Trajectory Generation and Tracking of Quadcopter

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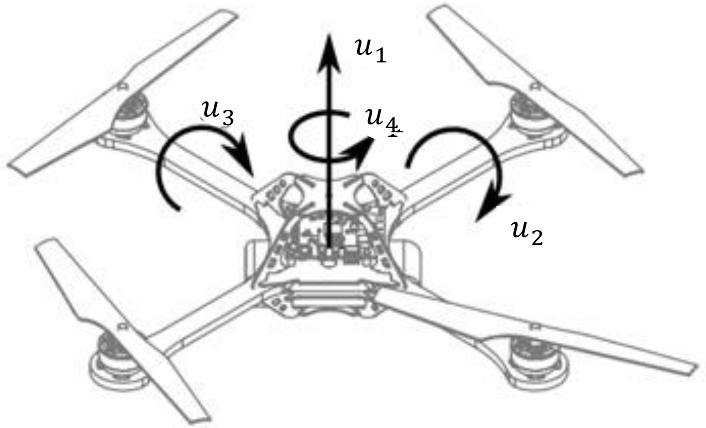
Quadcopter Dynamics

$$s = [x \quad y \quad z \quad \phi \quad \theta \quad \psi \quad \dot{x} \quad \dot{y} \quad \dot{z} \quad p \quad q \quad r]^T$$

$$\dot{s} = f(x) + g(x)u$$



$$f(x) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ p + q sin(\phi)\tan(\theta) + r cos(\phi)\tan(\theta) \\ q cos(\phi) - r sin(\phi) \\ q sin(\phi)/\cos(\theta) + r cos(\phi)/\cos(\theta) \\ 0 \\ 0 \\ g \\ (I_y - I_z)qr/I_x \\ (I_z - I_x)pq/I_y \\ (I_x - I_y)pq/I_z \end{bmatrix} \qquad g(x) = \begin{bmatrix} & & 0_{6*4} & \\ g_{71} & 0 & 0 & 0 \\ g_{81} & 0 & 0 & 0 \\ g_{91} & 0 & 0 & 0 \\ 0 & 1/I_x & 0 & 0 \\ 0 & 0 & 1/I_y & 0 \\ 0 & 0 & 0 & 1/I_z \end{bmatrix}$$

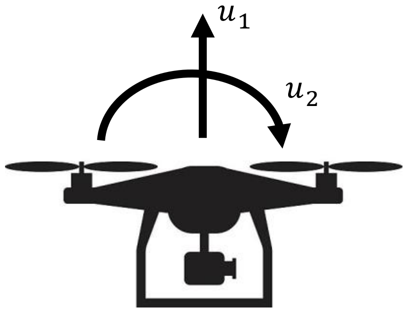
$$g_{71} = -\frac{1}{m}(\sin(\phi)\sin(\psi) + \cos(\phi)\cos(\psi)\cos(\theta))$$

$$g_{81} = -\frac{1}{m}(\sin(\phi)\cos(\psi) - \cos(\phi)\sin(\psi)\sin(\theta))$$

$$g_{91} = -\frac{1}{m}(\cos(\phi)\cos(\theta))$$

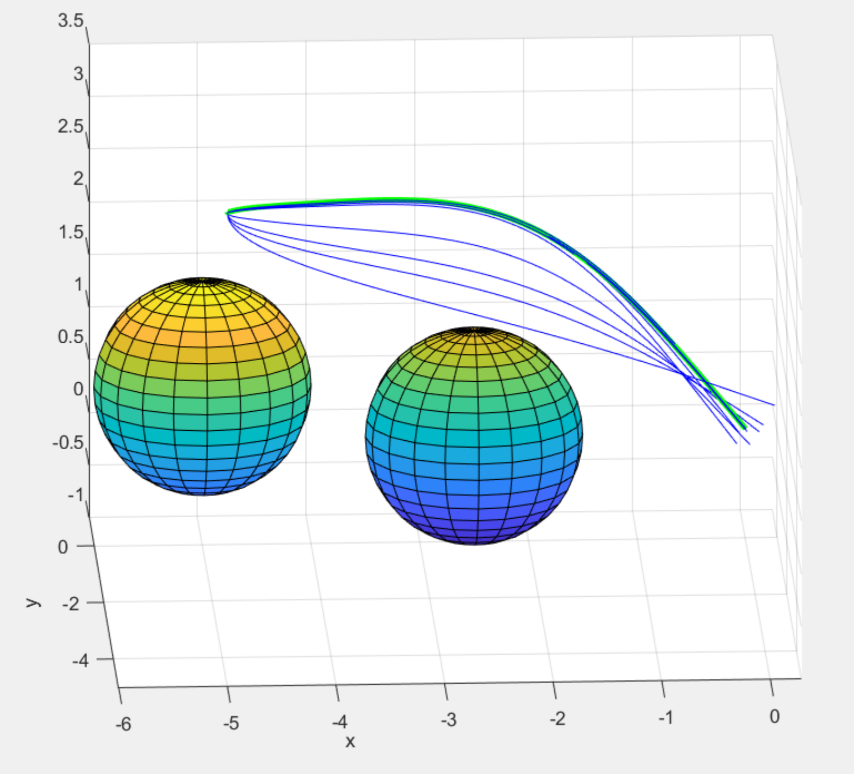
2D Simplified mode:

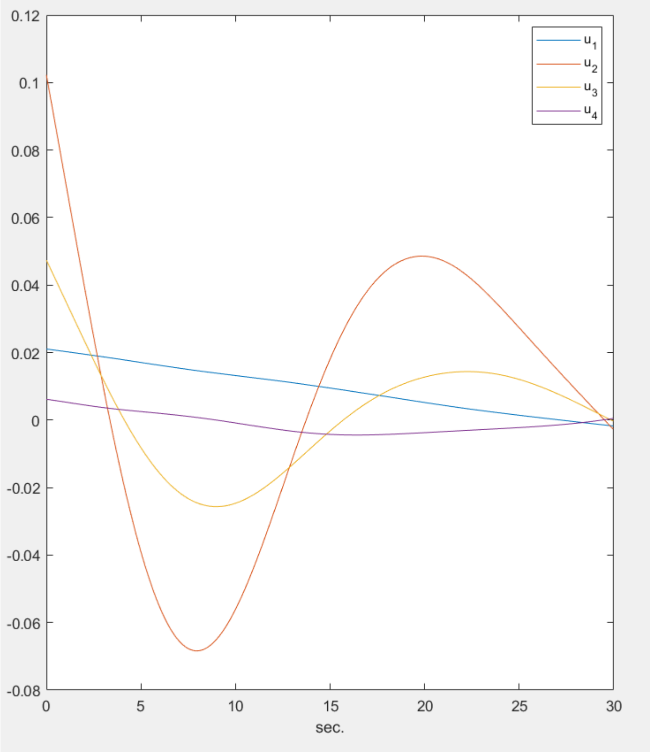
$$\frac{d}{dt}\begin{bmatrix} x \\ z \\ \theta \\ \dot{x} \\ \dot{z} \\ q \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{z} \\ q \\ \frac{R(\theta)}{m} * \begin{pmatrix} 0 \\ u_1 \end{pmatrix} \\ \frac{u_2}{J} \end{bmatrix}$$



Trajectory Generation using DDP

Generate trajectory using Differential Dynamic Programming → Convert discrete states into continuous desired states → Design a control law to track the trajectory



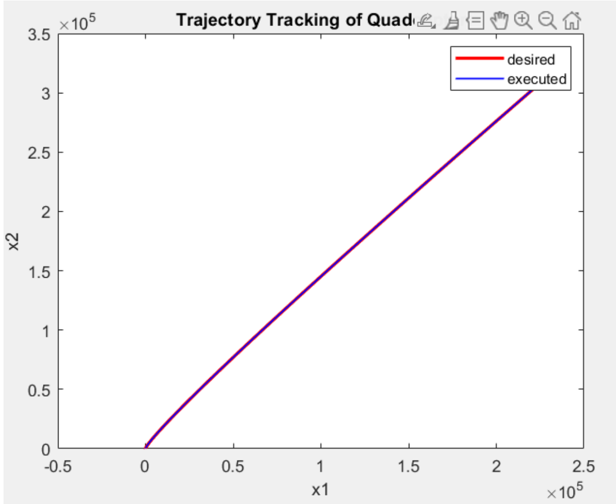


- Dealt with obstacles by augmenting the cost function.
- Discrete states converted into continuous states using polyfit.
- Passed this continuous desired state as the trajectory to be tracked by our feedback linearization controller.

Feedback Linearization

- Dynamic Compensator

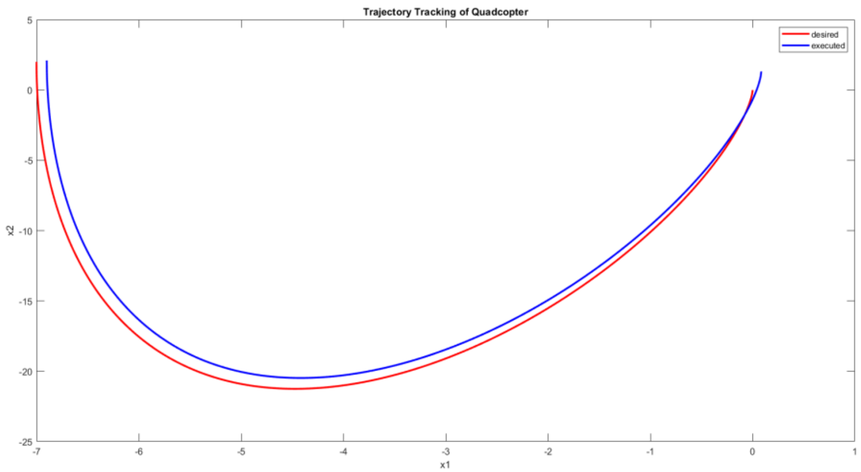
$$\xi = \begin{pmatrix} u_1 \\ \dot{u}_1 \end{pmatrix}$$

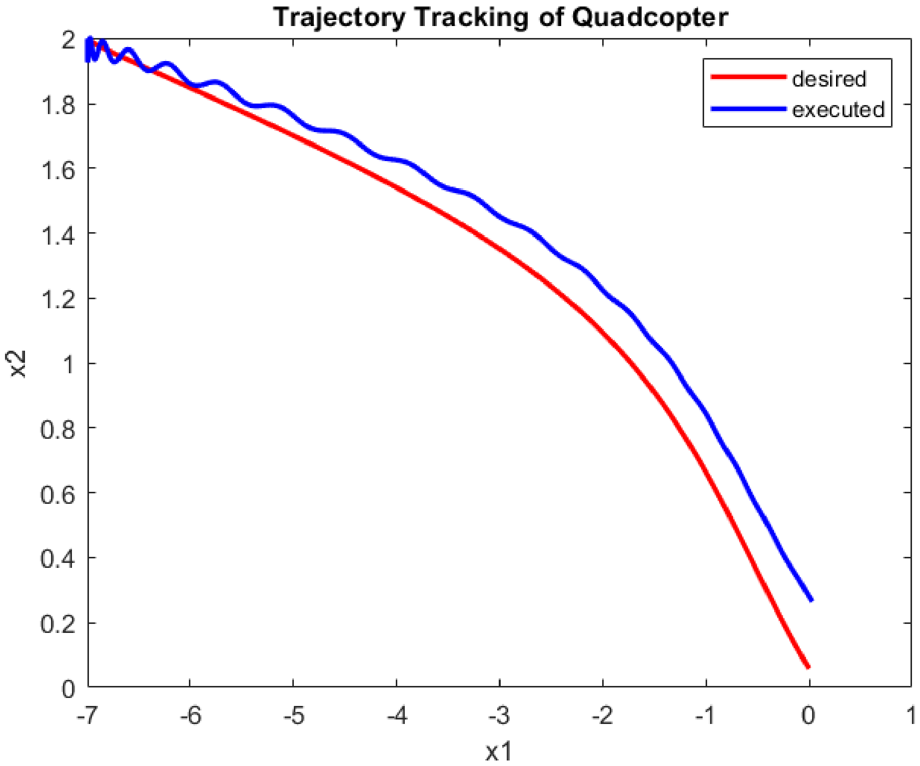


- Virtual Control

$$v = \begin{pmatrix} x_1^{(4)} \\ x_2^{(4)} \end{pmatrix}$$

$$\begin{pmatrix} u_2 \\ \ddot{u}_1 \end{pmatrix} = \begin{pmatrix} -J/\xi_1 & 0 \\ 0 & 1 \end{pmatrix} \left(mR^T(x_3)v - \begin{pmatrix} -2\xi_2 x_6 \\ -u_1 x_6^2 \end{pmatrix} \right)$$





Full Model Feedback Linearization

$$y = [h_1 \quad h_2 \quad h_3 \quad h_4]^T = [x \quad y \quad z \quad \psi]^T$$

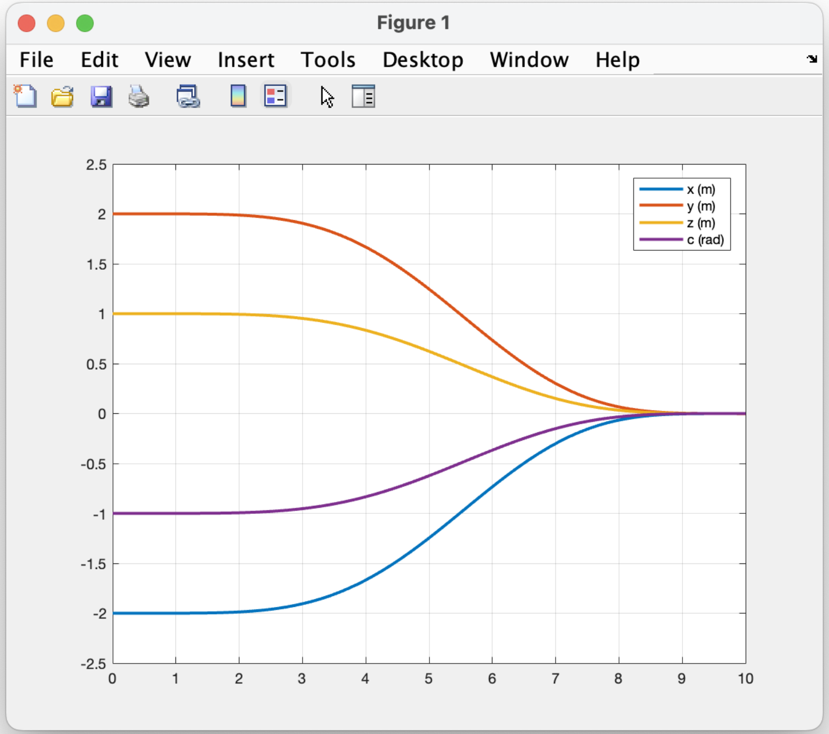
$$\begin{bmatrix} y_1^{(4)} \\ y_2^{(4)} \\ y_3^{(4)} \\ y_4^{(2)} \end{bmatrix} = \underbrace{\begin{bmatrix} L_f^{(4)} h_1 \\ L_f^{(4)} h_2 \\ L_f^{(4)} h_2 \\ L_f^{(2)} h_2 \end{bmatrix}}_{F} + \underbrace{\begin{bmatrix} L_{g1} L_f^{(4)} h_1 & L_{g2} L_f^{(4)} h_1 & L_{g3} L_f^{(4)} h_1 & L_{g4} L_f^{(4)} h_1 \\ L_{g1} L_f^{(4)} h_2 & L_{g2} L_f^{(4)} h_2 & L_{g3} L_f^{(4)} h_2 & L_{g4} L_f^{(4)} h_2 \\ L_{g1} L_f^{(4)} h_3 & L_{g2} L_f^{(4)} h_3 & L_{g3} L_f^{(4)} h_3 & L_{g4} L_f^{(4)} h_3 \\ L_{g1} L_f^{(2)} h_4 & L_{g2} L_f^{(2)} h_4 & L_{g3} L_f^{(2)} h_4 & L_{g4} L_f^{(2)} h_4 \end{bmatrix}}_{G} \begin{bmatrix} u_1^{(2)} \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

$$[\xi_1 \quad \xi_2]^T = \left[u_1 \quad u_1^{(1)}\right]^T$$

$$v_{1-3} = y_{d1-3}^{(4)} - K_0(y_{1-3} - y_{d1-3}) - K_1(y_{1-3}^{(1)} - y_{d1-3}^{(1)}) - K_2(y_{1-3}^{(2)} - y_{d1-3}^{(2)}) - K_3(y_{1-3}^{(3)} - y_{d1-3}^{(3)})$$

$$v_4 = y_{d4}^{(2)} - K_5(y_4 - y_{d4}) - K_5(y_4^{(1)} - y_{d4}^{(1)})$$

$$u_a = G^{-1}(v - F)$$

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G =

[-(sin(a)*sin(c) + cos(a)*cos(c)*sin(b))/m, -(u1*(cos(a)*sin(c) - cos(c)*sin(a)*sin(b)))/(Ix*m), -(u1*cos(b)*cos(c))/(Iy*m),                  0]
[-(cos(c)*sin(a) - cos(a)*sin(b)*sin(c))/m, -(u1*(cos(a)*cos(c) + sin(a)*sin(b)*sin(c)))/(Ix*m),  (u1*cos(b)*sin(c))/(Iy*m),                  0]
[                     -(cos(a)*cos(b))/m,                              (u1*cos(b)*sin(a))/(Ix*m),          (u1*sin(b))/(Iy*m),                  0]
[                                      0,                                                      0,          sin(a)/(Iy*cos(b)), cos(a)/(Iz*cos(b))]
F =

(Ix*Iy*p^2*u1*sin(a)*sin(c) - 2*Ix*Iy*du1*p*cos(a)*sin(c) - 2*Ix*Iy*du1*q*cos(b)*cos(c) + Ix*Iy*q^2*u1*sin(a)*sin(c) + Ix^2*p*r*u1*cos(b)*cos(c) - Iy^2*q*r*u1*cos(a)*sin(c) + 2*Ix*Iy*du1*p*cos(c)*sin(a)*sin(b
(2*Ix*Iy*du1*q*cos(b)*sin(c) - 2*Ix*Iy*du1*p*cos(a)*cos(c) + Ix*Iy*p^2*u1*cos(c)*sin(a) + Ix*Iy*q^2*u1*cos(c)*sin(a) - Iy^2*q*r*u1*cos(a)*cos(c) - Ix^2*p*r*u1*cos(b)*sin(c) - 2*Ix*Iy*du1*p*sin(a)*sin(b)*sin(c
```

Future work

- Block backstepping control

$$\frac{d}{dt}\begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \end{bmatrix} = 0_{7*1} + \begin{bmatrix} I_{3*3} & & 0_{3*3} & \\ & 1 & \sin\phi * tan\theta & cos\phi * tan\theta \\ 0_{3*3} & 0 & cos\phi & -sin\phi \\ & 0 & sin\phi/\cos\theta & cos\phi/\cos\theta \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ p \\ q \\ r \end{bmatrix}$$

$$\frac{d}{dt}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ g \\ (I_y - I_z)qr/I_x \\ (I_z - I_x)pr/I_y \\ (I_x - I_y)pq/I_z \end{bmatrix} + \begin{bmatrix} g_{71} & & & \\ g_{81} & & 0_{3*3} & \\ g_{91} & & & \\ & 1/I_x & 0 & 0 \\ 0_{3*1} & 0 & 1/I_x & 0 \\ & 0 & 0 & 1/I_x \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

- Discrete trajectory problems