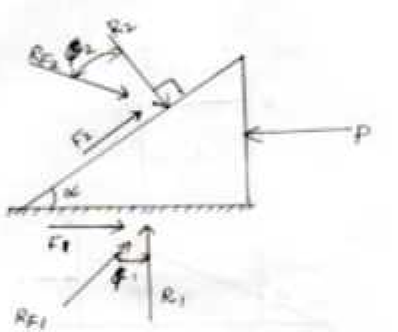


A wedge is a piece of wood or metal, usually of a triangular (or) trapezoidal shape in cross section used for Lifting Loads (or) for slight adjustment like tightening keys for shift

When the force P is applied, sliding take place on the edge AC, DG, and GF. Hence the reactive force components, normal reactions and the frictional forces are also developed on these sliding surface

Now Let as see the direction of these reactive forces, and draw the free body diagram of wedge and body.

Equilibrium of wedge



Normal force and Frictional force are combined to a single resultant force 'R_F'

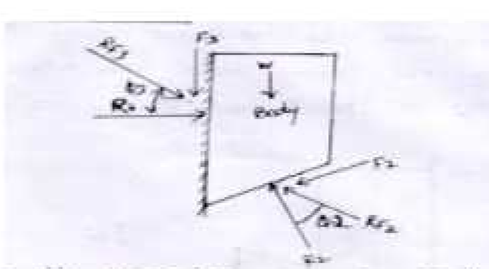
$R_{F1} \& R_{F2}$ *are drawn on wedge*

Where $R_{F1} = \sqrt{{F_1}^2 + {R_1}^2}$

$$R_{F2} = \sqrt{{F_2}^2 + {R_2}^2}$$

$F_1 \& F_2$ are Limiting friction $\emptyset_1, \emptyset_2$means of angle of friction

It is defined as the in b/w the angle of [Line of action of the normal reaction$N_1 \& N_2$] and Resultant force $R_{F_1} \& R_{F_2}$



Equilibrium of body

When the force 'P' push the wedge, the body tends to move upwards, hence the frictional force F_3 on the surface AD is acting download.

Then the normal reaction R_3 and F_3 are replaced by a single resultant R_{F3}, which makes on angle of $\emptyset_3$with line of action of normal reactionR_3

Concurrent forces$R_1 R_2$, self weight(w)

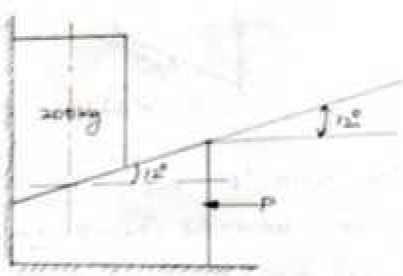
Note (i) always draw the free body diagram of wedge first then draw the force body diagram of the below

(ii) while solving, if the Load is given, solve free body of block first and if the force 'P' is given.solve the free body of wedge

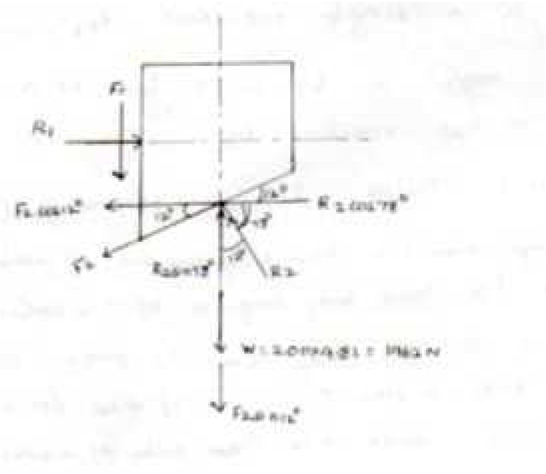
(iii) self-weight of the wedge is neglected

Problem: 1

Determine the horizontal force P required to raise the 200 kg block the efficient of friction for all the contact surface is 0.25



Free body Diagram of block



Sum of all X direction force

$\sum F_X = 0 \; + \longrightarrow \quad \longleftarrow -$

$R_1 - R_2 \cos 78 - F_2\, cos 12° = 0$

$R_1 - 0.207R_2 - \mu R_2 cos 12° = 0$

$R_1 - 0.207R_2 - 0.25 \times R_2 \times cos 12° = 0$

$R_1 - 0.207R_2 - 0.24R_2 = 0$

$R_1 = 0.207R_2 + 0.24R_2$

$R_1 = 0.45R_2$

Sum of all Y direction force

$\sum F_Y = 0 \;\; \downarrow - \; \uparrow +$

$-1962 - F_1 - F_2 sin 12 + R_2 sin 78 = 0$

$-1962 - 0.25R_1 - 0.25R_2 sin 12° + R_2 sin 78° = 0$

$-1962 - 0.25R_1 - 0.051R_2 + 0.97R_2 = 0$

$-1962 - 0.25 \times [0.45R_2] - 0.051R_2 + 0.97R_2 = 0$

$-1962 - 0.112R_2 - 0.051R_2 + 0.97R_2 = 0$

$-1962 + 0.807R_2 = 0$

$0.807R_2 = 1962$

$$R_2 = \frac{1962}{0.867}$$

$R_2 = 2431.22\ N$

R_2 Value sub in eqn (1)

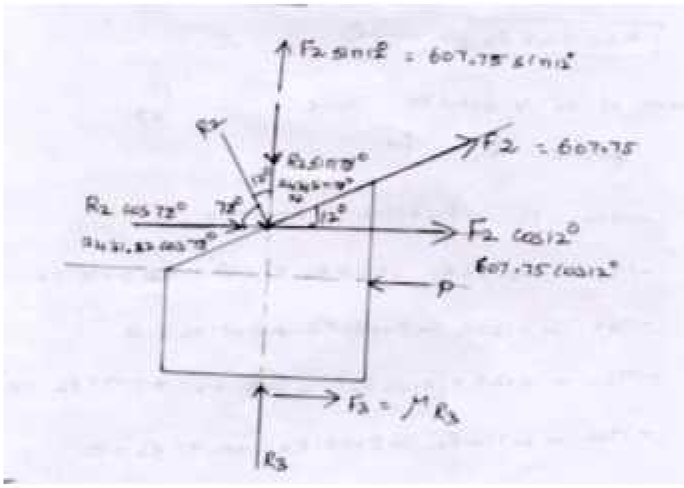
$R_1 = 0.45\ R_2$ ---→ 0.45×2431.22

$R_1 = 1094\ N$

$F_1 = \mu R_1 = 0.25 \times 1094 = 273.51\ N$

$F_2 = \mu R_2 = 0.25 \times 2431 = 607.75\ N$

Free body diagram of wedge



$\sum F_X = 0$

$-P + F_3 + 2431.22\, cos78° + 607.75\, cos12° = 0$

$-P + \mu R_3 + 278 + 594.46 = 0$

$-P = -\mu R_3 - 2378 - 594.46$

$-P = -[\mu R_3 + 2378 + 594.46]$

$P = 0.25 \times R_3 + 2378 + 594.46$-------------- (2)

$\sum F_Y = 0$

$R_3 - R_2 sin78° + 607.75\, sin12° = 0$

$R_3 - 2431.51 sin78° + 607.75\, sin12° = 0$

$R_3 - 505.47 + 126.35 = 0$

$R_3 = 505.47 - 126.35$

$R_3 = 379.11\ N$

R_3 vlauc sub in Eqn (2)

$P = 0.25 \times R_3 + 2378 + 594.46$

$P = 0.25 \times 379.11 + 2378 + 594.46$

$P = 3067.23\ N$

ENGINEERING MECHANICS CLASS NOTES ON

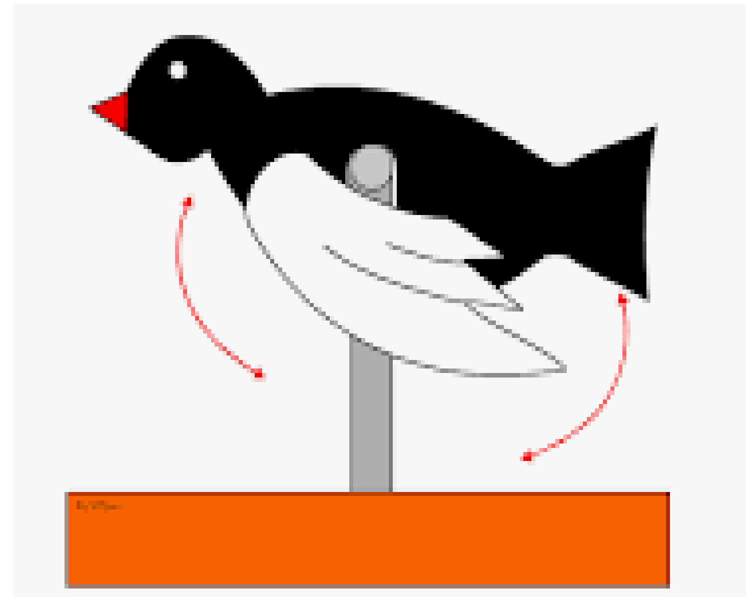
CENTRE OF GRAVITY AND CENTROID

PREPARED BY

V.JOSE ANANTH VINO

AP/MECH/B.I.H.E.R.,

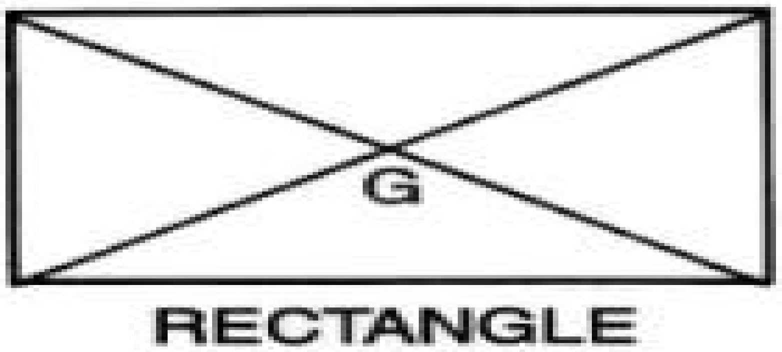
CENTER OF GRAVITY (C.G)



CENTRE OF GRAVITY : The point at which entire weight (mass) of body is assumed to be concentrated is known as centre of gravity.

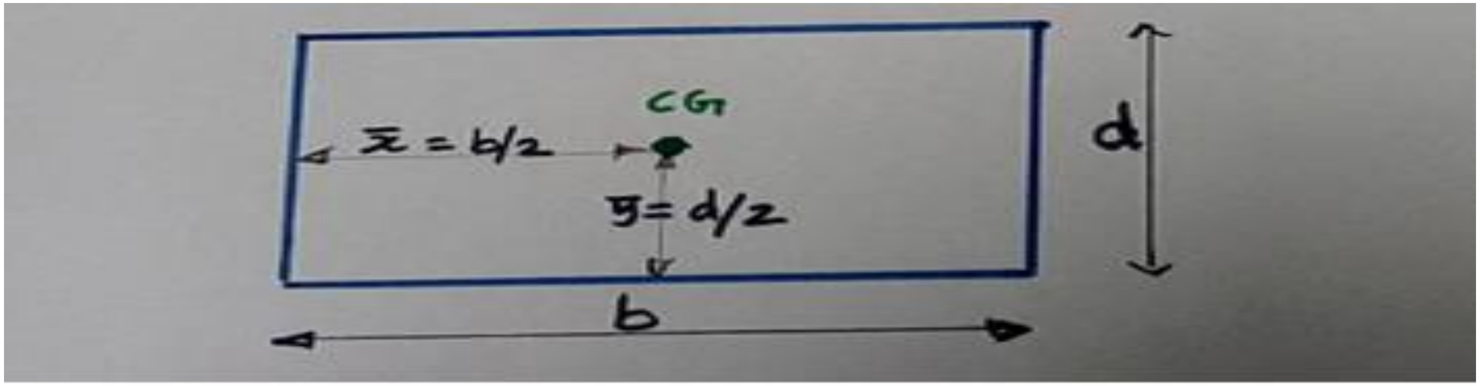
- CENTROID : The geometric centre of weightless laminas/plane areas is known as centroid

CENTROID OF RECTANGLE (by geometrical method)

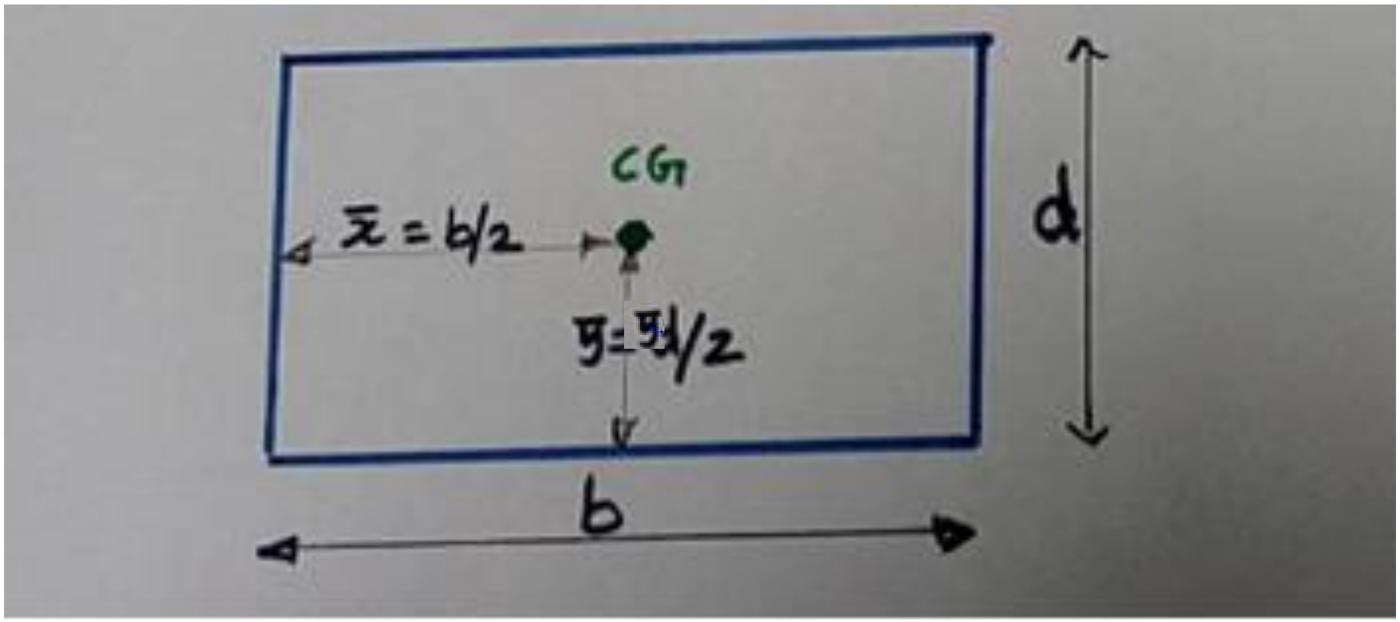


The diagonals of the rectangle intersects at centroid

CENTROID OF RECTANGLE (by analtical method)

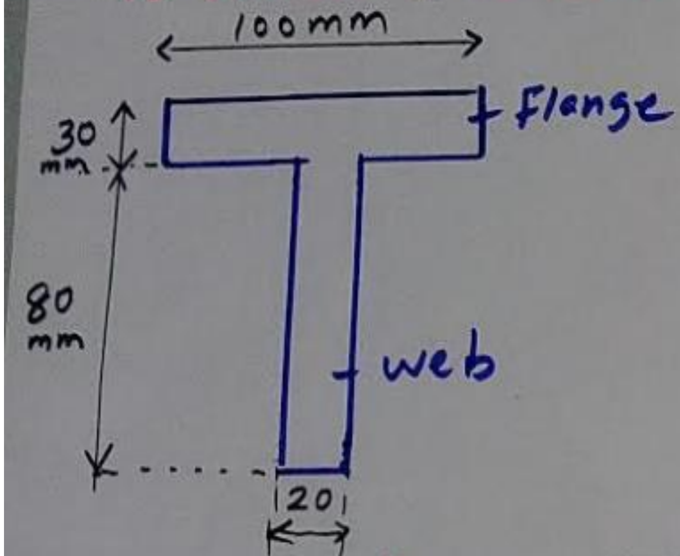


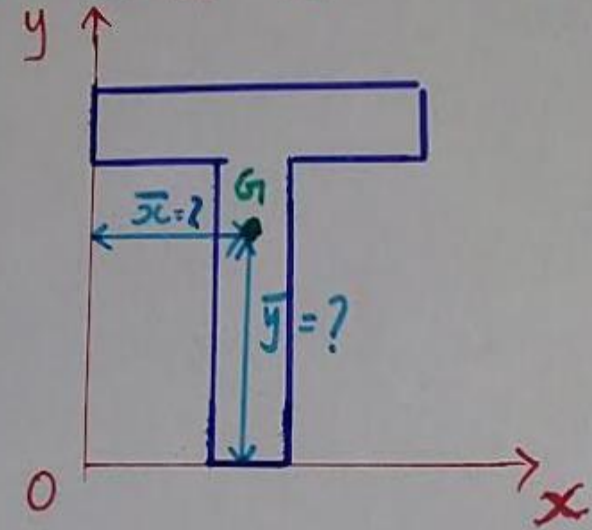
A rectangular laminae has base length 100 mm and has a depth of 60 mm. Find the perpendicular distance of centroid from the base length and also mention the distance of centroid from leftmost side.

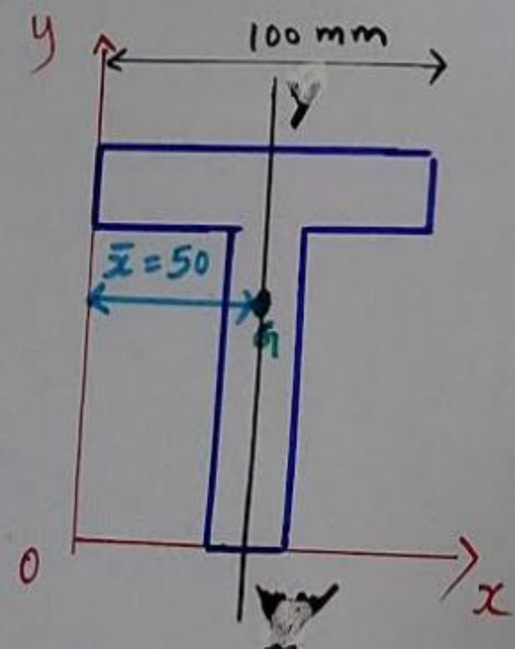


The perpendicular distance of centroid from base length($\bar{y}$) = d/2 =60/2 = 30 mm
The distance of centroid from left most side ($\bar{x}$) = b/2 = 100/2 = 50 mm

LOCATE THE CENTROID OF 'T' SECTION WHOSE FLANGE IS 100mm X 30 mm AND WEB IS 20mm X 80 mm





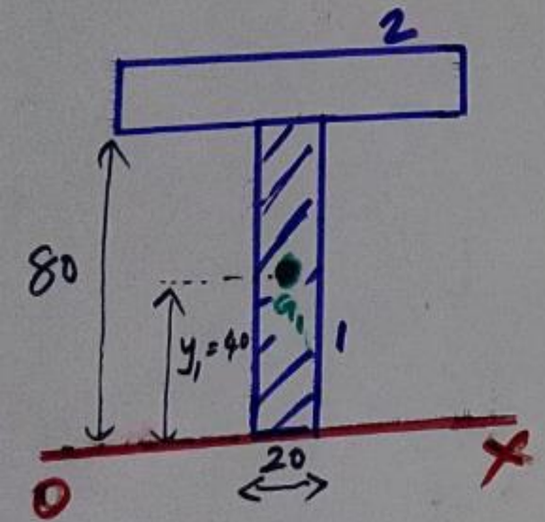


As the given 'T' section is symmetrical about yy axis, $\bar{x}$ is predictable

$$\bar{x} = \frac{100}{2} = 50 \text{ mm}$$

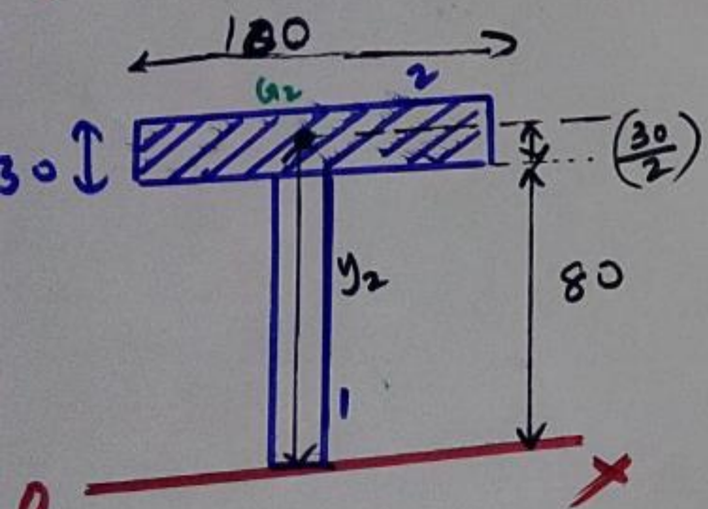
from oy axis.

$\bar{y}$ = ? [To be found using calculation]



$$a_1 = bh = 20 \times 80 = 1600 \text{ mm}^2$$

$$y_1 = \frac{80}{2} = 40$$



$$a_2 = 100 \times 30 = 3000 \text{ mm}^2$$

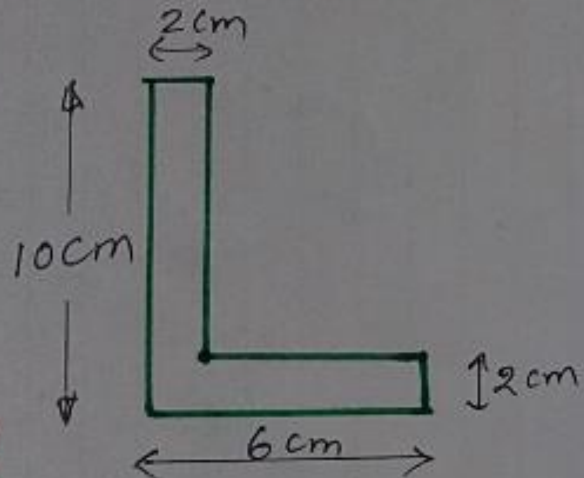
$$y_2 = 80 + \frac{30}{2} = 95 \text{ mm}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

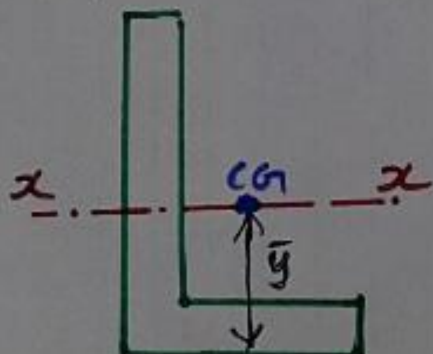
$$= \frac{(1600 \times 40) + (3000 \times 95)}{1600 + 3000}$$

$$= 75.87 \text{ mm}$$

LOCATE THE CENTROID OF L-SECTION SHOWN IN FIG.



SOLUTION



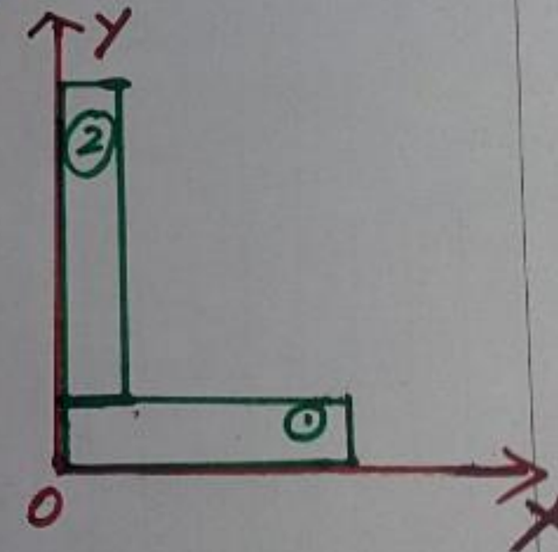
The given lamina not symmetrical w.r.to yy axis. Therfore $\bar{x}$ cannot be predicted

$$\boxed{\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}}$$

SOLUTION (continued)

1) Divide given 'L' section into two known rectangular lamina.

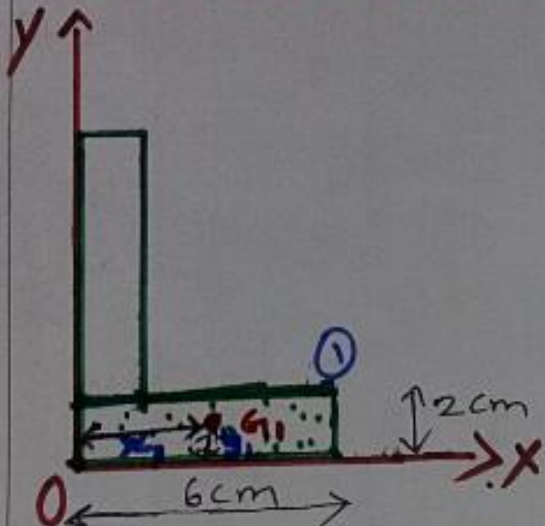
2) choose reference axis.



The given lamina is not symmetrical w.r.t xx axis. Therfore $\bar{y}$ cannot be predicted.

$$\boxed{\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}}$$

Considering ① section alone

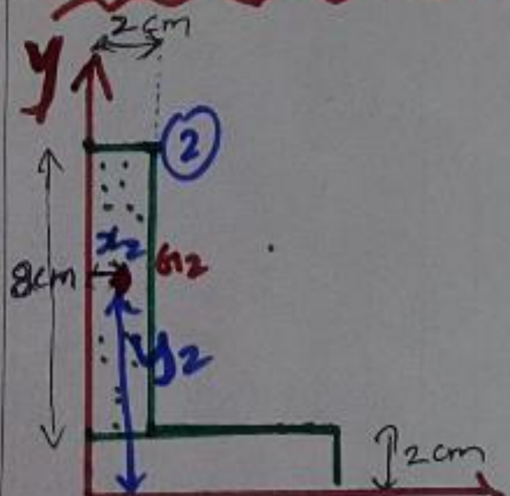


$$a_1 = bh$$
$$= 6 \times 2 = 12\,cm^2$$

$$x_1 = \frac{6}{2} = 3\,cm$$

$$y_1 = \frac{2}{2} = 1\,cm$$

Considering ② section alone



$$a_2 = bh$$
$$= 2 \times 8 = 16\,cm^2$$

$$x_2 = \frac{2}{2} = 1\,cm$$

$$y_2 = \left(\frac{8}{2}\right) + 2$$
$$= 6\,cm$$

$$\boxed{\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}}$$

$$= \frac{(12 \times 3) + (16 \times 1)}{12 + 16}$$

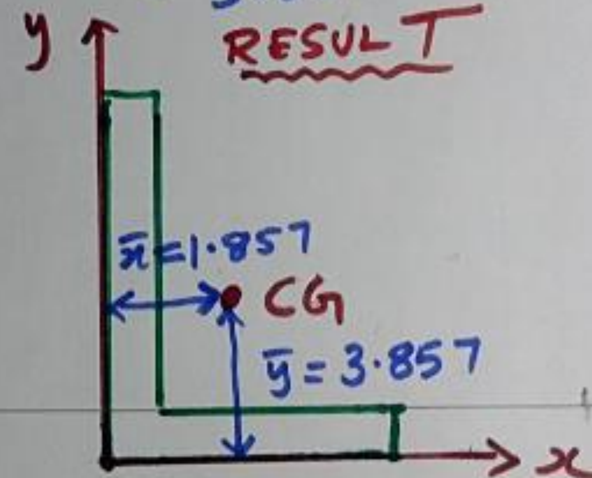
$$= 1.857$$

$$\boxed{\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}}$$

$$= \frac{(12 \times 1) + (16 \times 6)}{12 + 16}$$

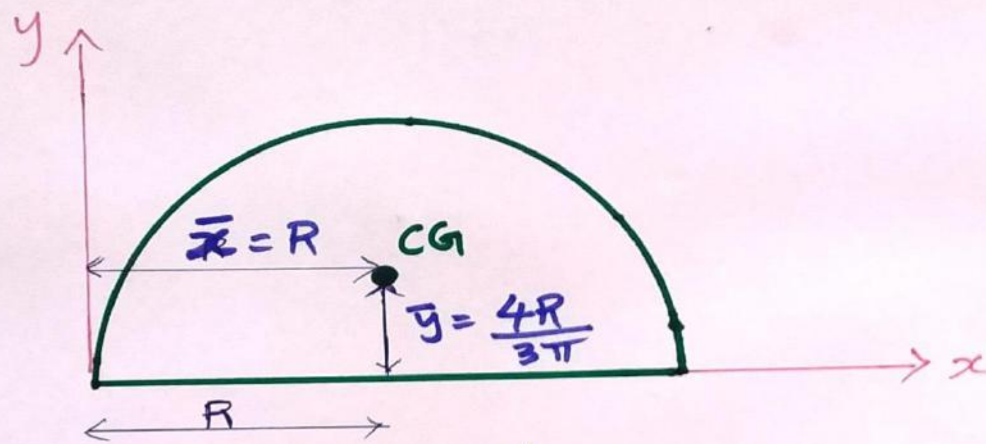
$$= 3.857\,cm$$

RESULT

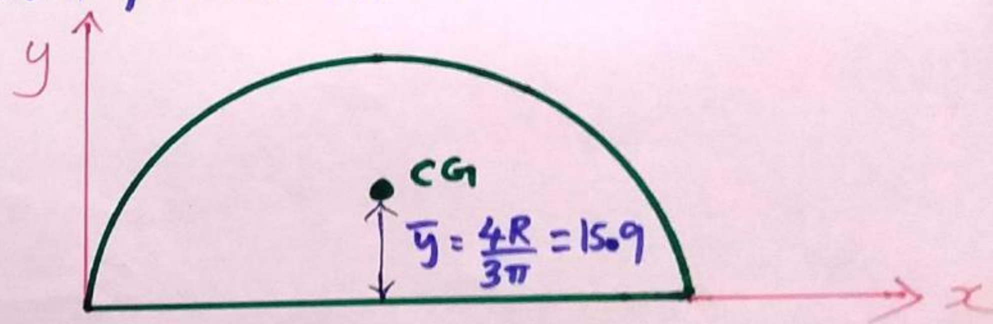


CENTROID OF SEMICIRCLE

CENTROID OF SEMICIRCLE



The diameter of semicircular section is 75mm. Find the distance of C.G of the section from the base diameter.



GIVEN

$D = 75\,mm$

$\Rightarrow R = D/2 = \frac{75}{2} = 37.5\,mm$

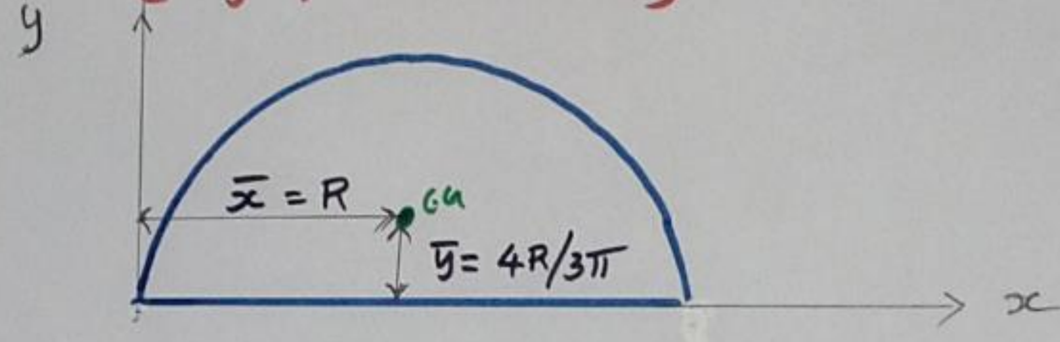
SOLUTION

$$\boxed{\bar{y} = \frac{4R}{3\pi}} = \frac{4 \times 37.5}{3 \times \pi}$$

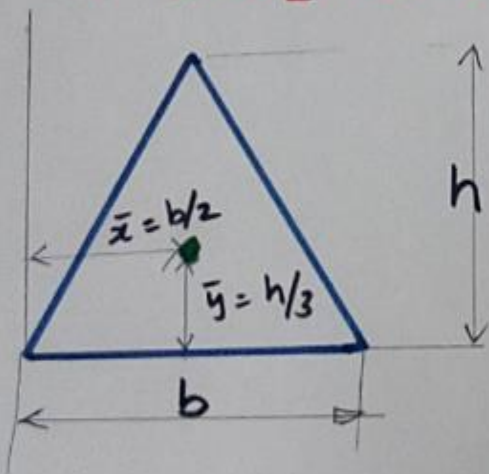
$$= 15.9\,mm$$

CENTROD OF SEMICIRCLIE

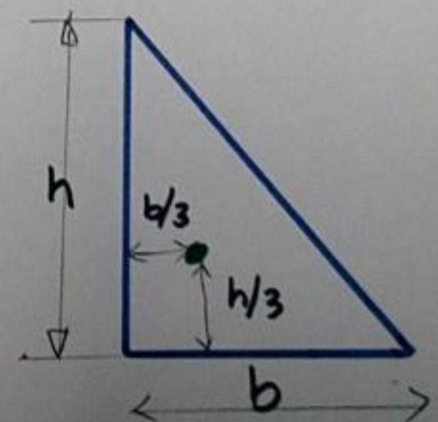
[c.g of semicircle].



CENTROID OF ISOCELES TRIANGLE

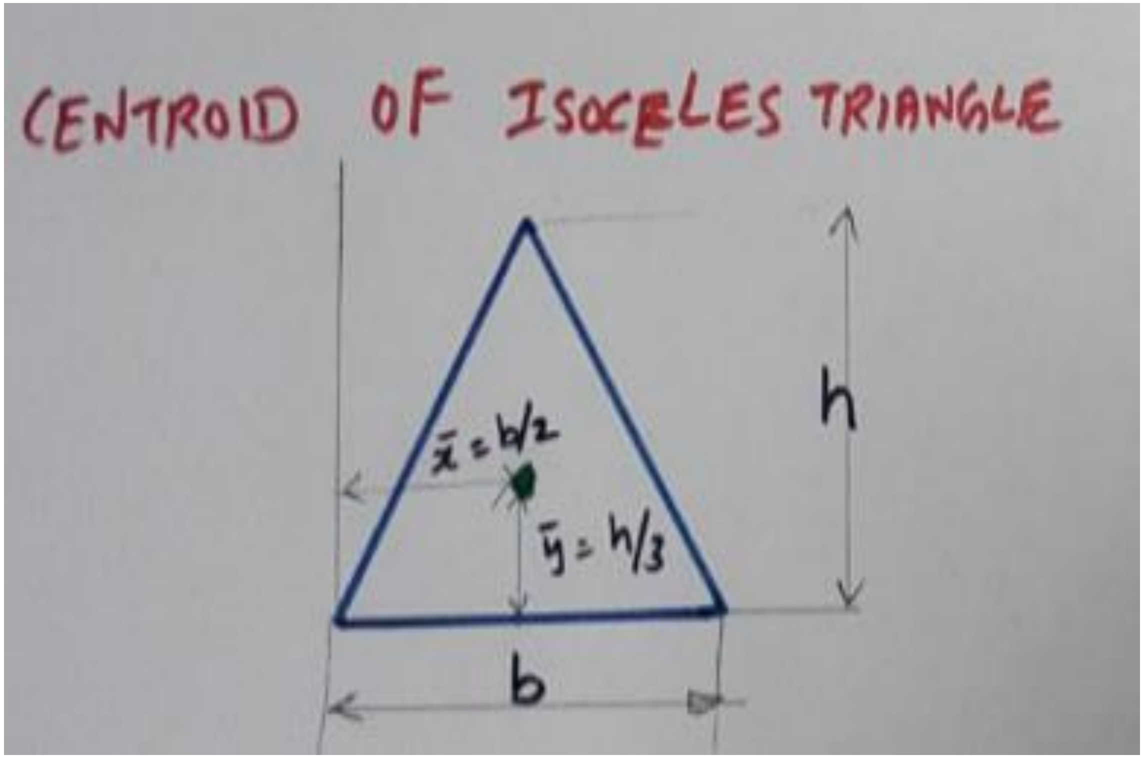


CENTROID OF RIGHT ANGLED TRIANGLE



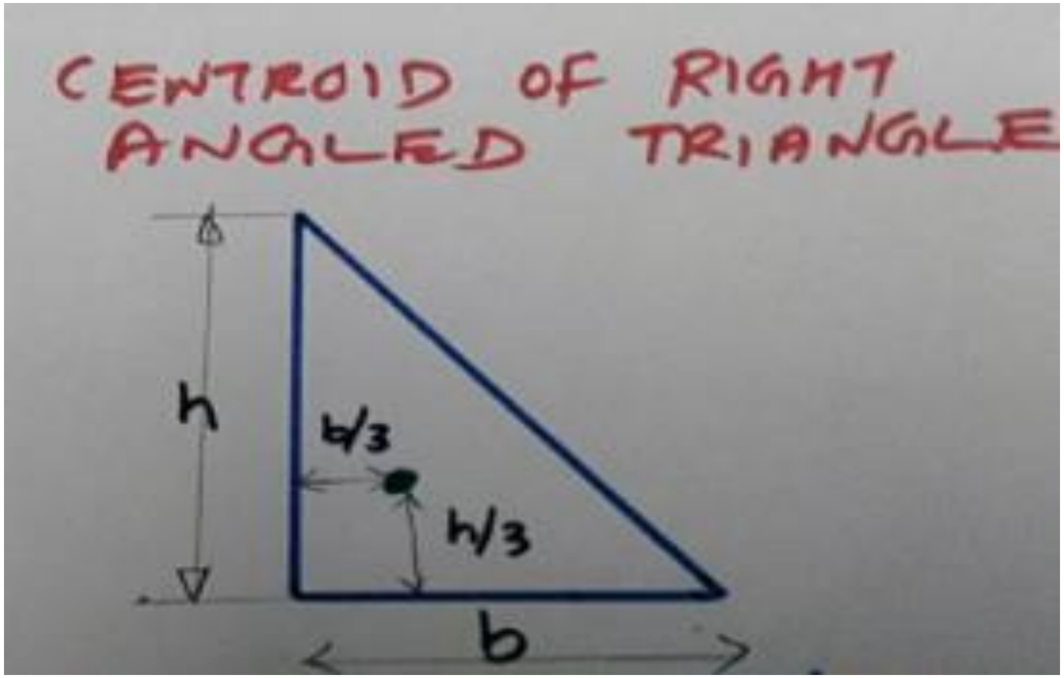
NOTE :- The centre of gravity of Large plane laminate may not be at its geomentrical centre if it is a right angle triangle [i.e, for Larger right angle triangle]

An isosceles triangle has base 20 mm and has height of 30 mm. Find the distance of centroid from the base of this triangle



The distance of centroid of the isosceles triangle from base of triangle = h/3=30/3 = 10 mm

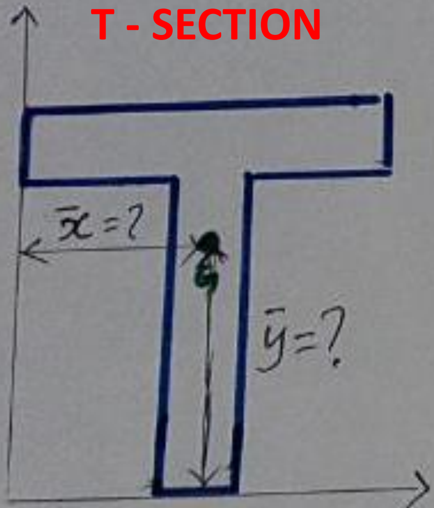
A right angled triangle has base 30 mm and has height 45 mm. Find the perpendicular distance of centroid of the triangle from the base of the triangle



The distance of centroid of the right angled triangle from base of triangle = h/3=45/3 = 15 mm

CENTROID OF COMPOSITE PLANE FIGURE

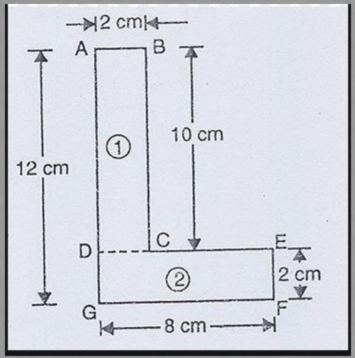
T - SECTION

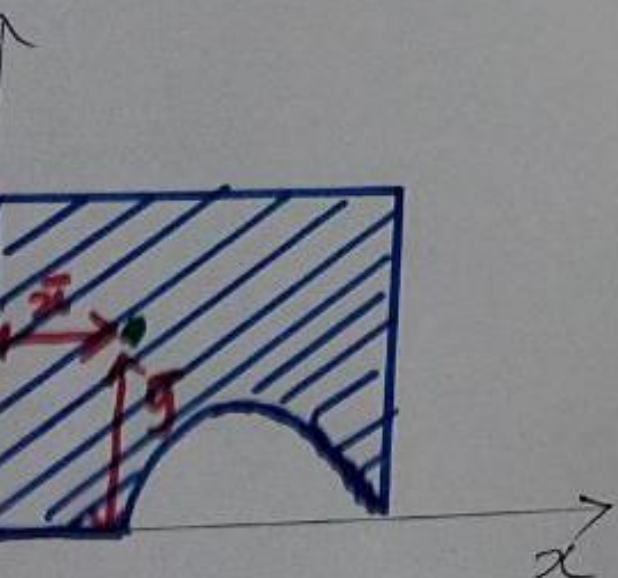


$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

L SECTION

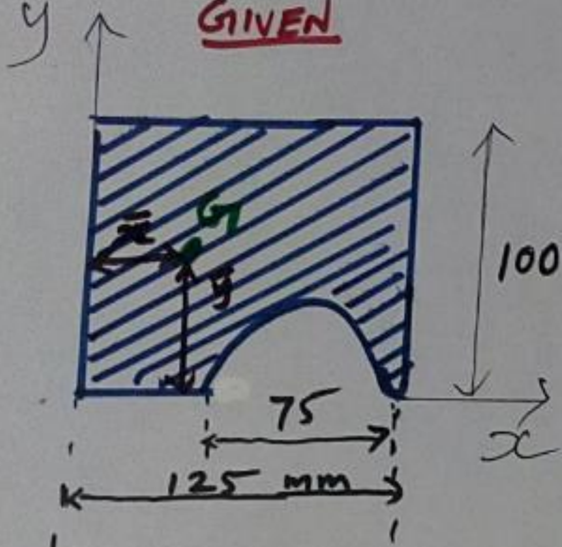




$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

LOCATE THE CENTROID OF LAMINA SHOWN BELOW



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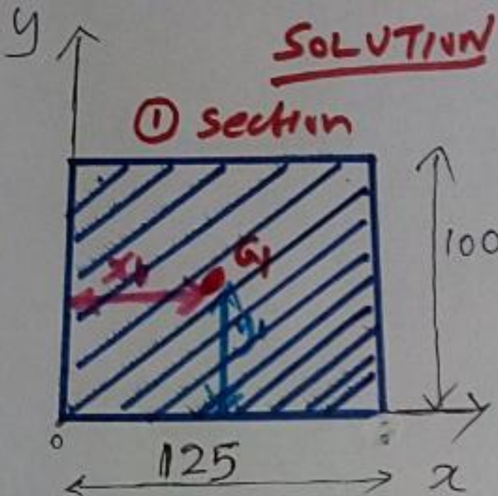

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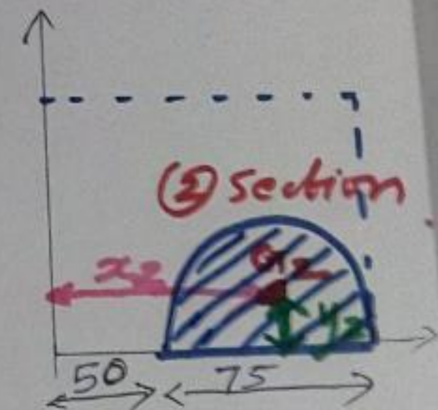

① section

$$a_1 = bh = 125 \times 100 = 12500\ mm^2$$

$$x_1 = \frac{125}{2} = 62.5\ mm$$

$$y_1 = 100/2 = 50\ mm$$

② section

$$a_2 = \frac{1}{2}\left[\frac{\pi}{4} D^2\right] = \frac{1}{2}\left[\frac{\pi}{4} 75^2\right] = 2208.93\ mm^2$$

$$x_2 = 50 + \frac{75}{2} = 87.5$$

$$y_2 = \frac{4R}{3\pi} = \frac{4 \times 37.5}{3 \times \pi} = 15.9$$

$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2} = \frac{12500 \times 62.5 - 2208 \times 87.5}{12500 - 2208} = 57.13\ mm$$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2} = \frac{12500 \times 50 - 2208 \times 15.9}{12500 - 2208} = 57.31\ mm$$

ANALYSIS OF FRICTION AND TRUSSES (UNIT II) CLASS NOTES

PREPARED BY

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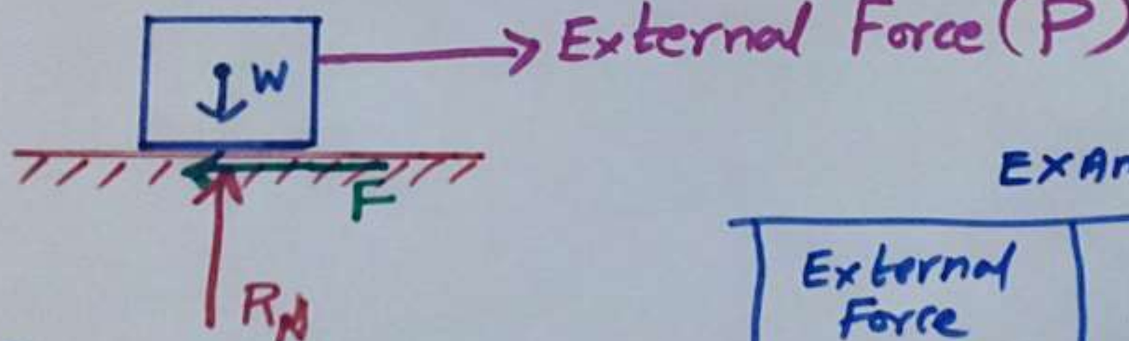
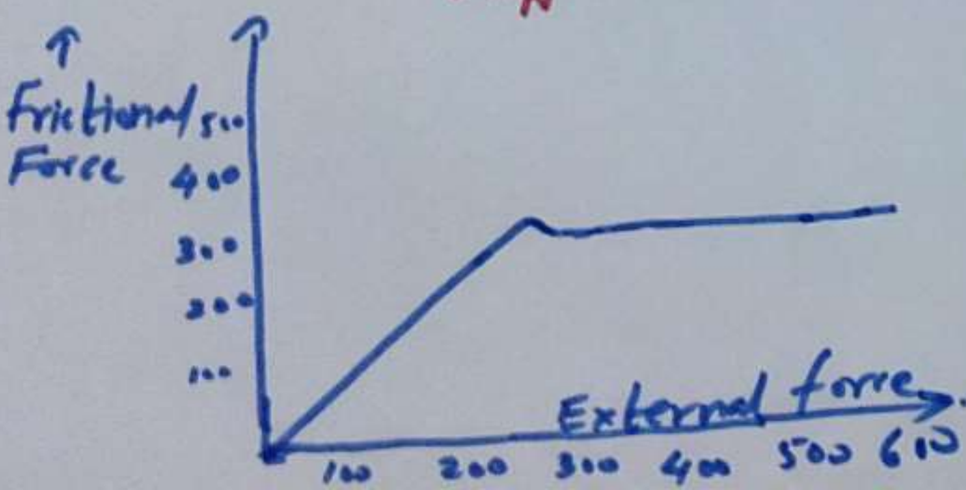
AP/MECH/B.I.H.E.R.,

FRICTION

FRICTION

The tangential resistive force developed at the contact surfaces against the direction in which one body moves/tends to move over another body is known as frictional force or friction.





EXAMPLE

External Force	Frictional force applied	
100 N	100 N	REST
200 N	200	
300 N	300	
400	290	MOTION
500	290	
600	290	

LIMITING FRICTION: (F_m or F)

The maximum frictional resistance developed at the contacting surfaces when one body just begins to move move another body is known as Limiting Friction

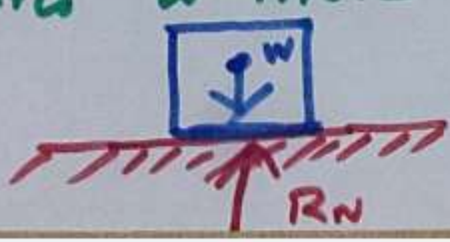
COEFFICIENT OF FRICTION (μ)

The ratio between maximum frictional force developed and Normal reaction is known as coefficient of friction.

$$\text{coefficient of friction} = \frac{\text{Limiting friction}}{\text{Normal reaction}}$$

$$\mu = \frac{F_m}{R_N}$$

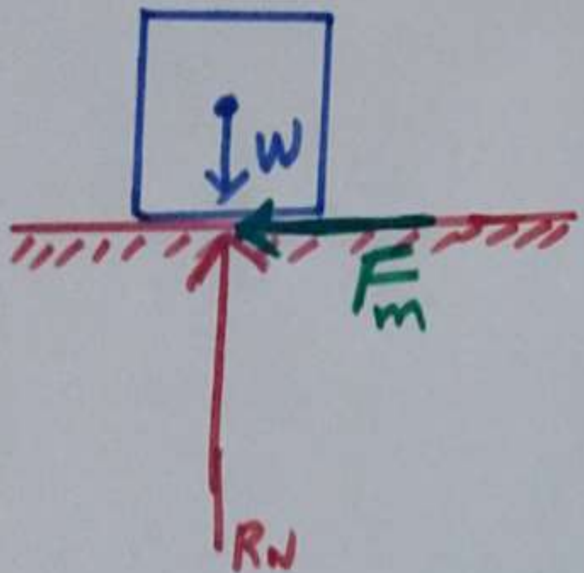
Example: A body of weight 1000 N rest on surface(h
If maximum frictional force developed when body just starts to move is 300 N, find coefficient of frict

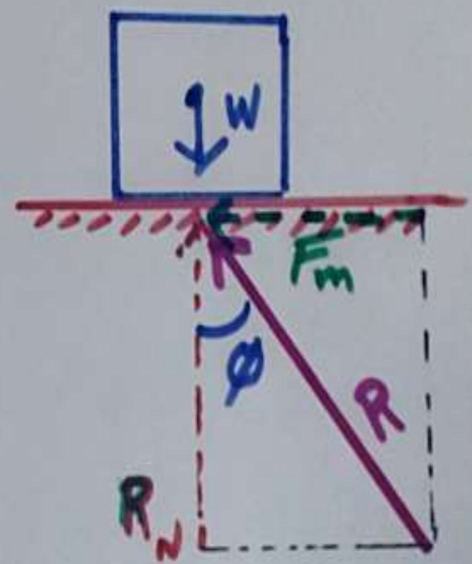


$R_N = W = 1000\,N$

$$\mu = \frac{F_m}{R_N} = \frac{300\,N}{1000\,N} = 0.3$$

ANGLE OF FRICTION (ϕ)





The angle made by Resultant force (of Normal reaction and limiting frictional force) with normal reaction is known as angle of friction

DRY FRICTION (**Coulomb friction**): **The friction(tangential resistive force developed at contact surface) developed between two dry solid surface when one tends to move/move over another is known as dry friction**

Types of dry friction:

1) Static friction: Static friction is the force of friction on an object that is not moving, but tends to move due to application of external force . If you push on a stationary block and it doesn't move, it is being held by static friction which is equal and opposite to your push.

2) Dynamic friction: The frictional force developed at contact surface when any body moves over another surface is known as dynamic friction.

LAWS OF FRICTION (COULOMB'S LAWS OF FRICTION)

LAWS OF STATIC FRICTION (THE FRICTIONAL FORCE ENCOUNTERED BEFORE COMMENCEMENT OF MOTION IS KNOWN AS STATIC FRICTION)

1) The frictional force always acts in the opposite direction to that direction in which the body tends to move
2) The value of frictional force developed is equal to the value of external force till the body is in rest state.
3) The value of frictional force developed automatically increases(as external force increases) upto maximum limit called limiting friction
4) The value of maximum limiting friction force developed is proportional to normal reaction developed .

$$F_m \propto R_N$$
$$F_m = \mu R_N$$

5) The value of maximum limiting friction force developed also depends on degree of roughness on the contacting surfaces and does not depends on shape and contact area between the bodies

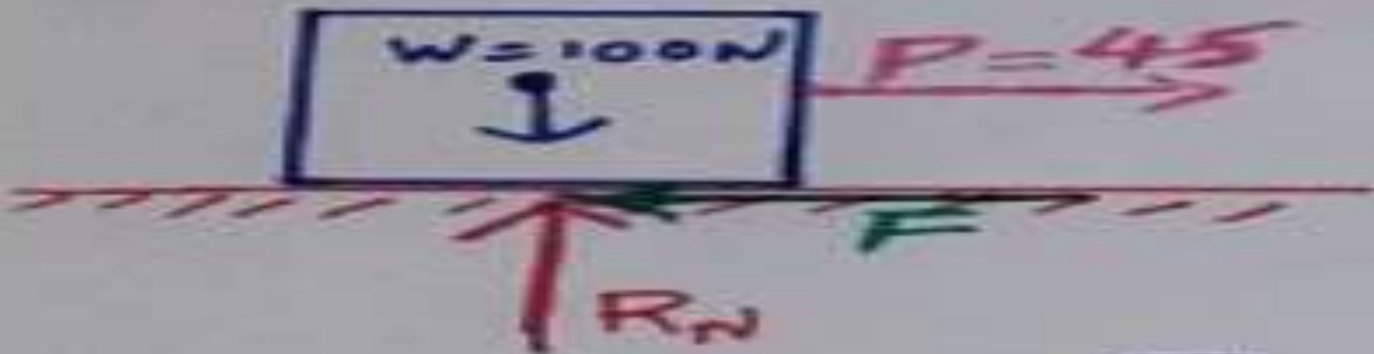
LAWS OF DYNAMIC FRICTION (THE FRICTIONAL FORCE ENCOUNTERED AFTER COMMENCEMENT OF MOTION IS KNOWN AS DYNAMIC FRICTION)

1) The frictional force always acts in the opposite direction to that direction in which the body moves.
2) The value of frictional force developed during motion of body is slightly less than maximum limiting friction developed in static friction and correspondingly value of coefficient of kinetic friction is marginally less than the value of coefficient of static friction

A body of weight 100 N is placed on three types of horizontal surface . When the object is placed on first horizontal surface, an horizontal force of 45 N force just caused the body to move. When the object is placed on second horizontal surface , a pulling type inclined force of magnitude 45 N inclined at 25 degree to horizontal , caused the body just to start to move. . When the object is placed on third horizontal surface , a pushing type inclined force of magnitude 45 N inclined at 25 degree to horizontal , caused the body just to start to move. Find the coefficient of friction in each cases.

CASE 1

[Horizontal force 45 N just moves the body]



$\Sigma F_V = 0$ [↑ = ↓]

$\uparrow R_N = 100 \downarrow$

$\Sigma F_H = 0$

$\overrightarrow{P} = \overleftarrow{F}$

$45 = \mu R_N$

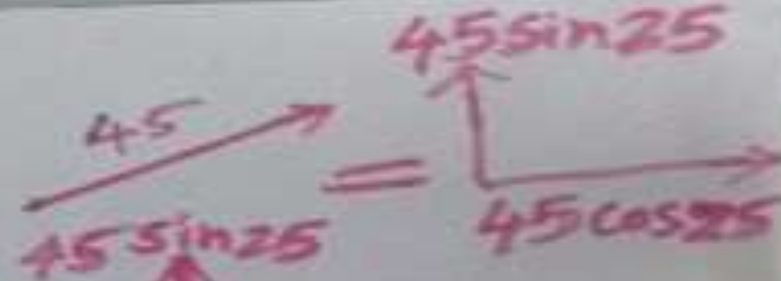
$45 = \mu\, 100$

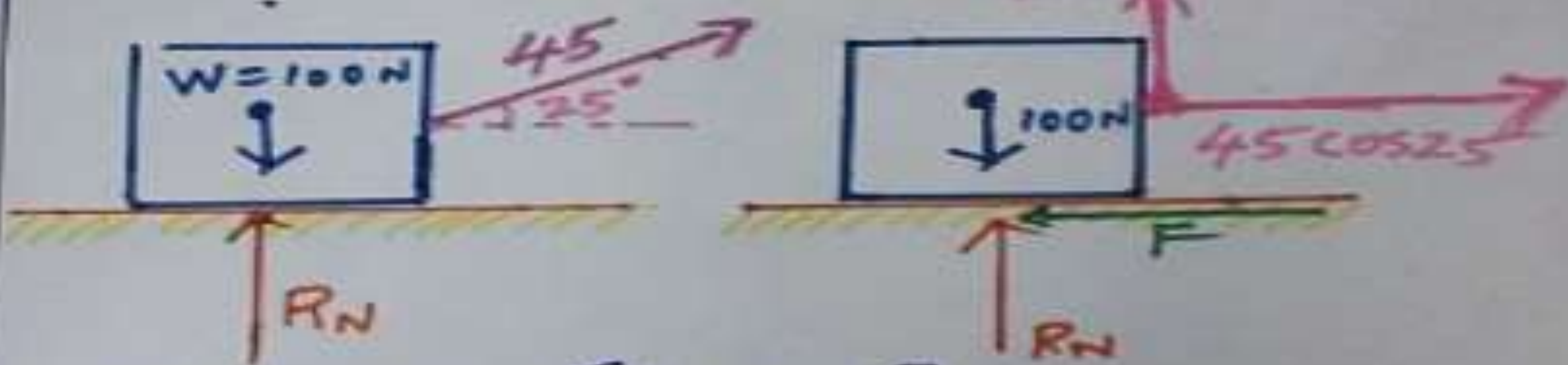
$\Rightarrow \mu = \frac{45}{100}$

$\Rightarrow \mu = 0.45$

$F = \mu R_N$

CASE 2
[inclined pulling force]





$\Sigma F_V = 0 \quad [\uparrow \; = \; \downarrow]$

$\uparrow R_N + 45 \sin 25 \uparrow = 100$

$\Rightarrow \quad R_N = 100 - 45 \sin 25 = 80.99\,N$

$F = \mu R_N$

$\Sigma F_H = 0$

$\overrightarrow{45 \cos 25} = \overleftarrow{F}$

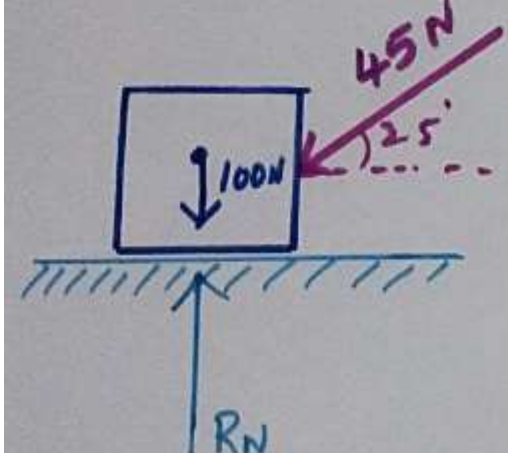
$45 \cos 25 = \mu R_N$

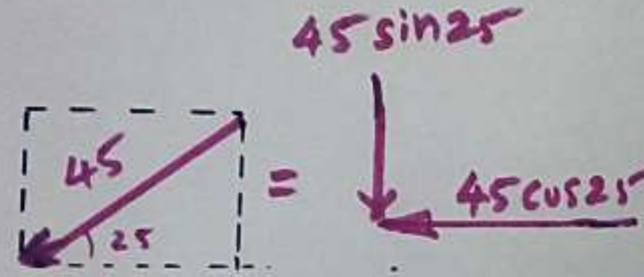
$\Rightarrow \quad \mu = \dfrac{45 \cos 25}{R_N} = \dfrac{45 \cos 25}{80.99} = 0.503$

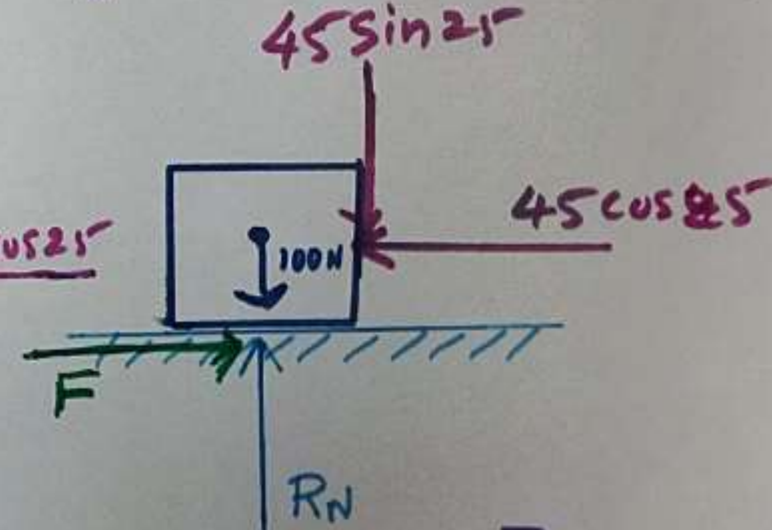
$= 0.503$

CASE - 3

[inclined pushing force 45 N just starts to move the body]







$\Sigma F_v = 0$ [$\uparrow = \downarrow$]

$\uparrow R_N = 100\downarrow + 45 \sin 25 = 119$ N

$\Sigma F_H = 0$

$\rightarrow \quad \leftarrow$

$F = 45 \cos 25$

$\Downarrow$

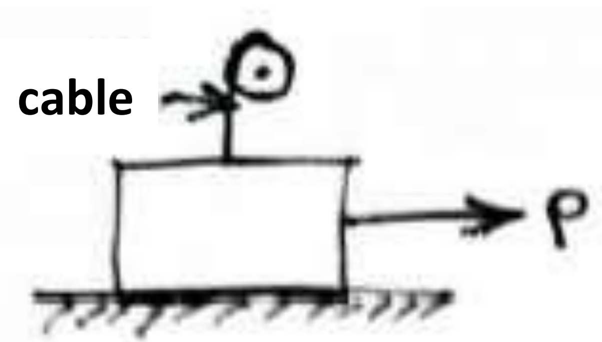
$\mu R_N = 40.78$

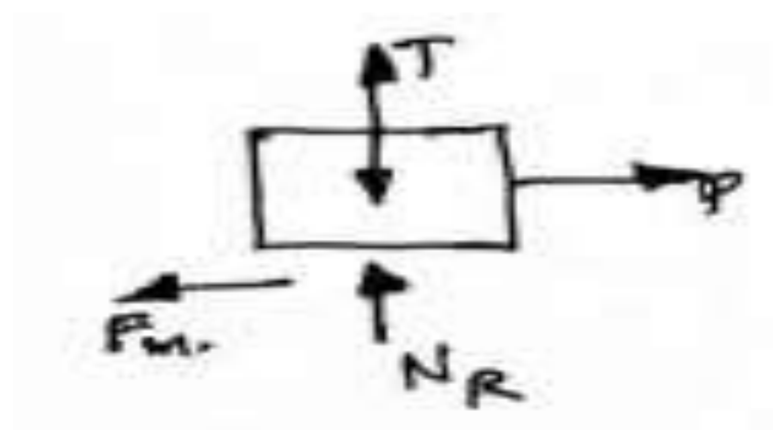
$\mu = \dfrac{40.78}{R_N} = \dfrac{40.78}{119} = 0.342$

$\boxed{F = \mu R_N}$

DRAW THE FREE BODY DIAGRAM

(of body suspended from crane tends to move on horizontal surface due application of force P)



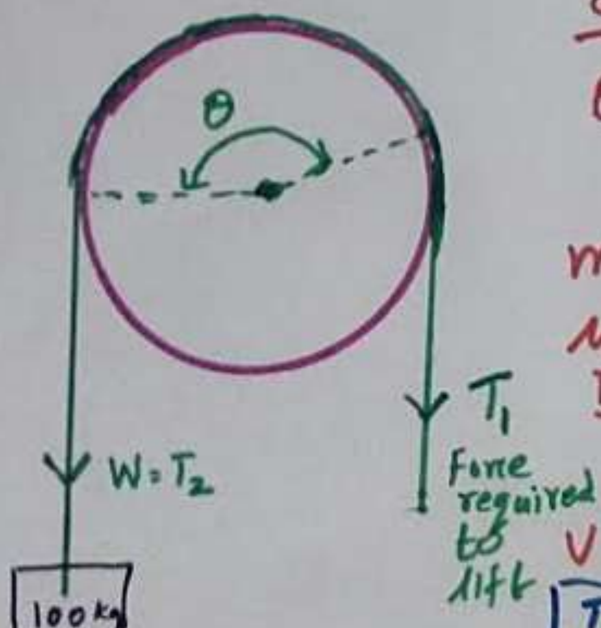


Free body diagram

BELT FRICTION

The frictional force developed between belt and pulley is used for transmiting power from one pulley to other, or for applying brakes, or for lifting load etc is known belt friction

A 100 kg mass is lifted by a rope rolling on a cylinder of 150 mm by applying force in vertically downward direction on the otherside. If coefficient of friction is 0.2 and angle of contact of rope and cylinder (pulley) is 180, Find the force required and power required if velocity of rope is 10 m/s



GIVEN

$\theta = 180^\circ = 180 \times \left(\frac{\pi}{180}\right)$ rad

$= \pi$ radian

$m = 100$ kg

$\mu = 0.2$

$D = 150$ mm

$= \frac{150}{1000} = 0.15$ m

$V = 10$ m/s

$\boxed{\text{Torque} = (T_1 - T_2) R = 36.3 \text{ Nm}}$

SOLUTION

$D = .15$ m

$R = D/2 = 0.075$ m

$W = mg$

$= 100 \times 9.81$

$= 981 \text{ N} = T_2$

$$\boxed{\frac{\text{Max. Tension}}{\text{Min. Tension}} = \frac{T_2}{T_1} = e^{\mu\theta}}$$

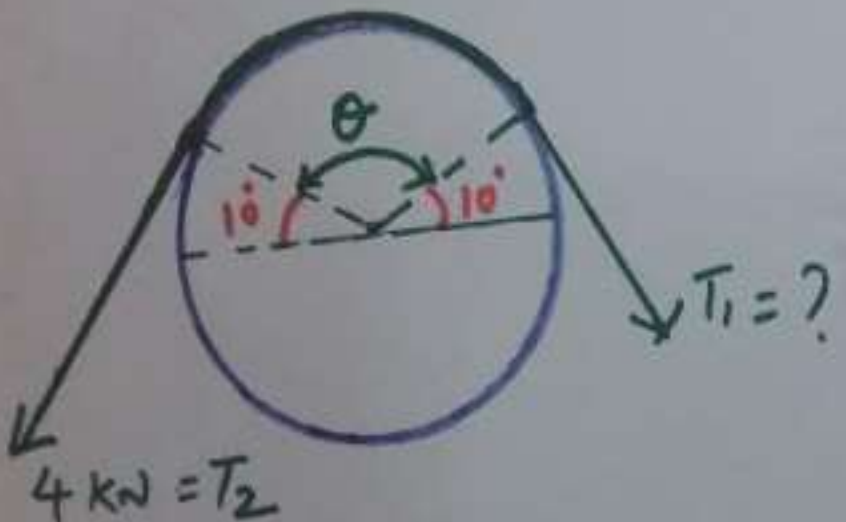
$$\Rightarrow T_1 = \frac{T_2}{e^{\mu\theta}} = \frac{981}{e^{0.2 \times \pi}} = 523.48\text{N}$$

$$\boxed{\text{power} = (T_2 - T_1) V}$$

$= (981 - 523)\ 30$

$= 13725$ watts

A rope is wound over a pulley as shown in figure. If the tension which pulls the belt on one end is 4 KN, determine the necessary tension on the other side of belt to resist. Take $\mu = 0.25$



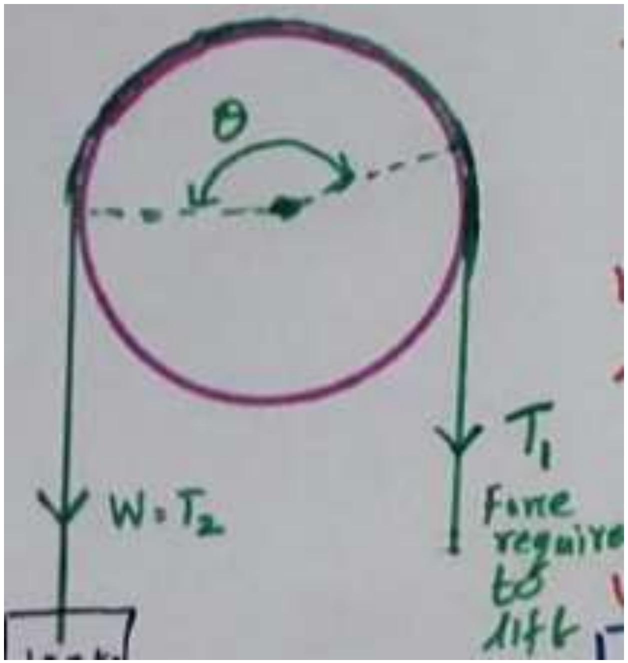
$$\theta = 180 - 10 - 10$$
$$= 160^\circ$$
$$= 160 \times \left(\frac{\pi}{180}\right)$$
$$= 2.792 \text{ radian.}$$

$$\boxed{\frac{T_2}{T_1} = e^{\mu\theta}}$$

$$\Rightarrow T_1 = \frac{T_2}{e^{\mu\theta}} = \frac{4}{e^{0.25 \times 2.792}}$$

$$\Rightarrow T_1 = 2 \text{ KN}$$

RATIO OF TENSION IN BELT DRIVE



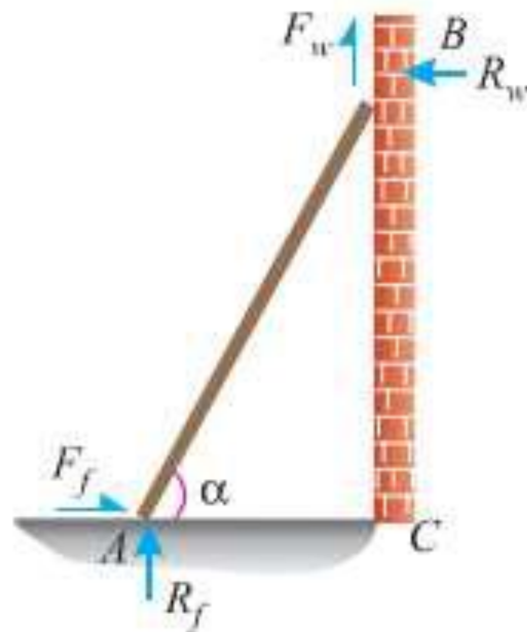
$$\boxed{\frac{\text{Max. Tension}}{\text{Min. Tension}} = \frac{T_2}{T_1} = e^{\mu\theta}}$$

$$T_2 = T_1 e^{\mu\theta}$$

FRICTION IN SMOOTH SURFACE:

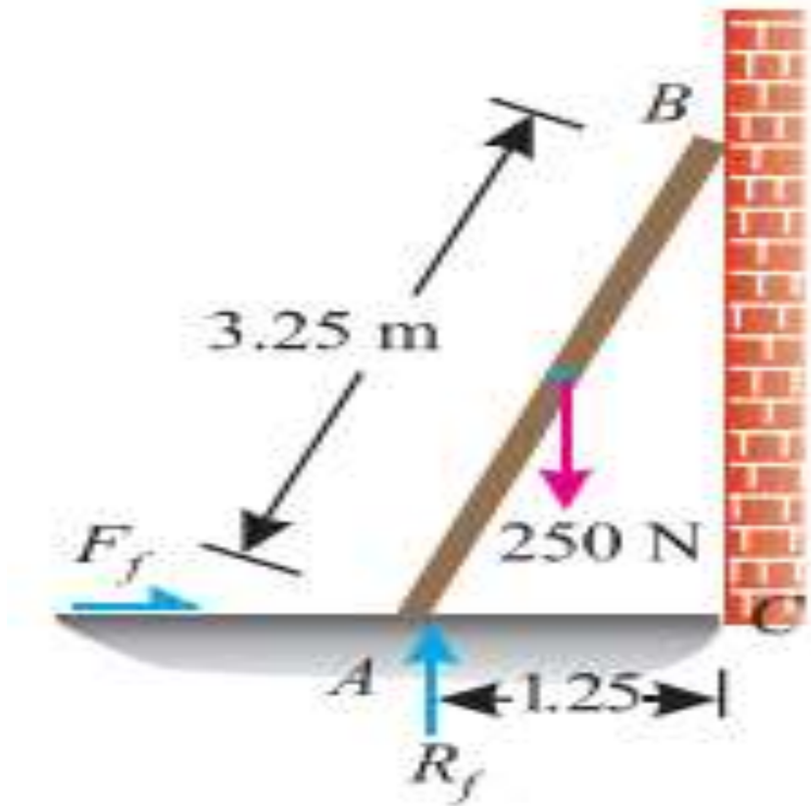
Generally smooth surface will develop only Negligible frictional force , which can be considered as zero friction.

LADDER FRICTION



A uniform ladder of length 3.25 m and weighing 250 N is placed against a smooth vertical wall with its lower end 1.25 m from the wall. The coefficient of friction between the ladder and floor is 0.3.

What is the frictional force acting on the ladder at the point of contact between the ladder and the floor? Show that the ladder will remain in equilibrium in this position.



Given: Length of the ladder (l) = 3.25 m; Weight of the ladder (w) = 250 N; Distance between the lower end of ladder and wall = 1.25 m and coefficient of friction between the ladder and floor (μ_f) = 0.3.

F_f = Frictional force acting on the ladder at the Point of contact between the ladder and floor, and

R_f = Normal reaction at the floor.

Since the ladder is placed against a smooth vertical wall, therefore there will be no friction at the point of contact between the ladder and wall.

Resolving the forces vertically,

$$R_f = 250 \text{ N}$$

From the geometry of the figure, we find that

$$BC = \sqrt{(3.25)^2 - (1.25)^2} \quad = 3.0 \text{ m}$$

Taking moments about B and equating the same,

$$F_f \times 3 = (R_f \times 1.25) - (250 \times 0.625) = (250 \times 1.25) - 156.3 \quad = 156.2 \text{ N}$$

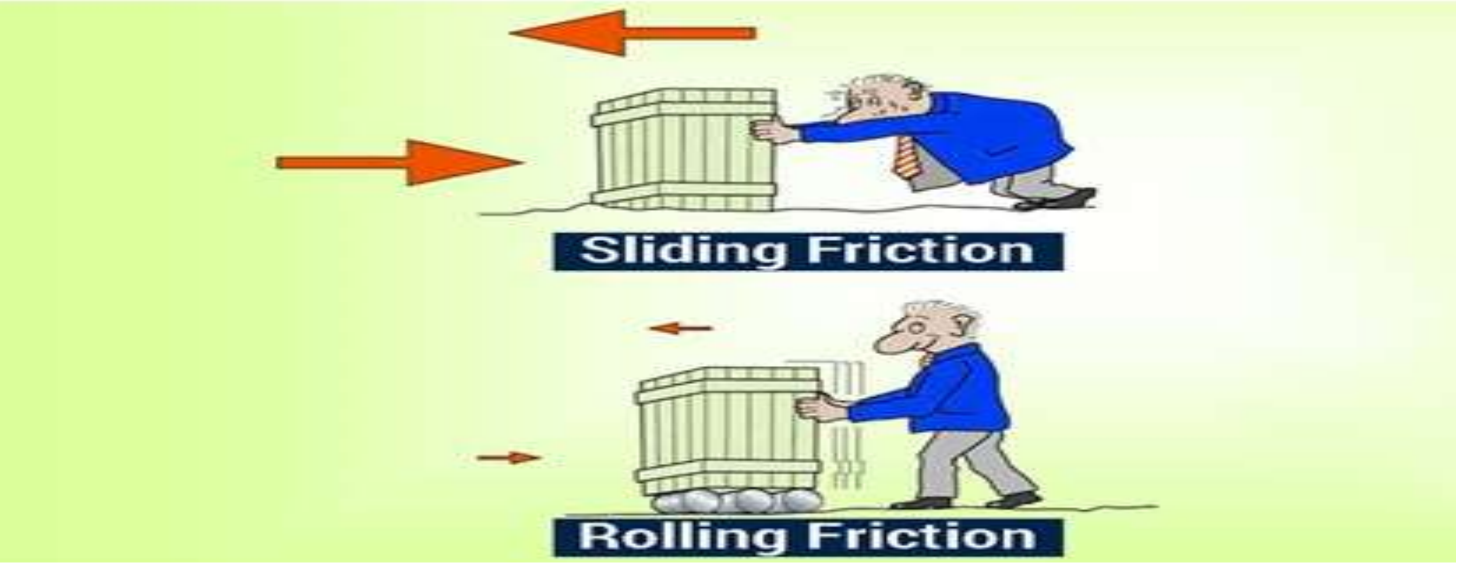
$$F_f = \frac{156.2}{3} = 52.1 \text{N}$$

The minimum frictional force required to be developed at floor surface for equilibrium of ladder is 52.1 N. If frictional force is lesser value , the ladder will slide down . But the maximum frictional force that can be generated at the floor surface is given by equation,

$$= \mu R_f = 0.3 \times 250 = 75 \text{ N}$$

Therefore the ladder will remain in equilibrium condition in this position

ROLLING FRICTION



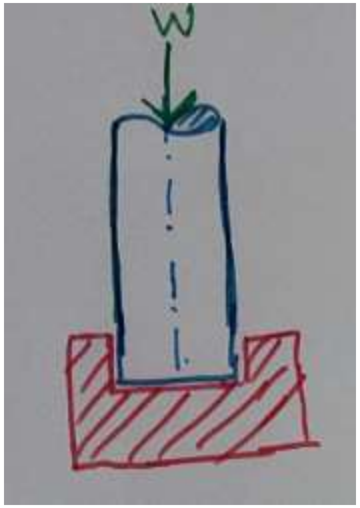
FRICTION DEVELOPED IN ROLLING FRICTION IS GENERALLY SMALLER THAN SLIDING FRICTION, STATIC FRICTION , FLUID FRICTION ETC

LUBRICANTS

- The substances which used to decrease the friction are known as lubricants.

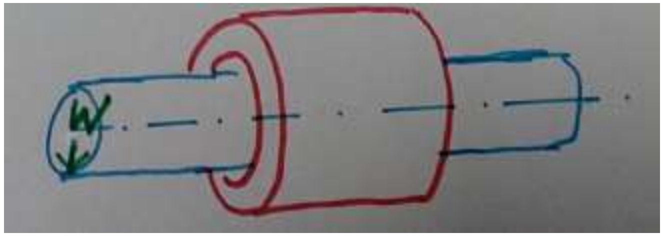
THRUST BEARINGS

The type of bearing used to support mainly axial load acting along the axis of rotation of shaft are known as thrust bearings



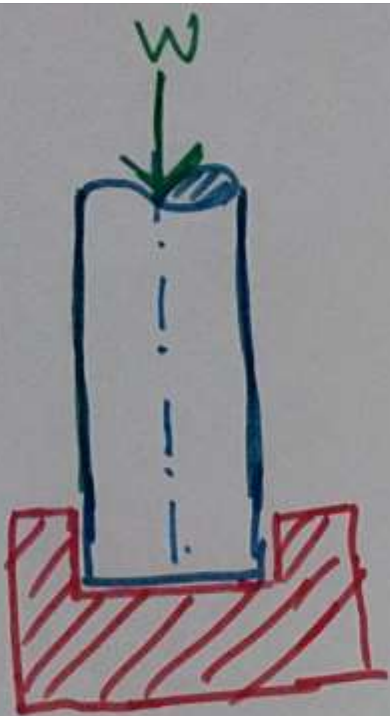
RADIAL BEARING :

The bearing which supports load acting perpendicular to axis of rotation of the shaft are known as radial bearings

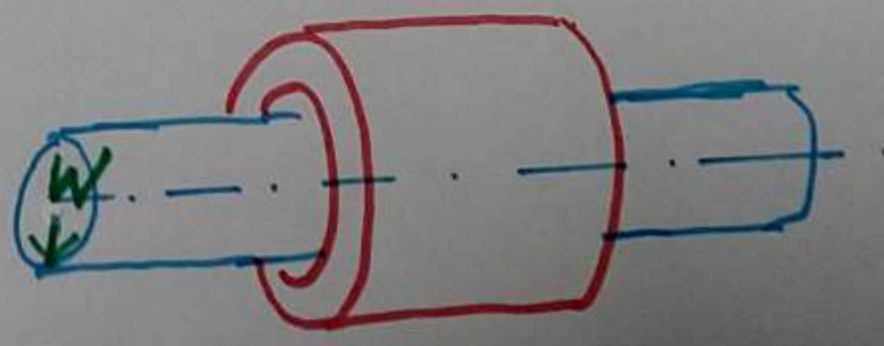


JOURNAL BEARINGS(ALSO KNOWN AS FRICTION BEARING) The type of plain bearing which has journal/shaft which rotates inside metal sleeve and is usually designed to carry radial loads only and therefore they carry load acting perpendicular to axis of shaft are known as journal bearing.

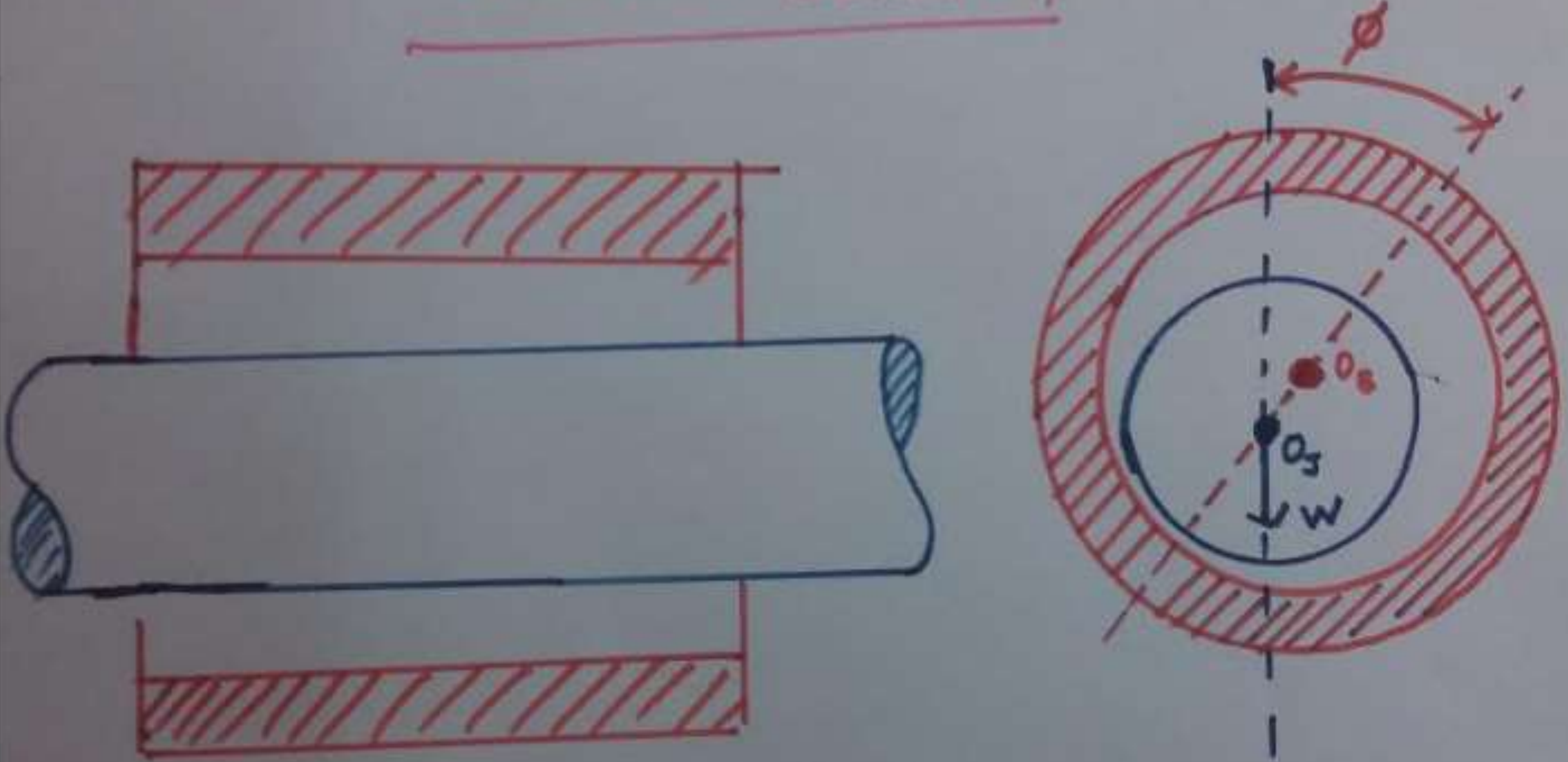
THRUST BEARING



JOURNAL BEARING

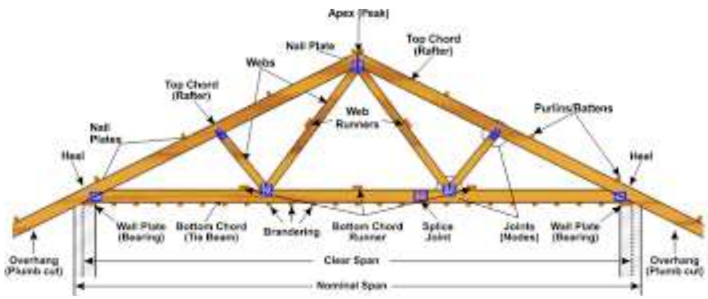


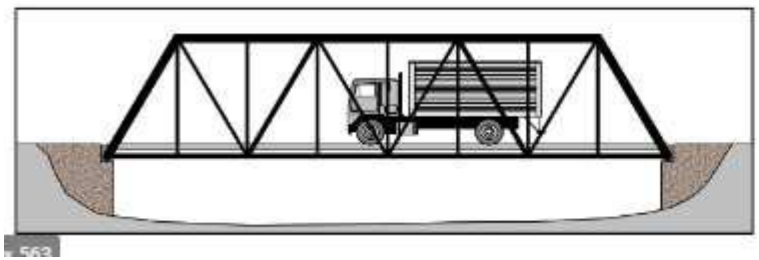
JOURNAL BEARING



ϕ = Attitude angle of bearing
[The location of Journal [shaft] may be located / measured by attitude angle]

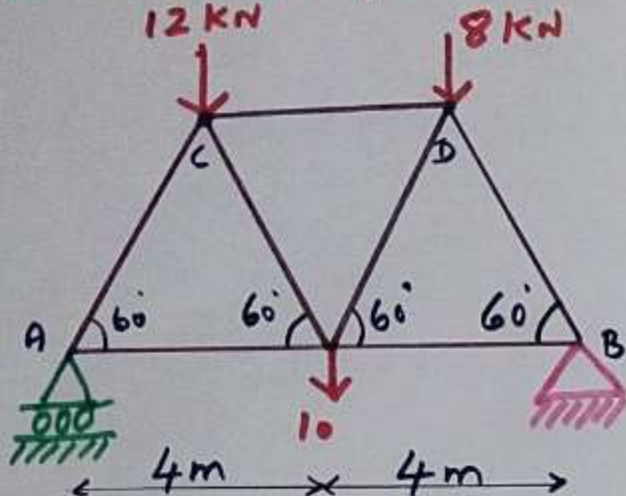
TRUSS

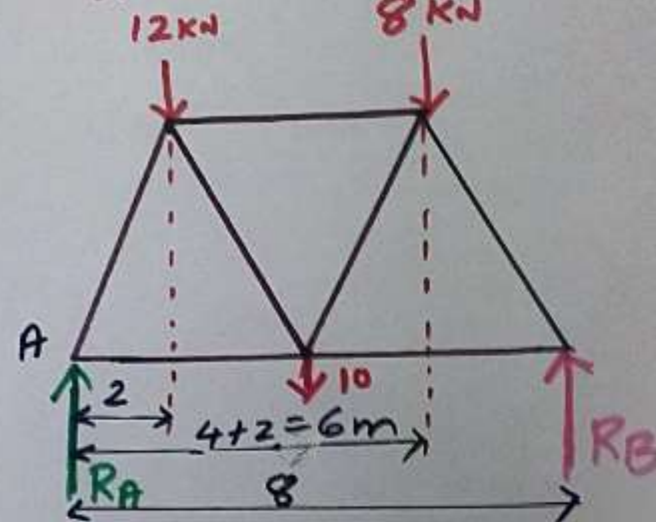




The framework/structure formed by joining several bars and is usually used for supporting roof, bridges etc are known as truss

A truss of 8m span is loaded as shown in figure. Find the support reactions.





$\underset{\sim\sim}{\Sigma F_v = 0}$ $[\uparrow = \downarrow]$

$\uparrow R_A + R_B \uparrow = 12 \downarrow + 8 \downarrow + 10$ ...... (1)

$R_A + R_B = 30$ kN

$\underset{\sim\sim}{\Sigma M_A = 0}$ $[\circlearrowleft = \circlearrowright]$

$(R_B \times 8) = (8 \times 6) + (12 \times 2) + (10 \times$

$8R_B = 112$

$\Rightarrow R_B = 112/8 = 14$ kN

substituting value of $R_B = 14$ kN in (1) $\Rightarrow R_A = 30 - R_B = 30 - 14 = 16$

ENGINEERING MECHANICS

(PORTION FOR CLA-3)

BY

V.JOSE ANANTH VINO

ASSISTANT PROFESSOR/MECH/B.I.H.E.R

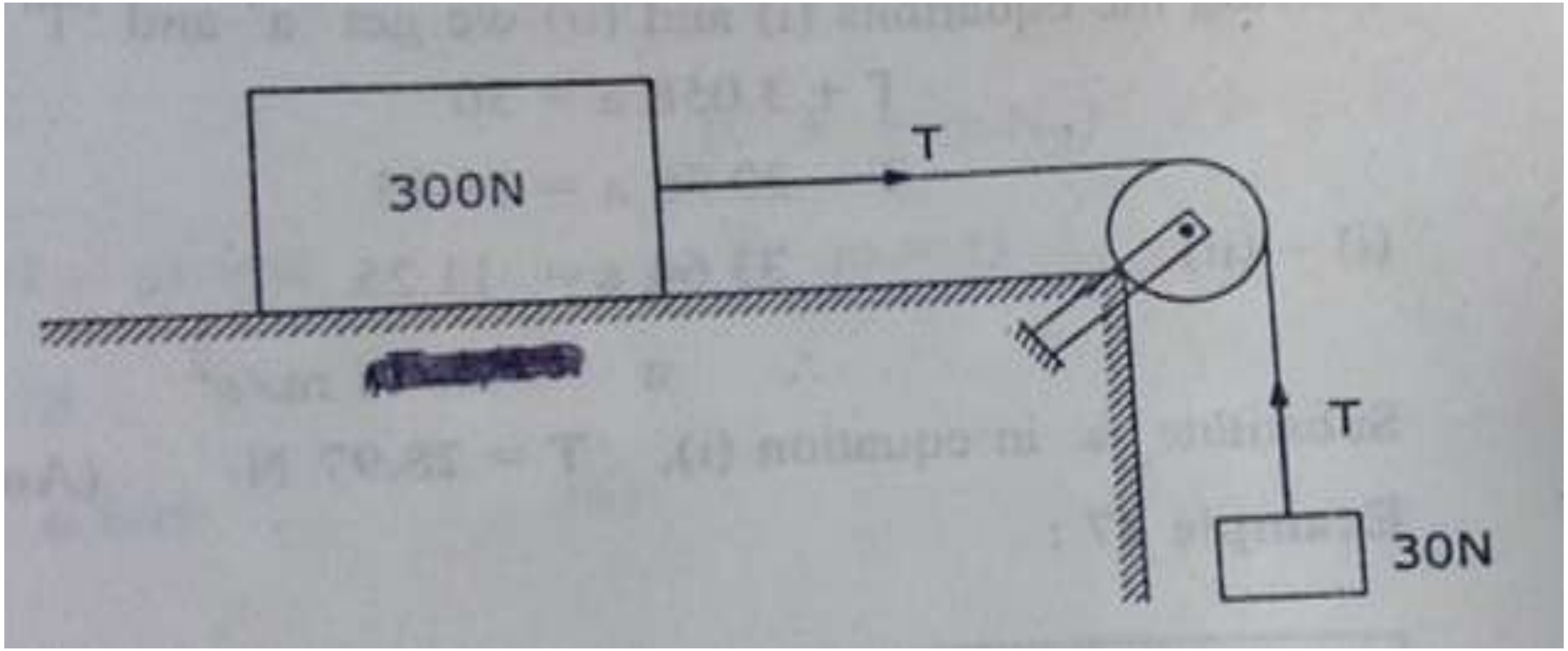
NEWTON’S SECOND LAW OF MOTION

The newton second law states that the acceleration(a) of an object as produced by a net force(F) is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass (m) of the object.

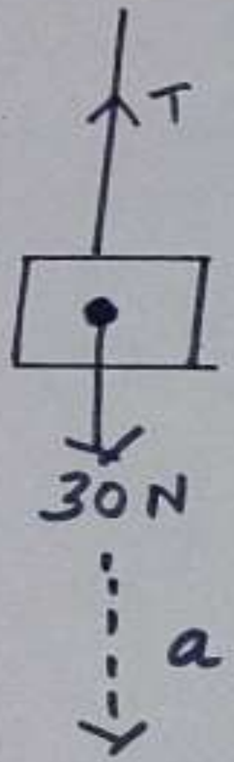
$$a = F/m$$

$$F = ma$$

A body of weight 300 N lies on a smooth horizontal plane. One end of string is attached to this body and a weight of 30 N is attached to other end of this string. Apply the laws of mechanics and find the acceleration of the system and tension in the string when the weight 30 N hangs vertically downwards through smooth pulley and moves downwards.



CONSIDERING DOWNWARD MOTION OF 30N WEIGHT



$$W_1 = 30\,N$$

$$\boxed{W = mg}$$

$$W_1 = m_1 g$$

$$m_1 = \frac{W_1}{g} = \frac{30}{9.81} = 3.06\ kg$$

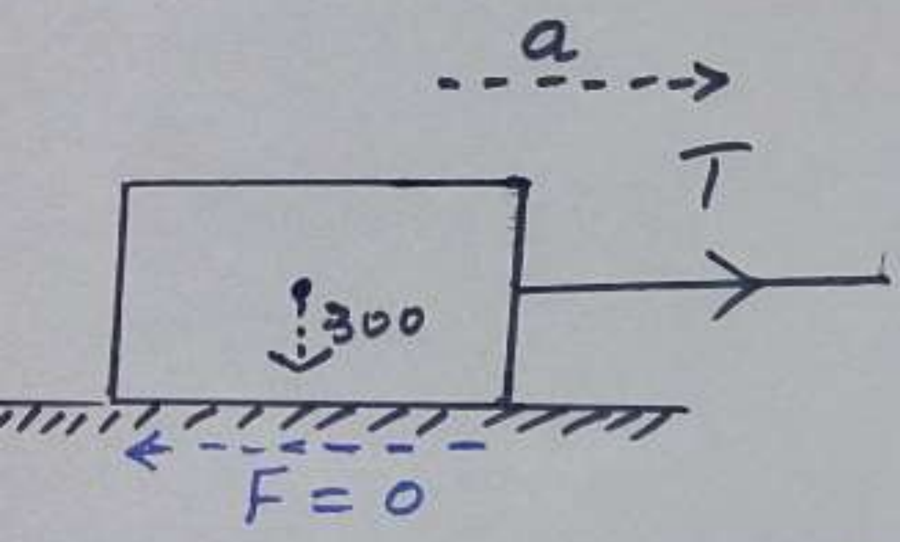
Applying Newton's second law

$$\underbrace{F}_{\text{net force}} = ma$$

$$(30\downarrow - T\uparrow) = 3.06 \times a$$

$$30 - T = 3.06\,a \quad \text{-------} \; (1)$$

CONSIDERING HORIZONTAL [RIGHTSIDE] MOTION OF 300N



$$W_2 = 300\,N$$

$$W_2 = m_2 g \Rightarrow m_2 = \frac{W_2}{g}$$

$$= \frac{300\,N}{9.81}$$

$$= 30.6\,N$$

Applying Newton's second law in Horizontal direction.

$$F = ma$$

$$T = m_2 a$$

$$T = 30.6\,a \quad \text{----} \quad (2)$$

Solving equation 1 & 2

$$30 - T = 3.06\,a \quad \text{------} \quad (1)$$
$$T = 30.6\,a \quad \text{------} \quad (2)$$

2 in 1

$$30 - 30.6a = 3.06a$$
$$30 = 3.06a + 30.6\,a$$
$$30 = a[3.06 + 30.6]$$
$$30 = 33.66\,a$$
$$\Rightarrow a = \frac{30}{33.66} = 0.89 \text{ m/s}^2$$

Substituting $a = 0.89 \text{ m/s}^2$ in equation 2

$$T = 30.6\,a$$
$$T = 30.6 \times 0.89 = 27.23 \text{ N}$$
$$T = 27.23 \text{N}$$

D' ALEMBERT'S PRINCIPLE

- **D'Alembert's principle, alternative form of Newton's second law of motion, stated by the 18th-century French polymath Jean le Rond d'Alembert. In effect, the principle reduces a problem in dynamics to a problem in statics. The second law states that the force F acting on a body is equal to the product of the mass m and acceleration a of the body, or F = ma; in d'Alembert's form, the force F plus the negative of the mass m times acceleration a of the body is equal to zero: F - ma = 0. In other words, the body is in equilibrium under the action of the real force F and the fictitious force -ma. The fictitious force is also called an inertial force and a reversed effective force.**

$$F = ma$$

$$F - ma = 0$$

$$F - \text{inertia force} = 0$$

IMPULSE

Impulse Defined

Impulse is defined as the product force acting on an object and the time during which the force acts. The symbol for impulse is *I*. So, by definition:

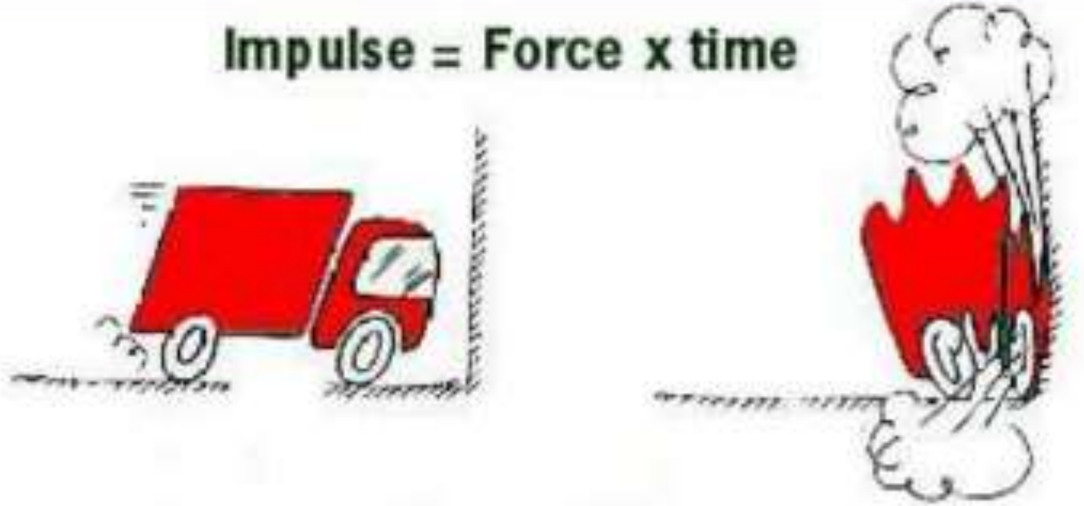
$$I = F\,t$$

Example: A 50 N force is applied to a 100 kg boulder for 3 s. The impulse of this force is $I = (50\ \text{N})\,(3\ \text{s}) = 150\ \text{N}\cdot\text{s}$.

Note that we didn't need to know the mass of the object in the above example.

Impulse Example

Impulse = Force x time



- **Example**: Wall exerts a force of 10,000 N on the van. The contact time is 0.01 s. What is the impulse?
- **Solve**: Impulse = F * t = 10,000 * 0.01
  - Impulse = 100 N-s

What is Momentum?

Momentum may be defined by its equation:

Momentum is a quantity defined as the product of the mass and velocity of an object.

$$\boxed{P = mv}$$

m = mass
v = velocity

Momentum Defined

Momentum p is defined as the product of mass and velocity, mv. *Units:* kg m/s

$$\boxed{p = mv}$$

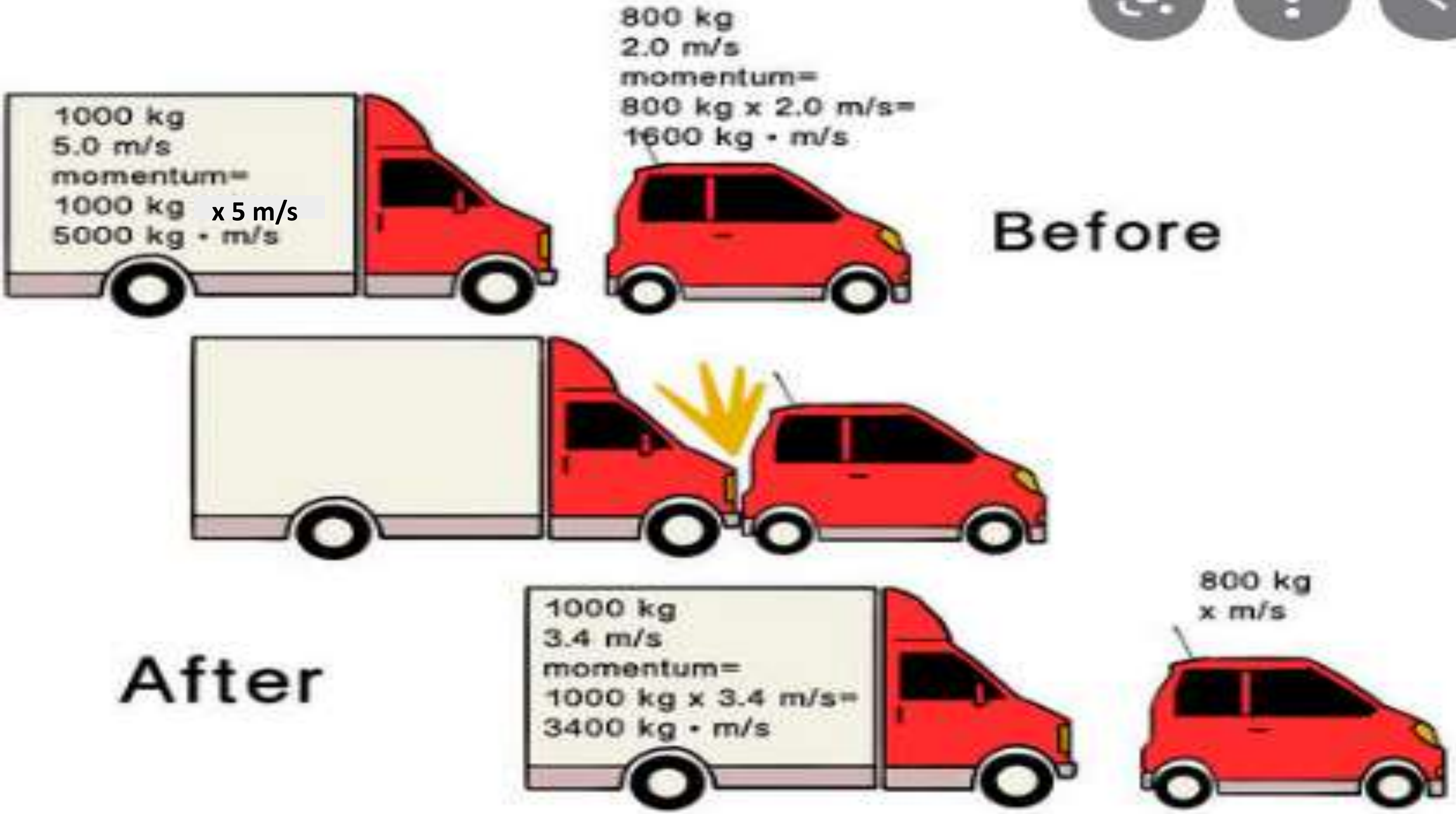
Momentum

m = 1000 kg

v = 16 m/s

p = (1000 kg)(16 m/s)

$$\boxed{p = 16{,}000 \text{ kg m/s}}$$


800 kg
2.0 m/s
momentum=
800 kg x 2.0 m/s=
1600 kg • m/s
1000 kg
5.0 m/s
momentum=
1000 kg x 5 m/s
5000 kg • m/s
Before
After
1000 kg
3.4 m/s
momentum=
1000 kg x 3.4 m/s=
3400 kg • m/s
800 kg
x m/s

LAW OF CONSERVATION OF MOMENTUM

LAW OF CONSERVATION OF MOMENTUM STATES THAT THE TOTAL MOMENTUM OF SYSTEM OF MASSES BEFORE COLLISION IS EQUAL TO TOTAL MOMENTUM OF SYSTEM OF MASSES AFTER COLLISION

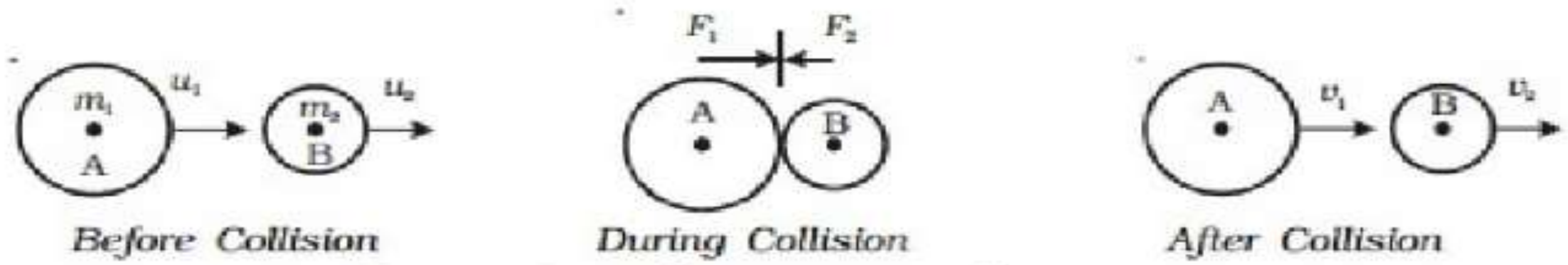


Fig.. Law of conservation of momentum

By the law of conservation of momentum,The total momentum of the system before collision = The total momentum of the system after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$

COEFFICIENT OF RESTITUTION (e)
OR
NEWTON'S LAW OF COLLISION

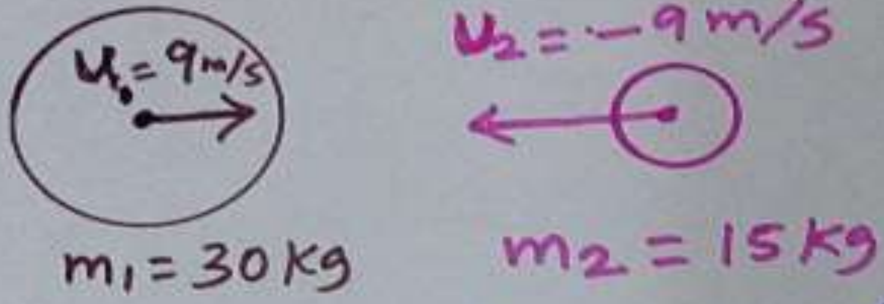
COEFFICIENT OF RESTITUTION (e) = RELATIVE VELOCITY OF APPROACH / RELATIVE VELOCITY OF SEPERATION

$$(e) = (u_1 - u_2) / (v_2 - v_1)$$

NEWTONS LAW OF COLLISION , $(e)\ (V_2 - V_1) = (u_1 - u_2)$

A body having mass 30 kg , moves with a velocity of 9 m/s strikes on an another body having mass 15 kg moving in opposite direction with the velocity of 9 m/s centrally. Taking the coefficient of restitution as 0.8 , analyse the motion of bodies and find the velocity of each body after the impact .

CONSIDERING MOTION BEFORE COLLISION

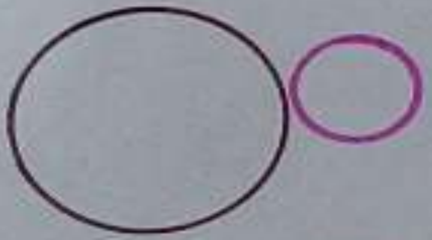


Total momentum before impact $= m_1 u_1 + m_2 u_2$

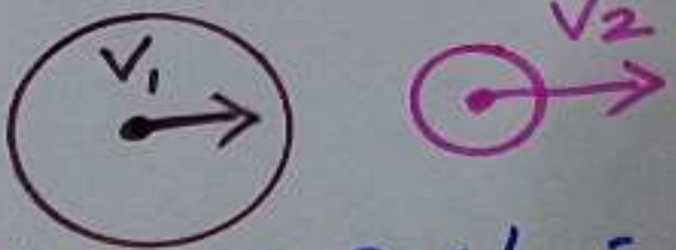
$= (30 \times 9) + (15 \times (-9))$

$= 135$ //

AT COLLISION



CONSIDERING MOTION AFTER COLLISON



Total momentum after impact $= m_1 V_1 + m_2 V_2$

$= 30 V_1 + 15 V_2$ //

Applying law of conservation of momentum

$$\left.\begin{array}{c}\text{Total momentum}\\ \text{before Impact}\end{array}\right\} = \begin{array}{c}\text{Total momentum}\\ \text{after impact}\end{array}$$

$$135 = 30V_1 + 15V_2.$$

$$\Rightarrow 30V_1 + 15V_2 = 135. \quad \text{--- --- (1)}$$

Applying Newton law of collision.

$$\begin{array}{c}\text{coefficient}\\ \text{of}\\ \text{restitution.}\\ (e)\end{array} \times \left.\begin{array}{c}\text{relative velocity}\\ \text{of approach}\end{array}\right\} = \begin{array}{c}\text{relative velocity}\\ \text{of seperation.}\end{array}$$

$$0.8 \times (u_1 - u_2) = (V_2 - V_1)$$

$$0.8 \times [9 - (-9)] = (V_2 - V_1)$$

$$0.8 \times 18 = V_2 - V_1$$

$$14.4 = V_2 - V_1$$

$$\Rightarrow V_2 - V_1 = 14.4 \quad \text{--- --- (2)}$$

$$30V_1 + 15V_2 = 135 \quad \text{----} (1)$$
$$V_2 - V_1 = 14.4 \quad \text{----} (2)$$
Solving

$$\Rightarrow V_2 = 14.4 + V_1 \quad \text{-----------} (3)$$

(3) in (1)

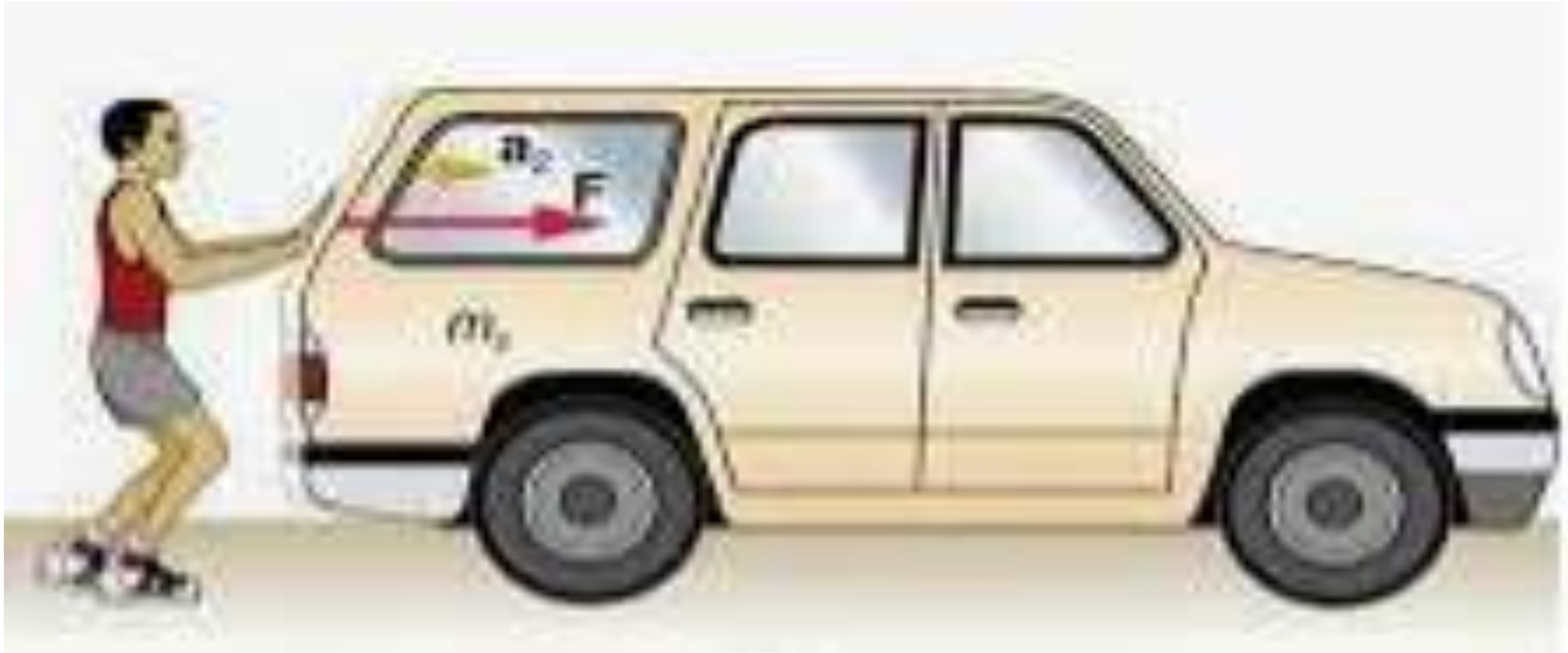
$$30V_1 + 15[14.4 + V_1] = 135$$
$$30V_1 + (15 \times 14.4) + (15V_1) = 135$$
$$30V_1 + 216 + 15V_1 = 135$$
$$216 + 30V_1 + 15V_1 = 135$$
$$30V_1 + 15V_1 = 135 - 216$$
$$45V_1 = -81$$
$$V_1 = -81/45 = -1.8 \text{ m/s} \quad \text{---} (4)$$

(4) in (3)

$$V_2 = 14.4 + V_1 = 14.4 - 1.8$$
$$V_2 = 13 \text{ m/s}$$

note: direction of V_1 is reversed i.e, opposite to the assumed direction.

WORKDONE



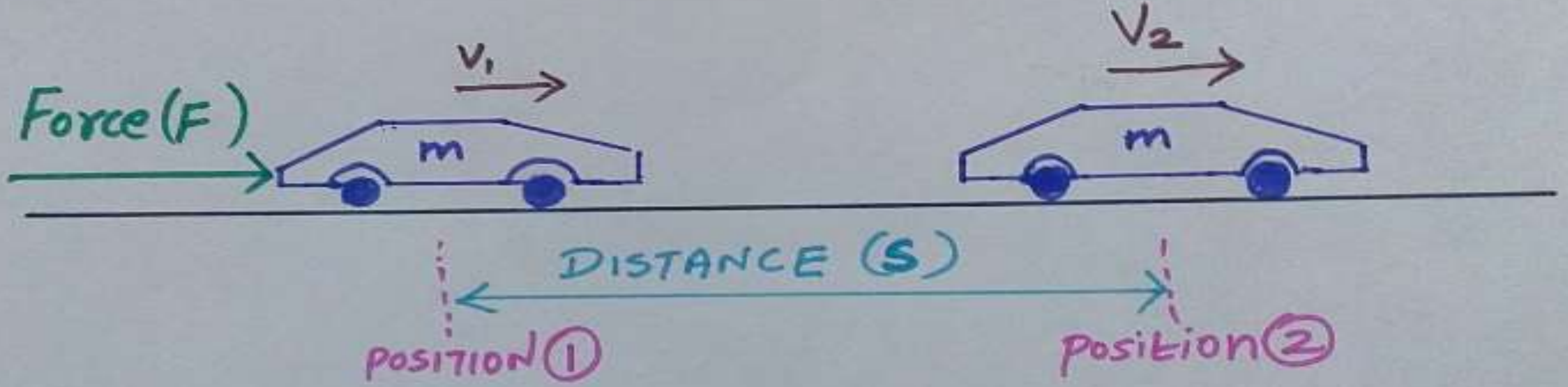
WORK DONE(energy)

- THE PRODUCT OF FORCE AND DISTANCE MOVED BY BODY IS KNOWN AS WORKDONE BY BODY.
- PROBLEM:FIND THE WORKDONE BY A CAR WHICH MOVES BY 5M DISTANCE DUE TO APPLICATION OF 1000 N FORCE
- ANSWER: WORKDONE = FORCE X DISTANCE

  = 1000 N x 5 M

  = 5000 NM

WORK - ENERGY PRINCIPLE



WORKDONE : Force × distance $= FS$

Kinetic energy at position (1) $= \frac{1}{2} m v_1^2$

Kinetic Energy at position (2) $= \frac{1}{2} m v_2^2$

According to work Energy principle, workdone = Change in Kinetic Energy

$$\boxed{FS = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2}$$

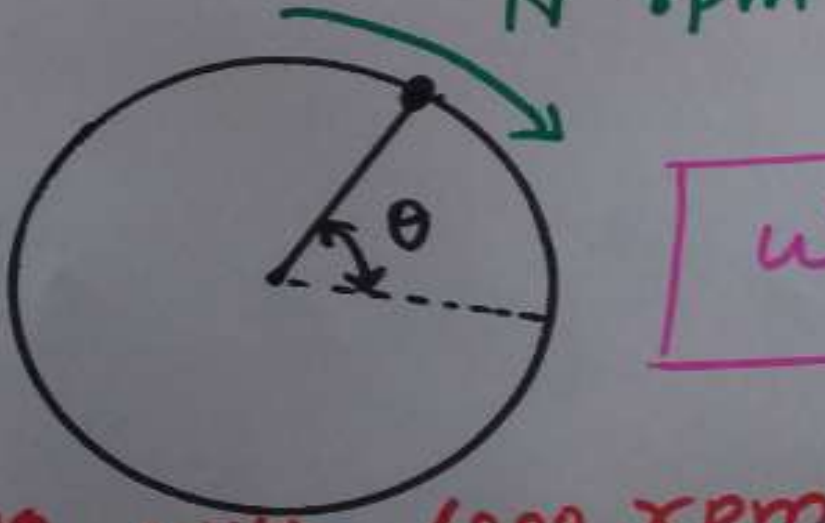
WORK ENERGY PRINCIPLE

According to the work-energy theorem, the net work on an object causes a change in the kinetic energy of the object. The formula for net work is

net work done= change in kinetic energy = final kinetic energy - initial kinetic energy.

ANGULAR VELOCITY & TANGENTIAL VELOCITY OF ROTATING BODY/PARTICLE

ANGULAR VELOCITY: The rate of change of angular displacement of body is known as angular velocity. Angular velocity is expressed in R.P.M (N) or in rad/second (ω)

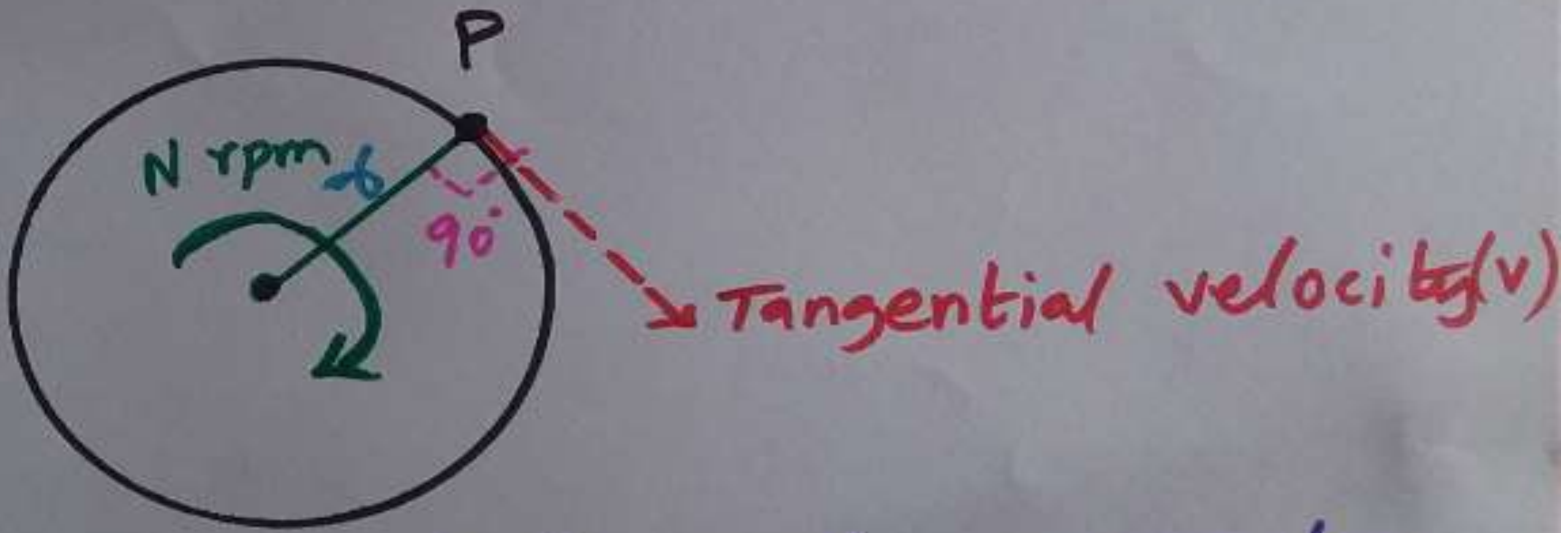


$$\boxed{\omega = \frac{2\pi N}{60}}$$

A body is rotating with 6000 r.p.m. Express its angular velocity in rad/second.

$$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 6000}{60} = 628\ \text{rad/s}$$

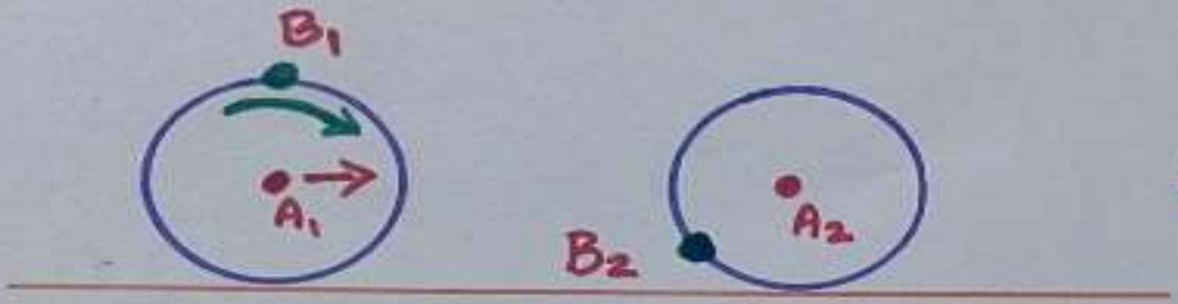
TANGENTIAL VELOCITY



when a particle is rotating along circular path, its corresponding linear velocity tends to lie perpendicular to the radial line of point is known as tangential velocity

Tangential velocity, $\boxed{V = r\omega}$

GENERAL PLANE MOTION



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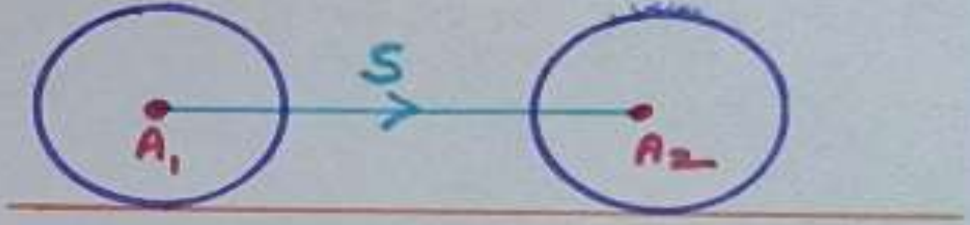
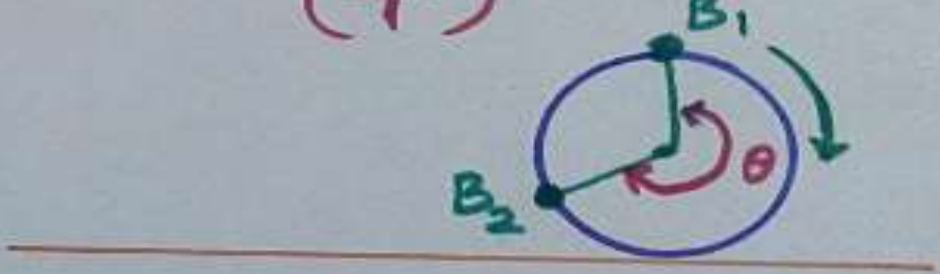


Translation
[distance moved 'S']

(+)



Rotation (θ)

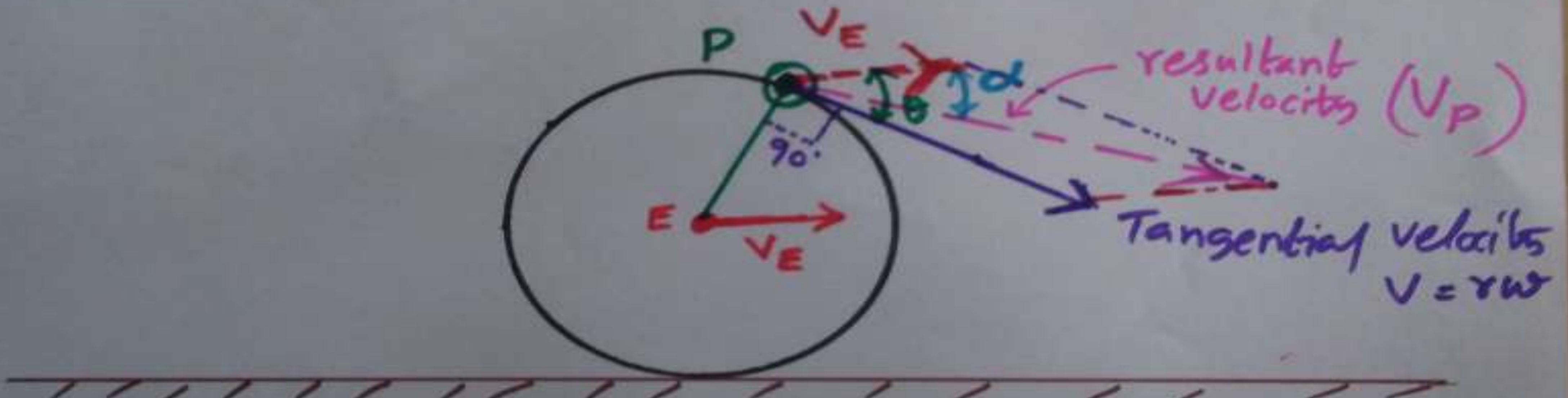
$\boxed{\text{General plane motion = Translation + Rotation.}}$

Example : motion of CAR WHEEL

"THE MOTION WHICH IS NEITHER PURE TRANSLATION NOR A PURE ROTATION, BUT COMBINATION OF TRANSLATION AND ROTATION IS KNOWN AS GENERAL PLANE MOTION"

GENERAL PLANE MOTION OF CAR WHEEL

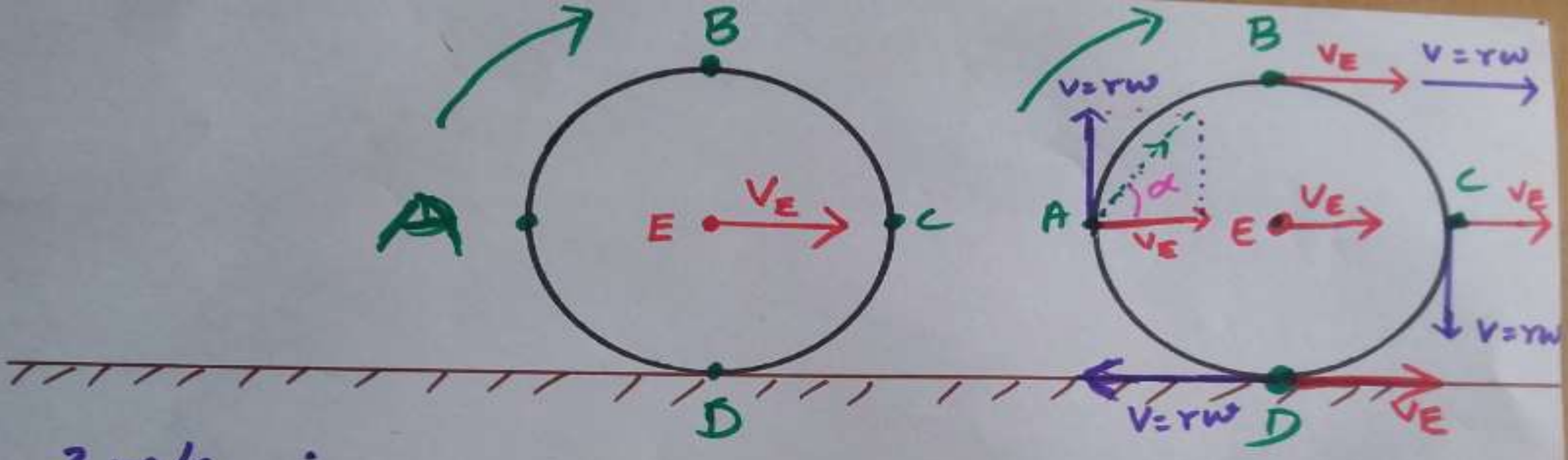
$$\boxed{\text{General plane motion} = \text{translation} + \text{rotation}}$$



resultant velocity } $V_P = \sqrt{(V_E)^2 + (V)^2 + 2 V_E V \cos\theta}$

Inclination of resultant velocity } $\alpha = \tan^{-1}\left(\dfrac{V}{V_E}\right)$

Analyze the general plane motion of a car wheel of radius 0.5 m which rolls without slipping along a horizontal plane .If, at certain instant the velocity of wheel at centre(E) is 3 m/s,then determine the velocity of points A,B,C and D which are located respectively at west,north,east and south corners of wheel.



$V_E = 3\,m/s$;

At point 'D'

$\boxed{V_D = 0}$

$$\left.\begin{array}{l}\text{resultant}\\ \text{velocity at 'D'}\end{array}\right\} = V_D = V_E - V$$

$$0 = 3 - r\omega$$

$$\Rightarrow r\omega = 3$$

$$\omega = \frac{3}{r} = \frac{3}{0.5} = 6\ rad/s$$

At point A

$$V_A = \sqrt{(V_E)^2 + (V)^2 + 2 V_E V \cos\theta}$$

$$= \sqrt{3^2 + (r\omega)^2 + (2 \times 3 \times r\omega \cos 90)}$$

$$= \sqrt{3^2 + (0.5 \times 6)^2 + 0} = 4.242 \text{ m/s}$$

$$\alpha = \tan^{-1}\left(\frac{V}{V_E}\right) = \tan^{-1}\left(\frac{r\omega}{V_E}\right) = \tan^{-1}\left[\frac{0.5 \times 6}{3}\right] = 45^\circ$$

At point 'B'

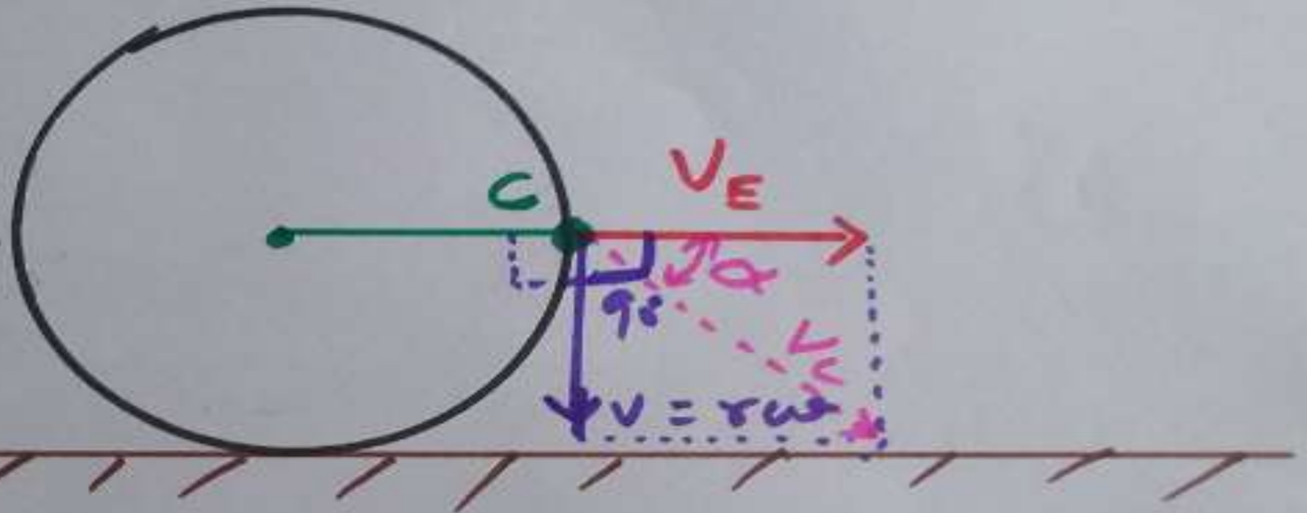
$$V_B = \sqrt{V_E^2 + V^2 + 2 V_E V \cos\theta}$$

$$= \sqrt{3 + (r\omega)^2 + 2 \times 3 \times r\omega \cos\theta}$$

$$= \sqrt{3^2 + (0.5 \times 6) + 2 \times 3 \times (0.5 \times 6) \cos\theta}$$

$$= 6 \text{ m/s}$$

AT POINT C



Resultant velocity at C $\Bigg\} = V_C = \sqrt{(V_E)^2 + (V)^2 + 2V_E V\cos\theta}$

$$= \sqrt{3^2 + (\overbrace{0.5\times 6}^{r\omega})^2 + 2V_E V\cos 90}$$

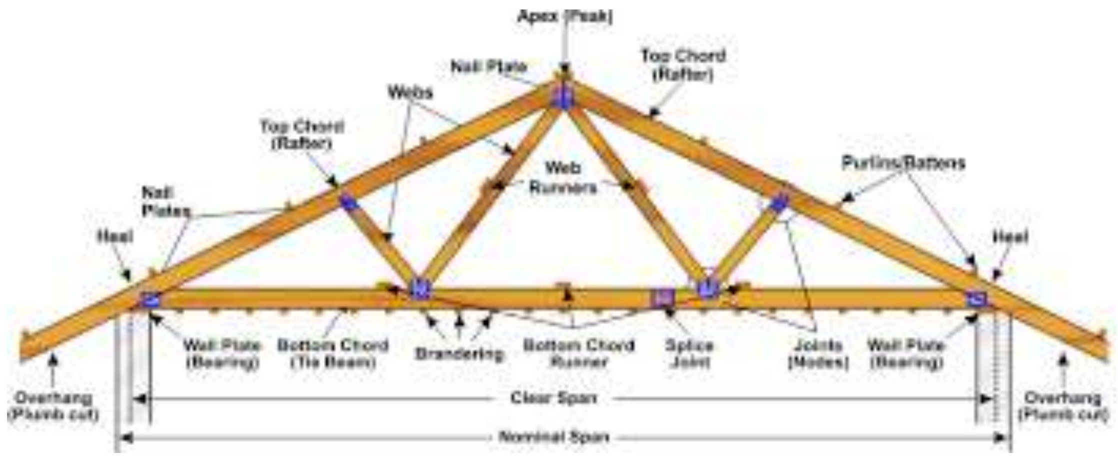
$$= \sqrt{3^2 + (0.5\times 6)^2 + 2\times 3\times(0.5\times 6)\cos 90}$$

$$= 4.242 \text{ m/s}$$

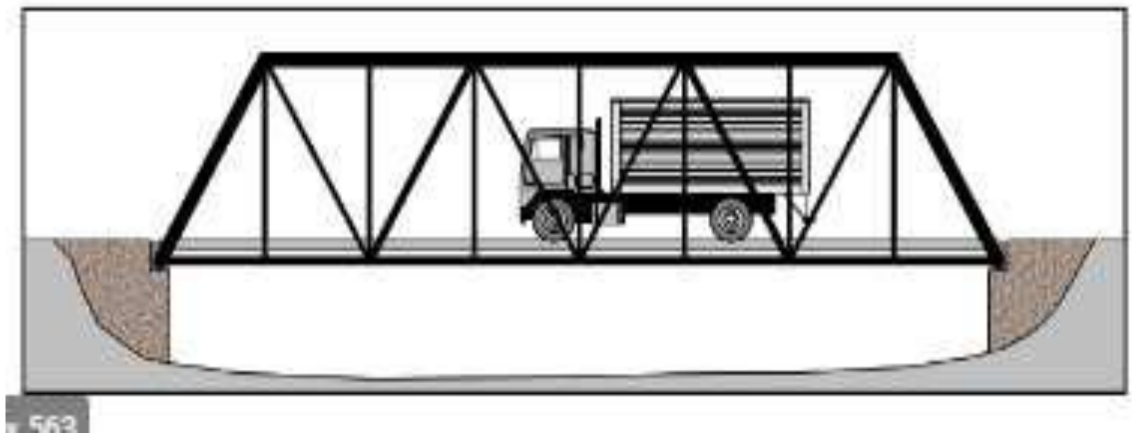
$$\alpha = \tan^{-1}\left[\left(\frac{V}{V_E}\right)\right] = \tan^{-1}\left[\frac{0.5\times 6}{3}\right] = 45^\circ$$

TRUSS

The framework/structure formed by joining several bars and is usually used for supporting roof, bridges etc are known as truss

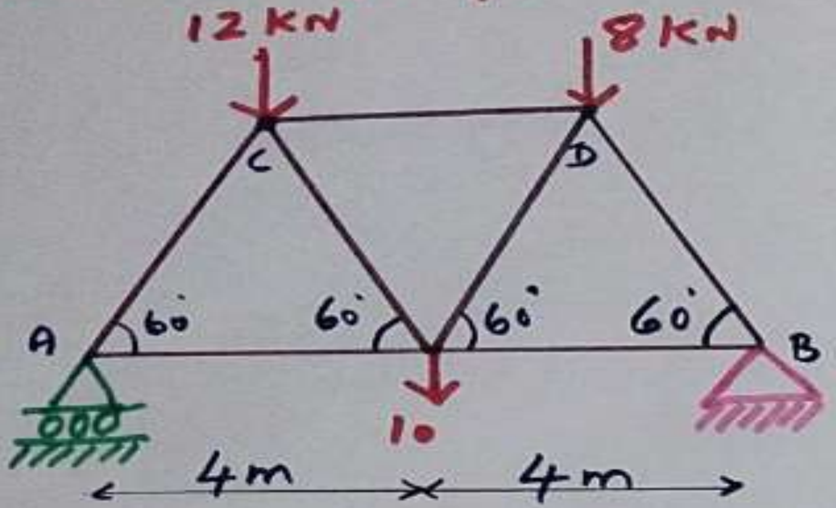


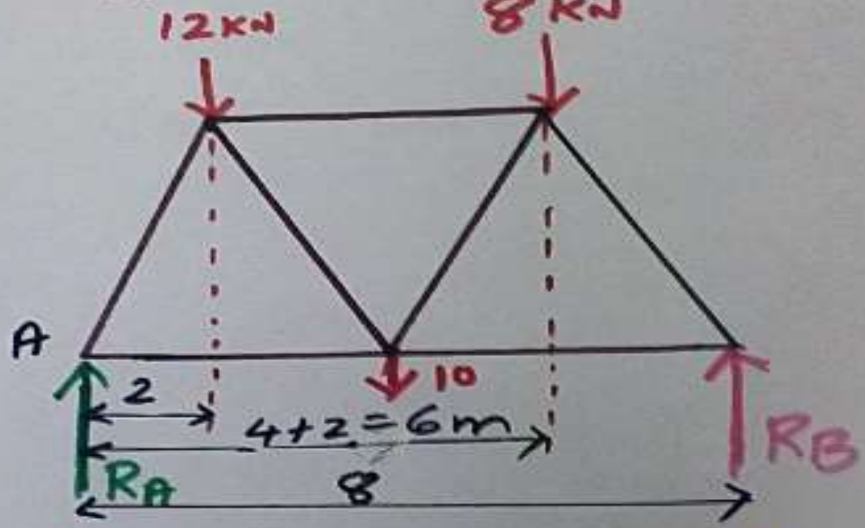
EXAMPLE 1 : ROOF TRUSS



EXAMPLE 2 : BRIDGE TRUSS

A truss of 8m span is loaded as shown in figure. Find the support reactions.





$\Sigma F_v = 0 \quad [\uparrow = \downarrow]$

$\uparrow R_A + R_B \uparrow = 12\downarrow + 8\downarrow + 10\downarrow \quad \text{.......(1)}$

$R_A + R_B = 30 \text{ kN}$

$\Sigma M_A = 0 \quad [\circlearrowleft = \circlearrowright]$

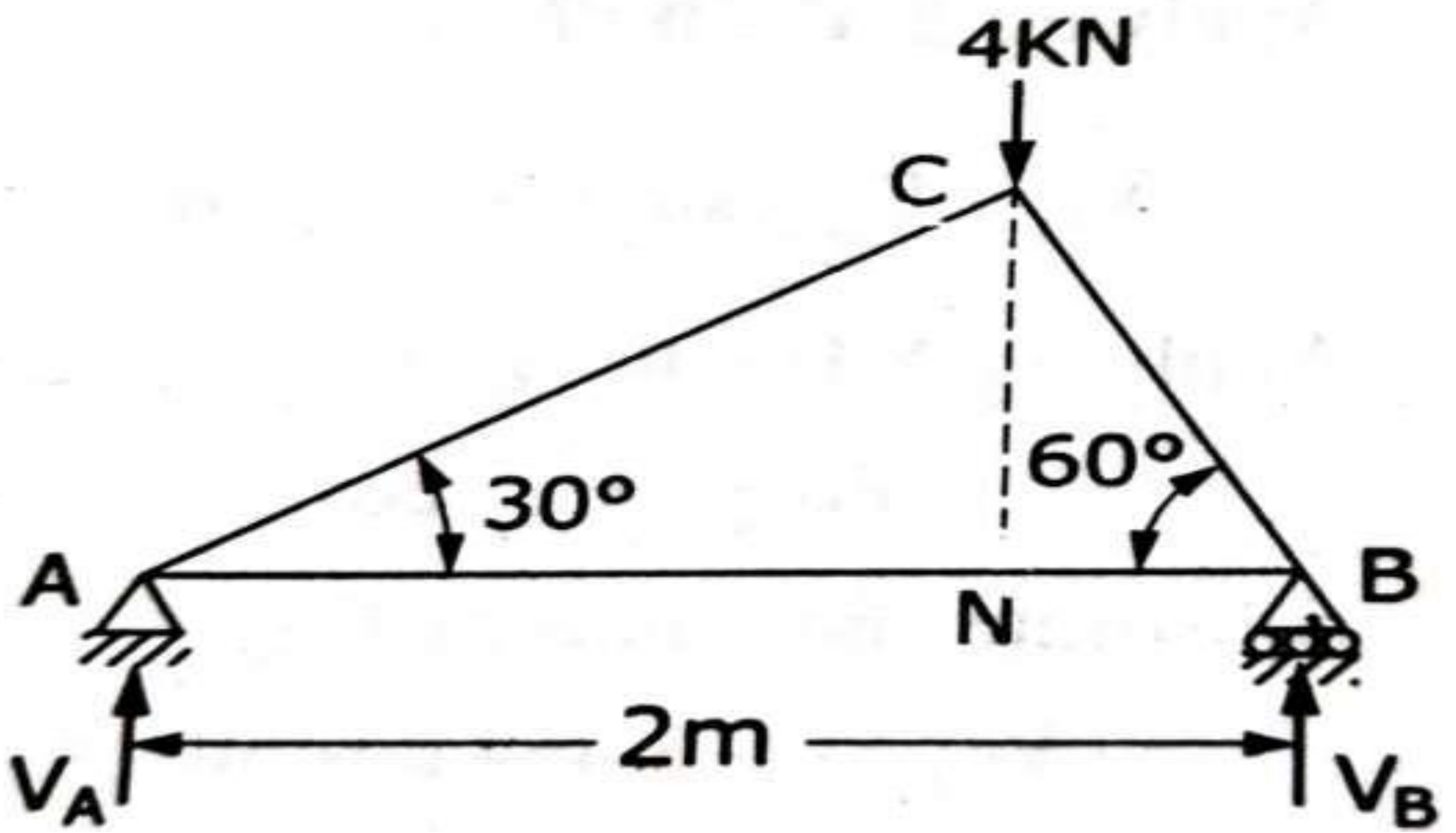
$(R_B \times 8) = (8 \times 6) + (12 \times 2) + (10 \times 4)$

$8R_B = 112$

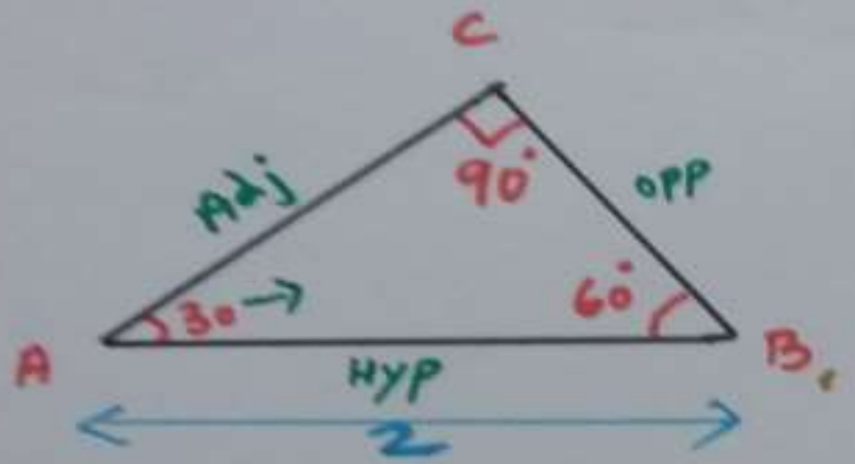
$\Rightarrow R_B = 112/8 = 14 \text{ kN}$

substituting value of $R_B = 14$ kN in (1) $\Rightarrow R_A = 30 - R_B = 30 - 14 = 16$

Apply the principles of resolution of forces and equilibrium condition to analyse and find the force transmitted to the members of truss shown below



CONSIDERING TRIANGLE ACB [right angled triangle]



$$\angle A + \angle B + \angle C = 180$$

$$\angle C = 180 - \angle A^\circ - \angle B^\circ$$

$$= 180 - 30^\circ - 60^\circ$$

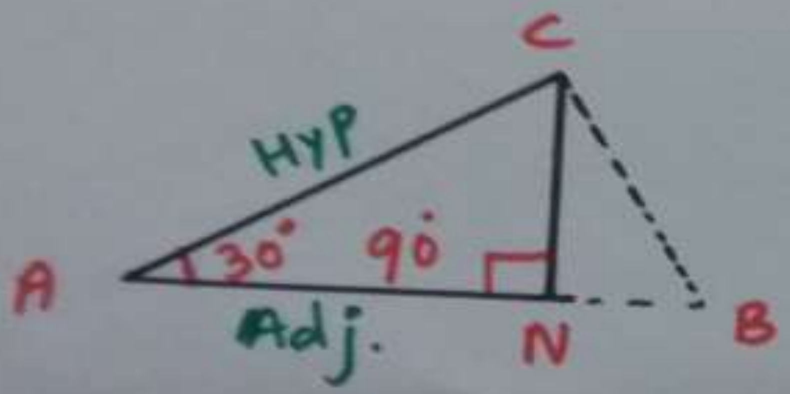
$$= 90^\circ$$ [right angled triangle]

$$\cos\theta = \frac{\text{Adj. side}}{\text{Hyp.}} = \frac{\text{adjasent side}}{\text{Hyp.}}$$

$$\cos 30 = \frac{AC}{2}$$

$$AC = 2\cos 30^\circ$$

CONSIDERING TRIANGLE ACN [right angled triangle]

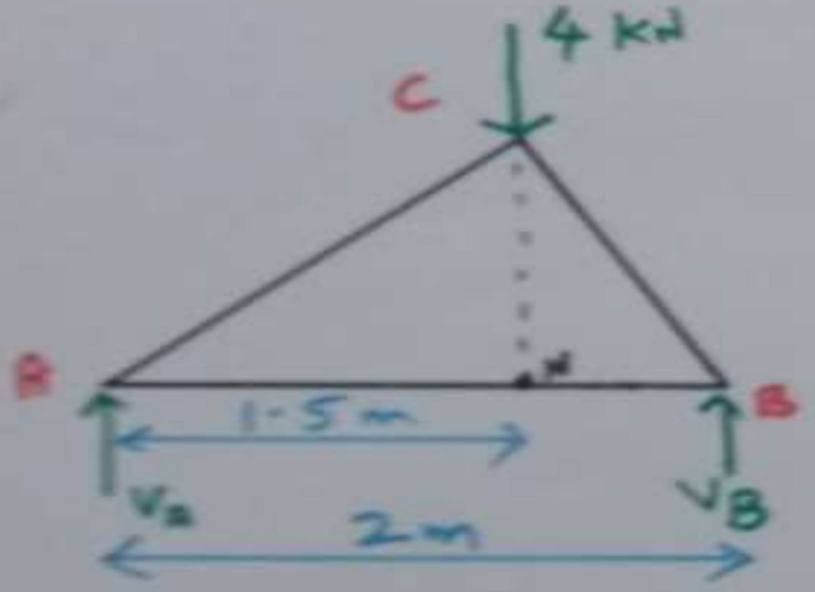


$$\cos\theta = \frac{\text{Adj. side}}{\text{Hyp.}}$$

$$\cos 30 = \frac{AN}{AC} = \frac{AN}{2\cos 30^\circ}$$

$$\Rightarrow AN = (2\cos 30^\circ)\cos 30^\circ = 1.5\,m.$$

TO FIND REACTION V_A & V_B



Equating the upward & downward forces

$$V_A \uparrow + V_B \uparrow = 4 \downarrow$$

$$V_A + V_B = 4 \quad \text{-----} \quad (1)$$

Taking moment from (A) and Equating the anticlockwise and clockwise moments.

$$(V_B \times 2) = (4 \times 1.5)$$

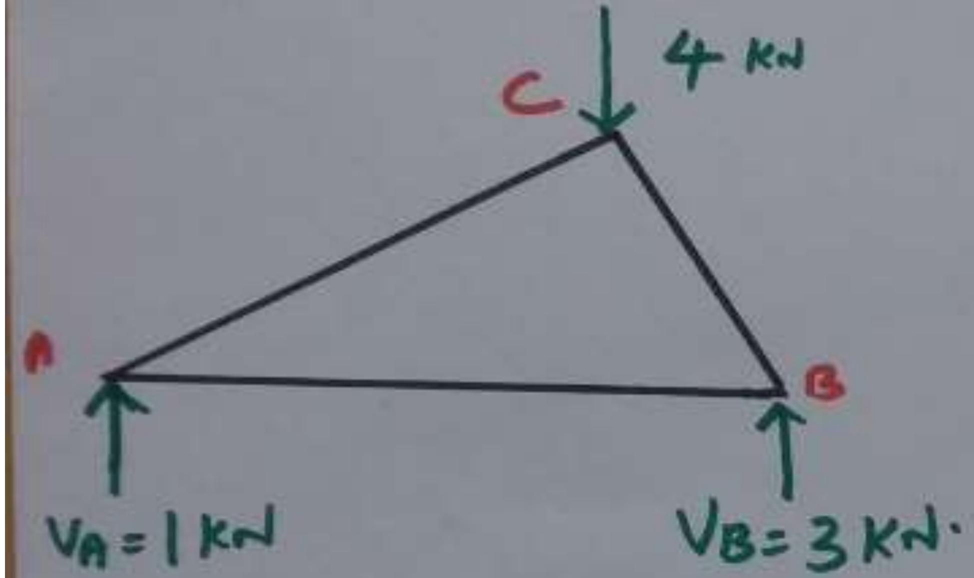
$$\Rightarrow V_B = \frac{4 \times 1.5}{2} = \underline{3 \text{ kN}} \quad \text{--} \quad (2)$$

(2) in (1)

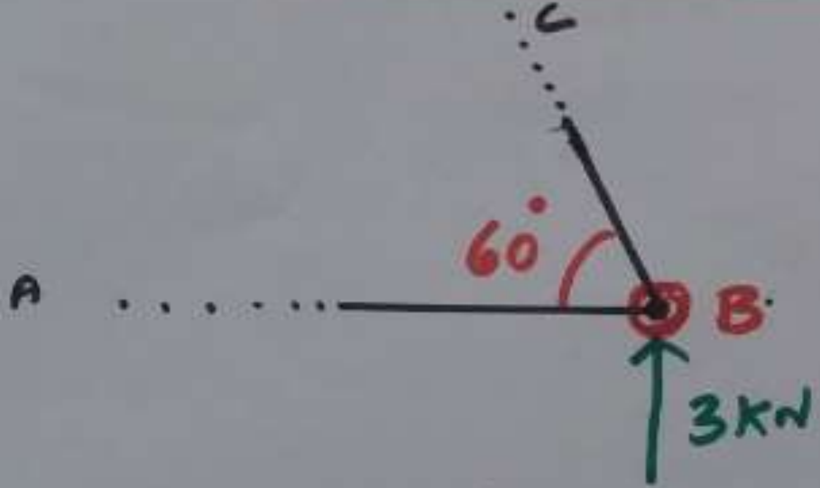
$$V_A + V_B = 4$$

$$V_A = 4 - V_B = 4 - 3 = 1 \text{ kN}$$

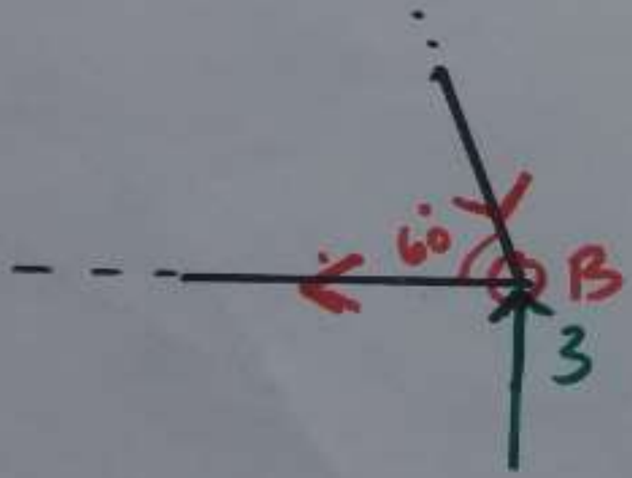
$$V_A = 1 \text{ kN}$$

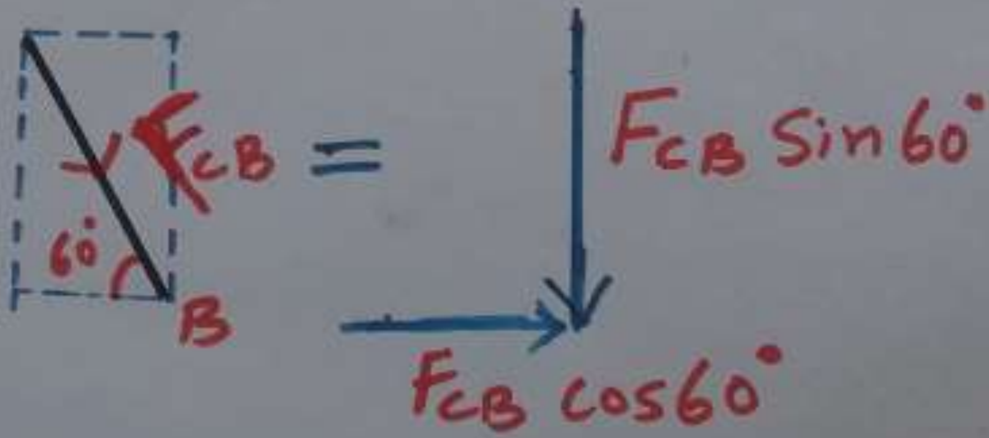


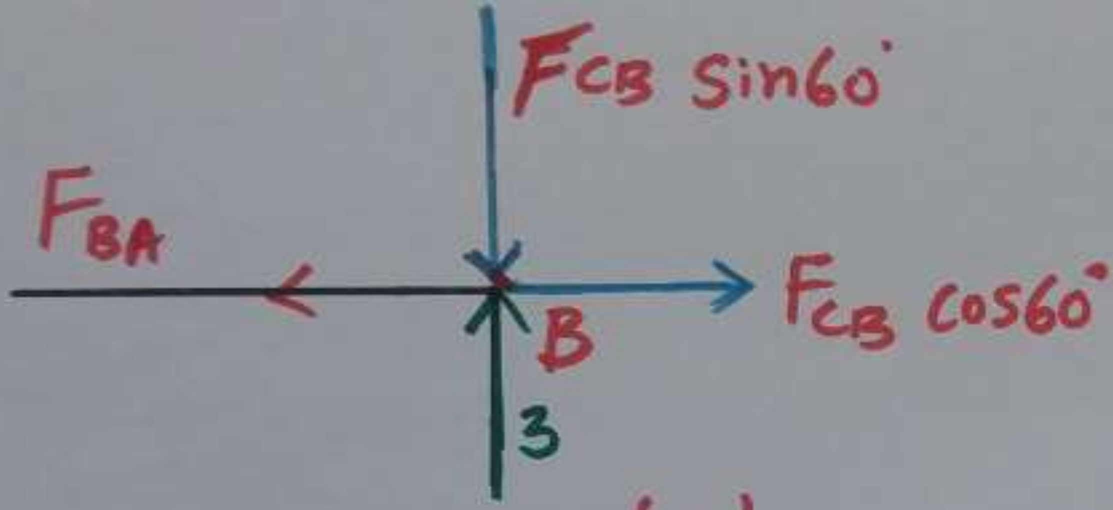
CONSIDERING EQUILIBRIUM OF JOINT 'B'



Assuming direction of forces logically







At. JOINT 'B'

Equating the upward forces and downward force $[\Sigma F_V = 0]$

$$3\uparrow \; = \; F_{CB}\sin 60^\circ \downarrow$$

$$\Rightarrow F_{CB} = \frac{3}{\sin 60^\circ} = 3.464 \text{ kN}$$

Equating the righthand side force to left side force $(\Sigma F_H = 0)$

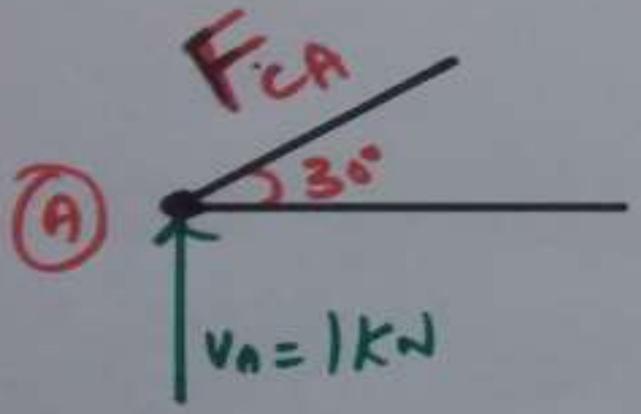
$$\overrightarrow{F_{CB}\cos 60} = \overleftarrow{F_{BA}}$$

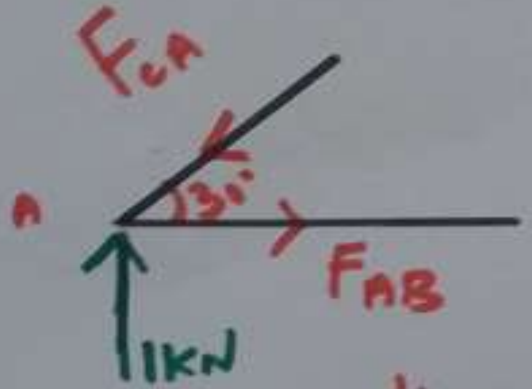
$$3.464\cos 60 = \overleftarrow{F_{BA}}$$

$$1.732 = F_{BA}$$

$$F_{BA} = 1.732 = F_{AB}$$

CONSIDERING EQUILIBRIUM OF JOINT A





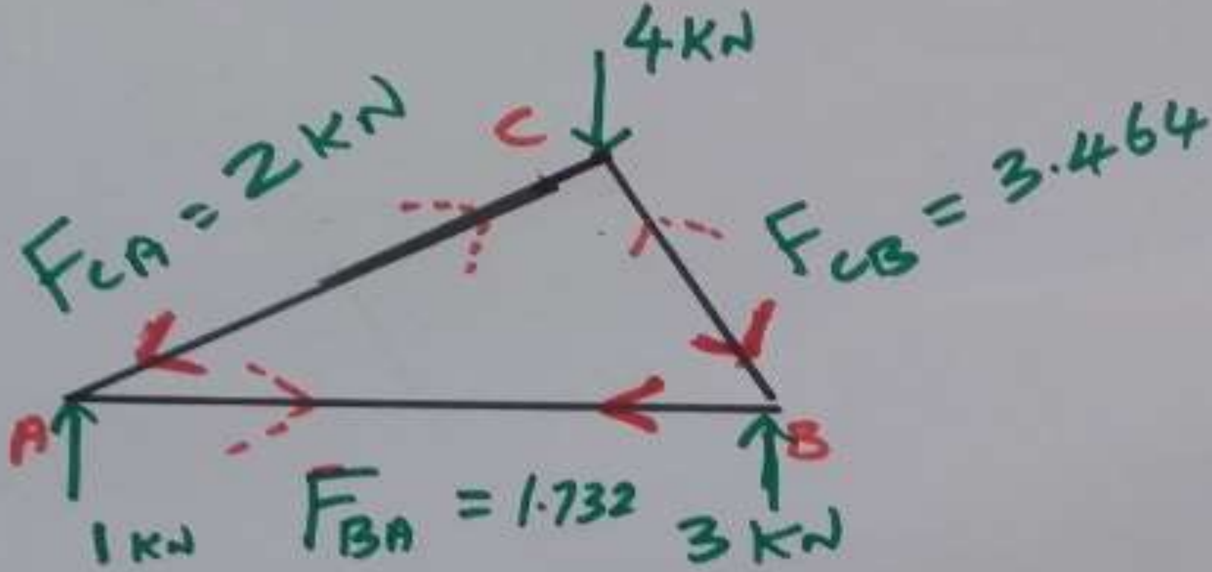
Assuming direction of Forces in members AB & CA

Equating the ~~horizontal~~ Vertical forces

$$\downarrow F_{CA}\sin 30 = 1\,kN$$

$$\Rightarrow F_{CA} = \frac{1}{\sin 30°} = 2\,kN$$

ANSWERS



S.N	MEMBER	Forces in KN	Nature
1	AB	$F_{BA} = F_{AB} = 1.732$	Tensile
2	BC	$F_{CB} = F_{BC} = 3.464$	compressive
3	CA	$F_{CA} = F_{AC} = 2$	compressive

THANK YOU STUDENTS.

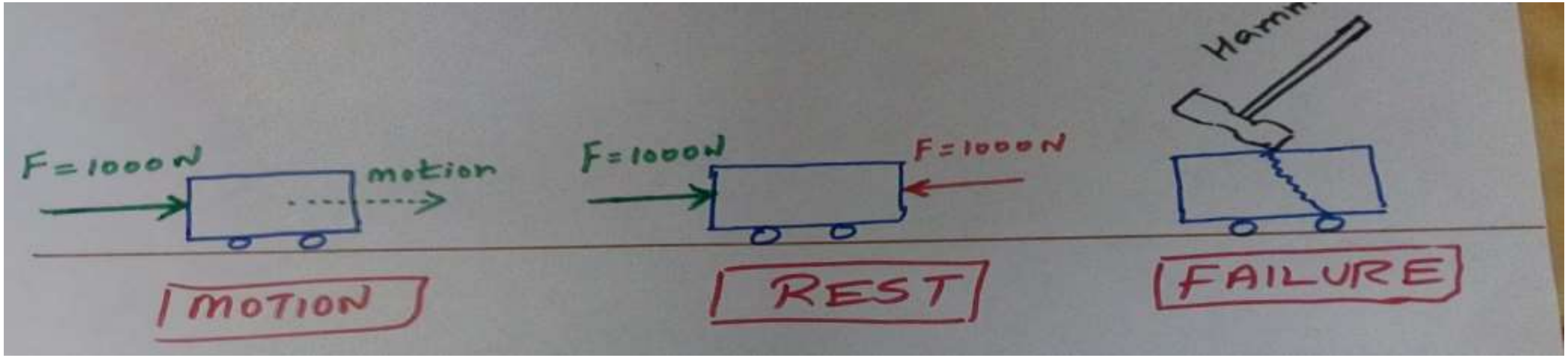
ENGINEERING MECHANICS CLASS NOTES FOR STATICS OF PARTICLES CHAPTER

BY

V.JOSE ANANTH VINO

ASSISTANT PROFESSOR/B.I.H.E.R

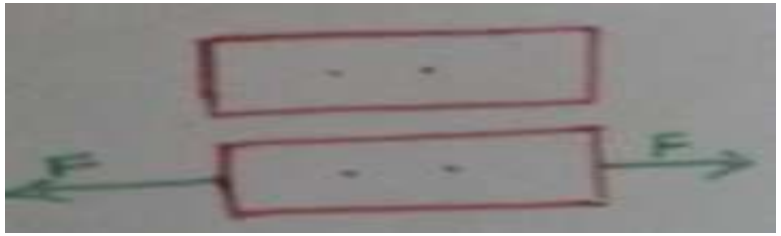
MECHANICS



THE BRANCH OF SCIENCE DEALING WITH FORCES AND EFFECT OF FORCES ON OBJECTS/BODIES IS KNOWN AS MECHANICS

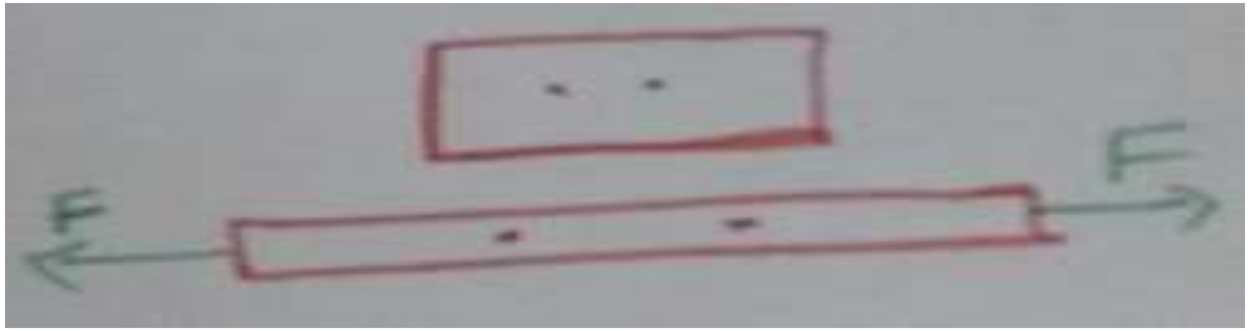
THE EFFECT OF FORCE ON A BODY MAY CAUSE MOTION OF BODY OR FAILURE OF BODY. THE SYSTEM OR GROUP OF FORCES MAY ALSO KEEP A BODY IN STATIC STATE.

RIGID BODY:



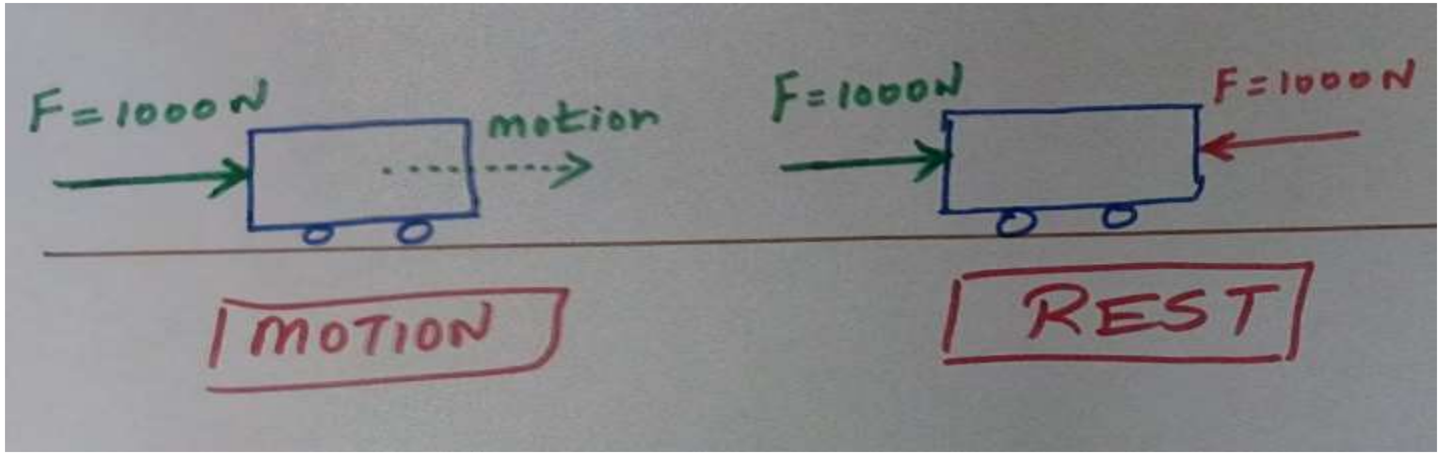
The solid body which does not undergo any deformations(no change in shape) under the application of forces is known as a rigid body . The distance between any two given points on a rigid body remains constant before and after applying force in rigid body.

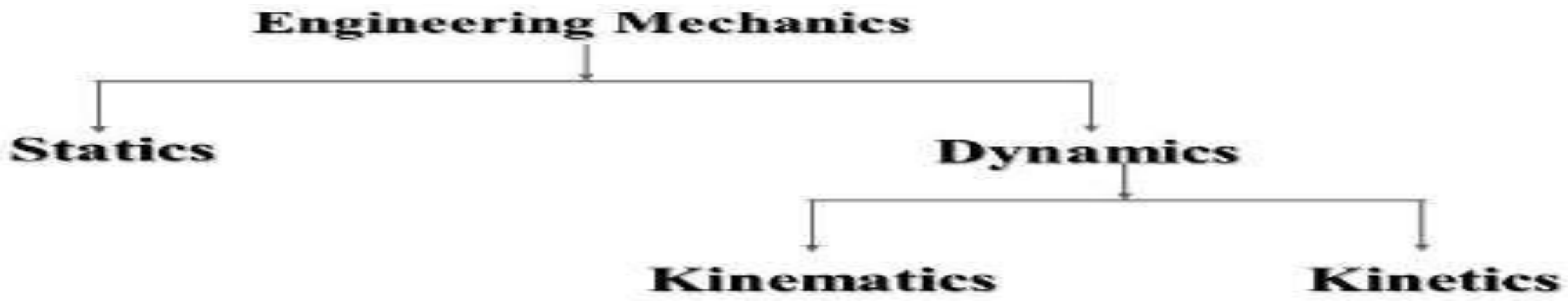
DEFORMABLE BODY



The solid body which deform(change in shape) under the application of force is known deformable body. The distance between any two given points on a deformable body changes before and after applying force.

ENGINEERING MECHANICS: The branch of mechanics dealing with forces and effect of forces on rigid bodies in causing motion state or static state for Engineering applications is known as Engineering mechanics





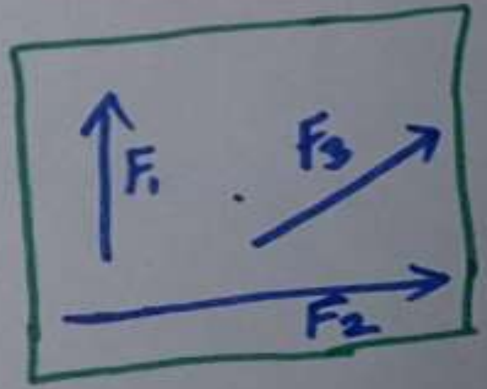
STATICS: The branch of Engineering Mechanics which deals with rigid bodies at rest during the system of forces acting on it is known as statics

DYNAMICS: The branch of Engineering Mechanics which deals with rigid bodies in motion during the system of forces acting on it is known as dynamics.

KINEMATICS: The branch of dynamics which deals with motion characteristics such as velocity, acceleration, distance travelled etc **without** considering the forces which cause motion is known as kinematics

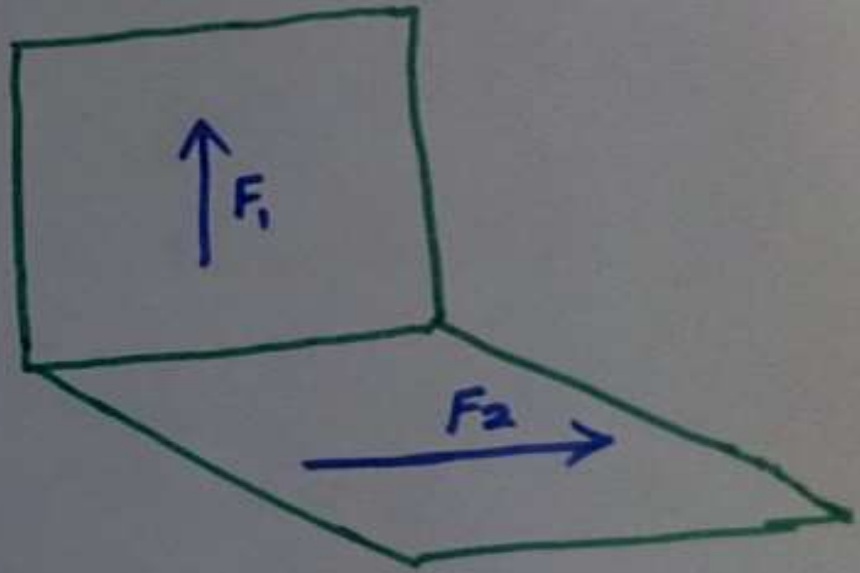
KINETICS: The branch of dynamics which deals with motion of rigid body by considering the forces causing motion is known kinetics. Kinetics relates force with motion like F= m a.

COPLANAR FORCES



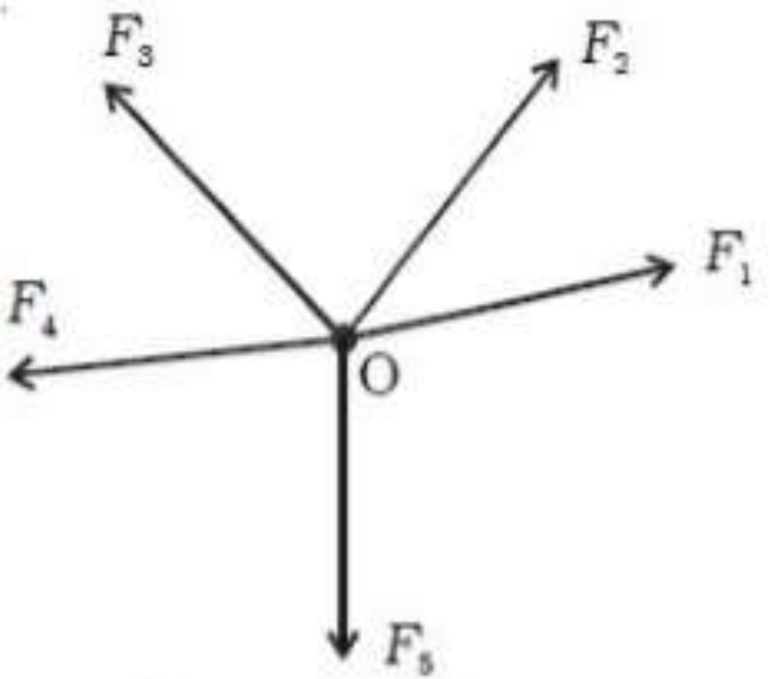
The system of forces which are lying in same plane is known as coplanar force system

NON COPLANAR FORCES

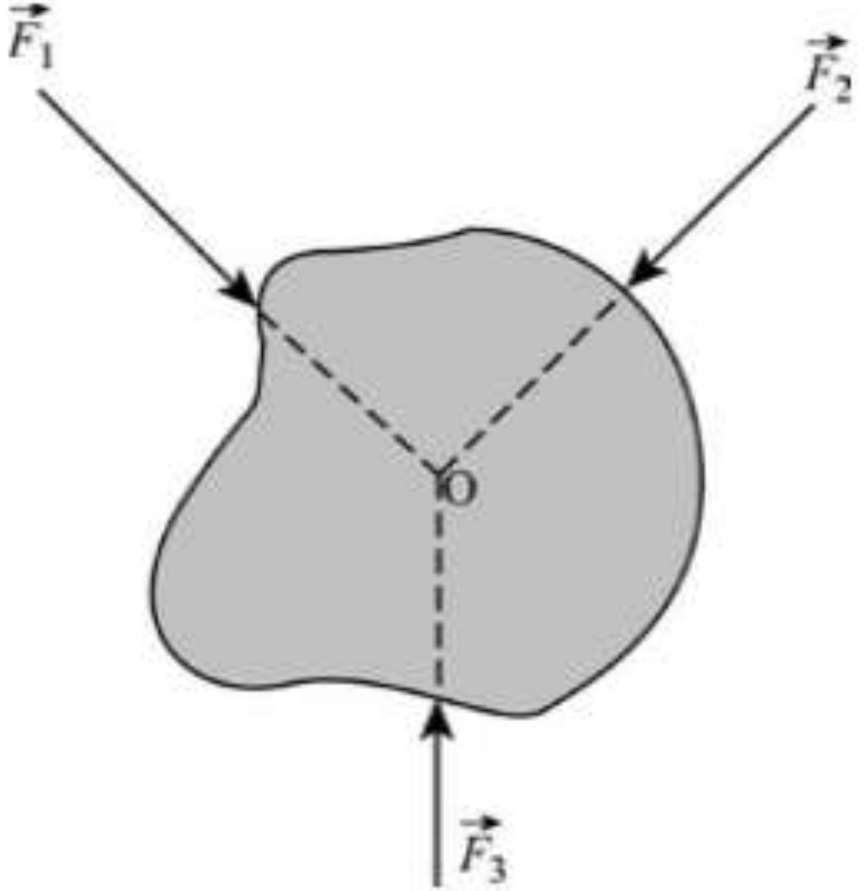


The system of forces which are lying in different plane is known as non coplaner force system.

CONCURRENT FORCES SYSTEM



Concurrent forces

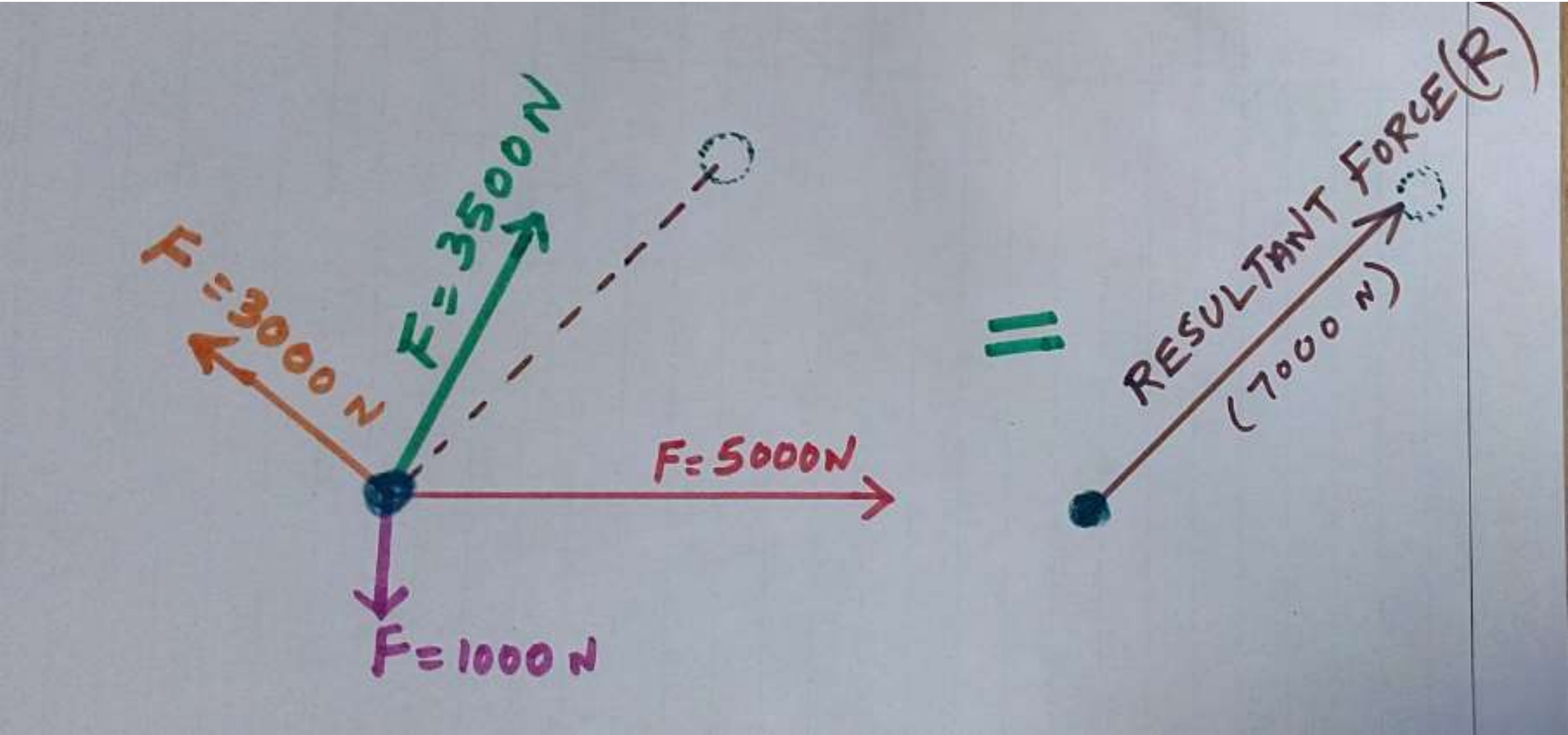


**E SYSTEM OF FORCES WHICH ORIGINATE FROM A COMMON
INT OR THE SYSTEM OF FORCES WHICH INTERSECTS AT
MMON POINT ARE KNOWN AS CONCURRENT FORCES SYSTEM**

RESULTANT FORCE

A SINGLE FORCE WHICH CAN PRODUCE SAME EFFORT AS THAT OF NET COMBINED EFFORT PRODUCED BY GROUP OF FORCES IS KNOWN AS RESULTANT FORCE

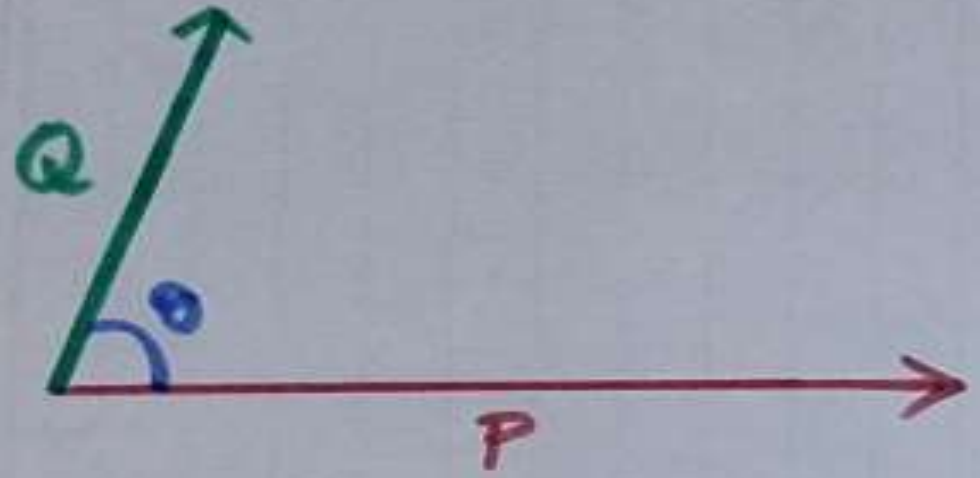
EXAMPLE FOR RESULTANT FORCE

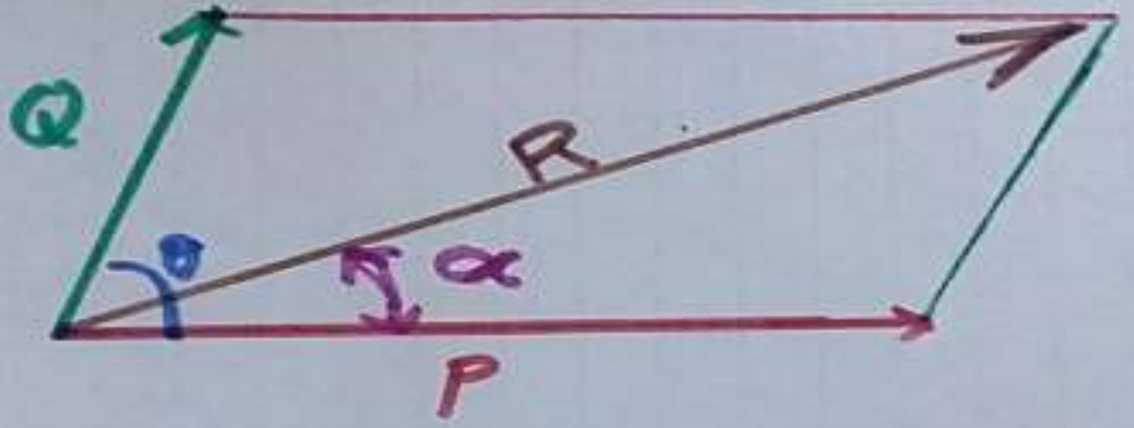


PARALLELOGRAM LAW OF FORCES

- If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram, their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection."

DIAGRAMATIC REPRESENTATIONS OF PARALLELOGRAM LAW OF FORCES





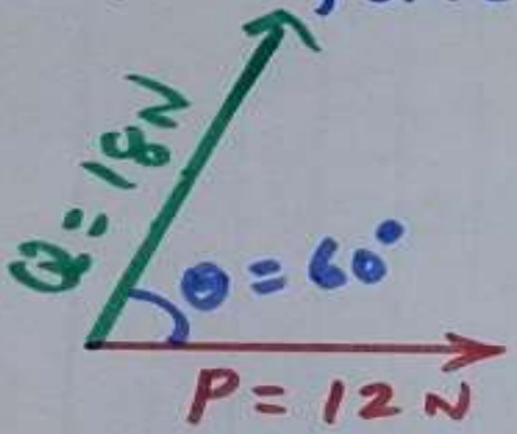
ANALTICAL FORMULAE FOR RESULTANT OF TWO FORCE (ARRIVED USING PARALLELOGRAM LAW OF FORCES)

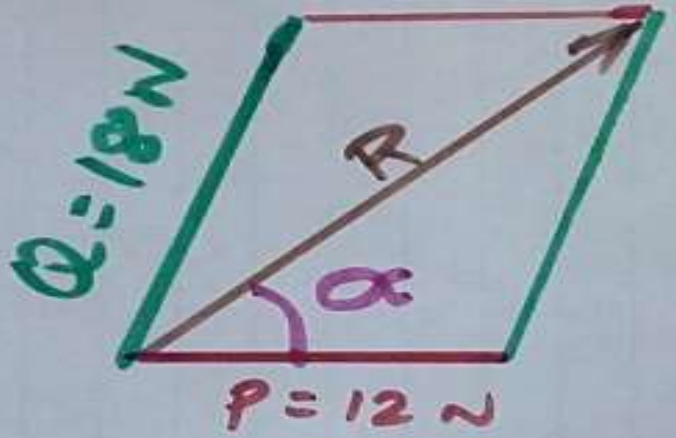
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

ANGLE MADE BY RESULTANT FORCE WITH FORCE P

$$\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right]$$

Two concurrent forces 12 N and 18 N are acting at angle of 60°. Find the resultant force. Also find the angle made by resultant force with 12 N force.





GIVEN

$P = 12\,N$

$Q = 18\,N$

$\theta = 60°$

FIND

$R = ?$

$\alpha = ?$

SOLUTION

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{12^2 + 18^2 + 2 \times 12 \times 18 \cos 60}$$

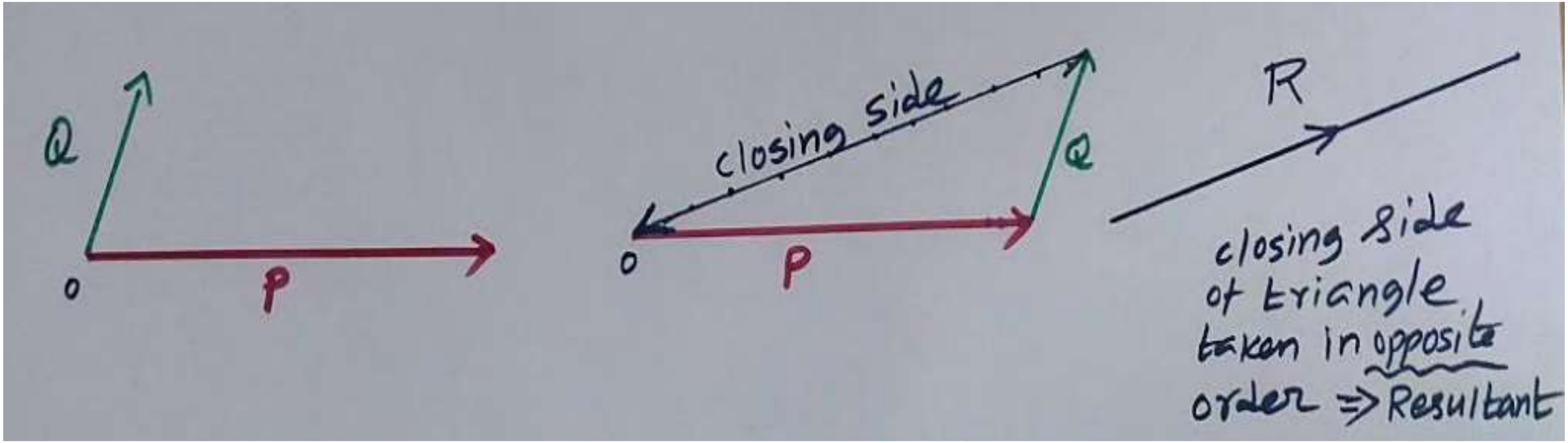
$$= 26.15\,N$$

$$\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right] = \tan^{-1}\left[\frac{18\sin 60}{12 + 18\cos 60}\right]$$

$$= 36°34'$$

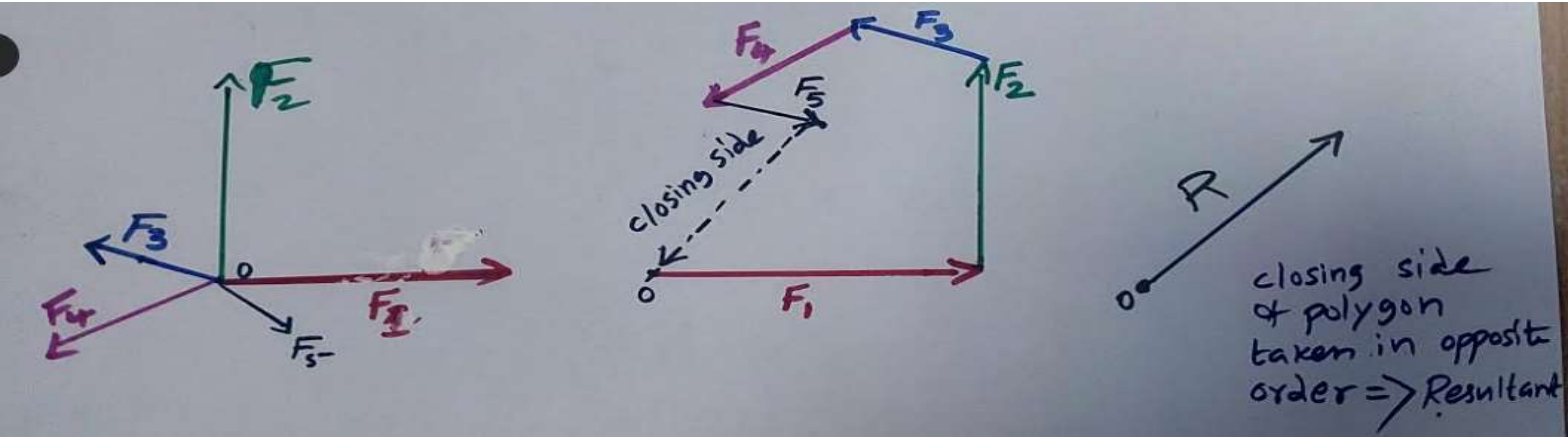
TRIANGULAR LAW OF FORCES:

If two forces acting at a point are represented in magnitude and direction by the two adjacent sides of a triangle taken in order, then the closing side of the triangle taken in the opposite order represents the resultant of the forces in magnitude and direction

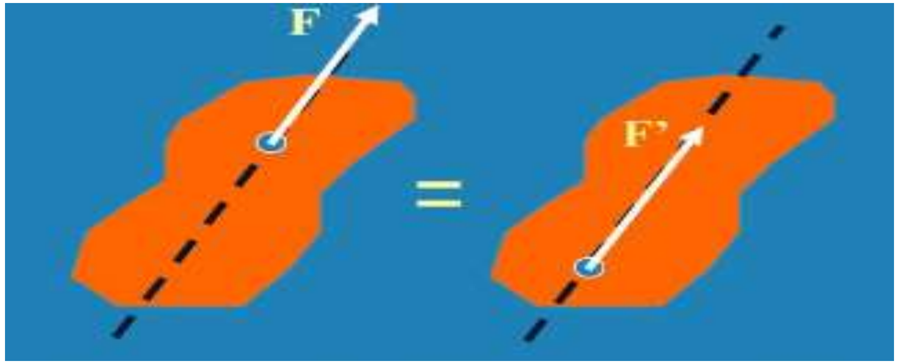


POLYGON LAW OF FORCES

The Law of Polygon of Forces states that – if any number of coplanar concurrent forces can be represented in magnitude and direction by the sides of a polygon taken in order; then their resultant will be represented by the closing side of the polygon taken in opposite order".



PRINCIPLE OF TRANSMISSIBILITY OF FORCES

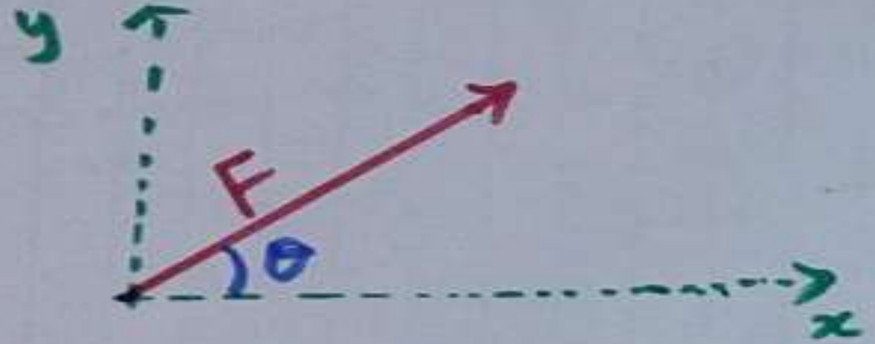


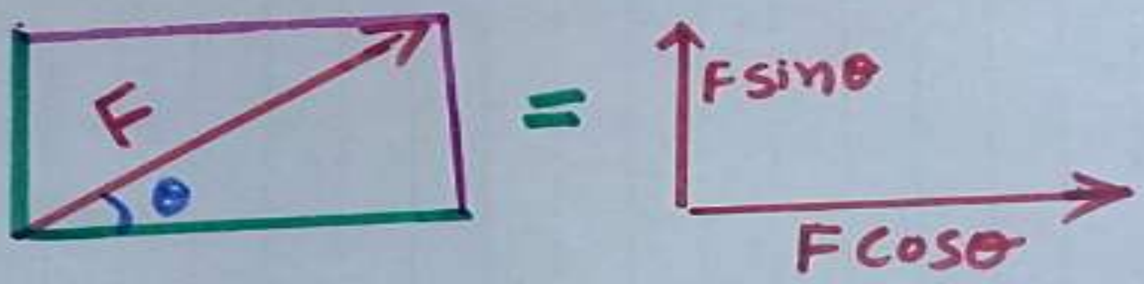
THE PRINCIPLE OF TRANSMISSIBILITY OF FORCES STATES THAT THE EFFECT OF FORCE REMAIN SAME EVENTHOUGH THE POINT OF APPLICATION OF FORCE IS CHANGED ALONG THE SAME LINE OF ACTION OF FORCE

RESOLUTION OF FORCES

It is defined as the process of splitting up the given force into a number of components , without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions.

PRINCIPLE OF RESOLUTION OF FORCES





EXAMPLE:- Resolve the force = 1000 N acting at an angle 60° with 'x' axis into two component force along 'x' and 'y' axis.

SOLUTION

$F = 1000\,N$

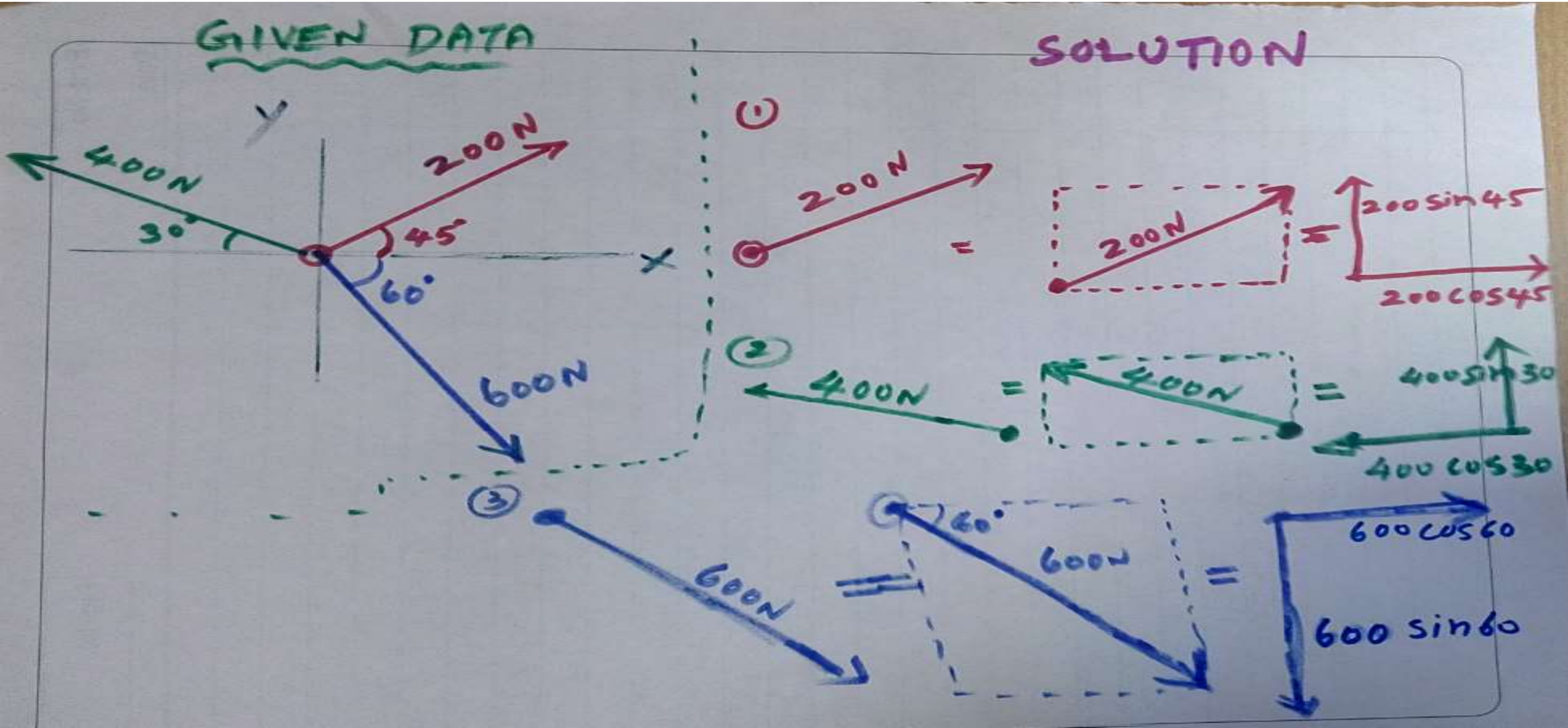
$\theta = 60^\circ$

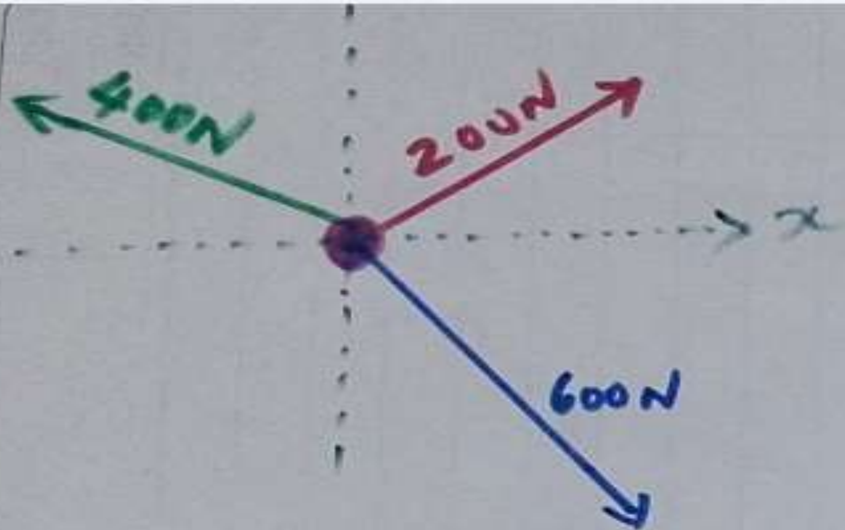
=

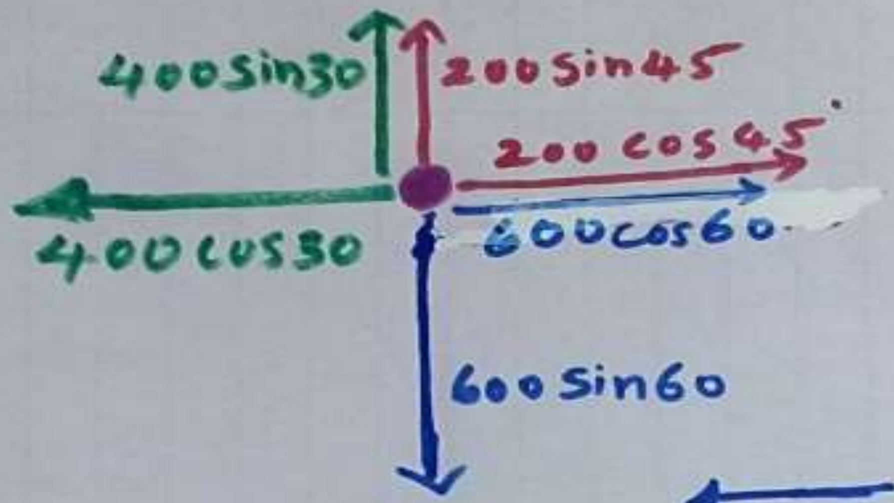
$F\sin\theta = 1000 \sin 60 = 866\ N$

$F\cos\theta = 1000 \cos 60 = 500N$

Three coplanar concurrent forces are acting at a point as shown in figure below .Find the magnitude and direction of resultant force







Algebraic sum of forces in horizontal 'x' direction $\Big\} = \Sigma F_H = 200\cos 45 - 400\cos 30 + 600\cos 60$

$= 95\ N$

Algebraic sum of forces in vertical direction (y axis) $\Big\} = \Sigma F_V = 200\sin 45 + 400\sin 30 - 600\sin 60$

$= -178.2\ N$

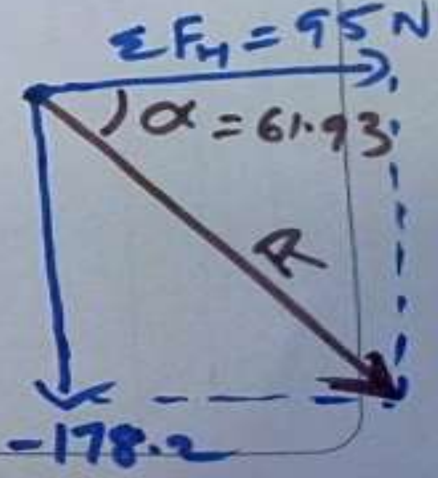
Resultant force $R = \sqrt{\Sigma F_H^2 + \Sigma F_V^2}$

$= \sqrt{95^2 + (-178.2)^2}$

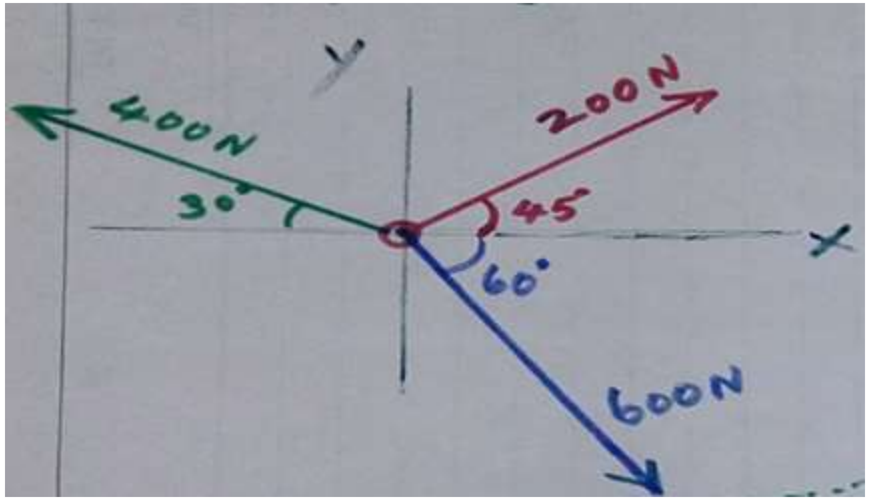
$= 201.95\ N$

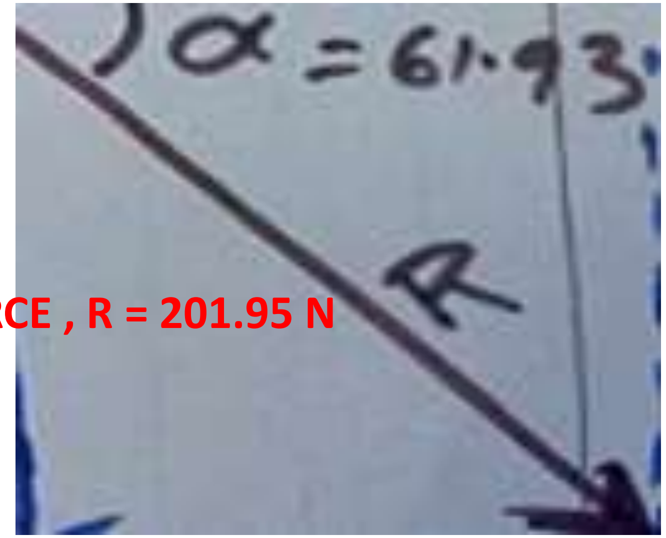
Inclination of Resultant force $\Big\}\ \alpha = \tan^{-1}\left(\dfrac{\Sigma F_V}{\Sigma F_H}\right) = \tan^{-1}\left[\dfrac{-178.2}{95}\right]$

$= 61.93^\circ$ with x axis



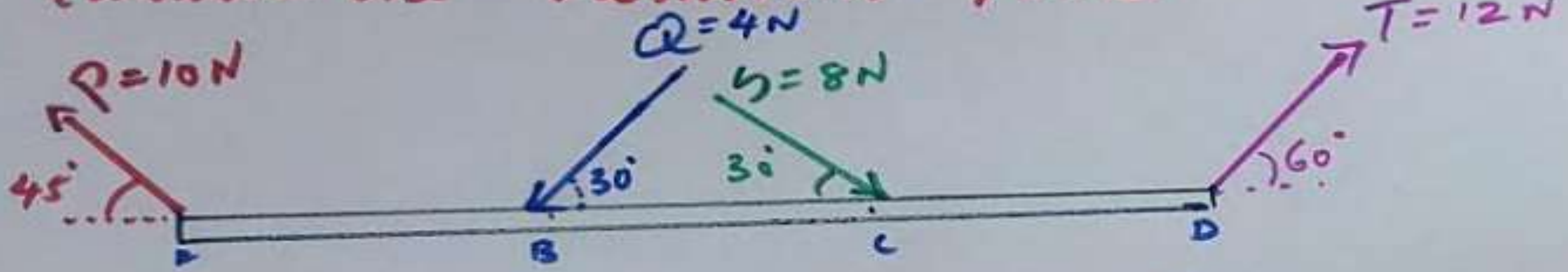
GIVEN FORCES AND RESULTANT FORCE

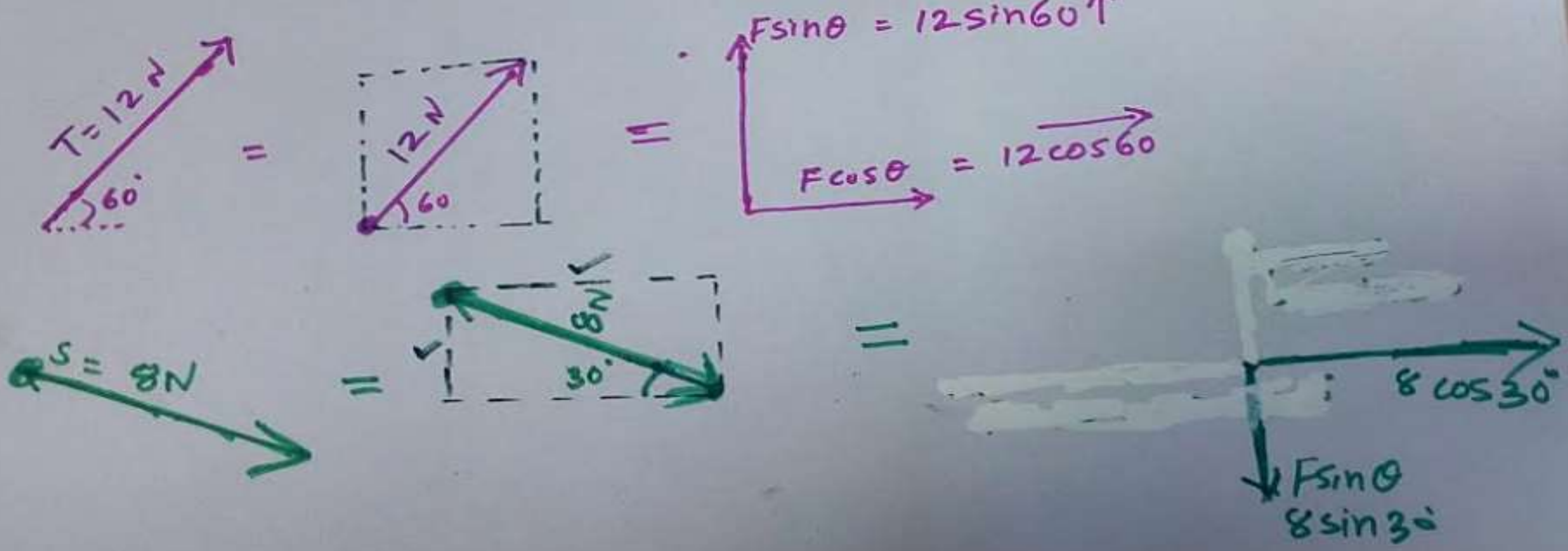


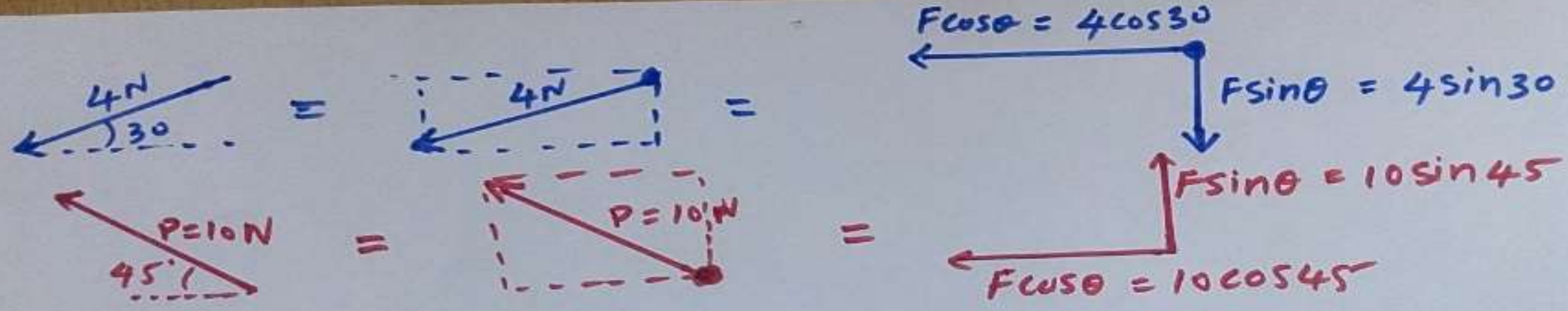


RESULTANT FORCE , R = 201.95 N

ABCD is a weight-less rod under the action of four forces P, Q, S, T as shown in figure. If $P = 10N$, $Q = 4N$, $S = 8N$, $T = 12N$, calculate the resultant force





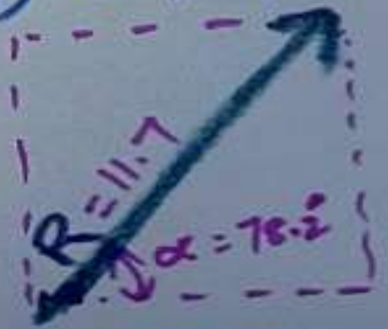


$$\Sigma F_H = \overrightarrow{12\cos 60} + \overrightarrow{8\cos 30} - \overleftarrow{4\cos 30} - \overleftarrow{10\cos 45} = 2.393\text{ N}$$

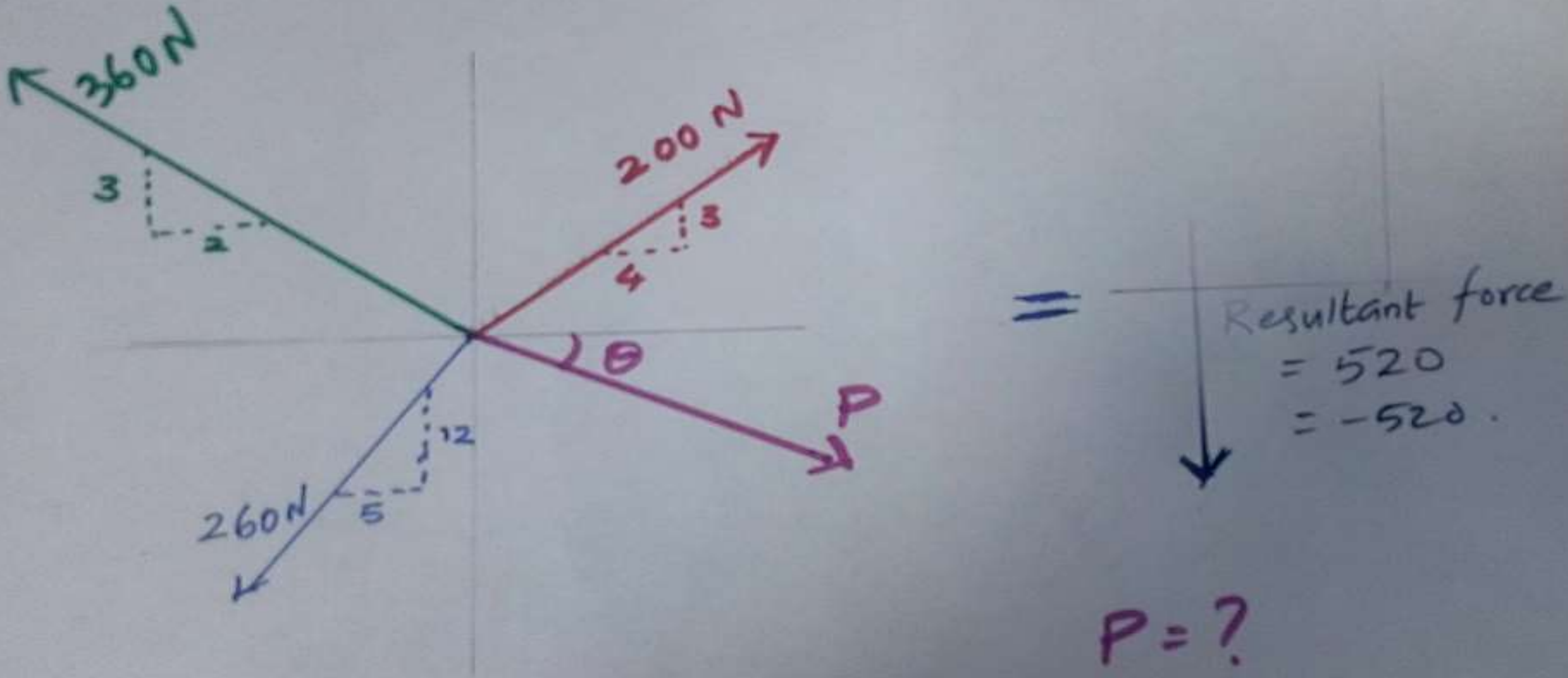
$$\Sigma F_V = 12\sin 60\uparrow - 8\sin 30\downarrow - 4\sin 30\downarrow + 10\sin 45\uparrow = 11.46\text{ N}$$

$$R = \sqrt{(\Sigma F_H)^2 + (\Sigma F_V)^2} = \sqrt{(2.39)^2 + (11.46)^2} = 11.7\text{ N}$$

$$\alpha = \tan^{-1}\left(\frac{\Sigma F_V}{\Sigma F_H}\right) = \tan^{-1}\left(\frac{11.46}{2.39}\right) = 78.2^\circ$$



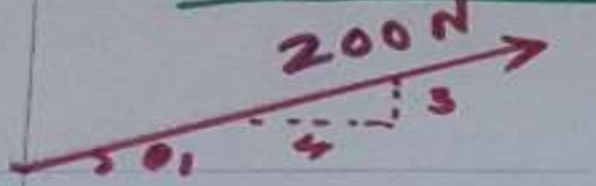
The resultant of force system shown below is 520 N acting along the negative direction of y-axis. Determine P and θ

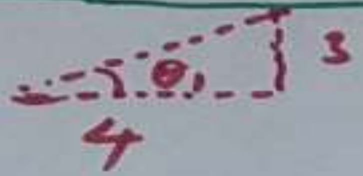


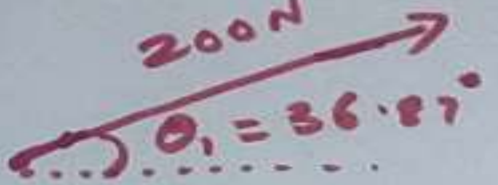
$P = ?$

$\theta = ?$

RESOLUTION OF INCLINED 200 N into components.





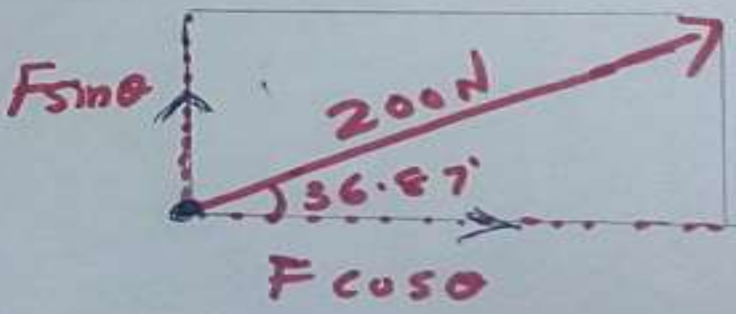


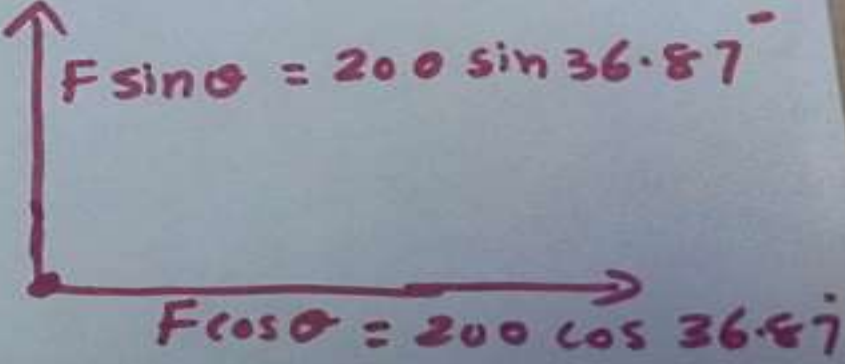
$$\tan\theta_1 = \frac{opp}{Adj}$$

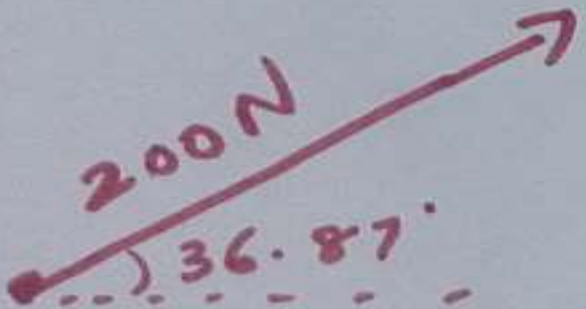
$$\tan\theta_1 = 3/4$$

$$\theta_1 = \tan^{-1} 3/4$$

$$\theta_1 = 36.87^\circ$$







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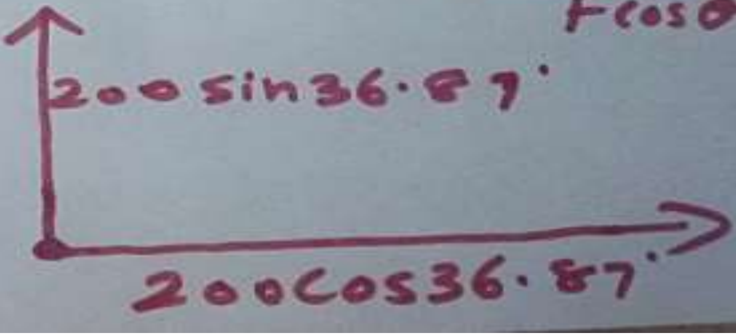
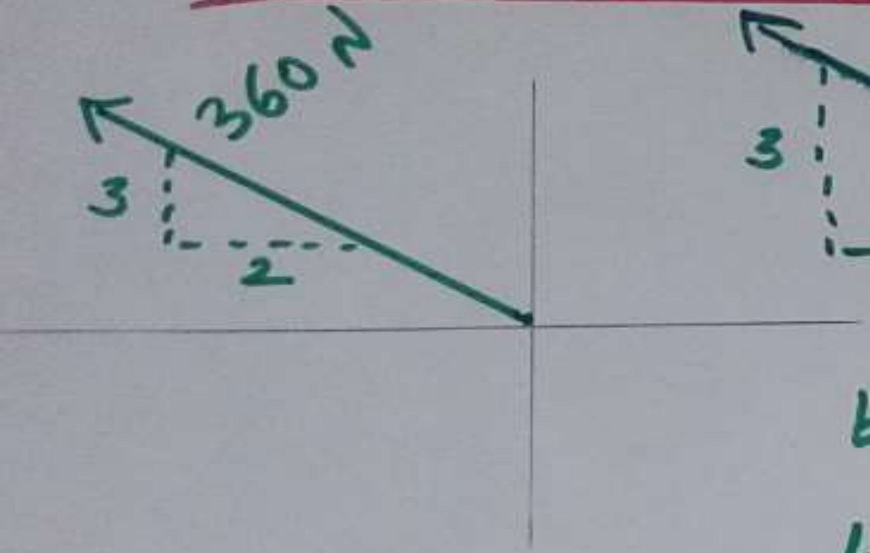
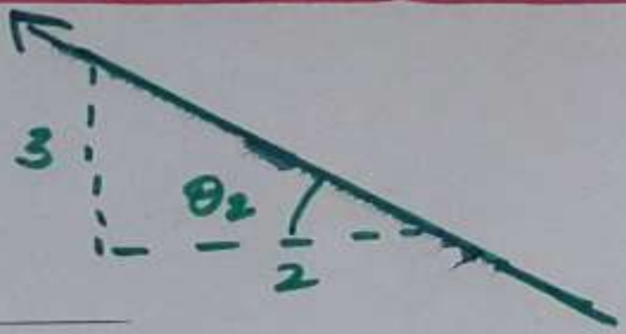
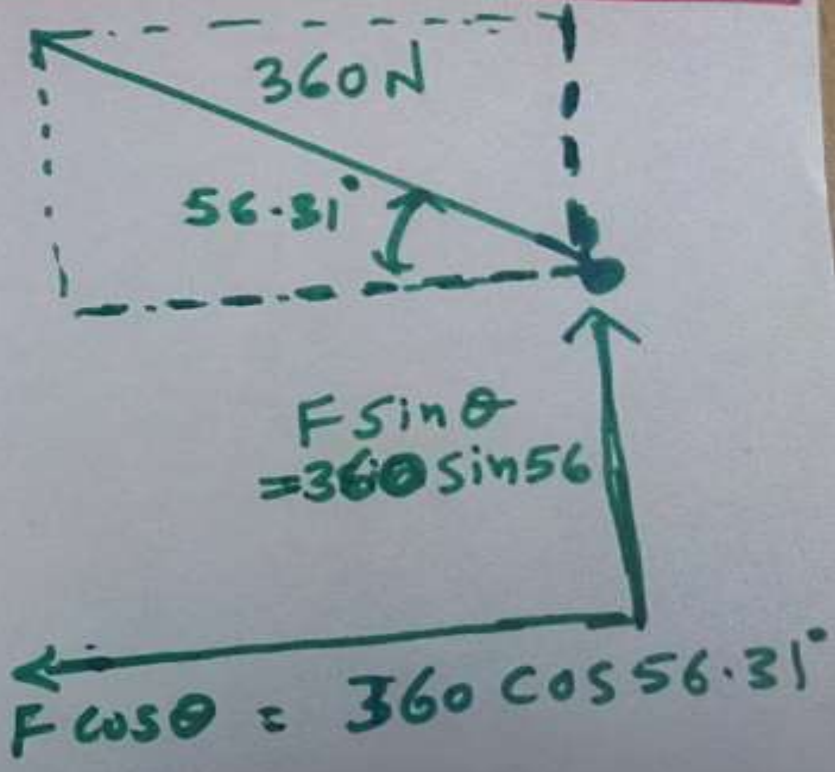


RESOLUTION OF INCLINED 360N INTO COMPONENT FORCES





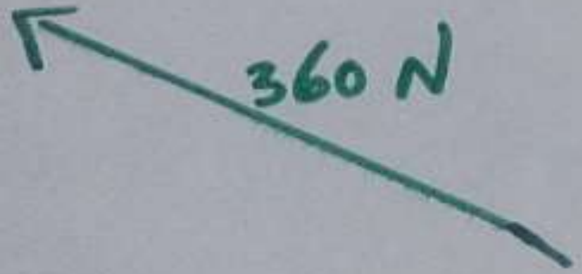


$$\tan\theta_2 = \frac{\text{opp}}{\text{Adj}}$$

$$\tan\theta_2 = \frac{3}{2}$$

$$\theta_2 = \tan^{-1}(3/2)$$

$$\theta_2 = 56.31^\circ$$



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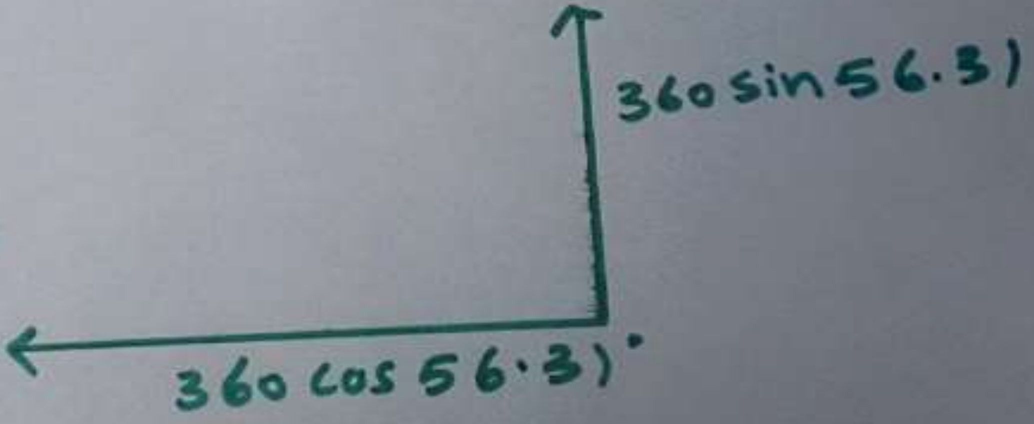
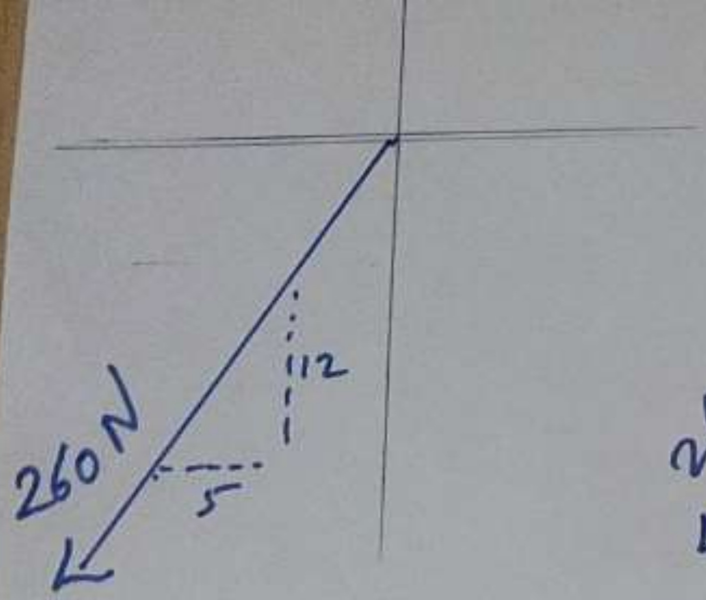
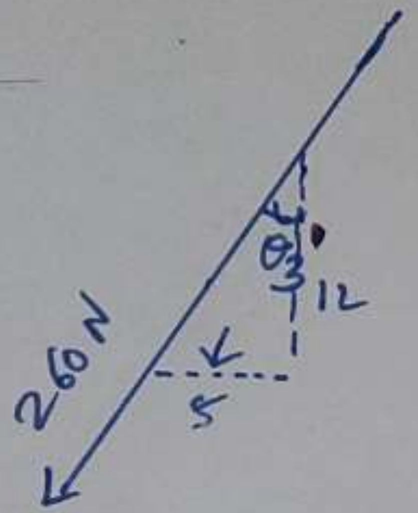






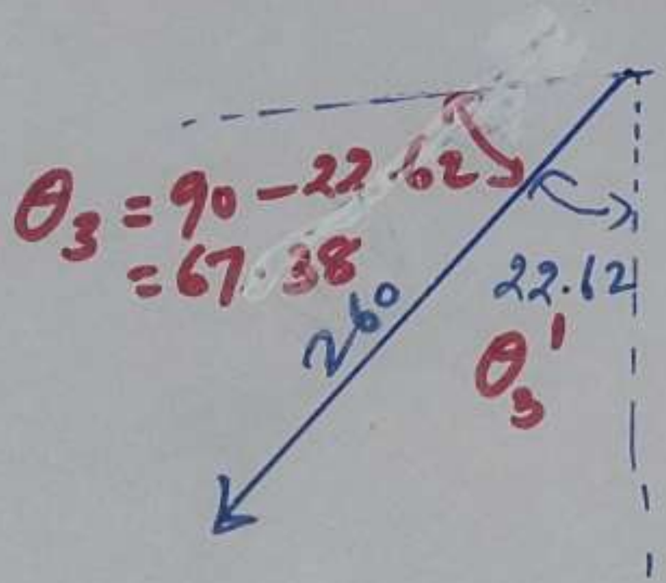
$$\tan\theta_3' = \frac{\text{opp}}{\text{Adj side}}$$

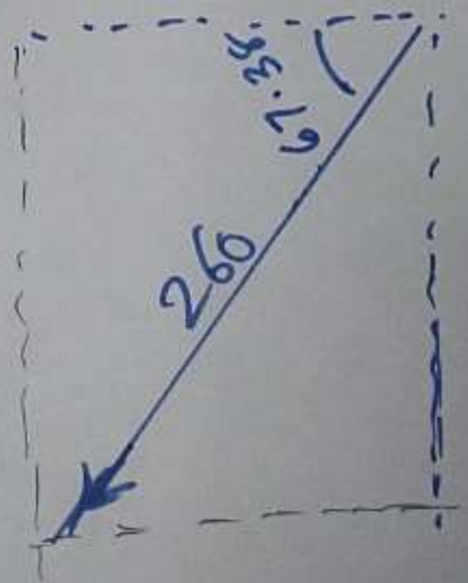
$$= \frac{5}{12}$$

$$\theta_3' = \tan^{-1}\left(\frac{5}{12}\right)$$

$$= \cancel{22.62}^{\circ}$$

$$= 22.62^{\circ}$$





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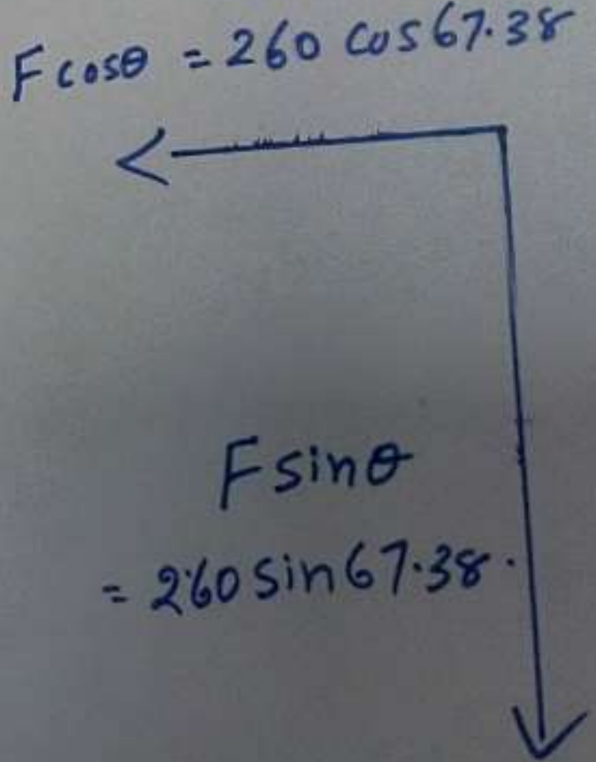
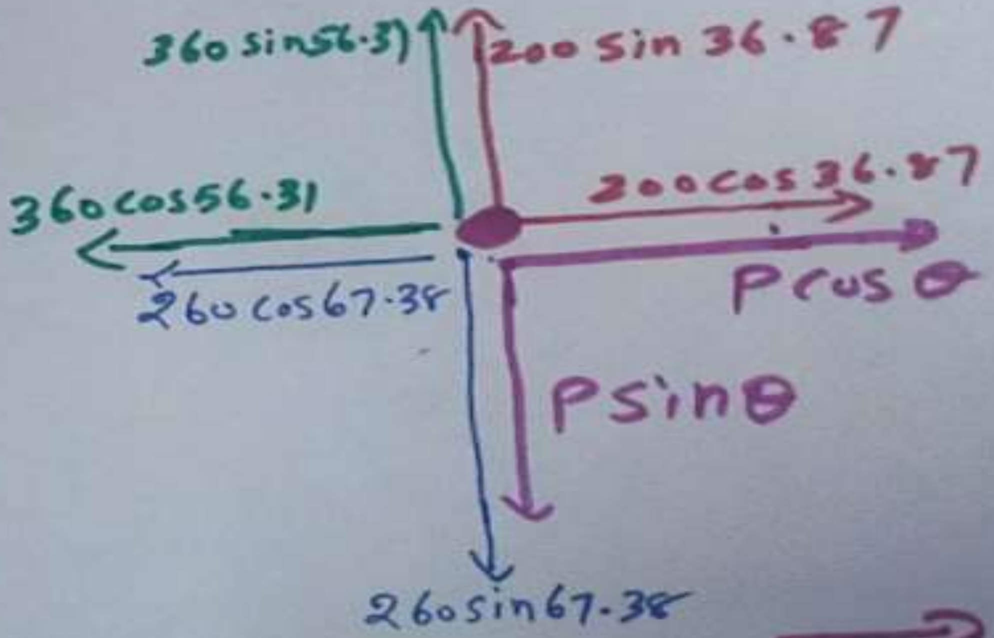


No horizontal component $\Rightarrow \sum F_H = 0$

R = Resultant Force [vertical]



$$\sum F_H = 200\cos 36.87 - 360\cos 56.31 - 260\cos 56.31 + P\cos\theta$$

$$\Downarrow$$

$$0 = P\cos\theta - 139.60$$

$$\Rightarrow \boxed{P\cos\theta = 139.60} \text{ - - - - } (1)$$

$$\Sigma F_V = 200\sin 36.87\uparrow + 360\sin 56.31\uparrow - 260\sin 67.38\downarrow - P\sin\theta$$

$$\Downarrow$$

$$-520 = 179.53 - P\sin\theta$$

$$\Rightarrow \boxed{P\sin\theta = 699.53 \quad --- \quad (2)}$$

Solving eqn (1) & (2) by (2)/(1)

$$\frac{P\sin\theta}{P\cos\theta} = \frac{699.53}{139.60}$$

$$\tan\theta = 5.007$$

$$\Rightarrow \theta = \tan^{-1}(5.007) = 78.7^\circ$$

From eqn (1)

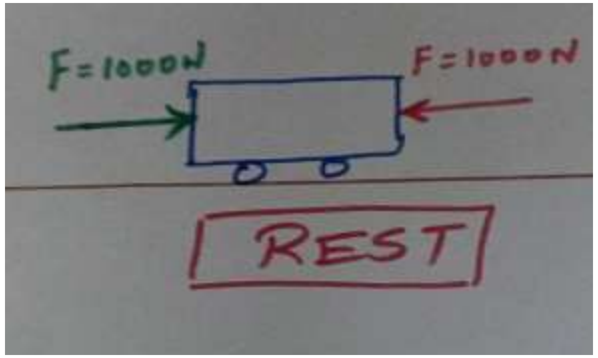
$$P\cos\theta = 139.60$$

$$P = \frac{139.60}{\cos 78.7} = 712.9\,N$$

EQUILIBIRIUM OF BODY

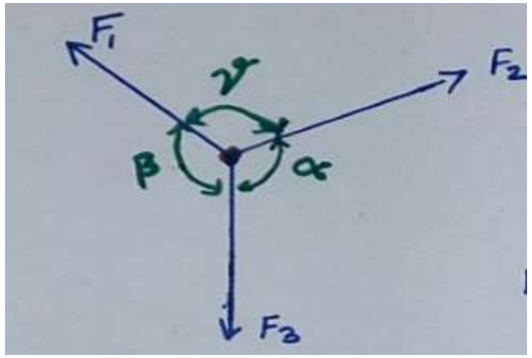
(REST STATE/UNIFORM MOTION)

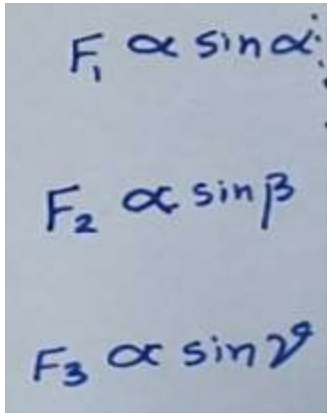
- **CONDITION FOR EQUILIBIRIUM OF BODY SUJECTED TO TWO FORCE SYSTEM**



BOTH THE FORCES MUST BE EQUAL AND OPPOSITE FOR TWO FORCE SYSTEM STATIC EQUILIBRIUM STATE

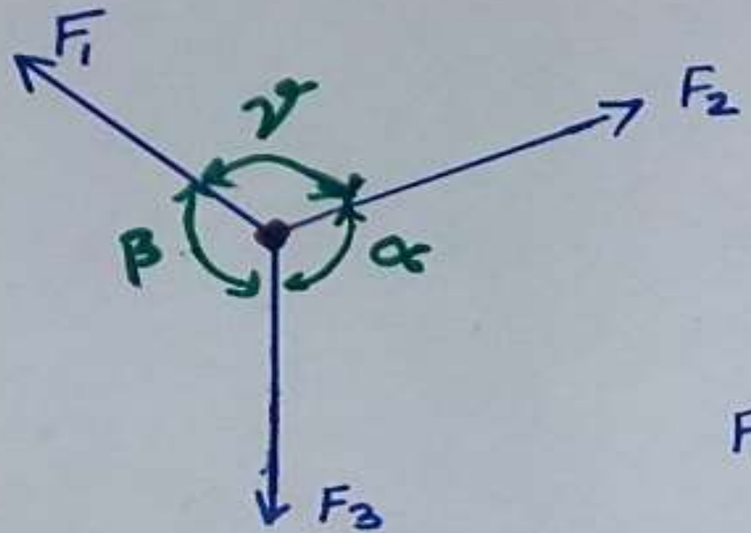
- **CONDITON FOR EQUILIBIRIUM OF BODY SUJECTED TO THREE FORCE SYSTEM**





EACH FORCE MUST BE PROPORTIONAL TO SINE OF ANGLE BETWEEN OTHER TWO FORCES FOR STATIC EQUILIBRIUM OF BODY SUBJECTED TO COPLANAR CONCURRENT THREE FORCE SYSTEM.

LAMI'S THEOREM



$F_1 \propto \sin\alpha \Rightarrow F_1 = \text{const.} \times \sin\alpha$

$\dfrac{F_1}{\sin\alpha} = \text{constant}$

$F_2 \propto \sin\beta \Rightarrow F_2 = \text{const} \times \sin\beta$

$\dfrac{F_2}{\sin\beta} = \text{constant}$

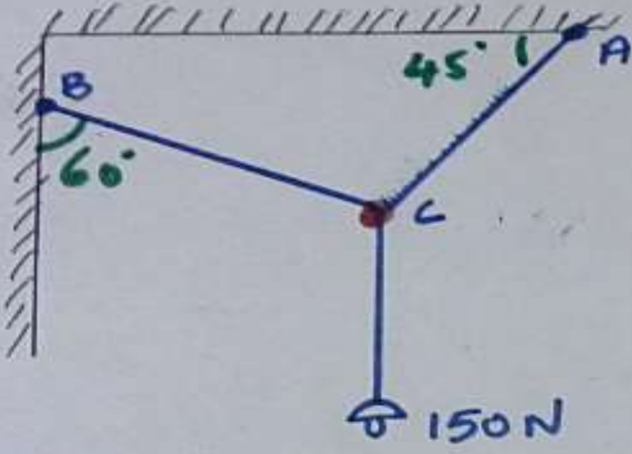
$F_3 \propto \sin\gamma \Rightarrow F_3 = \text{const} \times \sin\gamma$

$\dfrac{F_3}{\sin\gamma} = \text{consta}$

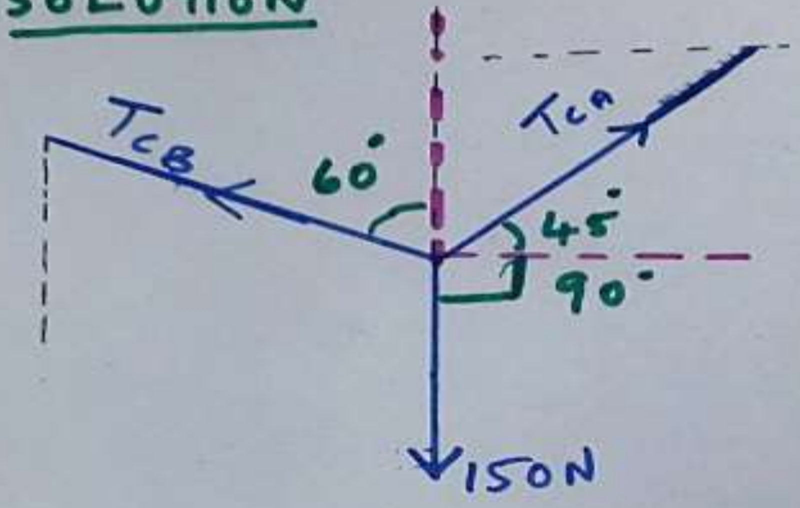
$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

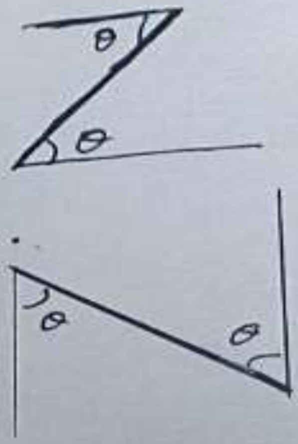
Lami's theorem states that if three coplanar forces acting at a point be in equilibrium, then each force is proportional to sine of angle between the other two forces.

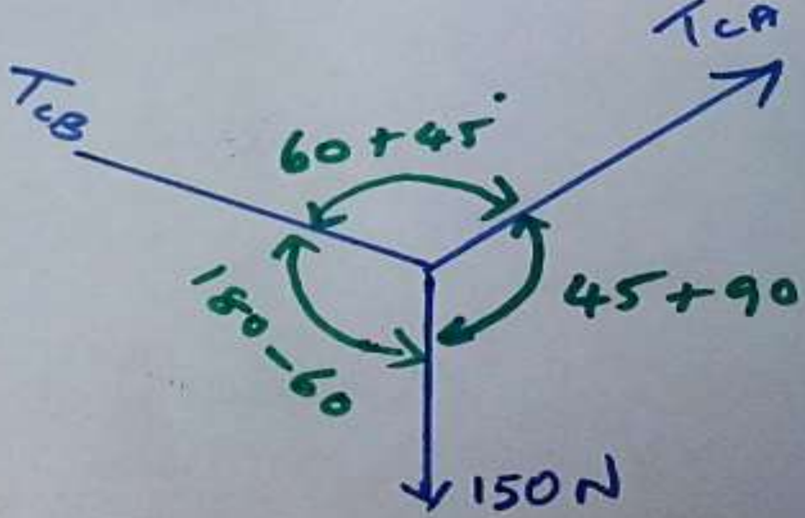
An electric light fixture weighing 150 Newtons hangs from point 'c' by two strings AC & BC as shown in figure below. Determine the forces in the strings AC & BC

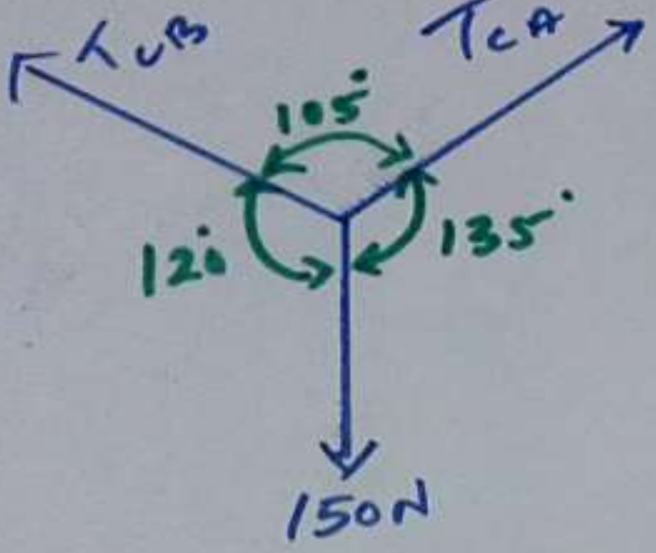


SOLUTION









By Applying Lami's theorem

$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

$$\frac{T_{CB}}{\sin 135} = \frac{T_{CA}}{\sin 120} = \frac{150}{\sin 105^\circ}$$

Taking first and last term

$$\frac{T_{CB}}{\sin 135} = \frac{150}{\sin 105}$$

$$T_{CB} = \frac{150}{\sin 105} \times \sin 135 = 109.81\,N$$

Taking second and last term

$$\frac{T_{CA}}{\sin 120} = \frac{150}{\sin 105^\circ}$$

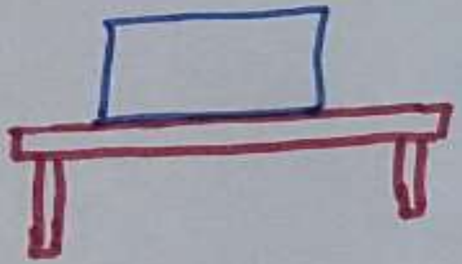
$$T_{CA} = \frac{150}{\sin 105} \times \sin 120 = 134.5\,N$$

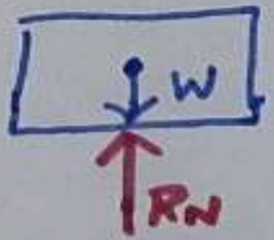
FREE BODY DIAGRAM

The free body diagram is a diagrammatic representation of single body [isolated from its surrounding] showing all the forces acting on the body

EXAMPLE 1

Real [Block resting on table]

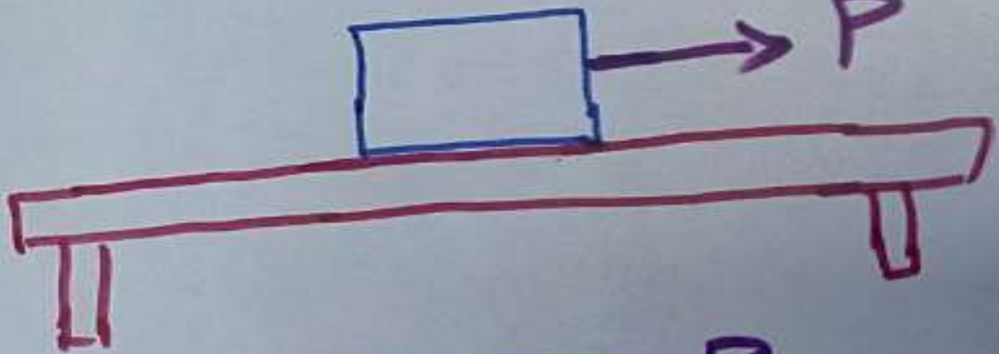


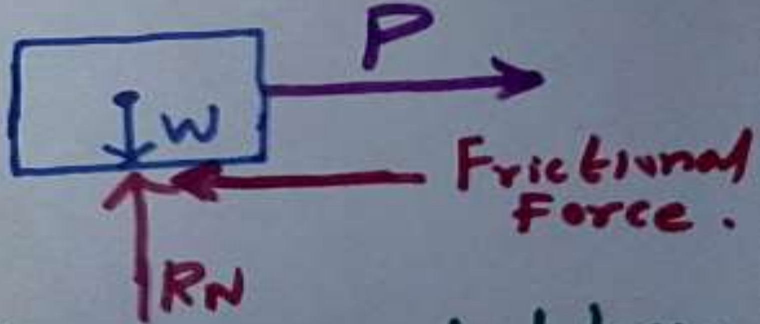


Free body diagram of block.

EXAMPLE 2

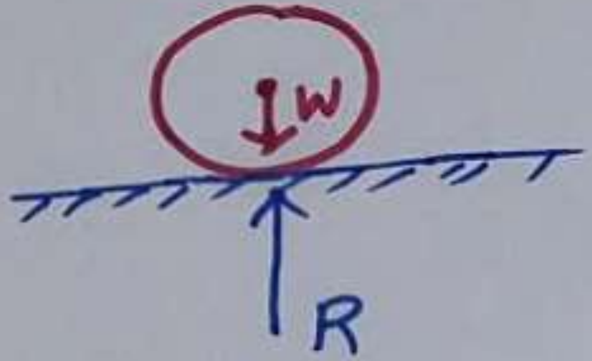
[Block moving on Table due to application of effort P]

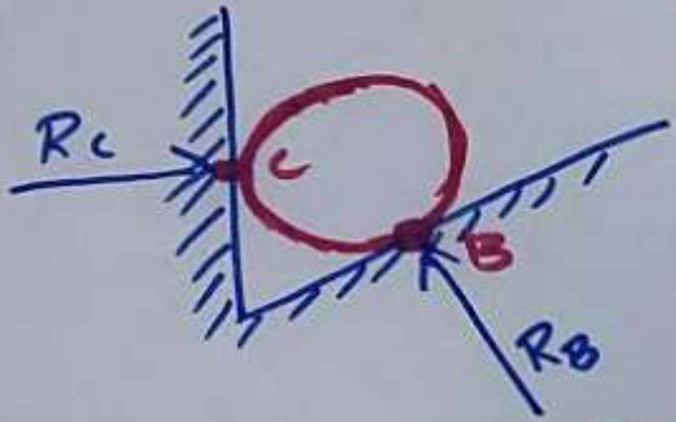




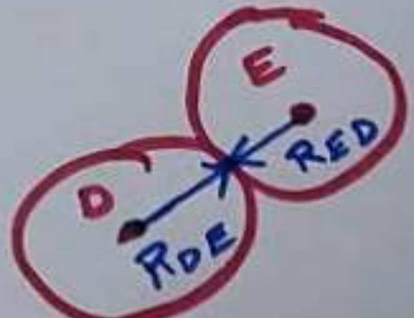
Free body diagram of block.

REACTIONS OF SMOOTH SURFACES WILL ALWAYS LIE PERPENDICULAR TO SURFACES AT THE POINT OF CONTACT



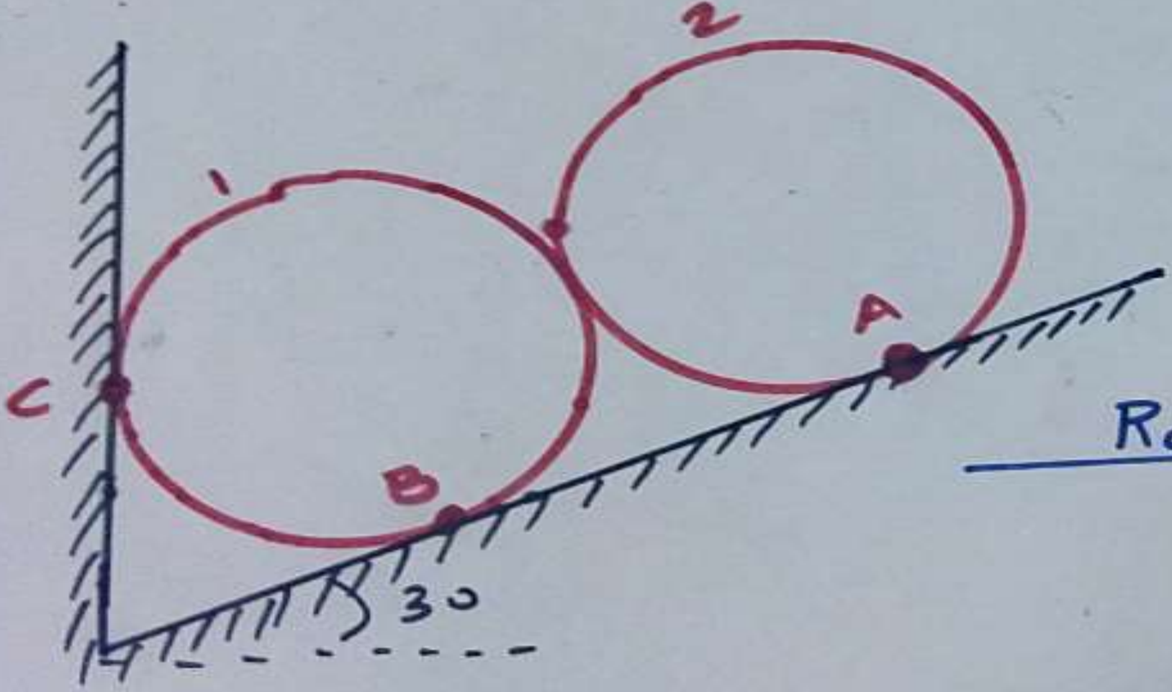


REACTION BETWEEN SMOOTH ROLLER WILL ALWAYS LIE ALONG CENTRE LINES BETWEEN ROLLERS.

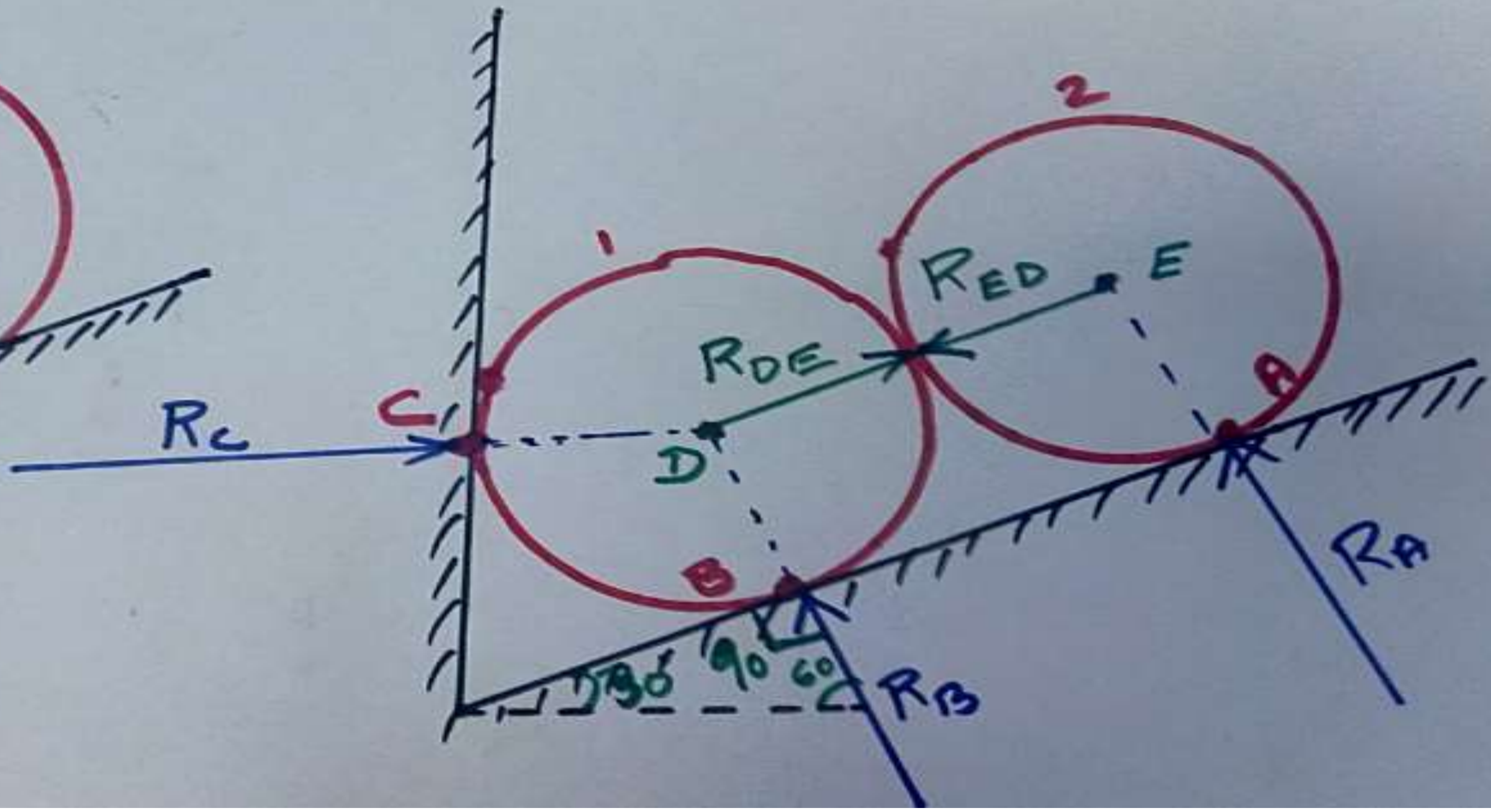


Two identical rollers, each of weight 50N, are supported by an inclined plane and a vertical plane wall as shown in figure below. Find the reactions at points of supports A, B, & C. Assume all the surfaces to be smooth.

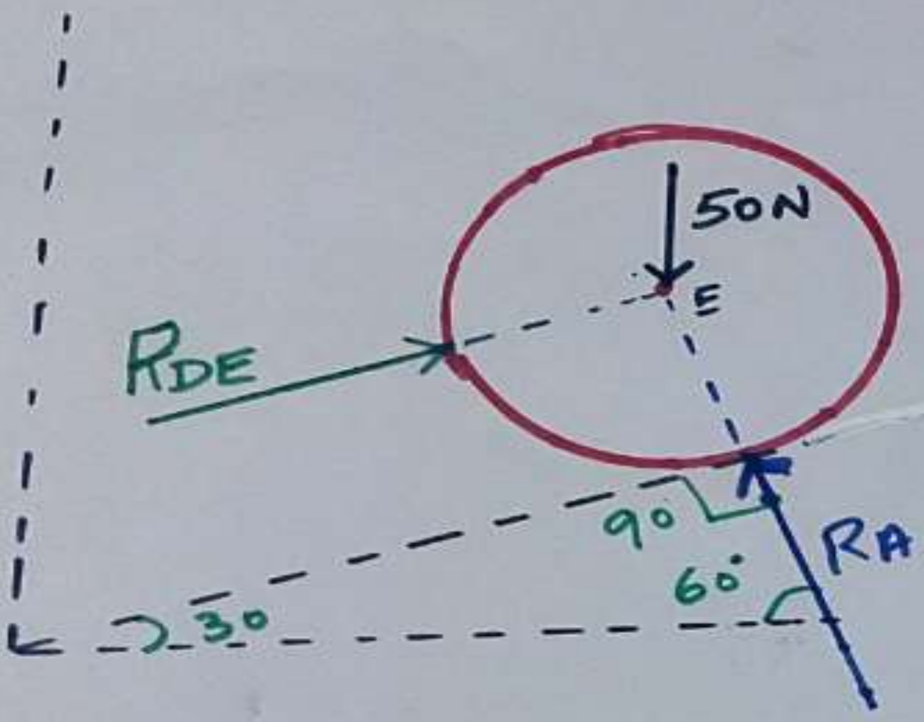
GIVEN

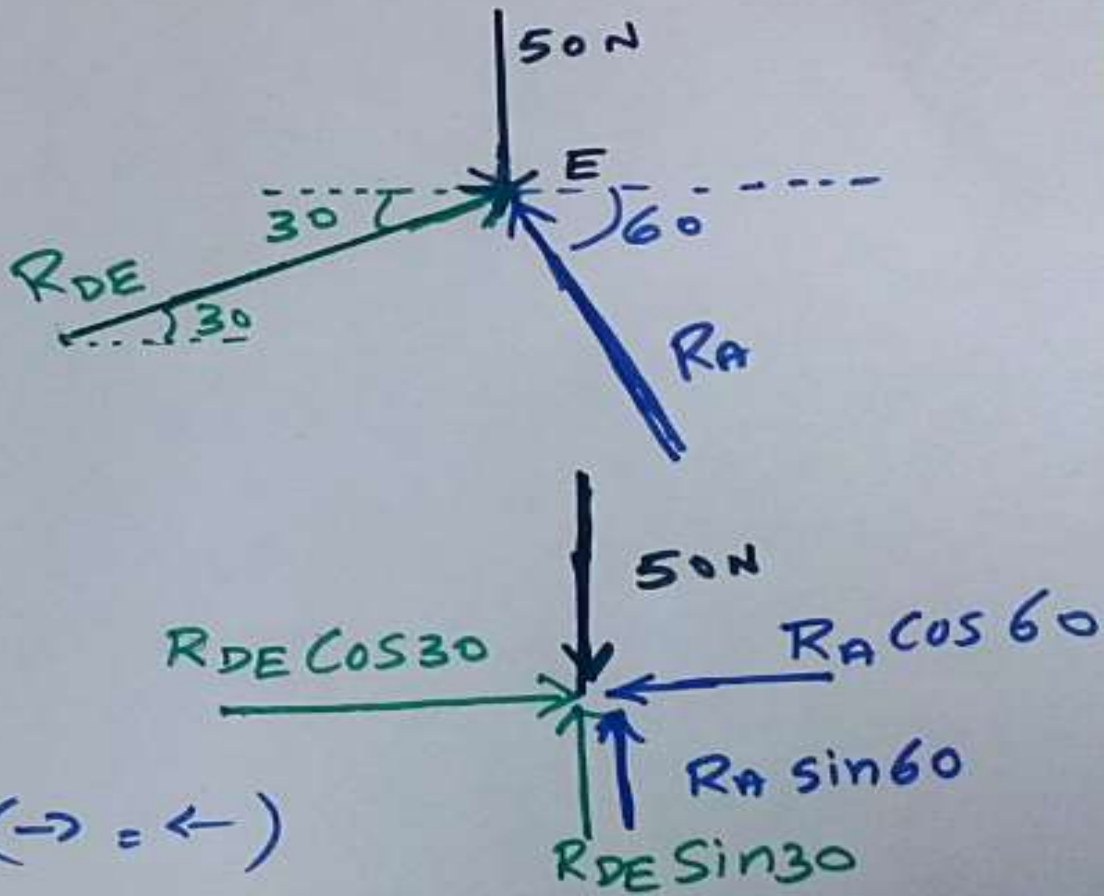


SOLUTION



CONSIDERING FREE BODY DIAGRAM OF ROLLER-2





Equating the horizontal forces (→ = ←)

$$\overrightarrow{R_{DE}\cos 30} = \overleftarrow{R_A \cos 60}$$

$$R_{DE} = \frac{R_A \cos 60}{\cos 30}$$

$$R_{DE} = 0.577\, R_A$$

Equating the vertical forces

$$\left[R_{DE} = 0.577 R_A \right]$$

$$R_{DE} \sin 30 + R_A \sin 60 = 50\,N$$

$$(0.577 R_A) \sin 30 + R_A \sin 60 = 50$$

$$1.154\, R_A = 50$$

$$R_A = \frac{50}{1.154}$$

$$= 43.32\,N$$

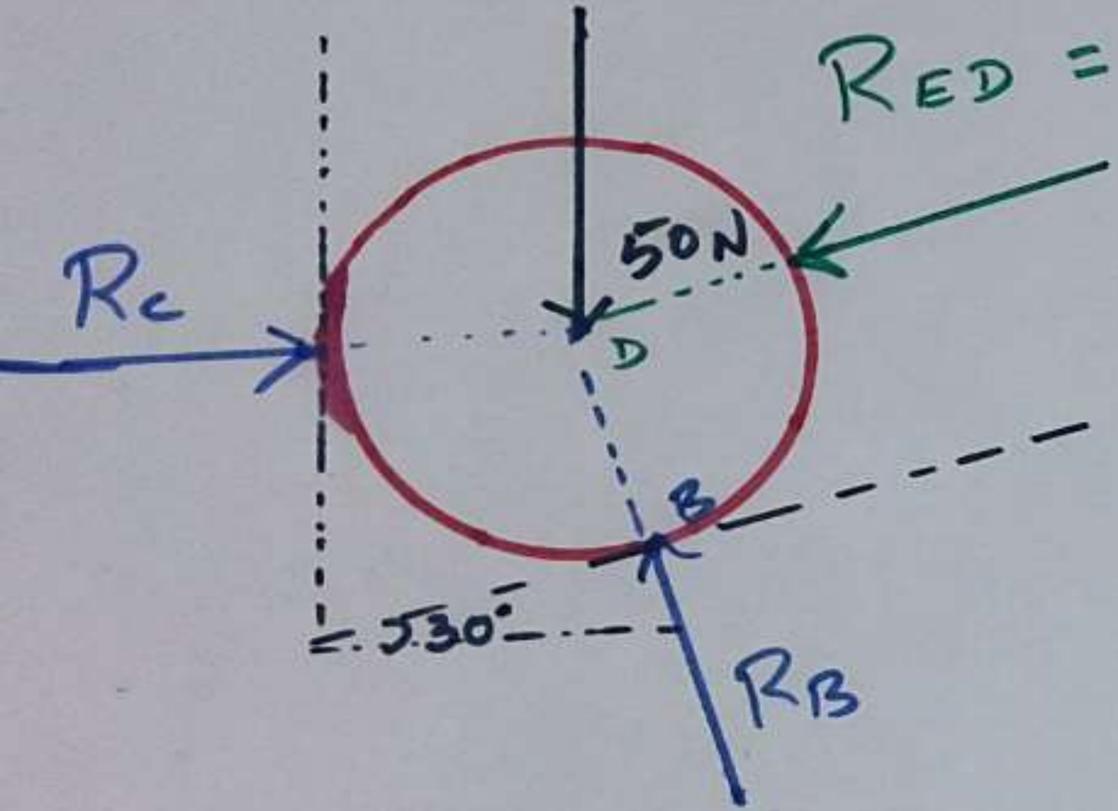
$$R_{DE} = (0.577 R_A)$$

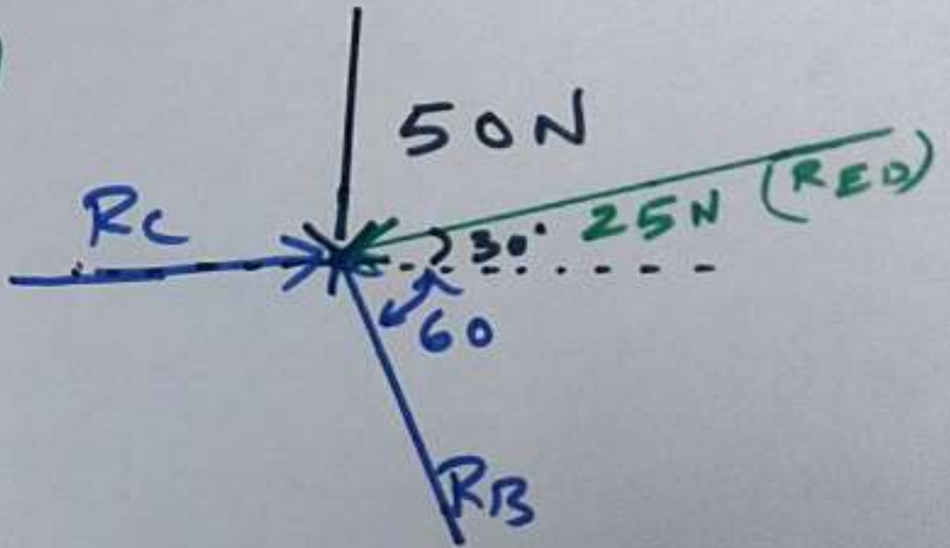
$$= 0.577 \times 43.32$$

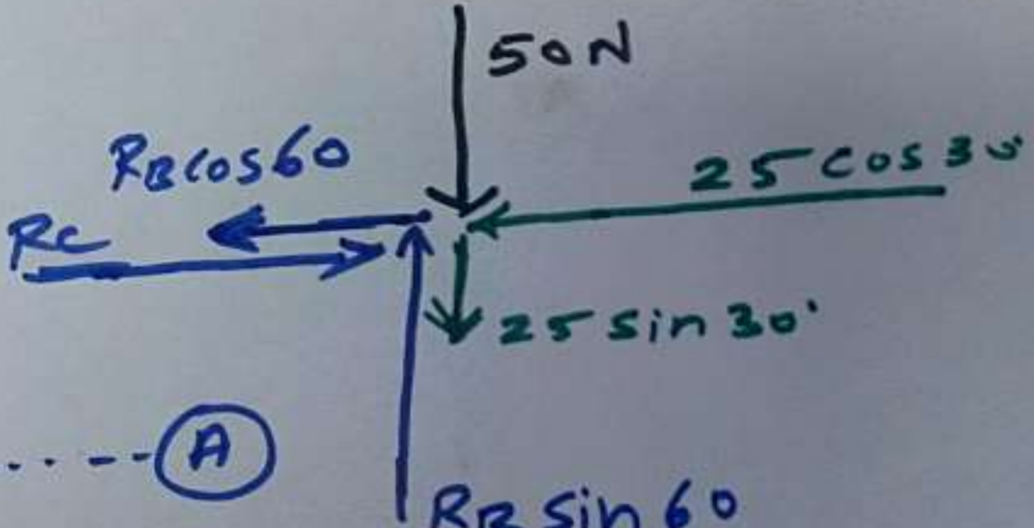
$$= 25\,N$$

Considering Free body diagram of Roller -1

$R_{ED} = R_{DE} = 25N$







EQUATING HORIZONTAL FORCES

$$\overrightarrow{R_C} = \overleftarrow{25\cos 30} + \overleftarrow{R_B \cos 60} \quad \cdots\cdots (A)$$

EQUATING THE VERTICAL FORCES

$$\uparrow R_B \sin 60 = 50 \downarrow + 25 \sin 30 \downarrow$$

$$R_B \sin 60 = 62.5$$

$$R_B = 62.5 / \sin 60 = 72.17\,N$$

$$R_B = 72.17\,N$$

substituting value of $R_B = 72.17$ in (A) eqn.

$$R_C = 25 \cos 30 + R_B \cos 60$$

$$R_C = 25 \cos 30 + 72.17 \cos 60$$

$$= 57.73\ N$$

ANS

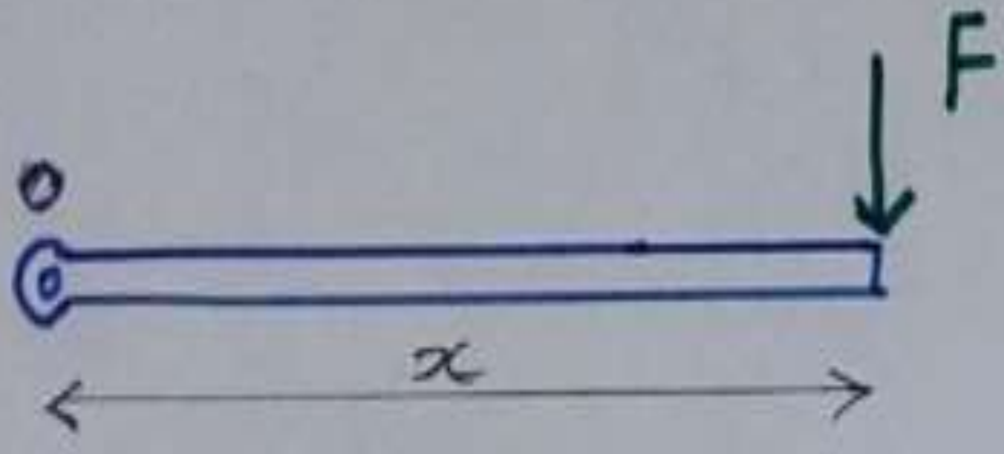
$$R_A = 43.32\,N$$

$$R_B = 72.17\,N$$

$$R_C = 57.73\,N$$

MOMENT OF FORCE
(TURNING MOMENT/TURNING EFFECT OF FORCE)

MOMENT OF FORCE

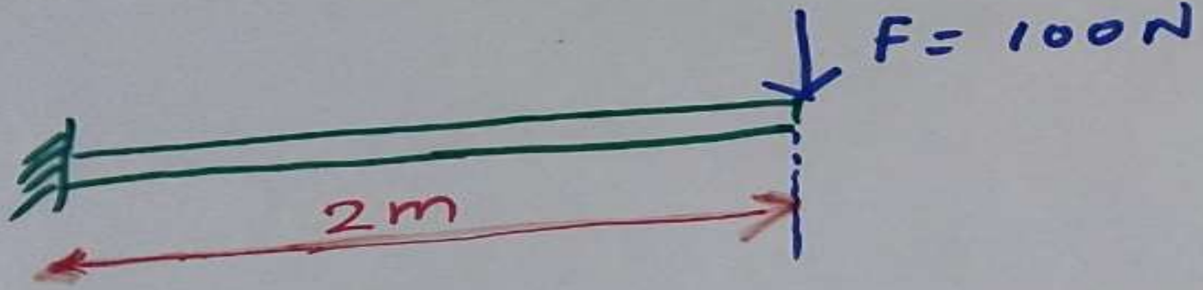


"Moment of force about a point is defined as the product of force and the perpendicular distance of line of action of the force from that point"

$$\boxed{\text{Moment of force about point O} = \text{Force} \times \text{perpendicular distance} = F \times x}$$

MOMENT OF FORCE

MOMENT OF FORCE = FORCE × perpendicular Distance



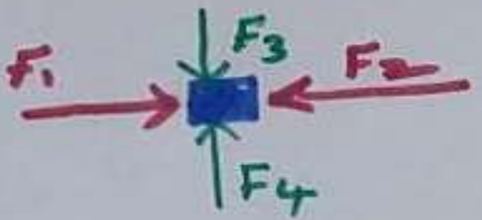
$\boxed{\text{Moment of force} = \text{Force} \times \text{distance [perpendicular distance]}}$

$= 100\ N \times 2\ m$

$= 200\ Nm$

CONDITIONS FOR EQUILIBRIUM OF PARTICLE IN TWO DIMENSION

$$\boxed{\begin{array}{l} (1)\ \Sigma F_x = 0 \\ (2)\ \Sigma F_y = 0 \end{array}}$$



(i) $\vec{F_1} = \overleftarrow{F_2}$

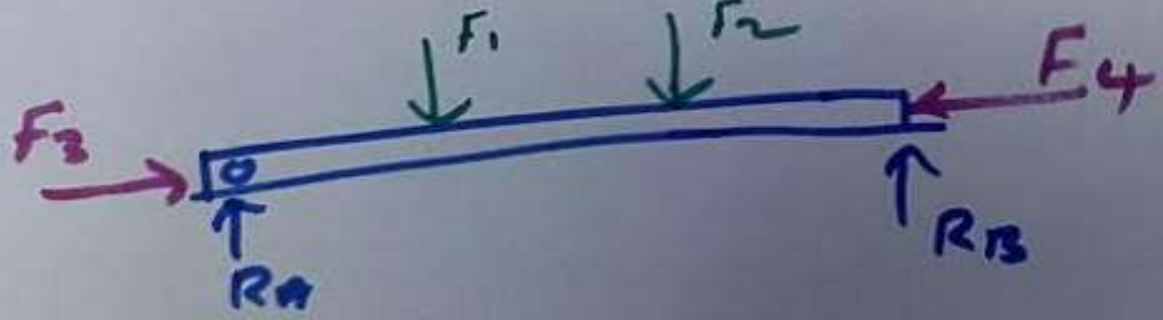
$\vec{F_1} - \overleftarrow{F_2} = 0$

(ii) $\uparrow F_4 = F_3 \downarrow$

$\uparrow F_4 - F_3 = 0$

CONDITIONS FOR EQUILIBRIUM OF RIGID BODY IN EQUILIBRIUM

$$\boxed{\begin{array}{l} (1)\ \Sigma F_x = 0 \\ (2)\ \Sigma F_y = 0 \\ (3)\ \Sigma M = 0 \end{array}}$$



TYPES OF SUPPORTS

- 1) ROLLER SUPPORT
- 2) HINGED SUPPORT
- 3) FIXED SUPPORT

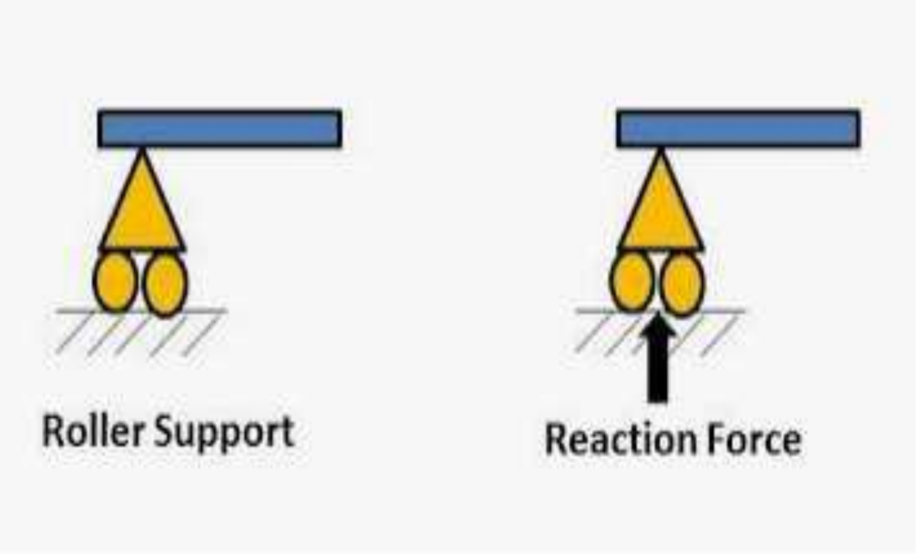
ROLLER SUPPORT

In this type of support,rollers are placed below beam and beam can slide over the rollers.

Reaction will be perpendicular to the surface on which rollers are supported.

This type of support is normally provided at the end of a bridge.

Due to breaking forces of vehicles and temperature forces,bridge slab can slide over the roller support and damage to bridge pier can be avoided.



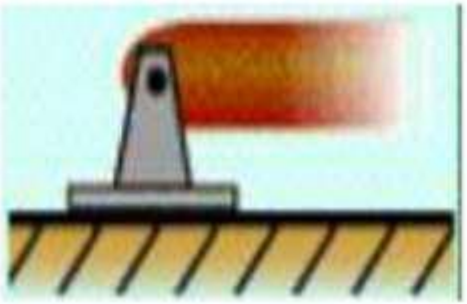
THE REACTION FORCE DEVELOPED BY ROLLER SUPPORT WILL ALWAYS LIES NORMAL TO THE PLANE OF ROLLERS

HINGED SUPPORT

Beam and support are connected by a hinge.

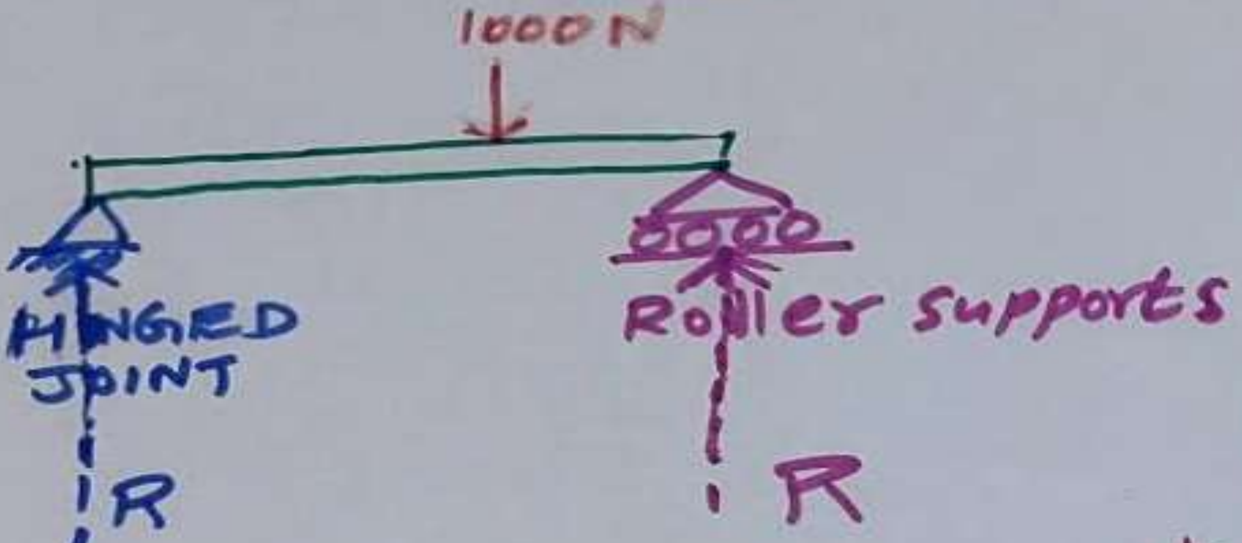
Beam can rotate about the hinge.

Reactions may be vertical, horizontal or inclined.

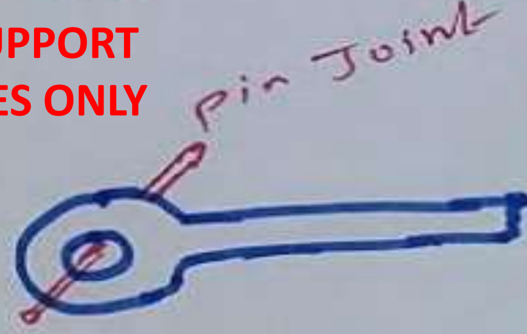


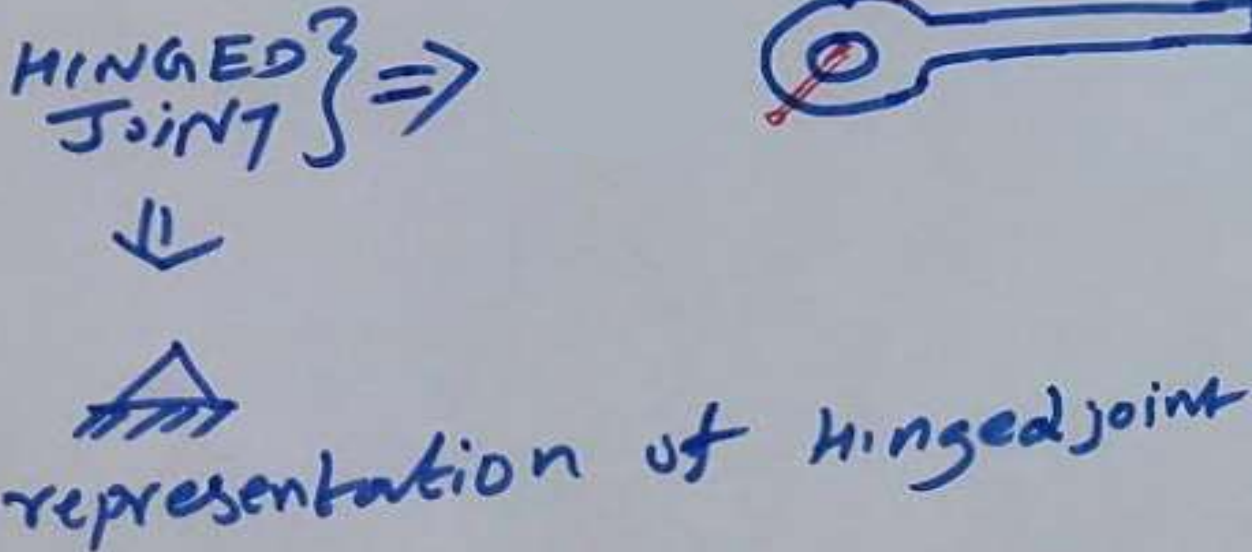
THE REACTION FORCE DEVELOPED BY HINGED SUPPORT WILL BE INCLINED WHEN THE LOAD ACTING ON THE BEAM IS INCLINED

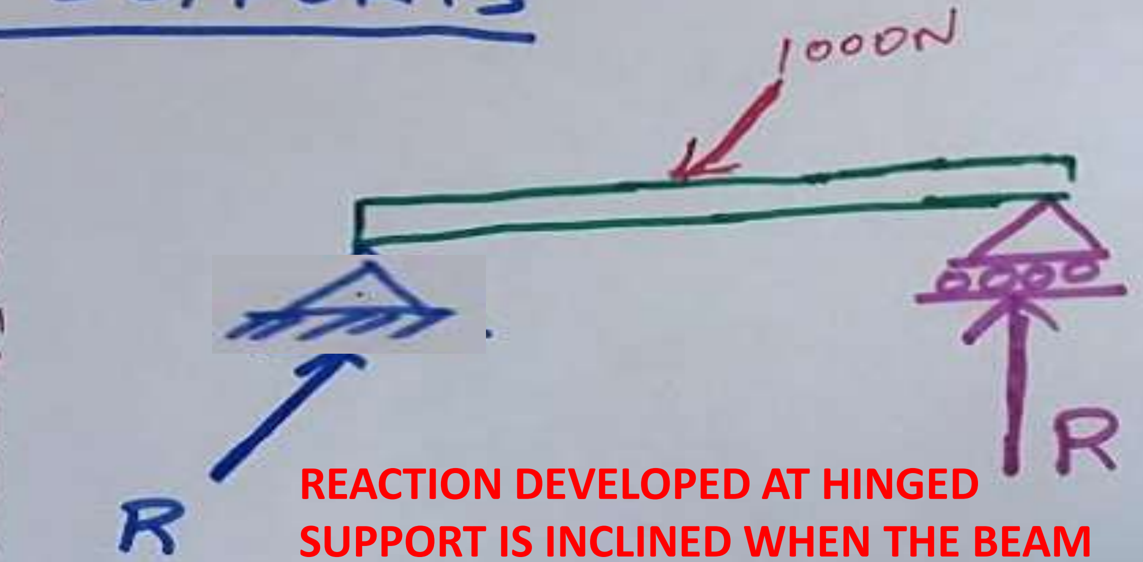
BEAMS AND SUPPORTS



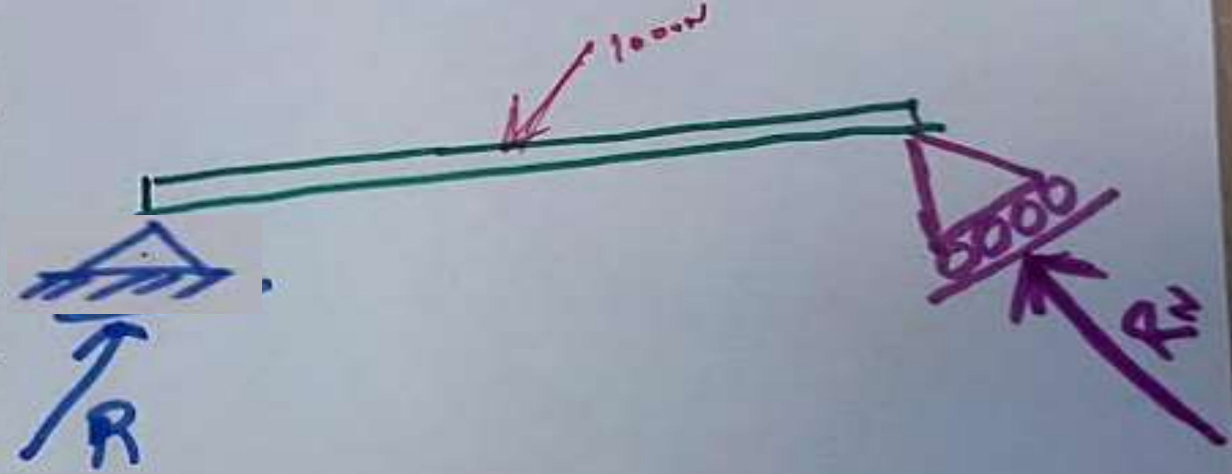
REACTION DEVELOPED AT HINGED SUPPORT IS VERTICAL WHEN THE BEAM CARRIES ONLY VERTICAL LOAD



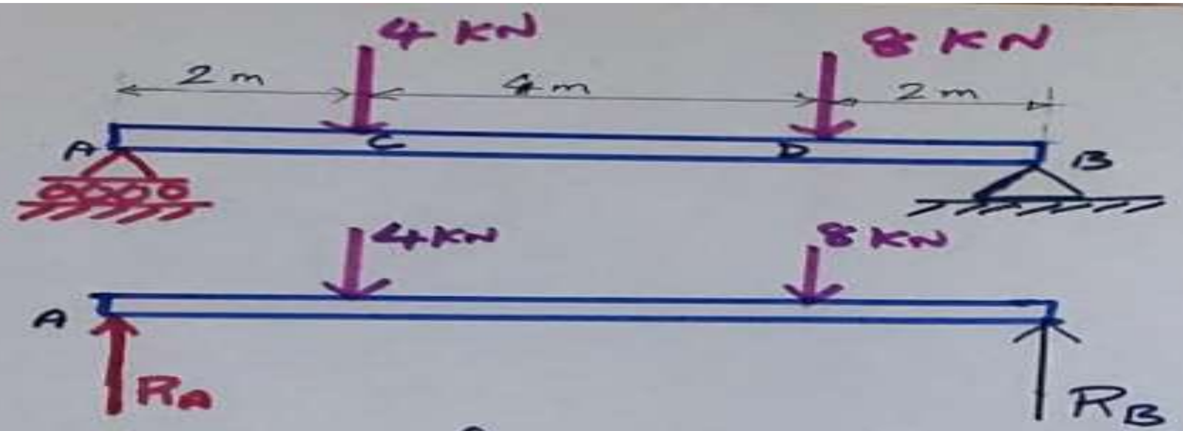




REACTION DEVELOPED AT HINGED SUPPORT IS INCLINED WHEN THE BEAM CARRIES INCLINED LOAD



Find the support reactions provided by roller support and hinged support when a beam of 8 m length is simply supported at its left end by roller support and its right end by hinged support and the beam carries point loads of 4 kN and 8 kN at a distance of 2 m and 6 m distance respectively from the left end.



Applying the conditions for equilibrium

(1) $\Sigma F = 0 \Rightarrow \Sigma F_v = 0 \; (\downarrow = \uparrow)$

$$R_A \uparrow + R_B \uparrow = 4 \downarrow + 8 \downarrow$$

$$R_A + R_B = 12 \quad \text{----} \quad \text{(1)}$$

(2) $\Sigma M_A = 0 \quad [\circlearrowleft = \circlearrowright]$

$$R_B \times 8 = 8 \times 6 + 4 \times 2$$

$$8 R_B = 56$$

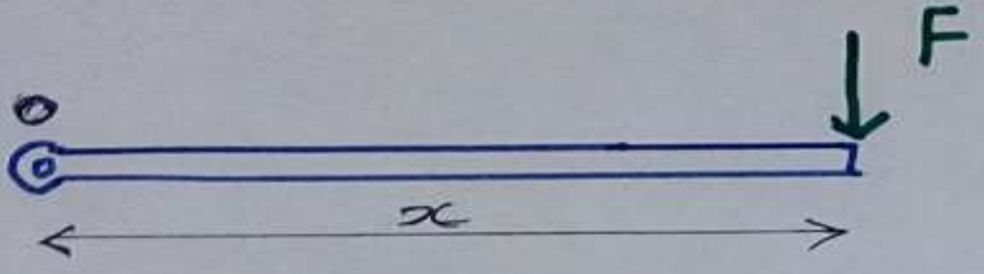
$$R_B = 56/8 = 7 \text{ kN} \quad \text{----} \quad \text{(2)}$$

(2) in (1)

$$R_A + 7 = 12$$

$$\Rightarrow R_A = 12 - 7 = 5 \text{ kN}$$

MOMENT OF FORCE



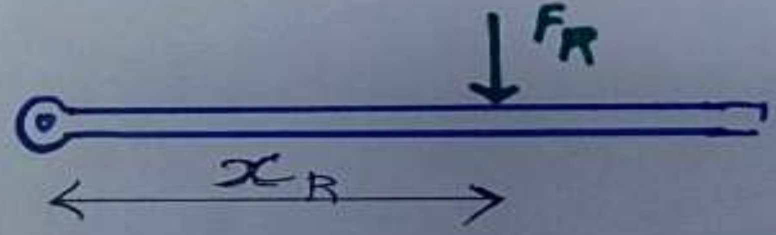
"Moment of force about a point is defined as the product of force and the perpendicular distance of line of action of the force from that point"

$$\boxed{\text{Moment of Force about point O} = \text{Force} \times \text{perpendicular distance} = F \times x}$$

VARIGON'S THEOREM :

"The algebraic sum of moments of any number of forces about any point in their plane is equal to moment of their resultant force about the same point"

=



$$(F_1 x_1) + (F_2 x_2) = (F_R \times x_R)$$

MOMENT OF INERTIA CHAPTER CLASS NOTES

PREPARED BY

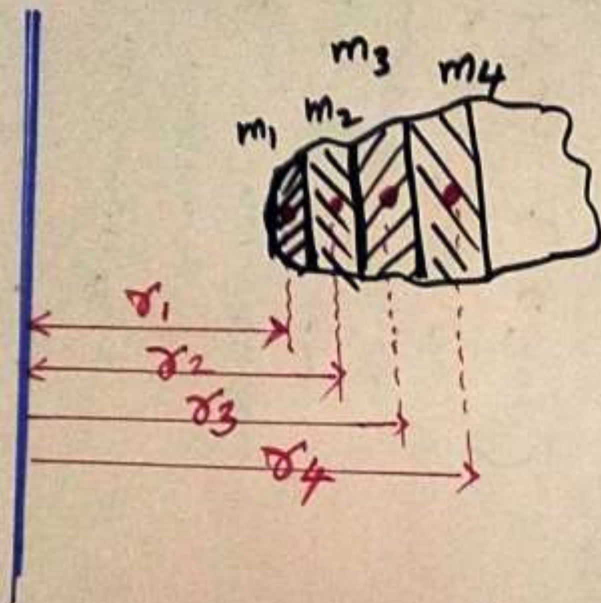
V.JOSE ANANTH VINO

AP/MECH/B.I.H.E.R.,

MOMENT OF INERTIA

Moment of inertia is the **inertia** of a rotating body with respect to its rotation about a particular axis. It is a rotating body's resistance to angular acceleration or deceleration, equal to the product of the mass and the square of its perpendicular distance from the axis of rotation.

MOMENT OF INERTIA

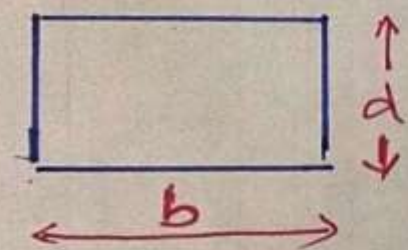
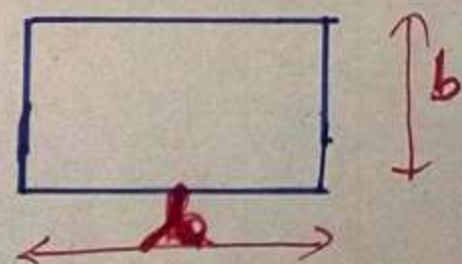
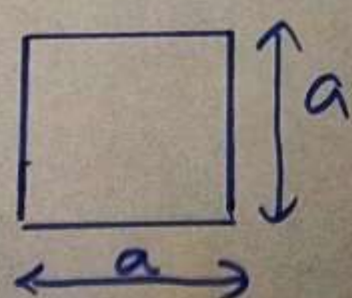


$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^4 + \dots$$

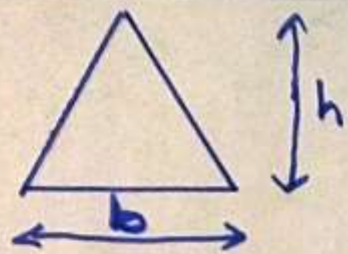
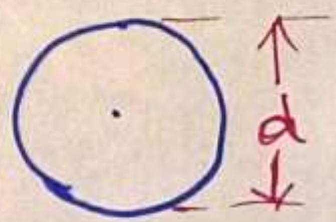
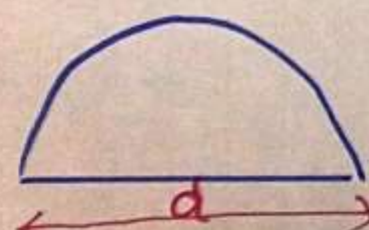
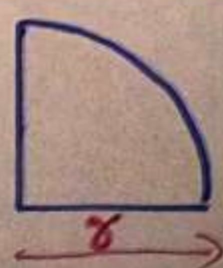
$$\boxed{I = \Sigma m r^2} \quad (\text{or}) \quad \boxed{I = \Sigma a_1 r_1^2}$$

"Moment of Inertia (I) about an axis is the algebric sum of products of its elements mass and square of distance of respect elements of mass from its axis."

FORMULAE FOR MOMENT OF INERTIA

S.N	NAME	SHAPE	I_{xx}	I_{yy}
1.	RECTANGLE		$\frac{bd^3}{12}$	$\frac{db^3}{12}$
			$\frac{lb^3}{12}$	$\frac{bl^3}{12}$
2.	SQUARE		$\frac{a^4}{12}$	$\frac{a^4}{12}$

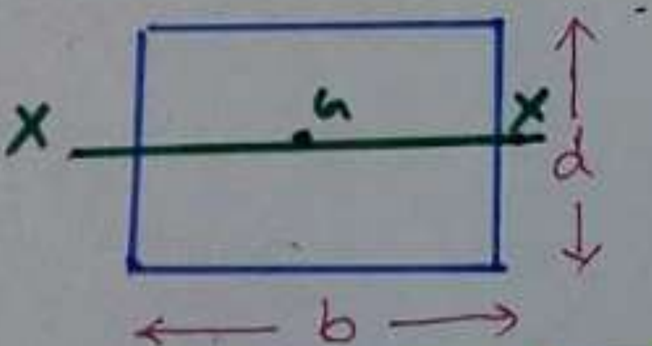
FORMULAE FOR MOMENT OF INERTIA

S.N	NAME	SHAPE	I_{xx}	I_{yy}
3	Triangle		$\frac{bh^3}{36}$	$\frac{hb^3}{48}$
4	circle		$\frac{\pi}{64} d^4$	$\frac{\pi}{64} d^4$
5	Semi circle		$0.0068\, d^4$	$\frac{\pi}{128} d^4$
6	Quadrant of circle.		$0.055\, r^4$	$0.055\, r^4$

FIND THE MOMENT OF INERTIA OF RECTANGLE ABOUT ITS CENTROIDAL HORIZONTAL AXIS (I_{xx}) IF $b = 50$ mm and $h = 40$ mm.

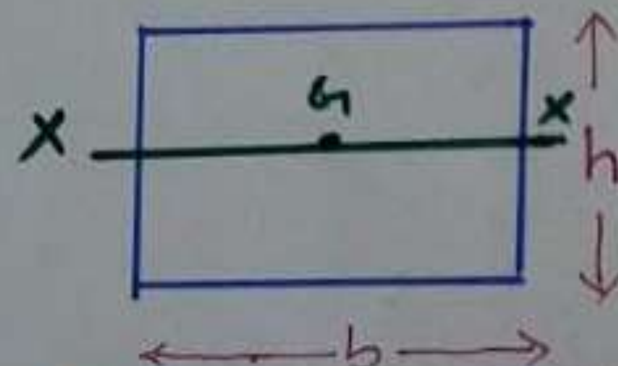
SOLUTION

If 'b' & 'd' are given



$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

when 'b' & 'h' are given



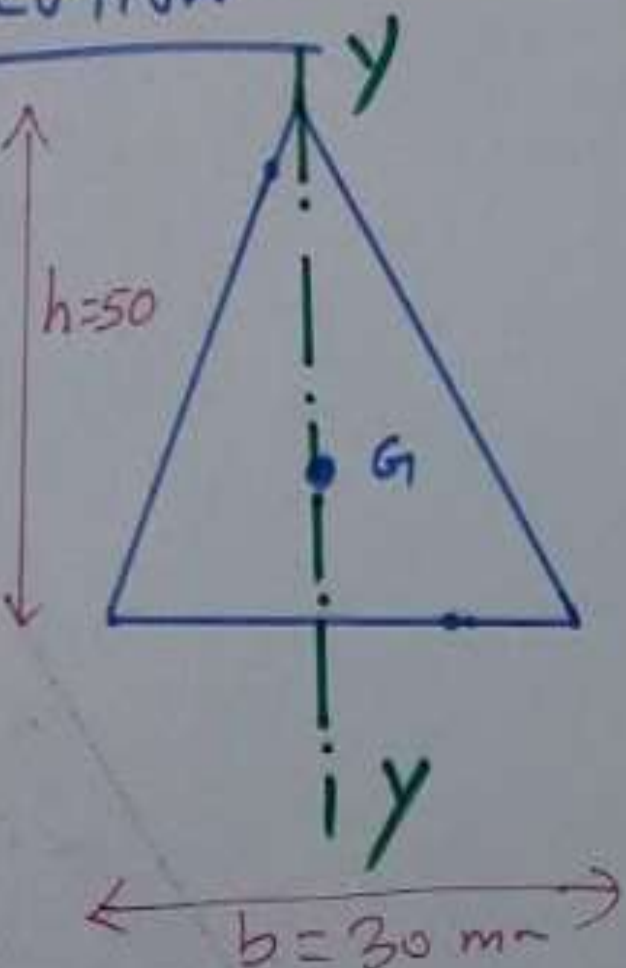
$$\boxed{I_{xx} = \frac{bh^3}{12}}$$

$$= \frac{50 \times 40^3}{12}$$

$$= 266666.66 \text{ mm}^4$$

Find the moment of Inertia of Triangle about vertical centroidal axis. [$I_{yy} = ?$] if $b = 30$ & $h = 50$ mm.

SOLUTION



$$\boxed{I_{yy} = \frac{hb^3}{48}}$$

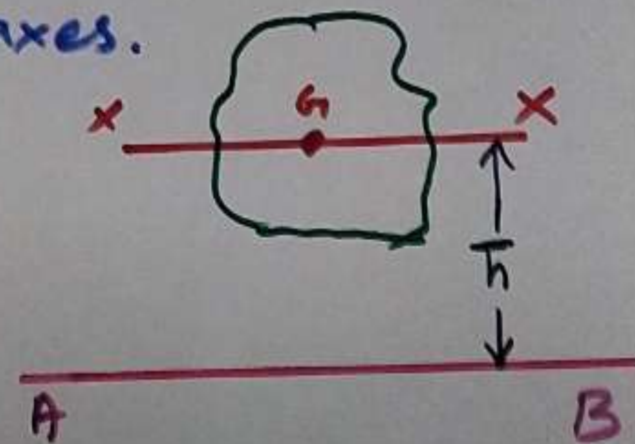
$$= \frac{50 \times 30^3}{48}$$

$$= 28125 \text{ mm}^4$$

PARALLEL AXIS THEOREM

Parallel axis theorem states that the moment of Inertia of a lamina about any axis in the plane of lamina is equal to sum of moment of Inertia about a parallel centroidal axis in the plane of lamina and product of area of lamina and square of distance between the two axes.

$$\boxed{I_{AB} = I_{xx} + A\bar{h}^2}$$



A = Area of lamina

$\bar{h}$ = distance between centroidal axis and new parallel axis

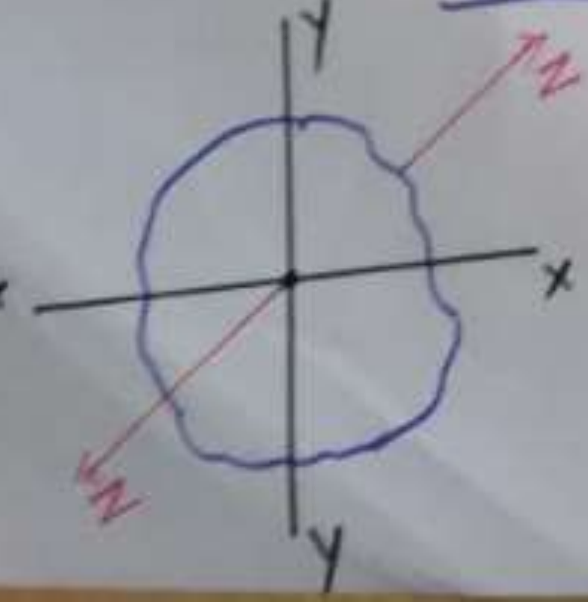
[NOTE: For finding $\bar{h}$ in composite plane figures $\bar{h} = \bar{y} - y$ (or) $y - \bar{y}$ may be used]

POLAR MOMENT of INERTIA

I_{zz} (or) J

The momoment of Inertia of Laminae about an axis passing perpendicular to the plane of the laminae is known polar Moment of Inertia.

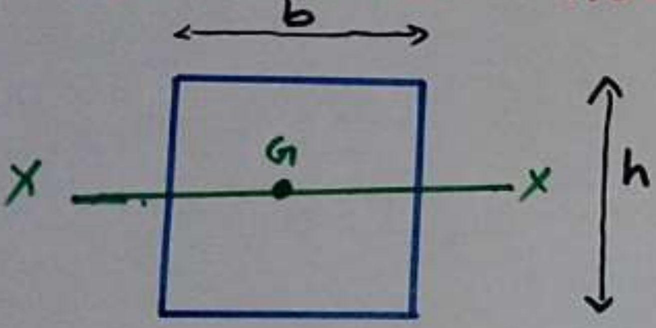
PERPENDICULAR AXIS THEOREM

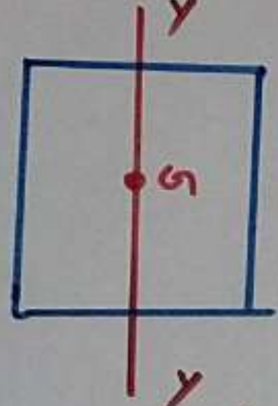


Perpendicular axis theorem states that the moment of Inertia of a laminae about an axis passing perpendicular to the plane of the laminae will be equal to sum of moment of Inertia of the laminae about two mutually perpendicular axis lying in the plane of the laminae

$$I_{zz} = I_{xx} + I_{yy}$$

MOMENT OF INERTIA OF RECTANGULAR LAMINAE ABOUT CENTROIDAL XX & YY AXIS.





$$I_{xx} = \frac{bh^3}{12} \qquad I_{yy} = \frac{hb^3}{12}.$$

Find Moment of Inertia of rectangle about centroidal horizontal XX axis and about centroidal vertical yy axis. The breadth and height of rectangle is 30 mm and 50 mm. Also Find its polar moment of Inertia.

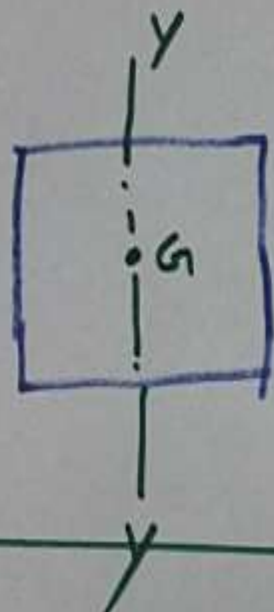
$$I_{xx} = \frac{bh^3}{12} = \frac{30 \times 50^3}{12} = 312500 \ MM^4$$

$$I_{yy} = \frac{hb^3}{12} = \frac{50 \times 30^3}{12} = 112500 \ MM^4$$

polar Moment of Inertia $= I_{zz} = I_{xx} + I_{yy}$

$I_{ZZ} = I_{XX} + I_{YY} = 312500 + 112500 = 425000$ MM^4

Find the moment of Inertia of rectangle about its vertical centroidal axis. The rectangle has 3 m breadth and 2 m depth (height)



$h = 2m$
$b = 3m$

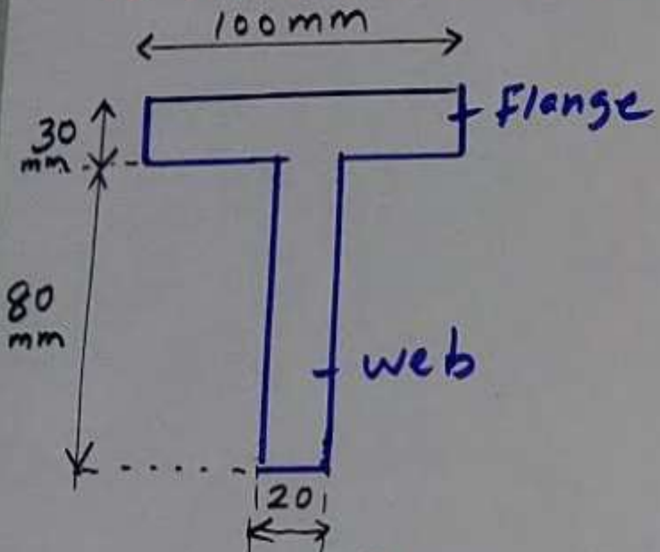
Formulae for M.I about vertical centroidal yy axis $\Big\} = \dfrac{hb^3}{12}$

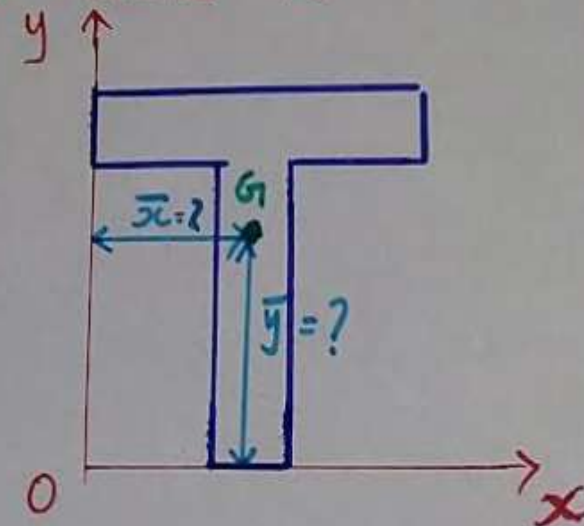
$$= \frac{2 \times 3^3}{12}$$

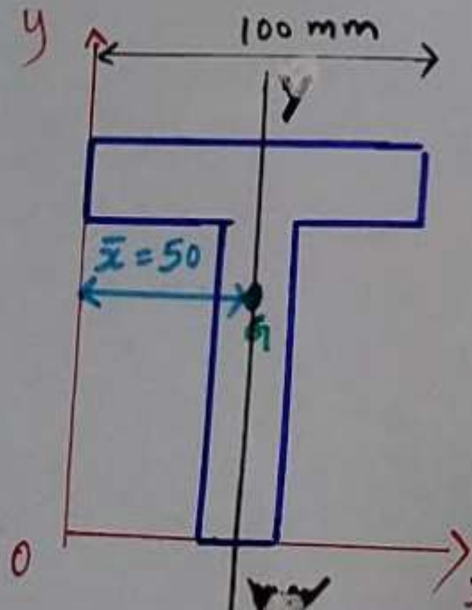
$$= 4.5\ m^4.$$

FIND THE MOMENT OF INERTIA OF A ‘T’ SECTION OF FLANGE 100 MM X 30 MM AND WEB 20 MM X 80 MM ABOUT ITS CENTROIDAL HORIZONTAL AXIS AND ALSO ABOUT ITS CENTROIDAL VERTICAL AXIS

LOCATE THE CENTROID OF 'T' SECTION WHOSE FLANGE IS 100mm X 30 mm AND WEB IS 20mm X 80 mm





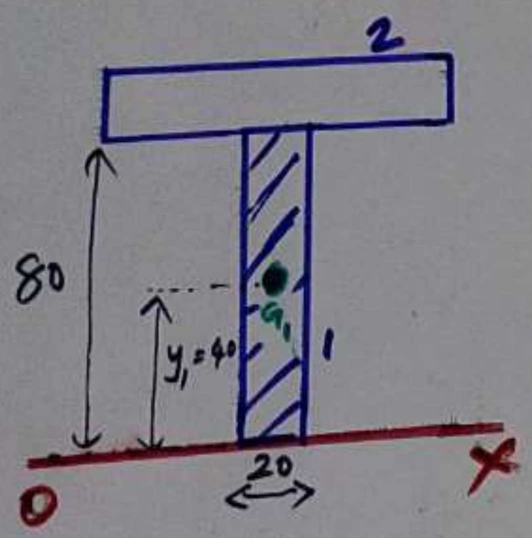


As the given 'T' section is symmetrical about yy axis, $\bar{x}$ is predictable

$$\bar{x} = \frac{100}{2} = 50 \text{ mm}$$

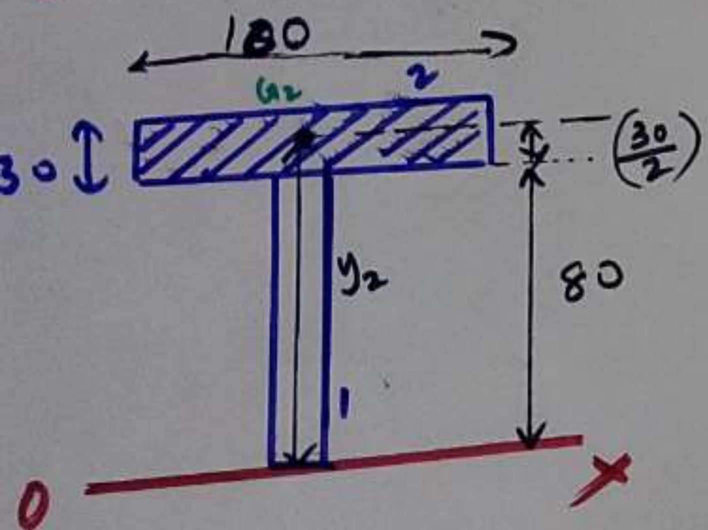
from oy axis.

$\bar{y} = ?$ [To be found using calculation]



$a_1 = bh = 20 \times 80 = 1600 \text{ mm}^2$

$y_1 = \frac{80}{2} = 40$



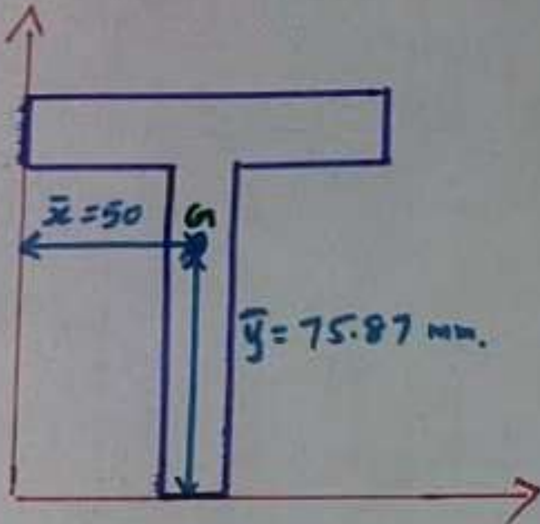
$a_2 = 100 \times 30 = 3000 \text{ mm}^2$

$y_2 = 80 + \frac{30}{2} = 95 \text{ mm}$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

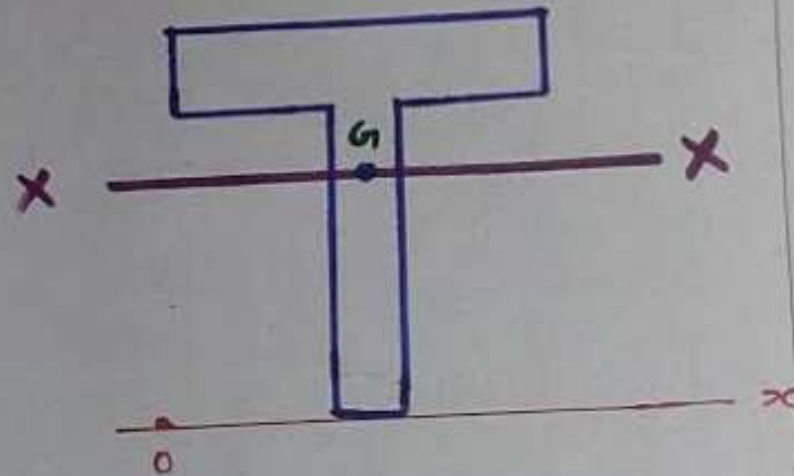
$$= \frac{(1600 \times 40) + (3000 \times 95)}{1600 + 3000}$$

$$= 75.87 \text{ mm}$$

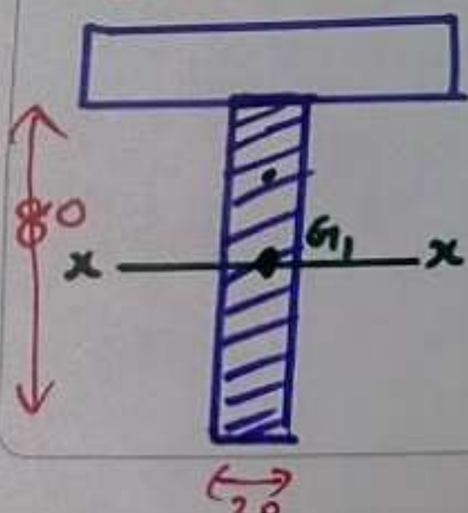


CENTROID OF 'T' SECTION

To FIND M.I about centroidal xx axis



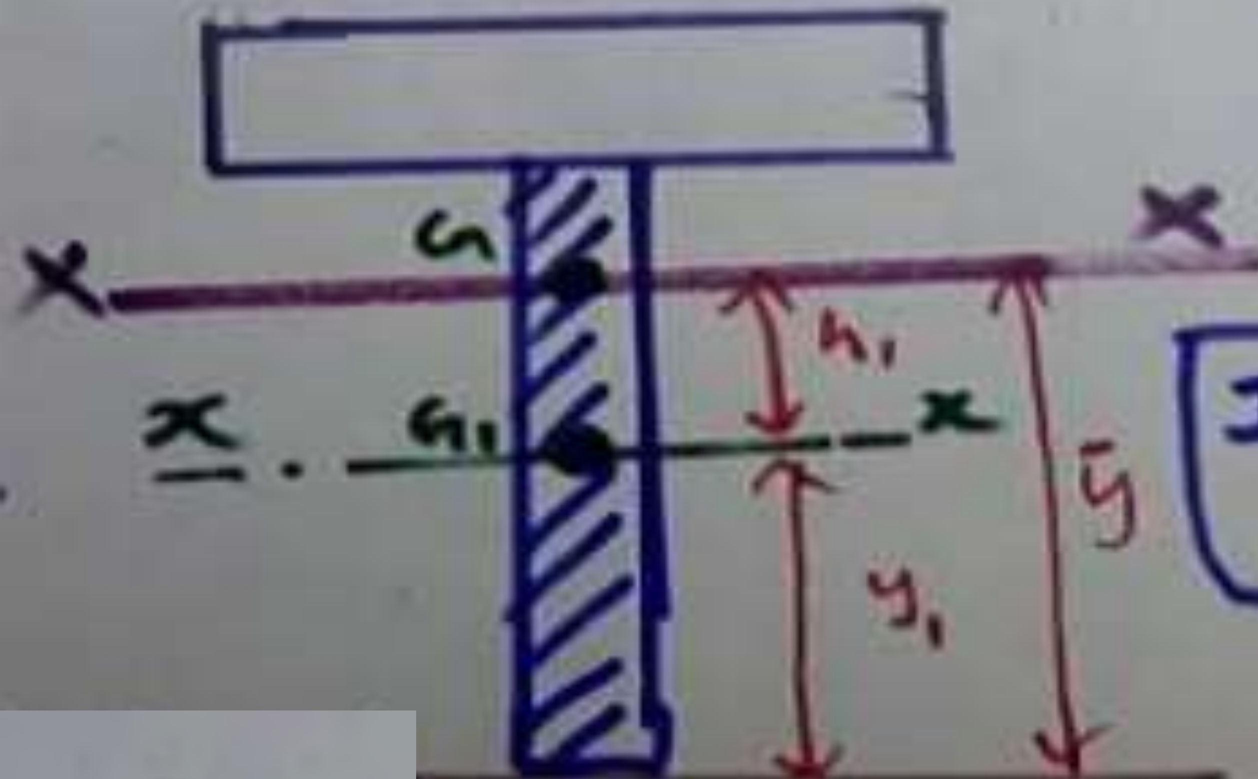
I SECTION



$$I_{G_1 xx} = \frac{bd^3}{12}$$

$$= \frac{20 \times 80^3}{12}$$

$$= 853333.33\ mm^4.$$



$h_1 = \bar{y} - y_1$

$= 75.87 - \left(\frac{80}{2}\right)$

$= 35.87$

$$\boxed{I_{1_{xx}} = I_{G_{1_{xx}}} + A_1 h_1^2}$$

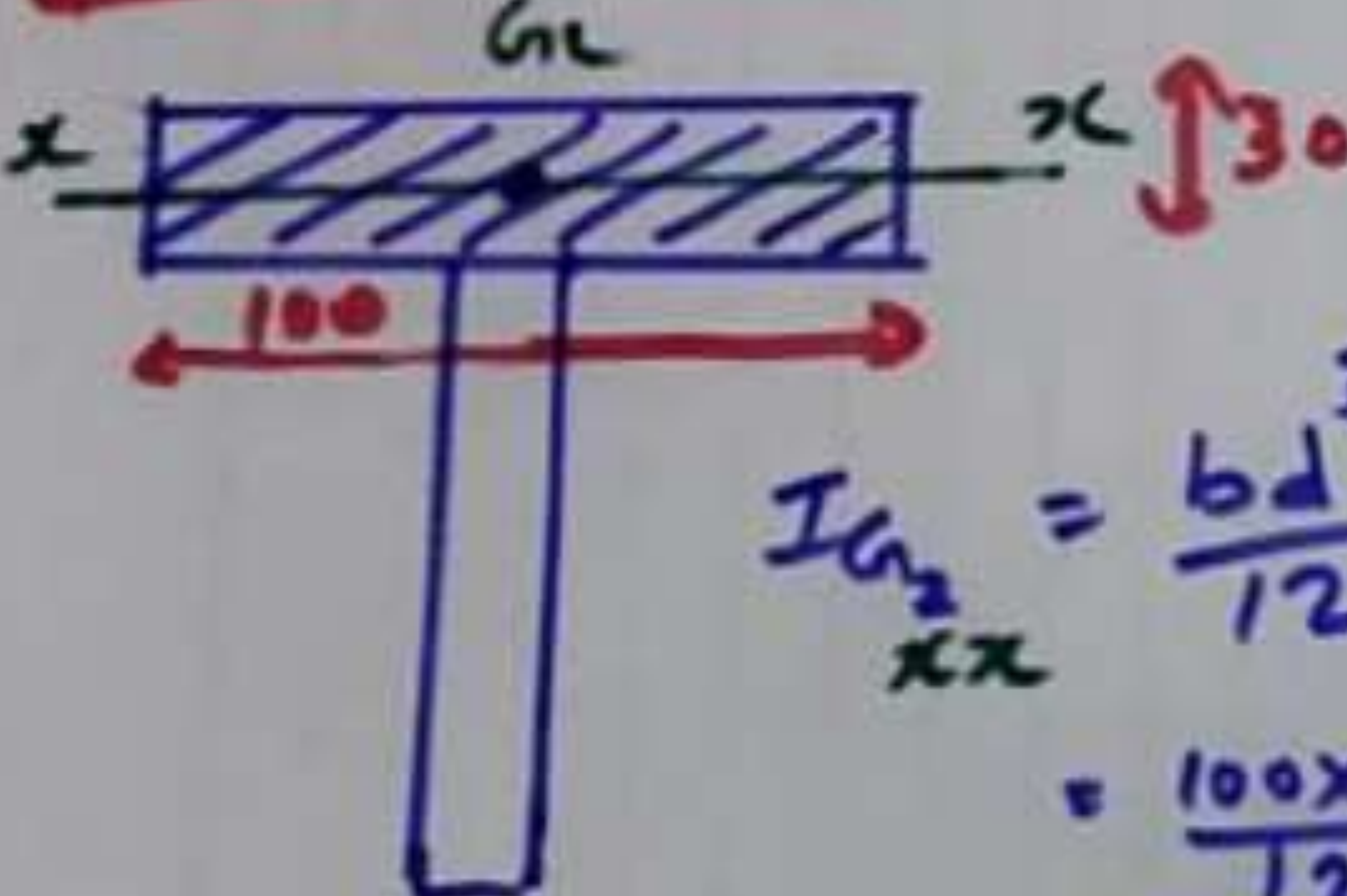
Applying parallel axis therrem in 'I' section

$$\boxed{I_{1_{xx}} = I_{G_{1_{xx}}} + A_1 h_1^2}$$

$= 853333.3 + (20 \times 80) \times 35.87^2$

$= 2.911 \times 10^6 \; mm^4$

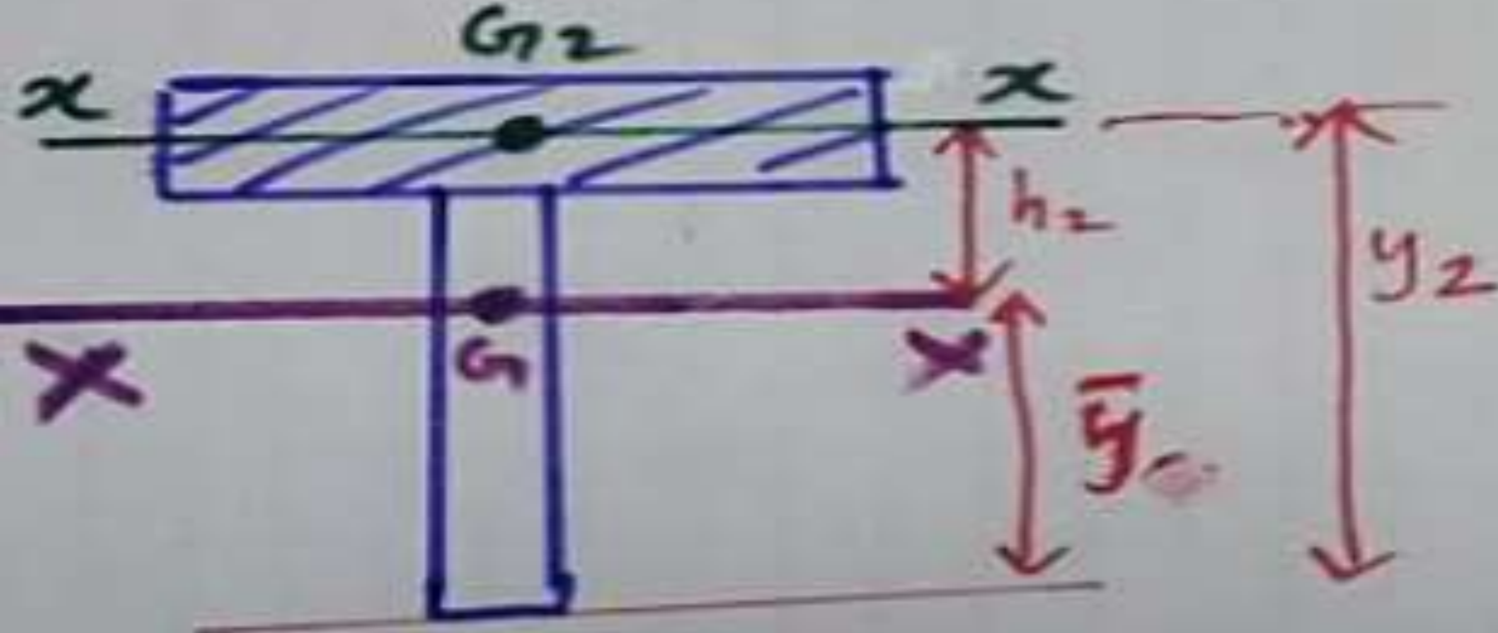
II SECTION



$$I_{G_2 xx} = \frac{bd^3}{12}$$

$$= \frac{100 \times 30^3}{12}$$

$$= 225000$$



mm^4

$$h_2 = y_2 - \bar{y} = 95 - 75.87$$
$$= 19.13 \text{ mm}$$

Applying parrallel axis theorem

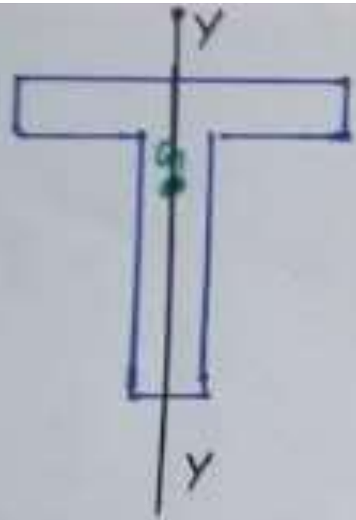
$$I_{2xx} = I_{G_2 xx} + A_2 h_2^2$$
$$= 225000 + (100 \times 30)\, 19.13^2$$
$$= 1.32 \times 10^6 \text{ mm}^4$$

$$I_{xx} = I_{1_{xx}} + I_{2_{xx}}$$

$$= 2.911 \times 10^6 + 1.32 \times 10^6 =$$

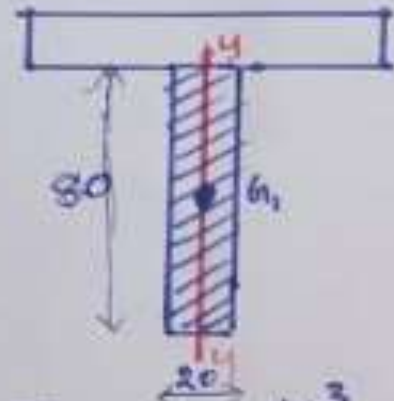
$$= 4.234 \times 10^6 \ mm^4$$

TO FIND MOMENT OF INERIAL ABOUT VERTICAL CENTROIDAL AXIS (I_{YY} = ?)



$I_{yy} = ?$

CONSIDERING I SECTION



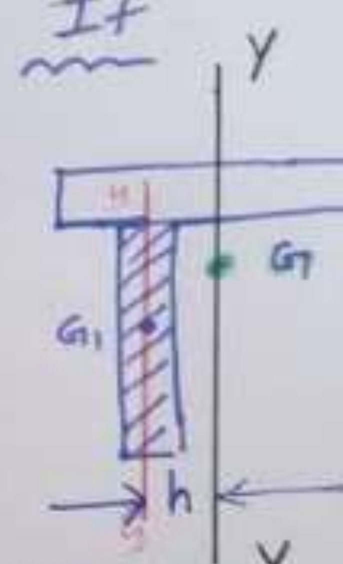
$$I_{G_1,yy} = \frac{db^3}{12}$$
$$= \frac{80\times 20^3}{12}$$
$$= 53333.33\ mm^4$$

$$\boxed{I_{1,yy} = I_{G_1,yy} + A_1 h_1^2}$$

$$= 53333.33 + (20\times 80)\,0^2$$
$$= 53333.33\ mm^4$$

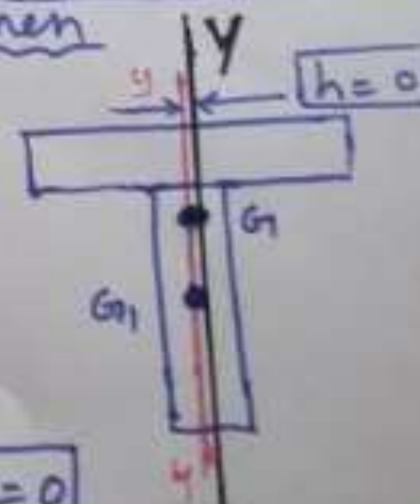
$\boxed{h_1 = 0}$

If



$\boxed{h = \text{some value}}$

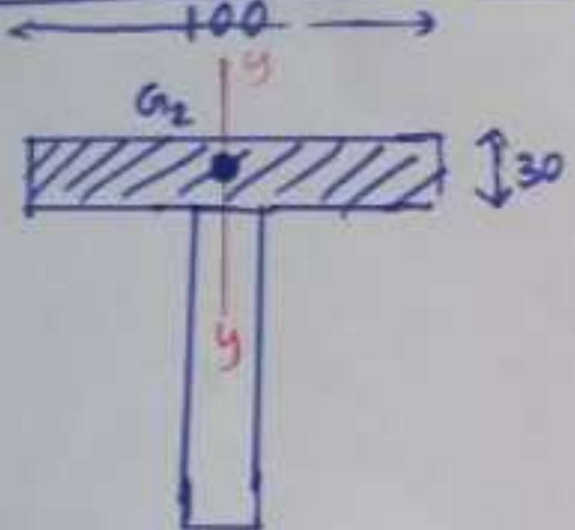
when

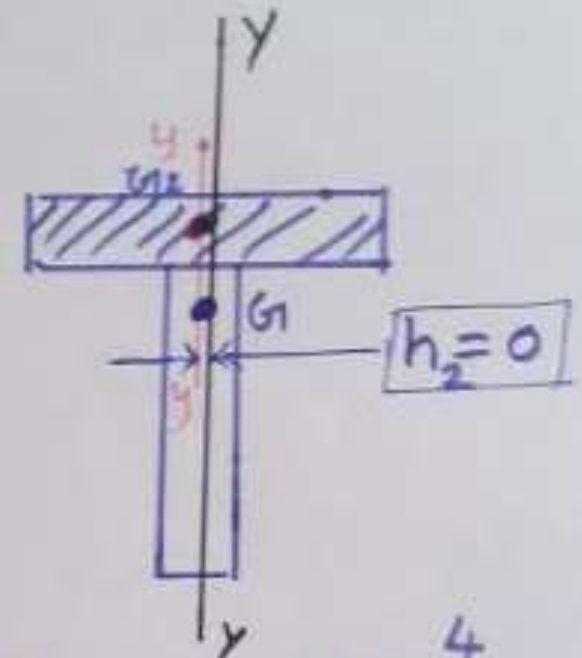


$\boxed{h = 0}$

$\boxed{h = 0}$

CONSIDERING II section





$$I_{G_{2yy}} = \frac{db^3}{12} = \frac{30 \times 100^3}{12} = 2500000 \text{ mm}^4$$

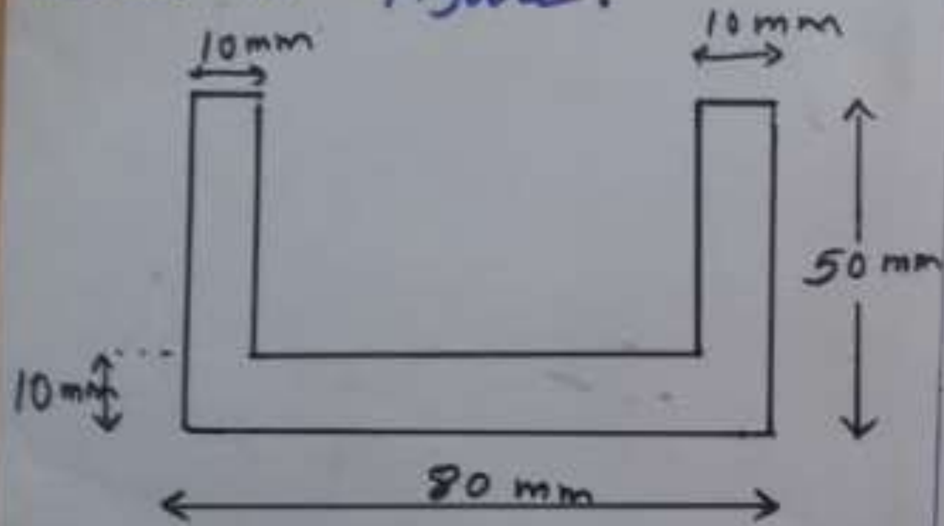
$$\boxed{I_{2yy} = I_{G_{2yy}} + A_2 h_2^2}$$

$$= 2500000 + (100 \times 30) \times 0^2 = 2500000 \text{ mm}^4$$

$$I_{yy} = I_{1yy} + I_{2yy} = 53333.33 + 2500000$$

$$= 253333 \text{ mm}^4$$

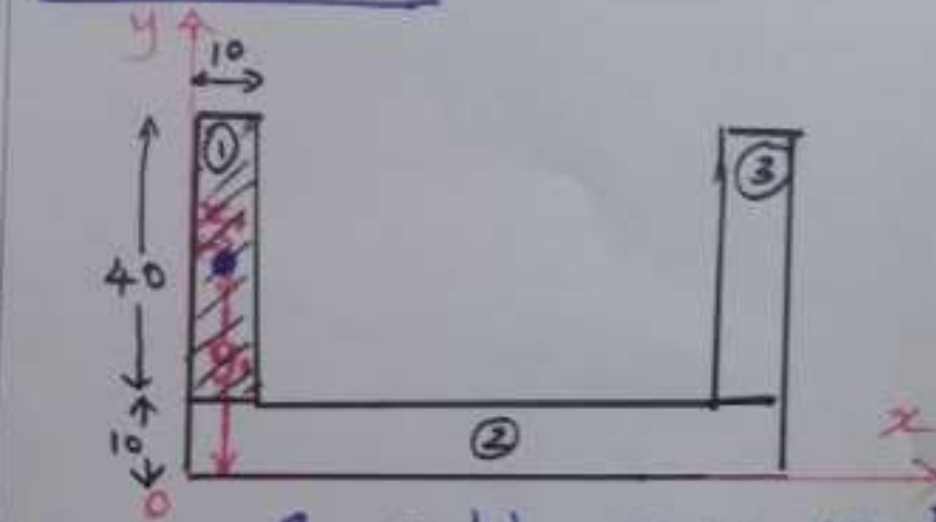
Find the Moment of inertia of channel section shown below in Figure.



As axis about which the moment of inertia is to found is not mentioned clearly , we can find the moment of inertia of the lamina about both centroidal horizontal axis and centroidal vertical axis after locating the centroid.

TO DETERMINE CENTROID FIRST

FIRST SECTION



$a_1 = bh = 10 \times 40 = 400 \text{ mm}^2$

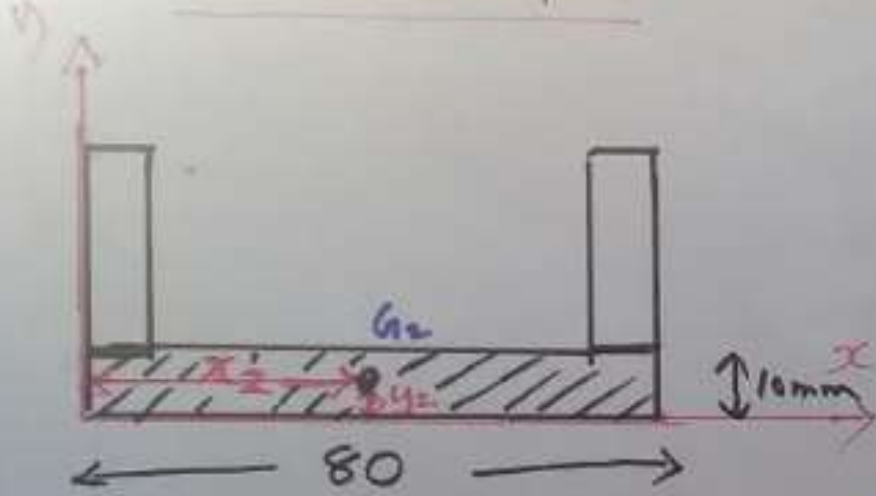
$x_1 = \frac{10}{2} = 5 \text{ mm}$

$y_1 = 10 + \left(\frac{40}{2}\right)$

$= 10 + 20$

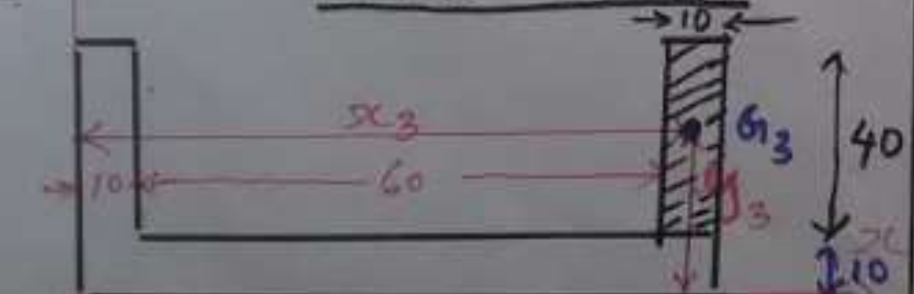
$= 30 \text{ mm}.$

SECOND SECTION



$a_2 = bh = 80 \times 10 = 800\,mm^2$
$x_2 = 80/2 = 40\,mm$
$y_2 = 10/2 = 5\,mm$

THIRD SECTION



$a_3 = bh = 10 \times 40 = 400$
$x_3 = 10 + 60 + \left(\frac{10}{2}\right) = 75$
$y_3 = 10 + \left(\frac{40}{2}\right) = 30\,mm$

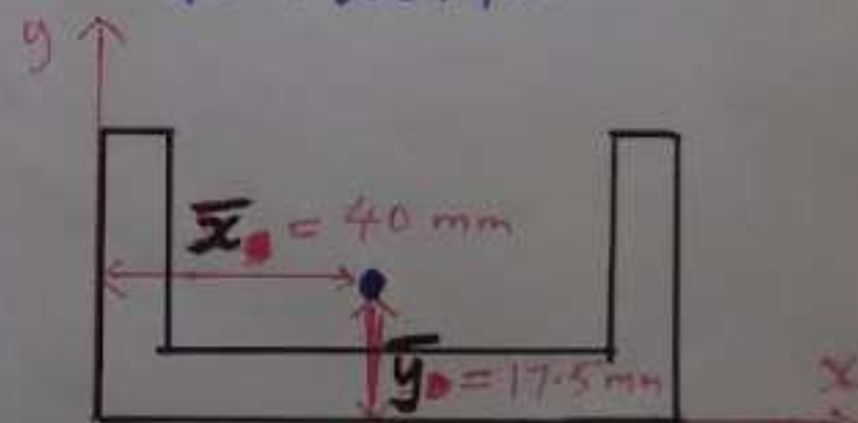
$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{(400 \times 5) + (800 \times 40) + (400 \times 75)}{400 + 800 + 400}$$

$$= 40\,mm$$

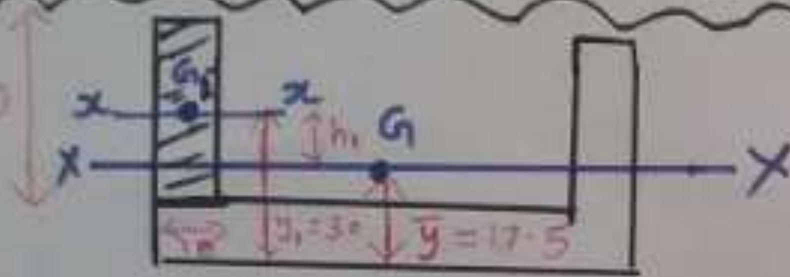
$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{(400 \times 30) + (800 \times 5) + (400 \times 30)}{400 + 800 + 400} = 17.5$$



$\bar{x} = 40\,mm$; $\bar{y} = 17.5\,mm$

TO DETERMINE I_{xx}

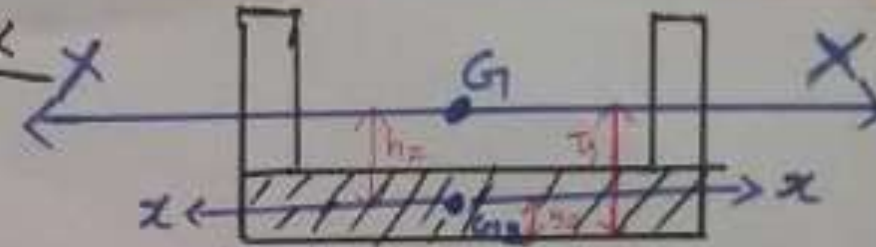


$$I_{G_1 xx} = \frac{bd^3}{12} = \frac{10 \times 40^3}{12} = 53333.33$$

$A_1 = 10 \times 40 = 400 \text{ mm}$

$h_1 = y_1 - \bar{y} = 30 - 17.5 = 12.5$

$$I_{1xx} = I_{G_1 xx} + A_1 h_1^2$$
$$= 115833.33 \text{ mm}^4$$

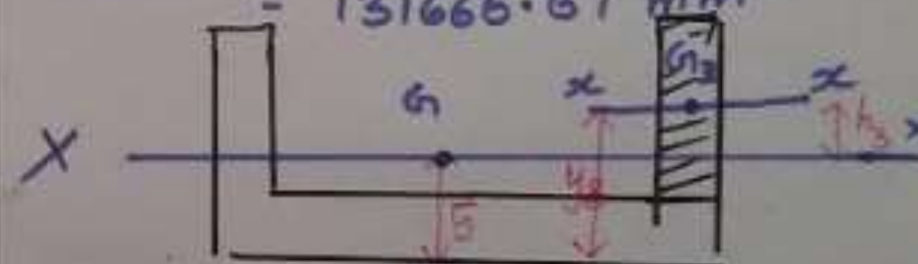


$$I_{G_2 xx} = \frac{bd^3}{12} = \frac{80 \times 10^3}{12} = 6666.67$$

$A_2 = 80 \times 10 = 800 \text{ mm}$

$h_2 = \bar{y} - y_2 = 17.5 - 5 = 12.5$

$$I_{2xx} = I_{G_2 xx} + A_2 h_2^2$$
$$= 131666.67 \text{ mm}^4$$



$$I_{G_3 xx} = bd^3/12 = \frac{10 \times 40^3}{12} = 53333.33$$

$A_2 = bh = 10 \times 40 = 400$

$h_1 = y_3 - \bar{y} = 30 - 17.5 = 12.5$

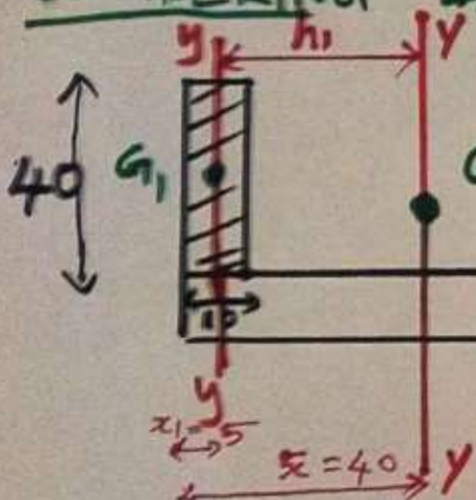
$$I_{3xx} = I_{G_3 xx} + A_3 h_3^2 = 115833.3$$

$$I_{xx} = I_{1xx} + I_{2xx} + I_{3xx}$$
$$= 363333.33$$

TO FIND MOMENT OF INERTIA ABOUT CENTROIDAL VERTICAL AXIS

$$\boxed{I_{yy} = ?}$$

CONSIDERING FIRST SECTION



$$I_{G_1 yy} = \frac{db^3}{12} = \frac{40 \times 10^3}{12} = 3333.33 \text{ mm}^4$$

$$h_1 = \bar{x} - x_1 = 40 - 5 = 35$$

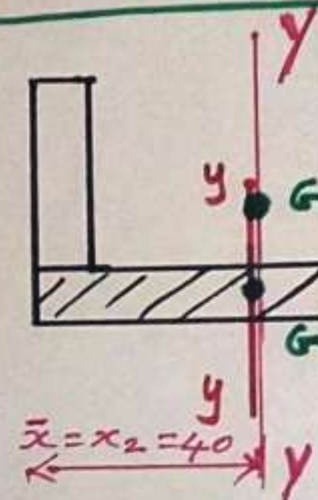
$$A_1 = 10 \times 40 = 400 \text{ mm}^2$$

$$\boxed{I_{1yy} = I_{G_1 yy} + A_1 h_1^2}$$

$$= 3333.33 + 400 \times 35^2$$

$$= 493333.33 \text{ mm}^4$$

CONSIDERING SECOND SECTION



$$I_{G_2 yy} = \frac{db^3}{12} = \frac{10 \times 80^3}{12} = 426666$$

$$h_2 = \bar{x} - x_2$$

$$= 40 - 40 = 0$$

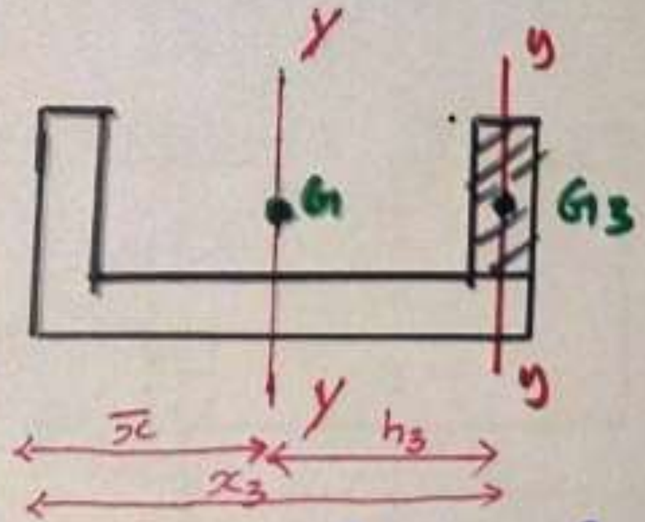
$$A_2 = bh = 80 \times 10 = 800$$

$$\boxed{I_{2yy} = I_{G_2 yy} + A_2 h_2^2}$$

$$= 426666 + (800 \times 0^2)$$

$$= 426666 \text{ mm}^4$$

Considering third section.



$$I_{G_3yy} = \frac{db^3}{12} = \frac{40\times10^3}{12} = 3333.33 \text{ mm}^4$$

$$h_3 = x_3 - \bar{x} = 75 - 40 = 30 \text{ mm}$$

$$I_{3yy} = I_{G_3yy} + A_3 h_3^2 = 3333.33 + [(40\times10)\times30^2]$$
$$= 493333.33$$

$$\boxed{I_{yy} = I_{1yy} + I_{2yy} + I_{3yy}}$$

$$= 493333.33 + 426666 + 493333.33$$
$$= 1413333.33 \text{ mm}^4$$

ENGINEERING MECHANICS UNIT- IV

DYNAMICS (portion upto CLA2)

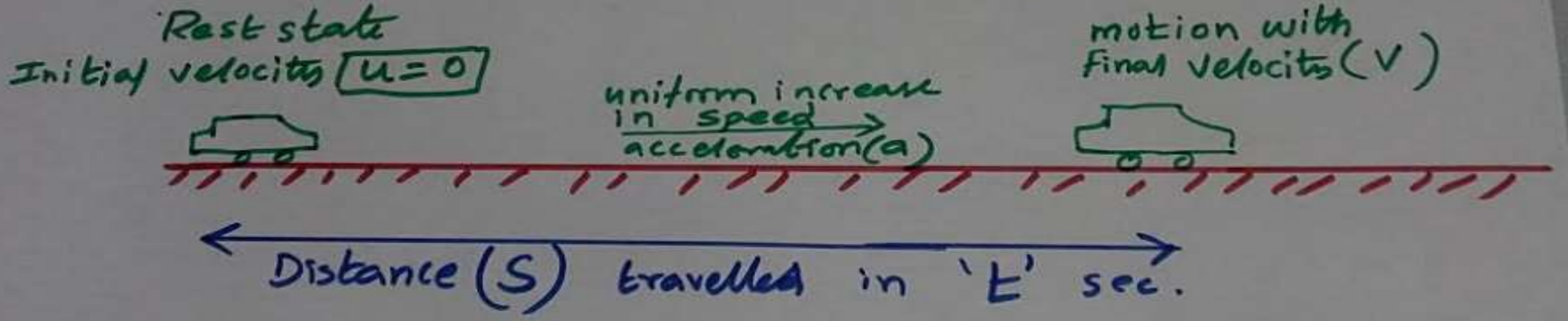
BY

V.JOSE ANANTH VINO

ASSISTANT PROFESSOR/MECH/B.I.H.E.R

EQUATIONS FOR MOTION OF VEHICLE/BODY WHICH MOVES WITH UNIFORM ACCELERATION

uniform acceleration (a) → uniform rate of increase of speed



(i) $\boxed{V = u + at}$

(ii) $\boxed{S = ut + \frac{1}{2}at^2}$

(iii) $\boxed{v^2 = u^2 + 2as}$

A Train starts from rest and attains a velocity of 45 km per hour in 2 minutes, with uniform acceleration. Calculate,
(i) acceleration *(ii) Distance travelled in this time, 2 min*
(iii) time required to reach a velocity of 36 km per hour

Solution

Given. Initial velocity, $u = 0$ ($\because$ starts from rest)

Final velocity, $v = 45$ km/hr

$$= \frac{45 \times 1000}{3600} \text{ m/s} = 12.5 \text{ m/s}$$

time taken, $t = 2$ minutes $= 2 \times 60 = 120$ sec

(i) Acceleration (a)

Using the equation, $v = u + at$

$$12.5 = 0 + (a \times 120)$$

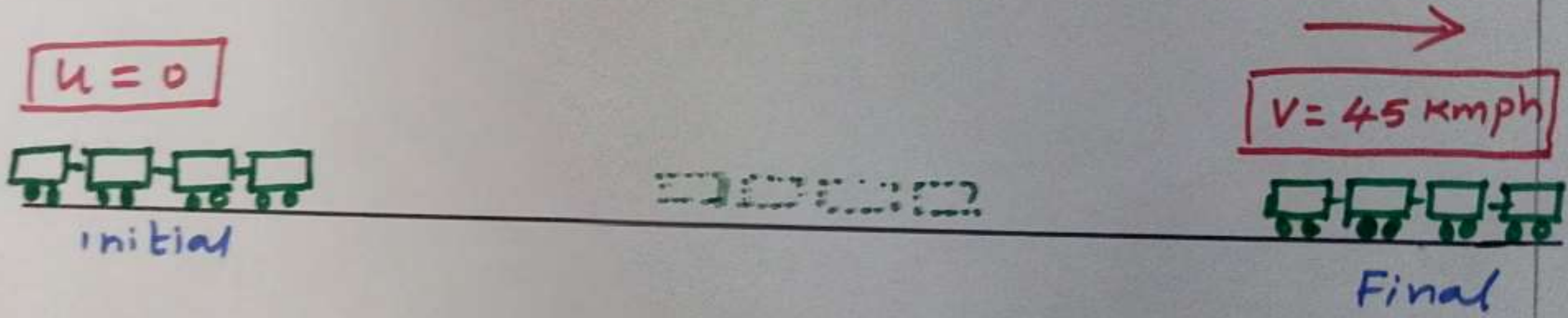
$$\therefore \quad a = \frac{12.5}{120} = 0.104 \text{ m/s}^2$$

(ii) Distance travelled (s) in 2 minutes

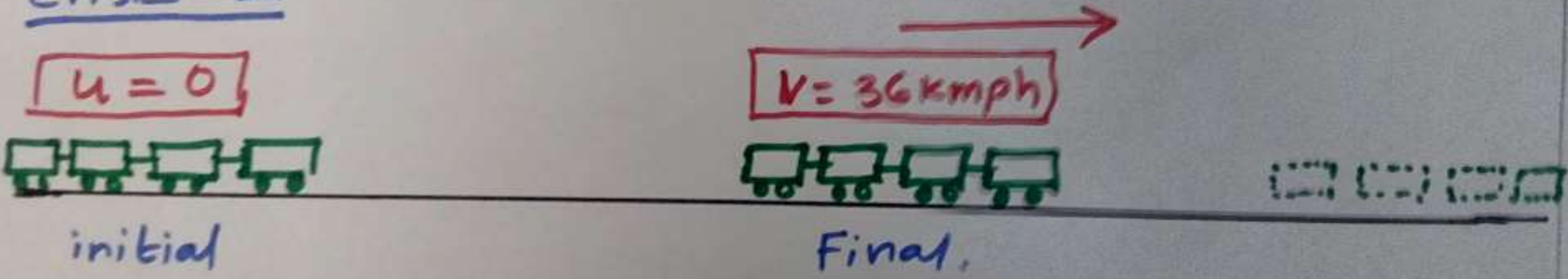
Using the eqn, $s = ut + \frac{1}{2}at^2 = (0 \times 120) + \left(\frac{1}{2} \times 0.104 \times 120^2\right)$

$$= 748.8 \text{ m}$$

CASE-1



CASE-2



(iii) Time required to attain velocity of 36 km/hr (taking case 2)

Let t be the time required by the particle, to attain velocity of 36 km/hr.

Again, initial velocity, $u = 0$

but, final velocity, $v = 36 \text{ km/hr} = \dfrac{36 \times 1000}{3600}$

$= 10 \text{ m/s}$

now, using the equation $v = u + at$

$$10 = 0 + (0.104 \times t)$$

$$\therefore \quad t = 96.15 \text{ sec}$$

A car is moving with a velocity of 20m/s. The car is brought to rest by applying brakes in 6 seconds.
Find (i) retardation (ii) Distance travelled by the car after applying the brakes.

Solution :

Given, Initial velocity, $u = 20$ m/s

Final velocity, $v = 0$ ($\because$ car is brought to rest) ; time, $t = 6$ sec

(i) Retardation

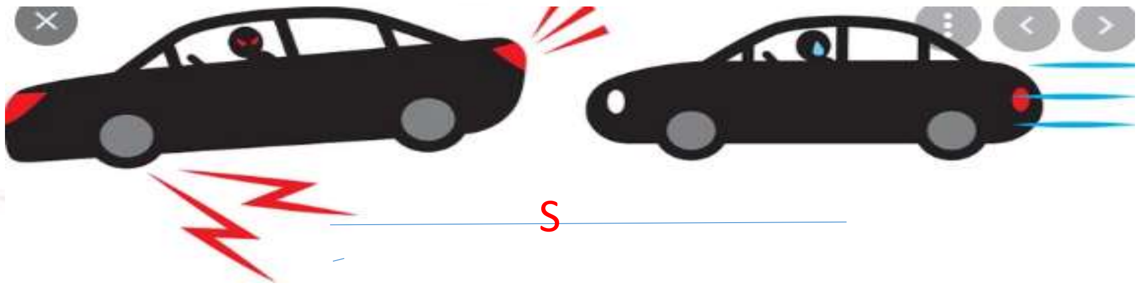
Retardation is the negative acceleration.

using the equation of motion, $v = u + at$

ie. $0 = 20 + (a \times 6)$

$\therefore \quad a = -3.33 \text{ m/s}^2$

$\therefore$ retardation $= 3.33 \text{ m/s}^2$



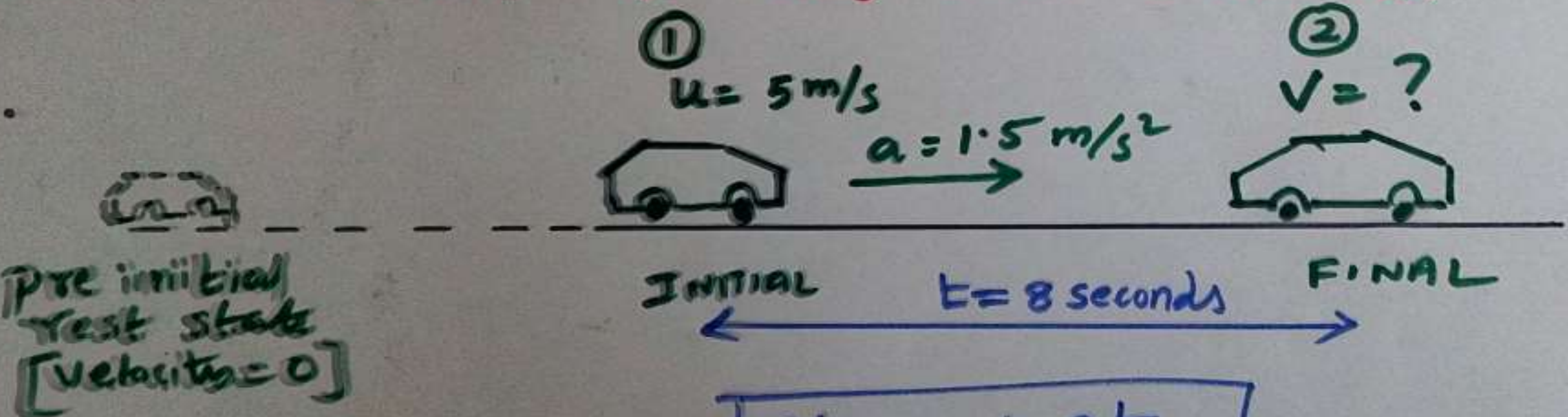
(ii) Distance travelled

Let s = Distance travelled by the car after applying the brakes

using the equation, $s = ut + \frac{1}{2} at^2$

$$s = (20 \times 6) + \frac{1}{2}(-3.33) \times 6^2 = 60 \text{ m}$$

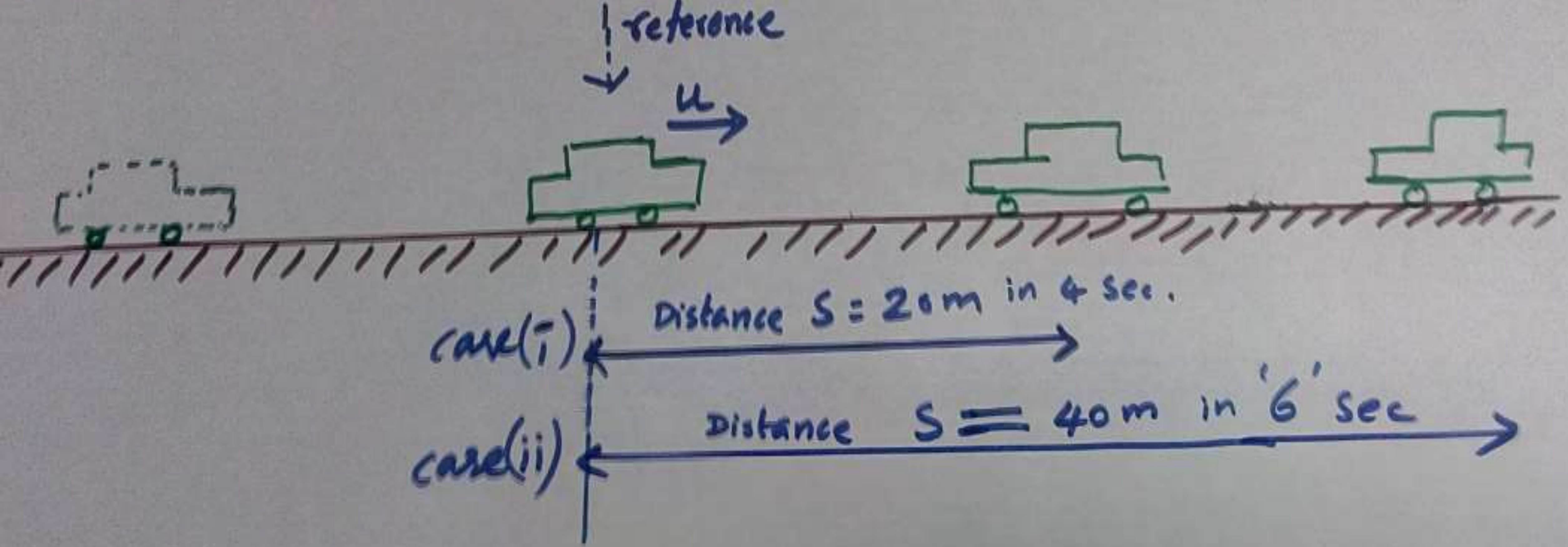
A body starts with an initial velocity of 5 m/s and moves with an uniform acceleration of $1.5\ m/s^2$. Find the velocity of body after 8 seconds



$$\boxed{V = u + at}$$

$$= 5 + (1.5 \times 8)$$
$$= 5 + 12$$
$$= 17\ m/s.$$

A motor(bike) moving with an acceleration [uniform accelerating] covers a distance of 20 m in 4 seconds and 40 m in 6 seconds. Find the uniform acceleration of the motor bike.



Given: Motor moving with an uniform acceleration

at $t = 4$ sec; $s = 20$m

and at $t = 6$ sec; $s = 40$m

acceleration = ?

Let u = Initial velocity (m/s)

a = Uniform acceleration (m/s^2)

Case (i) From equation, $s = ut + \frac{1}{2}at^2$

$$20 = u(4) + \frac{1}{2} \times a \times 4^2$$

$$20 = 4u + 8a \quad \ldots\ldots \quad (i)$$

Case (ii) Again from, $s = ut + \frac{1}{2}at^2$

$$40 = u(6) + \frac{1}{2} \times a \times 6^2$$

or $$40 = 6u + 18a \quad \ldots\ldots \quad (ii)$$

solving the equations (i) and (ii), we get

$$a = 1.667 \text{ m/s}^2$$

and $$u = 1.667 \text{ m/s}$$

$\therefore$ uniform acceleration of the motor is 1.667 m/s^2 (Ans)

NEWTON'S SECOND LAW OF MOTION

The newton second law states that the acceleration(a) of an object as produced by a net force(F) is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass (m) of the object.

$$a = F/m$$

$$F = ma$$

NEWTON'S SECOND LAW OF MOTION

- F = ma

 = m (change in velocity/time)

 = m ((v-u)/t)

 = (mv – mu)/t

 = (final momentum – initial momentum)/time

F = rate of change of momentum

Therefore newtons second law of motion can also be stated that the rate of change of momentum of a body is directly proportional to net force acting on the body

D' ALEMBERT'S PRINCIPLE

- **D'Alembert's principle, alternative form of Newton's second law of motion, stated by the 18th-century French polymath Jean le Rond d'Alembert. In effect, the principle reduces a problem in dynamics to a problem in statics. The second law states that the force F acting on a body is equal to the product of the mass m and acceleration a of the body, or F = ma; in d'Alembert's form, the force F plus the negative of the mass m times acceleration a of the body is equal to zero: F - ma = 0. In other words, the body is in equilibrium under the action of the real force F and the fictitious force -ma. The fictitious force is also called an inertial force and a reversed effective force.**

$$F = ma$$

$$F - ma = 0$$

$$F - \text{inertia force} = 0$$

IMPULSE

Impulse Defined

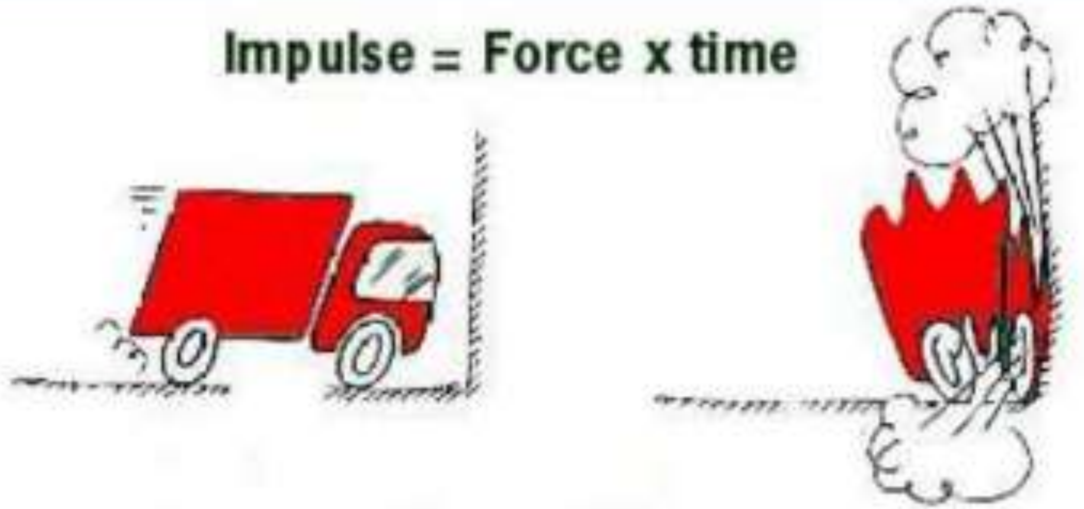
Impulse is defined as the product force acting on an object and the time during which the force acts. The symbol for impulse is *I*. So, by definition:

$$I = F\,t$$

Example: A 50 N force is applied to a 100 kg boulder for 3 s. The impulse of this force is $I = (50\text{ N})\,(3\text{ s}) = 150\text{ N}\cdot\text{s}$.

Note that we didn't need to know the mass of the object in the above example.

Impulse Example



- **<u>Example</u>**: Wall exerts a force of 10,000 N on the van. The contact time is 0.01 s. What is the impulse?
- **<u>Solve</u>**: Impulse = F * t = 10,000 * 0.01
  - Impulse = 100 N-s