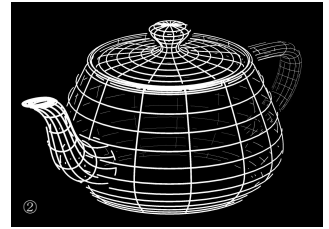
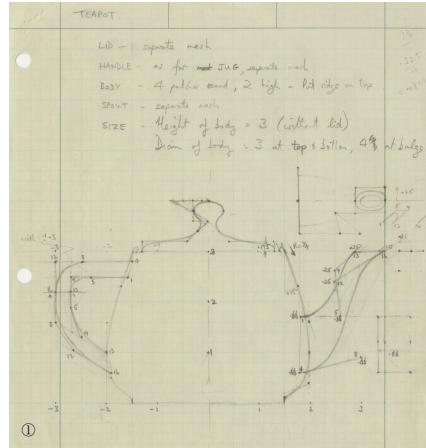


# Computer Graphics



CSE 4303 / CSE 5365  
Clipping / Euler Angles, 2019 Spring

- ① <http://www.computerhistory.org/revolution/computer-graphics-music-and-art/15/206/556>
- ② <http://www.cs.technion.ac.il/~gershon/site/img/gallery/gallery-pic-cat3-depth-cueing-2-big.jpg>
- ③ [http://www.omnigraphica.com/gallery/maingallery/original/Utah\\_teapot\\_1.png](http://www.omnigraphica.com/gallery/maingallery/original/Utah_teapot_1.png)
- ④ <http://unfold.be/assets/images/000/113/719/large-utahalog3.jpg>

## Clipping

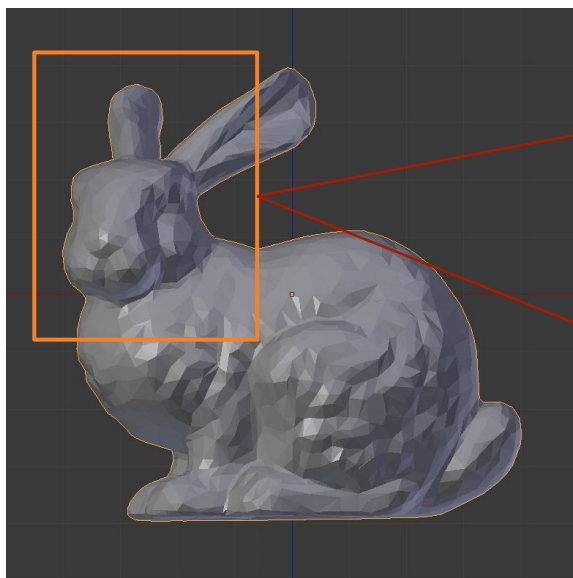


# Clipping

- *Clipping* is the identification of objects or parts of objects as either *inside* or *outside* a specified region.
- *Interior* clipping is the saving of what's *inside* the region.
  - For example, *copy* a piece of a picture.
- *Exterior* clipping is the saving of what's *outside* the region.
  - For example, *clear* a piece of a picture



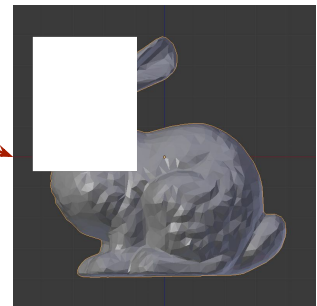
## Clipping



*Interior Clip*



*Exterior Clip*



## Clipping

- In CG, clipping is primarily used to decide which objects or *parts* of objects should be visible when a scene is rendered.
- Why clip?
  - Don't waste time on objects that can't be seen.
    - Or even an unseeable *part* of an object.
  - Avoid degenerate cases that might cause divide-by-0 or overflow conditions.

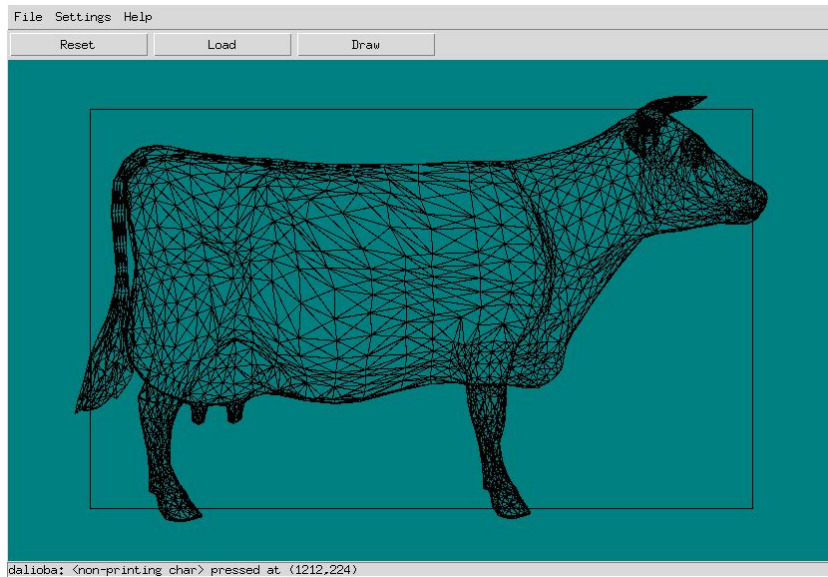


## Clipping

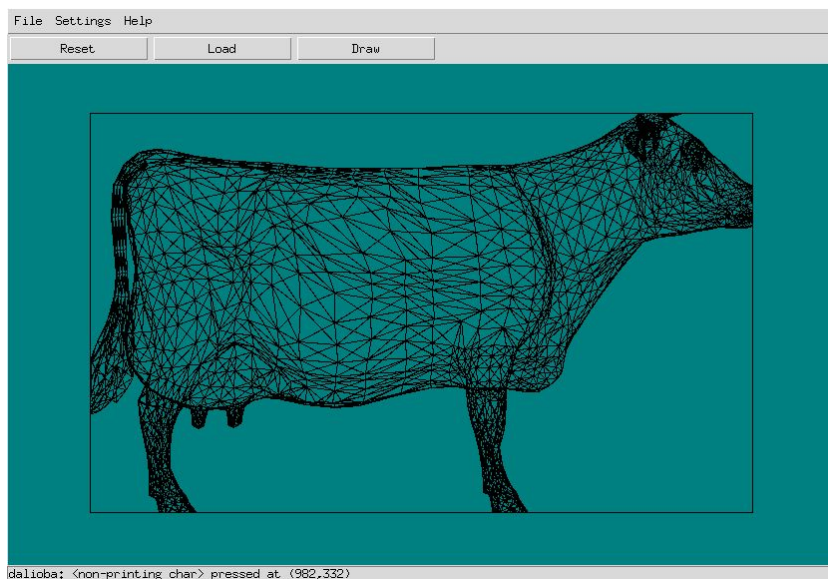
- Clipping may be done at various points in the rendering pipeline.
  - Each point has its own way to specify the clipping region.
  - In 3D, it's a volume. In 2D, it's a region, usually rectangular.
- Different kinds of clipping include
  - *Point*: Keep point only if inside.
  - *Line*: Keep portion of line that's inside, if any.
  - *Polygon*: Make a new polygon that's the portion inside, if any.
- Since we are drawing lines at present, we will start with 2D clipping of lines against the view window.



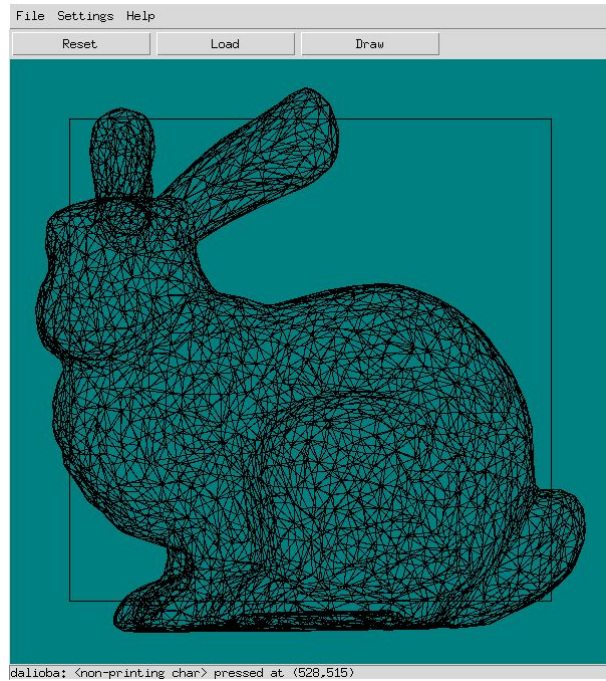
# Clipping



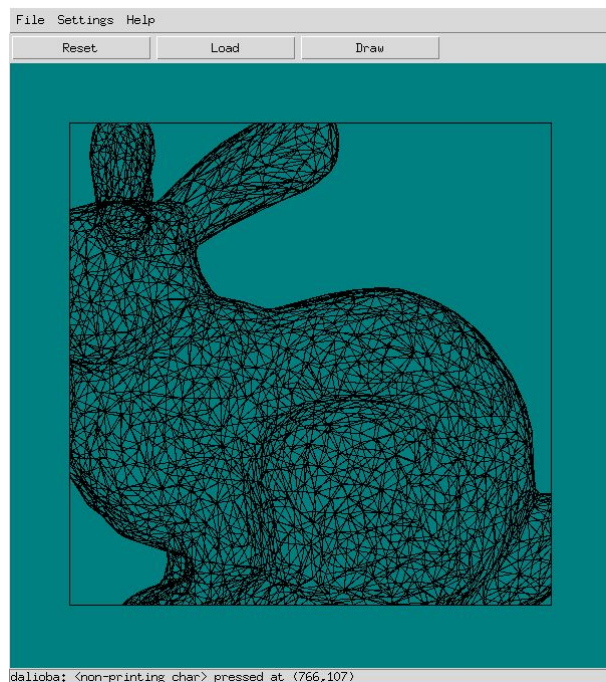
# Clipping



# Clipping

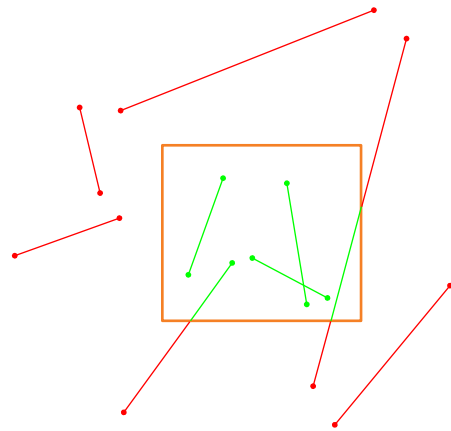


# Clipping



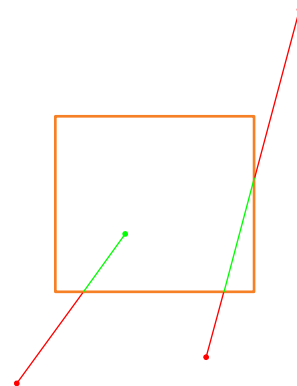
## Clipping

- Consider a viewport that we want to clip against.
- Some lines are clearly *inside* the region and should be drawn.
- Some lines are clearly *outside* the region and should *not* be drawn.
- Others are both inside *and* outside.
- *Part* of the line should be drawn.



## Clipping

- Notice there are two kinds of *partial* lines.
  - One of the points is *inside* the clipping region.
  - Both of the points are *outside* the clipping region.
- We cannot eliminate a line just because both of its points are *outside*.



## Line Clipping

- There are many, many methods for line clipping.
  - They all have various claims to fame, application area, capability, speed, simplicity, etc.
- (One of) the earliest is the Cohen-Sutherland method.
  - Invented by Danny Cohen and Ivan Sutherland in 1967 while working on a flight simulator.
- It's a simple algorithm.
  - Quickly accepts completely inside lines. Quickly rejects certain categories of completely outside lines.
  - Uses iteration to make a decision on the rest.



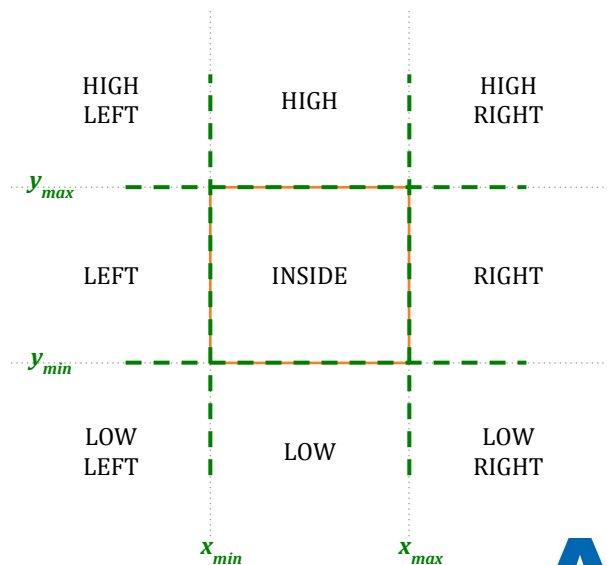
## [ *Quick Drawing Review* ]

- Objects are defined as a set of vertices and faces.
  - The  $v_x y z$  lines specify the vertex's position in world space.
  - The  $f_{v_1 v_2 v_3}$  lines specify which vertices make up each face.
- The positions of the vertices are transformed from  $x, y, z$  world coordinates into pixel coordinates.
- Three lines are drawn for each face.
  - ①  $v_1$  to  $v_2$  ②  $v_2$  to  $v_3$  ③  $v_3$  to  $v_1$
- Because a vertex may end up outside the viewport region, each of these lines may need to be clipped.



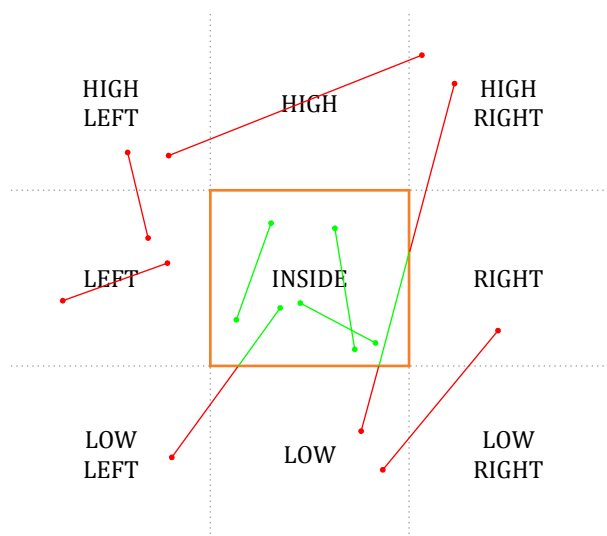
## Cohen-Sutherland Line Clipping

- Divides the viewport space into nine areas.
- The central area is the *inside* space that the user sees.
- All other areas are *outside* and are not seen.
- INSIDE is bounded by the lines  $x_{min}$ ,  $x_{max}$ ,  $y_{min}$ ,  $y_{max}$ .



## Cohen-Sutherland Line Clipping

- Step one in clipping a line is to determine in which of the nine regions its end points fall.
- This is easy! :)





# Cohen-Sutherland Line Clipping

- Starting with the point's  $x$  and  $y$  coordinates ...
- Compare  $x$  against  $x_{min}$  and  $x_{max}$  to determine if the point is LEFT or RIGHT.
- Compare  $y$  against  $y_{min}$  and  $y_{max}$  to determine if the point is BELOW or ABOVE.
- Done! :)

```

INSIDE = 0
LEFT  = 1
RIGHT = 2
BELOW = 4
ABOVE = 8
    
```

These are mutually exclusive powers of 2, so each is a unique bit.

```

def outcode( x, y, xMin, yMin, xMax, yMax ) :
    code = INSIDE
    
```

```

    if ( x < xMin ) :
        code = code | LEFT
    elif ( x > xMax ) :
        code = code | RIGHT
    
```

```

    if ( y < yMin ) :
        code = code | BELOW
    elif ( y > yMax ) :
        code = code | ABOVE
    
```

```

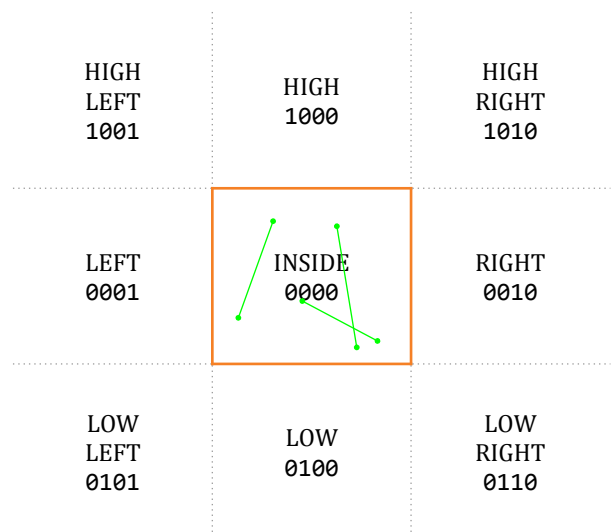
    return code
    
```

Bit-wise OR operations, so no bit interferes with another..



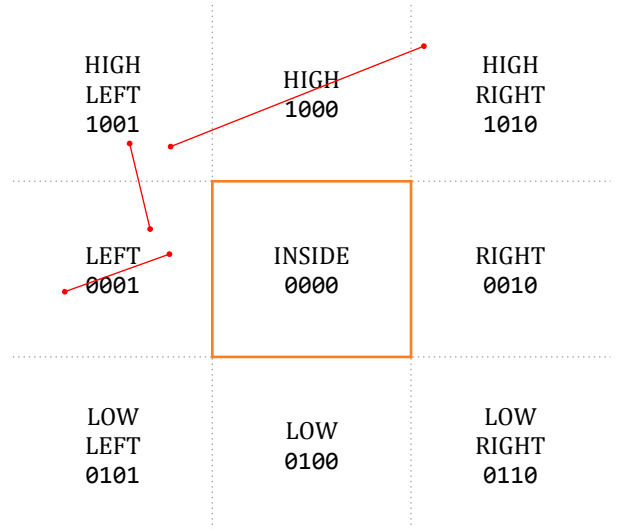
# Cohen-Sutherland Line Clipping

- The result will be a 4-bit code corresponding to which area the point is in.
- Notice that INSIDE's code ends up being 0000.
- This makes it trivial to accept a line that is completely INSIDE.
- Both points will have code 0000.
- Easy to detect!



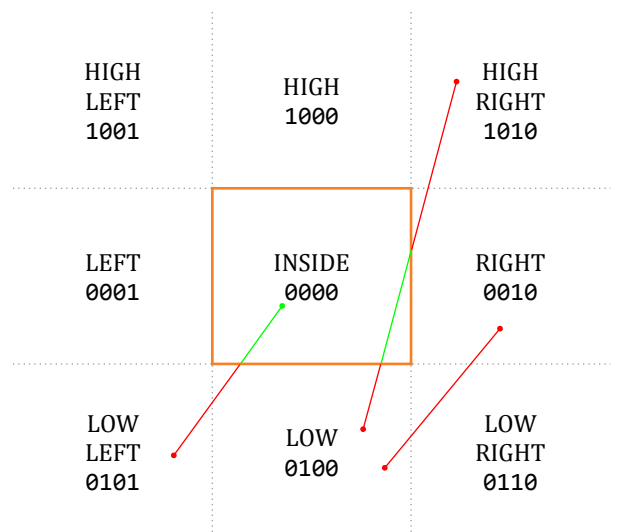
## Cohen-Sutherland Line Clipping

- What about trivial rejects?
- Lines whose points are both in the same region that is *not* INSIDE can be rejected.
  - Line is entirely in unseen region.
- Lines whose points are on the same side of INSIDE can be rejected.
  - Line cannot intersect INSIDE region so nothing to draw.
- How to compute these relationships?
  - Bitwise AND of codes will be non-zero.
- Easy to detect!



## Cohen-Sutherland Line Clipping

- What about the *mixed* or *both outside* cases?
- We have to determine which *portion* of the line *if any* is to be drawn.
- The algorithm is relatively simple, moving one point or the other along the line to its bounding line.



# Cohen-Sutherland Line Clipping

- Pick a point that is *not* INSIDE.
  - There has to be one, otherwise the line would be a trivial accept.
- Move that point to the spot on the line that removes (one of) its out-of-bounds problems.
  - If ABOVE, move to  $y_{max}$  along the line.
  - If BELOW, move to  $y_{min}$  along the line.
  - If RIGHT, move to  $x_{max}$  along the line.
  - If LEFT, move to  $x_{min}$  along the line.
- Even after moving one point, the line might still be non-trivial to accept or reject, so iterate.
  - Replace the point with the new x, y and recompute its code first.

```

p1Out = outcode( p1x, p1y, xMin, yMin, xMax, yMax )
p2Out = outcode( p2x, p2y, xMin, yMin, xMax, yMax )

anOutCode = p2Out if p1Out == INSIDE else p1Out

if ( anOutCode & ABOVE ) :
    # Move point along the line down to Y max.
    x = p1x + ( p2x - p1x )*( yMax - p1y )/( p2y - p1y )
    y = yMax

elif ( anOutCode & BELOW ) :
    # Move point along the line up to Y min.
    x = p1x + ( p2x - p1x )*( yMin - p1y )/( p2y - p1y )
    y = yMin

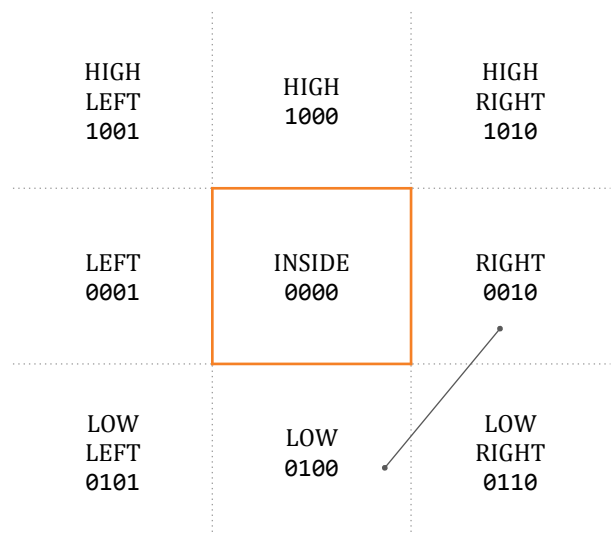
elif ( anOutCode & RIGHT ) :
    # Move it along the line over to X max.
    x = xMax
    y = p1y + ( p2y - p1y )*( xMax - p1x )/( p2x - p1x )

elif ( anOutCode & LEFT ) :
    # Move it along the line over to X min.
    x = xMin
    y = p1y + ( p2y - p1y )*( xMin - p1x )/( p2x - p1x )
    
```



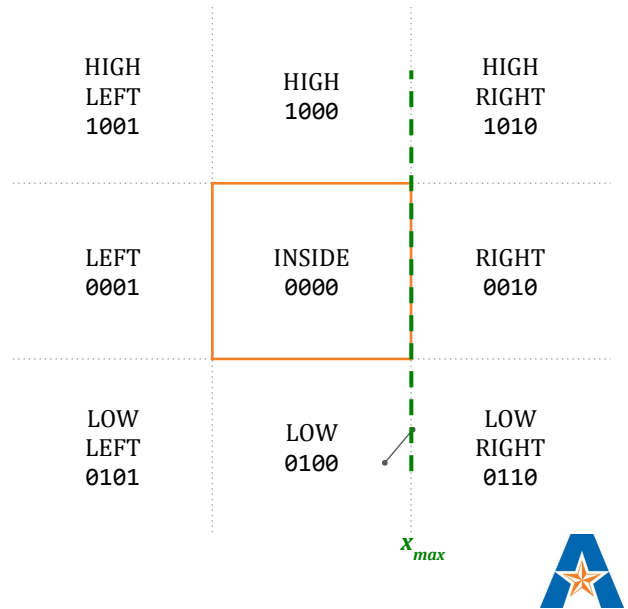
# Cohen-Sutherland Line Clipping

- For example, both points of this line are outside the INSIDE region.



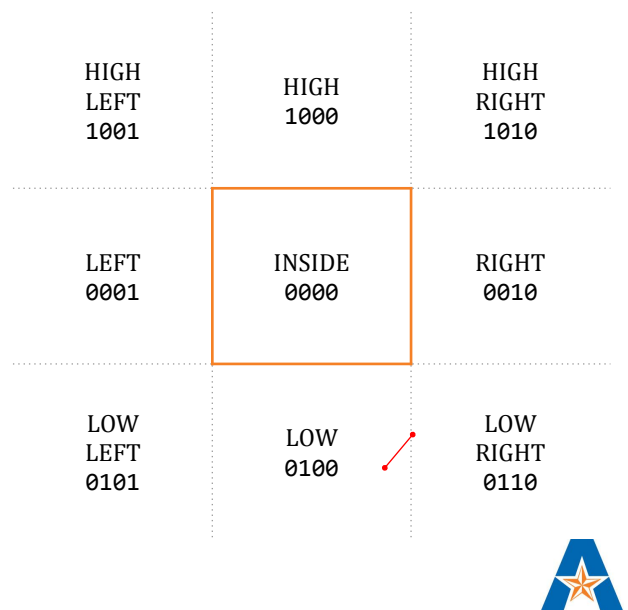
## Cohen-Sutherland Line Clipping

- For example, both points of this line are outside the INSIDE region.
- If we manipulate the point in RIGHT, it gets moved to the  $x_{max}$  line.



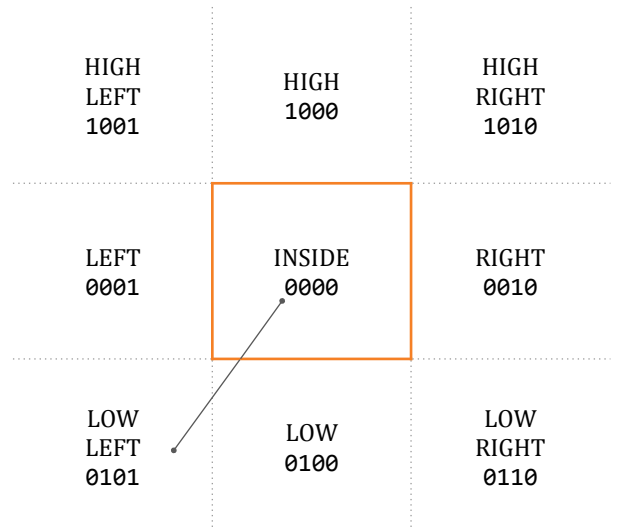
## Cohen-Sutherland Line Clipping

- For example, both points of this line are outside the INSIDE region.
- If we manipulate the point in RIGHT, it gets moved to the  $x_{max}$  line.
- When we recompute its code, both points will be LOW, so trivial reject.



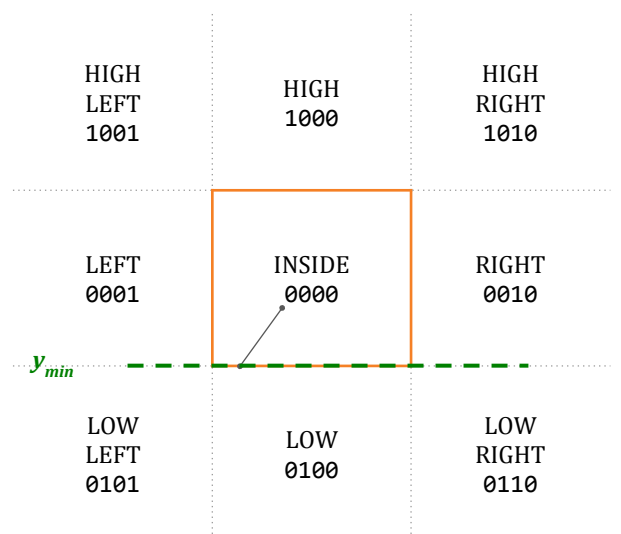
## Cohen-Sutherland Line Clipping

- Another example. This line has one point INSIDE and one point outside the INSIDE region.
- The point in LOW LEFT will be manipulated.



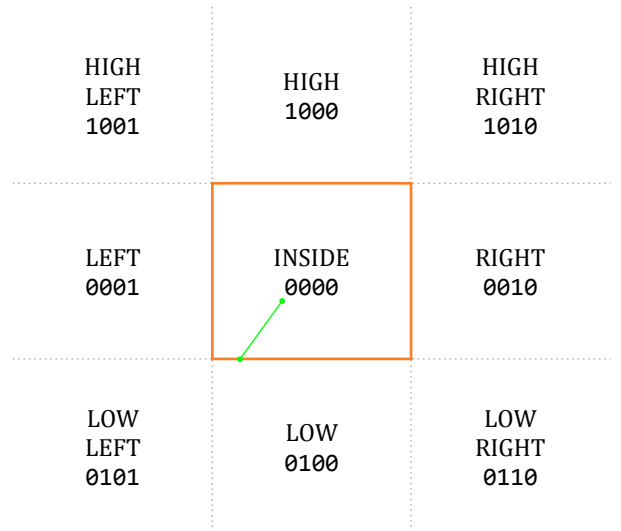
## Cohen-Sutherland Line Clipping

- Another example. This line has one point INSIDE and one point outside the INSIDE region.
- The point in LOW LEFT will be manipulated.
- Since it is LOW, it will get moved to the  $y_{min}$  line.



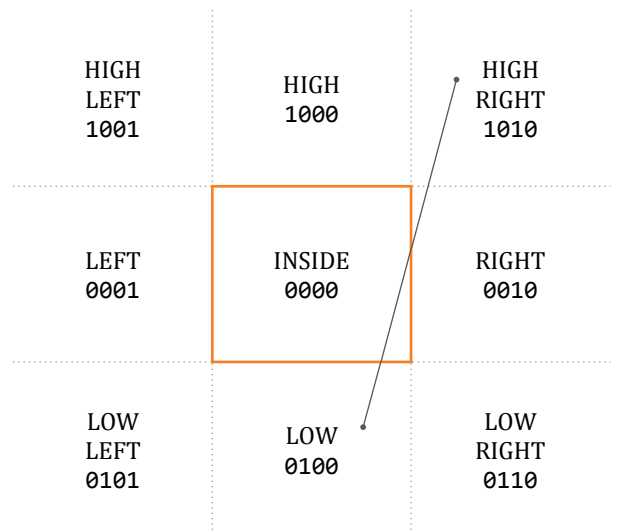
## Cohen-Sutherland Line Clipping

- Another example. This line has one point INSIDE and one point outside the INSIDE region.
- The point in LOW LEFT will be manipulated.
- Since it is LOW, it will get moved to the  $y_{min}$  line.
- When we recompute its code, both points are INSIDE so trivial accept.



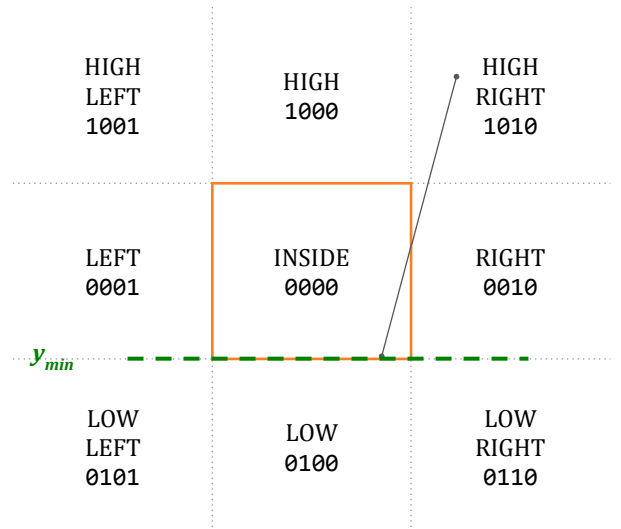
## Cohen-Sutherland Line Clipping

- Final example. This line has both points outside the INSIDE region.
- The point in LOW will be manipulated.



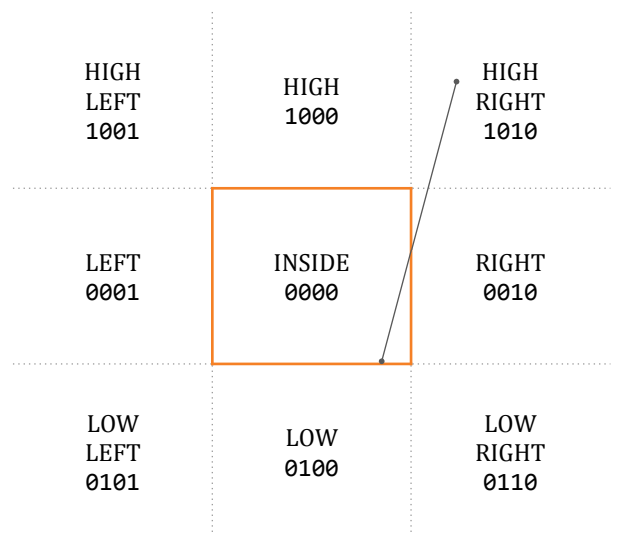
## Cohen-Sutherland Line Clipping

- Final example. This line has both points outside the INSIDE region.
- The point in LOW will be manipulated.
- Since it is LOW, it will get moved to the  $y_{min}$  line.



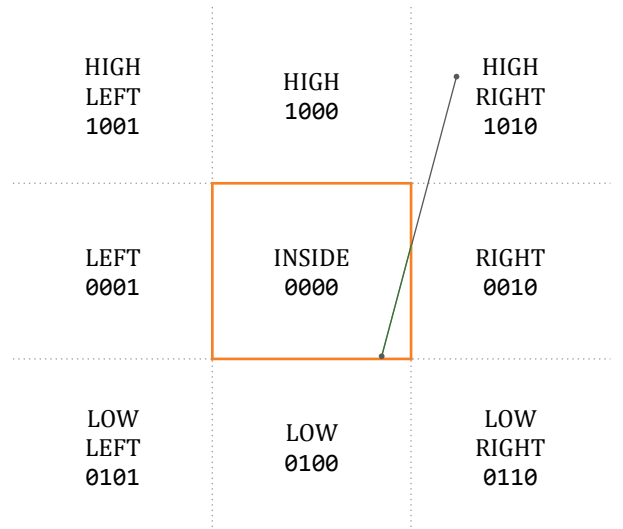
## Cohen-Sutherland Line Clipping

- Final example. This line has both points outside the INSIDE region.
- The point in LOW will be manipulated.
- Since it is LOW, it will get moved to the  $y_{min}$  line.
- When we recompute its code, it is now INSIDE, but no easy accept or reject since the other point is still outside.



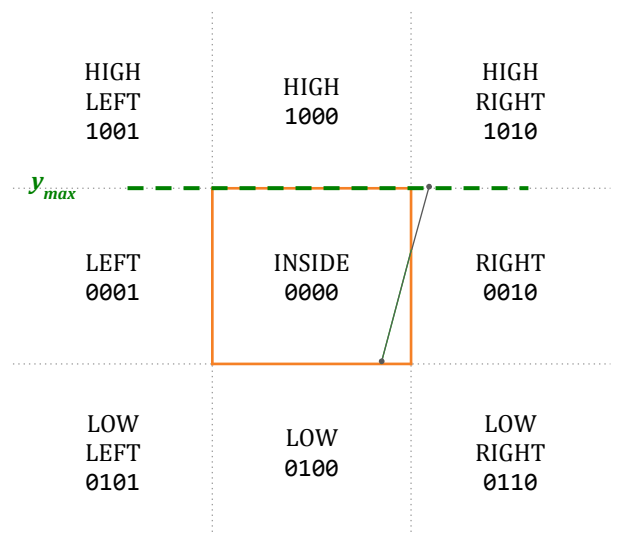
## Cohen-Sutherland Line Clipping

- We next consider the point in HIGH RIGHT.



## Cohen-Sutherland Line Clipping

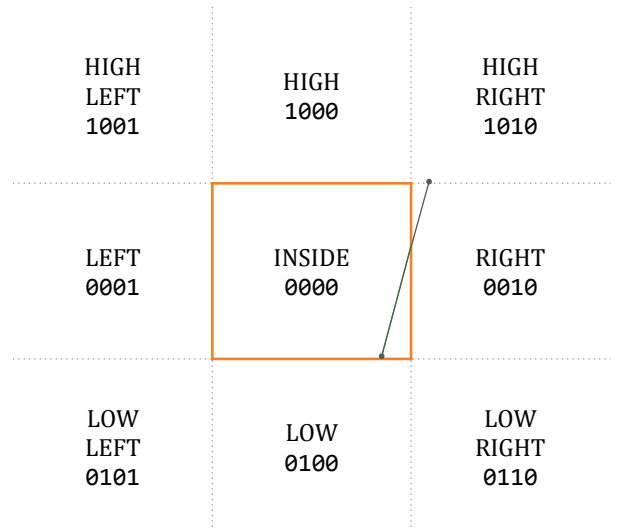
- We next consider the point in HIGH RIGHT.
- Since it is HIGH, we move it to the  $y_{max}$  line.





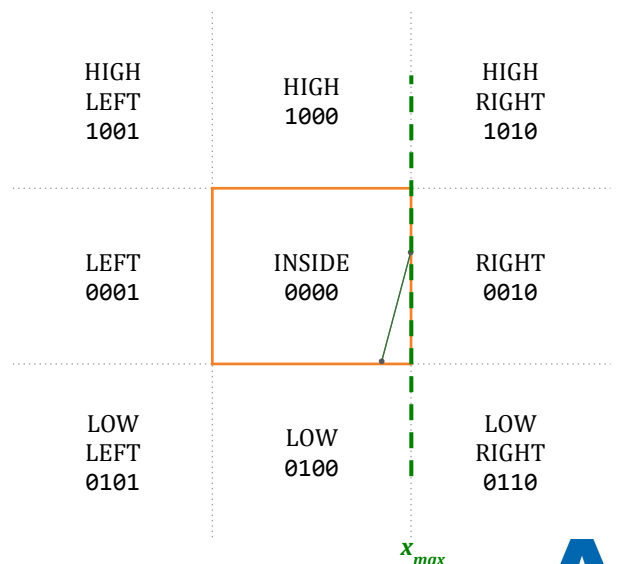
## Cohen-Sutherland Line Clipping

- We next consider the point in HIGH RIGHT.
- Since it is HIGH, we move it to the  $y_{max}$  line.
- Its recomputed code is RIGHT.
- There is no trivial accept or reject.
- We consider the point again.



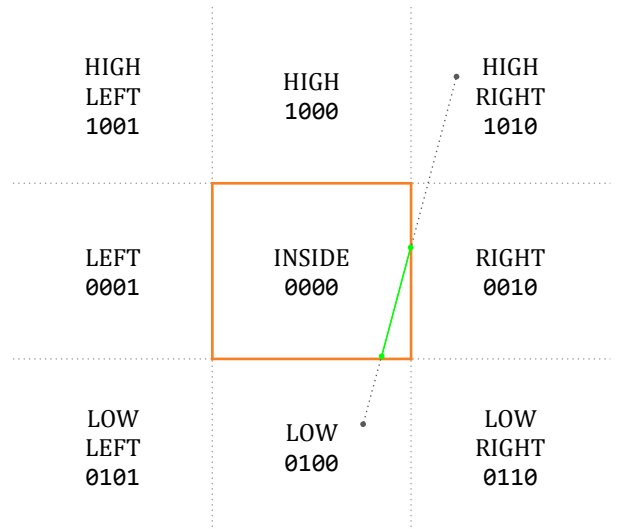
## Cohen-Sutherland Line Clipping

- We next consider the point in HIGH RIGHT.
- Since it is HIGH, we move it to the  $y_{max}$  line.
- Its recomputed code is RIGHT.
- There is no trivial accept or reject.
- We consider the point again.
- Since it is RIGHT, we move it to the  $x_{max}$  line.



## Cohen-Sutherland Line Clipping

- After recomputing its code again, it is now INSIDE.
- Since both points are now INSIDE, trivial accept.
- Notice that neither of the two original end points are being used to draw the line.



## Euler Angle Rotation *Summary*



# Euler Angle Rotation

- Euler Angles are used to orient a *rigid body* with respect to a fixed coordinate system.
  - Introduced by Leonhard Euler in 1776, yet another mathematical idea that's been around for centuries.
- Tait and Bryan extended Euler's concept (about 1910).
- We will use  $\psi$  for *yaw*,  $\theta$  for *pitch*,  $\phi$  for *roll*.
- As usual, everyone has their own notation, so be careful.



## Euler Angles

Since we have three axes, we can rotate about three directions at once.

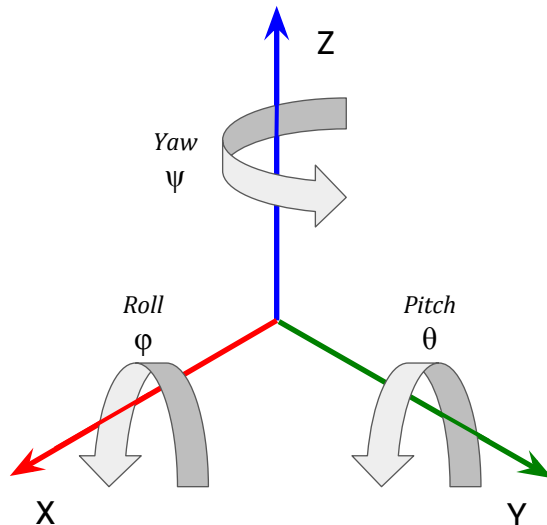
The rotations are commonly known as **Yaw**, **Pitch**, and **Roll**. Generally,

**Yaw**  $\psi$  is about the **Z** axis.

**Pitch**  $\theta$  is about the **Y** axis.

**Roll**  $\phi$  is about the **X** axis.

Commonly Yaw is designated by  $\psi$ , Pitch by  $\theta$ , and Roll by  $\phi$ . As usual, there are many variances in notation.

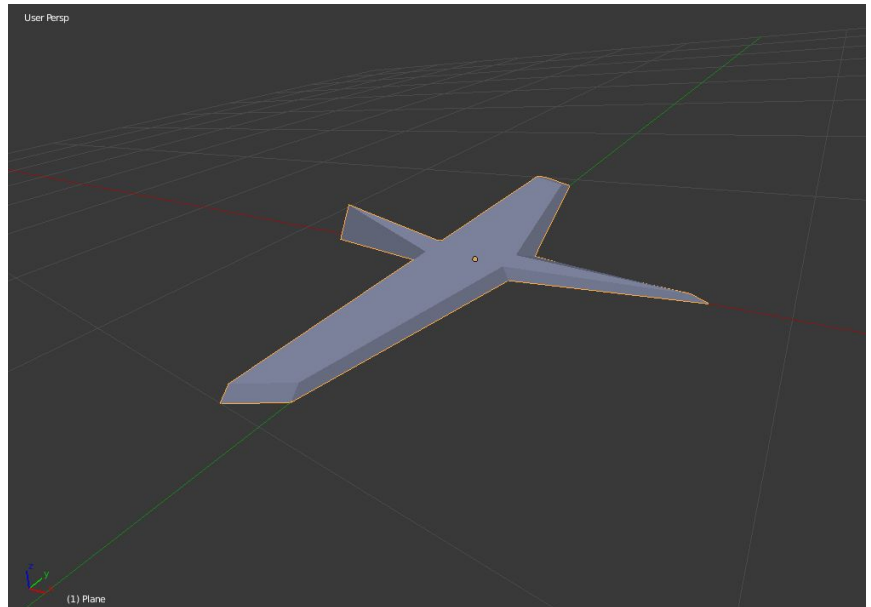


# Simple Plane Model

*The nose points in the positive X direction.*

*The wings point along the positive and negative Y directions.*

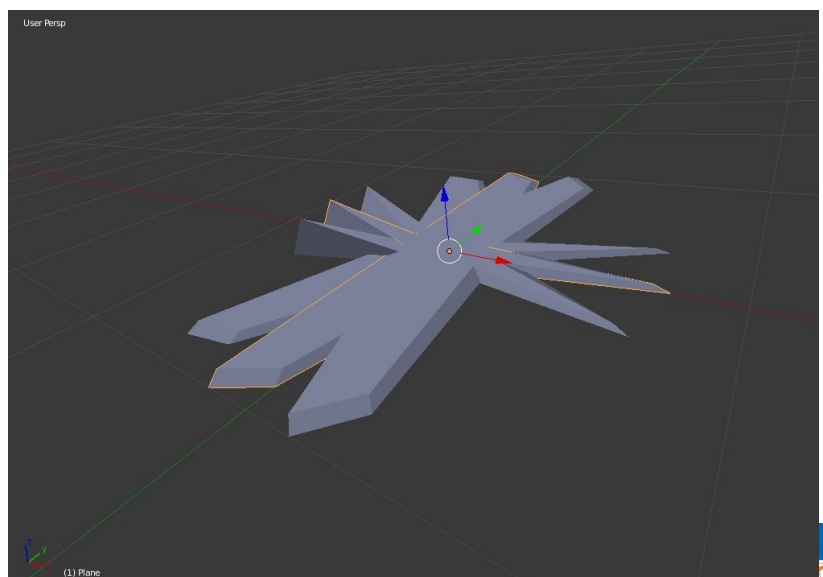
*The rudder sticks up in the positive Z direction.*



## Yaw

*Yaw is about the Z axis.*

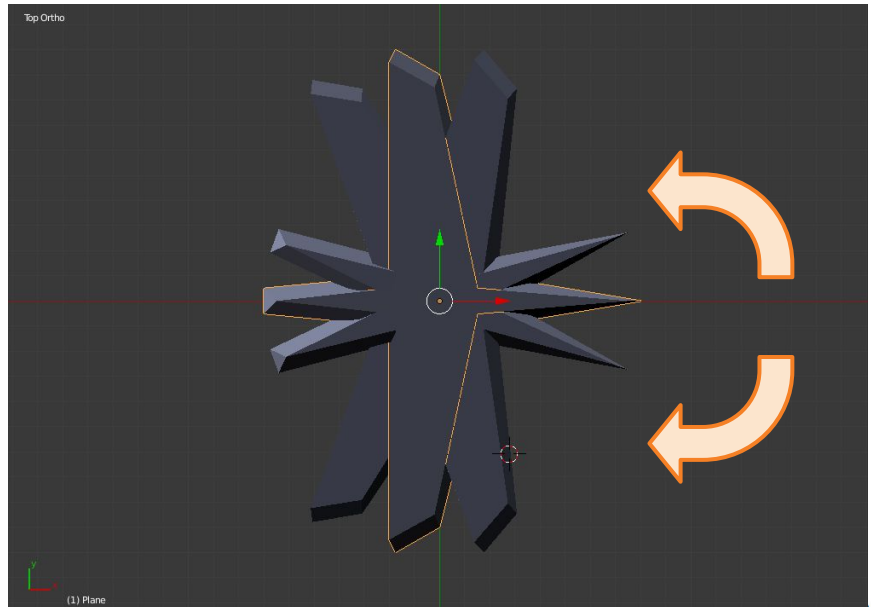
*The nose and tail go side to side as the body of the plane yaws.*



# Yaw

*Yaw is about the Z axis.*

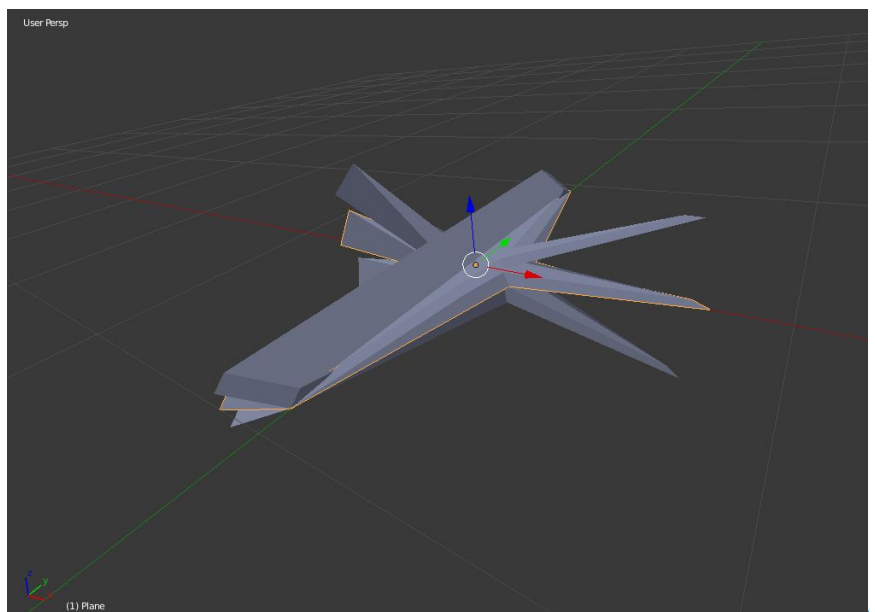
*The nose and tail go side to side as the body of the plane yaws.*



# Pitch

*Pitch is about the Y axis.*

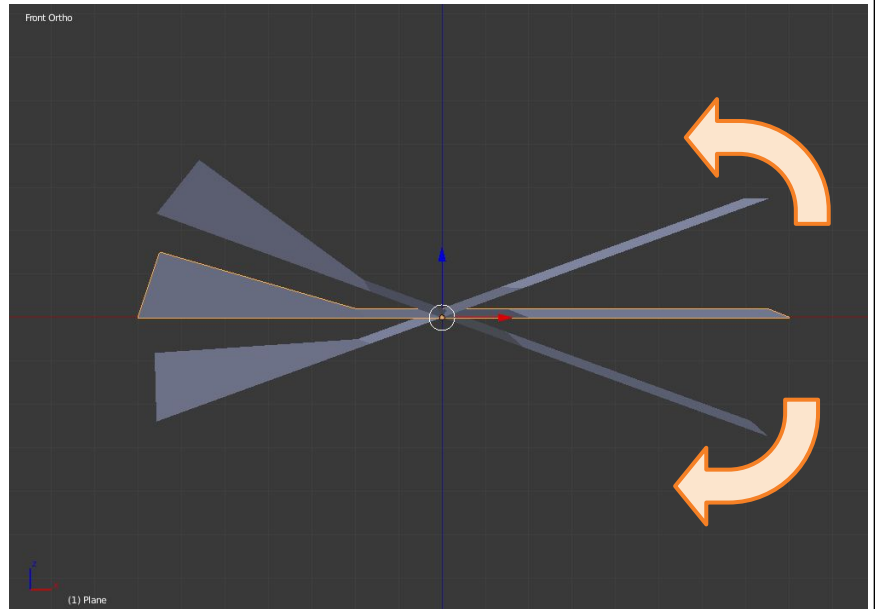
*The nose and tail go up and down as the body of the plane pitches.*



# Pitch

*Pitch is about the Y axis.*

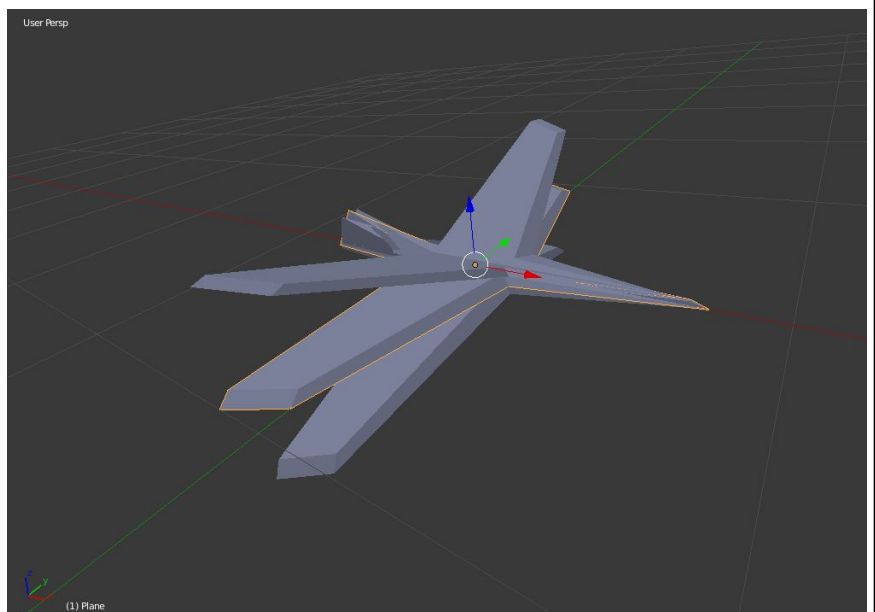
*The nose and tail go up and down as the body of the plane pitches.*



# Roll

*Roll is about the X axis.*

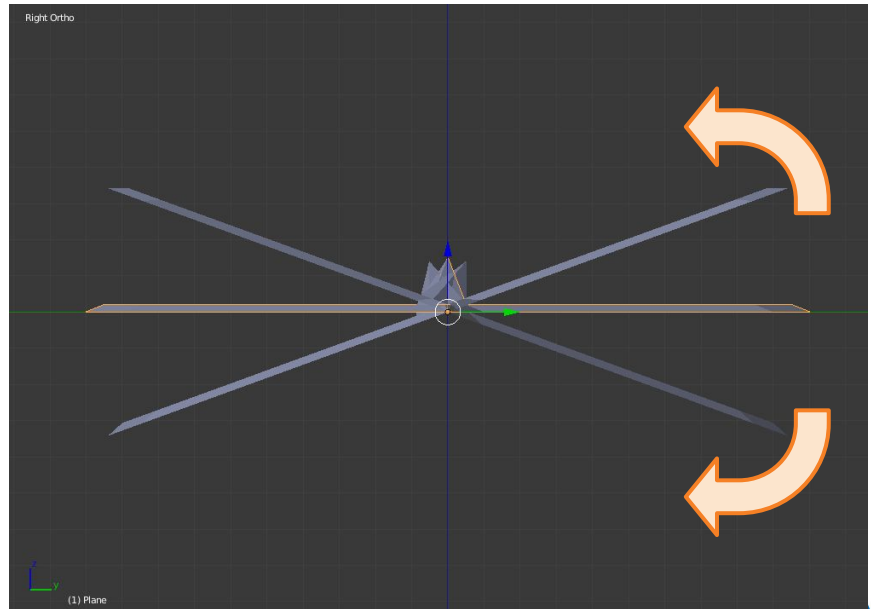
*The wing tips go up and down as the body of the plane rolls.*



# Roll

*Roll is about the X axis.*

*The wing tips go up and down as the body of the plane rolls.*



## Euler Angle Rotation

- Note that Euler angles must be applied in a particular order.
  - There are twelve ways to apply the angles, considering all of Euler's and Tait-Bryan's combinations.
  - *Euler*: z-x-z, x-y-x, y-z-y, z-y-z, x-z-x, y-x-y
  - *Tait-Bryan*: x-y-z, y-z-x, z-x-y, x-z-y, z-y-x, y-x-z
- Notice that in Euler's set, there's always a repeated axis and in Tait-Bryan's set, there's no repeated axis. Why?



## Euler Angle Rotation

- We will be using the fairly common sequence z-y-x.
  - So technically it's a *Tait-Bryan* rotation, not an *Euler* rotation.
- This must be taken into account when you construct the transformation matrix sequence.

