23/08/2022

Proposition:- A sentence either true or false but not both is called proposition.

Eg:-

1. 2 is a prime number (T)
2. The sun rises in the East (T)
3. Two is a odd number (F)
4. Five is an odd number. (T)

compound proposition:- By adding the connectivities like $\sim$ (Negation), $\wedge$ (and), $\vee$ (or), $\rightarrow$ (implies), $\Leftrightarrow$ (By implies) of two proposition then it forms a compound proposition.

P: 2 is a prime number (T)

$\sim$P: 2 is not a prime number (F)

Eg:- $\sim P$, $P \rightarrow q$, $P \leftrightarrow q$, $P \wedge q$, $P \vee q$.

conjunction proposition:- If p, q are two propositions by adding a connectivity like "and" ($\wedge$) then the compound proposition p and q (or) $p \wedge q$ is said to be conjunction proposition. It is denoted by $p \wedge q$ reads as p and q.

The truth value of p and q is true if both p, q are true otherwise it is a false

truth table of $(p \wedge q)$:-

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction proposition :- If p, q are two propositions by adding a connectivity "or" $(\vee)$ then it forms disjunction (or)

The compound proposition $p \vee q$ reads as p or q said to be disjunction proposition

It is denoted by $p \vee q$

* The truth table value of p or q is False when both p, q are False otherwise true

truth table (of $p \vee q$) :-

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional proposition :- If p and q are two propositions by adding a connectivity → (implies to) p and q then the compound proposition $p \to q$ is said to be conditional proposition.

* It is denoted by $p \to q$ read as

p implies q or if p then q

* The truth value of $p \to q$ is false when p is true then q is false.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Byconditional proposition :- If p, q are two propositions by adding a connectivity like "By implies" ($\Leftrightarrow$) the two propositions p and q, then the compound proposition p by implies q is said to be byconditional proposition.

* It is denoted by $p \Leftrightarrow q$ reads as

p by implies q or p if and only if q

* The truth value of $p \Leftrightarrow q$ is False, when p is false q is true then $p \Leftrightarrow q$ is false.

* If q is false p is true then $p \Leftrightarrow q$ is false

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

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Well-formed formula:- By adding connectivities like ~(Negation), ∧ (and), ∨ (or), → (implies), ↔ (by implies) between two compound propositions then it forms the well-formed formula

Eg:- $(p \wedge q) \rightarrow (q \wedge p)$

$(p \wedge q) \leftrightarrow (q \vee r)$

Tautology:- The truth values of given well-formed formula all values are true's then it is said to be tautology.

Eg:- $p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

The resulting well-formed formula all values are true's

∴ Given well formed formula $p \to (p \vee q)$ is tautology.

② $\sim(p \wedge q) \vee q$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim(p \wedge q) \vee q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

The resulting well-formed formula all values are true's

∴ Given well formed formula $\sim(p \wedge q) \vee q$ is tautology

Q:- verify the well-formed formula $\sim(p \wedge q) \vee q$

contradiction :- The truth values of given well-formed formula all values are false then it is said to be contradiction.

Eg :- $p \wedge (q \wedge \sim p)$

p	q	$\sim p$	$q \wedge \sim p$	$p \wedge (q \wedge \sim p)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

all values of resulting well-formed formula is false

Given well-formed formula is contradiction.

Contigency :- A well formed formula which neither tautology nor contradiction then it is said to be contigency.

Eg :- $(p \rightarrow q) \rightarrow r$

p	q	r	$p \rightarrow q$	$(p \rightarrow q) \rightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

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Find the truth table for the following well formed formula

① $p \vee \sim p$

② $(p \wedge q) \rightarrow p$

③ $\sim p \rightarrow (p \rightarrow q)$

④ $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

⑤ $(p \wedge q) \rightarrow (p \rightarrow q)$

⑥ $(\sim p \rightarrow r) \wedge (p \Leftrightarrow q)$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
T	T	T
T	F	F

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

① $p \vee \sim p$

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

② $(p \wedge q) \rightarrow p$

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

③ $\sim p \rightarrow (p \rightarrow q)$

p	q	$\sim p$	$p \rightarrow q$	$\sim p \rightarrow (p \rightarrow q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

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④ $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	Φ
T	T	T	T	T	T	T	T
T	T	F	F	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

⑤ $(p \wedge q) \to (p \to q)$

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

∴ The given well formed formula is a tautology

⑥ $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$

p	q	r	$\sim p$	$\sim p \rightarrow r$	$p \leftrightarrow q$	Q
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	F

26/03/2022

Logically Equivalent:-

Let A, B be two statements formulas and let small $p_1, p_2, p_3, \ldots\ldots, p_n$ denotes all the variables occuring in both A & B. Consider these an assignment of truth values to $p_1, p_2, p_3, \ldots\ldots, p_n$ and resulting truth values of A and B. If the truth value of A is equal to the truth value of B for every one of the 2^n possible sets of truth values assigned to $p_1, p_2, p_3, \ldots\ldots, p_n$ then A and B are said to

Logically equivalent. ($\equiv$)

Eg:- show that the following pairs of propositions are logically equivalent $\equiv$

① $\sim(p \wedge q)$ and $\sim p \vee \sim q$

p	q	$p \wedge q$	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

(or)

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

$\therefore \sim(p \wedge q) \equiv \sim p \vee \sim q$

② show that $p \wedge q$ and $q \wedge p$ are logically equivalent (commutative law).

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$\therefore p \wedge q \equiv q \wedge p$

$\frac{0}{1} = 0$

$1^0 = 1$

$a^0 = 1$

$e^0 = 1$

$\frac{1}{0} = \infty$

$\frac{1}{\infty} = 0$

$e^\infty = \infty$

$e^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0$

③ $(p \vee q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$

(Demorgans law)

p	q	r	$(p \vee q)$	$(p \vee q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

④ $(p \to q) \Leftrightarrow (\neg p \vee q)$

p	q	$\sim p$	$p \to q$	$\sim p \vee q$	$(p \to q) \Leftrightarrow (\sim p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Given well formed formula is tautology

Every tautology is a logically equivalent

$$p \to q \equiv \sim p \vee q$$

Algebra propositions:-

1. Idempotent law:-

ⓐ $p \vee p \equiv p$

ⓑ $p \wedge p \equiv p$

2. Associative law:-

ⓐ $(p \vee q) \vee r \equiv p \vee (q \vee r)$

ⓑ $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

3. Distributive law:-

ⓐ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

ⓑ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

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④ Identity law :-

ⓐ $p \vee F \equiv p$

ⓑ $p \vee T \equiv T$

ⓒ $p \wedge T \equiv p$

ⓓ $p \wedge F \equiv F$

⑤ complement law :-

ⓐ $p \vee \sim p \equiv T$

ⓑ $\sim T \equiv F$

ⓒ $p \wedge \sim p \equiv F$

ⓓ $\sim F \equiv T$

⑥ Involution law :-

$\sim(\sim p) \equiv p$

⑦ Demorgan's law :-

ⓐ $\sim(p \vee q) \equiv \sim p \wedge \sim q$

ⓑ $\sim(p \wedge q) \equiv \sim p \vee \sim q$

$p \Leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

bles

$\wedge(p \vee r)$
T
T
T
T
T
=
=
=

by using

prove distributive law using truth tables

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

prove that $p \vee (p \wedge q) \equiv p$ or $p \wedge (p \vee q) \equiv p$ by using laws

$p \vee (p \wedge q) \equiv (p \wedge T) \vee (p \wedge q)$ $\qquad$ $(p \vee p) \wedge (p \vee q)$

$\equiv p \wedge (T \vee q)$ $\qquad$ $p \wedge (p \vee q)$

$\equiv p \wedge T$ $\quad$ ($\because$ Identity law) $\qquad$ $(p \wedge p) \vee (p \wedge q)$

$= p$ $\qquad$ $(p \wedge p) \vee (p \wedge q)$

$p \vee (p \wedge q)$

S.NO	symbol used	connective word	nature of the compound statement	symbolic form
1	$\sim, \backsim, \neg, N$	Not	Negation	$\sim(P)$
2	$\wedge$	and	conjunction	$P \wedge q$
3	$\vee$	or	disjunction	$P \vee q$
4	$\rightarrow$	if.....then	conditional	$P \rightarrow q$
5	$\leftrightarrow$	if and only if	Bi-conditional	$P \leftrightarrow q$

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P: paris is in France ⇒ write $\sim P$

$\sim P$: paris is not in France

Normally, it can be written as NOT P: paris is not in France and also it can be written as

$\sim P$: It is not the case that paris is in France

<u>Note</u>: The following propositions all have the same meaning

p: all people are intelligent

q: every person is intelligent

r: each person is intelligent

s: any person is intelligent

④ Write all following negation of the proposition

p: all students are intelligent.

Sol: $\sim P$: some students are not intelligent

$\sim P$: There exist a student who is not intelligent

$\sim P$: atleast one student is not intelligent

② q: no student is intelligent

$\sim q$: some students are intelligent

Negation:-

$\sim(\sim p) \equiv p$

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

$\sim(p \rightarrow q) \equiv \sim p \vee q \;/\; p \wedge \sim q$

$\sim(p \leftrightarrow q) \equiv [p \wedge (\sim q)] \vee [q \wedge (\sim p)]$

Methods of proofs:-

1. **Direct proof:-** We are typically faced with a set of Hypothesis $H_1, H_2, H_3, \ldots\ldots, H_n$ from which we want to in for a conclusion 'c'.

One of the most natural sorts of proof is, The direct proof in which we show

$H_1 \wedge H_2 \wedge H_3 \wedge \ldots\ldots\ldots H_n \longrightarrow c$

Eg:- Prove that the product of 2 odd integers is an odd integer

$p: 2n+1$

$q: 2m+1$

$pq: (2n+1)(2m+1)$

$= 4mn + 2n + 2m + 1$

$= 2(\underbrace{2mn + m + n}_{m}) + 1$

$= 2m + 1 \longrightarrow$ odd

Hence the product of two odd integers is an odd integers.

Let $p = 2n-1$ & $q = 2m-1$ are two odd integers then their products.

$$pq = (2n-1)(2m-1)$$

$$= 4mn - 2n - 2m + 1$$

$$= 2\underbrace{(2m-n-m)}_{m} + 1$$

$$= 2m+1 \Rightarrow \text{odd}$$

Thus the product of two odd integers is an odd integer

$p = 2m$, $q = 2n \rightarrow$ even integers

<u>Note</u>:- The product of 2 even integers is an even integers.

Indirect methods:-

The proofs which are not direct is called indirect proofs.

Indirect proofs are 2 types

1. contradiction
2. contrapositive

Example for contradiction:-

prove that $\sqrt{2}$ is an irrational number by using contradiction proof.

proof:- Let us Assume $\sqrt{2}$ is an rational

$\sqrt{2} = \frac{a}{b}$ $b \neq 0$, a & b not have common factor

squaring on both sides.

$$2 = \frac{a^2}{b^2}$$

$$a^2 = 2b^2 \rightarrow (1)$$

it is multiple of 2

a^2 is a Even

a is also Even

$\therefore a = 2k$

from (1) $\Rightarrow (2k)^2 = 2b^2$

$$\cancel{4}^{2} k^2 = \cancel{2} b^2$$

$$b^2 = 2k^2$$

b^2 is an Even

b is also even

$b = 2k$

a and b have a common factor that is 2

$\therefore \sqrt{2}$ is rational is false

$\therefore \sqrt{2}$ is an irrational.

Example for contrapositive:-

prove that if $m+n >= 73$ than $m \geq 73$ or $n \geq 37$

proof:- here m, n are two integers & it seems to number

here prove the given fact directly

In straight can one proof by taking the contrapositive "not".

i-e $m \geq 73$ or $n \geq 37 \Rightarrow$ "$m+n \geq 73$"

by demorgans laws ~~the~~

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

not $n \geq 37$ means that $m \leq 36$ & $n \leq 36$

so that the contrapositive proposition is

if $m \leq 36$ than $m+n \leq 72$. This follower immediately from the property of any quality.

if $a \leq c$ & $b \leq d \Rightarrow a+b \leq c+d$.

for all real numbers a, b, c & d.

31/03/2022

Converse, contrapositive, & inverse:-

converse:- The converse of $p \rightarrow q$ is $q \rightarrow p$

contrapositive:- The contrapositive of $p \rightarrow q$ is

$$\sim q \longrightarrow \sim p.$$

Inverse:- The inverse of $p \to q$ is $\sim p \to \sim q$

proposition p: It rains

q: The crops will grow.

conditional proposition $p \to q$ is

If it rains then the crop will grow

converse: $q \to p$

If crops grow then their has been rained

contrapositive: $\sim q \to \sim p$

If crops do not grow then their has been no rain.

Inverse: $\sim p \to \sim q$

If doesn't rain then the crops will not grow.

p	q	$p \to q$	$q \to p$	$\sim p$	$\sim q$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Note: 1. $p \to q \equiv \sim q \to \sim p$ (conditional preposition and contrapositive logically equivalent)

$q \to p \equiv \sim p \to \sim q$.

(converse proposition and Inverse is logically equivalent)

prove that if x^2 is divisible by 4 then 'x' is even

Let p & q are proposition such that p is x^2 is divisible by 4 & q is 'x' is even.

∴ The conditional proposition of the form is $p \to q$ & contrapositive is $\sim q \to \sim p$ that is

If x is odd then x^2 is not divisible by 4.

x is odd then $\to 2m+1$

then $x^2 = (2m+1)^2$

$= (2m)^2 + 2(2m)(1) + 1$

$= 4m^2 + 4m + 1$

$= 4(m^2 + m + 1/4)$

$m^2 + m$ an integer but $m^2 + m + 1/4$ is not integer x^2 is not divisible by 4.

Normal forms :-

Normal forms are 2 types

1. disjunctive normal form (DNF)
2. conjunctive normal form

1. Disjunctive normal form (DNF) :-

- In an logical expression a product of variables and their negations is called elementary Prod

Eg:- $p \wedge \sim q$, $\sim p \wedge q$, $\sim p \wedge \sim q$ are elementary products in two variables.

A sum of variables and their negations is called an elementary sum.

Eg:- $p \vee \sim q$, $\sim p \vee q$, $\sim p \vee \sim q$

A logical apression is said to be in DNF "if it is the sum of elementary products"

Eg:- $p \vee (q \wedge \sim r)$

procedure :-

step 1:- remove all $\rightarrow$ and $\leftrightarrow$ an equivalent expression containing the connectivities $\vee, \wedge, \sim$ only

step 2:- eliminate negation before sums and products using the double negation. or using demorgan's law

Step 3: apply the distributive law until a sum of elementary product is obtain

Egs: Obtain the DNF of the following

ⓐ $p \wedge (p \to q)$

ⓑ $p \vee (\sim p \to (q \vee (q \to \sim r)))$

ⓒ $p \to [(p \to q) \wedge \sim(\sim q \vee \sim p)]$

ⓐ $p \wedge (p \to q)$

$p \wedge (\sim p \vee q)$ $\quad [\because p \to q \equiv \sim p \vee q$ or $p \wedge (\sim q)]$

$(p \wedge \sim p) \vee (p \wedge q)$ $\quad$ ($\because$ distributive law)

which is required DNF

ⓑ $p \vee (\sim p \to (q \vee (q \to \sim r)))$

$p \vee (\sim p \to (q \vee (\sim q \vee \sim r)))$ $\quad (p \to q = \sim p \vee q)$

$p \vee (\sim(\sim p)) \vee (q \vee (\sim q \vee \sim r))$

$p \vee p \vee q \vee \sim q \vee \sim r$

$p \vee q \vee \sim q \vee \sim r$

③ $p \to [(p \to q) \wedge \sim(\sim q \vee \sim p)]$

$\sim p \vee [(p \to q) \wedge \sim(\sim q \vee \sim p)]$

$\sim p \vee [(\sim p \vee q) \wedge \sim(\sim q \vee \sim p)]$

$\sim p \vee [(\sim p \vee q) \wedge (q \wedge p)]$

$\sim p \vee [(\sim p \wedge (q \wedge p)) \vee (q \wedge (q \wedge p))]$

$\sim p \vee [((\sim p \wedge q) \wedge p) \vee ((q \wedge q) \wedge p)]$

$= \sim p \vee [F \vee (q \wedge p)]$

$= \sim p \vee [F \vee (p \wedge q)]$

$= \sim p \vee (p \wedge q)$

06/04/2023

Q:- $p \wedge \sim(q \wedge r)$

$p \wedge (\sim q \vee \sim r)$ Demorgans law

$(p \wedge \sim q) \vee (p \wedge \sim r)$ (Distributive law)

Which is required DNF form

Q:- $(p \to q) \wedge (q \to r)$

$\underbrace{(\sim p \vee q)}_{A} \wedge \underbrace{(\sim q \vee r)}_{B \quad C}$

$\times \to (\sim p \wedge (\sim q \vee r)) \vee (q \wedge (\sim q \vee r))$

$((\underbrace{\sim p \vee q}_{A}) \wedge \underbrace{\sim q}_{B}) \vee (\underbrace{(\sim p \vee q) \wedge r}_{C})$

$(A \vee B) \wedge C \equiv (A \wedge C) \vee (B \wedge C)$

$((\sim p \wedge \sim q) \vee (q \wedge \sim q)) \vee ((\sim p \wedge r) \vee (q \wedge r))$

Which is required DNF.

Q:- $\neg(p \vee q) \leftrightarrow (p \wedge q)$ $\qquad p \leftrightarrow q \equiv$

$$\boxed{p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)}$$

$$\underbrace{\sim(p \vee q)}_{p} \leftrightarrow \underbrace{(p \wedge q)}_{q}$$

$= (\sim(p \vee q) \wedge (p \wedge q)) \vee ((p \vee q) \wedge \sim(p \wedge q))$

$$\boxed{\begin{aligned} \sim(p \vee q) &= \sim p \wedge \sim q \\ \sim(p \wedge q) &= \sim p \vee \sim q \end{aligned} \quad \text{Demorgans law}}$$

$((\sim p \wedge \sim q) \wedge (p \wedge q)) \vee ((\underset{A}{p \vee q}) \wedge (\underset{B}{\sim p} \vee \underset{C}{\sim q}))$

$((\sim p \wedge \sim q) \wedge (p \wedge q)) \vee [(p \vee q \wedge \sim p) \vee (p \vee q \wedge \sim q)]$

$= [(\sim p \wedge \sim q) \wedge (p \wedge q)] \vee ((p \wedge \sim p) \vee (q \wedge \sim p)) \vee ((p \wedge \sim q) \vee (q \wedge \sim q))$

$(\sim p \wedge \sim q \wedge (p \vee q)) \vee (p \wedge \sim p) \vee (q \wedge \sim p) \vee (p \wedge \sim q) \vee (q \wedge \sim q)$

Q2 $\sim(p \vee q)$

$(p \rightarrow q) \wedge \sim(\sim q \vee \sim p)$

$(\sim p \vee q) \wedge \sim(\sim q \vee \sim p)$

$\underset{A}{(\sim p \vee q)} \wedge \underset{C}{(q \wedge p)}$

$= \underset{A}{(q \wedge p)} \wedge \underset{B}{(\sim p \vee q)}$

$= ((q \wedge p) \wedge \sim p) \vee ((q \wedge p) \wedge q)$

$= (q \wedge p \wedge \sim p) \vee (q \wedge p \wedge q)$

principle disjunctive Normal form (PDNF):-

Let P, Q are two statement variables then $P \wedge Q$, $P \wedge \sim Q$, $\sim P \wedge Q$, $\sim P \wedge \sim Q$ are called "min terms" of P and Q for given two variables, there exist the possible min terms are 2^n i.e $2^2 = 4$.

Suppose P, Q, R are three statement variables then their exist the possible minterms are $2^n = 2^3 = 8$ those are

$(P \wedge Q \wedge R)$, $(P \wedge Q \wedge \sim R)$, $(P \wedge \sim Q \wedge R)$, $(\sim P \wedge Q \wedge R)$

$(\sim P \wedge \sim Q \wedge R)$, $(\sim P \wedge Q \wedge \sim R)$, $(P \wedge \sim Q \wedge \sim R)$, $(\sim P \wedge \sim Q \wedge \sim R)$

Q:- $\sim(p \vee q)$

$(p \to q) \wedge \sim(\sim q \vee \sim p)$

$(\sim p \vee q) \wedge \sim(\sim q \vee \sim p)$

$\underset{A}{(\sim p \vee q)} \wedge \underset{B}{(q \wedge p)}$

$= \underset{A}{(q \wedge p)} \wedge \underset{B}{(\sim p \vee q)}$

$= ((q \wedge p) \wedge \sim p) \vee ((q \wedge p) \wedge q)$

$= (q \wedge p \wedge \sim p) \vee (q \wedge p \wedge q)$

principle disjunctive Normal form (PDNF):-

Let P, Q are two statement variables then $P \wedge Q$, $P \wedge \sim Q$, $\sim P \wedge Q$, $\sim P \wedge \sim Q$ are called "min terms" of P and Q for given two variables, there exist the possible min terms are 2^n i.e $2^2 = 4$.

Suppose P, Q, R are three statement variables then their exist the possible minterms are $2^n = 2^3 = 8$ those are

$(P \wedge Q \wedge R)$, $(P \wedge Q \wedge \sim R)$, $(P \wedge \sim Q \wedge R)$, $(\sim P \wedge Q \wedge R)$

$(\sim P \wedge \sim Q \wedge R)$, $(\sim P \wedge Q \wedge \sim R)$, $(P \wedge \sim Q \wedge \sim R)$, $(\sim P \wedge \sim Q \wedge \sim R)$

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PNDF :-

p	q	minterms
T	T	$p \wedge q$
T	F	$p \wedge \sim q$
F	T	$\sim p \wedge q$
F	F	$\sim p \wedge \sim q$

P, Q are two statement variables then conjunction operation is

p	q	minterms
T	T	$p \wedge q$
T	F	$p \wedge \sim q$
F	T	$\sim p \wedge q$
F	F	$\sim p \wedge \sim q$

Methods to find PDNF :-

1. Using truthtable

Q Find PDNF of $p \rightarrow q$

p	q	$p \rightarrow q$	minterms
T	T	T	$p \wedge q$
T	F	F	-
F	T	T	$\sim p \wedge q$
F	F	T	$\sim p \wedge \sim q$

PDNF is disjunction of minterms.

PDNF is $(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$.

Q:- Find PDNF of $(\underset{x}{p \wedge Q}) \vee (\underset{y}{\sim p \wedge R}) \vee (\underset{z}{Q \wedge R})$

p	Q	R	$\sim p$	$p \wedge Q$	$\sim p \wedge R$	$Q \wedge R$	$x \vee y \vee z$	minterms
T	T	T	F	T	F	T	T	$p \wedge Q \wedge R$
T	T	F	F	T	F	F	T	$p \wedge Q \wedge \sim R$
T	F	T	F	F	F	F	F	–
T	F	F	F	F	F	F	F	–
F	T	T	T	F	T	T	T	$\sim p \wedge Q \wedge R$
F	T	F	T	F	F	F	F	–
F	F	T	T	F	T	F	T	$\sim p \wedge \sim Q \wedge R$
F	F	F	T	F	F	F	F	–

PDNF is disjunctions of minterms

PDNF is $(p \wedge Q \wedge R) \vee (p \wedge Q \wedge \sim R) \vee (\sim p \wedge Q \wedge R) \vee (\sim p \wedge \sim Q \wedge R)$

method :-

procedure:-

1. If the statement contain implication ($\rightarrow$), By implication $\leftrightarrow$, we have to replace them by using equivalent connectivity by using $\wedge, \vee, \sim$ only.

Step ②: If Negation is part of the problem or entire problem then use demorgan's law.

Step ③: Apply distributive law to obtain DNF

Step ④: If elementary product contains contradiction then we have to drop that product

Step ⑤: If any proposition is missing then we have to add proposition by using complementary law with help of $(P \vee \sim P)$ or $(Q \vee \sim Q)$ or $(R \vee \sim R)$ .....

Step ⑥: Remove Identical minterms.

Q: Find ~~$P \rightarrow Q$~~ PDNF of $P \rightarrow Q$ without using it.

P	Q	$P \rightarrow Q$	minterms
T	T	T	$P \wedge Q$
T	F	F	—
F	T	T	$\sim P \wedge Q$
F	F	T	$\sim P \wedge \sim Q$

Given $P \rightarrow Q$

$\Leftrightarrow \sim P \vee Q$

$\Leftrightarrow (\sim P \wedge T) \vee (Q \wedge T)$

$\Leftrightarrow (\underset{A}{\sim P} \wedge \underset{B}{(Q \vee \sim Q)}) \vee (\underset{C}{Q \wedge (P \vee \sim P)})$

$\Leftrightarrow (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (Q \wedge P) \vee (Q \wedge \sim P)$

$\Leftrightarrow (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q) \vee (\sim P \wedge Q)$

$\Leftrightarrow (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (P \wedge Q)$

Q:- Find PDNF of $(P\wedge Q)\vee(\sim P\wedge R)\vee(Q\wedge R)$ without ##

Given $(P\wedge Q)\vee(\sim P\wedge R)\vee(Q\wedge R)$

$\Leftrightarrow (P\wedge Q\wedge(R\vee\sim R))\vee(\sim P\wedge R\wedge(Q\vee\sim Q))\vee(Q\wedge R\wedge(P\vee\sim$

$\Leftrightarrow [(P\wedge Q\wedge R)\vee(P\wedge Q\wedge\sim R)]\vee[(\sim P\wedge R\wedge Q)\vee(\sim P\wedge R\wedge\sim Q)]\vee$

$[(Q\wedge R\wedge P)\vee(Q\wedge R\wedge\sim P)]$

$\Leftrightarrow (P\wedge Q\wedge R)\vee(P\wedge Q\wedge\sim R)\vee(\sim P\wedge Q\wedge R)\vee(\sim P\wedge\sim Q\wedge R)$

$\vee(P\wedge Q\wedge R)\vee(\sim P\wedge Q\wedge R)$

$\Leftrightarrow (P\wedge Q\wedge R)\vee(P\wedge Q\wedge\sim R)\vee(\sim P\wedge Q\wedge R)\vee(\sim P\wedge\sim Q\wedge R)$

Q:- Find PDNF of $P\vee(\sim P\rightarrow(\sim Q\rightarrow R))$

Given, $P\vee(\sim P\rightarrow(\sim Q\rightarrow R))$

$\Leftrightarrow P\vee(\sim P\rightarrow\sim Q\rightarrow R))$

$\Leftrightarrow P\vee(\sim P\rightarrow(Q\vee R))$

$\Leftrightarrow P\vee(P\vee(Q\vee R))$

$\Leftrightarrow (P\vee P)\vee(Q\vee R)$

$\Leftrightarrow P\vee Q\vee R$

$\Leftrightarrow (P\wedge T\wedge T)\vee(Q\wedge T\wedge T)\vee(R\wedge T\wedge T)$

case ①

$P\wedge T\wedge T\equiv(P\wedge(Q\vee\sim Q)\wedge(R\vee\sim R))$

$\equiv(P\wedge Q)\vee(P\wedge\sim Q)$

$\equiv(P\wedge Q\wedge(R\vee\sim R))\vee(P\wedge\sim Q\wedge(R\vee\sim R))$

$\equiv (P\wedge Q\wedge R)\vee(P\wedge Q\wedge\sim R)\vee(P\wedge\sim Q\wedge R)\vee$

$(P\wedge\sim Q\wedge\sim R)$

<u>case ②</u> :-

$Q\wedge T\wedge T$

$\equiv Q\wedge T$

$\Leftrightarrow Q\wedge(P\vee\sim P)$

$\Leftrightarrow ((Q\wedge P)\vee(Q\wedge\sim P))$

$\Leftrightarrow ((Q\wedge P\wedge T)\vee(Q\wedge\sim P\wedge T))$

$\Leftrightarrow (Q\wedge P\wedge(R\vee\sim R))\vee(Q\wedge\sim P\wedge(R\vee\sim R))$

$\Leftrightarrow (Q\wedge P\wedge R)\vee(Q\wedge P\wedge\sim R)\vee(Q\wedge\sim P\wedge R)\vee(Q\wedge\sim P\wedge\sim R)$

$\Leftrightarrow (P\wedge Q\wedge R)\vee(P\wedge Q\wedge\sim R)\vee(\sim P\wedge Q\wedge R)\vee(\sim P\wedge Q\wedge\sim R)$

<u>case ③</u> :-

$R\wedge T\wedge T$

$R\wedge T$

$\Leftrightarrow R\wedge(P\vee\sim P)$

$\Leftrightarrow (R\wedge P)\vee(R\wedge\sim P)$

$\Leftrightarrow (R\wedge P\wedge T)\vee(R\wedge\sim P\wedge T)$

$\Leftrightarrow (R\wedge P\wedge(Q\vee\sim Q))\vee(R\wedge\sim P\wedge(Q\vee\sim Q))$

$\Leftrightarrow (R\wedge P\wedge Q)\vee(R\wedge P\wedge\sim Q)\vee(R\wedge\sim P\wedge Q)\vee(R\wedge\sim P\wedge\sim Q)$

$\Leftrightarrow (P\wedge Q\wedge R)\vee(P\wedge\sim Q\wedge R)\vee(\sim P\wedge Q\wedge R)\vee(\sim P\wedge\sim Q\wedge R)$

$P\vee Q\vee R \equiv (P\wedge Q\wedge R)\vee(P\wedge Q\wedge\sim R)\vee(P\wedge\sim Q\wedge R)\vee(P\wedge\sim Q\wedge\sim R)$

$\vee(P\wedge Q\wedge R)\vee(P\wedge Q\wedge\sim R)\vee(\sim P\wedge Q\wedge R)\vee(\sim P\wedge Q\wedge\sim R)$

$\vee(P\wedge Q\wedge R)\vee(P\wedge\sim Q\wedge R)\vee(\sim P\wedge Q\wedge R)\vee(\sim P\wedge\sim Q\wedge R)$

$\equiv (P\wedge Q\wedge R)\vee(P\wedge Q\wedge\sim R)\vee(P\wedge\sim Q\wedge R)\vee(\sim P\wedge Q\wedge R)\vee$

$(P\wedge\sim Q\wedge\sim R)\vee(\sim P\wedge Q\wedge\sim R)\vee(\sim P\wedge\sim Q\wedge R)$

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$P \vee (\sim P \rightarrow (\sim Q \rightarrow R))$

P	Q	R	$\sim P$	$\sim Q$	$\sim Q \rightarrow R$	$\sim P \rightarrow (\sim Q \rightarrow R)$	$P \vee (\sim P \rightarrow (\sim Q \rightarrow R))$	minterms
T	T	T	F	F	T	T	T	$P \wedge Q \wedge R$
T	T	F	F	F	T	T	T	$P \wedge Q \wedge \sim R$
T	F	T	F	T	T	T	T	$P \wedge \sim Q \wedge R$
T	F	F	F	T	F	T	T	$P \wedge \sim Q \wedge \sim R$
F	T	T	T	F	T	T	T	$\sim P \wedge Q \wedge R$
F	T	F	T	F	T	T	T	$\sim P \wedge Q \wedge \sim R$
F	F	T	T	T	T	T	T	$\sim P \wedge \sim Q \wedge R$
F	F	F	T	T	F	F	F	—

PDNF is $(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge \sim Q \wedge R)$

$\vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge Q \wedge \sim R)$

$\vee (\sim P \wedge \sim Q \wedge R)$

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dnf
pdnf
with tt — without truth table
$p_1 \vee p_2 \vee p_3 \vee \ldots$

cnf
pcnf
TT — w.TT
$p_1 \wedge p_2 \wedge p_3 \wedge \ldots$

CNF :- Conjunctive Normal form :- A logical expression is said to be conjunctive normal form if it consists of a product of elementary sums

Obtain a Conjunctive Normal form of the following

① $p \wedge (p \rightarrow q)$

$p \wedge (\sim p \vee q)$

Which is required CNF

② $[q \vee (p \wedge r)] \wedge \sim[(p \vee r) \wedge q]$

$\cancel{[(q \vee p) \wedge (q \vee r)] \wedge \sim[(p \vee q) \wedge (p \vee q)]}$

Given,

$[q \vee (p \wedge r)] \wedge \sim[(p \vee r) \wedge q)]$

$\Leftrightarrow [q \vee (p \wedge r)] \wedge [(\sim(p \vee r) \vee \sim q)]$

$\Leftrightarrow [(q \vee p) \wedge (q \vee r)] \wedge (\sim p \wedge \sim r \vee \sim q)$

$A \wedge C \vee A$

$(q \vee p) \wedge (q \vee r) \wedge (\underset{A}{\sim p} \wedge \underset{B}{\sim r}) \vee \underset{\substack{C \\ A}}{\sim q})$

$(q \vee p) \wedge (q \vee r) \wedge (\sim q \vee \sim p) \wedge (\sim q \vee \sim r)$

$(q \vee p) \wedge (q \vee r) \wedge (\sim p \vee \sim q) \wedge (\sim q \vee \sim r)$

PCNF

principle of conjunctive Normal form :-

The conjunction of max terms is called PCNF.

max term:- disjunction of statement variables

let P, Q are two statement variables then max terms are $P \vee Q$, $P \vee \sim Q$, $\sim P \vee Q$, $\sim P \vee \sim Q$

P	q	maxterms
T	T	$\sim P \vee \sim Q$
T	F	$\sim P \vee Q$
F	T	$P \vee \sim Q$
F	F	$P \vee Q$

Q:- Find PCNF of $P \leftrightarrow Q$ using truth table

P	q	$P \leftrightarrow Q$	maxterms
T	T	T	—
T	F	F	$\sim P \vee Q$
F	T	F	$P \vee \sim Q$
F	F	T	—

Therefore PCNF is $(\sim P \vee Q) \wedge (P \vee \sim Q)$

Q: Find PCNF of $(\sim P \to R) \wedge (Q \leftrightarrow P)$

P	Q	R	$\sim P$	$\sim P \to R$	$Q \leftrightarrow P$	$(\sim P \to R) \wedge (Q \leftrightarrow P)$	maxterms
T	T	T	F	T	T	T	—
T	T	F	F	T	T	T	—
T	F	T	F	T	F	F	$\sim P \vee Q \vee \sim R$
T	F	F	F	T	F	F	$\sim P \vee Q \vee R$
F	T	T	T	T	F	F	$P \vee \sim Q \vee \sim R$
F	T	F	T	F	F	F	$P \vee \sim Q \vee R$
F	F	T	T	T	T	T	—
F	F	F	T	F	T	F	$P \vee Q \vee R$

∴ The required PCNF is

$(\sim P \vee Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R)$

Q: Find PCNF of $P \leftrightarrow Q$ without using truthtable

Given $P \leftrightarrow Q$

$\equiv (P \wedge Q) \vee (\sim P \wedge \sim Q)$

$A \vee B \quad C \Rightarrow (A \vee B) \wedge (A \vee C)$

$\equiv ((P \wedge Q) \vee \sim P) \wedge ((P \wedge Q) \vee \sim Q)$

$\equiv (P \vee \sim P) \wedge (\sim P \vee Q) \wedge (P \vee \sim Q) \wedge (Q \vee \sim Q)$

$\equiv T \wedge (\sim P \vee Q) \wedge (P \vee \sim Q) \wedge T$

$\equiv (\sim P \vee Q) \wedge (P \vee \sim Q)$

Q: Find PCNF of $(\sim P \to R) \wedge (Q \leftrightarrow P)$

Given $(\sim P \to R) \wedge (Q \leftrightarrow P)$

$(\sim(\sim P) \vee R) \wedge [(Q \wedge P) \vee (\sim Q \wedge \sim P)]$

$(P \vee R) \wedge [(Q \wedge P) \vee (\sim Q \wedge \sim P)]$

Monday:

1:15 11/04/2022

PCNF without truth table

Step①:- If the statement contains $\to$, $\leftrightarrow$ then replace them by using equivalent connectivities $\wedge, \vee, \sim$ only.

Step②:- If negation is the part of the problem or entire of the problem then use demorgan's law.

Step③:- Apply distributive law upto CNF

Step④:- If elementary sum contains tautology or contradiction then we have to drop them

Step⑤:- If any proposition is missing then we have to add with the help of $P \wedge \sim P$

Step⑥:- Identify identical max-terms.

Q: Find PCNF of $P \Leftrightarrow Q$ without truth table

$P \Leftrightarrow Q$

$\Leftrightarrow (P \to Q) \wedge (Q \to P)$

$\Leftrightarrow (\sim P \vee Q) \wedge (\sim Q \vee P)$

$\Leftrightarrow (\sim P \vee Q) \wedge (P \vee \sim Q)$

Q: Find PCNF of $(\sim P \to R) \wedge (Q \Leftrightarrow P)$

Given $(\sim P \to R) \wedge (Q \Leftrightarrow P)$

$\Leftrightarrow (\sim(\sim P) \vee R) \wedge (Q \to P) \wedge (P \to Q)$

$\Leftrightarrow (P \vee R) \wedge (\sim Q \vee P) \wedge (\sim P \vee Q)$

$\Leftrightarrow (P \vee R) \wedge (P \vee \sim Q) \wedge (\sim P \vee Q)$

$\Leftrightarrow (P \vee R \vee F) \wedge (P \vee \sim Q \vee F) \wedge (\sim P \vee Q \vee F)$

$\Leftrightarrow (P \vee R \vee (Q \wedge \sim Q)) \wedge (P \vee \sim Q \vee (R \wedge \sim R)) \wedge (\sim P \vee Q \vee (R \wedge \sim R))$

$\Leftrightarrow (P \vee R \vee Q) \wedge (P \vee R \vee \sim Q) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$

$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$

16/04/2020

Predicate:- A declarative sentence describing the properties on object or relation among the objects is called predicate.

Eg:- "Ravi is a bachelor"
Ravi → subject
bachelor → predicate

"Ramu is a bachelor"
Ramu → subject
bachelor → predicate.

* With Ravi and Ramu are the same property & being bachelor

* In the proposition logic we require two different symbol to express them and these symbol don't reveal the common property of these two proposition in predicate characters.

* These two proposition can be replaced by a single proposition. X is a bachelor

* A predicate is a symbolized by a capital letter which is followed by the list of variables. The list of variables enclosed in parenthesis.

* If p stands for the predicate is a bachelor the P(X) stands for x is a bachelor, where x is a predicate variable.

Eg:① Ramu is a good boy.

Ramu → subject, good boy → predicate

② $x > 3$

x → subject, > 3 → predicate

2. X is a student, S(X): X is a student

S → predicate, X → variable

S(X) → preposition function or statement function

4. X is a man

statement function :- M(x)

M → predicate, x → variable

5. X is a women

statement function :- W(x)

W → predicate, x → variable

$M(x) \wedge W(x)$, $m(x) \vee w(x)$

$\neg m(x)$, $M(x) \rightarrow w(x)$

} compound statement or compound preposition function

Quantifiers :-

① universal quantifiers $\forall$ (for all)

② Existential quantifiers $\exists$ (there exist)

* If the sentance contain phrases like for all, for every, for each, every thing, for any then we use the universal quantifiers ($\forall$)

* If the sentance contain phrases like there exist, for some, there is atleast one, there exist some then we have the Existential quantifier ($\exists$)

Represent following sentance in symbolic form

1.

ⓐ something is good $(\exists x)(G(x))$

ⓑ Everything is good $(\forall x)(G(x))$

ⓒ something is not good $(\exists x)(\sim G(x))$

ⓓ Nothing is good $(\forall x)(\sim G(x))$

2

ⓐ All men are mortal

ⓑ Every apple is red

ⓒ Any Integer is either positive or Negative

ⓐ $M(x)$: x is a man
$L(x)$: x is mortal
$\Big\}$ $(\forall x)(M(x) \rightarrow L(x))$

ⓑ $A(x)$: x is a apple
$R(x)$: x is a Red
$\Big\}$ $(\forall x)(A(x) \rightarrow R(x))$

ⓒ $I(x)$: x is a integer
$P(x)$: x is a either positive or Negative
$\Big\}$ $(\forall x)(I(x) \rightarrow P(x))$

③

ⓐ There exist a man $(\exists x)(M(x))$

ⓑ some men are clever $(\exists x)(M(x) \wedge C(x))$

ⓒ some real numbers are rational $(\exists x)(R(x) \wedge L(x))$

$M(x)$: X is a men
$C(x)$: X is a clever
$\Big\}$ $(\exists x)(M(x) \wedge C(x))$

$R(x)$: X is a real number
$L(x)$: X is a rational
$\Big\}$ $(\exists x)(R(x) \wedge C(x))$

11:10

18/04/2022

Rules of Inference :-

	Rule of Inference	Tautological form	Name
1	a. $\dfrac{p}{\therefore p \vee q}$ b. $\dfrac{q}{\therefore p \vee q}$	ⓐ $p \to (p \vee q)$ ⓑ $q \to (p \vee q)$	Addition
2	ⓐ $\dfrac{p \wedge q}{\therefore p}$ ⓑ $\dfrac{p \wedge q}{\therefore q}$	ⓐ $p \wedge q \to p$ ⓑ $p \wedge q \to q$	simplication
3	$\dfrac{\begin{array}{c} p \\ q \end{array}}{\therefore p \wedge q}$	$[(p) \wedge (q)] \to (p \wedge q)$	conjunction
4	$\dfrac{\begin{array}{c} p \to q \\ p \end{array}}{\therefore q}$	$[(p \to q) \wedge p] \to q$	modus ponens

5	$p \to q$ $\sim q$ $\therefore \sim p$	$[(p \to q) \wedge (\sim q)] \to \sim p$	modus tollens
6	$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)]$ $\to (p \to r)$	Hypothetical syllogism
7	$p \vee q$ $\sim p$ $\therefore q$	$[(p \vee q) \wedge \sim p] \to q$	Disjunction syllogism
8	$(p \to q) \wedge (r \to s)$ $p \vee r$ $\therefore q \vee s$	$[(p \to q) \wedge (r \to s) \wedge$ $(p \vee r)] \to (q \vee s)$	constructive dilemma
9	$(p \to q) \wedge (r \to s)$ $\sim q \vee \sim s$ $\therefore \sim p \vee \sim r$	$[(p \to q) \wedge (r \to s) \wedge$ $(\sim q \vee \sim s)] \to$ $\sim p \vee \sim r$	distructive dilemma

show that t is a valued conclusion from the primises

$p \to q,\ q \to r,\ r \to s,\ \sim s,\ p \vee t$

① $p \to q$ (given premises)

② $q \to r$ (given premises)

③ $r \to s$ (given premises)

④ $p \to r$ (using ① and ② & Hypothetical syllogism)

⑤ $p \to s$ (using ④ and ③ & Hypothetical syllogism)

⑥ $\sim s$ (given premises)

⑦ $\sim p$ (using 5 & 6 & modus tollens)

⑧ $p \vee t$ (given premises)

⑨ t (using 7 & 8 & disjunction syllo sym)

$\therefore$ t is a conclusion which is valued function

② show that s is valued conclusion from the primises

$p \to q, p \to r, \sim(q \wedge r); s \vee p$

① $p \to q$ (Given premises)

② $p \to r$ (Given premises)

③ $(p \to q) \wedge (p \to r)$ (using ① and ②)

④ $\sim(q \wedge r)$ (Given premises)

⑤ $\sim q \vee \sim r$ (Demorgans law)

⑥ $\sim p \vee \sim p$ (3 & 5 distructive Dilemma)

⑦ $\sim p$ [Idempotent law]

⑧ $s \vee p$ (given premises)

⑨ s (7 & 8 disjunction syllosym)

Q:- prove the validity of the following arguments

if I get the job and work hard,
then I will get promotted,
then I will be happy.
I will not happy therefore either
I will not get the job or I will not
work hard

p: I get the job
q: I work hard
r: I get promotted
s: I will be happy

① $(p \wedge q) \rightarrow r$
② $r \rightarrow s$
③ $\sim s$

① $(p \wedge q) \rightarrow r$ (Given premises)
② $r \rightarrow s$ (Given premises)
③ $(p \wedge q) \rightarrow s$ (from ① and ②, Hypothetical
④ $\sim s$ (Given premises)
⑤ $\sim (p \wedge q)$ (3 & 4 modus tollens)
⑥ $\sim p \vee \sim q$